Ncert exempler

KUNWAR DUSHYANT SINGH EE22BTECH11031

Question 12.13.3.38

A and B throw a pair of dice alternately. A wins the game if he gets a total of 6 and B wins if she gets a total of 7. It A starts the game, find the probability of winning the game by A in third throw of the pair of dice.

Solution: Let state defined be

State	description
X_0	A rolls dice
X_1	B rolls dice
X_2	A wins
X_3	B wins
X_4	game stops

TABLE I
STATES

Markov chain

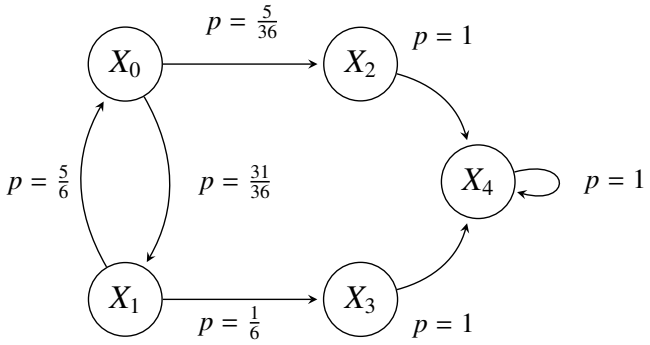

Fig. 1. State diagram generated using LatexTikZ

Initial state vector

$$S_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{1}$$

Transition matrix

$$P = \begin{pmatrix} 0 & \frac{31}{36} & \frac{5}{36} & 0 & 0 \\ \frac{5}{6} & 0 & 0 & \frac{1}{6} & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2}$$

Probablity of A winning in third throw given A starts first

$$S_3 = S_0^\top P^3 \tag{3}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}^\top \begin{pmatrix} \frac{4805}{7776} & \frac{775}{7776} & 0 & \frac{61}{216} & \\ \frac{775}{1296} & 0 & 0 & \frac{155}{1296} & \frac{61}{216} \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{4}$$

$$S_3 = \begin{pmatrix} 0 \\ 0.617 \\ 0.099 \\ 0 \end{pmatrix} \tag{5}$$

$$S_3[2] = 0.099 \tag{6}$$