# Ncert exempler

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**Question 12.13.3.38**
A and B throw a pair of dice alternately. A wins the game if he gets a total of 6 and B wins if she gets a total of 7. It A starts the game, find the probability of winning the game by A in third throw of the pair of dice.
**Solution:** Let state defined be

| State | description |
|---|---|
| $X_0$ | A rolls dice |
| $X_1$ | B rolls dice |
| $X_2$ | A wins |
| $X_3$ | B wins |
| $X_4$ | game stops |

TABLE I: States

Markov chain

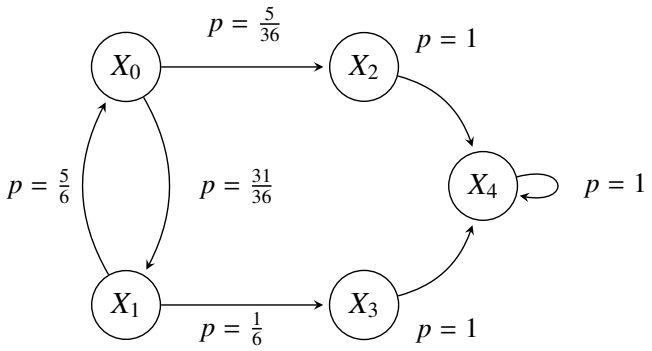


Fig. 1: State diagram generated using LatexTikZ

Initial state vector

$$\mathbf{S}_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{1}$$

Transition matrix

$$\mathbf{P} = \begin{pmatrix} 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{31}{36} & 0 & 0 & 0 & 0 \\ \frac{5}{36} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \tag{2}$$

Probablity of A winning in third throw given A starts first

$$\mathbf{S}_1 = \mathbf{P}\mathbf{S_0} \tag{3}$$

$$\mathbf{S_2} = \mathbf{P}\mathbf{S_1} \tag{4}$$

$$\mathbf{S_3} = \mathbf{P}\mathbf{S_2} \tag{5}$$

$$= \mathbf{P^3}\mathbf{S_0} \tag{6}$$

$$= \begin{pmatrix} \frac{775}{1296} & 0 & 0 & 0 & 0 \\ \frac{4805}{7776} & 0 & 0 & 0 & 0 \\ \frac{775}{7776} & 0 & 0 & 0 & 0 \\ 0 & \frac{155}{1296} & 0 & 0 & 0 \\ \frac{61}{216} & \frac{61}{216} & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

$$\mathbf{S_3} = \begin{pmatrix} 0.597 \\ 0.617 \\ 0.099 \\ 0 \\ 0.282 \end{pmatrix} \tag{8}$$

$$\mathbf{S_3}[2] = 0.099 \tag{9}$$