

Ncert exemplar

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Question 12.13.3.38

A and B throw a pair of dice alternately. A wins the game if he gets a total of 6 and B wins if she gets a total of 7. It A starts the game, find the probability of winning the game by A in third throw of the pair of dice.

Solution: Let state defined be

| State | description |
|-------|--------------|
| X_0 | A rolls dice |
| X_1 | B rolls dice |
| X_2 | A wins |
| X_3 | B wins |
| X_4 | game stops |

TABLE I
STATES

Markov chain

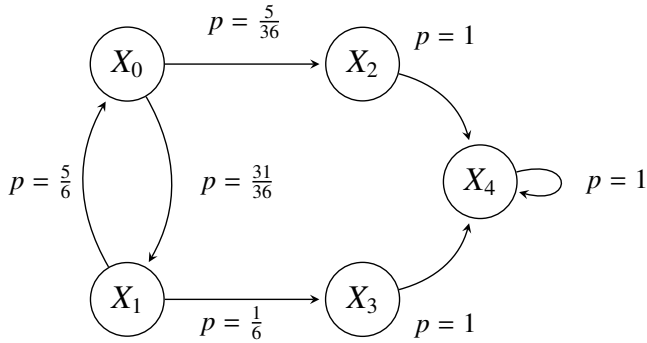


Fig. 1. State diagram generated using LatexTikZ

Initial state vector

$$S_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (1)$$

Transition matrix

$$P = \begin{pmatrix} 0 & \frac{5}{6} & 0 & 0 & 0 \\ \frac{31}{36} & 0 & 0 & 0 & 0 \\ \frac{5}{36} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix} \quad (2)$$

Probability of A winning in third throw given A starts first

$$S_1 = PS_0 \quad (3)$$

$$S_2 = PS_1 \quad (4)$$

$$S_3 = PS_2 \quad (5)$$

$$= P^3 S_0 \quad (6)$$

$$= \begin{pmatrix} \frac{775}{1296} & 0 & 0 & 0 & 0 \\ \frac{4805}{7776} & 0 & 0 & 0 & 0 \\ \frac{775}{7776} & 0 & 0 & 0 & 0 \\ 0 & \frac{155}{1296} & 0 & 0 & 0 \\ \frac{61}{216} & \frac{61}{216} & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} S_3 = \begin{pmatrix} 0.597 \\ 0.617 \\ 0.099 \\ 0 \\ 0.282 \end{pmatrix} \quad (7)$$

$$S_3[2] = 0.099 \quad (8)$$