Assignment

dushyant — EE22BTECH11031

Question 9.3.3 On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct?
Solution:

TABLE 1: Random Variables

Variable	Value	Description
X	$0 \le X \le 5$	Number of correct questions

$$p = \frac{1}{3} \tag{1}$$
$$n = 5 \tag{2}$$

Binomial

$$\implies P(X \ge 4) = \sum_{k=4}^{5} {}^{5}C_k \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{5-k} \tag{3}$$
$$= \frac{11}{243} \tag{4}$$
$$= 0.04526 \tag{5}$$

Gaussian

$$X \sim Bin(n, p) \tag{6}$$
$$\sim Bin\left(5, \frac{1}{3}\right) \tag{7}$$

Mean and Varience of X are

$$\mu_X = n \times p = \frac{5}{3} \tag{8}$$
$$\sigma_X^2 = np(1-p) = \frac{10}{9} \tag{9}$$
$$\tag{10}$$

Let, Z be a random variable with mean $\mu_Z = 0$ and $\sigma_Z = 1$ such that,

$$Z \approx \frac{X - \mu_X + 0.5}{\sigma_X} \tag{11}$$
$$\approx 1.74604 \tag{12}$$

0.5 is added for correction.
Z converges to normal distribution for large value of n

$$f(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} \tag{13}$$

And the Q funtion is defined as

$$Q(x) = \Pr(X > x) \tag{14}$$

We need

$$\Pr(Z < x) = 1 - \Pr(Z > x) \tag{15}$$
$$= 1 - Q(x) \tag{16}$$

Upon computation for $Z = 1.74604$

$$\Pr(Z < 1.74604) = 1 - 0.9596 \tag{17}$$
$$= 0.040402 \tag{18}$$

Therefore the Gaussian approximation for the given question is 0.040402

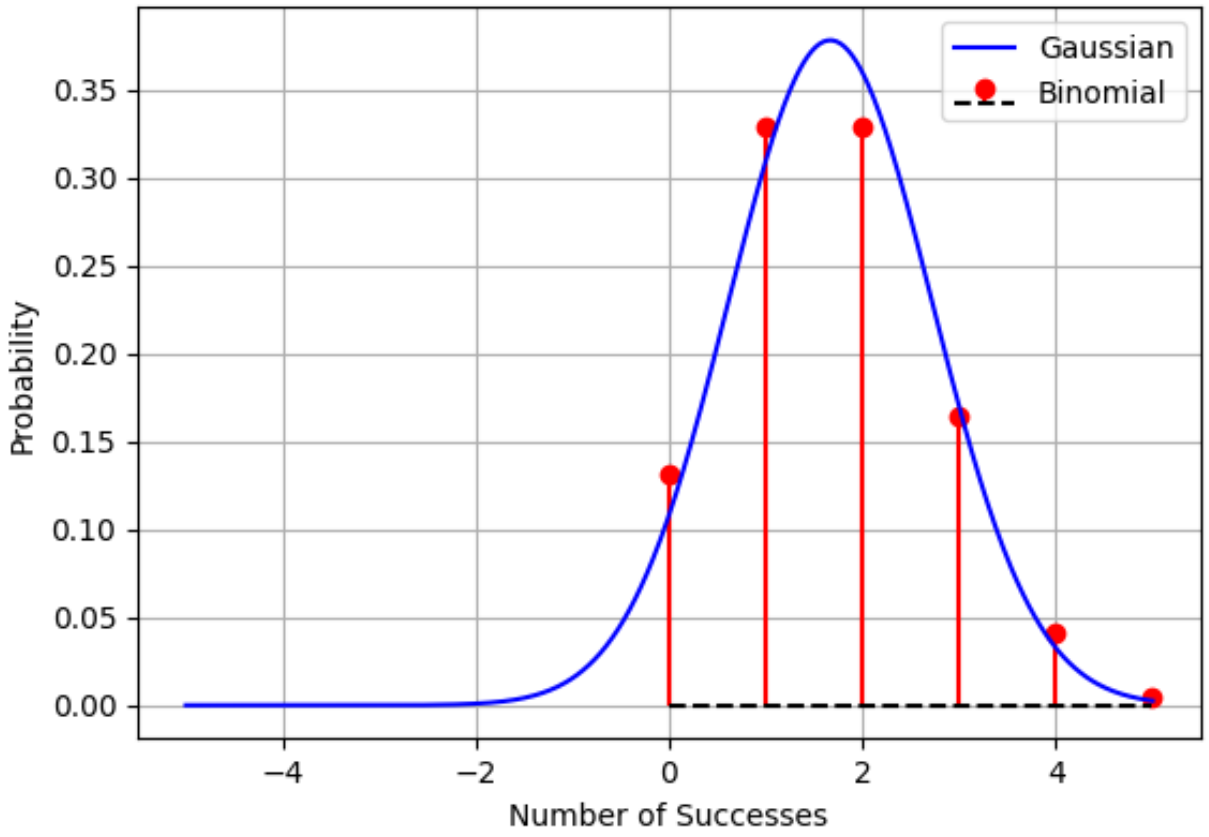

Fig. 1: Binomial vs guassian