Assignment

dushyant — EE22BTECH11031

Question 9.3.3 On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct?

Solution:

Binomial

$$p = \frac{1}{3} \tag{1}$$

$$n = 5 \tag{2}$$

Let X be a Binomial random variable with parameters p and n

$$\Pr(X = k) = {}^{n}C_k p^k (1-p)^{n-k} \tag{3}$$

$$= {}^{10}C_k \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{5-k} \tag{4}$$

CDF of X

$$F_X(n) = \Pr(X \geq n) \tag{5}$$

$$= \sum_{k=0}^{n} \Pr(X = k) \tag{6}$$

$$= \sum_{k=0}^{n} {}^{5}C_k \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{5-k} \tag{7}$$

Since, according to question n here equals,

$$\implies F_X(X \geq 4) = \sum_{k=4}^{5} {}^{5}C_k \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{5-k} \tag{8}$$

$$= \frac{11}{243} \tag{9}$$

$$= 0.04526 \tag{10}$$

Gaussian

$$X \sim Bin(n, p) \tag{11}$$

$$\sim Bin\left(5, \frac{1}{3}\right) \tag{12}$$

Mean and Varience of X are

$$\mu_X = np \tag{13}$$

$$= \frac{5}{3} \tag{14}$$

$$\sigma_X^2 = np(1-p) \tag{15}$$

$$= \frac{10}{9} \tag{16}$$

Let, Z be a random variable with mean $\mu_Z = 0$ and $\sigma_Z = 1$ such that,

$$Z = \frac{X - \mu_X + 0.5}{\sigma_X} \tag{17}$$

$$\approx 1.74604 \tag{18}$$

0.5 is added for correction.
Z converges to normal distribution for large value of n

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{19}$$

And the Q funtion is defined as,

$$Q(x) = \Pr(X > x) \tag{20}$$

We need

$$\Pr(Z < x) = 1 - \Pr(Z > x) \tag{21}$$

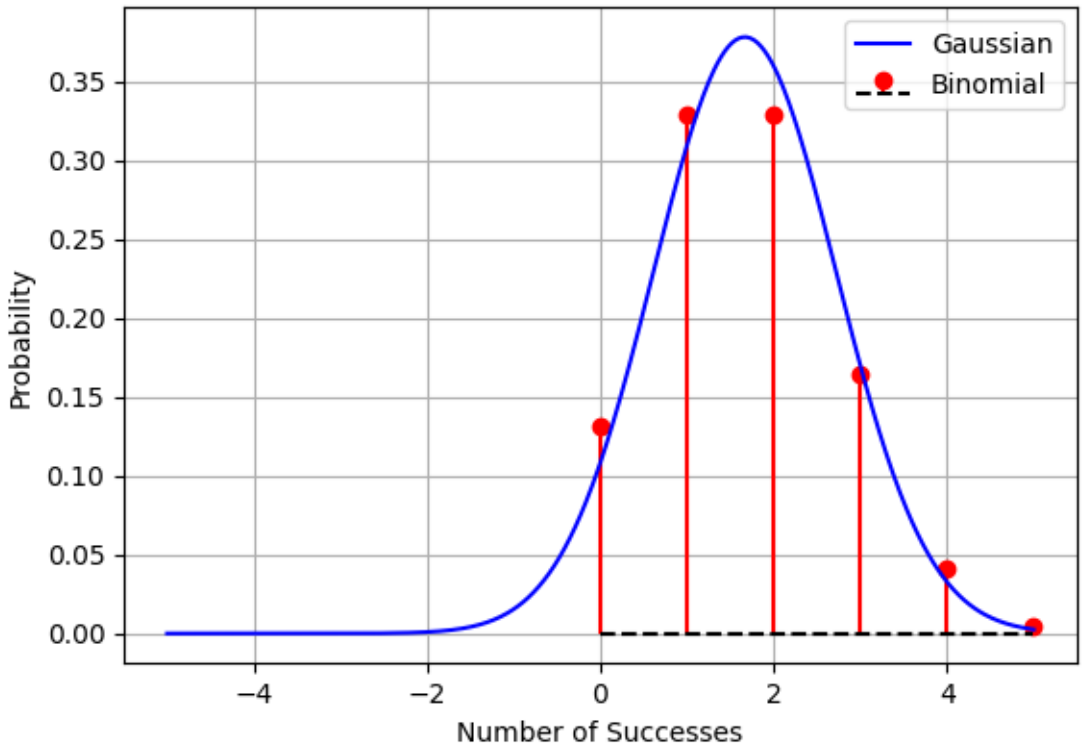
$$= 1 - Q(x) \tag{22}$$

Upon computation for $Z = 1.74604$

$$\Pr(Z < 1.74604) = 1 - 0.9596 \tag{23}$$

$$= 0.040402 \tag{24}$$

Therefore the Gaussian approximation for the given question is 0.040402


Gaussian
Binomial
Probability
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−4
−2
0
2
4
Number of Successes