

Assignment

dushyant — EE22BTECH11031

Question 9.3.3 On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct?

Solution:

TABLE 1: Random Variables

Variable	Value	Description
X	$0 \leq X \leq 5$	Number of correct questions

X has binomial distribution with parameters

$$p = \frac{1}{3}, \quad n = 5 \quad (1)$$

(2)

Pmf of X for $0 \leq k \leq 5$ is

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \quad (3)$$

(4)

Probability that 4 or more are correct

$$\Rightarrow P(X \geq 4) = \sum_{k=4}^5 {}^5C_k \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{5-k} \quad (5)$$

$$= \frac{11}{243} \quad (6)$$

$$= 0.04526 \quad (7)$$

Let Y be a gaussian variable

Mean and Variance of X are

$$\mu_X = n \times p = \frac{5}{3} \quad (8)$$

$$\sigma_X^2 = np(1-p) = \frac{10}{9} \quad (9)$$

(10)

Using normal distribution at $X \geq 4$

$$Z \approx \frac{X - \mu_X + 0.5}{\sigma_X} \quad (11)$$

$$\approx 1.74604 \quad (12)$$

0.5 is added for correction.
For pdf calculation

$$f_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (13)$$

$$P(Y \geq 4) = 1 - P(Z = 1.74604) \quad (14)$$

$$(15)$$

Upon computation for $Z = 1.74604$

$$P(Y \geq 4) = 1 - 0.9596 \quad (16)$$

$$= 0.040402 \quad (17)$$

Therefore the Gaussian approximation for the given question is 0.040402

$$P_X \approx P_Y \quad (18)$$

$$(19)$$

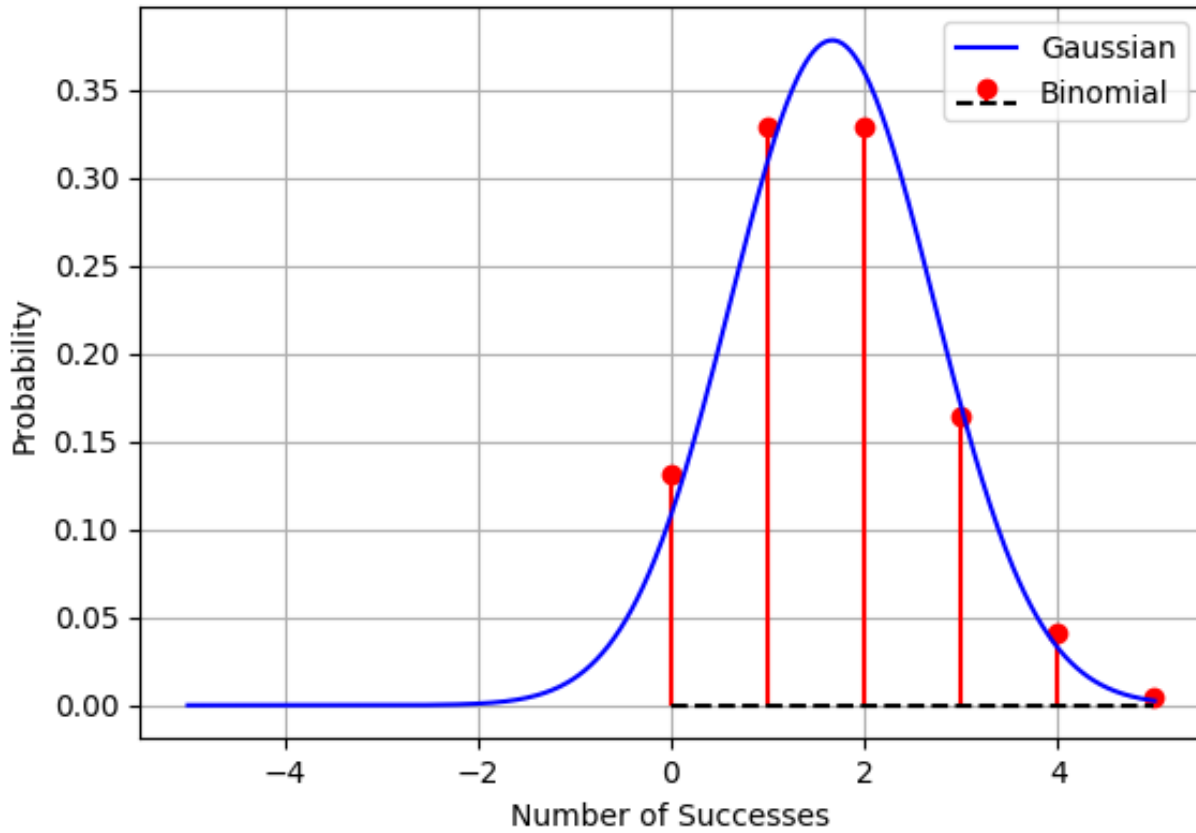


Fig. 1: Binomial vs gaussian