Assignment

dushyant — EE22BTECH11031

Question 9.3.3 On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct?

Solution:

Gaussian:

TABLE 1: Variables

Variable	Value	Description
n	5	Number of questions
p	$\frac{1}{3}$	probability of question being correct
$\mu = np$	$\frac{5}{3}$	mean of distribution
$\sigma = \sqrt{npq}$	$\sqrt{\frac{10}{9}}$	variance of distribution
X	$0 \le X \le 5$	Number of correct questions

$$Y \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{1}$$

CDF of Y is defined as:

$$F_Y(x) = \Pr(Y \le x) \tag{2}$$
$$= \Pr\left(\frac{Y-\mu}{\sigma} \le \frac{X-\mu}{\sigma}\right) \tag{3}$$
$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}(0,1) \tag{4}$$
$$= 1 - \Pr\left(\frac{Y-\mu}{\sigma} > \frac{X-\mu}{\sigma}\right) \tag{5}$$
$$= 1 - Q\left(\frac{X-\mu}{\sigma}\right) \tag{6}$$

1) Without correction:

$$\Pr(Y \ge 4) = 1 - \Pr(Y \le 4) \tag{7}$$
$$= 1 - F_Y(4) \tag{8}$$
$$\implies \Pr(Y \ge 4) = Q\left(\frac{X-\mu}{\sigma}\right) \tag{9}$$
$$= Q(2.22286) \tag{10}$$
$$\Pr(Y \ge 4) = 0.013113 \tag{11}$$

2) With a 0.5 correction:

$$\Pr(Y \ge 4) = Q\left(\frac{X-\mu+0.5}{\sigma}\right) \tag{12}$$
$$= Q(1.74604) \tag{13}$$
$$\implies \Pr(Y \ge 4) = 0.040402 \tag{14}$$

Binomial:

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \tag{15}$$
$$\tag{16}$$

Probablity that 4 or more are correct

$$\implies P(X \geq 4) = \sum_{k=4}^{5} {}^{5}C_k \left(\frac{1}{3}\right)^k \left(\frac{2}{3}\right)^{5-k} \tag{17}$$

$$= \frac{11}{243} \tag{18}$$

$$= 0.04526 \tag{19}$$

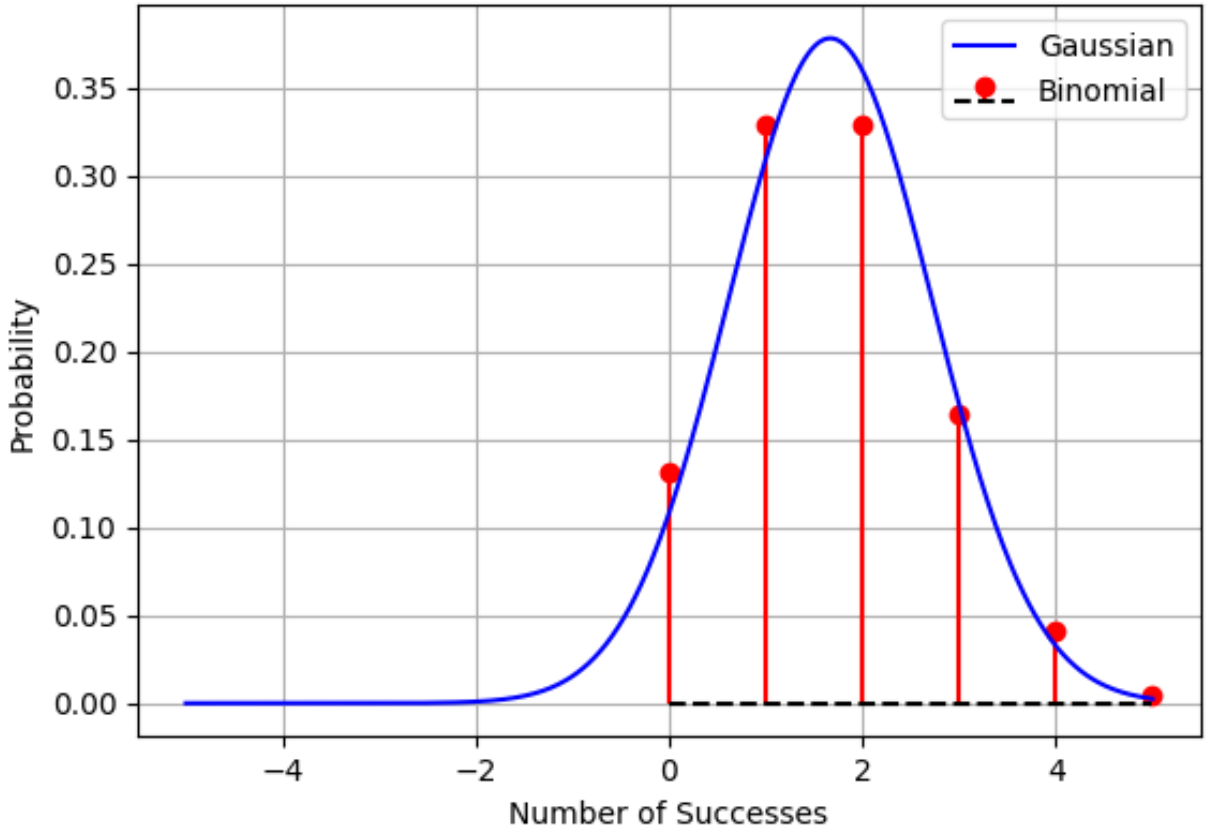

Fig. 1: Binomial vs guassian