Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -4 \\ -6 \end{pmatrix} \tag{1}$$

1 Vectors

parameters	values	description
$\mathbf{m_1}$	$\begin{pmatrix} -1 \\ -8 \end{pmatrix}$	AB
$\mathbf{m_2}$	$\begin{pmatrix} -7 \\ -1 \end{pmatrix}$	BC
$\mathbf{m_3}$	$\begin{pmatrix} 8 \\ 9 \end{pmatrix}$	CA
$\|\|A-B\|\|$	8.06	length of AB
$\|\|B-C\|\|$	7.07	length of BC
$\|\|C-A\|\|$	12.04	length of CA
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$	3	non collinear
$\mathbf{n_1}$	$\begin{pmatrix} -8 \\ 1 \end{pmatrix}$	AB
$\mathbf{n_2}$	$\begin{pmatrix} -1 \\ 7 \end{pmatrix}$	BC
$\mathbf{n_3}$	$\begin{pmatrix} -9 \\ 8 \end{pmatrix}$	CA
Area	27.5	Area of Triangle
$\angle A$	34.5°	Angles
$\angle B$	105.2°	
$\angle C$	40.2°	

2 Median

parameters	value	description
$\mathbf{D}$	$\begin{pmatrix} -0.5 \\ -5.5 \end{pmatrix}$	BC midpoint
$\mathbf{E}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	CA midpoint
$\mathbf{F}$	$\begin{pmatrix} 3.5 \\ -1 \end{pmatrix}$	AB midpoint
$\mathbf{m_4}$	$\begin{pmatrix} 4.5 \\ 8.5 \end{pmatrix}$	AD
$\mathbf{n_4}$	$\begin{pmatrix} -8.5 \\ 4.5 \end{pmatrix}$	
$\mathbf{m_5}$	$\begin{pmatrix} 3 \\ -3.5 \end{pmatrix}$	BE
$\mathbf{n_5}$	$\begin{pmatrix} 3.5 \\ 3 \end{pmatrix}$	
$\mathbf{m_6}$	$\begin{pmatrix} -7.5 \\ -5 \end{pmatrix}$	CF
$\mathbf{n_6}$	$\begin{pmatrix} -5 \\ 7.5 \end{pmatrix}$	
$\mathbf{G}$	$\begin{pmatrix} 1 \\ -2.66 \end{pmatrix}$	Centroid
$\frac{BG}{GE}$	2	Division ratio by $\mathbf{G}$
$\frac{CG}{GF}$		
$\frac{AG}{GD}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{A} & \mathbf{D} & \mathbf{G} \end{pmatrix}$	2	collinear
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{B} & \mathbf{E} & \mathbf{G} \end{pmatrix}$		
$\text{rank}\begin{pmatrix} 1 & 1 & 1 \\ \mathbf{C} & \mathbf{F} & \mathbf{G} \end{pmatrix}$		

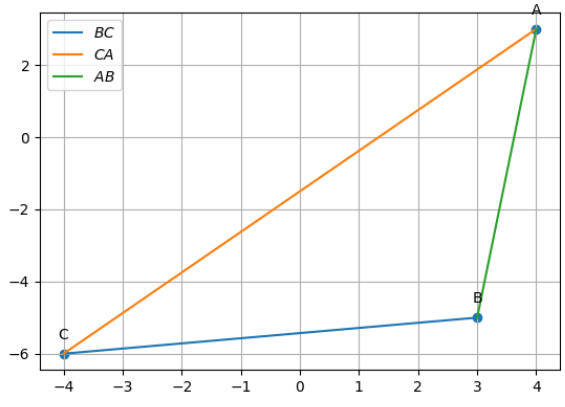

Fig. 1: sides

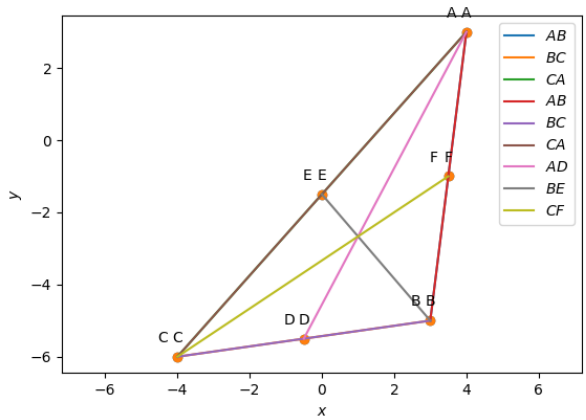

Fig. 2: Midpoints

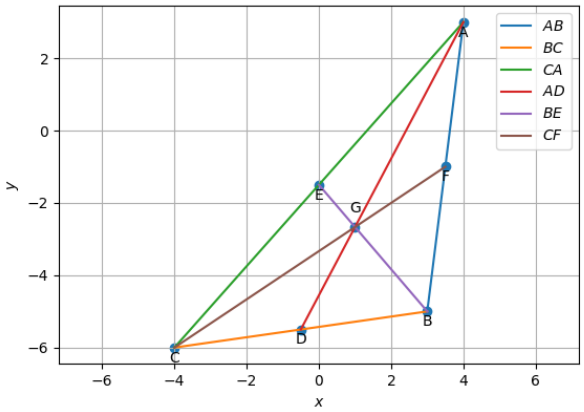

Fig. 3: centroid

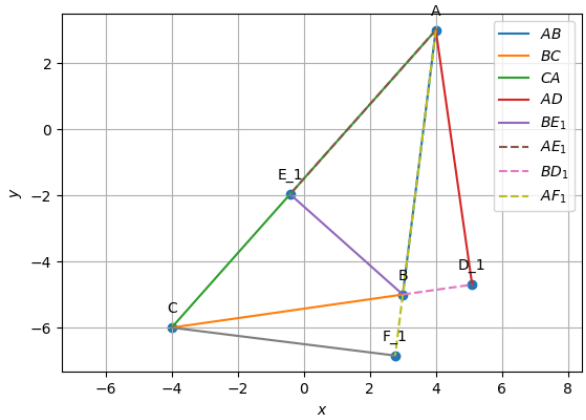

Fig. 4: altitudes

3 Altitude

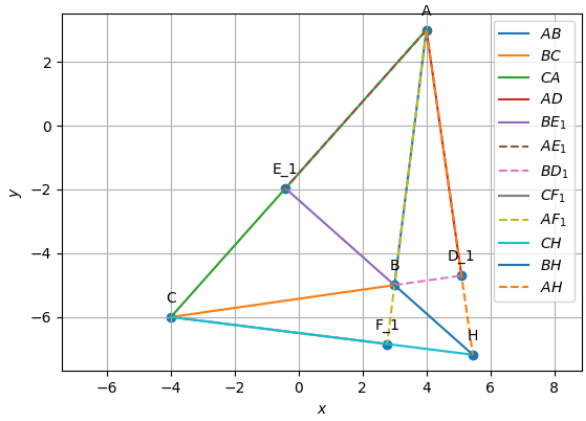

Fig. 5: orthocentre

parameters	value	description
$\mathbf{D_1}$	$\begin{pmatrix}5.14.7\end{pmatrix}$	Foot of altitude from **A**
$\mathbf{E_1}$	$\begin{pmatrix}-0.41,-1.96\end{pmatrix}$	Foot of altitude from **B**
$\mathbf{F_1}$	$\begin{pmatrix}2.7\\-6.8\end{pmatrix}$	Foot of altitude from **C**
$\mathbf{m_7}$	$\begin{pmatrix}1.1\\-4.7\end{pmatrix}$	AD_1
$\mathbf{n_7}$	$\begin{pmatrix}4.7\\1.1\end{pmatrix}$	
$\mathbf{m_8}$	$\begin{pmatrix}-3.41\\3.03\end{pmatrix}$	BE_1
$\mathbf{n_8}$	$\begin{pmatrix}3.03\\3.41\end{pmatrix}$	
$\mathbf{m_9}$	$\begin{pmatrix}6.76\\-0.84\end{pmatrix}$	CF_1
$\mathbf{n_9}$	$\begin{pmatrix}0.84\\6.76\end{pmatrix}$	
H	$\begin{pmatrix}5.45\\-7.18\end{pmatrix}$	Orthocentre

4 Perpendicular bisector

parameters	value	description
O	$\begin{pmatrix}-1.22\\-0.40\end{pmatrix}$	Circumcentre
$\|\mathbf{O}-\mathbf{A}\|$	6.24	$OA = OB = OC = R$
$\|\mathbf{O}-\mathbf{B}\|$		
$\|\mathbf{O}-\mathbf{C}\|$		
R		
$\angle BOC$	69.01°	$\angle BOC = 2\angle BAC$
$\angle BAC$	34.5°	

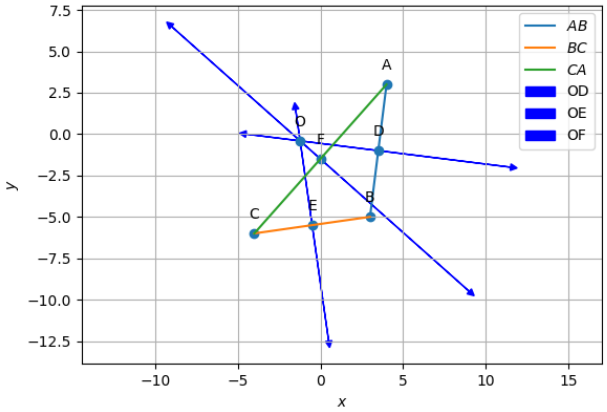

Fig. 6: perpendicular bisector

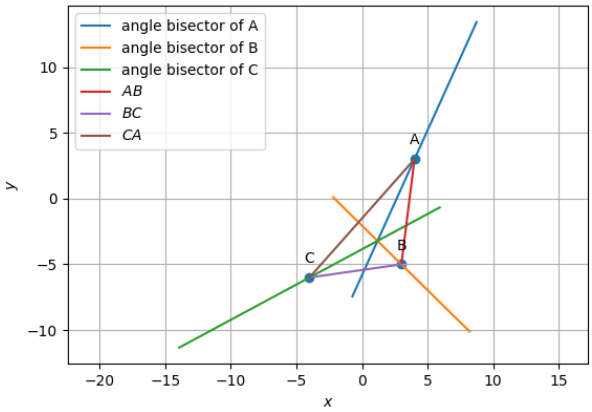

Fig. 8: angle bisector

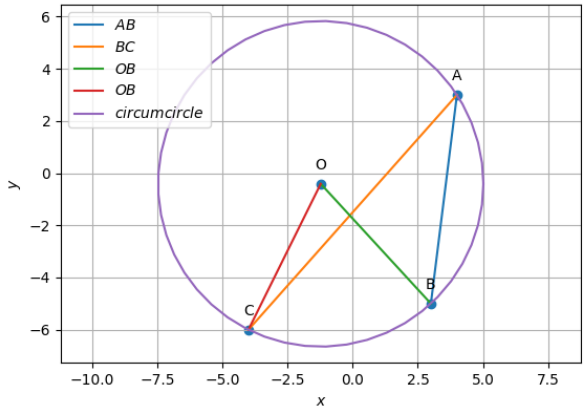

Fig. 7: circumcircle

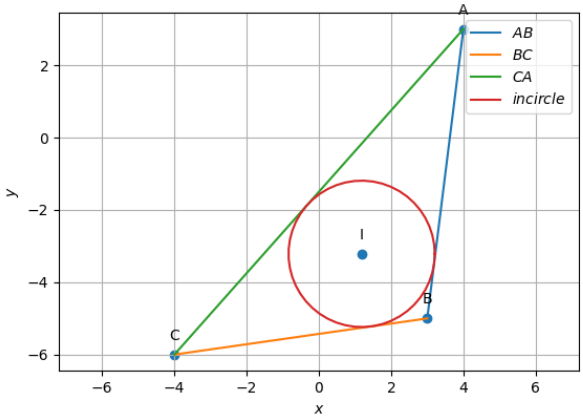

Fig. 9: incircle

5 Angle bisector

parameters	value	description
$\mathbf{I}$	$\begin{pmatrix} 1.18 \\ -3.21 \end{pmatrix}$	Incentre
$\mathbf{D_3}$	$\begin{pmatrix} 1.46 \\ -5.21 \end{pmatrix}$	Point of contact with BC
$\mathbf{E_3}$	$\begin{pmatrix} 3.19 \\ -3.46 \end{pmatrix}$	Point of contact with AC
$\mathbf{F_3}$	$\begin{pmatrix} -0.32 \\ -1.87 \end{pmatrix}$	Point of contact with AB
$\|\mathbf{I} - \mathbf{D_3}\|$	2.02	$ID_3 = IE_3 = IF_3 = r$
$\|\mathbf{I} - \mathbf{E_3}\|$		
$\|\mathbf{I} - \mathbf{F_3}\|$		
r		
$\angle BAI$	17.25°	$\angle BAI = \angle CAI$
$\angle CAI$