Norwegian University of Science and Technology

Assignment Title

Assignment 3

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Propositional and Predicate Logics

Course

TDT4136 Introduction to Artificial Intelligence

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Submitted by

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Task 1: Models and Entailment in Propositional Logic

1. For each statement below, determine whether the statement is true or false by building complete model table

(a) A ⋀ ¬B |= A ⋁ B (True)

A	B
T	T
T	F
F	T
F	F

A∧¬B	A∨B
F	T
T	T
F	T
F	F

(b) A ⋁ B |= A ⋀ ¬B (false)

A	B
T	T
T	F
F	T
F	F

A∧¬B	A∨B
T	F
T	T
T	F
F	F

(c) A ⇔ B |= A ⇒ B (True)

A	B
T	T
T	F
F	T
F	F

A⇔B	A⇒B
T	T
F	F
F	T
T	T

(d) (A ⇔ B) ⇔ C |= A ⋁ ¬B ⋁ ¬C (True)

A	B	C
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

(A⇔B)⇔C	A∨¬B∨¬C
T	T
F	T
F	T
T	T
F	F
T	T
T	T
F	T

(e) $(\neg A \wedge B) \wedge (A \Rightarrow B)$ is satisfiable (True)

A	B	$(\neg A \wedge B) \wedge (A \Rightarrow B)$
T	T	F
T	F	F
F	T	T
F	F	F

(f) $(\neg A \wedge B) \wedge (A \Leftrightarrow B)$ is satisfiable (False) - it is unsatisfiable because its and contradiction

A	B	$(\neg A \wedge B) \wedge (A \Leftrightarrow B)$
T	T	F
T	F	F
F	T	F
F	F	F

2. Consider a logical knowledge base with 100 variables, A_1,A_2 ... A_{100}. This will have Q = 2^{100} possible models. For each logical sentence below, give the number of models that satisfy it.

(a) $A_{31} \wedge A_{76}$

1 - (1- ½ * ½)= <u>¼ Q</u>

(b) $A_{44} \wedge A_{49} \wedge A_{78}$

1 - (1 - ½ * ½ * ½) = <u>⅛ Q</u>

(c) $A_{44} \vee A_{49} \vee A_{78}$

$(1 - (½)^3)$ = <u>⅞ Q</u>

(d) $A_{70} \Rightarrow \neg A_{92}$

$\neg A_{70} \vee \neg A_{92}$

$(1 - (½)^2)$ = <u>¾ Q</u>

(e) $(A_7 \Leftrightarrow A_{72}) \wedge (A_{83} \Leftrightarrow A_{84})$

$(A_7 \Rightarrow A_{72}) \wedge (A_{72} \Rightarrow A_7) \wedge (A_{83} \Rightarrow A_{84}) \wedge (A_{84} \Rightarrow A_{83})$

$(\neg A_7 \vee A_{72}) \wedge (\neg A_{72} \vee A_7) \wedge (\neg A_{83} \vee A_{84}) \wedge (\neg A_{84} \vee A_{83})$

$(1 - (½)^2)(1 - (½)^2)(1 - (½)^2)(1 - (½)^2) = \underline{(3/4)}^4$ Q

(f) $\neg A_9 \wedge \neg A_{19} \wedge A_{37} \wedge A_{50} \wedge A_{68} \wedge A_{73} \wedge A_{79} \wedge A_{81}$

1 - (1- ½ * ½ * ½ * ½ * ½ * ½ * ½ * ½)= <u>1/256 Q</u>

Task 2: Resolution in Propositional Logic

1. Convert each of the following sentences to Conjunctive Normal Form (CNF).

(a) ¬A (B ∧ C)
distributivity of ∨ over ∧:
(¬A ∨ B) ∧ (¬A ∨ C)

(b) ¬(A ⇒ B) ∧ ¬(C ⇒ D)
Implication elimination:
=> ¬(¬A ∨ B) ∧ ¬(¬C ∨ D)
Double negation elimination and De Morgan:
=> A ∧ ¬B ∧ C ∧ ¬D

(c) ¬(A ⇒ B) ∨ ¬(C ⇒ D)
Implication elimination:
=> ¬(¬A ∨ B) ∨ ¬(¬C ∨ D)
Double negation elimination and De Morgan:
=> (A ∧ ¬B) ∨ (C ∧ ¬D)
distributivity of ∨ over ∧:
=> (A ∨ C) ∧ (A ∨ ¬D) ∧ (¬B ∨ C) ∧ (¬B ∨ ¬D)

(d) (A ⇒ B) ⇔ C
Biconditional elimination:
=> ((A ⇒ B) ⇒ C) ∧ (C ⇒ (A ⇒ B))
Biconditional elimination:
=> ((¬A ∨ B) ⇒ C) ∧ (C ⇒ (¬A ∨ B))
Biconditional elimination:
=> (¬(¬A ∨ B) ∨ C) ∧ (¬C ∨ (¬A ∨ B))
Double negation elimination, De Morgan, and associativity of ∨:
=> (¬A ∨ B) ∨ C) ∧ (¬A ∨ B ∨ ¬C)
Distributivity of ∨ over ∧
=> (A ∨ C) ∧ (¬B ∨ C) ∧ (¬A ∨ B ∨ ¬C)

2. Consider the following Knowledge Base (KB):

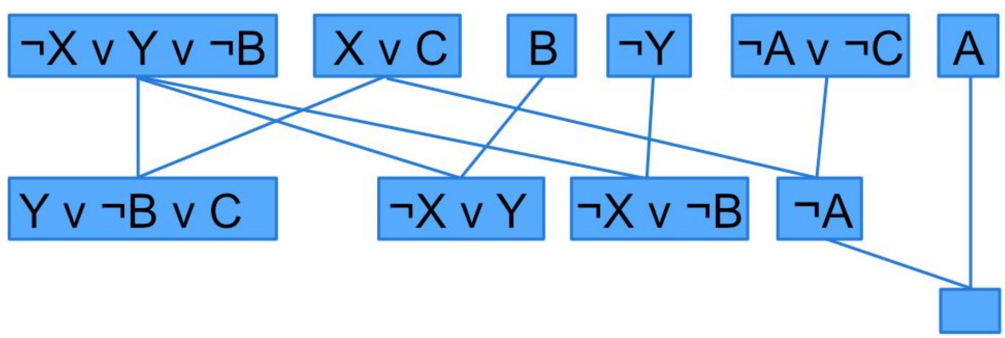
- $(X \land \neg Y) \Rightarrow \neg B$
- $\neg X \Rightarrow C$
- $B \land \neg Y$
- $A \Rightarrow \neg C$

Use resolution to show that KB |= ¬A

By converting the KB to CNF, using logical equivalence, we can use resolution to show that KB |= ¬A. In this knowledge base we only need "implication elimination" and "double-negation elimination" to make it into CNF.

- $(X \land \neg Y) \Rightarrow \neg B$
  - $\neg X \lor Y \lor \neg B$
- $\neg X \Rightarrow C$
  - $X \lor C$
- $B \land \neg Y$
- $A \Rightarrow \neg C$
  - $\neg A \lor \neg C$

This, in addition to the negation of ¬A, gives us:
$(\neg X \lor Y \lor \neg B) \land (X \lor C) \land (\neg A \lor \neg C) \land (B) \land (\neg Y) \land (A)$



We end up in a contradiction, which means that the negation of the query is not possible. Hence, just the positive of the query is possible, which means that it's <u>true</u>.

3. Do exercise 7.18 from the textbook (“Consider the following sentence. . . ”), but with the following the sentence instead of the one in the textbook:
$[(\text{Food} \lor \text{Drinks}) \Rightarrow \text{Party}] \Rightarrow [\neg\text{Party} \Rightarrow \neg\text{Food}]$

(a) Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid),or unsatisfiable.
The sentence is a tautology, hence it is true for all models

FOOD	PARTY	DRINKS	FOOD ∨ DRINKS	((FOOD ∨ DRINKS) ⇒ PARTY)	¬PARTY	¬FOOD	(¬PARTY ⇒ ¬FOOD)
T	T	T	T	T	F	F	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	F	T
T	F	F	T	F	T	F	F
F	T	T	T	T	F	T	T
F	T	F	F	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

(b) Convert the left-hand and right-hand sides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a).

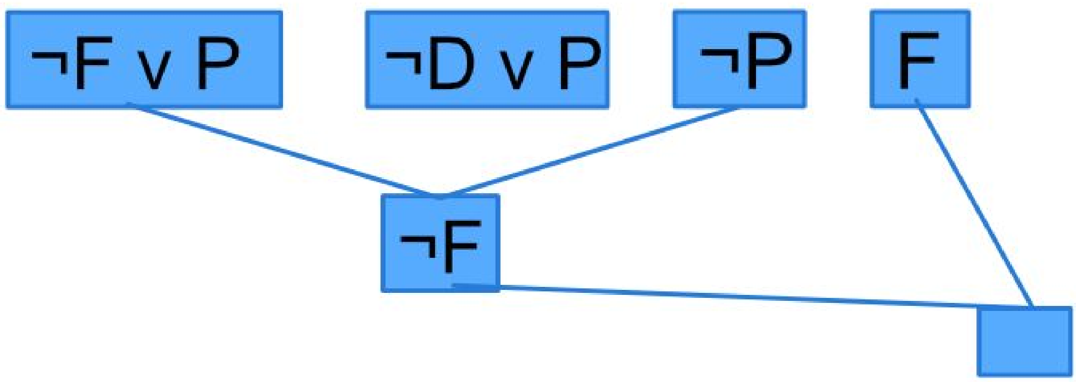
$(F \lor D) \Rightarrow P$
Implication elimination:
$\neg(F \lor D) \lor P$
De Morgan:
$(\neg F \land \neg D) \lor P$
Distributivity of ∧ over ∨
$(\neg F \lor P) \land (\neg D \lor P)$

$\neg P \Rightarrow \neg F$
Implication elimination:
$\neg(\neg P) \lor \neg F$
Double negation elimination
$(P \lor \neg F)$

Since we have the same on both sides, we know that it is a valid statement, and this confirms the answer to **a)**.

(c) Prove your answer to (a) using resolution.

Need to prove: ¬(P ∨ ¬F), which is: <u>¬P ∧ F</u>



Task 3: Representation is First-Order-Logic

1. Consider a first-order logical knowledge base that describes worlds containing movies, actors, directors and characters. The vocabulary contains the following symbols:

- PlayedInMovie(a,m): predicate. Actor/person a played in the movie m
  - = PIM(a,m)
- PlayedCharacter(a,c): predicate. Actor/person a played character c
  - = PC(a,c)
- CharacterInMovie(c,m): predicate. Character c is in the movie m.
  - = CIM(c,m)
- Directed(p,m): person p directed movie m.
  - = D(p,m)
- Constants related to the name of the movie, person or character with obvious meaning (to simplify you may use the surname or abbreviation).

Express the following statements in First-Order Logic:

(a) The character "Batman" was played by Christian Bale, George Clooney and Val Kilmer.

PC(Bale,Batman) ∧ PC(Clooney,Batman) ∧ PC(Kilmer, Batman)

(b) Heath Ledger and Christian Bale did not play the same characters.

$\forall x,y$ PC(Ledger,x) ∧ PC(Bale,y) ⇒ ¬(x = y)

(c) Christian Bale played in all "Batman" movies directed by Christopher Nolan
$\forall m$ (PIM(Bale,m) ∧ PC(Bale,Batman)) ⇒ D(Nolan,m)

(d) "The Joker" and "Batman" are characters that appear together in some movies.
$\exists m$ CIM(The Joker,m) ∧ CIM (Batman,m)

(e) Kevin Costner directed and starred in the same movie.
$\exists m$ PIM(Costner,m) ∧ D(costner,m)

(f) George Clooney and Tarantino never played in the same movie and Tarantino never directed a film that George Clooney played.
$\forall m, n$ PIM(Clooney,m) ∧ PIM(Tarantino,n) ∧ D(Tarantino,n) ⇒ ¬(m = n)

(g) Uma Thurman played a character in some movies directed by Tarantino.
$\exists m$ PIM(Thurman,m) ∧ D(Tarantino,m)

2. Arithmetic assertions can be written using FOL. Use the predicates ($<, \leq, \neq, =$), usual arithmetic operations as function symbol (+,-,x,/), biconditionals to create new predicates, and integer number constants to express the following statements in FOL:

(a) An integer number x is divisible by y if there is some integer z less than x such that x = z * y (in other words, define the predicate Divisible(x, y)).
$\forall x, y, z$ Divisible(x,y) ⇒ ((z < x) ∧ (x = z*y))

(b) A number is even if and only if it is divisible by 2 (define the predicate Even(x)).
$\forall x$ Even(x) ⇔ (x/2)

(c) The result of summing an even number with 1 is an odd number (define the predicate Odd(x)).
$\forall x$ Odd(x) ⇒ (Even(x) + 1)

(d) A prime number is divisible only by itself.
$\forall x, y$ Prime(x) ⇒ ((x/y) ⇒ (x = y))

Task 4: Resolution in First-Order-Logic

1. Find the unifier (⊖) – if possible – for each pair of atomic sentences. Philosopher(x), StudentOf(y, x), Write(x, z), Read(y, z) and Book(z) are predicates, while TeacherOf(y) is a function that maps a philosophy student to its teacher name and Author(z) maps a book to its author.

(a) Philosopher(x) ... Philosopher(Plato)

⊖ = {x/Plato}

(b) Write(Plato, TheRepublic) ... Write(Plato, y)

⊖ = {y/TheRepublican}

(c) Read(x, Metaphysics) ... Read(Peter, y)

⊖ = {x/Peter, y/Metaphysics}

(d) Write(x, Fear And Trembling) ... Write(Kierkegaard, x)

⊖ = {}

(e) Write(Kant,CritiqueOfPureReason) ... Write(Author(y),y)

⊖ = {x/Kant, y/CritiqueOfPureReason}

2. Using the same predicates of the previous question perform skolemization with the following expressions:

P(x) = Philosopher(x), StudentOf(y,x) = SO(y,x), Book(z) = B(z), Write(x,z) = W(y,z), Read(y,z) = R(y,z)

(a) $\exists x \exists y$ **[Philosopher(x) ∧ StudentOf(y,x)]**

Substitute x by a and y by b

P(a) ∧ SO(a,b)

(b) $\forall y, x$ **[Philosopher(x) ∧ StudentOf(y,x) ⇒ [** $\exists z$ **: Book(z) ∧ Write(x,z) ∧ Read(y,z)]]**

Eliminate Implication:

$\forall x, y$ [¬P(x) ∨ ¬SO(y,x) ∨ $\exists z$ [B(z) ∧ W(x,z) ∧ R(y,z)]]

Substitue z by g(x,y)

$\forall x, y$ [¬P(x) ∨ ¬SO(y,x) ∨ [B(g(x,y)) ∧ W(x,g(x,y)) ∧ R(y,g(x,y))]]

3. Use resolution to prove SuperActor(Tarantino) given the information below. You must first convert each sentence into CNF. Feel free to show only the applications of the resolution rule that lead to the desired conclusion. For each application of the resolution rule, show the unification bindings, . We are using in this case the same predicates of Exercise 3.1 (movies, actors, etc).

1. $\forall x$ [SuperActor(x)⇔ $\exists m$ [PlayedInMovie(x,m) ∧ Directed(x,m)]]
2. $\forall m$ [Directed(Tarantino,m)⇔PlayedInMovie(UmaThurman,m)]
3. $\exists m$ [PlayedInMovie(UmaThurman,m) ∧ PlayedInMovie(Tarantino,m)]

(a) Show all the steps in the proof (or the diagram).

1. $\forall x$ **[SuperActor(x)⇔** $\exists m$ **[PlayedInMovie(x,m) ∧ Directed(x,m)]]**

Biconditional elimination:

$\forall x$ [(SuperActor(x) ⇒ $\exists m$ [PlayedInMovie(x,m) ∧ Directed(x,m)] ∧ $\exists m$ [PlayedInMovie(x,m) ∧ Directed(x,m)] ⇒ (SuperActor(x)]

Implication elimination:

$\forall x$ [(¬(SuperActor(x) ∨ $\exists m$ [PlayedInMovie(x,m) ∧ Directed(x,m)]) ∧ (¬ $\exists n$ [PlayedInMovie(x,n) ∧ Directed(x,m)] ∨ (SuperActor(x))]

Move negation inwards (and De Morgans rule):

$\forall x$ [¬(SuperActor(x) ∨ $\exists m$ [PlayedInMovie(x,m) ∧ Directed(x,m)] ∧ $\forall x$ [¬PlayedInMovie(x,n) ∨ ¬Directed(x,n)] ∨ (SuperActor(x)]

Substitute m by g(x) (Skolemize):

$\forall x$ [¬(SuperActor(x) ∨[PlayedInMovie(*x*,*g(x)*) ∧ Directed(*x*,*g(x)*)] ∧ [¬PlayedInMovie(x,*n*) ∨ ¬Directed(*x*,*n*)] ∨ (SuperActor(*g(x)*)]

Drop the universal quantifier:

[(¬SuperActor(x) ∨ [PlayedInMovie(*x*,*g(x)* ∧ Directed(*x*,*g(x)*)]) ∧ ([¬PlayedInMovie(x,*n*) ∨ ¬Directed(*x*,*n*)] ∨ (SuperActor(*g(x)*))]

Distribute ∨ over ∧:

[(¬SuperActor(x) ∨ PlayedInMovie(*x*,*g(x)*) ∧ (¬SuperActor(x) ∨ Directed(*x*,*g(x)*)) ∧ (¬PlayedInMovie(x,*n*) ∨ ¬Directed(*x*,*n*) ∨ SuperActor(*g(x)*))]

2. $\forall m$ **[Directed(Tarantino,m) ⇔ PlayedInMovie(UmaThurman,m)]**

Biconditional elimination:
$\forall m$ [(Directed(Tarantino,m) ⇒ PlayedInMovie(UmaThurman,m)) ∧ (PlayedInMovie(UmaThurman,m) ⇒ Directed(Tarantino,m))]

Implication Elimination and Drop universal quantifiers:
(¬Directed(Tarantino,m) ∨ PlayedInMovie(UmaThurman,m)) ∧
(¬ PlayedInMovie(UmaThurman,m) ∨ Directed(Tarantino,m))

3. $\exists m$ **[PlayedInMovie(UmaThurman,m) ∧ PlayedInMovie(Tarantino,m)]**

Skolemize:
PlayedInMovie(UmaThurman,m) ∧ PlayedInMovie(Tarantino,m)

KB:
0.1: ¬SuperActor(x) ∨ (PlayedInMovie(*x*,*g(x))*
0.2: ¬SuperActor(x) ∨ Directed(*x*,*g(x)*)
0.3: ¬PlayedInMovie(x,*n*) ∨ ¬Directed(*x*,*n*) ∨ SuperActor(*g(x))*
0.4: ¬Directed(Tarantino,m) ∨ PlayedInMovie(UmaThurman,m)
0.5: ¬PlayedInMovie(UmaThurman,m) ∨ Directed(Tarantino,m)
0.6: PlayedInMovie(UmaThurman,m)
0.7: PlayedInMovie(Tarantino,m)

¬Q: ¬ SuperActor(Tarantino)

Resolution:
1: KB(0.5 + 0.3):
¬PlayedInMovie(UmaThurman,m) ∨ Directed(Tarantino,m) | ¬PlayedInMovie(x,*g(x)*) ∨ ¬Directed(*x*,*g(x)*) ∨ (SuperActor(*g(x))*
$\ominus$ = {x/Tarantino, g(x)/m}
SUB:
¬PlayedInMovie(UmaThurman,m) ∨¬PlayedInMovie(Tarantino,*m*) ∨ SuperActor(*Tarantino)*

2: KB(1 + 0.6):
¬PlayedInMovie(UmaThurman,m) ∨¬PlayedInMovie(Tarantino,*m*) ∨ (SuperActor(*Tarantino)* |
PlayedInMovie(UmaThurman,m)
SUB:
¬PlayedInMovie(Tarantino,*m*) ∨ SuperActor(*Tarantino)*

3: KB(2 + 0.7):
¬PlayedInMovie(Tarantino,*m*) ∨ (SuperActor(*Tarantino)* |
PlayedInMovie(Tarantino,m)

SUB:
SuperActor(Tarantino)

4: KB(3 + ¬Q):
SuperActor(Tarantino) | ¬ SuperActor(Tarantino)
⊥

(b) Translate the information given in FOL into English (or Norwegian) and describe in high level the reasoning you could apply in English to have the same result (in other words, describe a proof of the result in natural language).

1.
X is a super-actor if, and only if, there is at least one movie where X has played and directed the same movie.

2.
Uma Thurman has <u>only</u> played in the movies that are directed by Tarantino.

3.
There is at least one movie where Uma Thurman and Tarantino has played in the same movie.