Post-Evaluation Model of - Multiple Award Schedule (MAS) Using Fuzzy Theory

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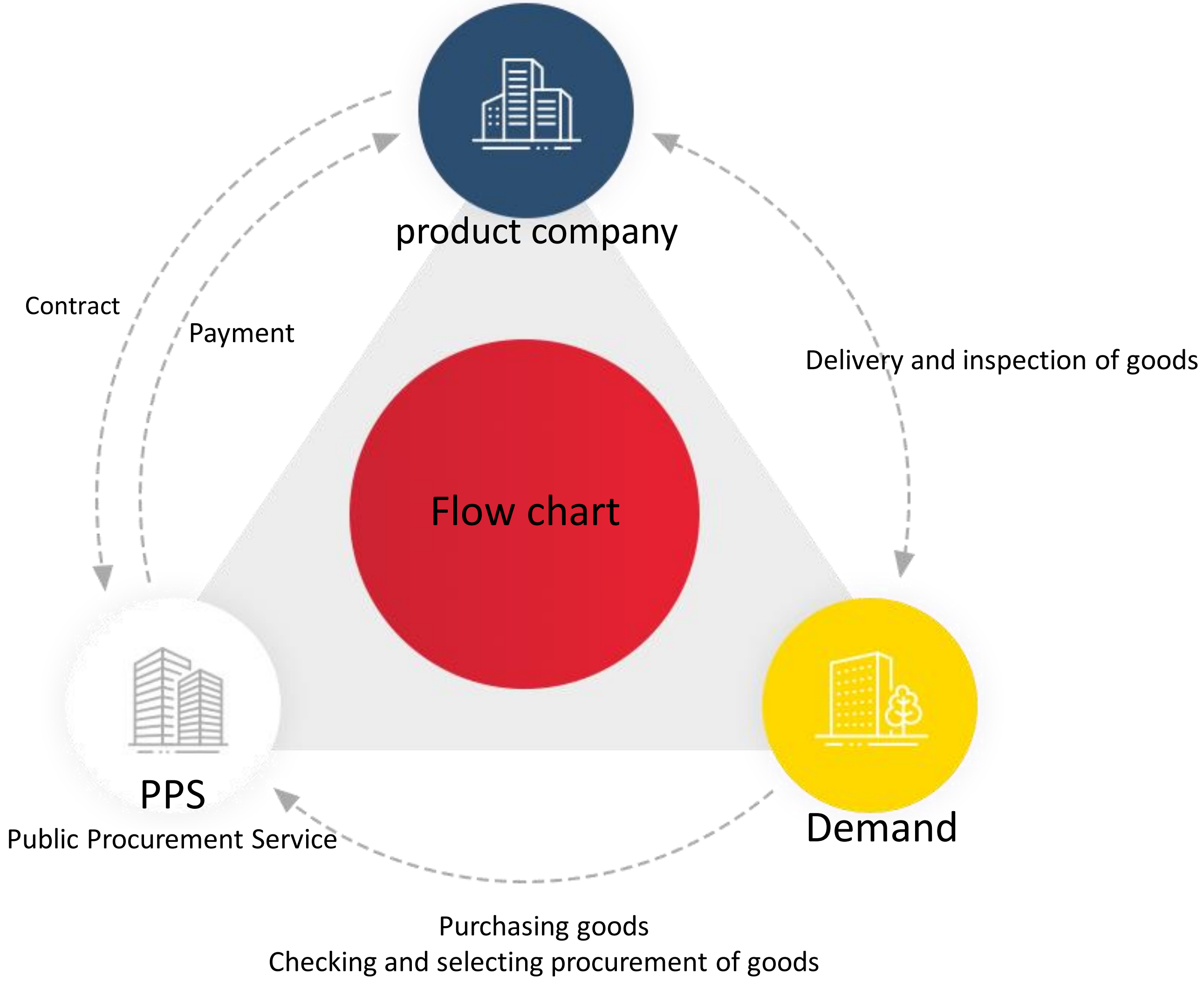
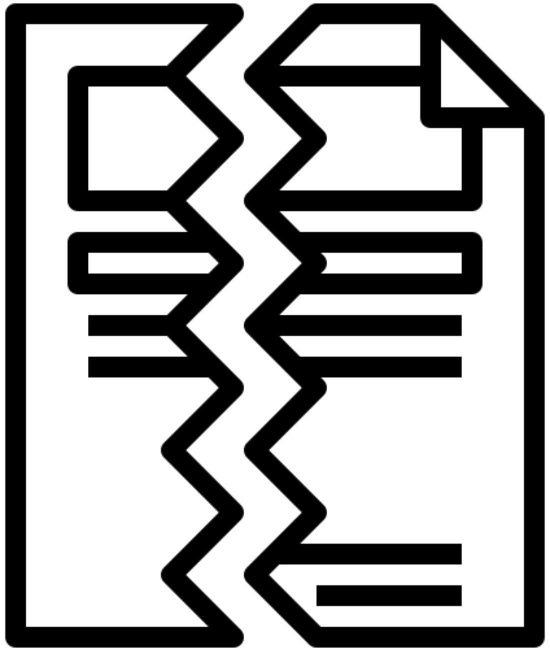
PART 01

Introduction

MAS: Multiple Award Schedule





ambiguous information can be lost.

PART 02

Fuzzy numbers

Definition 1 (Fuzzy number)

For the fuzzy membership function $\mu_A: R \to [0,1]$ of a fuzzy set $\tilde{A}$, $\tilde{A}$ is a fuzzy number if satisfies following properties:

1. (Normality) There exist which satisfies $\mu_{\tilde{A}}(x_0) = 1$.

2. (Fuzzy Convexity) For given $x, y \in R$, and $t \in [0,1] \subset R$,
$$\mu_{\tilde{A}}(tx + (1-t)y) \geq \min\left(\mu_A(x), \mu_A(y)\right)$$

3. (Upper semi-continuity) For any $x_0 \in R$, if $\mu_{\tilde{A}}(x_0) \geq \lim_{x \to x_0^+} \mu_{\tilde{A}}(x)$, then $\tilde{A}$ is a fuzzy number.

Zadeh's extension principle

Suppose that $X = X_1 \times \cdots \times X_n$ is a cartesian product and μ_i is a fuzzy set in X_i, respectively, and $f: X \to Y$ is a mapping. Then the extension principle allows us to define a fuzzy set v in Y by

$$v(y) = \begin{cases} \sup\limits_{(x_1,\ldots,x_n)\in f^{-1}(y)} \min\{\mu_1(x_1), \ldots, \mu_n(x_n)\} & ,\text{if } f^{-1}(y) \neq \emptyset \\ 0 & ,\text{if } f^{-1}(y) = \emptyset \end{cases}$$

In the case of $n = 1$, the extension principle reduces to a fuzzy set $v = f(\mu)$ defined by

$$v(y) = \begin{cases} \sup\limits_{x\in f^{-1}(y)} \mu(x) & ,if\ f^{-1}(y) \neq \emptyset \\ 0 & ,if\ f^{-1}(y) = \emptyset \end{cases}$$

Reference : [11]L. A. Zadeh. Fuzzy sets. Information and Control, 8:338–353, 1965.

Membership functions for triangular and trapezoidal fuzzy numbers

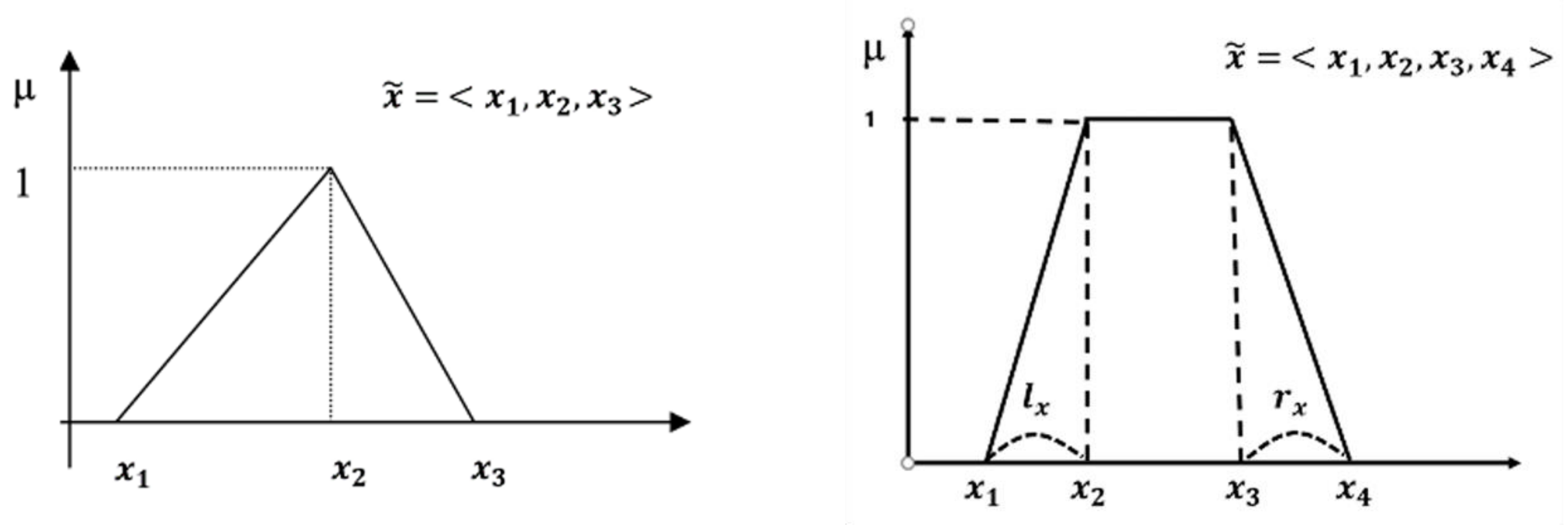


Figure 1. Membership functions for triangular and trapezoidal fuzzy numbers

The triangular fuzzy number and the trapezoidal fuzzy

$(x_1, x_2, x_3) =< l_x, x, r_x >$, where $l_x = x_2 - x_1, x = x_2, r_x = x_3 - x_2$

$(x_1, x_2, x_3, x_4) =< l_x, x_2, x_3, r_x >$, where $l_x = x_2 - x_1, r_x = x_4 - x_3$

PART 03

Post-evaluation function for fuzzy data

Proposed One-sided Fuzzy Reliability Function

$$\tilde{d}_i^j = \begin{cases} 0, & \tilde{Y}_i^j \le \tilde{Y}_{min}^j \\ \frac{\tilde{Y}_i^j - \tilde{Y}_{\min}^j}{\tilde{Y}_{\max}^j - \tilde{Y}_{\min}^j}, & \tilde{Y}_{\min}^j < \tilde{Y}_i^j < \tilde{Y}_{\max}^j \\ 1, & \tilde{Y}_i^j \ge \tilde{Y}_{\max}^j \end{cases}$$

$\tilde{Y}_i^j$: fuzzy satisfaction with j-th item of the i-th company (or fuzzy evaluation score)

$\tilde{Y}_{min}^j$: minimum fuzzy satisfaction with j-th item (or fuzzy evaluation score)

$\tilde{Y}_{max}^j$: maximum fuzzy satisfaction with j-th item (or fuzzy evaluation score) $(i = 1, \ldots, n, j = 1, \ldots, m)$

Proposed Two-sided Fuzzy Reliability Function

$$\tilde{d}_i^j = \begin{cases} \dfrac{\tilde{Y}_i^j - \tilde{Y}_{\min}^j}{\tilde{C}_i^{\,j} - \tilde{Y}_{\min}^j}, & \tilde{Y}_{\min}^j < \tilde{Y}_i^j < \tilde{C}_i^{\,j} \\ \dfrac{\tilde{Y}_i^j - \tilde{Y}_{\max}^j}{\tilde{C}_i^{\,j} - \tilde{Y}_{\max}^j}, & \tilde{C}_i^{\,j} \le \tilde{Y}_i^j < \tilde{Y}_{\max}^j \\ 0, & \tilde{Y}_i^{\,j} \le \tilde{Y}_{min}^j \ or\ \tilde{Y}_i^{\,j} \ge \tilde{Y}_{\max}^j \end{cases}$$

$\tilde{Y}_i^{\,j}$: fuzzy satisfaction with j-th item of the i-th company (or fuzzy evaluation score)

$\tilde{Y}_{min}^j$: minimum fuzzy satisfaction with j-th item (or fuzzy evaluation score)

$\tilde{Y}_{max}^j$: maximum fuzzy satisfaction with j-th item (or fuzzy evaluation score)

$\tilde{C}_i^{\,j}$: specific objective value for the j-th item of the i-th company $(i = 1, \dots, n, j = 1, \dots, m)$

Post-evaluation function using fuzzy scale

a *Post*-evaluation function $\widetilde{D}_i$ using a fuzzy scale in which the result of fuzzy satisfaction evaluation $\tilde{d}_i^j$ the importance of item j of the i-th company is defined as $w_i^j (i = 1, \ldots, n, j = 1, \ldots, m)$

$$\widetilde{D}_i = \sum_{j=1}^{m} w_i^j \tilde{d}_i^j, \text{ where } \sum_{j=1}^{m} w_i^j = 1.$$

Definition 2 (Operation of Triangular fuzzy number using function principle)

triangular fuzzy numbers: Let $\tilde{X} = (X_l, X, X_r)$ and $\tilde{Y} = (Y_l, Y, Y_r)$

(1) Addition: $\tilde{X} + \tilde{Y} = (X_l + Y_l, X + Y, X_r + Y_r)$

(2) Subtraction: $\tilde{X} - \tilde{Y} = (X_l - Y_r, X - Y, X_r - Y_l)$

(3) Multiplication: $\tilde{X} \cdot \tilde{Y} = (\min(X_l Y_l, X_l Y_r, X_r Y_l, X_r Y_r)\,, XY, \max(X_l Y_l, X_l Y_r, X_r Y_l, X_r Y_r))$

(4) Division: $\frac{\tilde{X}}{\tilde{Y}} = (\min\left(\frac{X_l}{Y_l}, \frac{X_l}{Y_r}, \frac{X_r}{Y_l}, \frac{X_r}{Y_r}\right), \frac{X}{Y}, \max\left(\frac{X_l}{Y_l}, \frac{X_l}{Y_r}, \frac{X_r}{Y_l}, \frac{X_r}{Y_r}\right))$

Definition 2 (Operation of Trapezoidal fuzzy number using function principle)

trapezoidal fuzzy numbers: Let $\tilde{X} = (X_l, X_1, X_2, X_r)$ *and* $\tilde{Y} = (Y_l, Y_1, Y_2, Y_r)$

(1) Addition: $\tilde{X} + \tilde{Y} = (X_l + Y_l, X_1 + Y_1, X_2 + Y_2, X_r + Y_r)$

(2) Subtraction: $\tilde{X} - \tilde{Y} = (X_l - Y_r, X_1 - Y_2, X_2 - Y_1, X_r - Y_l)$

(3) Multiplication: $\tilde{X} \cdot \tilde{Y} = (\min(X_l Y_l, X_l Y_r, X_r Y_l, X_r Y_r), X_1 Y_1, X_2 Y_2, \max(X_l Y_l, X_l Y_r, X_r Y_l, X_r Y_r))$

(4) Division: $\frac{\tilde{X}}{\tilde{Y}} = (\min\left(\frac{X_l}{Y_l}, \frac{X_l}{Y_r}, \frac{X_r}{Y_l}, \frac{X_r}{Y_r}\right), \frac{X_1}{Y_1}, \frac{X_2}{Y_2}, \max\left(\frac{X_l}{Y_l}, \frac{X_l}{Y_r}, \frac{X_r}{Y_l}, \frac{X_r}{Y_r}\right))$

Definition 3 (New operations for triangular fuzzy numbers using fixed spread)

Let $\tilde{X} =< l_X, X, r_X >$ and $\tilde{Y} =< l_Y, Y, r_Y >$ be two triangular fuzzy numbers

And where $l_X = X - X_l,\ l_Y = Y - Y_l, r_X = X_r - X,$ and $r_Y = Y_r - Y$

$$(1)\ \tilde{X} + \tilde{Y} = < l, X + Y, r >$$

$$(2)\ \tilde{X} - \tilde{Y} = < l, Y - X, r >$$

$$(3)\ \tilde{X} \cdot \tilde{Y} = < l, XY, r >$$

$$(4)\ \frac{\tilde{X}}{\tilde{Y}} = \tilde{X} \cdot \frac{1}{\tilde{Y}} \left(\frac{1}{\tilde{Y}} = < l, \frac{1}{Y}, r >, \qquad Y_1 > 0, Y_2 > 0 \right)$$

$$(5)\ k < l, X, r > = < l, kX, r > \quad (k > 0)$$

For symmetric fuzzy numbers, we define $l = r$

Definition 3 (New operations for trapezoidal fuzzy numbers using fixed spread)

Let $\tilde{X} =< l_X, X_1, X_2, r_X >$ and $\tilde{Y} =< l_Y, Y_1, Y_2, r_Y >$ be two trapezoidal fuzzy numbers

And where $l_X = X_1 - X_l,\ l_Y = Y_1 - Y_l, r_X = X_r - X_2,$ and $r_Y = Y_r - Y_2$

$$(1)\ \tilde{X} + \tilde{Y} = < l, X_1 + Y_1, X_2 + Y_2, r >$$

$$(2)\ \tilde{X} - \tilde{Y} = < l, Y_1 - X_2, Y_2 - X_1, r >$$

$$(3)\ \tilde{X} \cdot \tilde{Y} = < l, \min\{X_1Y_1, X_1Y_2, X_2Y_1, X_2Y_2\}, \max\{X_1Y_1, X_1Y_2, X_2Y_1, X_2Y_2\}, r >$$

$$(4)\ \frac{\tilde{X}}{\tilde{Y}} = \tilde{X} \cdot \frac{1}{\tilde{Y}} \left(\frac{1}{\tilde{Y}} = < l, \frac{1}{Y_2}, \frac{1}{Y_1}, r >, \qquad Y_1 > 0, Y_2 > 0 \right)$$

$$(5)\ k < l, X_1, X_2, r > = < l, kX_1, kX_2, r > \quad (k > 0)$$

Remark (Modification of fuzzy operations)

For the case when the left or right end points are not located in the defined domain, they are modified as follows:

If (x_1, x_2, x_3), $x_1 < 0$ or $x_3 > 1$, then the result is defined by $(\max\{x_1, 0\}, x_2, \min\{x_3, 1\})$

If (x_1, x_2, x_3, x_4), $\mathrm{x}_1 < 0$ or $x_4 > 1$, then the result is defined by $(\max\{x_1, 0\}, x_2, x_3 \min\{x_4, 1\})$

PART 04

Data Analysis using Fuzzy Post-Evaluation Model

Fuzzy Post-Evaluation Model for PPS of South Korea using one-sided fuzzy reliability function

Table 1. Status of MAS in South Korea

	2017	2018	2019	2020	2021
Total amount (Billions of KRW)	11,408	11,755	13,473	20,408	23,082
# of contracts (Thousands)	1,825	1,895	2,236	2,805	3,010

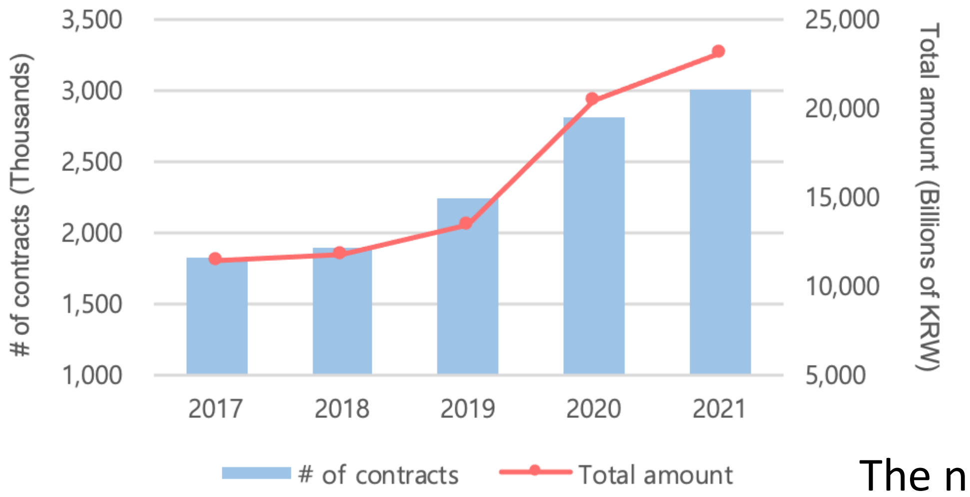


Figure 2. Status of MAS in South Korea

The number of contracts increased from 1.8 million to 3 million.

To evaluate the performance of the proposed model, we generated 5 sample data

Table 3. Sample data for MAS

Company Name	Delivery Observation	Delivery Delay	Defect Care	Quality Sat.	Price Sat.	Service Sat.	Post-Sat.	Supply Ratio	Defect Care Period	Fraud Penalty	Trade Penalty
A	75	10	5	5	4	3	3	76	9	30	45
B	90	4	1	1	5	6	5	90	3	3	6
C	35	20	8	8	1	0	1	43	15	120	130
D	82	7	2	2	5	4	4	87	6	15	30
E	45	17	7	7	3	2	2	50	12	90	80

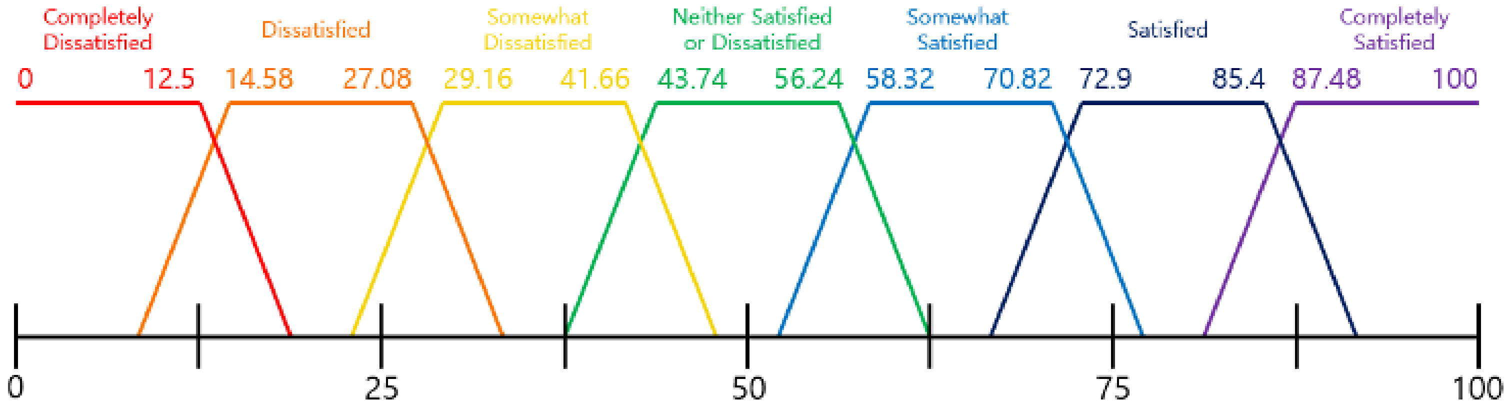


Figure 3. Satisfaction score using trapezoidal fuzzy number

We used 7 point Likert scale for Qualitative categories
like ‘Quality satisfaction’, ‘Price satisfaction’, ‘Service satisfaction’, ‘Post-satisfaction’,
and they are surveyed as percentages of satisfaction

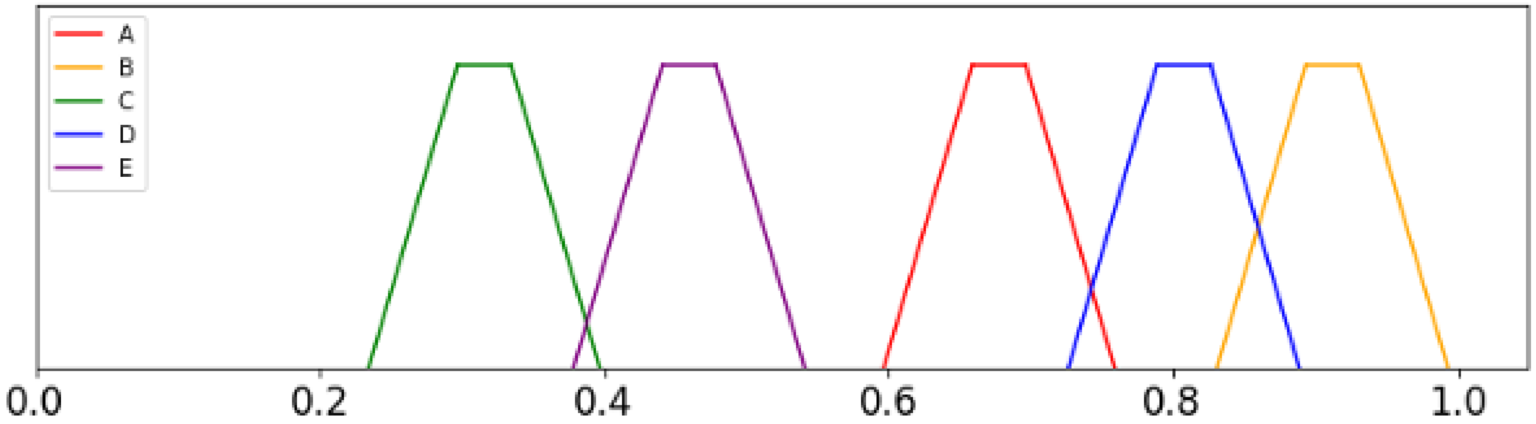


Figure 4. Post-evaluation score using trapezoidal fuzzy numbers

Table 4. Post-evaluation score of 5 companies

Company Name	Fuzzy Score
A	(0.596, 0.659, 0.696, 0.759)
B	(0.831, 0.893, 0.931, 0.993)
C	(0.234, 0.297, 0.334, 0.397)
D	(0.726, 0.789, 0.826, 0.889)
E	(0.378, 0.441, 0.478, 0.541)

Also each quantitative category is considered as an interval instead of one number for interval operation with fuzzy numbers

Fuzzy Post-Evaluation Model for using two-sided fuzzy reliability function

Table 5. Sample Data for Delivery Service

Store	Arrival Time1	Score1	Arrival Time 2	Score 2	ArrivalTime3	Score 3	Arrival Time 4	Score 4	Arrival Time 5	Score 5
A	11:50	20	11:42	12	12:24	54	12:11	41	11:45	15
B	12:01	31	12:03	33	11:54	24	12:08	38	11:57	27
C	12:24	54	12:30	60	11:31	1	11:33	3	12:29	59
D	11:47	17	11:52	22	12:01	31	11:58	28	12:11	41
E	12:14	44	11:39	9	11:30	0	12:28	58	11:54	24

Let's consider the delivery system. If a customer instructs them to deliver some items at a certain time, the customer will be less satisfied if they ship earlier or later than that time.

To compare the results using two different operations
we generated five sample data

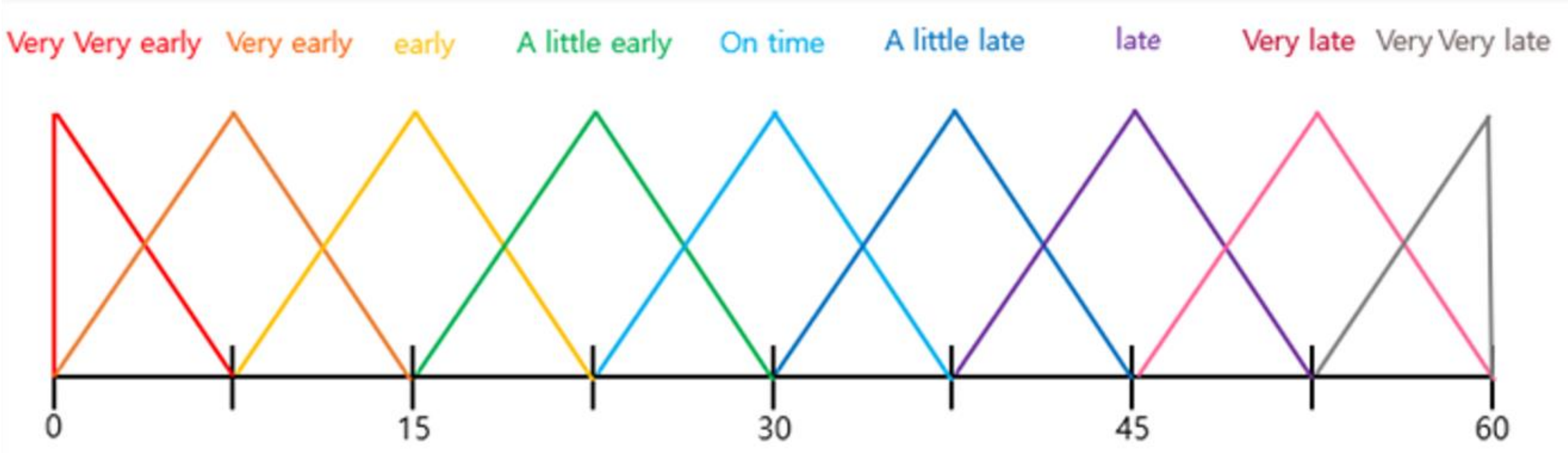


Figure 5. Post-Evaluation score using triangular fuzzy numbers

Results using fuzzy operations with function principle

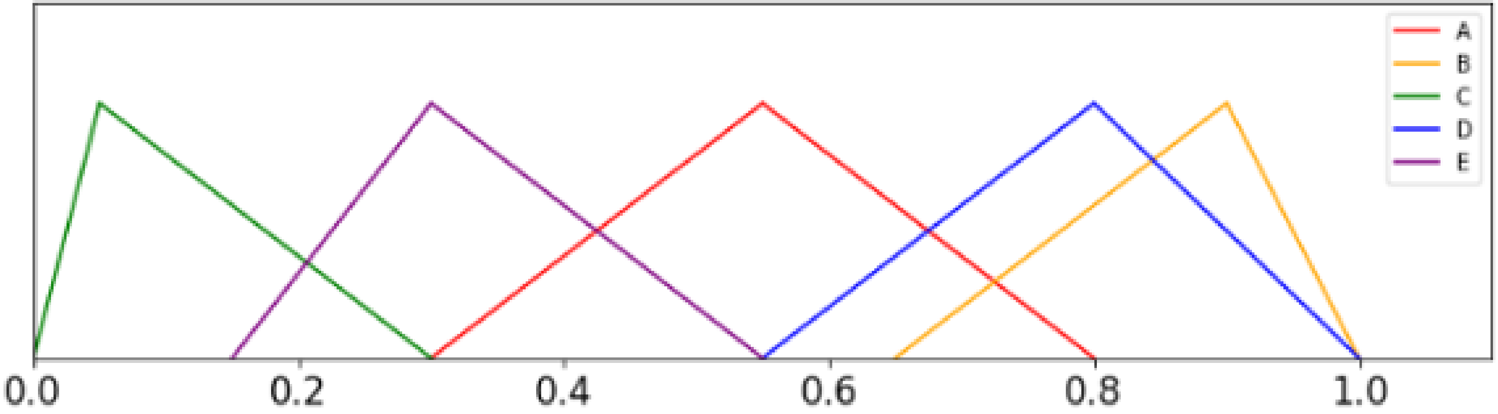


Figure 6. Fuzzy post-evaluation scores of the delivery service data calculated by function principle

Table 6. Fuzzy post-evaluation scores of the delivery service data calculated by function principle

Store	Fuzzy post-evaluation score
A	[0.3, 0.55, 0.8]
B	[0.65, 0.9, 1.0]
C	[0.0, 0.05, 0.3]
D	[0.55, 0.8, 1.0]
E	[0.15, 0.3, 0.55]

the delivery service of each store is not always the same
some differences can occur from situation to situation
the fuzzy post-evaluation score shows the attributes of the given data better

Results using fuzzy operations with fixed spread

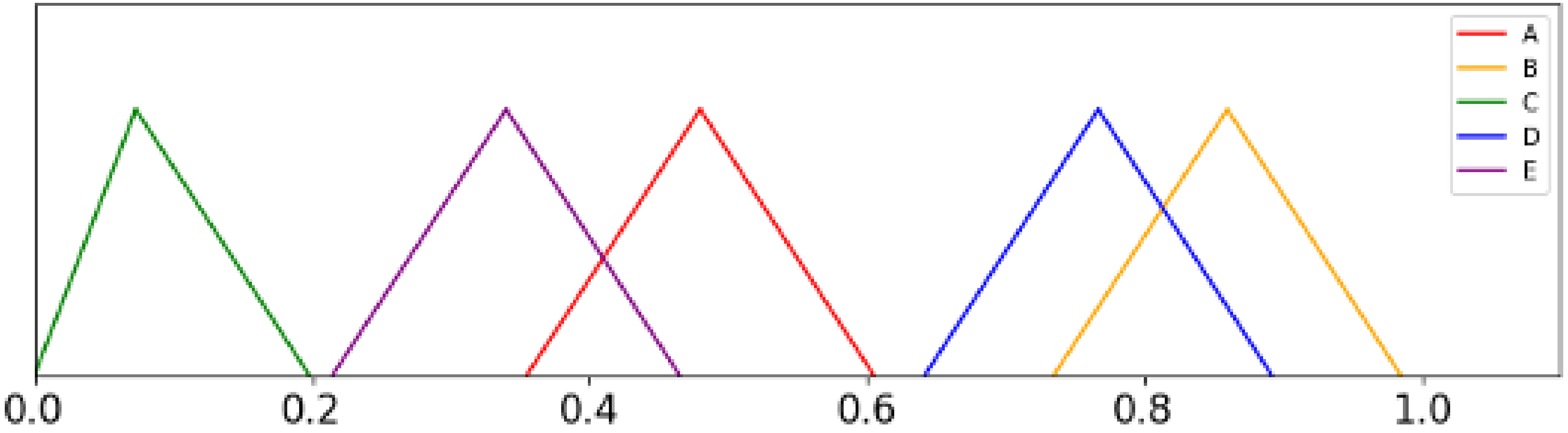


Figure 9. Fuzzy post-evaluation scores of the delivery service data calculated by fixed spread

Table 7. Fuzzy post-evaluation scores of the delivery service data calculated by fixed spread

Store	Fuzzy post-evaluation score
A	(0.355, 0.48, 0.605)
B	(0.735, 0.86, 0.985)
C	(0.0, 0.073, 0.198)
D	(0.642, 0.767, 0.892)
E	(0.215, 0.340, 0.465)

fixed spread symmetric triangular fuzzy numbers
The results using fuzzy operations with fixed spread
reflect the characteristics of variables better than those of function principle

PART 05

Conclusions

Conclusion

- When comparing data,
  Our newly defined fuzzy operations are more helpful than the existing defined fuzzy operations

- If qualitative indicators are fuzzified with our post-valuation function,
  it is possible to evaluate indicators without loss of information

- This study can be applied in all situations where this post-evaluation system is required

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