Post-Evaluation Model using Fuzzy Theory

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Abstract

Companies make transactions based on signed contracts. There are several contracting methods for product distribution, such as the Multiple Award Schedule (MAS). MAS can be employed to provide high-quality goods to consumers by inducing competition among suppliers. After the contract is implemented, it is necessary to evaluate the contract content or product service. Therefore, post-evaluation is performed after the contract is implemented, such as whether the goods have been delivered according to the contract and the quality of the goods is assessed. As an evaluation method, satisfaction assessments, including price satisfaction or service satisfaction, are commonly used. In general satisfaction evaluations, qualitative indicators are mainly used. However, these indicators are ambiguous when the opinion of a person is being expressed. Furthermore, for instance, in the case of satisfaction, data can be grouped by a 5-point scale, such as, one ranging from 'very satisfied' and 'very dissatisfied'. When the data are grouped and summarized in this way, ambiguous information loss occurs. To solve this, we employed the fuzzy theory in this study. Fuzzy theory expresses ambiguous human language as one that computers can understand. Using the proposed fuzzy reliability function, we can set the data in some fuzzy intervals, and manage all data without loss of information. In this study, we proposed a new post-evaluation model using fuzzy numbers to reflect the situation well in the evaluation results. Therefore, we defined two fuzzy reliability functions for post-evaluation. Two different types of fuzzy operations were applied. One is fuzzy operation using the function principle, and the other is a new fuzzy operation that has a fixed spread. Furthermore, we performed data analysis using the model with triangular and trapezoidal fuzzy numbers. In the analysis, we generated some sample data. One type of data, which were used by a one-sided fuzzy reliability function, was grouped on a 7-point Likert scale for qualitative categories such as post-satisfaction. The other data, which were used by two-sided fuzzy reliability function, was about the delivery service. Because the model can be applied to assessments with quantitative indicators, it is expected to contribute to reliable results in various fields. Additionally, because the fuzzy theory was applied, satisfaction can be compared by utilizing ambiguous information.

keyword

post-evaluation model; satisfaction; fuzzy reliability function; Multiple Award Schedule (MAS); fuzzy theory; fuzzy numbers

1. Introduction

Several users supply or purchase goods in the procurement market. Therefore, for the smooth distribution of the market, the use of some rules is necessary. Based on the rules at the national level, there are various contracts created for management and supervision among companies. Among the contracts, the Multiple Award Schedule (MAS) has shown the best performance. Therefore, the number of contractors who use MAS is increasing every year.

MAS was designed to improve the problems of quality degradation and lack of diversity faced by other similar systems. In addition, it is a consumer-centered contract method. This method is widely used in the United States, Canada, and South Korea (www.pps.go.kr).

MAS focuses on inducing fair competition and supplying quality goods to consumers by selecting multiple suppliers. In addition, as the e-commerce market grows, the suitability of this method is being embraced in some countries that operate MAS.

In South Korea, it is used by the public procurement service (PPS). PPS is an organization within South Korea government that is currently serving as an intermediary for sale and purchasing companies at Korea ON-line e-Procurement System (KONEPS).

The names of institution in charge of government procurement and evaluation of items in procurement vary from country to country. For example, in Germany, the name is the Federal Ministry for Economic Affairs and Energy, and in Canada, the institution is Public Services and Procurement Canada (PSPC) agency. Therefore, in

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this study, we focused on the PPS of South Korea government procurement, which operates MAS.

The post-evaluation items of MAS are divided into period of delivery, quality, satisfaction of demand institutions, service, and sincerity in fulfilling the contract. Furthermore, each item is subdivided into several items, and there are 11 evaluation indicators (www.gmas.or.kr).

The allocation of each evaluation item is different, and the score of evaluation is measured by applying different scores between grades for each evaluation indicators.

Currently, evaluations of procurement companies are conducted three times, including pre- and post-evaluation, and in some cases, interim inspections are also conducted. In the pre-evaluation, the reasons for disqualification of the procurement company, and whether the company qualifies to participate in the bidding are evaluated through eligibility evaluation for contractor selection.

In the post-evaluation, the PPS evaluates the quality, period of delivery, and service of procurement twice a year, and in the case of the MAS phase 2 competition system, it is reflected in the company's next contract. However, the evaluation process of MAS is still insufficient.

Among the post-evaluation items, indicators, such as satisfaction of demand institutions, are evaluated by the person in charge; thus, they are not quantitative. If these qualitative indicators are calculated in the same manner as other quantitative indicators, inaccuracy may be arise. Therefore, in this study, a new post-evaluation model was proposed that follows the evaluation index calculation using the concept of fuzzy numbers.

Several studies have been conducted on the post-evaluation model. In 2007, Yoon presented a satisfaction evaluation function, a model for post-supplier evaluation in a public procurement system based on the MAS and used the reliability function proposed by Derringer and Suich (1980). The reliability function can have various shapes between 0 and 1, depending on the unique characteristics of the indicator using a simple linear function or a general exponential function.

The reliability function used in this study is key to quantifying the measurement results of the evaluation measurement index and deriving the total score. The application process derives the current satisfaction level for each indicator through a survey.

Quantitative evaluation indicators employ measured values without changing them; however, qualitative evaluation indicators should be converted into quantitative values for statistical analyses. To this end, a method has been used in which the evaluators could significantly judge the difference in satisfaction by applying the theory that the range of short-term memory limits in humans exists within 7±2, and constructing it in the form of 5, 7, and 9 scales.

In addition, this study introduces an evaluation method to improve the post-evaluation model based on the problems associated with the measurement of ambiguous data.

Studies on satisfaction in the supply system using fuzzy scales were studied by Kumar et al. in 2004 and 2006. In these studies, using the reliability function suggested by Derringer and Suich (1980), the result of satisfaction in the evaluation of a particular item was expressed as a function value of the fuzzy membership function.

In particular, a post-model representing the data itself as a fuzzy number was proposed, rather than representing only the result of satisfaction as a fuzzy membership function value.

The fuzzy number is a special type of a fuzzy set suggested by Zadeh (1965). It allows ambiguous or linguistic expressions during quantification to be expressed mathematically, thereby helping us to use the properties of ambiguous information when analyzing data.

For example, satisfaction is evaluated by dividing the criteria into 'satisfied,' 'normal,' and 'dissatisfied;' however, this is a subjective criterion and is difficult to express quantitatively. Specifically, the results may vary depending on the evaluator's criteria. Therefore, the concept of fuzzy numbers that can be quantified while reducing information loss is introduced into the post-evaluation model. The fuzzy theory can solve the problem of ambiguity, because it uses a membership function that represents the degree of belonging to the part when expressing numbers. For this reason, the fuzzy theory has been applied to many fields so far such as regression, mediation analysis, time series, ANOVA (analysis of variance), and chaos theory where ambiguous data can be observed

However, in this field, the post-evaluation model applying a fuzzy theory has not been previously investigated. Therefore, we improved the evaluation accuracy of the post-evaluation model.

The rest of this paper is organized as follows: in Section 2, we present some literature review and fuzzy theory. In Section 3, we propose new fuzzy reliability functions for the post-evaluation of each item and operations of triangular and trapezoidal fuzzy numbers. Furthermore, we present a method of calculating post-satisfaction using the fuzzy scale. In Section 4, we introduce data analysis using a fuzzy post-evaluation model. Finally, the conclusions are drawn in Section 5.

2. Fuzzy numbers

Recently, research on artificial intelligence, which can function similar to a human brain, has been actively conducted. For an artificial intelligence to accurately understand and perform human needs, it must be able to process all languages used by humans. There are accurate expressions, such as "two" and "10" degrees, in human language; however, ambiguous expressions, such as "a few" and "about 10 degrees" can be a challenge. Therefore, fuzzy theory provides a theoretical basis for addressing these ambiguous human languages mathematically.

This theory, first introduced by Zadeh (1965), plays a key role in reducing the loss of information in the process of expressing imprecise data. He first derived the fuzzy theory from the idea of how the degree of beauty can be expressed.

One of the evaluation indicators of the post-evaluation model, 'consumer satisfaction', is also measured by vaguely expressed data. Therefore, to obtain more accurate post-evaluation results, it is necessary to express and measure evaluation indicators using fuzzy theory.

To illustrate a fuzzy set, consider a group of people of 'young age.' Generally, 'young age' is subjective and can have various ranges, such as 0 to 30 years and 0 to 40 years, depending on how a person perceives it.

In general set theory, what corresponds to 'young age' is not called a set because the degree of belonging is unclear. However, the fuzzy set can express this ambiguous degree of belonging by assigning a degree to the difference. For example, many people usually consider a person between the age of 20 and 30 as young, but it is certain that 20 is the youngest. Specifically, this range is not enough to simply express the people as 'young'; therefore, in some cases, it is necessary to distinguish the level of youth. Here, the fuzzy set provides a degree to the difference.

Suppose that the membership function of the fuzzy set of 'young age' is expressed as shown in Figure 1. Therefore, the value of membership function represents the degree of belonging to the fuzzy set, and in the following Figure 1, a 15-year-old person is likely to be included in the 'young age' 1, 30 years old is 0.87, and 55 years old or older is zero.

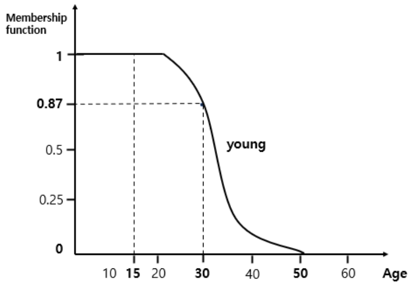

Figure 1. Fuzzy set 'young age'

A fuzzy number is a special type of fuzzy set, which is defined as follows:

Definition 1. Fuzzy number

For the fuzzy membership function $\mu_A: R \to [0,1]$ of a fuzzy set $\tilde{A}$, $\tilde{A}$ is a fuzzy number if $\tilde{A}$ satisfies the following properties:

(Normality) There exist x_0, which satisfies $\mu_{\tilde{A}}(x_0) = 1$.

(Fuzzy Convexity) For particular $x, y \in R$, and $t \in [0,1] \subset R$

if $\mu_{\tilde{A}}(x_0) \geq \lim_{x \to x_0^+} \mu_{\tilde{A}}(x)$,

$\mu_{\tilde{A}}(tx + (1-t)y) \geq \min(\mu_A(x), \mu_A(y))$.

(Upper semi-continuity) For any $x_0 \in R$,

then $\tilde{A}$ is a fuzzy number

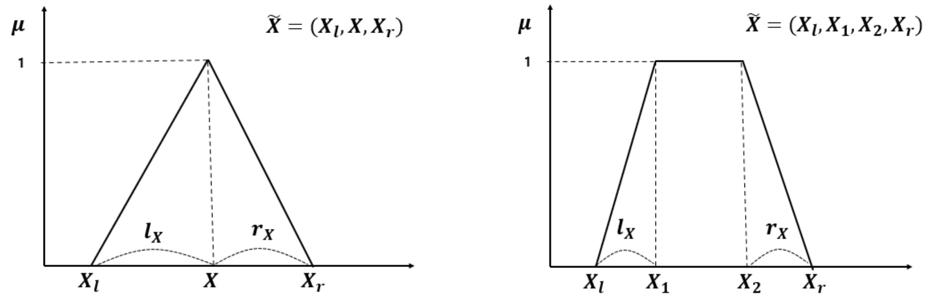

Figure 2. Membership functions for triangular and trapezoidal fuzzy numbers

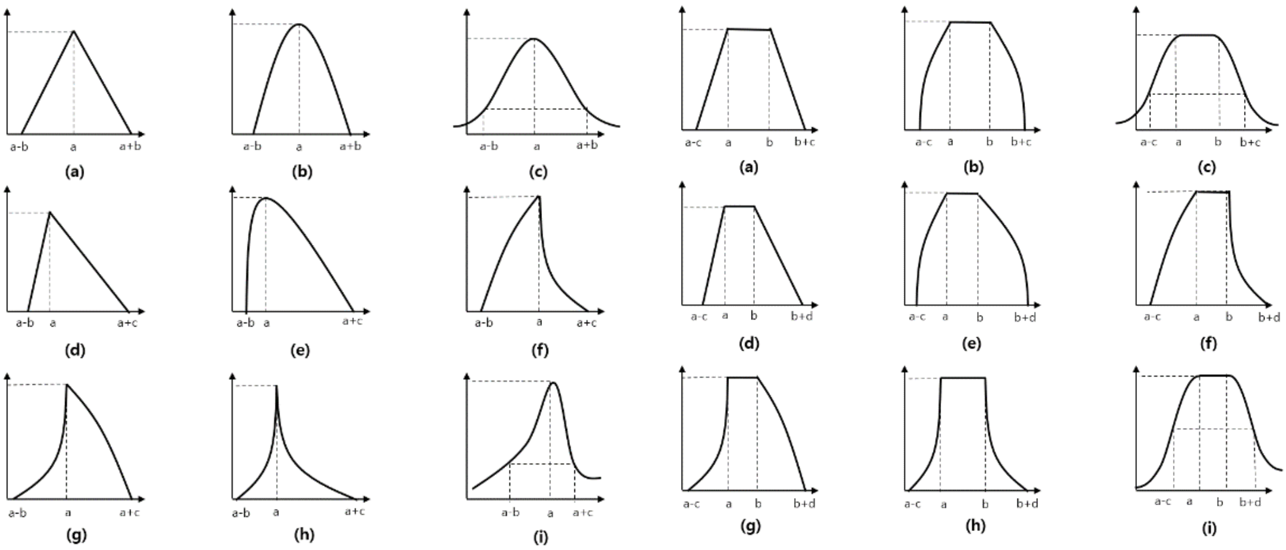

Figure 3. Various types of fuzzy numbers with extended triangular and trapezoidal fuzzy numbers

There are many types of operations of fuzzy numbers. The operations based on Zadeh's extension Principle (1965) is widely used. The extension principle is expressed as follows:

Suppose that $X = X_1 \times \ldots \times X_n$ is a cartesian product, μ_i is a fuzzy set in X_i, respectively, and $f: X \to Y$ is a mapping, then the extension principle allows us to define a fuzzy set v in Y as follows:

$$v(y) = \begin{cases} \sup\limits_{(x_1,\ldots,x_n)\in f^{-1}(y)} \min\{\mu_1(x_1),\ldots,\mu_n(x_n)\} & ,\text{if } f^{-1}(y) \neq \emptyset \\ 0 & ,\text{if } f^{-1}(y) = \emptyset \end{cases}$$

In the case of $n = 1$, the extension principle reduces to a fuzzy set $v = f(\mu)$ defined by

$$v(y) = \begin{cases} \sup\limits_{x\in f^{-1}(y)} \mu(x) & ,if\ f^{-1}(y) \neq \emptyset \\ 0 & ,if\ f^{-1}(y) = \emptyset \end{cases}$$

There are numerous forms of fuzzy number. Among them, the most commonly used is triangular fuzzy number, and is easy to use because it can be expressed in three points. In addition, there are various types of triangular and trapezoidal fuzzy numbers as shown in Figure 2.

The triangular and trapezoidal fuzzy numbers are denoted by (X_l, X, X_r) and (X_l, X_1, X_2, X_r), respectively. In addition, they can be represented by using spreads as follows:

$$(X_l, X, X_r) =< l_X, X, r_X >,$$

$$\text{where } l_X = X - X_l, r_X = X_r - X$$

$$\text{and } (X_l, X_1, X_2, X_r) =< l_X, X_1, X_2, r_X >,$$

$$\text{where } l_X = X_1 - X_l, r_X = X_r - X_2$$

In this study, we employed triangular fuzzy numbers and trapezoidal fuzzy numbers.

When the satisfaction was measured highest at a particular value, triangular fuzzy number was employed. (see Figure 2.) Conversely, when measuring the satisfaction of each evaluation item measured by the survey with fuzzy data, trapezoidal fuzzy number was employed. (see Figure 2.) Although the triangular fuzzy number is most commonly used in fuzzy data analysis, the trapezoidal fuzzy number is the most appropriate as a fuzzy scale that expresses a person's mind. This is because it is better for the point indicating the largest degree of membership degree to be expressed in a range rather than a single number.

3. Post-evaluation function for fuzzy data

3.1. Proposed Fuzzy Reliability Function for Post-evaluation of Each Item

3.1.1 Proposed One-sided Fuzzy Reliability Function

Here, we consider the satisfaction evaluation function applicable to fuzzy data using the reliability function proposed by Derringer and Suich (1980) introduced.

Suppose the total number of companies to be surveyed is n, and the total number of items evaluated is m.
If $\tilde{d}_i^j$ is the satisfaction evaluation function for $j(j = 1, \dots, m)$-th item of the $i(i = 1, \dots, n)$-th company, it is expressed as follows.

$$\tilde{d}_i^j = \begin{cases} 0, & \tilde{Y}_i^j \le \tilde{Y}_{min}^j, \\ \frac{\tilde{Y}_i^j - \tilde{Y}_{\min}^j}{\tilde{Y}_{\max}^j - \tilde{Y}_{\min}^j}, & \tilde{Y}_{\min}^j < \tilde{Y}_i^j < \tilde{Y}_{\max}^j, \\ 1, & \tilde{Y}_i^j \ge \tilde{Y}_{\max}^j, \end{cases} \quad [1]$$

where

$\tilde{Y}_i^j$: fuzzy satisfaction with j-th item of the i-th company,

$\tilde{Y}_{min}^j$: minimum fuzzy satisfaction with j-th item, and

$\tilde{Y}_{max}^j$: maximum fuzzy satisfaction with j-th item.

$(i = 1, \dots, n, j = 1, \dots, m)$

This is an indicator that normalizes the fuzzy satisfaction (or fuzzy score) measured by the fuzzy scale and evaluates it on the same basis regardless of unit or size.

3.1.2 Proposed Two-sided Fuzzy Reliability Function.

When the response has a specific objective value for evaluating the satisfaction, the two-sided fuzzy reliability function is used. The fuzzy reliability function for evaluating satisfaction obtained by fuzzy data is expressed as:

$$\tilde{d}_i^j = \begin{cases} \frac{\tilde{Y}_i^j - \tilde{Y}_{\min}^j}{\tilde{C}_i^j - \tilde{Y}_{\min}^j}, & \tilde{Y}_{\min}^j < \tilde{Y}_i^j < \tilde{C}_i^j \\ \frac{\tilde{Y}_i^j - \tilde{Y}_{\max}^j}{\tilde{C}_i^j - \tilde{Y}_{\max}^j}, & \tilde{C}_i^j \le \tilde{Y}_i^j < \tilde{Y}_{\max}^j \\ 0, & \tilde{Y}_i^j \le \tilde{Y}_{min}^j \text{ or } \tilde{Y}_i^j \ge \tilde{Y}_{\max}^j \end{cases} \quad [2]$$

$\tilde{Y}_i^j$: fuzzy satisfaction with j-th item of the i-th company
$\tilde{Y}_{min}^j$: minimum fuzzy satisfaction with j-th item
$\tilde{Y}_{max}^j$:maximum fuzzy satisfaction with j-th
$\tilde{C}_i^j$: specific objective value for the j-th item of the i-th company

3.2. Proposed operations for trapezoidal fuzzy numbers

Typically, the operation of the fuzzy number is calculated using Zadeh's Extension Principle (1965). However, this method increases complexity in multiplying or dividing operations, and results in many cases. Therefore, in this study, we proposed an operation that transforms the interval operation in the calculation of the satisfaction evaluation function. The basic fuzzy operation has crucial drawback that if we repeat the operations then the spreads of fuzzy numbers increase more than those of their original forms. Therefore, we employed two different fuzzy operations. Definition 2 is the fuzzy operations using function principle by Gani (2012). Definition 3 is the proposed new operations using fixed spreads.

Definition 2. (Operation of Triangular and trapezoidal fuzzy number using function principle)

The following four operations can be performed on triangular fuzzy numbers: Let $\tilde{X} = (X_l, X, X_r)$ and $\tilde{Y} = (Y_l, Y, Y_r)$,

(1) Addition: $\tilde{X} + \tilde{Y} = (X_l + Y_l, X + Y, X_r + Y_r)$

(2) Subtraction: $\tilde{X} - \tilde{Y} = (X_l - Y_r, X - Y, X_r - Y_l)$

(3) Multiplication: $\tilde{X} \cdot \tilde{Y} = (\min\{X_lY_l, X_lY_r, X_rY_l, X_rY_r\}, XY, \max\{X_lY_l, X_lY_r, X_rY_l, X_rY_r\})$

(4) Division: $\frac{\tilde{X}}{\tilde{Y}} = (\min\{\frac{X_l}{Y_l}, \frac{X_l}{Y_r}, \frac{X_r}{Y_l}, \frac{X_r}{Y_r}\}, \frac{X}{Y}, \max\{\frac{X_l}{Y_l}, \frac{X_l}{Y_r}, \frac{X_r}{Y_l}, \frac{X_r}{Y_r}\}$

Similarly, the following four operations can be performed on trapezoidal fuzzy numbers:

Let $X = (X_l, X_1, X_2, X_r)$, $Y = (Y_l, Y_1, Y_2, Y_r)$,

(1) Addition: $\tilde{X} + \tilde{Y} = (X_l + Y_l, X_1 + Y_1, X_2 + Y_2, X_r + Y_r)$

(2) Subtraction: $\tilde{X} - \tilde{Y} = (X_l - Y_r, X_1 - Y_2, X_2 - Y_1, X_r - Y_l)$

(3) Multiplication: $\tilde{X} \cdot \tilde{Y} =$
$(\min\{X_lY_l, X_lY_r, X_rY_l, X_rY_r\}, X_1Y_1, X_2Y_2,$

$\max\{X_lY_l, X_lY_r, X_rY_l, X_rY_r\})$

(4) Division: $\frac{\tilde{X}}{\tilde{Y}} =$
$(\min\left\{\frac{X_l}{Y_l}, \frac{X_l}{Y_r}, \frac{X_r}{Y_l}, \frac{X_r}{Y_r}\right\}, \min\{\frac{X_1}{Y_1}, \frac{X_2}{Y_2}\}, \max\left\{\frac{X_1}{Y_1}, \frac{X_2}{Y_2}\right\}, \max\{\frac{X_l}{Y_l}, \frac{X_l}{Y_r}, \frac{X_r}{Y_l}, \frac{X_r}{Y_r}\})$

Next, the new operations using fixed spreads were provided.

Definition 3. (New operations for symmetric fuzzy numbers**)**

Let $\tilde{X} =< l_X, X, r_X >$ and $\tilde{Y} =< l_Y, Y, r_Y >$ be two triangular fuzzy numbers, where $l_X = X - X_l$, $l_Y = Y - Y_l, r_X = X_r - X,$ and $r_Y = Y_r - Y$, then fuzzy operations for symmetric triangular fuzzy numbers with fixed spreads are expressed as follows:

(1) $\tilde{X} + \tilde{Y} =< l, X + Y, r >$

(2) $\tilde{X} - \tilde{Y} =< l, Y - X, r >$

(3) $\tilde{X} \cdot \tilde{Y} =< l, XY, r >$

(4) $\frac{\tilde{X}}{\tilde{Y}} = \tilde{X} \cdot \frac{1}{\tilde{Y}} \left(\frac{1}{\tilde{Y}} =< l, \frac{1}{Y}, r >, \quad Y_1 > 0, Y_2 > 0\right)$

(5) $k < l, X, r > =< l, kX, r > \quad (k > 0)$

Moreover, the following four operations can be performed on symmetric trapezoidal fuzzy numbers.

Let $\tilde{X} =< l_X, X_1, X_2, r_X >$ and $\tilde{Y} =< l_Y, Y_1, Y_2, r_Y >$ be two trapezoidal fuzzy numbers, where $l_X = X_1 - X_l$, $l_Y = Y_1 - Y_l, r_X = X_r - X_2,$ and $r_Y = Y_r - Y_2$. Then fuzzy operations for symmetric trapezoidal fuzzy numbers with fixed spreads are expressed as follows:

(1) $\tilde{X} + \tilde{Y} =< l, X_1 + Y_1, X_2 + Y_2, r >$

(2) $\tilde{X} - \tilde{Y} =< l, Y_1 - X_2, Y_2 - X_1, r >$

(3) $\tilde{X} \cdot \tilde{Y} =$
$< l, \min\{X_1Y_1, X_1Y_2, X_2Y_1, X_2Y_2\}, \max\{X_1Y_1, X_1Y_2, X_2Y_1, X_2Y_2\}, r >$

(4) $\frac{\tilde{X}}{\tilde{Y}} = \tilde{X} \cdot \frac{1}{\tilde{Y}}$,

where $\frac{1}{\tilde{Y}} =< l, \frac{1}{Y_2}, \frac{1}{Y_1}, r >, \quad Y_1 > 0, Y_2 > 0$

(5) $k < l, X_1, X_2, r > =< l, kX_1, kX_2, r > \quad (k > 0)$

Remark. (Modification of fuzzy operations)

For the case when the left or right end points are not located in the defined domain, they are modified as follows:

If (x_1, x_2, x_3), $x_1 < 0$ or $x_3 > 1$, then the result is defined by $(\max\{x_1, 0\}, x_2, \min\{x_3, 1\})$

If (x_1, x_2, x_3, x_4), $x_1 < 0$ or $x_4 > 1$, then the result is defined by $(\max\{x_1, 0\}, x_2, x_3 \min\{x_4, 1\})$

3.3. Post-evaluation function using fuzzy scale

In this section, we propose a Post-evaluation function $\widetilde{D}_i$ using a fuzzy scale in which the result of fuzzy satisfaction evaluation $\tilde{d}_i^j$ of each item is fused into one using the above calculation. The weight considering the importance of item j-th of the i-th company is defined as $w_i^j (0 < w_i^j < 1)$ and the post-evaluation function for each company is defined as follows:

$$\widetilde{D}_i = \sum_{j=1}^{m} w_i^j \tilde{d}_i^j, \text{ where } \sum_{j=1}^{m} w_i^j = 1, \qquad [3]$$

where $i = 1, \dots, n$ and $j = 1, \dots, m$.

Data Analysis using Fuzzy Post-Evaluation Model for PPS of South Korea

Fuzzy Post-Evaluation Model for PPS of South Korea using one-sided fuzzy reliability function

First, the data from PPS (https://data.g2b.go.kr/) were employed to check the status of MAS in South Korea, as shown in Figure 4.

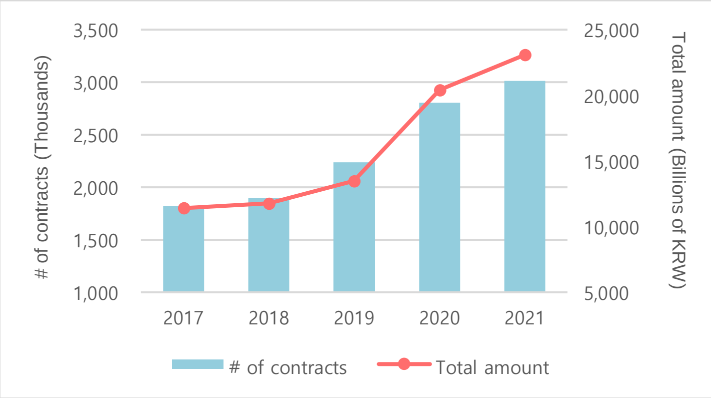

Figure 4. Status of MAS in South Korea

Table 1 . Sample data for MAS

Company Name	Delivery Observation	Delivery Delay	Defect Care	Quality Sat.	Price Sat.	Service Sat.	Post-Sat.	Supply Ratio	Defect Care Period	Fraud Penalty	Trade Penalty
A	75	10	5	5	4	3	3	76	9	30	45
B	90	4	1	1	5	6	5	90	3	3	6
C	35	20	8	8	1	0	1	43	15	120	130
D	82	7	2	2	5	4	4	87	6	15	30
E	45	17	7	7	3	2	2	50	12	90	80

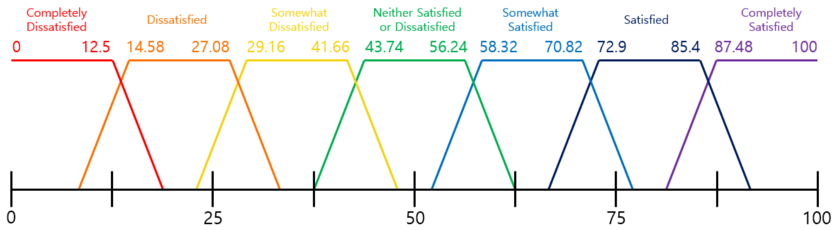

Figure 5. Satisfaction score using trapezoidal fuzzy number

Thereafter, to evaluate the performance of the proposed model, we generated 5 sample data on Table 1. Evaluation categories and allocation weights were the same as the current MAS system (http://www.gmas.or.kr). We used 7 point Likert scale for Qualitative categories like 'Quality satisfaction', 'Price satisfaction', 'Service satisfaction', and 'Post-satisfaction', and they were surveyed as percentages of satisfaction. Thereafter, we fuzzified the response shown in Figure 5 to calculate post-evaluation score.

In addition, each quantitative category was considered as an interval instead of one number for interval operation with fuzzy numbers. For example, the crisp value 10 was fuzzified by trapezoidal fuzzy number (10, 10, 10, 10). Data of each category was converted, and then calculated as a score between 0 and 1. Finally, we obtained post-evaluation score, weighted mean of those scores using the allocation points. We employed one-sided fuzzy reliability function [1] and post-evaluation function [3] to evaluate the companies. To calculate post-evaluation score, fuzzy operation from Definitions 2 and 3 were applied. In the process of the operation, a value might be less than 0 or greater than 1. Thus, a **Remark** is provided to correct it because the value should be in (0, 1).

Post-evaluation scores of 5 sample data using function principle of Definition 2 are listed in Table 2, and are shown as a fuzzy membership functions in Figure 6. Consequently, the score was shown in the order of companies B, D, A, E and C. The fuzzy post-evaluation score of company B after correction is (0.848, 1.000, 1.000, 1.000), which is the highest score, while that of company C is (0.000, 0.000, 0.046, 0.171), which the lowest score. The result of new operations of Definition 3 is in Table 3 and Figure 7. The rank of the companies is the same as the previous one using the function principle. Company B, one with the highest score, receives (0.845, 0.907, 1.000, 1.000). The score of company C is (0.000, 0.000, 0.046, 0.171), which is the highest score among the 5 companies.

The spreads of the score with function principle becomes wider than those with new operations. However, as shown in Tables 2 and 3, it can be observed that the sizes of spreads are reduced when using operations with fixed spread. Thus, it can be used to achieve more precise comparison.

Moreover, if the analysis was conducted without fuzzy theory, the scores would be a single number, and would not reflect uncertainty and vagueness of the data, resulting in loss of information.

Table 2. Post-evaluation score of 5 companies with function principle.

Company Name	Fuzzy Post-evaluation Score	
	Before correction	After correction
A	(0.502, 0.600, 0.645, 0.903)	(0.502, 0.600, 0.645, 0.903)
B	(0.848, 1.000, 1.000, 1.326)	(0.848, 1.000, 1.000, 1.000)
C	(-0.171, -0.068, 0.046, 0.171)	(0.000, 0.000, 0.046, 0.171)
D	(0.692, 0.820, 0.841, 1.148)	(0.692, 0.820, 0.841, 1.000)
E	(0.117, 0.199, 0.278, 0.471)	(0.117, 0.199, 0.278, 0.471)

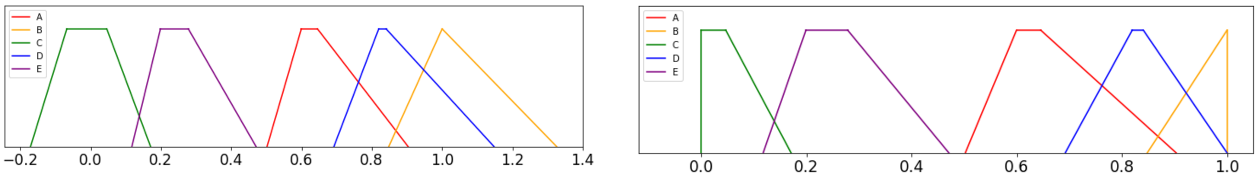

Figure 6. Fuzzy post-evaluation score with function principle before correction(left) and the same one after correction(right)

Table 3. Post-evaluation score of 5 companies with new operations.

Company Name	Fuzzy Post-evaluation Score	
	Before correction	After correction
A	(0.490, 0.553, 0.735, 0.797)	(0.490, 0.553, 0.735, 0.797)
B	(0.845, 0.907, 1.135, 1.198)	(0.845, 0.907, 1.000, 1.000)
C	(-0.130, -0.068, 0.068, 0.130)	(0.000, 0.000, 0.068, 0.130)
D	(0.686, 0.748, 0.955, 1.017)	(0.686, 0.748, 0.955, 1.000)
E	(0.123, 0.186, 0.334, 0.397)	(0.123, 0.186, 0.334, 0.397)

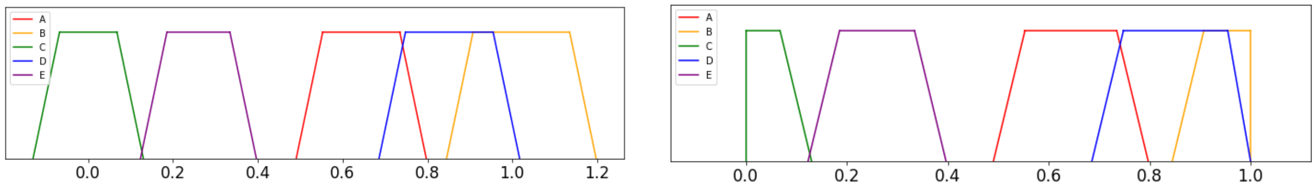

Figure 7. Fuzzy post-evaluation score with new operations before correction(left) and the same one after correction(right)

4.2. Fuzzy Post-Evaluation Model for delivery service using two-sided fuzzy reliability function

Let us consider a delivery system. When a customer orders some items to be delivered at a specific time, then if delivery is made earlier or later than that time, customer satisfaction will be decreased. Therefore, in this case, it is reasonable to use the two-sided fuzzy reliability function introduced above. For data analysis, we generated the following fuzzy data.

A sample data is proposed in Table 4 to analyze fuzzy post-evaluation model using two-sided fuzzy reliability function. Table 4 shows a sample data for delivery service of 5 stores.

Table 4. Sample Data for Delivery Service

Store	Arrival Time1	Score 1	Arrival Time 2	Score 2	Arrival Time3	Score 3	Arrival Time 4	Score 4	Arrival Time 5	Score 5
A	11:50	20	11:42	12	12:24	54	12:11	41	11:45	15
B	12:01	31	12:03	33	11:54	24	12:08	38	11:57	27
C	12:24	54	12:30	60	11:31	1	11:33	3	12:29	59
D	11:47	17	11:52	22	12:01	31	11:58	28	12:11	41
E	12:14	44	11:39	9	11:30	0	12:28	58	11:54	24

When the customer ordered some items to be delivered at 12:30 pm, if it is delivered earlier then 12:30 or later than 12:30, then the customer will not be satisfied with this delivery service. Let us consider only the time interval from 12:00 pm to 13:00 pm. Therefore, the time interval that we have considered is [0, 60] based on minutes.

Because the scores in Table 4 are based on the arrival time, it is clear that a store can obtain the highest score when they deliver some items exactly at 12:30 pm; thus, it is better to use triangular fuzzy numbers than trapezoidal fuzzy numbers in this case. Using the interval [0, 60], the fuzzified arrival time is expressed in Figure 8. The arrival time is fuzzified using triangular fuzzy numbers with spread 7.5.

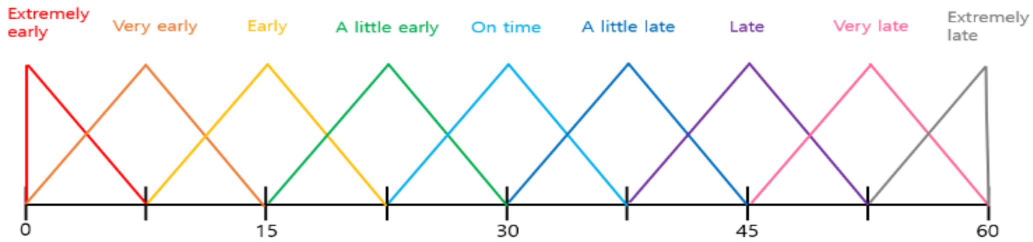

Figure 8. Post-Evaluation score using triangular fuzzy numbers

The fuzzy post-evaluation score of the given data using [2] and [3] is provided in Table 5. To avoid having large spread, the function principle in Definition 2 was applied.

Table 5. Fuzzy post-evaluation scores of the delivery service data calculated by function principle

Store	Fuzzy post-evaluation score
A	(0.3, 0.55, 0.8)
B	(0.65, 0.9, 1.0)
C	(0.0, 0.05, 0.3)
D	(0.55, 0.8, 1.0)
E	(0.15, 0.3, 0.55)

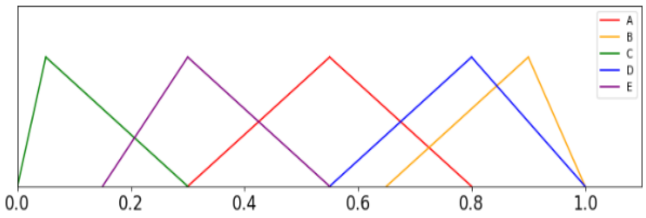

Figure 9. Fuzzy post-evaluation scores of the delivery service data calculated by Definition 2

Figure 9 shows the comparison of each fuzzy post-evaluation scores. B shows the best score, and C shows the lowest score. In addition, the fuzzified data are a symmetric triangular fuzzy number; however, using function principle they can be expressed as an asymmetric triangular fuzzy number, as shown in Figure 9. This result shows that the fuzzy post-evaluation score better shows the properties of the particular data, considering that the delivery service of each store is not always the same, and some differences can occur depending on the situations.

The other fuzzy post-evaluation score of the provided data using [2] and [3] is listed in Table 6. The newly defined operations method, Definition 3, was applied. Figure 10 shows similar results compared to Figure 9. However, in this case, fuzzy post-evaluation scores are also represented by symmetric triangular fuzzy numbers using a fixed spread considering post evaluation score. As an exception, Store C is not symmetric because the fuzzy post-valuation score is (-0.052, 0.073, 0.198), and is adjusted by **Remark** to a value between 0 and 1.

Considering that the spread of fuzzy post-evaluation scores is narrower in Figure 10, it can be observed that the New operations is more helpful method than function principle, which evaluates the delivery service of each score when comparing data.

To summarize the results of the analysis by function principle (Definition 2) and new operations (Definition 3), function principle has the disadvantage that the degree of ambiguity is considerably expressed; thus, it cannot be distinguished between each company. Accordingly, it has trouble in differentiating and evaluating companies.

Conversely, when using new operations (Definition 3), it can be observed that the spread ranges are narrower than those of the results by function principle (Definition 2). When using this definition, discrimination between companies is differentiated between companies, allowing us to determine the best company.

5. Conclusions

There are several contracting methods for product distribution. In general, the data used in contract methods have slight vagueness and ambiguity. Fuzzy theory is used to manage ambiguous data. Therefore, we employed fuzzy numbers to express vague information for our data analysis.

In this study, a fuzzy post-evaluation function was proposed. One-sided and two-sided fuzzy reliability functions were defined. The reliability function were used differently depending on the best characteristic obtained from either large or nominal data. With the fuzzy reliability functions, each item was evaluated. The post-evaluation function was a weighted average of the fuzzy reliability function values. Here, each weight considered the importance of each item. For the data analysis, two simulated datasets were provided for the one-sided and two sided fuzzy reliability functions. For the calculation of fuzzy reliability functions, two different operations were applied. The first one is the fuzzy operations based on the function principle, which is widely used. However, this operation has a drawback of making the results wider than expected. Therefore, fuzzy operations using fixed spreads were proposed to obtain reasonable spreads.

According to our data analysis, our newly defined fuzzy operations are more helpful when comparing data than the existing fuzzy operations. Currently, we have applied this post-evaluation model to only sampled data, including MAS data; however, it can be applied to all situations where a post-evaluation system or satisfaction survey is required. Therefore, in our further research, there will be some more applications with various measurements to evaluate the satisfaction for quality control using fuzzy theory.

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