Diagrammatic Categories in Representation Theory
Honours Thesis
(Draft)

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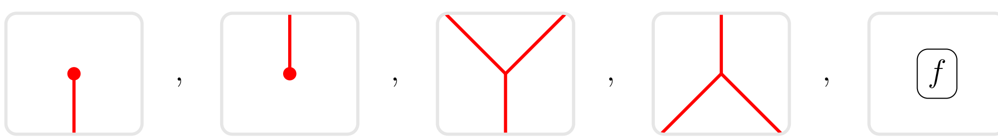
One-colour Diagrammatics

3.1 One-colour Diagrammatic Hecke Category

In this section, we describe one-colour diagrammatics for morphisms in $\mathbb{BSB}\mathrm{im}$. The morphisms in this category have a presentation in terms of generators and relations.

The generators are the following univalent and trivalent vertices, along with boxes where f is a homogeneous polynomial in R.



(3.1.1)

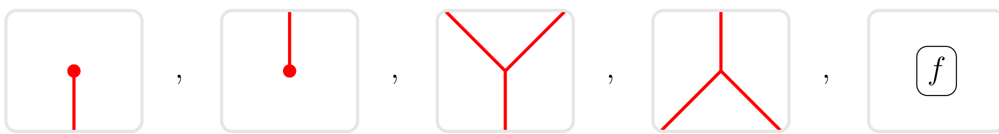
These are the unit, multiplication, counit and comultiplication maps from the Frobenius algebra structure of $B_s \in \mathbb{BSB}\mathrm{im}$.

3.2 Diagrammatic $\mathcal{O}(\mathrm{SL}(2))$

We will describe one-colour diagrammatics for $\mathcal{O}(\mathrm{SL}(2))$ via generators and relations up to isotopy.

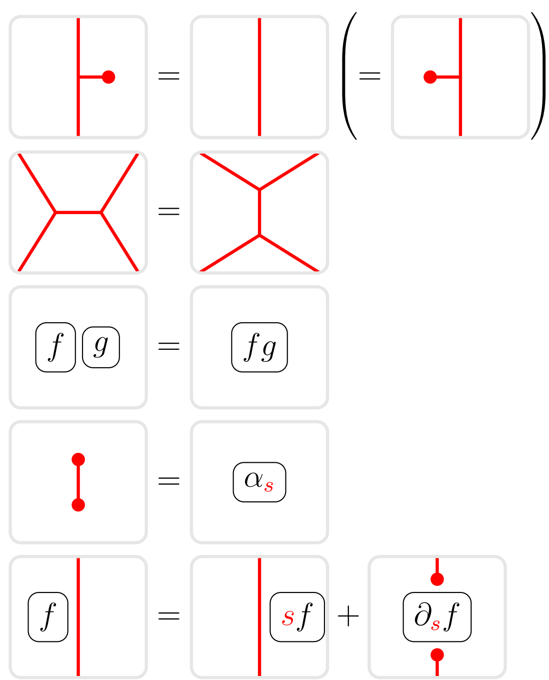
The elements of this category are generated by taking tensor products of an element s, coloured red.

Similar to one colour diagrammatics for $\mathbb{BSB}$im, the morphisms in this category are generated by horizontal concatenation, vertical concatenation, and sums of the following univalent and trivalent vertices, along with boxes where f is a homogeneous polynomial in ??.



(3.2.1)

The morphisms are subject to the following local relations, up to isotopy.



(3.2.2a)

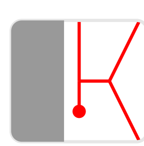
(3.2.2b)

(3.2.2c)

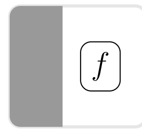
(3.2.2d)

(3.2.2e)

Additionally, we must impose a right R-module relation following from the lack of a left action. Instead of embedding in the double-sided strip $[0,1] \times \mathbb{R}$, we embed the diagrams in the one-sided strip $[0,1] \times \mathbb{R}_{>0}$ (Check the inequality), where the left side is an imaginary wall. For example a morphism may be

.

Then, diagrams are related to the wall by

 $= 0 \qquad (3.2.3)$

where f is a homogeneous polynomial in R with non-zero degree. That is, if a diagram has a non-constant homogeneous polynomial on its far left, then the entire diagram dies.

Chapter 4

Two-colour Diagrammatics

4.1 Two-colour Diagrammatic Hecke Category

Blah

4.2 Diagrammatic $\mathrm{Tilt}(\mathrm{SL}(2))$

Blah