Diagrammatic Categories in Representation Theory

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Diagrams

Motivation from Representation theory

Talk Overview

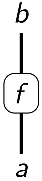
Monoidal Categories

A *monoidal category* is a category with an associative multiplication $\otimes$ for objects and morphisms, and a unit object $\mathbb{1}$, such that the multiplication works well with composition.

Let $\mathcal{C}$ be a monoidal category.

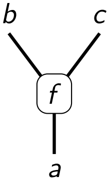
Monoidal Categories

$$f : a \to b$$



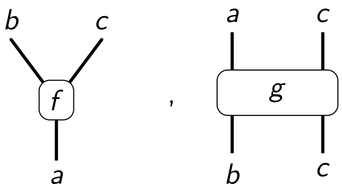
Monoidal Categories

$$f : a \to b \otimes c$$



Monoidal Categories

$$f : a \to b \otimes c$$
$$g : b \otimes c \to a \otimes c$$



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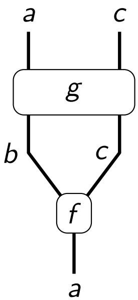

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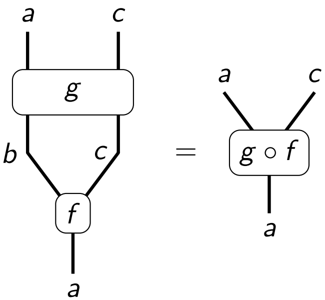
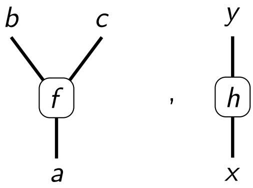


Monoidal Categories

$$\mathrm{id}_a : a \to a$$

Monoidal Categories

$$f : a \to b \otimes c$$
$$h : x \to y$$



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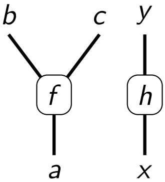

Monoidal Categories

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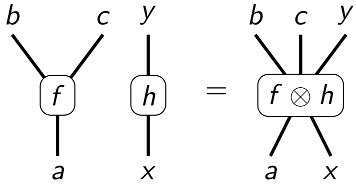
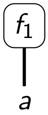


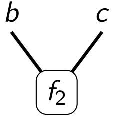
Monoidal Categories

$$f_1 : a \to \mathbb{1}$$



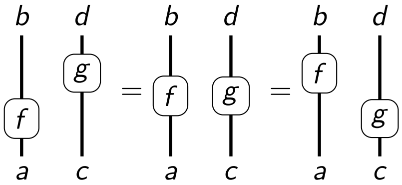
Monoidal Categories

$$f_2 : \mathbb{1} \to b \otimes c$$

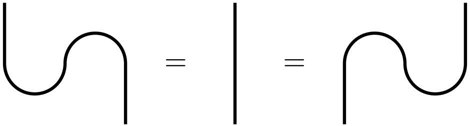


Monoidal Categories

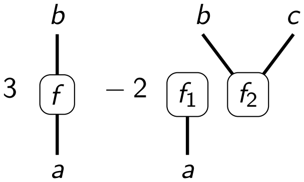
Interchange Law



Isotopy

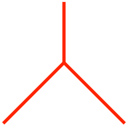
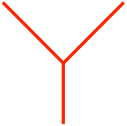


$\mathbb{Z}$-linear Monoidal Category



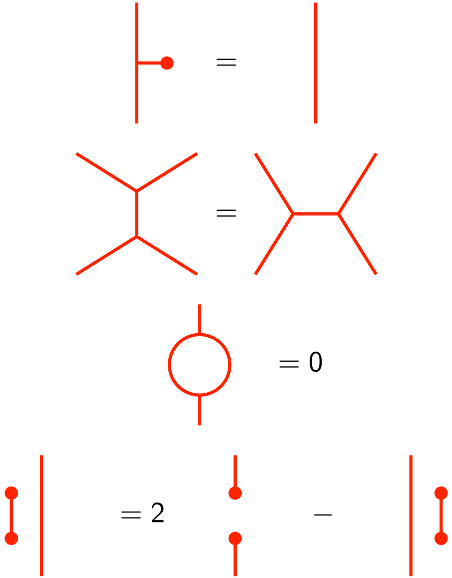
Diagrammatic Soergel Bimodules

A $\mathbb{Z}$-linear monoidal category $\mathcal{H}$ with:
Generating object |.
Generating morphisms

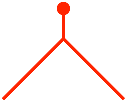
 ,  ,  , 

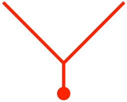
and local relations...

Diagrammatic Soergel Bimodules

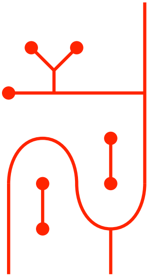


Diagrammatic Soergel Bimodules

 $:=$ 

 $:=$ 

Example 1



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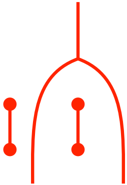

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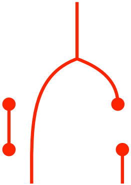
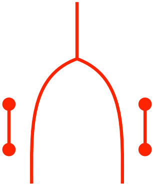
2  − 

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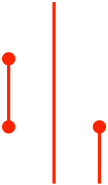
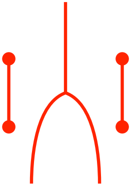
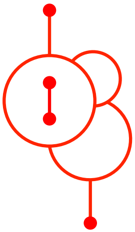
2  − 

Example 2



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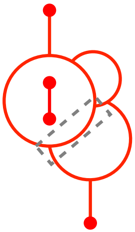

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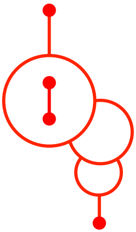

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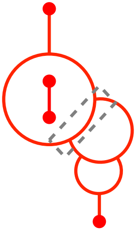

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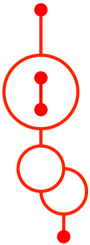

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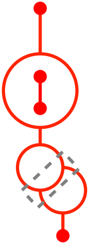

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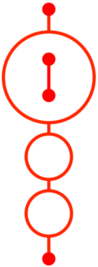

$= 0$

Soergel Bimodules

Theorem (Elias-Williamson, 2013)

The diagrammatic category $\mathsf{Kar}^{\oplus}(\mathcal{H})$ *and the category of Soergel Bimodules* $\mathbb{S}\mathsf{Bim}$ *over* S_2 *are equivalent as graded* $\mathbb{C}$*-linear monoidal categories.*

Generalisations

Applications