Diagrammatic Categories

in Representation Theory

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Kazhdan–Lusztig Conjecture

(Motivation)

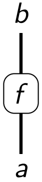
Talk Overview

Monoidal Categories

A *monoidal category* is a category with an associative multiplication $\otimes$ for objects and morphisms, and a unit object $\mathbb{1}$, such that the multiplication works well with composition.

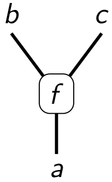
Monoidal Categories

$$f : a \to b$$



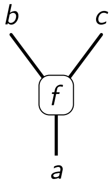
Monoidal Categories

$$f : a \to b \otimes c$$



Monoidal Categories: Composition

$$f : a \to b \otimes c$$
$$g : b \otimes c \to a \otimes c$$



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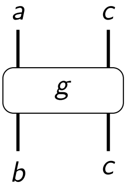

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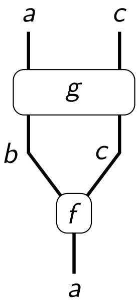

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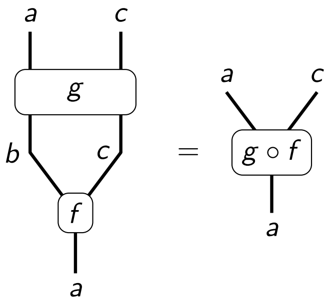
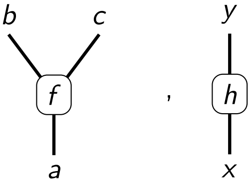


Monoidal Categories: Identity

$$\mathrm{id}_a : a \to a$$

Monoidal Categories: Tensor

$$f : a \to b \otimes c$$
$$h : x \to y$$



Monoidal Categories: Tensor

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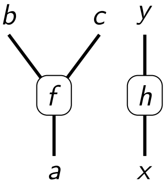
$$h : x \to y$$



Monoidal Categories: Tensor

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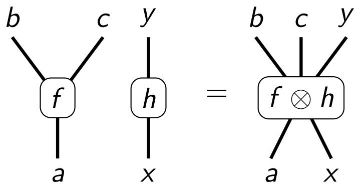
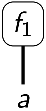


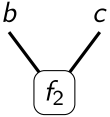
Monoidal Categories: Unit

$$f_1 : a \to \mathbb{1}$$



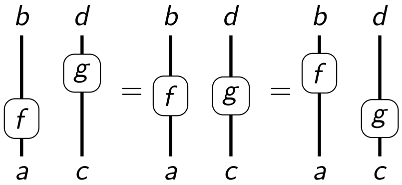
Monoidal Categories: Unit

$$f_2 : \mathbb{1} \to b \otimes c$$

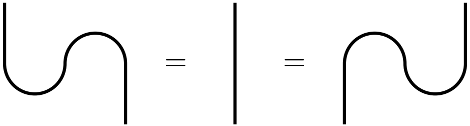


Monoidal Categories

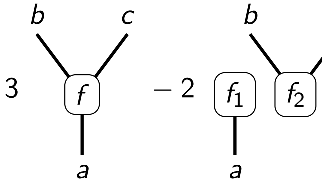
Interchange Law



Isotopy

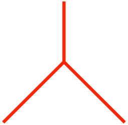
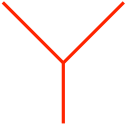


$\mathbb{Z}$-linear Monoidal Category



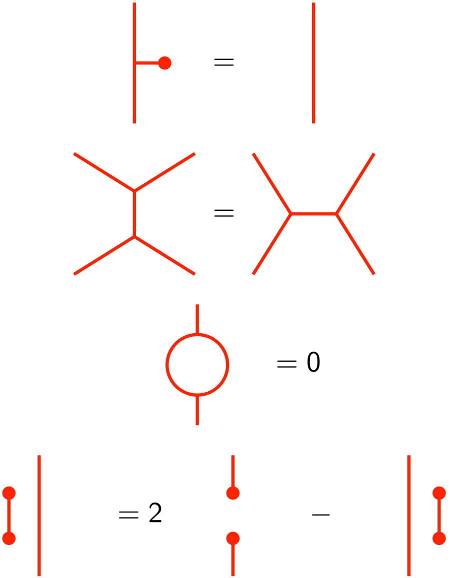
Diagrammatic Soergel Bimodules

A $\mathbb{Z}$-linear monoidal category $\mathcal{H}$ with:
Generating object |.
Generating morphisms

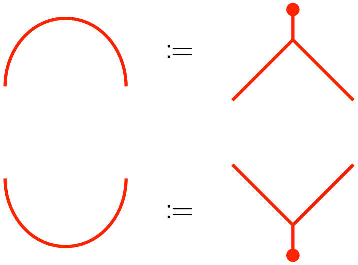
 ,  ,  , 

and local relations...

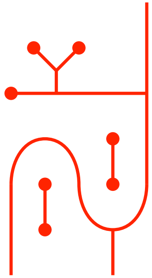
Relations



Relations



Example 1



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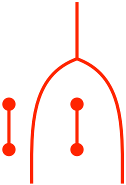

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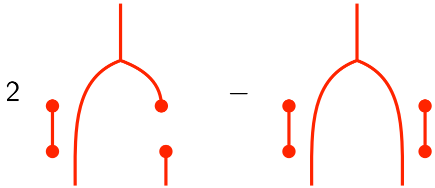

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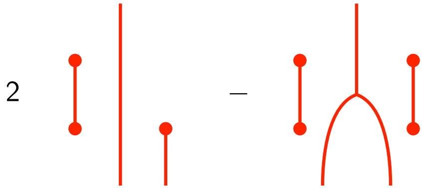
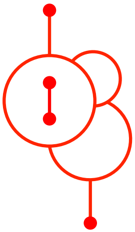


Example 2



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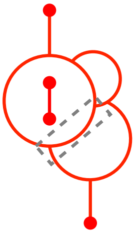

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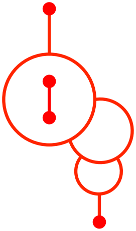

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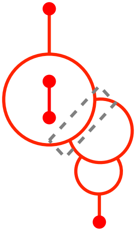

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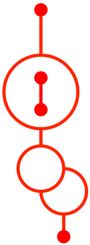

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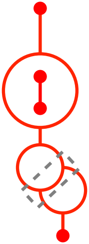

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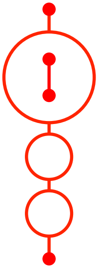

$= 0$

Soergel Bimodules

Theorem (Elias–Williamson, 2013)

The diagrammatic category $\mathsf{Kar}^{\oplus}(\mathcal{H})$ *and the category of Soergel Bimodules* $\mathbb{S}\mathsf{Bim}$ *over* S_2 *are equivalent as graded* $\mathbb{C}$*-linear monoidal categories.*

Generalisations

- Coxeter groups e.g. S_n, D_n
- Other categories

Further Applications

- Category $\mathcal{O}$
- Characteristic p