Diagrammatic Categories in Representation Theory
Honours Thesis
(Draft)

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March 9, 2023

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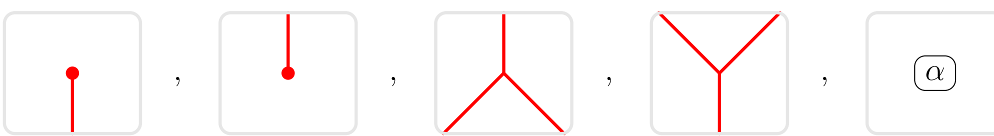
3.1 One-colour Diagrammatic Hecke Category

The first one-colour diagrammatic we explore is the one-colour diagrammatic Hecke category $\mathcal{H}$ for the symmetric group $S_2 = \langle s \mid s^2 = 1 \rangle$.

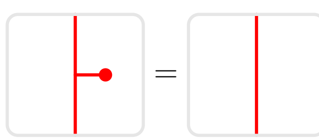
The objects of this category are generated by taking formal tensor products of the non-identity element $s \in S_2$. For example the tensor product of four s's which denote with the expression (s, s, s, s).

The morphisms in this category have a presentation in terms of generators and relations. For convenience, we will describe them up to isotopy. The generators are the following univalent and trivalent vertices, along with boxes with some symbol α. Should we just ditch the boxes and use barbells? If not, maybe mention that the boxes help us easier read linear combinations of barbells

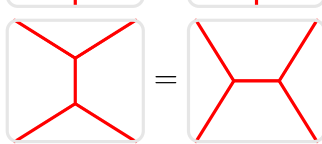


(3.1.1)

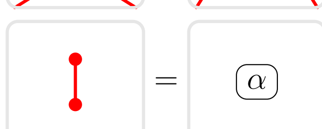
Because we have isotopy, we dont need the 'co-' maps. We can flip around the unit and multiplication using cups and caps. These morphisms are subject to the following local relations.



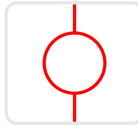
(3.1.2a)

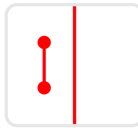
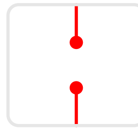
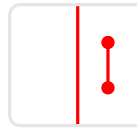


(3.1.2b)

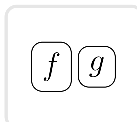


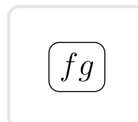
(3.1.2c)

 $= 0$ (3.1.2d)

 $= 2$  $-$  (3.1.2e)

To simplify the boxes, we can write



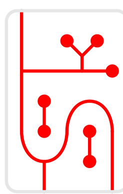
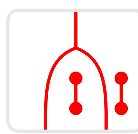
$=$


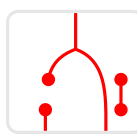
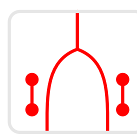
(3.1.3)

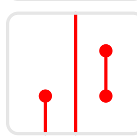
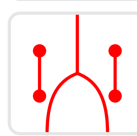
where f, g and fg are polynomials in α. Polynomials with what coefficients?

The first four generators with the relations (3.1.2a) and (3.1.2b) describes a Frobenius algebra object structure on the object s. Here the generators correspond to the unit, counit, multiplication and comultiplication maps respectively.

Example 3.1.4. Let us use the relations in (3.1.2) to simplify the following morphism in $\mathrm{Hom}((s, s), (s))$.

 $=$ 

$= 2$  $-$ 

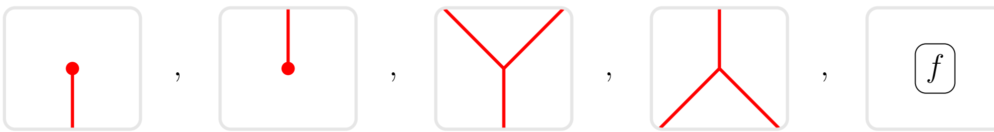
$= 2$  $-$  .

3.2 Diagrammatic $\mathcal{O}(\mathrm{SL}(2))$

We will describe one-colour diagrammatics for $\mathcal{O}(\mathrm{SL}(2))$ via generators and relations up to isotopy.

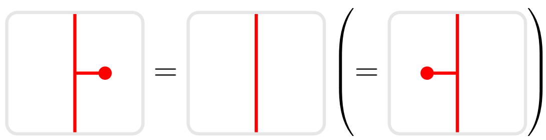
The elements of this category are generated by taking tensor products of an element s, coloured red.

Similar to one colour diagrammatics for $\mathbb{BSB}\mathrm{im}$, the morphisms in this category are generated by horizontal concatenation, vertical concatenation, and sums of the following univalent and trivalent vertices, along with boxes where f is a homogeneous polynomial in ??.

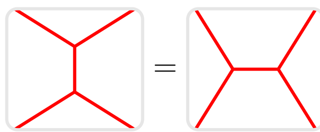


(3.2.1)

The morphisms are subject to the following local relations, up to isotopy.

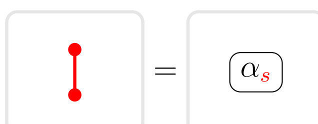


(3.2.2a)

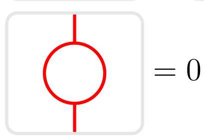


(3.2.2b)

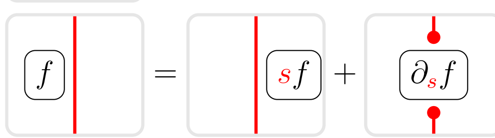
(3.2.2c)



(3.2.2d)

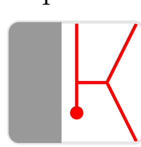


(3.2.2e)

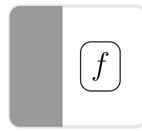


(3.2.2f)

Additionally, we must impose a right R-module relation following from the lack of a left action. Instead of embedding in the double-sided strip $[0,1] \times \mathbb{R}$, we embed the diagrams in the one-sided strip $[0,1] \times \mathbb{R}_{>0}$ (Check the inequality), where the left side is an imaginary wall. For example a morphism may be

.

Then, diagrams are related to the wall by



$$= 0 \tag{3.2.3}$$

where f is a homogeneous polynomial in R with non-zero degree. That is, if a diagram has a non-constant homogeneous polynomial on its far left, then the entire diagram dies.

Chapter 4

Two-colour Diagrammatics

4.1 Two-colour Diagrammatic Hecke Category

Blah

4.2 Diagrammatic $\mathrm{Tilt}(\mathrm{SL}(2))$

Blah