Diagrammatic Categories in Representation Theory
Honours Thesis
(Draft)

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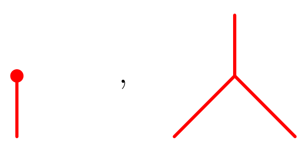
One-colour Diagrammatics

3.1 One-colour Diagrammatic Hecke Category

The first one-colour diagrammatic we explore is the one-colour diagrammatic Hecke category $\mathcal{H}(S_2)$ for the symmetric group $S_2 = \langle s \mid s^2 = 1 \rangle$.

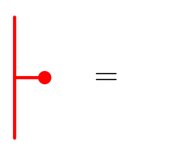
The objects of this category are generated by taking formal tensor products of the non-identity element $s \in S_2$. For example the tensor product of four s's which denote with the expression (s, s, s, s).

The morphisms in this category have a presentation in terms of generators and relations. For convenience, we will describe them up to isotopy. The generators are the following univalent and trivalent vertices, which can be rotated and flipped vertically using isotopy.

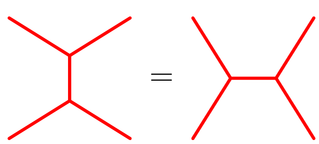


(3.1.1)

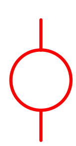
These morphisms are subject to the following local relations.



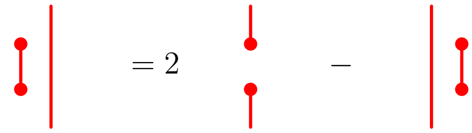
(3.1.2a)



(3.1.2b)



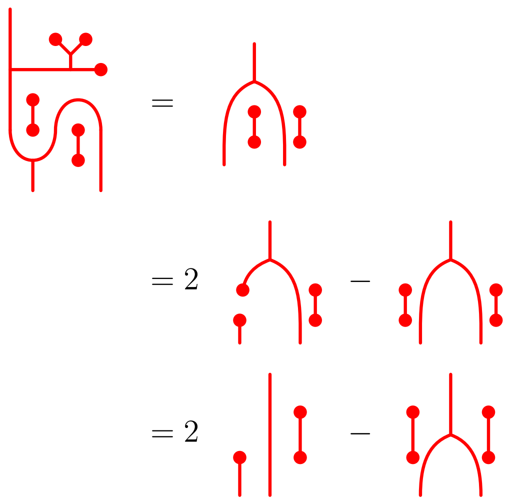
$= 0$ (3.1.2c)



(3.1.2d)

Remark 3.1.3. The first four generators with the relations (3.1.2a) and (3.1.2b) describes a Frobenius algebra object structure on the object s. Here the generators correspond to the unit, counit, multiplication and comultiplication maps respectively.

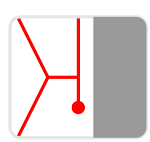
Example 3.1.4. Let us use the relations in (3.1.2) to simplify the following morphism in $\mathrm{Hom}((s,s),(s))$.



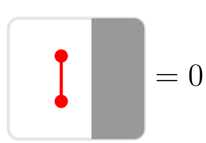
3.2 Diagrammatic $\mathcal{O}(\mathrm{SL}(2))$

With the diagrammatic category $\mathcal{H}(S_2)$, we can describe diagrammatics for the category $\mathcal{O}(\mathrm{SL}(2))$. In particular, we define a modular category *[what do we call this cat?]* over $\mathcal{H}(S_2)$.

This module category has elements copied from $\mathcal{H}(S_2)$ and morphisms are generated by the empty diagram $\varnothing$, with $\mathcal{H}(S_2)$ acting on the left by left concatenation on objects and morphisms. Additionally, the morphisms have one new relation, where diagrams collapse to 0 when there are barbells on the right. To depict this we add a wall on the right of the diagram, i.e. embedding the diagrams in the one-sided strip $[0,1] \times \mathbb{R}_{\geq 0}$ instead of in the double-sided strip $[0,1] \times \mathbb{R}$. For example a morphism may be



Then, diagrams are related to the wall by



(3.2.1)

Chapter 4

Two-colour Diagrammatics

4.1 Two-colour Diagrammatic Hecke Category

Blah

4.2 Diagrammatic $\mathrm{Tilt}(\mathrm{SL}(2))$

Blah