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HOMEWORK 6

1.

(a) $A \Leftrightarrow B$:

$\Leftrightarrow (A \Rightarrow B) \land (B \Rightarrow A)$

$\Leftrightarrow (\neg A \lor B) \land (\neg B \lor A)$

(b) $(A \land B) \Leftrightarrow (A \lor B)$

$\Leftrightarrow (((A \land B) \Rightarrow (A \lor B)) \land ((A \lor B) \Rightarrow (A \land B)))$

$\Leftrightarrow ((\neg(A \land B)) \lor (A \lor B)) \land (\neg(A \lor B) \lor (A \land B)))$

$\Leftrightarrow ((\neg A \lor \neg B \lor A \lor B) \land ((\neg A \land \neg B) \lor (A \land B)))$

$\Leftrightarrow ((\neg A \land \neg B) \lor (A \land B))$

$\Leftrightarrow ((\neg A \land \neg B) \lor A) \land ((\neg A \land \neg B) \lor B)$

$\Leftrightarrow (A \lor \neg B) \land (\neg A \lor B)$

(c) $A \land (A \Rightarrow B) \Rightarrow B$

$\Leftrightarrow \neg(A \land (\neg A \lor B)) \lor B$

$\Leftrightarrow (\neg A \lor \neg (\neg A \lor B)) \lor B$

$\Leftrightarrow (\neg A \lor (A \land \neg B)) \lor B$

$\Leftrightarrow (\neg A \lor \neg B) \lor B$

$\Leftrightarrow \neg A \lor 1$

⇔1

2.

KB = {A ∧ B ⇒ C, D ∧ E ⇒ B, F ⇒ A, E ⇒ A, E, D}

Prove that the sentence C is entailed by KB by using

a) Forward chaining

R1: A ∧ B ⇒ C

R2: D ∧ E ⇒ B

R3: F ⇒ A R4:

E ⇒ A

F1: E

F2: D

- Execute:
  - F3: A(F1,R4)
  - F4: B(F1,F2,R2)
  - F5: C(R1,F3,F4)

⇨ We infer C

⇨ KB := C

b) Backward chaining

We have current goal is C.

Using the Backward Chaining rule, we have a new goals, A and B.

Using the Backward chaining rule for A as it conclusion. We find R3. R3 is such a rule, which only fire if it is satisfied D and E, so we have the new current goals is F include of A.

But we don't find any rule which F as this solution so this goal failed.

Using the Backward chaining rule for A as it conclusion. We find R4. R4 is such a rule, which only fire if it is satisfied E, so we can reach this goal success.

Using the Backward chaining rule for B as it conclusion. We find R2. R2 is such a rule, which only fire if it is satisfied D and E, so we can reach this goal success.

Now, we have A and B so we will get goal is C.

- ⇨ We infer C
- ⇨ KB |= C

3:

Formalization:

+ Accomplice: A

+ Key: K

+ Car: C

KB = $\{(A \Rightarrow C) \land [(\neg A \land \neg K) \lor (A \land K)] \land K\}$

(*): $A \Rightarrow C \equiv \neg A \lor C$

(*): $[(\neg A \land \neg K) \lor (A \land K)] \equiv [(\neg A \land \neg K) \lor A] \land [(\neg A \land \neg K) \lor K]$

$$\equiv (\neg K \lor A) \land (\neg A \lor K)$$

#	Clause
1	$\neg A \lor C$
2	$(\neg K \lor A)$
3	$(\neg A \lor K)$
4	K

A1: $\neg A \lor C$

A2: $(\neg K \lor A)$

A3: $(\neg A \lor K)$

A4: K

A5: A (A4, A3)

A6: C (A5, A1)

$\Rightarrow KB \models C$

=> The criminal came in a car

4.

a) $\forall x \forall y, (Cat(x) \wedge Mouse(y)) => Chase(x,y)$

b) $\forall x \exists y, Cat(x) => (Mouse(y) \wedge Chase(x,y))$

c) $\exists x \forall y, Cat(x) \wedge (Mouse(y) => Chase(x,y))$

d) $\exists x \exists y, Cat(x) \wedge Mouse(y) \wedge Chase(x,y)$

e) $\exists y \forall x, Mouse(y) \wedge (Cat(x) => Chase(x,y))$

f) $\forall y \exists x, Mouse(y) => Cat(x) \wedge Chase(x,y)$

5.

a)

- Every child loves Santa :
  - $\forall x$, Child(x) => Loves(x,Santa)
- Everyone who loves Santa loves any reindeer : ○ $\forall x \forall y$, Loves(x,Santa) => Reindeer(y) => Loves(x,y)
- Rudolph is a reindeer, and Rudolph has a red nose: ○ Reindeer(Rudolph) ∧ RedNose(Rudolph)
- Anything which has a red nose is weird or is a clown: ○ $\forall x$, RedNose(x) => Weird(x) ∨ Clown(x)
- No reindeer is a clown :
  - ¬∃x, Reindeer(x) ∧ Clown(x)
- Scrooge does not love anything which is weird:
  - $\forall x$, Clown(x) => ¬Loves(Scrooge,x) b)

- Every child loves Santa :
  - $\forall x$[Child(x) => Loves(x,Santa)] ○ ≡ $\forall x$ [¬ Child(x)∨ Loves(x,Santa)] ○ CNF: [¬ Child(x) ∨ Loves(x,Santa)]
- Everyone who loves Santa loves any reindeer :
  - $\forall x$ Loves(x,Santa) => $\forall y$Reindeer(y) => Loves(x,y)
  - ≡$\forall x$[Loves(x,Santa) => ($\forall y$ Reindeer(y) => Loves(x,y))] ○ ≡$\forall x$[¬Loves(x,Santa) ∨ ($\forall y$ ¬Reindeer(y) ∨ Loves(x,y)]
  - ≡$\forall x$[¬Loves(x,Santa) ∨ (¬Reindeer(F(x)) ∨ Loves(x,F(x))]
  - ≡ [¬Loves(x,Santa) ∨ (¬Reindeer(F(x)) ∨ Loves(x,F(x))] ○ CNF: [¬Loves(x,Santa) ∨ ¬Reindeer(F(x)) ∨ Loves(x,F(x))]
- Rudolph is a reindeer, and Rudolph has a red nose: ○ Reindeer(Rudolph) ∧ RedNose(Rudolph) ○ CNF : Reindeer(Rudolph) ∧ RedNose(Rudolph)

- Anything which has a red nose is weird or is a clown: ○ ∀x, RedNose(x) => Weird(x) ∨ Clown(x)
    - ≡ ∀x[¬RedNose(x) ∨ Weird(x) ∨ Clown(x)] ○ ≡[¬RedNose (x) ∨ Weird(x) ∨ Clown(x)] ○ CNF: ¬RedNose(x) ∨ Weird (x) ∨ Clown(x)
- No reindeer is a clown :
    - ¬∃x, Reindeer(x) ∧ Clown(x)
    - ≡ ∀x[¬(Reindeer(x) ∧ Clown(x))] ○ ≡∀x[¬Reindeer(x) ∨ ¬Clown(x)] ○ ≡[¬Reindeer(x) ∨ ¬Clown(x)] ○ CNF: [¬Reindeer(x) ∨ ¬Clown(x)]
- Scrooge does not love anything which is weird: ○ ∀x, Weird(x) => ¬Loves(Scrooge,x)
    - ≡ ∀x[¬Weird(x) ∨ ¬Loves(Scrooge,x)] ○ ≡[¬Weird(x) ∨ ¬Loves(Scrooge,x)] ○ CNF: ¬ Weird (x) ∨ ¬Loves(Scrooge,x)

c)

#	Clause
1	¬ Child(x) ∨ Loves(x,Santa)
2	¬Loves(x,Santa) ∨ ¬Reindeer(F(x)) ∨ Loves(x,F(x))
3	Reindeer(Rudolph)
4	RedNose(Rudolph)
5	¬RedNose(x) ∨ Weird(x) ∨ Clown (x)
6	¬Reindeer(x) ∨ ¬Clown (x)
7	¬ Weird (x) ∨ ¬Loves(Scrooge,x)

A1: ¬ Child(x) ∨ Loves(x,Santa)

A2: ¬Loves(x,Santa) ∨ ¬Reindeer(F(x)) ∨ Loves(x,F(x))

A3: Reindeer(Rudolph)

A4: RedNose(Rudolph)

A5: ¬RedNose(x) ∨ Weird(x) ∨ Clown(x)

A6: ¬Reindeer(x) ∨ ¬Clown(x)

A7: ¬ Weird (x) ∨ ¬Loves(Scrooge,x)

A8: ¬Clown(Rudolph)(A3,A6)

A9: Weird(Rudolph) (A8,A5,A4)

A10: ¬Loves(Scrooge, Rudolph) (A9,A7)

A11: ¬Loves(Scrooge,Santa) (A10,A2,A3)

A12: ¬ Child(Scrooge)(A11,A1)

- ⇨ Scrooge is not a child