Problems for Week 4: Sampling

1) Use the m-script provided (HW_4_1.m) to calculate the effect of ice surface roughness and dielectric constant on the coherence of ice penetrating radar data. The script contains a simple model that returns the phase distribution for a plane-wave returned from a flat ice/bed interface after passing though an ice/air interface of variable rms-height and dielectric constant twice. You should only need to modify two lines of code (defining the rms-height and dielectric constant to do this problem).
    a. [Using this model], sketch the phase distribution of the bed return for a perfectly flat ice/air interface.
    b. [Without using this model], sketch what the distribution of recorded phase for the same bed return would be if the radar were not coherent.
    c. [Using this model] and a dielectric constant of 3.2, describe the effect of increasing the surface rms-height on coherence of bed returns.
    d. At what rms-height value would you consider the coherence of the bed return to be "lost"?
    e. How does reducing the dielectric constant affect this level?

2) Assume that you hope to detect the ice/bed interface in the presence of noise and that you are trying to assess the improvement in the probability of detecting the bed that is achievable by stacking. Assume that the level is fixed, normally distributed, and has a standard deviation equal to its mean. Assume that initially (with no stacking) the signal-to-noise ratio (taken as the ratio of the signal to the mean of the noise) is 0.01 and that the signal strength increases as N^0.5 where N is the number of stacks.
    a. For the case where there is no stacking (N=1), what is the probability that more of the returned energy will come from signal than from noise?
    b. After how many stacks would the probability that more returned energy would come from signal than noise be 50%?
    c. After how many stacks would the probability that more returned energy would come from signal than noise be 90%?
    d. After how many stacks would the probability that more returned energy would come from signal than noise be 99%?
    e. Sketch this probability as a function of N.
    f. What does this tell you about the gain in information about bed echo energy as a function of stacking?

3) Assume that you have an aircraft with a survey velocity of 130 m/s and a survey height of 500 m above the ice surface.
   a. If you have an ice penetrating radar with a center frequency of 60 MHz, a PRF of 6 kHz, and a stacking depth of 30, what is the folding frequency for this radar?
   b. What is the range of Doppler frequencies (just the shifts in Hz) that could be produced by the surface for this system?
   c. Sketch the output (in range-Doppler space) that would result from an azimuth FFT on the surface return for this system (if the surface returned equal energy over entire range of Doppler frequencies)?
   d. If you have an ice penetrating radar with a center frequency of 150 MHz, a PRF of 10 kHz, and a stacking depth of 250, what is the folding frequency for this radar?
   e. What is the range of Doppler frequencies (just the shifts in Hz) that could be produced by the surface for this system?
   f. Sketch the output (in range-Doppler space) that would result from an azimuth FFT on the surface return for this system (if the surface returned equal energy over entire range of Doppler frequencies)?

4) Sketch each of the following waveforms and determine what digitizer sampling frequency is required to meet the Nyquist sampling requirement on each (where H is the Heavyside step function)?
   a. $f(t) = 60*(H(t)-H(t-10))$, where f is in MHz and t is in μs
   b. $f(t) = 60*(H(t)-H(t-1))$, where f is in MHz and t is in μs
   c. $f1(t) = (60+15*(t-0.5))*(H(t)-H(t-1))$, where f is in MHz and t is in μs
   d. $f1(t) = (60+1500*(t-0.005))*(H(t)-H(t-0.01))$, f is in MHz and t is in μs

5) Assume that, at your IPR wavelength, the ice surface has a Doppler bandwidth of ~5 Hz, englacial layers have Doppler bandwidths of ~1 Hz, and the bed had a Doppler bandwidth of ~ 10 Hz. Plot the output of an azimuth FFT for:
   a. The ice surface
   b. A horizontal englacial layer
   c. A sloped englacial layer
   d. The bed