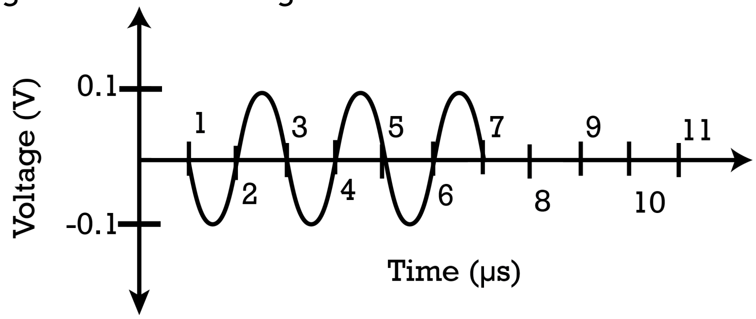
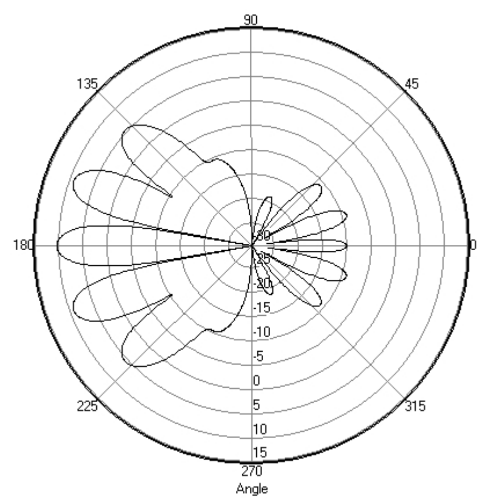
Problems for Week 3: The Radar Equation

1) Your radar digitizes the following waveform from a 50-ohm co-axial cable:



   a. What is the digitized voltage at t = 2.5 **μs**? (in V)
   b. What is the received power at t = 2.5 **μs**? (in mW)
   c. What is the digitized voltage at t = 9 **μs**? (in V)
   d. What is the received power at t = 9 **μs**? (in mW)
   e. What is the digitized voltage at t = 3 **μs**? (in V)
   f. What is the received power at t = 3 **μs**? (in mW)
   g. What is the total received power? (in J)

2) The gain for an optimal horn-antenna is $G = 1/2*4\pi/\lambda^2*(A*B)$, where λ is the wavelength in air and A and B are the horn edge lengths (in m). Assume that you are building a radar with a horn antenna that must fit in a 0.7m x 0.7m x 0.7m cube and has a center frequency of 150 MHz.
   a. What is the maximum possible gain for this antenna?
   b. What is the nadir-gain for a two antenna array of plate dipoles mounted ¼ wavelength beneath the wing of a DC3 (beam pattern shown below)



   c. If both systems (Horn and Dipoles) use the same antennas for transmit and receive, how many time weaker is the weakest detectable signal for the Dipoles compared to the Horn? (assuming all other radar system parameters are the same)

3) Assume you are looking at some radar data and see that the return from Point A is 250 times stronger than from Point B. Your current hypothesis about the region containing these points is supported the existence of liquid water at Point A and frozen bed at Point B so you quickly write and submit a paper called "The watery wonders of Point A" claiming that there is liquid water at Point A and include this figure from Peters 2005 to support your interpretation.

Table 1. Dielectric Permittivity (ε_{3r}, $\tan\delta_3$), Smooth Interface Power Reflection Coefficient, $|\tilde{R}_{23}|^2$, and Reflected Field Phase Shift, $\angle\tilde{R}_{23}$, for Possible Subglacial Materials[a]

Subglacial Material	ε_{3r}	$\tan\delta_3$	$\lvert\tilde{R}_{23}\rvert^2$, dB	$\angle\tilde{R}_{23}$	Source
Seawater	77	11.3	−1	175°	*Neal* [1979]
Groundwater (gw)	80	1.4	−2	172°	*Keller* [1966]
Fresh water	80	0.002	−3	180°	*Boithias* [1987]
Unfrozen till (40% gw)	18	0.82	−6	164°	*van Beek* [1967][b]
Unfrozen bedrock (15% gw)	6.6	0.41	−13	155°	*van Beek* [1967][b]
Frozen till (40% gw ice)	2.8	0.035	−30	13°	*van Beek* [1967][b,c]
Frozen bedrock (15% gw ice)	2.7	0.022	−28	6°	*van Beek* [1967][b,c]
Marine ice	3.43	0.05	−33	151°	*Neal* [1979]

[a]Permittivity of glacial ice near the subglacial interface was assumed to be $\varepsilon_{3r} = 3.17$ and $\tan\delta_2 = 0.0062$ [*Glen and Paren*, 1975].
[b]Looyenga's mixing formula [*van Beek*, 1967] with sand was used ($\varepsilon_r = 2.6$, $\tan\delta = 0.015$) from *Keller* [1966].
[c]Permittivity for groundwater ice was taken as $\varepsilon_r = 3.17$, $\tan\delta = 0.062$.

In review, you receive the following comments. Respond to them.

a. Point A is 2 times closer to the aircraft than Point B. Could this account for the observed difference in echo strengths?
b. Also, Point A is beneath 200 m of ice and Point B is beneath 900m of ice. The region you are studying has ice very similar to the Siple Dome core for which the attenuation has been well characterized (see figure from MacGregor 2007 below). Could this account for the observed difference in echo strengths?

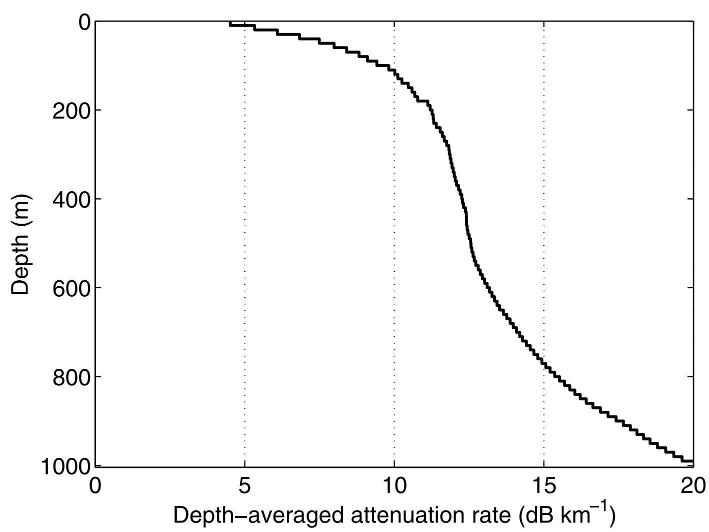


Figure 4. Depth-averaged modeled attenuation rate profile (N_a^*) at Siple Dome calculated using (11) at all depths.

c. Are you wrong?

4) Assume that you are planning a survey flight over an iceberg and intend to identify both the bottom corner of the iceberg and water-filled basal crevasses from unfocused radar data. You plan to approximate the bottom corner of the iceberg as a dihedral 2D corner reflector with a radar cross-section of:

$$\sigma = \begin{cases} 8\pi a^4/\lambda^2 & \text{for } 0° < \theta < 90° \\ 0 & \text{for } 90° < \theta \end{cases}$$

where a is the depth, width, and height of the 2D corner reflector and θ is the angle of incidence for the incoming radar (with 0° being horizontal). You also plan to approximate a water-filled basal crevasse as the edge of a rectangular sheet, which has a radar cross-section of $\sigma = L^2/\pi$, where L is the length of the edge.

a. Sketch what the radar record (in fast-time and slow-time) would look like for the return from the bottom corner of the iceberg.
b. Sketch what the radar record (in fast-time and slow-time) would look like for the return from a water-filled basal crevasse.
c. How will you distinguish the returns from these two features?

5) Assume that you are using an ice penetrating radar on a space-craft that is operating 200 km above the surface of an icy planetary body. The radar system operates 9 MHz with a 3 MHz bandwidth, has a transmit power of 10 W, an antenna gain 1.8 dB, and a noise floor of 100 nW at the input of the receiver. You are hoping to detect hypothesized regions in the ice, which contain a large number of spherical water bodies with radii that range from 1 cm to 1 m and vertical densities that range from 0.05 to 0.2 bodies/m. The radar cross section of a sphere (with circumference less than the radar wavelength) is:

$$\sigma = (\pi r^2)*(7.11*(k*r)^4)$$

where r is the radius of the sphere, and k is $2\pi/\lambda$.

a) What is the range resolution of this radar (in m)?
b) Fill in the table below with the returned power you'd expect in one range resolution cell that includes spheres with the following sizes and densities.

Radius (in m)	Density (bodies/m)	Returned Power (mW)	SNR
0.01	0.05		
0.01	0.2		
0.1	0.05		
0.1	0.2		
1.0	0.05		
1.0	0.2		

c) Under what conditions would you be optimistic about detecting the hypothesized spherical water bodies?