Problems for Week 6: Scattering

1) A square conducting plate (with side length a) has a scattering function with a central lobe that with an angular width (W) of:

$$W = 30°/(a/\lambda)$$

where λ is the radar wavelength. The nose-on radar cross section is:

$$\sigma = 4\pi a^4/\lambda^2$$

   a. How will the strength of the radar return from one of these features vary as a function of their dimension?
   b. For what size of plate would the width of the central lobe be most sensitive to the dimension of the plate?
   c. If you wanted to constrain the size of plate-like surface features based on the width of the central lobes of radar returns, how would you choose the wavelength (relative to the dimensions of those features) of the radar system to use?

2) The effect of roughness on the strength of an single nadir-directed radar return is described by the following equation from Peters 2005:

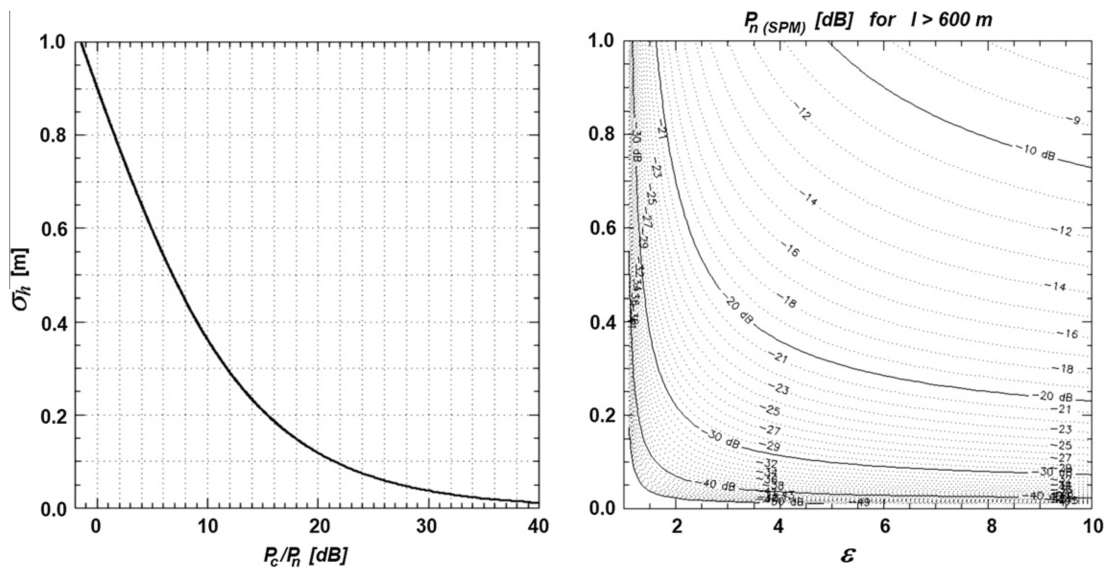
$$\rho = \exp(-g^2)I_0^2(g^2/2)$$

where $g = 4\pi S/\lambda$, S is the rms height of the surface, λ is the in-ice radar wavelength, and I_0 is the zero order modified Bessel function.

   a. If your radar system has a central frequency of 60 MHz, how many times stronger would the return from a surface with an rms height of 1 cm be than the return from a surface with an rms height of 2 cm?
   b. If your radar system has a central frequency of 60 MHz, how many times stronger would the return from a surface with an rms height of 50 cm be than the return from a surface with an rms height of 100 cm?
   c. How large a difference (in dB) in bed reflectivity could be explained by differences in bed roughness? How could this affect the interpretation bed conditions (i.e. between wet and dry beds)?
   d. What impact does focusing have on this phenomenon?

3) Assume an anomalously bright radar return was observed from a planetary ice surface that was observed from an angle of 30° off-nadir. Explain how that anomalously bright return could have been caused by each of the following mechanisms and make a back-of-the-envelope estimate for strongest anomaly (in dB) that each mechanism is likely to produce.
   a. Surface roughness
   b. Surface dielectric constant
   c. Surface conductivity
   d. Volume scattering

4) The figure below (from Grima 2012) shows the relationship between surface roughness (σ_h) and the ratio between the coherent echo power (P_c) and the incoherent echo power (P_n) (left) and the relationship between incoherent echo power, surface roughness, and dielectric constant (ε).



a. What is the rms height and dielectric constant of a surface with incoherent echo power of -20 dB and coherent echo power of -10 dB?
b. Is this technique more sensitive to large or small rms height values?
c. Is this technique more sensitive to large or small dielectric constants?
d. What is the range of rms heights and dielectric constants for which you would be confident in using this technique?

5) Sketch following surfaces:
a. A smooth surface
b. A rough surface
c. The same rms height as (b), but twice the auto-correlation length
d. The same auto-correlation length as (b), but twice the rms height
e. The same rms slope as (b), but twice the rms height
f. Large rms deviation at a short lag but small rms deviation at a large lag
g. Small rms deviation at a short lag but large rms deviation at a large lag