Dustin Newman

CS 181, Discussion 1C

Campbell, Mathur

Assignment Five

We prove that L_1 is not regular by contradiction.

Assume that L_1 is regular. Therefore, the pumping lemma applies to L_1. Therefore, there exists some constant $p > 0$ such that for all strings $w \in L_1$, $|w| \geq p$. For every w, w can be partitioned into substrings x, y, z such that $w := xy^i z$ for all $i \geq 0$. Per the pumping lemma, $|xy| \leq p$ and $|y| > 0$.

Consider the case when $w := \mathrm{b}^{\mathrm{p}}\mathrm{cc}^{\mathrm{p}}$. Clearly, $|w| \geq p$, in fact $|w| = 2p + 1$. $w \in L_1$ since more than half of the symbols in w are c's i.e. $\#(\mathrm{c}, w) = \frac{1}{2}|w| + 1 > \frac{1}{2}|w|$.

Since $|xy| \leq p$, x, y must consist of all b's, since it is not possible for the length of y to exceed p. Therefore, it is more accurate to rewrite w as $w := \mathrm{b}^m \mathrm{b}^n \mathrm{b}^{p-m-n} \mathrm{cc}^p$ where $m := |x|$ and $n := |y|$.

No matter the specific assignment of x, y, z, we know that x, y must consist of all b's. We also know that the pumping lemma holds for *all* values of i.

Consider the case where $i := 3$. We then rewrite w as $w := \mathrm{b}^m \mathrm{b}^{3n} \mathrm{b}^{p-m-n} \mathrm{cc}^p$. The pumping lemma states that $w \in L_1$. However, $w \notin L_2$ because more than half of its symbols are not c's:

$$m + 3n + p - m - n > p + 1$$
$$2n + p > p + 1$$

The above is impossible since $n \geq 1$. We therefore arrive at a contradiction since $\forall i(w := \mathrm{b}^p \mathrm{cc}^p = xy^i z \in L_1)$ does not hold when $i := 3$.

$\therefore L_1$ is not regular.