

# CS 181 Spring 2020 Homework Weeks 8-9

Assigned Thursday 5/21; Due via GradeScope Thursday 5/28 1:00pm

1. Let  $L_p$  be a recursively enumerable language, and let  $L_a$  be a recursive language. Show that  $L_p \cap L_a$  is recursively enumerable. The approach you must use is to describe how you would construct a new TM procedure for the intersection by modifying a TM procedure diagram for  $L_p$  and a TM algorithm diagram for  $L_a$  and combining them into a single TM procedure diagram that recognizes the intersection.
2. Consider the following language over alphabet  $\Sigma = \{\#, 0, 1\}$ :

$$L_{10} = \{w \mid w \text{ is a valid encoding of a TM, } M, \text{ and on input } 0^{10} \text{ } M \text{ writes} \\ \text{at least 10 consecutive 0's on one of its work tapes}\}.$$

Decide whether  $L_{10}$  is: *recursive* or *recursively enumerable and not recursive*. Justify your choice by *briefly* describing how you could use the Universal TM (UTM) to construct an *algorithm* or *procedure* (respectively) for  $L_{10}$ . Include a *very* brief explanation of why your constructed TM would be an algorithm or why it would be a procedure.

3. If  $L$  is a language that is *recursively enumerable and not recursive*, what can we say  $\bar{L}$  which is the complement of  $L$ ? Briefly justify your answer using the closure properties of the four families of languages which we discussed in lecture: *recursive*, *recursively enumerable* (RE), *recursively enumerable and not recursive*, and *non-recursively enumerable* (NRE).