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CS 181, Discussion 1C
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Assignment Six

1

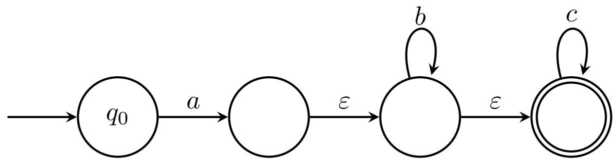
The language described by this PDA is:

$$L := \{1^2 0^m 1 0^n \mid m = 2n\}$$

Basically, any string which begins with exactly two ones, has some even number of zeroes m, exactly one one, and then exactly half of m number of zeroes again, is in the language. Note the interesting parallel where there is twice the number of characters in the $1^2 0^m$ substring as there is in the $1^1 0^n$ substring.

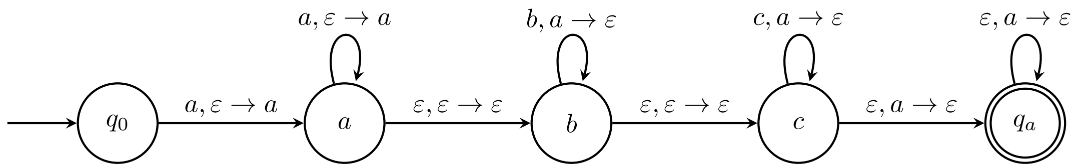
2

Assume that L_2 *is* a finite-state language. Since finite-state languages are closed under intersection, then $L_2 \cap ab^*c^* = \{ab^ic^j \mid 0 \leq i \leq j\}$. However, this is exactly $\{ab^mc^k \mid 0 \leq m \leq k\}$, which we know not to be finite-state. Since ab^*c^* is known to be finite-state (the NFA is given below), L_2 cannot be regular.



3

$L_3 := \{a^i b^j c^k \mid i > j + k\}$ is a non-FSL CFL. The (N)PDA is given below.



The PDA above ensures that at least one a is in the input string (since even if $j = k = 0$, $i \geq 1$ since $0 \not> 0$). After that, we push an a onto the stack for each input symbol a. We allow epsilon-transitions for the b and c states since it is possible for $j = k = 0$. For each b or c encountered, we pop an a from the stack. If at any point, we read a b or c and there are no more a's on the stack, then we block and fail to accept. Additionally, to get to the accepting state q_a, we require there be at least one a on the stack remaining, since $i \neq j + k$. We loop forever and accept as long as there are remaining a's, since we can have unbounded occurrences of a.

4

L_4 is not a CFL. To prove by contradiction, assume that L_4 is a CFL. Per the pumping lemma for CFLs, there is some pumping length p such that for any string s such that $|s| \geq p$, $s := uvxyz$. $\forall(i \geq 0)(uv^ixy^iz \in L_4)$, $|vy| > 0$, and $|vxy| \leq p$. Consider the string $s_p = a^{2p}b^pc^{2p}$. $|s| = 5p > p$ and $s \in L_4$ since the number of a's and the number of c's are equal and twice the number of b's. Since $|vxy| \leq p$ and $|vy| > 0$, we know that there are two broad possibilities to consider: either vy consists of the same symbol or it does not. If vy consists of the same symbol, then when $i = 0$, the resulting string $s' \notin L_4$. To show this, consider all possible cases from Σ:

- a - When $i = 0$, $s' = uxz = a^{2p-|vy|}b^pc^{2p}$ i.e. $\#(a, s') \neq \#(c, s')$.
- b - When $i = 0$, $s' = uxz = a^{2p}b^{p-|vy|}c^{2p}$ i.e. $\#(a, s') \neq 2 \cdot \#(b, s')$
- c - When $i = 0$, $s' = uxz = a^{2p}b^pc^{2p-|vy|}$ i.e. $\#(c, s') \neq \#(a, s')$.

If either v or y consists of different symbols, notice it is not possible for vy to consist of more than two symbols since $|vy| \leq p$ and the substrings are all of at least length p. Therefore, there are three cases once again for the value of vy:

- No c's - Thus, vy consists of a's and b's. Because of this, consider when $i = 0$, then $s' = uxz$ and $\#(a, s') \neq \#(c, s')$ because vy consists of some non-zero number of a's and cannot be long enough to consist of any amount of c's.
- No a's - Thus, vy consists of b's and c's. Similar logic as above follows i.e. $\#(c, s') \neq \#(a, s')$ because there is some non-zero amount of c's removed when $i = 0$.
- No b's - This option is not possible since $|vy| \leq p$ and since there are p occurrences of $b \in s'$, it is not possible for both a and c to be in vy.

Thus, as no possible partition of s' holds for *all* values of i, $s' \notin L_4$ and thus L_4 cannot be a CFL since it does not obey the pumping lemma.