

CS 181 Spring 2020 Homework Week 1
Assigned Tue 3/31; Due via GradeScope Mon 4/6 6:00pm

0: Briefly explain the system used in the Sipser textbook to number the sections, subsections, exercises, problems, figures, examples, theorems, etc..

1: Let G be a connected acyclic undirected graph (i.e., an undirected tree). Prove that adding exactly one edge to G always results in a graph (call it G') which contains a cycle. Hint: Why is it important that G be connected?
If this is too easy, you may prove (for no extra credit whatsoever) that G' will always have *exactly one* cycle.

Inspired by Sipser Exercises: pp 25-27:

2: Let X be the set $\{x, y, z\}$, and let B be the set $\{0, 1\}$.

- a. List the elements of the Cartesian product $B \times (X \times B)$
- b. List the elements of the Cartesian product $(B \times X) \times B$
- c. List the elements of the Cartesian product $B \times X \times B$
- d. What is the cardinality of the power set $\mathcal{P}(B \times X)$?

3. Let alphabet $\Sigma = \{a, b, c, d\}$. Let language over Σ $L_3 = \{aa, a, ad\}$.

- a. What is the language concatenation $L_3 \cdot \{a, c, aa\}$?
- b. What is the language concatenation $L_3^+ \cdot \{\}$?
- c. What is the Cartesian Product $\{\varepsilon\} \times L_3$?
- d. What is the Cartesian Product $\{\} \times L_3^*$?
- e. What is the language concatenation $\{\varepsilon\} \cdot L_3^+$? Does it contain ε ?

Inspired by Sipser Exercises: p 84:

4: Let alphabet $\Sigma = \{a, b\}$. Show a DFA which recognizes the following language over Σ . Show the DFA as a *fully specified state diagram*. Be sure to clearly indicate your initial state and accepting state(s).

$$L_4 = \{w \in \Sigma^+ \mid w \text{ contains 3 consecutive a's and does not contain 3 consecutive b's}\}$$

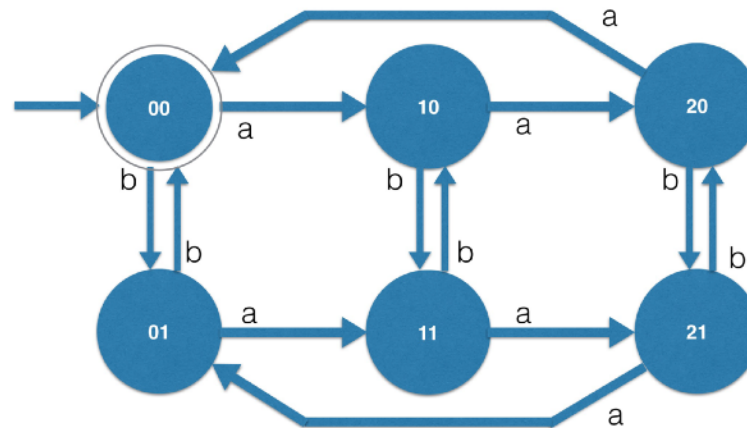
Briefly describe how your design works.

5: Let alphabet $\Sigma = \{c, d, e\}$. Consider the following language over Σ :

$$L_5 = \{w \in \Sigma^+ \mid w = yx, \text{ where } y \in \Sigma^+ \text{ \& } x \in \Sigma, \text{ and } w \text{ contains substring } xxxx\}$$

- Show two examples of strings in L_5 .
- Show two examples of strings not in L_5 .
- Briefly describe L_5 in plain, precise English.

6: Briefly describe in English the language over $\Sigma = \{a, b\}$ accepted by this DFA.
The accepting state is "00".



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