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CS 181, Discussion 1A

Campbell, Mathur

Assignment Four

1

L_1 is a finite state (FS) language which can be represented with the regular expression below.

$$R_1 := (b \mid c)^* \cdot [a \cdot (b \mid c) \cdot (b \mid c)^* \cdot a \cdot (b \mid c) \cdot (b \mid c)^*]^* \cdot (b \mid c)^*$$

2

L_2 is **not** a finite state language, but is a context-free language. Below, I prove by contradiction that L_2 is not a regular language and then present a context-free grammar for the language.

Assume that L_2 is regular. By the pumping lemma, there exists some pumping length $p > 0$ such that for every string $w \in L_2$ of at least length p (i.e. $|w| \geq p$), w can be partitioned in the form $w := xy^iz$. Per the pumping lemma, we know that (1) $|xy| \leq p$, that (2) $|y| > 0$, and that (3) all these properties hold for all values of $i \geq 0$. Effectively, this allows us to "pump" (up or down) w and still have the string remain in L_2.

Let our $w := a^{p+1}b^{p+1}c^0$. $w \in L_2$ because our predicate $k = m + n$ holds when $k = p + 1$ and $m = p + 1$ (i.e. $p + 1 = p + 1 + 0$).

$|w| \geq p$ because it contains one more "a" and one more "b" than each value of $p \geq 0$ and thus has a specific length of $2p + 2 > p$.

Since $|xy| \leq p$, we know that xy consists of some fraction of the "a"s in w. Maximally, there must be at least one "a" not in the xy partition.

More formally, we say that $x := a^\alpha$ and $y := a^\beta$ and $z := a^{p+1-\alpha-\beta}b^{p+1}$ where $(\alpha + \beta) \leq p$ and $\beta > 0$.

No matter the partition of xy^iz, consider the case when $i := 2$. Then we have $w_2 := xyyz := a^\alpha a^\beta a^\beta a^{p+1-\alpha-\beta}b^{p+1}$.

However, $w_2 \notin L_2$ because the number of occurrences of "a" does not equal the number of occurrences of "b" i.e. $\#(a, w) \neq \#(b, w)$. We can see that illustrated below:

$$\alpha + \beta + \beta + p + 1 - \alpha - \beta = p + 1$$
$$\beta + p + 1 = p + 1$$

This can only hold when $\beta := 0$. However, since the length of y must be greater than 0, $\beta > 0$. Thus, the above does not hold and $w_2 \notin L_2$.

This is a contradiction since, by the pumping lemma, $w_2 \in L_2$ since it is of the form xy^iz.

$\therefore L_2$ is not a regular language.

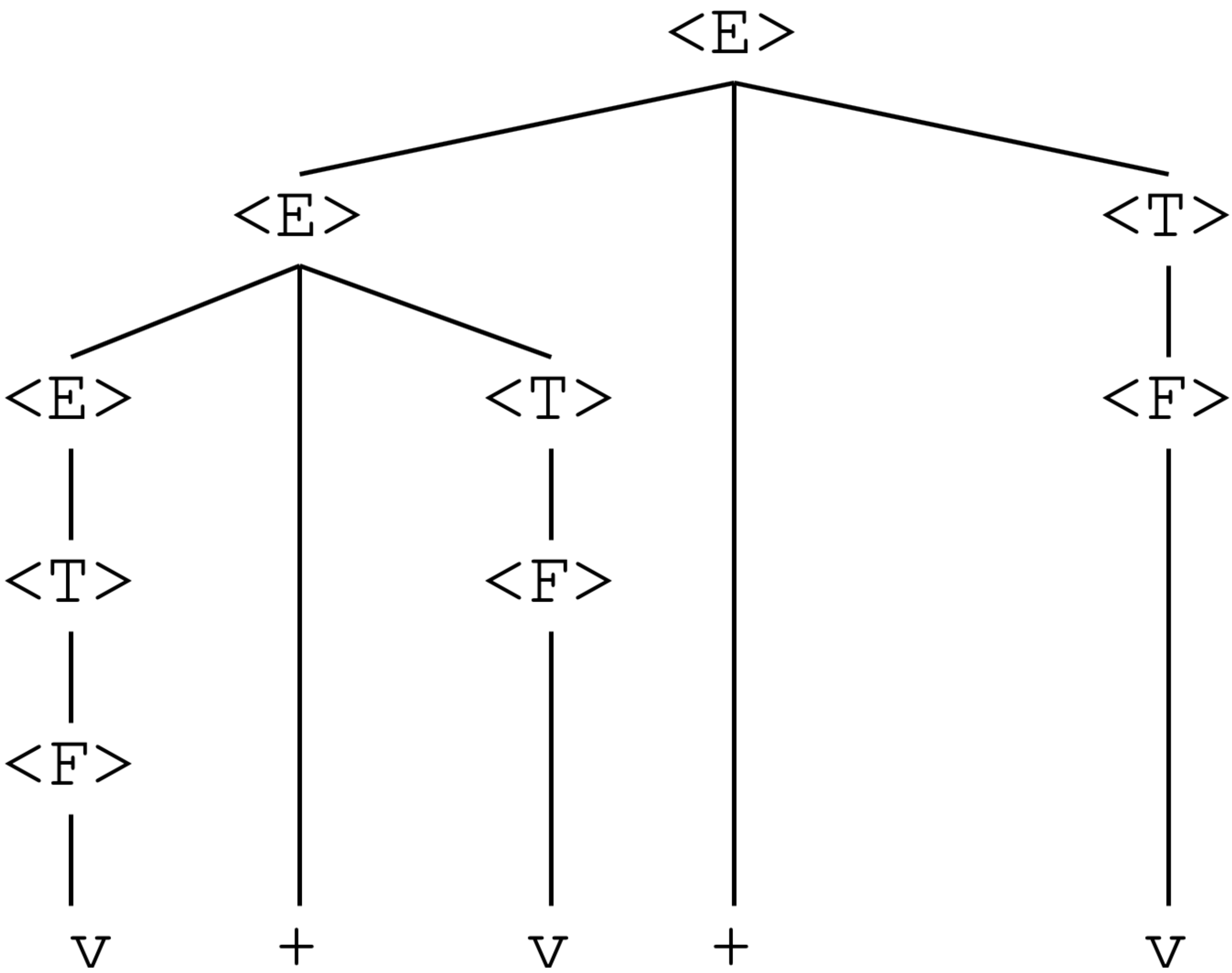
The context-free grammar is given below:

$$S \to XY$$
$$X \to aXb \mid \varepsilon$$
$$Y \to bYc \mid \varepsilon$$

3

a

The parse tree:



b

The right-most derivation:

$$\begin{gathered}
\underline{E} \Rightarrow E + T \\
E + \underline{T} \Rightarrow E + F \\
E + \underline{F} \Rightarrow E + v \\
\underline{E} + v \Rightarrow E + T + v \\
E + \underline{T} + v \Rightarrow E + F + v \\
E + \underline{F} + v \Rightarrow E + v + v \\
\underline{E} + v + v \Rightarrow T + v + v \\
\underline{T} + v + v \Rightarrow F + v + v \\
\underline{F} + v + v \Rightarrow v + v + v
\end{gathered}$$

c

The left-most derivation:

$$
\begin{gathered}
\underline{E} \Rightarrow E + T \\
\underline{E} + T \Rightarrow E + T + T \\
\underline{E} + T + T \Rightarrow T + T + T \\
\underline{T} + T + T \Rightarrow F + T + T \\
\underline{F} + T + T \Rightarrow v + T + T \\
v + \underline{T} + T \Rightarrow v + F + T \\
v + \underline{F} + T \Rightarrow v + v + T \\
v + v + \underline{T} \Rightarrow v + v + F \\
v + v + \underline{F} \Rightarrow v + v + v
\end{gathered}
$$