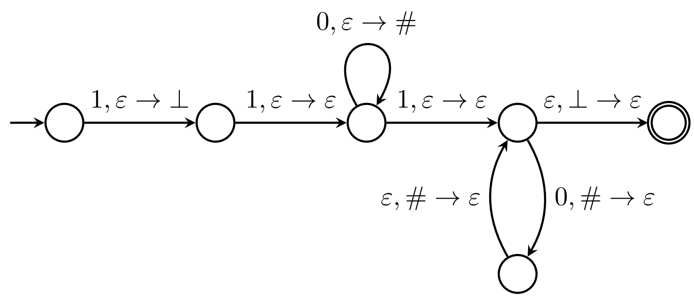
# CS 181 Spring 2020 Homework Week 6

Assigned Tue 5/5; Due via GradeScope Mon 5/11 6:00pm

1. Let $\Sigma = \{0, 1\}$ be the alphabet. Consider the following PDA:




Describe precisely and clearly the language recognized by this PDA.

2. Consider the following language over the alphabet $\Sigma = \{a, b, c\}$.

$$L_2 = \{a^n b^m c^k \mid n, m, k \geq 0; \text{ and if } n > 0, \text{ then } m \leq k\}.$$

In other words, $L_2$ consists of words of the form $a^n b^m c^k$, and if a word starts with $a$, then the number of $c$'s is at least the number of $b$'s. But if there are no $a$'s, then the "if $n > 0$, then $m \leq k$" in the definition does not apply, so $m$ and $k$ can be any arbitrary combination.

$L_2$ is not finite-state but trying to prove this using the pumping lemma fails. For any string, $s$, there will always be a partition, $s = xyz$, with $y$ being the first symbol, which satisfies the lemma. This language demonstrates that there are some languages which are not finite state, but they happen to obey the pumping lemma.

Instead, prove that $L_2$ is not finite-state using *closure properties*.

You may assume that any of the following languages are not finite state, without proof:

- $\{a^n b^m c^k \mid n \geq 0, 0 \leq m \leq k\}$,
- $\{a b^m c^k \mid 0 \leq m \leq k\}$,
- $\{a^n b^m c^k \mid n, m, k \geq 0, m \neq k\}$.

Remember, for your proof to be valid, you must transform $L_2$ into one of the given non-FS languages using only combinations of closure properties and languages which are known to be finite state (not the reverse!). And you cannot combine $L_2$ with any other non-FS languages as part of your proof.

Let alphabet $\Sigma = \{a, b, c\}$. For questions 3 and 4, decide ("classify") whether a given language is: a *finite-state language* (FSL), a *context free language* (CFL) and *not finite-state*, or *not a context free language*. If it is a FSL, prove it by any means. If it is a non-FSL CFL, give a PDA for it. (If it is a non-FSL CFL, you do not need to prove that it is a non-FSL.) If it is a non-CFL, prove that using the *pumping lemma for CFLs*.

3. $L_3 = \{a^i b^j c^k, \text{ where } i > j + k\}$.

4. $L_4 = \{w \in \Sigma^* \mid \#(a, w) = 2 \cdot \#(b, w) = \#(c, w)\}$.

As always, any model you give should be accompanied by a brief justification of its correctness.