# CS 181 Spring 2020 Homework Week 1

Assigned Tue 3/31; Due via GradeScope Mon 4/6 6:00pm

0: Briefly explain the system used in the Sipser textbook to number the sections, subsections, exercises, problems, figures, examples, theorems, etc..

Not graded. As long as your answer makes sense to you, thinking it through may help you later in the course.

1: Let G be a connected acyclic undirected graph (i.e., an undirected tree). Prove that adding exactly one edge to G always results in a graph (call it G') which contains a cycle. Hint: Why is it important that G be connected?
If this is too easy, you may prove (for no extra credit whatsoever) that G' will always have *exactly one* cycle.

Call the new edge in G' e=(u,v).
There are two cases:
If e is a self-loop (u=v), then that is a cycle.
If u ≠ v, then there must be path in G between u and v since G was connected. That path also exists in G', since we didn't take away any edges. Then the path in G' from u to v and then from v to u via e forms a cycle in G'.
Since G' contains a cycle in all cases. *qed*.

## Inspired by Sipser Exercises: pp 25-27:

2: Let Let $\mathrm{X}$ be the set { x, y, z }, and let $\mathrm{B}$ be the set { 0, 1 }.

a. List the elements of the Cartesian product $\mathrm{B} \times (\mathrm{X} \times \mathrm{B})$

{ (0,(x,0)), (0,(x,1)), (0,(y,0)), (0,(y,1)), (0,(z,0)), (0,(z,1)),
(1,(x,0)), (1,(x,1)), (1,(y,0)), (1,(y,1)), (1,(z,0)), (1,(z,1)) }

b. List the elements of the Cartesian product $(\mathrm{B} \times \mathrm{X}) \times \mathrm{B}$

{ ((0,x),0), ((0,y),0), ((0,z),0), ((0,x),1), ((0,y),1), ((0,z),1),
((1,x),0), ((1,y),0), ((1,z),0), ((1,x),1), ((1,y),1), ((1,z),1) }

c. List the elements of the Cartesian product $\mathrm{B} \times \mathrm{X} \times \mathrm{B}$

{ (0,x,0)), (0,x,1)), (0,y,0)), (0,y,1)), (0,z,0)), (0,z,1)),
(1,x,0)), (1,x,1)), (1,y,0)), (1,y,1)), (1,z,0)), (1,z,1)) }

d. What is the cardinality of the power set $\mathscr{P}(\mathrm{B} \times \mathrm{X})$ ?

$2^{(\,|\mathrm{B}| \bullet |\mathrm{X}|\,)} = 2^{(\,2 \bullet 3\,)} = 2^{6} = 2^{64}$

3. Let alphabet $\Sigma = \{ a, b, c, d \}$. Let language over $\Sigma$ $L_3 = \{ aa, a, ad \}$.

a. What is the language concatenation $L_3 \bullet \{ a, c, aa \}$ ?

{ aaa, aa, ada, aac, ac, adc, aaaa, adaa }

b. What is the language concatenation $L_3^+ \bullet \{\}$ ?

{}

c. What is the Cartesian Product $\{ \varepsilon \} \times L_3$ ?

$\{ (\varepsilon, aa), (\varepsilon, a), (\varepsilon, ad) \}$

d. What is the Cartesian Product $\{\} \times L_3^*$ ?

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e. What is the language concatenation $\{ \varepsilon \} \bullet L_3^+$ ? Does it contain $\varepsilon$ ?

$\{ \varepsilon \} \bullet L_3^+ = L_3^+$. The set $\{ \varepsilon \}$ is the identity for the operation of language concatenation.

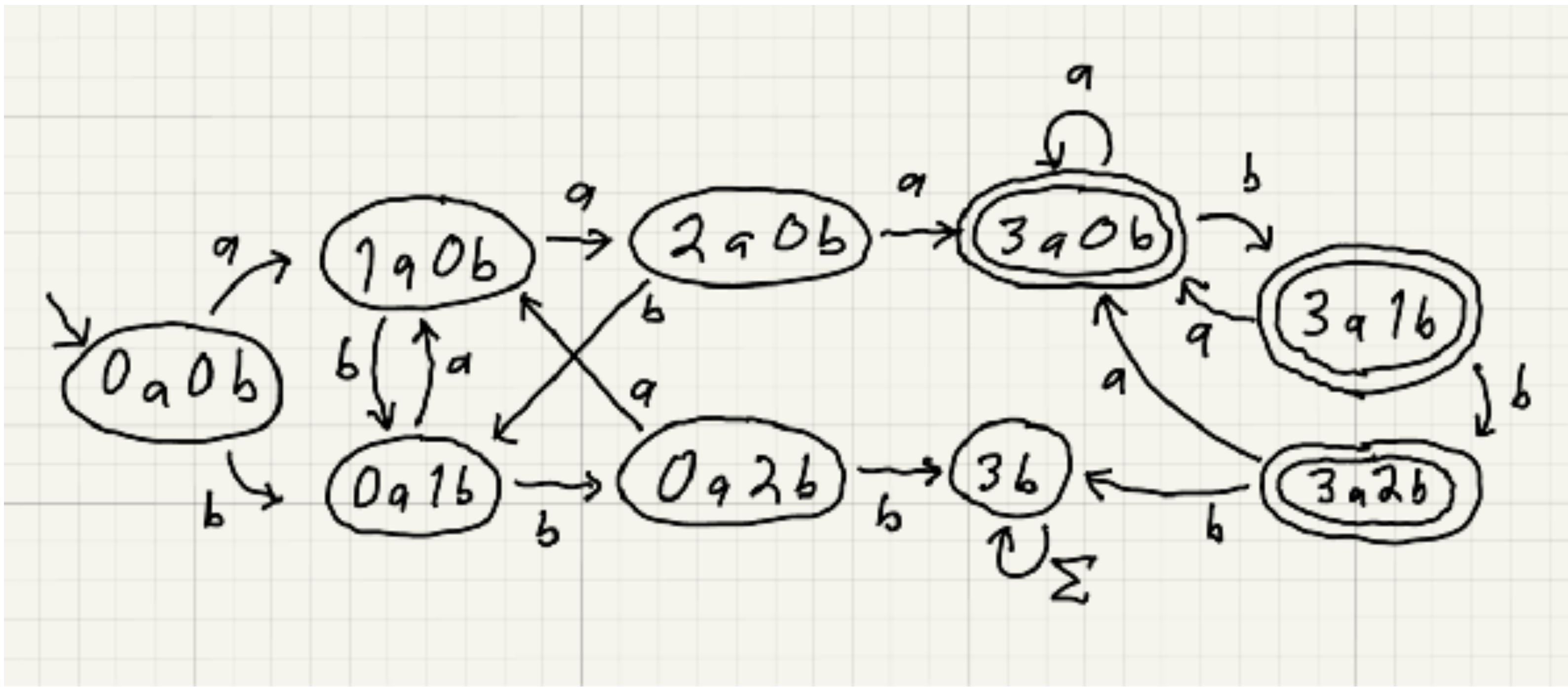
It will contain $\varepsilon$ if and only if $L_3$ contains $\varepsilon$ .

Inspired by Sipser Exercises: p 84:

4: Let alphabet $\Sigma$ = { a, b }. Show a DFA which recognizes the following language over $\Sigma$. Show the DFA as a *fully specified* state diagram. *Be sure to* clearly *indicate your initial state and accepting state(s).*

$L_4$ = { w ∈ $\Sigma^+$ | w contains 3 consecutive a's and does not contain 3 consecutive b's }

Briefly describe how your design works.




State names indicate consecutive symbols seen. Top path counts consecutive a's; bottom path counts consecutive b's. Can only reach the set of three accepting states after seeing 3 consecutive a's. Once we see 3 consecutive a's, the only way out of the accepting states is by seeing 3 consecutive b's. If we ever see 3 consecutive b's we end up in state "3b" which is essentially a dead state. Other transitions reset the counters depending on last symbol seen. Fully specified because every state has exactly two out-going edges, one for each symbol of $\Sigma$.

5: Let alphabet $\Sigma = \{ c, d, e \}$. Consider the following language over $\Sigma$:

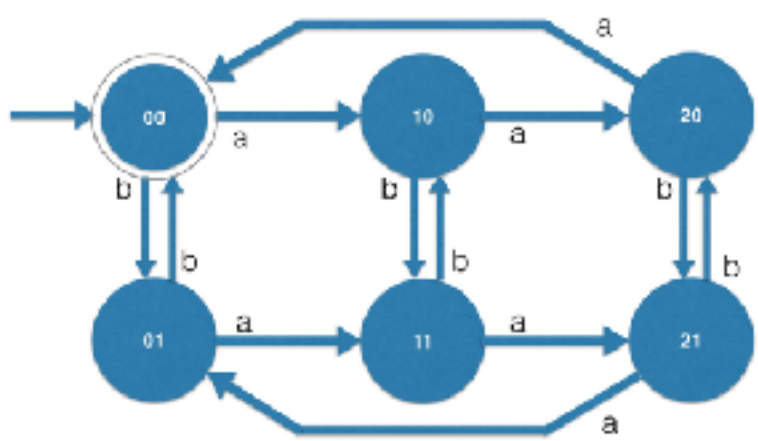
$$L_5 = \{ w \in \Sigma^+ \mid w = yx, \text{ where } y \in \Sigma^+ \ \& \ x \in \Sigma, \text{ and } w \text{ contains substring } xxxx \}$$

a. Show two examples of strings in $L_5$.
w=bbbb Note that w does not have to contain "xxxx" as a *proper* substring; so it can be the whole string.
w=accccbc

b. Show two examples of strings not in $L_5$.
w=acccca
w=bbb

c. Briefly describe $L_5$ in plain, precise English.
All strings over $\Sigma$ where the last symbol occurs at least 4 times consecutively at least once in the string. (Granted, "over $\Sigma$" is not exactly plain English.)

6: Briefly describe in English the language over $\Sigma = \{ a, b \}$ accepted by this DFA.
The accepting state is "00".




All strings over $\Sigma$ such that it has an even number of b's *and* a number of a's divisible by 3 in any order (including $\varepsilon$).

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