CS 181 Spring 2020 Homework Week 4

Assigned Tue 4/21; Due via GradeScope Mon 4/27 6:00pm

Let $\Sigma = \{a, b, c\}$. For problems 1 and 2, decide whether each of the following languages over Σ is *finite state* (FS) or *context free* (CF) but not *finite state*. If a language is FS, give a *regular expression* for it. Otherwise, prove that it is not FS using the *pumping lemma*, and show that it is CF by giving a *context free grammar* (CFG) for it.

1. L_1 is the set of all words in which there are an even number of a's and every a is immediately followed by at least one b. For instance *abcababcab*, ε, c, and *bbbabbab* are in the language; strings not in the language include *bbaab*, *abbabbcab*, and *aabb*.

2. $L_2 = \{w \in \Sigma^* \mid w = a^m b^k c^n,\ k = m + n\}$. For instance this language includes ε, *aaabbbbbcc*, *ab*, and *bc*; this language does not include *abc*, *b*, or *ccbbba*.

3. Consider the following *context free grammar*, which is a variation of Sipser's Example 2.1 on p. 154 with shorter variable names and a different terminal symbol "v" instead of "a". It produces strings that resemble simplified expressions:

$$\begin{aligned} E &\to E + T \mid T \\ T &\to T \times F \mid F \\ F &\to (E) \mid v. \end{aligned}$$

   This grammar generates the string $e = v + v + v$ *unambiguously*.

   (a) Show *the* parse tree in the grammar for e.

   (b) Show *the* right-most derivation in the grammar for e.

   (c) Show *the* left-most derivation in the grammar for e.

   As part of each of your derivations, you must underline the variable that is replaced at each step.