Week One, Lecture Two

Proof - Combination of mathematical notation and human language that provides a convincing argument for the truth of a statement.

Models of Computations

Strings

To define strings, we need an alphabet where an **alphabet** is defined as any nonempty finite set of elements called **symbols**, which you - as a reader - must set aside from your conceptions of meaning. For example, the Greek letter pi which usually means the irrational number 3.1415... here is just another written symbol.

A **string** is a finite sequence of symbols.

Assume our alphabet is `{0,1}`. What are all the possible strings you can create from this alphabet?

- 0, 1, 00, 01, 10, ...
- All the possible strings from an alphabet (denoted by Σ^+ (reads "sigma plus")) form an *infinite* set of *finite* strings.

Note that the sigma plus of an alphabet is *not* the same as the power set because sigma plus is a *sequence* of all possible strings. If it were the power set, then "00" would be the same as "0" as they are comprised of the same elements.

Concatenation is represented by a dot •.

A **language** is a set of strings over some alphabet. For example, a language over the binary alphabet earlier can be `{00, 01, 10, 11}`.

You can even concatenate languages! If we have two languages L1 and L2, then:

```
L1 • L2 = { x • y | x ∈ L1, y ∈ L2}
```

If L1 = `{00, 01, 10, 11}` and L2 = `{0, 1}`, then L1 • L2 is `{000, 010, 100, 110, 001, 011, 101, 111}`. In this example, L1 • L2 = L2 • L1 i.e. it is commutative, HOWEVER this is NOT usually the case and is definitely not strictly true. Despite the shared symbol, concatenation is not commutative like multiplication.

- The empty set is similar to zero in multiplication however, in that L1 • Ø = Ø. L1 • Ø is NOT L1.
- If the empty set is like zero, then do we have an equivalent to 1? Such that _ • W = W?
- Yes! It's called epsilon ε or **the empty string**. It has *length* zero however.

How is epsilon different from the empty set? First, we define **sigma star** as sigma plus union with epsilon.

```
∑+ = {all possible strings over an alphabet}
∑* = ∑+ U {ε}
```

Epsilon is the *identity* operation for string concatenation; empty set is the *anihilator*.

Say we have a language L which comprises the set of all American currency strings. For this, we consider 1.99,17.00, \$0.99, and \$0 to be valid and leading zeroes (except the special \$0 case) (e.g. 01.99),, 9, \$.99 to be INVALID.

Write an FSA which models this language.