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Homework 2

Chapter 2.1

2.1.9 Compute each Ax by dot products of the rows with the column vector:

$$(a)\begin{bmatrix}1&2&4\\-2&3&1\\-4&1&2\end{bmatrix}\begin{bmatrix}2\\2\\3\end{bmatrix} \qquad (b)\begin{bmatrix}2&1&0&0\\1&2&1&0\\0&1&2&1\\0&0&1&2\end{bmatrix}\begin{bmatrix}1\\1\\1\\2\end{bmatrix}$$

$$(a)\begin{bmatrix}1&2&4\\-2&3&1\\-4&1&2\end{bmatrix}\begin{bmatrix}2\\2\\3\end{bmatrix}=\begin{bmatrix}18\\5\\0\end{bmatrix} \qquad \begin{bmatrix}2&1&0&0\\1&2&1&0\\0&1&2&1\\0&0&1&2\end{bmatrix}\begin{bmatrix}1\\1\\1\\2\end{bmatrix}=\begin{bmatrix}3\\4\\5\\5\end{bmatrix}$$

2.1.10 Compute each Ax in Problem 9 as a combination of the columns:

$$9(a) \text{ becomes} \qquad Ax = 2\begin{bmatrix}1\\-2\\-4\end{bmatrix}+2\begin{bmatrix}2\\3\\1\end{bmatrix}+3\begin{bmatrix}4\\1\\2\end{bmatrix}=\begin{bmatrix}2+4+12\\-4+6+3\\-8+2+6\end{bmatrix}=\begin{bmatrix}18\\5\\0\end{bmatrix}$$

How many separate multiplications for Ax, when the matrix is "3 by 3"?

9 seperate multiplications

2.1.12 Multiply A times x to find three components of Ax :

$$\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}z\\y\\x\end{bmatrix} \qquad \begin{bmatrix}2&1&3\\1&2&3\\3&3&6\end{bmatrix}\begin{bmatrix}1\\1\\-1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \qquad \begin{bmatrix}2&1&3\\1&2&3\\3&3&6\end{bmatrix}\begin{bmatrix}1\\1\\-1\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

2.1.17 Find the matrix P that multiplies (x, y, z) to give (y, z, x). Find the matrix Q that multiplies (y, z, x) to bring back (x, y, z).

$$\begin{bmatrix}0&1&0\\0&0&1\\1&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}y\\z\\x\end{bmatrix} \qquad \begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}\begin{bmatrix}y\\z\\x\end{bmatrix}=\begin{bmatrix}x\\y\\z\end{bmatrix}$$

2.1.19 What 3 by 3 matrix E multiplies (x, y, z) to give $(x, y, z+x)$? What matrix E^{-1} multiplies (x, y, z) to give $(x, y, z-x)$? If you multiply $(3, 4, 5)$ by E and then multiply by E^{-1}, the two results are (_____) and (_____).

$$E \rightarrow \begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}x\\y\\z+x\end{bmatrix} \qquad E^{-1} \rightarrow \begin{bmatrix}1&0&0\\0&1&0\\-1&0&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}x\\y\\z-x\end{bmatrix}$$

$$E\begin{bmatrix}3\\4\\5\end{bmatrix}=\begin{bmatrix}3\\4\\8\end{bmatrix} \qquad E^{-1}\begin{bmatrix}3\\4\\8\end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix}$$

2.1.33 **Multiplying by A is a "linear transformation"**. Those important words mean: If w is a combination of u and v, then Aw is the same combination of Au and Av. It is this "***linearity***" $Aw = cAu+dAv$

that gives us the name *linear algebra*. Problem: If $u = \begin{bmatrix}1\\0\end{bmatrix}$ and $v = \begin{bmatrix}0\\1\end{bmatrix}$ then Au and Av are the columns of A. Combine $w = cu + dv$. **If $\boldsymbol{w} = \begin{bmatrix}\mathbf{5}\\\mathbf{7}\end{bmatrix}$ how is $\boldsymbol{Aw}$ connected to $\boldsymbol{Au}$ and $\boldsymbol{Av}$?**

$$w = cu + dv \to \left[\begin{smallmatrix}5\\7\end{smallmatrix}\right] = c\left[\begin{smallmatrix}1\\0\end{smallmatrix}\right] + d\left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]$$
Upon inspection $c = 5$, $d = 7$
$Aw = cAu + dAv$ then becomes $Aw = 5Au + 7Av$

Chapter 2.2

2.2.5 Choose a right side which gives no solution and another right side which gives infinitely many solutions. What are two of those solutions?

Singular system $\quad$ $3x + 2y = 10$
$6x + 4y =$

A right hand side of 20 would yield infinitely many solutions, any other right hand side would result in no solution. Since the equations are parallel.

2.2.6 Choose a coefficient b that makes this system singular. Then choose a right side g that makes it solvable. Find two solutions in that singular case.

$$2x + by = 16$$
$$4x + 8y = g$$

If $b = 4$, then the system is singular becuase the equations are parallel. Then $g = 32$ and there are infinitely many solutions such as (0, 4) and (6, 1).

2.2.11 (Recommended) A system of linear equations can't have exactly two solutions. *Why*?

(a) If (x, y, z) and (X, Y, Z) are two solutions, what is another solution?

$(\frac{1}{2}x, \frac{1}{2}y, \frac{1}{2}z)$ There are infinitely many solutions along (x, y, z).

(b) If 25 planes meet at two points, where else do they meet?

Along an entire line through those points.

2.2.14 Which number d forces a row exchange, and what is the triangular system (not singular) for that d? Which d makes this system singular (no third pivot)?

$$2x + 5y + z = 0$$
$$4x + dy + z = 2$$
$$y - z = 3$$

If $d = 10$ a row exchange is forced since the first multiplier is 2 and $10 - (2)5 = 0$
If $d = 10$ the system becomes:
$$\begin{aligned} 2x + 5y + z &= 0 \\ -z &= 2 \\ y - z &= 3 \end{aligned}$$
If $d = 11$ the system is singular.

2.2.18 Construct a 3 by 3 example that has 9 different coefficients on the left side, but rows 2 and 3 become zero in elimination. How many solutions to your system with $\boldsymbol{b} = (1, 10, 100)$ and how many with $\boldsymbol{b} = (0, 0, 0)$?

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 6 & 9 & 12 \\ 10 & 15 & 20 \end{bmatrix}$$
If $Ax = b$ and $b = (1, 10, 100)$ there are no solutions.
If $b = (0, 0, 0)$ there are infinitely many.

2.2.21 Find the pivots and the solution for both systems ($Ax = b$ and $Kx = b$):

$$\begin{aligned} 2x + y &= 0 \\ x + 2y + z &= 0 \\ y + 2z + t &= 0 \\ z + 2t &= 5 \end{aligned} \qquad \begin{aligned} 2x - y &= 0 \\ -x + 2y - z &= 0 \\ y + 2z - t &= 0 \\ -z + 2t &= 5. \end{aligned}$$

For $Ax = b$, the pivots are $2, \frac{3}{2}, \frac{4}{4}$ and

$$\begin{aligned} 2x + y &= 0 \\ x + 2y + z &= 0 \\ y + 2z + t &= 0 \\ z + 2t &= 5 \end{aligned} \qquad \begin{aligned} 2x + y &= 0 \\ \tfrac{3}{2}y + z &= 0 \\ y + 2z + t &= 0 \\ z + 2t &= 5 \end{aligned} \qquad \begin{aligned} 2x + y &= 0 \\ \tfrac{3}{2}y + z &= 0 \\ \tfrac{4}{3}z + t &= 0 \\ z + 2t &= 5 \end{aligned} \qquad \begin{aligned} 2x + y &= 0 \\ \tfrac{3}{2}y + z &= 0 \\ \tfrac{4}{3}z + t &= 0 \\ \tfrac{5}{4}t &= 5 \end{aligned}$$

Through back subsititution $t = 4, z = -3, y = 2, x = -1$.

$$\begin{aligned} 2x - y &= 0 \\ -x + 2y - z &= 0 \\ -y + 2z - t &= 0 \\ -z + 2t &= 5 \end{aligned} \qquad \begin{aligned} 2x - y &= 0 \\ \tfrac{3}{2}y - z &= 0 \\ -y + 2z - t &= 0 \\ -z + 2t &= 5 \end{aligned} \qquad \begin{aligned} 2x - y &= 0 \\ \tfrac{3}{2}y - z &= 0 \\ \tfrac{4}{3}z - t &= 0 \\ -z + 2t &= 5 \end{aligned} \qquad \begin{aligned} 2x - y &= 0 \\ \tfrac{3}{2}y - z &= 0 \\ \tfrac{4}{3}z - t &= 0 \\ \tfrac{5}{4}t &= 5 \end{aligned}$$

Through back subsititution $t = 4, z = 3, y = 2, x = 1$.

2.2.25 For which three numbers a will elimination fail to give three pivots?

$$A = \begin{bmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{bmatrix} \text{ is singular for three values of } a.$$

Elimination fails to give three pivots when $a = 2, 3, 0$.

Chapter 2.3

2.3.3 Which three matrices E_{21}, E_{31}, E_{32} put A into triangular form U?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} \qquad \text{and} \qquad E_{32}E_{31}E_{21}A = u.$$

Multiply those E's to get one matrix M that does elimination: $MA = U$.

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \quad E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} \quad M = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{bmatrix}$$

2.3.7 Suppose E subtracts 7 times row 1 from row 3.

(a) To *invert* that step you should _____ 7 times row _____ to row _____

To invert that step you should add 7 times row 1 to row 3.

(b) What "inverse matrix" E^{-1} takes that reverse step (so $E^{-1}E = I$)?

$$E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 7 & 0 & 1 \end{bmatrix}$$

(c) If the reverse step is applied first (and then E) show that $EE^{-1} = I$.

$$E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -7 & 0 & 1 \end{bmatrix} \qquad EE^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -7 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 7 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.3.10 (a) What 3 by 3 matrix E_{13} will add row 3 to row 1?

$$E_{13} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) What matrix adds row 1 to row 3 and at the same time row 3 to row 1?

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(c) What matrix adds row 1 to row 3 and then adds row 3 to row 1?

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

2.3.17 The parabola $y = a + bx + cx^2$ goes through the points $(x, y) = (1, 4)$ and $(2, 8)$ and $(3, 14)$. Find and solve a matrix equation for the unknowns (a, b, c).

With the given parabola and points we can create a matrix equation and then use elimination to solve the system :

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 14 \end{bmatrix}$$

Subtract row 1 from rows 2 & 3.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 2 & 8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 10 \end{bmatrix}$$

Then subtract 2 times row 2 from 3.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix}$$

Through back substitution $c = 1, b = 1$, and $a = 2$.

2.3.18 Multiply these matrices in the orders EF and FE:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & 0 & 1 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix}$$

Also compute $E^2 = EE$ and $F^3 = FFF$. You can guess F^{100}.

$$EF = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \qquad FE = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ ac+b & c & 1 \end{bmatrix} \qquad E^2 = \begin{bmatrix} 1 & 0 & 0 \\ 2a & 1 & 0 \\ 2b & 0 & 1 \end{bmatrix} \qquad F^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2c & 1 \end{bmatrix}$$

$$F^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3c & 1 \end{bmatrix} \qquad F^{100} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 100c & 1 \end{bmatrix}$$

2.3.19 Multiply these row exchange matrices in the orders PQ and QP and P^2 :

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad Q = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Find another non-diagonal matrix whose square is $M^2 = I$.

$$PQ = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad QP = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

2.3.21 If E adds row 1 to row 2 and F adds row 2 to row 1, does EF equal FE?

$$EF = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad FE = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2.3.24 Apply elimination to the 2 by 3 augmented matrix $[A \;\; b]$. What is the triangular system $Ux = c$? What is the solution x?

$$Ax = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 17 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & & 1 \\ 4 & 1 & & 17 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} 2 & 3 & & 1 \\ 0 & -5 & & 15 \end{bmatrix} \qquad \begin{aligned} 2x_1 + 3x_2 &= 1 \\ -5x_2 &= 15 \end{aligned}$$

$$\text{x=} \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

Chapter 2.4

2.4.5 Compute A^2 and A^3. Make a prediction for A^5 and A^n:

$$A = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2b \\ 0 & 1 \end{bmatrix} \quad A^3 = \begin{bmatrix} 1 & 3b \\ 0 & 1 \end{bmatrix} \quad A^5 = \begin{bmatrix} 1 & 5b \\ 0 & 1 \end{bmatrix} \quad A^n = \begin{bmatrix} 1 & nb \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2^2 & 2^2 \\ 0 & 0 \end{bmatrix} \quad A^3 = \begin{bmatrix} 2^3 & 2^3 \\ 0 & 0 \end{bmatrix} \quad A^5 = \begin{bmatrix} 2^5 & 2^5 \\ 0 & 0 \end{bmatrix} \quad A^n = \begin{bmatrix} 2^n & 2^n \\ 0 & 0 \end{bmatrix}$$

2.4.6 Show that $(A + B)^2$ is different from $A^2 + 2AB + B^2$, when

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}.$$

Write down the correct rule for $(A + B)(A + B) = A^2 + \underline{\quad\quad} + B^2$.

$$(A + B)(A + B) = AA + \underline{AB + BA} + BB \qquad AB = \begin{bmatrix} 7 & 0 \\ 0 & 0 \end{bmatrix} \qquad BA = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$$

$AB \neq BA$ therefore $AB + BA$ cannot be combined into $2AB$.

2.4.13 Which of the following matrices are guaranteed to equal $(A - B)^2$: $A^2 - B^2, (B - A)^2, A^2 - 2AB + B^2, A(A - B) - B(A - B), A^2 - AB - BA + B^2$?

$$(A - B)^2 = (A - B)(A - B) = \underline{A(A - B) - B(A - B)} = \underline{A^2 - AB - BA + B^2}$$

2.4.14 True or false:

(a) If A^2 is defined then A is necessarily square.
(b) If AB and BA are defined then A and B are square.
(c) If AB and BA are defined then AB and BA are square.
(d) If $AB = B$ then $A = I$.

(a) True
(b) False - If A is an m x n matrix and B is an n x m matrix then AB is defined and results in an m x m matrix and BA is also defined, resulting in a n x n matrix.
(c) True
(d) False - B could be a zero matrix.

2.4.22 By trial and error find real nonzero 2 by 2 matrices such that

$$A^2 = -I \qquad BC = 0 \qquad DE = -ED \text{ (not allowing } DE = 0).$$

If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ then $A^2 = -I$. If $B = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ then $BC = 0$.

If $D = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ and $E = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ then $DE = -ED$.

2.4.23 (a) Find a nonzero matrix A for which $A^2 = 0$.
(b) Find a matrix that has $A^2 \neq 0$ but $A^3 = 0$.

(a) If $A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$ then $A^2 = -I$.

(b) If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ then $A^2 \neq 0$ but $A^3 = 0$.

2.4.29 Which matrices E_{21} and E_{31} produce zeros in the $(2,1)$ and $(3,1)$ positions of $E_{21}A$ and $E_{31}A$?

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -2 & 0 & 1 \\ 8 & 5 & 2 \end{bmatrix}.$$

Find the single matrix $E = E_{31}E_{21}$ that produces both zeros at once. Multiply EA.

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \qquad EA = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

Chapter 2.5

2.5.7 (Important) If A has row 1+ row 2 = row 3, show that A is not invertible:
(a.) Explain why $Ax = (1, 0, 0)$ cannot have a solution.
(b.) Which right sides (b_1, b_2, b_3) might allow a solution to $Ax = b$?
(c.) What happens to row 3 in elimination?

(a) Equation 1 + equation 2 - equation 3 should equal zero, but it equals 1.
(b) Right side must be $b_1 + b_2 = b_3$.
(c) Row three becomes a row of zeros. Therefore there is no third pivot and A is not invertible.

2.5.13 If the product $M = ABC$ of three square matrices is invertible, then B is invertible. (So are A and C.) Find a formula for B^{-1} that involves M^{-1} and A and C.

$$M = ABC \quad \rightarrow \quad A^{-1}M = BC \quad \rightarrow \quad B^{-1}A^{-1}M = C \quad \rightarrow \quad B^{-1}A^{-1} = CM^{-1} \quad \rightarrow \quad B^{-1} = CM^{-1}A$$

2.5.29 True or false (with a counterexample if false and a reason if true):
(a) A 4 by 4 matrix with a row of zeros is not invertible.
(b) Every matrix with 1's down the main diagonal is invertible.
(c) If A is invertible then A^{-1} and A^2 are invertible.

(a) True - a pivot would be missing
(b) False - the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ is not invertible.
(c) True - the inverse of A^{-1} is A and the inverse of A^2 is $(A^{-1})^2$.

2.5.30 For which three numbers c is this matrix not invertible, and why not?

$$A = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$$

$c = 0$, there is a row of zeros.
$c = 2$, row one and two would be the same.
$c = 7$, columns 2 and 3 would be the same.

2.5.31 Prove that A is invertible if $a \neq 0$ and $a \neq b$ (find the pivots or A^{-1}):

$$A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$$

$$\begin{bmatrix} a & b & b \\ 0 & a-b & 0 \\ 0 & a-b & a-b \end{bmatrix} \rightarrow \begin{bmatrix} a & b & b \\ 0 & a-b & 0 \\ 0 & 0 & a-b \end{bmatrix}$$

There are 3 pivots, the matrix must be invertible.

Chapter 2.6

2.6.7 What three elimination matrices E_{21}, E_{31}, E_{32} put A into its upper triangular form $E_{21}, E_{31}, E_{32}A = U$? Multiply by E_{32}^{-1}, E_{31}^{-1} and E_{21}^{-1} to factor A into L times U:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix} \quad L = E_{32}^{-1}E_{31}^{-1}E_{21}^{-1}$$

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \quad E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$E_{21}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_{31}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \quad E_{32}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$A = LU \quad \rightarrow \quad \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

2.6.8 Suppose A is already lower triangular with 1's on the diagonal. Then $U = I$!

$$A = L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$$

The elimination matrices E_{21}, E_{31}, E_{32} contain -a then -b then -c.
(a) Multiply E_{32}, E_{31}, E_{21} to find the single matrix E that produces $EA = I$.
(b) Multiply $E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}$ to bring back L (nicer than E).

$$\text{(a)} \quad E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -b & 0 & 1 \end{bmatrix} \quad E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -c & 1 \end{bmatrix}$$

$$E_{32} * E_{31} * E_{21} = E = \begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ ac-b & -c & 1 \end{bmatrix}$$

$$\text{(b)} \quad E_{21}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_{31}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ b & 0 & 1 \end{bmatrix} \quad E_{32}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix}$$

$$E_{32}^{-1} * E_{31}^{-1} * E_{21}^{-1} = L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$$

2.6.13 Compute L and U for the symmetric matrix A:

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

Find four conditions on a, b, c, d to get $A = LU$ with four pivots.

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}$$

$$a \neq 0, b \neq a, c \neq b, d \neq c.$$

2.6.16 Solve $L\boldsymbol{c} = \boldsymbol{b}$ to find $\boldsymbol{c}$. Then solve $U\boldsymbol{x} = \boldsymbol{c}$ to find $\boldsymbol{x}$. **What was** A?

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \boldsymbol{b} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} = A$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \text{ Using back substitution: } c_1 = 4, c_2 = 1, c_3 = 1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} \text{ Using back substitution: } x_3 = 1, x_2 = 0, x_1 = 3$$

Chapter 2.7

2.7.3 (a) The matrix $((AB)^{-1})^T$ comes from $(A^{-1})^T$ and $(B^{-1})^T$. *In what order*?

$(A^{-1})^T(B^{-1})$

(b) If U is upper triangular then $(U^{-1})^T$ is ____ triangular.

lower

2.7.6 The transpose of a block matrix $M = \left[\begin{smallmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{smallmatrix}\right]$ is $M^T =$ ____. Test an example.
Under what conditions on A, B, C, D is the block matrix symmetric?

$M^T = \begin{bmatrix} A^T & C^T \\ B^T & D^T \end{bmatrix}$
The block matrix is symmetric when $A^T = A, B^T = C$ and $D^T = D$

2.7.16 If $A = A^T$ and $B = B^T$, which of these matrices are certainly symmetric?
(a) $A^2 - B^2$ (b) $(A+B)(A-B)$ (c) ABA (d) $ABAB$.

$A^2 - B^2$ is symmetric, along with ABA and $ABAB$. $(A+B)(A-B)$ is not symmetric.

2.7.31 Producing x_1 trucks and x_2 planes needs $x_1 + 50x_2$ tons of steel, $40x_1 + 1000x_2$ pounds of rubber, and $2x_1 + 50x_2$ months of labor. If the unit costs y_1, y_2, y_3 are \$700 per ton, \$3 per pound, and \$3000 per month, what are the values of one truck and one plane? Those are the components of $A^T\boldsymbol{y}$.

$$Ax = \begin{bmatrix} 1 & 50 \\ 40 & 1000 \\ 2 & 50 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad A^T y = \begin{bmatrix} 1 & 40 & 2 \\ 50 & 1000 & 50 \end{bmatrix} \begin{bmatrix} 700 \\ 3 \\ 3000 \end{bmatrix} = \begin{bmatrix} 6820 \\ 188000 \end{bmatrix} \text{1 truck} = 6820, \text{1 plane} = 188000$$

2.7.40 Suppose $Q^T equals\ Q^{-1}$ (transpose equals inverse, so $Q^TQ = I$).

$$Q^TQ = I \rightarrow \begin{bmatrix} q_1 & q_3 \\ q_2 & q_4 \end{bmatrix} \begin{bmatrix} q_1 & q_2 \\ q_3 & q_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(a) Show that the columns $q_1, \cdots, q_n$ are unit vectors: $\|\boldsymbol{q}_i\|^2 = 1$.

From the above equation $q_1^2 + q_3^2 = 1, q_2^2 + q_4^2 = 1$ so, $\left[\begin{smallmatrix} q_1 \\ q_3 \end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} q_2 \\ q_4 \end{smallmatrix}\right]$ are unit vectors.

(b) Show that every two columns of Q are perpendicular: $\boldsymbol{q}_1^2\boldsymbol{q}_2 = 0$.

From the above equation $q_1^T q_2 + q_3^T q_4 = 0, q_3^T q_1 + q_4^T q_2 = 0$ so, $\left[\begin{smallmatrix} q_1 \\ q_3 \end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} q_2 \\ q_4 \end{smallmatrix}\right]$ are perpendicular.

(c) Find a 2 by 2 example with first entry $q_{11} = cos\theta$.

$$\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$$