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March 20, 2017

Homework 5

Chapter 5.1

5.1.3 True or false, with a reason if true or a counterexample if false:

(a) The determinant of $I + A$ is $1 + \det A$.

False, $A = \begin{bmatrix} 7 & 1 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad A + I = \begin{bmatrix} 8 & 1 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & 2 \end{bmatrix}$ $\quad 1 + |A| = 29 \neq |I + A| = 80$

(b) The determinant of ABC is $|A||B||C|$.

True, because of property #9.

(c) The determinant of $4A$ is $4|A|$.

False, $\det\left(4\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right) = 8 * 8 \neq 4\begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = 4 * 4.$

(d) The determinant of $AB - BA$ is zero. Try an example with $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

False, $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$AB - BA = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ which is invertible, meaning the determinant is not zero.

5.1.24 Elimination reduces A to U. Then $A = LU$:

$$A = \begin{bmatrix} 3 & 3 & 4 \\ 6 & 8 & 7 \\ -3 & 5 & -9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 4 & 1 \end{bmatrix} \begin{bmatrix} 3 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & -1 \end{bmatrix} = LU.$$

Find the determinants of $L, U, A, U^{-1}, L^{-1}, and U^{-1}L^{-1}A$.

If one reduces L to its reduced row echelon form it becomes I. So $|L| = 1$

$|U| = 3 * 2 * 1 = -1$

$|A| = |U| = -6$

$|U^{-1}L^{-1}| = \frac{1}{|U|} * \frac{1}{|L|} = -\frac{1}{6}$

$|U^{-1}L^{-1}A| = |A| = 1$

5.1.27 Compute the determinants of these matrices by row operations:

$$A = \begin{bmatrix} 0 & a & 0 \\ 0 & 0 & b \\ c & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \\ d & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} a & a & a \\ a & b & b \\ a & b & c \end{bmatrix}.$$

A: Two row swaps are required to get A in the form $\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ so $|A| = (-1)(-1)abc = abc$.

B: Three row swaps are required to get B in the form $\begin{bmatrix} d & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \end{bmatrix}$

so $|B| = (-1)(-1)(-1)abcd = abcd$.

C: You get RREF(C) as $\begin{bmatrix} a & 0 & 0 \\ 0 & b-a & 0 \\ 0 & 0 & c-b \end{bmatrix}$ so $|C| = a(b-a)(c-b)$.

5.1.28 True or false (give a reason if true or a 2 by 2 example if false):

(a) If A is not invertible then AB is not invertible.

True, det(AB)=det(A)∗det(B)=0∗det(B)=0.

(b) The determinant of A is always the products of its pivots.

False, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ requires a row swap therefore the product is the products of its pivots times -1.

(c) The determinant of $A - B$ equals det(A) - det(B).

False, $\left| \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \right| = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \neq \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - \begin{vmatrix} 0 & -1 \\ 0 & 0 \end{vmatrix} = 1 - 0 = 1$

(d) AB and BA have the same determinant.

True, since multiplication is commutative. $|AB| = |A||B| = |B||A| = |BA|$

Chapter 5.2

5.2.9 Show that 4 is the largest determinant for a 3 by 3 matrix of 1's and -1's.

5.2.23 With 2 by 2 blocks in 4 by 4 matrices, you cannot always use block determinants:

$$\begin{vmatrix} A & B \\ 0 & D \end{vmatrix} = |A|\,|D| \qquad \begin{vmatrix} A & B \\ C & D \end{vmatrix} \neq |A|\,|D| - |C|\,|B|.$$

(a) Why is the first statement true? Somehow B doesn't enter.

(b) Show by example that equality fails (as shown) when C enters.

(c) Show by example that the answer det($AD - CB$) is also wrong.

5.2.33 The symmetric Pascal matrices have determinant 1. If I subtract 1 from the n, n entry, why does the determinant become zero? (Use rule 3 or cofactors.)

$$\det \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix} = 1 \text{ (known)} \qquad \det \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & \mathbf{19} \end{bmatrix} = \mathbf{0} \text{ (to explain).}$$