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Homework 1

Chapter 1.1

1.1.24 Which vectors are combinations of u and v, and *also* combinations of v and w?

Combinations of u and v fill one plane, and v and w another plane. Those planes meet at a line: av

1.1.26 What combinations $c\left[\begin{smallmatrix}1\\2\end{smallmatrix}\right] + d\left[\begin{smallmatrix}3\\1\end{smallmatrix}\right]$ produces $\left[\begin{smallmatrix}14\\8\end{smallmatrix}\right]$? Express this question as two equations for the coefficients c and d in the linear combination.

$c\left[\begin{smallmatrix}1\\2\end{smallmatrix}\right] + d\left[\begin{smallmatrix}3\\1\end{smallmatrix}\right] = \left[\begin{smallmatrix}14\\8\end{smallmatrix}\right]$ so, $c = 2$ and $d = 4$. Then $2\left[\begin{smallmatrix}1\\2\end{smallmatrix}\right] + 4\left[\begin{smallmatrix}3\\1\end{smallmatrix}\right] = \left[\begin{smallmatrix}14\\8\end{smallmatrix}\right]$

1.1.28 Find vectors v and w so that $v + w = (4, 5, 6)$ and $v - w = (2, 5, 8)$. This is a question with ____ unknown numbers, and an equal number of equations to find those numbers.

$$\begin{bmatrix} X_v \\ Y_v \\ Z_v \end{bmatrix} + \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$
$$\begin{bmatrix} X_v \\ Y_v \\ Z_v \end{bmatrix} - \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$$

Solve for the six unknowns using the system of equations. Add to find $2v = (6, 10, 14)$ so, $v = (3, 5, 7)$ and $w = (1, 0, -1)$

Chapter 1.2

1.2.5 Find unit vectors $\boldsymbol{u_1}$ and $\boldsymbol{u_2}$ in the directions of $v = (3, 1)$ and $w = (2, 1, 2)$.
Find unit vectors $\boldsymbol{U_1}$ and $\boldsymbol{U_2}$ that are perpendicular to $\boldsymbol{u_1}$ and $\boldsymbol{u_2}$.

$\boldsymbol{u_1} = (\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}})$ $\quad \boldsymbol{u_2} = (\frac{2}{3}, \frac{2}{3}, \frac{2}{3})$
$v = (3, 1)$ $\quad w = (2, 1, 2)$

$$\boldsymbol{u_1} = \begin{bmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \end{bmatrix} \cdot \left[\begin{smallmatrix}x\\y\end{smallmatrix}\right] = 0$$

$x = \frac{1}{\sqrt{10}}$, $y = \frac{-3}{\sqrt{10}}$

$$\boldsymbol{u_2} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x = \frac{1}{\sqrt{2}}$, $y = 0$, $z = \frac{-1}{\sqrt{2}}$

1.2.8 True or false (give a reason if true or a counterexample if false):
(a). If $\boldsymbol{u}$ is perpendicular (in three dimensions) to $\boldsymbol{v}$ and $\boldsymbol{w}$, those vectors $\boldsymbol{v}$ and $\boldsymbol{w}$ are parallel.

False. $\boldsymbol{v}$ and $\boldsymbol{w}$ are any vectors in the plane perpendicular to $\boldsymbol{u}$

(b.) If $\boldsymbol{u}$ is perpendicular to $\boldsymbol{v}$ and $\boldsymbol{w}$, then $\boldsymbol{u}$ is perpendicular to $\boldsymbol{v} + 2\boldsymbol{w}$.

True. $\boldsymbol{u} \cdot (\boldsymbol{v} + 2\boldsymbol{w}) = \boldsymbol{u} \cdot \boldsymbol{v} + 2\boldsymbol{u} \cdot \boldsymbol{w} = 0$

(c.) If $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular unit vectors then $\|\boldsymbol{u} - \boldsymbol{v}\| = \sqrt{2}$

True. $\|\boldsymbol{u} - \boldsymbol{v}\|^2 = (\boldsymbol{u} - \boldsymbol{v}) \cdot (\boldsymbol{u} - \boldsymbol{v})$ seperates into $\boldsymbol{u} \cdot \boldsymbol{u} + \boldsymbol{v} \cdot \boldsymbol{v} = 2$ when $\boldsymbol{u} \cdot \boldsymbol{v} = \boldsymbol{v} \cdot \boldsymbol{u} = 0$

1.2.13 Find two vectors $\boldsymbol{v}$ and $\boldsymbol{w}$ that are perpendicular to $(1, 0, 1)$ and to each other.

$\boldsymbol{v} = (1, 0, -1)$ and $\boldsymbol{w} = (0, 1, 0)$.

1.2.22 The Schwarz inequality $|\boldsymbol{v}+\boldsymbol{w}| \leq \|\boldsymbol{v}\|\|\boldsymbol{w}\|$ by algebra instead of trigonometry:
(a) Multiply out both sides of $(v_1w_1+v_2w_2)^2 \leq (v_1^2+v_2^2)(w_1^2+w_2^2)$.
(b) Show that the difference between those two sides equals $(v_1w_2-v_2w_1)^2$. This cannot be negative since it is a square - so the inequality is true.

$v_1^2w_1^2+2v_1w_1v_2w_2+v_2^2w_2^2 \leq v_1^2w_1^2+v_1^2w2^2+v_2^2w_1^2+v_2^2w_2^2$
$2v_1w_1v_2w_2 \leq v_1^2w_2^2+v_2^2w_2^2$
$v_1^2w_2^2+v_2^2w_1^2-2v_1w_1v_2w_2 \geq 0$
$(v_1w_2-v_2w_1)^2 \geq 0$

1.2.27 If $\|\boldsymbol{v}\|=5$ and $\|\boldsymbol{w}\|=3$, what are the smallest and largest values of $\|\boldsymbol{v}-\boldsymbol{w}\|$? What are the smallest and largest values of $\boldsymbol{v}\cdot\boldsymbol{w}$?

$\|\boldsymbol{v}-\boldsymbol{w}\|$ is between 2 and 8.
$\boldsymbol{v}\cdot\boldsymbol{w}$ is between -15 and 15 by the Schwartz inequality.

Chapter 1.3

1.3.3 Solve these three equations for y_1, y_2, y_3 in terms of B_1, B_2, B_3:

$$Sy=\boldsymbol{B} \qquad \begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=\begin{bmatrix}B_1\\B_2\\B_3\end{bmatrix}$$

Write the solution y as a matrix $A=S^{-1}$ times the vector $\boldsymbol{B}$. Are the columns of S independent or dependent?

$\beta_1 = y_1$
$\beta_2 = y_1+y_2 \qquad y_2=\beta_2-\beta_1$
$\beta_3 = y_1+y_2+y_3 \qquad y_3=\beta_3-\beta_2$
There are independent columns in A and S.

1.3.4 Find a combination $x_1\boldsymbol{w_1}+x_2\boldsymbol{w_2}+x_3\boldsymbol{w_3}$ that gives the zero vector:

$$w_1=\begin{bmatrix}1\\2\\3\end{bmatrix} \qquad w_2=\begin{bmatrix}4\\5\\6\end{bmatrix} \qquad w_3=\begin{bmatrix}7\\8\\9\end{bmatrix}$$

Those vectors are (independent)(dependent). The three vectors lie in a ____. The matrix W with those columns is *not invertible.*

$$=\begin{bmatrix}2\\4\\6\end{bmatrix} \qquad =\begin{bmatrix}8\\10\\12\end{bmatrix} \qquad =\begin{bmatrix}-14\\-16\\-18\end{bmatrix}$$
$$x_1=1 \qquad x_2=-2 \qquad x_3=1$$

The vectors are dependent, they lie in a plane.

1.3.8 Moving to a 4 by 4 difference equation $Ax=\boldsymbol{b}$, find the four components x_1, x_2, x_3, x_4. Then write this solution as $x=S\boldsymbol{b}$ to find the inverse matrix $S=A^{-1}$:

$$Ax=\begin{bmatrix}1&0&0&0\\-1&1&0&0\\0&-1&1&0\\0&0&-1&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}b_1\\b_2\\b_3\\b_4\end{bmatrix}=\boldsymbol{b}$$

$x_1-0=b_1 \qquad x_1=b_1$
$x_2-x_1=b_2 \qquad x_2=b_1+b_2$
$x_3-x_2=b_3 \qquad x_3=b_1+b_2+b_3$
$x_4-x_3=b_4 \qquad x_4=b_1+b_2+b_3+b4$

$$=\begin{bmatrix}1&0&0&0\\1&1&0&0\\1&1&1&0\\1&1&1&1\end{bmatrix}\begin{bmatrix}b_1\\b_2\\b_3\\b_4\end{bmatrix}=A^{-1}b$$