Interpretation of Effects in Multivariate Multiple Regression

These notes focus on how to **interpret regression effects** when a model has multiple predictors and **multiple correlated response variables**.

---

1. What an "Effect" Means in the Multivariate Setting

In the multivariate linear model

$$\mathbf{Y} = \mathbf{X}\mathbf{B} + \mathbf{E},$$

each predictor has a **vector of coefficients** rather than a single scalar. An "effect" refers to how a predictor changes the **joint distribution** of the response vector.

Key implication: - Effects are **directional in response space**, not tied to any single outcome.

---

2. Interpreting a Predictor Effect

For predictor x_k, the coefficient vector is

$$\boldsymbol{\beta}_k = \begin{pmatrix} \beta_{k1} \\ \beta_{k2} \\ \vdots \\ \beta_{kq} \end{pmatrix}.$$

Interpretation: - A one-unit increase in x_k shifts the mean response vector by $\boldsymbol{\beta}_k$. - The magnitude and direction of this shift must be interpreted **relative to the covariance structure** of the responses.

---

3. Joint vs Marginal Interpretation

Marginal (Univariate) View

- Examine β_{kj} for each response Y_j separately.
- Easy to communicate but ignores correlation among responses.

Joint (Multivariate) View

- Focus on whether the predictor changes the response vector **as a whole**.
- Captured by multivariate test statistics (Wilks, Pillai, etc.).

Important distinction: - A predictor may have **no individually significant coefficients**, yet still have a **significant joint effect**.

---

4. Linear Combinations of Responses

Multivariate inference is fundamentally about linear combinations:

$$\mathbf{a}^\top \mathbf{Y}.$$

Interpretation: - The predictor affects some weighted combination of outcomes. - Weights are determined implicitly by the eigenstructure of $\mathbf{E}^{-1}\mathbf{H}$.

This explains why: - multivariate tests can detect subtle but coordinated effects, - interpretation often requires dimension reduction.

---

5. Canonical Variates Perspective

Canonical variates are linear combinations of responses that: - maximize separation attributable to predictors, - correspond to eigenvectors of $\mathbf{E}^{-1}\mathbf{H}$.

Interpretation: - Effects are strongest along the first canonical dimension. - Later dimensions represent weaker or orthogonal patterns.

Caution: - Canonical directions are **data-driven** and may not have direct substantive meaning.

---

6. Effect Size in Multivariate Models

Common multivariate effect size measures: - Partial Wilks' Lambda - Pillai's Trace (interpretable as explained variance in response space)

General interpretation: - Larger values indicate a greater proportion of joint response variation explained by predictors.

Effect sizes should be interpreted **comparatively**, not absolutely.

---

7. Interpreting Categorical Predictors

For factor predictors: - Effects represent **differences in mean response vectors** between groups. - Interpretation parallels MANOVA.

Useful tools: - Group mean profile plots, - contrasts among factor levels, - confidence regions for mean vectors.

---

8. Role of the Response Covariance

The covariance matrix $\mathbf{\Sigma}$ determines: - which response combinations matter most, - how effects are weighted across outcomes.

Key insight: - Strongly correlated responses amplify joint effects. - Weakly correlated responses behave closer to separate regressions.

---

9. Follow-up Interpretation After a Significant Test

A significant multivariate test answers: > "Is there a joint effect?"

It does **not** answer: - which responses drive the effect, - how large the effect is for each outcome.

Typical follow-ups: - univariate confidence intervals with multiplicity control, - examination of dominant canonical variates, - scientifically motivated contrasts.

---

10. Common Misinterpretations to Avoid

- Interpreting individual coefficients without considering covariance.
- Treating multivariate significance as "all outcomes are affected".
- Over-interpreting canonical directions without subject-matter grounding.
- Using post hoc univariate tests as primary evidence.

---

11. Key Takeaways

- Effects are **vectors**, not scalars.
- Interpretation is inherently **geometric**.
- Multivariate significance reflects changes in **response space**, not necessarily in any single outcome.
- Clear interpretation often requires structured follow-up analysis.