微积分第九版附录答案

Appendix Solutions of Calculus Early Transcendentals Ninth Edition

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创建日期：**2023-06-15**
数据抓取日期：**2023-06-25**
版本：**V 0.03**

前言

最近准备学习 Calculus Early Transcendentals Ninth Edition, 发现 Appendix 部分网上没有答案（有答案的网站一般是收费网站，即使是不收费的网站，一般也都是一个网页只包含一道题，如果只是要检查自己的答案是否正确的话，最好还是有一份汇总这些答案的 PDF 文档，)，基于这个原因，想要制作一份这样的 PDF 文档，顺便学习如何使用 LaTeX 制作 PDF 文档。要学习微积分，使用 LaTeX 输入数学符号也是有必要学习的。

答案整理自网站 https://www.gradesaver.com/，起始网页为 https://www.gradesaver.com/textbooks/math/calculus/CLONE-1669a839-2df3-45c5-8bce-02a5db7c0ba8

可能会有部分错误，以及部分公式格式转化存在问题。Appendix F 没有习题，所以没有包含这部分内容。Appendix H 本身就是答案部分，所以也不需要包含。这个网站有答案的习题，我都包含进去了。没有答案的习题，会显示 This answer hasn't been written yet!

目录

第一章　Appendix A: Numbers, Inequalities, and Absolute Values

1.1 Answer

1. 18
2. -18
3. π
4. $\pi - 2$
5. $5 - \sqrt{5}$
6. 1
7. $2 - x$
8. $x - 2$
9. $x + 1$ or $-x - 1$
10. $|2x - 1| = 2x - 1 \geq 0.5$

    and

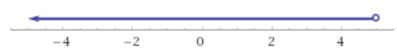
    $-(2x - 1) < 0.5$
11. $x^2 + 1$ for all real numbers x.
12. $|1 - 2x^2| = (1 - 2x^2), x \in (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

    And

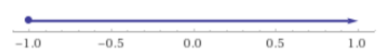
    $|1 - 2x^2| = -(1 - 2x^2), x \in (-\infty, -\frac{1}{\sqrt{2}}) \cup x \in (\frac{1}{\sqrt{2}}, \infty)$
13. $x > -2$

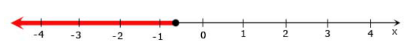
14. $x < 5$

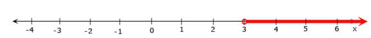
15. $x \geq -1$

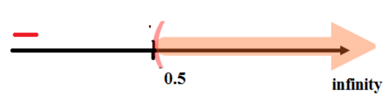
16. $x \leq -\frac{2}{3}$

17. $3 < x$

18. $(1/2, \infty)$

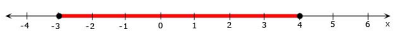
    

19. $2 < x < 6$

20. $-1 < x \leq 4$

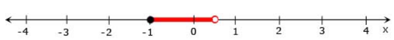
21. $0 < x \leq 1$

22. $-3 \leq x \leq 4$



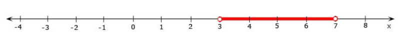
23. $-1 \leq x < \frac{1}{2}$

The solution set is $[-1, \frac{1}{2})$



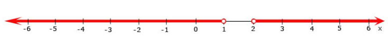
24. $3 < x < 7$

The solution set is $(3, 7)$



25. $x < 1$ or $x > 2$

The solution set is $(-\infty, 1) \cup (2, \infty)$

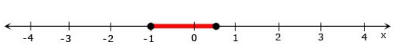


26. $x \leq -\frac{3}{2}$ or $x \geq 1$

The solution set is $(-\infty, -\frac{3}{2}] \cup [1, \infty)$

27. $-1 \leq x \leq \frac{1}{2}$

The solution set is $[-1, \frac{1}{2}]$



28. $-2 < x < 4$

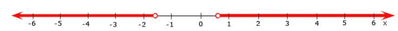
The solution set is $(-2, 4)$

29. $-\infty < x < \infty$

The solution set is $(-\infty, \infty)$

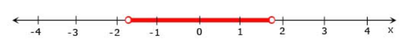
30. $x < \frac{-1-\sqrt{5}}{2}$ or $x > \frac{-1+\sqrt{5}}{2}$

The solution set is $(-\infty, \frac{-1-\sqrt{5}}{2}) \cup (\frac{-1+\sqrt{5}}{2}, \infty)$



31. $-\sqrt{3} < x < \sqrt{3}$

The solution set is $(-\sqrt{3}, \sqrt{3})$

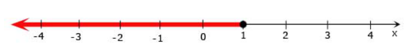


32. $x \leq -\sqrt{5}$ or $x \geq \sqrt{5}$

The solution set is $(-\infty, -\sqrt{5}] \cup [\sqrt{5}, \infty)$

33. $x \leq 1$

The solution set is $(-\infty, 1]$



34. $-3 \leq x \leq -1$ or $x \geq 2$

The solution set is $[-3, -1] \cup [2, \infty)$

35. $-1 < x < 0$ or $x > 1$

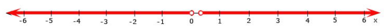
The solution set is $(-1, 0) \cup (1, \infty)$

36. $x < 0$ or $1 < x < 3$

The solution set is $(-\infty, 0) \cup (1, 3)$

37. $x < 0$ or $x > \frac{1}{4}$

The solution set is $(-\infty, 0) \cup (\frac{1}{4}, \infty)$



38. $x < -\frac{1}{3}$ or $x \geq 1$

The solution set is $(-\infty, -\frac{1}{3}) \cup [1, \infty)$

39. $10 \leq C \leq 35$

40. $68 \leq F \leq 86$

41. (a) $T°C = 20°C - 10h$

(b)$-30°C \leq T \leq 20°C$

42. $[0, 3]$

43. $x = \pm\frac{3}{2}$

44. $x = -\frac{4}{3}, -2$

45. $x = 2, -\frac{4}{3}$

46. $x = -4, -\frac{2}{5}$

47. The solution set is $(-3, 3)$

48. $(-\infty, -3] \cup [3, \infty)$

49. The solution set is $(3, 5)$

50. The solution set is $(5.9, 6.1)$

51. $(-\infty, -7] \cup [-3, \infty)$

52. $(-\infty, -4] \cup [2, \infty)$

53. $[1.3, 1.7]$

54. The solution set is $(-\frac{4}{5}, \frac{8}{5})$

55. $[-4, -1] \cup [1, 4]$

56. The solution set is $(\frac{9}{2}, 5) \cup (5, \frac{11}{2})$

57. $x \geq c(\frac{a+b}{ab})$

58. $\frac{a-c}{b} \leq x < \frac{2a-c}{b}$

59. $x > \frac{c-b}{a}$

The solution set is $(\frac{c-b}{a}, \infty)$

60. $x \leq$(bc-b)/a

61. $|(x+y) - 5| <$0.05

62. If $|x+3| < \frac{1}{2}$ then $|4x+13| < 3$

63. $a < (a+b)/2 < b$

64. $1/a > 1/b$

65. $|ab| = |a||b|$

66. $|a/b| = |a|/|b|$

67. $a^2 < b^2$

68. $|x-y| \geq |x| - |y|$

69. The sum, difference, and product of rational numbers are rational numbers.

70. (a) NOT

(b) NOT

1.2 Step by Step

1. $|5 - 23|$

Perform subtraction: $|-18|$.

Compute distance from 0: 18.

2. $|5| - |-23|$

Compute distance from 0 for each absolute value term: $5 - 23$.

Subtract: -18.

3. Start with absolute value: $|-\pi|$

Compute distance from 0 of $-\pi$: π.

4. Start with absolute value expression: $|\pi - 2|$.

   Since $\pi - 2$ is positive, the absolute value does not change it.

   Therefore, $|\pi - 2| = \pi - 2$.

5. Start with absolute value expression: $|\sqrt{5} - 5|$.

   Since $\sqrt{5} - 5$ is negative, we must find the negative of it to find the absolute value:

   $|\sqrt{5} - 5| = -(\sqrt{5} - 5) = 5 - \sqrt{5}$.

6. Start with absolute value expression: $||-2| - |-3||$.

   Find inner absolute values: $|2 - 3|$.

   Since $2 - 3 = -1$ is negative, we must find the negative of it to find the absolute value:

   $|-1| = -(-1) = 1$.

7. Start with expression given: $|x-2|, x < 2$.

   Since the expression inside is negative, we must take the negative of $x - 2$:

   $|x - 2| = -(x - 2) = 2 - x$.

8. Start with expression given: $|x - 2|, x > 2$

   Since the expression inside is positive, we leave what is inside the absolute value alone:

   $|x - 2| = x - 2$.

9. Start with given expression: $|x + 1|$

   Since $x + 1$ could be positive or negative, the absolute value is both the positive and negative version of it:

   $|x+1| = x+1$ or $|x+1| = -(x+1) = -x-1$

10. For $x = 0$ or $x > 0, |x| = x$

    For $x < 0, |x| = -x$

    Now, apply the above definition for the expression $|2x - 1|$.

    $|2x - 1| > 0$ if $x > 0.5$ and $2x - 1 < 0$ if $x < 0.5$.

    Therefore,

    $|2x - 1| = 2x - 1 \geq 0.5$

    and

    $-(2x - 1) < 0.5$

11. Start with given expression: $|x^2 + 1|$.

    Since $x^2 + 1$ must be positive for all x, the absolute value is the same as what is inside the absolute value for all x:

    $|x^2 + 1| = x^2 + 1$ for all x.

12. For $x = 0$ *or* $x > 0, |x| = x$

    For $x < 0, |x| = -x$

    Now, apply above definition for the expression $|1 - 2x^2|$.

    $|1 - 2x^2| > 0$ if $x \in (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

    And

    $|1 - 2x^2| < 0$ if $x \in (-\infty, -\frac{1}{\sqrt{2}}) \cup x \in (\frac{1}{\sqrt{2}}, \infty)$

    Therefore,

    $|1 - 2x^2| = (1 - 2x^2), x \in (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

    And

    $|1 - 2x^2| = -(1 - 2x^2), x \in (-\infty, -\frac{1}{\sqrt{2}}) \cup x \in (\frac{1}{\sqrt{2}}, \infty)$

13. For inequality equations we consider:

    $a > b = a - c > b - c$ and $a > b = \frac{a}{c} > \frac{b}{c}$

    Using that we solve:

    $2x + 7 > 3$

    $2x + 7 - 7 > 3 - 7$

    $2x > -4$

    Then we divide each side of the equation by 2

$\frac{2x}{2} > \frac{-4}{2}$

$x > -2$

We look in the graphic for all the numbers greater than -2:

14. $3x - 11 < 4$

First, we put the -11 to the other side, changing the sign

$3x < 4 + 11$

Then, solve the sum

$3x < 15$

After that, we have:

$x < 5$

15. Considering the inequality rules:

$-a \geq b = a \leq -b$

and if $a > b$ then $a - c > b - c$

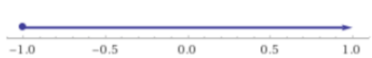
then:

$1 - x \leq 2$

$= 1 - 1 - x \leq 2 - 1$

$= -x \leq 1$

$= x \geq -1$



16. $4 - 3x \geq 6$

$-3x \geq 2$

$x \leq -\frac{2}{3}$

17. $2x + 1 < 5x - 8$

$9 < 3x$

$3 < x$

18. Solve the inequality $1 + 5x > 5 - 3x$ in terms of intervals and illustrate the solution set on the real number line.

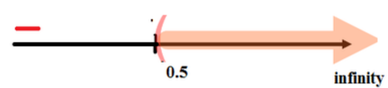
$1 + 5x > 5 - 3x$

$5x + 3x > 5 - 1$

$x > 1/2$

Hence, the solution set is $(1/2, \infty)$.

The solution set is depicted on the real number line below:



19. $-1 < 2x - 5 < 7$

$4 < 2x < 12$

$2 < x < 6$

20. $1 < 3x + 4 \leq 16$

$-3 < 3x \leq 12$

$-1 < x \leq 4$

21. $0 \leq 1 - x < 1$

$-1 \leq -x < 0$

$1 \geq x > 0$

$0 < x \leq 1$

22. $-5 \leq 3 - 2x \leq 9$

    $-8 \leq -2x \leq 6$

    $4 \geq x \geq -3$

    $-3 \leq x \leq 4$

23. $4x < 2x + 1 \leq 3x + 2$

    $4x < 2x + 1 \quad \text{and} \quad 2x + 1 \leq 3x + 2$

    Then:

    $4x < 2x + 1$

    $2x < 1$

    $x < \frac{1}{2}$

    and

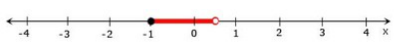
    $2x + 1 \leq 3x + 2$

    $-1 \leq x$

    Solution:

    $-1 \leq x < \frac{1}{2}$

    The solution set is $[-1, \frac{1}{2})$



24. $2x - 3 < x + 4 < 3x - 2$

    $2x - 3 < x + 4 \quad \text{and} \quad x + 4 < 3x - 2$

    Then:

    $2x - 3 < x + 4$

    $x < 7$

    and

    $x + 4 < 3x - 2$

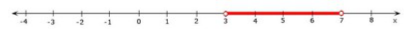
    $6 < 2x$

    $3 < x$

    Solution:

    $3 < x < 7$

    The solution set is $(3, 7)$



25. $(x-1)(x-2) > 0$

    When $x = 1$ or $x = 2$, then $(x-1)(x-2) = 0$

    When $x < 1$, then $(x-1)(x-2) > 0$

    When $1 < x < 2$, then $(x-1)(x-2) < 0$

    When $x > 2$, then $(x-1)(x-2) > 0$

    Solution:

    $x < 1$ or $x > 2$

    The solution set is $(-\infty, 1) \cup (2, \infty)$

26. $(2x+3)(x-1) \geq 0$

    When $x = -\frac{3}{2}$ or $x = 1$, then $(2x+3)(x-1) = 0$

    When $x < -\frac{3}{2}$, then $(2x+3)(x-1) > 0$

    When $-\frac{3}{2} < x < 1$, then $(2x+3)(x-1) < 0$

    When $x > 1$, then $(2x+3)(x-1) > 0$

    Solution:

    $x \leq -\frac{3}{2}$ or $x \geq 1$

    The solution set is $(-\infty, -\frac{3}{2}] \cup [1, \infty)$

27. $2x^2 + x \leq 1$

    $2x^2 + x - 1 \leq 0$

    We can use the quadratic formula to find the values of x such that $2x^2 + x - 1 = 0$:

    $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

    $x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-1)}}{(2)(2)}$

    $x = \frac{-1 \pm \sqrt{9}}{4}$

    $x = \frac{-1 \pm 3}{4}$

    $x = -1, \frac{1}{2}$

When $x = -1$ or $x = \frac{1}{2}$, then $2x^2 + x - 1 = 0$

When $x < -1$, then $2x^2 + x - 1 > 0$

When $-1 < x < \frac{1}{2}$, then $2x^2 + x - 1 < 0$

When $x > \frac{1}{2}$, then $2x^2 + x - 1 > 0$

Solution:

$-1 \leq x \leq \frac{1}{2}$

The solution set is $[-1, \frac{1}{2}]$

28. $x^2 < 2x + 8$

$x^2 - 2x - 8 < 0$

$(x-4)(x+2) < 0$

When $x = -2$ or $x = 4$, then $(x-4)(x+2) = 0$

When $x < -2$, then $(x-4)(x+2) > 0$

When $-2 < x < 4$, then $(x-4)(x+2) < 0$

When $x > 4$, then $(x-4)(x+2) > 0$

Solution:

$-2 < x < 4$

The solution set is $(-2, 4)$

29. $x^2 + x + 1 > 0$

We can use the quadratic formula to find the values of x such that $x^2 + x + 1 = 0$:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{(2)(1)}$

$x = \frac{-1 \pm \sqrt{-3}}{2}$

Since there is a negative value inside the square root, there are no values of x such that $x^2 + x + 1 = 0$

When $x = 1$, then $x^2 + x + 1 = 3 > 0$

Since $x^2 + x + 1$ is continuous, and there are no values of x such that $x^2 + x + 1 = 0$, then $x^2 + x + 1 > 0$ for all values of x

Solution:

$-\infty < x < \infty$

The solution set is $(-\infty, \infty)$

30. $x^2 + x > 1$

$x^2 + x - 1 > 0$

We can use the quadratic formula to find the values of x such that $x^2 + x - 1 = 0$:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{(2)(1)}$

$x = \frac{-1 \pm \sqrt{5}}{2}$

$x = \frac{-1 - \sqrt{5}}{2}, \frac{-1 + \sqrt{5}}{2}$

When $x = \frac{-1-\sqrt{5}}{2}$ or $x = \frac{-1+\sqrt{5}}{2}$, then $x^2 + x - 1 = 0$

When $x < \frac{-1-\sqrt{5}}{2}$, then $x^2 + x - 1 > 0$

When $\frac{-1-\sqrt{5}}{2} < x < \frac{-1+\sqrt{5}}{2}$, then $x^2 + x - 1 < 0$

When $x > \frac{-1+\sqrt{5}}{2}$, then $x^2 + x - 1 > 0$

Solution:

$x < \frac{-1-\sqrt{5}}{2}$ or $x > \frac{-1+\sqrt{5}}{2}$

The solution set is $(-\infty, \frac{-1-\sqrt{5}}{2}) \cup (\frac{-1+\sqrt{5}}{2}, \infty)$

31. $x^2 < 3$

$|x| < \sqrt{3}$

$-\sqrt{3} < x < \sqrt{3}$

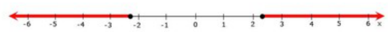
The solution set is $(-\sqrt{3}, \sqrt{3})$

32. $x^2 \geq 5$

    $|x| \geq \sqrt{5}$

    $x \leq -\sqrt{5}$ or $x \geq \sqrt{5}$

    The solution set is $(-\infty, -\sqrt{5}] \cup [\sqrt{5}, \infty)$



33. $x^3 - x^2 \leq 0$

    $x^2(x-1) \leq 0$

    When $x = 0$ or $x = 1$, then $x^2(x-1) = 0$

    When $x < 0$, then $x^2(x-1) < 0$

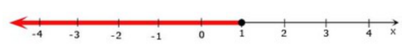
    When $0 < x < 1$, then $x^2(x-1) < 0$

    When $x > 1$, then $x^2(x-1) > 0$

    Solution:

    $x \leq 1$

    The solution set is $(-\infty, 1]$



34. $(x+1)(x-2)(x+3) \geq 0$

    When $x = -1$, $x = 2$, or $x = -3$, then $(x+1)(x-2)(x+3) = 0$

    When $x < -3$, then $(x+1)(x-2)(x+3) < 0$

    When $-3 < x < -1$, then $(x+1)(x-2)(x+3) > 0$

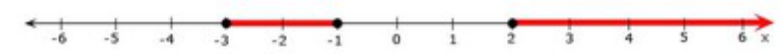
    When $-1 < x < 2$, then $(x+1)(x-2)(x+3) < 0$

    When $x > 2$, then $(x+1)(x-2)(x+3) > 0$

    Solution:

    $-3 \leq x \leq -1$ or $x \geq 2$

    The solution set is $[-3, -1] \cup [2, \infty)$



35. $x^3 > x$

    Case 1: $x < 0$

    $x^3 > x$

    $x^2 < 1$

    $|x| < 1$

    $-1 < x < 0$

    Case 2: $x > 0$

    $x^3 > x$

    $x^2 > 1$

    $x > 1$

    Solution:

    $-1 < x < 0$ or $x > 1$

    The solution set is $(-1, 0) \cup (1, \infty)$

36. $x^3 + 3x < 4x^2$

    $x^3 - 4x^2 + 3x < 0$

    $x(x^2 - 4x + 3) < 0$

    $(x)(x-3)(x-1) < 0$

    When $x = 0$, $x = 1$, or $x = 3$, then $(x)(x-3)(x-1) = 0$

    When $x < 0$ then $(x)(x-3)(x-1) < 0$

    When $0 < x < 1$, then $(x)(x-3)(x-1) > 0$

    When $1 < x < 3$, then $(x)(x-3)(x-1) < 0$

    When $x > 3$, then $(x)(x-3)(x-1) > 0$

    Solution:

    $x < 0$ or $1 < x < 3$

    The solution set is $(-\infty, 0) \cup (1, 3)$

37. $\frac{1}{x} < 4$

$0 < \frac{1}{x} < 4 \quad$ or $\quad x < 0$

$x > \frac{1}{4} \quad$ or $\quad x < 0$

The solution set is $\quad (-\infty, 0) \cup (\frac{1}{4}, \infty)$

38. $-3 < \frac{1}{x} \leq 1$

Case 1: $x < 0$

$-3 < \frac{1}{x}$

$-\frac{1}{3} > x$

Case 2: $x > 0$

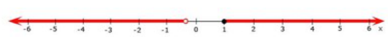
$\frac{1}{x} \leq 1$

$x \geq 1$

Solution:

$x < -\frac{1}{3} \quad$ or $\quad x \geq 1$

The solution set is $\quad (-\infty, -\frac{1}{3}) \cup [1, \infty)$



39. The interval on the Celsius scale corresponds to the temperature range. $50 \leq F \leq 95$ can be calculated as follows:

Given: $50 \leq F \leq 95$

$50 - 32 \leq F - 32 \leq 95 - 32$

$18 \leq F - 32 \leq 63$

$\frac{5}{9}(18) \leq \frac{5}{9}(F - 32) \leq \frac{5}{9}(63)$

$10 \leq \frac{5}{9}(F - 32) \leq 35$

$10 \leq C \leq 35$

Therefore, the interval on the Celsius scale corresponds to the temperature range. $50 \leq F \leq 95$ is $10 \leq C \leq 35$.

40. The interval on the Celsius scale corresponds to the temperature range. $20 \leq C \leq 30$ can be calculated as follows:

Given: $C = \frac{5}{9}(F - 32)$

$9C = 5(F - 32)$

We get, $F = \frac{9C}{5} + 32$

Further,

$20 \leq C \leq 30$

$\frac{9}{5}(20) \leq \frac{9}{5}C \leq \frac{9}{5}(30)$

$36 \leq \frac{9}{5}C \leq 54$

$36 + 32 \leq \frac{9}{5}C + 32 \leq 54 + 32$

$68 \leq \frac{9}{5}C + 32 \leq 86$

$68 \leq F \leq 86$

Therefore, the interval on the Celsius scale corresponds to the temperature range $68 \leq F \leq 86$.

41. As dry air moves upward, it expands and in doing so cools at a rate of about $1°C$ for each 100-m rise, up to about 12 km.

(a) If the ground temperature is $20°C$, then write a formula for the temperature at height h.

The temperature at height h km is $T°C = 20°C - 10h$

Here, $0 \leq h \leq 12$

(b) The range of temperature can be expected if a plane takes off and reaches a maximum height of 5 km.

The temperature at height 5 km is $T°C = 20°C - 10(5)°C = -30°C$

Here, range of the temperature is

$-30°C \leq T \leq 20°C$

42. Solve for the inequality

$h \geq 32$

Substitute

$128 + 16t - 16t^2 \geq 32$

Simplify on one side

Identify key numbers by equating

Intervals : $[0, 3]$, $[3, \infty)$

$[0, 3]$

43. Solve the inequality $|2x| = 3$.

The absolute value is $2x = \pm 3$

Divide by 2.

$x = \pm\frac{3}{2}$

44. Solve the inequality $|3x + 5| = 1$

$3x + 5 = \pm 1$

Subtract 5.

$3x = \pm 1 - 5$

This implies

$3x = 1 - 5$ or $3x = -1 - 5$

$3x = -4$ or $3x = -6$

Divide by 3.

Hence, $x = -\frac{4}{3}, -2$

45. Solve $|x + 3| = |2x + 1|$.

$\pm(x + 3) = (2x + 1)$

It can be simplified as follows:

$(x+3) = (2x+1)$ and $-(x+3) = (2x+1)$

$2x - x = 3 - 1$ and $-x - 3 = 2x + 1$

$x = 2$ and $x = -\frac{4}{3}$

Hence, $x = 2, -\frac{4}{3}$

46. Given: $|\frac{2x-1}{x+1}| = 3$

Solve the given inequality as follows:

$\frac{2x-1}{x+1} = \pm 3$

$2x - 1 = \pm 3(x + 1)$

$2x - 1 = 3(x + 1)$

This implies

$2x - 3x = 3 + 1$

$x = -4$

or

$2x - 1 = -3(x + 1)$

This implies

$2x + 3x = -3 + 1$

$5x = -2$

$x = -\frac{2}{5}$

Hence, the solution are $x = -4, -\frac{2}{5}$

47. $|x| < 3$

$-3 < x < 3$

The solution set is $(-3, 3)$

48. The inequality$|x| \geq 3$ can be calculated as follows:

We know that if$|x| \geq a$, then

$x \leq -a, x \geq a \cdots (1)$

Now, apply equation (1) for expression

$|x| \geq 3$ to get

$x \leq -3, x \geq 3$

Hence, the solution set is$(-\infty, -3] \cup [3, \infty)$.

49. $|x - 4| < 1$

$-1 < x - 4 < 1$

$3 < x < 5$

The solution set is $(3, 5)$

50. $|x - 6| < 0.1$

$-0.1 < x - 6 < 0.1$

$5.9 < x < 6.1$

The solution set is $(5.9, 6.1)$

51. The inequality$|x+5| \geq 2$ can be calculated as follows:

    If$|x| \geq a$, then

    $x \leq -a, x \geq a \cdots (1)$

    Now, apply equation (1) for the expression

    $|x+5| \geq 2$ to get

    $x+5 \leq -2, x+5 \geq 2$

    This implies

    $x \leq -7, x \geq -3$

    Hence, the solution set is$(-\infty - 7] \cup [-3, \infty)$.

52. The inequality$|x+1| \geq 3$ can be calculated as follows:

    If$|x| \geq a$ then:

    $x \leq -a, x \geq a \cdots (1)$

    Now, apply equation (1) for expression$|x+1| \geq 3$ to get

    $x+1 \leq -3, x+1 \geq 3$

    This implies

    $x \leq -4, x \geq 2$

    Hence, the solution set is$(-\infty, -4] \cup [2, \infty)$.

53. The inequality$|2x-3| \leq 0.4$ can be calculated as follows:

    If$|x| \geq a$ then:

    $x \leq -a, x \geq a \cdots (1)$

    Now, apply equation (1) for expression

    $|2x-3| \leq 0.4$ to get:

    $-0.4 \leq 2x - 3 \leq 0.4$

    This implies

    $3 - 0.4 \leq 2x \leq 0.4 + 3$

    $2.6 \leq 2x \leq 3.4$

    $1.3 \leq x \leq 1.7$

    Hence, the solution set is$[1.3, 1.7]$.

54. $|5x-2| < 6$

    $-6 < 5x - 2 < 6$

    $-4 < 5x < 8$

    $-\frac{4}{5} < x < \frac{8}{5}$

    The solution set is $(-\frac{4}{5}, \frac{8}{5})$

55. The inequality$1 \leq |x| \leq 4$ can be calculated as follows:

    If$|x| \geq a$ then:

    $x \leq -a, x \geq a \cdots (1)$

    And

    if $|x| \leq a$ then:

    $-a \leq x \leq a \cdots (2)$

    Now, apply equation (1) for expression $1 \leq |x| \leq 4$.

    It can be written as $|x| \geq 1, |x| \leq 4$

    From equation (1), we have

    $|x| \geq 1$ then $x \leq -1, x \geq 1$

    From equation (2), we have

    $|x| \leq 4$ then $-4 \leq x \leq 4$

    Hence, the solution set is$[-4, -1] \cup [1, 4]$.

56. $0 < |x-5| < \frac{1}{2}$

    Case 1:

    $-\frac{1}{2} < x - 5 < 0$

    $\frac{9}{2} < x < 5$

    Case 2:

    $0 < x - 5 < \frac{1}{2}$

    $5 < x < \frac{11}{2}$

    The solution set is $(\frac{9}{2}, 5) \cup (5, \frac{11}{2})$

57. Given: $a(bx - c) \geq bc$

    The solution set for x can be calculated as follows:

    $bx \geq \frac{bc}{a} + c$

$x \geq c(\frac{1}{a} + \frac{1}{b})$

$x \geq c(\frac{a+b}{ab})$

Hence, the solution set is $x \geq c(\frac{a+b}{ab})$.

58. Solve the inequality $a \leq bx + c < 2a$ for x, assuming a, b, and c are positive constants.

    Thus,

    Subtract c:

    $a - c \leq bx < 2a - c$

    Divide b:

    $\frac{a-c}{b} \leq x < \frac{2a-c}{b}$

    Hence, $\frac{a-c}{b} \leq x < \frac{2a-c}{b}$

59. Note that a, b, and c are negative constants.

    $ax + b < c$

    $ax < c - b$

    $x > \frac{c-b}{a}$

    The solution set is $(\frac{c-b}{a}, \infty)$

    Note that the inequality changes from $<$ to $>$ when we divide by a because $a < 0$

60. $(ax + b)/c \leq$b

    $ax + b \geq$bc (since $c < 0$)

    $ax \geq$bc-b

    $x \leq$(bc-b)/a (since $a < 0$)

61. $a = x - 2$

    $b = y - 3$

    Plug in the values

    Final answer is

    $|(x + y) - 5| <$0.05

    because $0.01 + 0.04 = 0.05$

    The substitution allows for the $(x + y) - 5$ in the absolute value form.

62. Suppose that $|x + 3| < \frac{1}{2}$

    Then:

    $|x + 3| < \frac{1}{2}$

    $-\frac{1}{2} < x + 3 < \frac{1}{2}$

    $-2 < 4x + 12 < 2$

    $-1 < 4x + 13 < 3$

    $-3 < -1 < 4x + 13 < 3$

    $|4x + 13| < 3$

63. Add a to both sides

    Divide by 2

    Then

    Add b on both sides

    Divide by 2

    Combine the inequalities and result is $a < (a + b)/2 < b$

64. Use the "rules for inequalities"

    Prove rule 5

    Multiply equation by 1/ab

    Final result is

    $1/a > 1/b$

65. $|ab| = \sqrt{(ab)^2}$

    $|ab| = \sqrt{(a)^2(b)^2}$

    $|ab| = \sqrt{a}^2\sqrt{b}^2$

    $|ab| = |a||b|$

66. Substitute a with a/b

    Rewrite and then solve:

    $|a/b| = |a|/|b|$

67. Multiply a on both sides.

    Multiply b on both sides.

    Combine inequalities and we get:

    $a^2 < b^2$

68. Solution:

We have to prove $|x-y| \geq |x|-|y|$ using triangular inequality $|a+b| \leq |a|+|b|$.

Use $a = x-y$ and $b = y$

Therefore,

$|x| \geq |(x-y)+y|$

$|(x-y)|+|y| \geq |x|$

$(x-y) \geq |x|-|y|$

Hence, the result is proved.

69. Show that the sum, difference, and product of rational numbers are rational numbers.

1. Take a, b two rational numbers.

By the definition of rational numbers $a = \frac{m_1}{n_1}$

Here, m_1 and n_1 are integers and $n_1 \neq 0$.

Also, $b = \frac{m_2}{n_2}$

Here, m_2 and n_2 are integers and $n_2 \neq 0$

2. Take the sum of two rational numbers.

$a + b = \frac{m_1}{n_1} + \frac{m_2}{n_2} = \frac{m_1 n_2 + m_2 n_1}{n_1 n_2}$

Here, $(m_1 n_2 + m_2 n_1)$ are integers and $n_1 n_2 \neq 0$

Because m_1, m_2, n_1 and n_2 are integers and $n_1 \neq 0$, $n_2 \neq 0$, the sum $a + b$ is also a rational number.

3. Take the product of two rational numbers.

$ab = \frac{m_1}{n_1}\frac{m_2}{n_2} = \frac{m_1 m_2}{n_1 n_2}$

Here, $m_1 m_2$ are integers and $n_1 n_2 \neq 0$

Because m_1, m_2, n_1 and n_2 are integers and $n_1 \neq 0$, $n_2 \neq 0$, the sum ab is also a rational number.

4. Take the differnce of two rational numbers.

$a - b = \frac{m_1}{n_1} - \frac{m_2}{n_2} = \frac{m_1 n_2 - m_2 n_1}{n_1 n_2}$

Here, $(m_1 n_2 - m_2 n_1)$ are integers and $n_1 n_2 \neq 0$

Because m_1, m_2, n_1 and n_2 are integers and $n_1 \neq 0$, $n_2 \neq 0$, the sum $a - b$ is also a rational number.

70. (a) The sum of two irrational numbers is not always an irrational number.

Consider the below two irrational numbers $2 + \sqrt{3}, 2 - \sqrt{3}$. Their sum is 4 which is not an irrational number.

Hence, the sum of two irrational numbers is always not an irrational number.

(b) The product of two irrational numbers is always not an irrational number.

Consider the below two irrational numbers $2 + \sqrt{3}, 2 - \sqrt{3}$. Their product is 1 which is not an irrational number.

Hence, the product of two irrational numbers is not always an irrational number.

第二章　Appendix B: Coordinate Geometry and Lines

2.1 Answer

1. 5
2. $2\sqrt{29}$
3. $\sqrt{74}$
4. $\sqrt{13}$
5. $2\sqrt{37}$
6. $\sqrt{2a^2 - 4ab + 2b^2}$
7. 2
8. $-\frac{9}{5}$
9. $m = -\frac{9}{2}$
10. $m = \frac{4}{7}$
11. The triangle with vertices

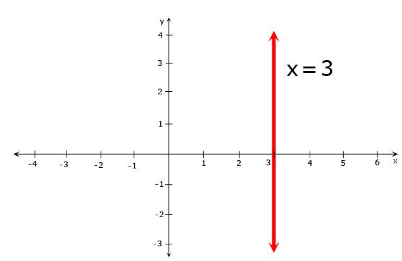
    $A(0, 2)$, $B(-3, -1)$ and $C(-4, 3)$

    is isosceles.
12. (a) The triangle with vertices $A(6, -7)$, $B(11, -3)$ and $C(2, -2)$ is a right triangle by using the converse of the Pythagorean Theorem.

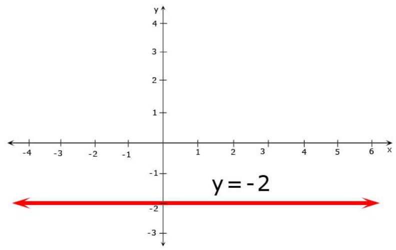
    (b) The triangle with vertices $A(6, -7)$, $B(11, -3)$ and $C(2, -2)$ is a right triangle.

    (c) Area = 20.5 square units
13. The points $(-2, 9)$, $(4, 6)$ and $(1, 0)$, $(-5, 3)$ are vertices of a square.
14. $a)|AB| + |BC| = |AC|$

    $b)m_1 = m_2 = m3$
15. The given points are vertices of a parallelogram because they form two pairs of congruent opposite sides
16. If ABCD is a Rectangle, than d(AB) = d(CD), d(DA) = d(BC) and AB is perpendicular to DA (a parallelogram with two adjacent perpendicular sides)
17. We can see the graph of the equation $x = 3$ below.

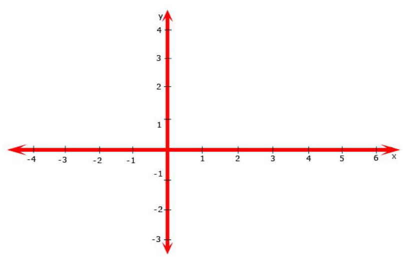


18. We can see the graph of the equation $y = -2$ below.

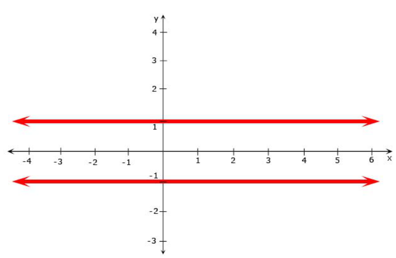


19. The graph of the equation $xy = 0$ is a horizontal line that passes through the point $(0, 0)$ and a vertical line that passes through the point $(0, 0)$

    We can see the graph of the equation $xy = 0$ below.



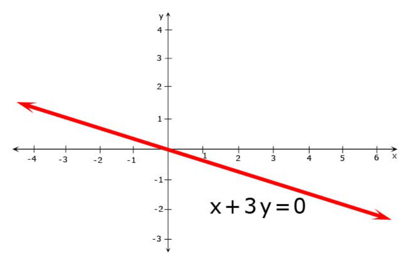
20. We can see the graph of the equation $|y| = 1$ below.



21. $y = 6x - 15$
22. $y = -3x + 1$
23. $3y = 2x + 19$
24. $2y = -7x - 31$
25. $y = -5x + 11$
26. $y = x - 1$
27. $y = 3x - 2$
28. $y = \frac{2}{5}x + 4$
29. $y = 3x - 3$
30. $y = \frac{3}{4}x + 6$
31. $y = 5$
32. $x = 4$
33. $y = -\frac{1}{2}x - \frac{11}{2}$
34. $y = -\frac{2}{3}x + 6$
35. $y = \frac{5}{2}x + \frac{1}{2}$
36. $y = -2x + \frac{1}{3}$
37. The slope is $-\frac{1}{3}$

    The y-intercept is 0

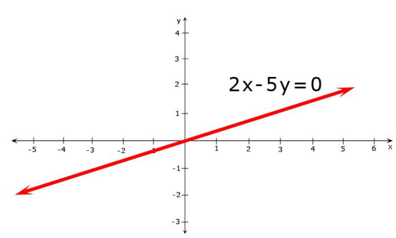
    We can see the graph of the line $x + 3y = 0$:



38. The slope is $\frac{2}{5}$

    The y-intercept is 0

    We can see the graph of the line $2x - 5y = 0$:



39. $m = 0$

    $b = -2$
40. $m = 2/3$

    $b = 2$

41. $m = 3/4$

    $b = -3$

42. $m = -4/5$

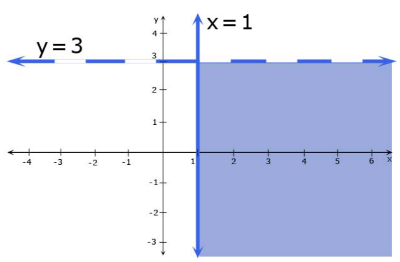
    $b = 2$

43. It will be to the LEFT of the line.

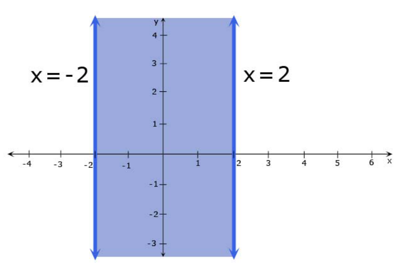
44. It will be ABOVE the line.

45. Quadrants II and IV.

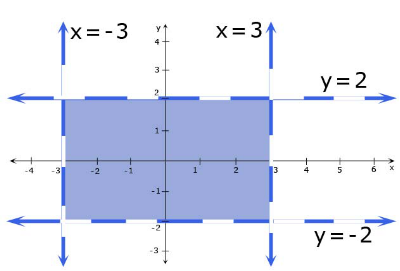
46. The region includes the points which are to the right of the vertical line $x = 1$, (as well as the points on this line), and below the horizontal line $y = 3$, (but not including the points on this horizontal line).



47. The region includes all the points between the vertical lines $x = -2$ and $x = 2$, as well as the points on these vertical lines.

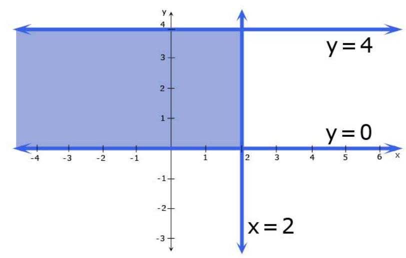


48. The region includes all the points where $-3 < x < 3$ and $-2 < y < 2$



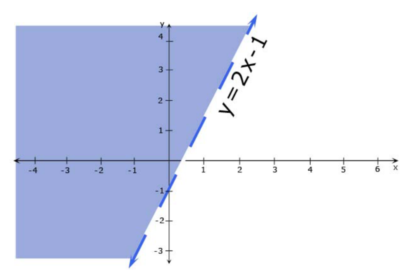
49. Thus, the region includes all the points which are between the horizontal lines $y = 0$ and $y = 4$, and to the left of the vertical line $x = 2$

    Note that the points on these three lines are also included in the region.



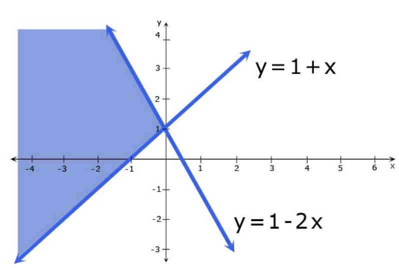
50. The region includes all the points that are above the line $y = 2x - 1$

    Note that the points on this line are not included in the region.



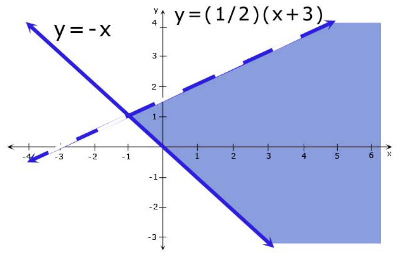
51. The region includes all the points that are above the line $y = 1 + x$ and below the line $y = 1 - 2x$

    Note that the points on these two lines are also included in the region.



52. The region includes all the points that are above the line $y = -x$ and below the line $y = \frac{1}{2}(x + 3)$

Note that the points on the line $y = -x$ are also included in the region, but the points on the line $y = \frac{1}{2}(x+3)$ are not included in the region.



53. The point on the y-axis is $(0, -4)$

54. Substitute the points P1 and P2 to the midpoint formula

55. (a) The midpoint is $(4, 9)$

    (b) The midpoint is $(\frac{7}{2}, -3)$

56. $M_1 = \sqrt{37}; M_2 = \frac{\sqrt{145}}{2}; M_3 = \frac{\sqrt{109}}{2}$

57. Since these two lines do not have equal slopes, they are not parallel.

    The point of intersection is $(1, -2)$

58. The product of the slopes of the two lines is -1, so these two lines are perpendicular.

    The point of intersection is $(2, 5)$

59. $y = x - 3$

60. This answer hasn't been written yet!

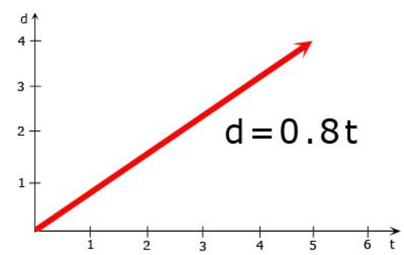
61. (a) $\frac{x}{a} + \frac{y}{b} = 1$

    (b) $4x - 3y - 24 = 0$

62. (a) $d = 0.8\, t$ (with t in minutes)

    (b) We can see the graph below.

    (c) The slope of the line is $0.8\ mi/min$ (or $48\ mi/hr$) and this represents the speed of the car.



2.2 Step by Step

1. To find the distance between $(1, 1)$ and $(4, 5)$,

   we use the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

   We plug in the given x and y values: $d = \sqrt{(4-1)^2 + (5-1)^2}$.

   We then simplify to arrive at the answer: $d = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

2. To find the distance between $(1, -3)$ and $(5, 7)$,

   we use the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

   We plug in the given x and y values: $d = \sqrt{(5-1)^2 + (7-(-3))^2}$.

   We then simplify to arrive at the answer: $d = \sqrt{4^2 + 10^2} = \sqrt{116} = 2\sqrt{29}$.

3. To find the distance between $(6, -2)$ and $(-1, 3)$,

   we use the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

   We plug in the given x and y values: $d = \sqrt{(-1-6)^2 + (3-(-2))^2}$.

   We then simplify to arrive at the answer: $d = \sqrt{(-7)^2 + 5^2} = \sqrt{74}$.

4. To find the distance between $(1, -6)$ and $(-1, -3)$,

we use the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

We plug in the given x and y values: $d = \sqrt{(-1-1)^2 + (-3-(-6))^2}$.

We then simplify to arrive at the answer: $d = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$.

5. To find the distance between $(2, 5)$ and $(4, -7)$,

   we use the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

   We plug in the given x and y values: $d = \sqrt{(4-2)^2 + (-7-5)^2}$.

   We then simplify to arrive at the answer: $d = \sqrt{2^2 + 12^2} = \sqrt{148} = 2\sqrt{37}$.

6. To find the distance between (a, b) and (b, a),

   we use the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

   We plug in the given x and y values: $d = \sqrt{(b-a)^2 + (a-b)^2}$.

   We then simplify to arrive at the answer: $d = \sqrt{b^2 - 2ab + a^2 + a^2 - 2ab + b^2} = \sqrt{2a^2 - 4ab + 2b^2}$.

7. To find the slope of the line passing through $(1, 5)$ and $(4, 11)$,

   we use the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

   Plug in the given values of x and y: $m = \frac{11-5}{4-1}$.

   Simplify to arrive at the answer: $m = \frac{6}{3} = 2$.

8. Find the slope (m) of the line through P and Q.

   $P(-1, 6)$

   $Q(4, -3)$

   $m = \frac{y_2 - y_1}{x_2 - x_1}$

   $m = \frac{-3-6}{4-(-1)} = -\frac{9}{5}$

9. To find the slope of the line passing through $(-3, 3)$ and $(-1, -6)$,

   we use the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

   Plug in the given values of x and y: $m = \frac{-6-3}{-1-(-3)}$.

   Simplify to arrive at the answer: $m = -\frac{9}{2}$.

10. To find the slope of the line passing through $(-1, -4)$ and $(6, 0)$,

    we use the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

    Plug in the given values of x and y: $m = \frac{0-(-4)}{6-(-1)}$.

    Simplify to arrive at the answer: $m = \frac{4}{7}$.

11. Given: The triangle with vertices $A(0, 2)$, $B(-3, -1)$ and $C(-4, 3)$ is isosceles.

    An isosceles is a triangle that having at least two sides of equal length.

    Consider AB, BC and AC are three sides of a triangle.

    Use the distance formula

    $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

    to calculate each side.

    Therefore,

    For side AB:

$$\begin{aligned} AB &= \sqrt{(-3-0)^2 + (-1-2)^2} \\ &= \sqrt{9+9} \\ &= 3\sqrt{2} \end{aligned}$$

    For side BC:

$$\begin{aligned} BC &= \sqrt{(-4-(-3))^2 + (3-(-1))^2} \\ &= \sqrt{1+16} \\ &= \sqrt{17} \end{aligned}$$

For side AC:

$$\begin{aligned} AC &= \sqrt{(-4-0)^2+(3-2)^2} \\ &= \sqrt{16+1} = \sqrt{17} \end{aligned}$$

From the above calculations, we conclude that $BC = AC$

Hence, the triangle with vertices $A(0,2)$, $B(-3,-1)$ and $C(-4,3)$ is isosceles.

12. (a) As we are given the triangle with vertices $A(6,-7)$, $B(11,-3)$ and $C(2,-2)$.

Consider AB, BC and AC are three sides of a triangle.

Use distance formula

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

For side AB:

$AB = \sqrt{(11-6)^2+(-3-(-7))^2} = \sqrt{16+25} = \sqrt{41}$

For side BC:

$BC = \sqrt{(2-11)^2+(-2-(-3))^2} = \sqrt{81+1} = \sqrt{82}$

For side AC:

$AC = \sqrt{(2-6)^2+(-2-(-7))^2} = \sqrt{25+16} = \sqrt{41}$

For a triangle using the converse of the Pythagorean Theorem it must satisfy the condition $BC^2 = AB^2 + AC^2$

$(\sqrt{41})^2 + (\sqrt{41})^2 = (\sqrt{82})^2$

$82 = 82$

Hence, the result is proved.

(b) For a triangle to be right angle triangle, the corresponding sides must be perpendicular.

In order to find this use slope formula $m = \frac{y_2-y_1}{x_2-x_1}$

For side AB:

$m = \frac{-3-(-7)}{11-6} = \frac{4}{5}$

For side BC:

$m = \frac{-2-(-3)}{2-11} = -\frac{1}{9}$

For side CA:

$m = \frac{-2-(-7)}{2-6} = -\frac{5}{4}$

Thus, product of slopes of AB and AC is -1 this implies that the corresponding sides are perpendicular.

Hence, the triangle is a right triangle.

(c) Area of a triangle= $\frac{1}{2}(AB)(BC)$

$= \frac{1}{2}(\sqrt{41})(\sqrt{41})$

Hence, Area=20.5 square units

13. We are given the square with vertices $A(-2,9)$, $B(4,6)$ and $C(1,0)$, $D(-5,3)$.

Consider AB, BC, CD, and AD are four sides of a square.

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$m = \frac{y_2-y_1}{x_2-x_1}$

For side AB:

$$\begin{aligned} AB &= \sqrt{(4-(-2))^2+(6-9)^2} \\ &= \sqrt{36+9} \\ &= \sqrt{45} \end{aligned}$$

$m = \frac{6-9}{4-(-2)} = -\frac{1}{2}$

For side BC:

$$\begin{aligned} BC &= \sqrt{(1-4)^2+(0-6)^2} \\ &= \sqrt{9+36} \\ &= \sqrt{45} \end{aligned}$$

$m = \frac{0-6}{1-4} = 2$

For side CD:

$$\begin{aligned} CD &= \sqrt{(-5-1)^2 + (3-0)^2} \\ &= \sqrt{36+9} \\ &= \sqrt{45} \end{aligned}$$

$m = \frac{3-0}{-5-1} = -\frac{1}{2}$

For side AD:

$AD = \sqrt{(-5-(-2))^2 + (3-9)^2} = \sqrt{9+36} = \sqrt{45}$

$m = \frac{3-9}{-5-(-2)} = 2$

All sides are equal, and their consecutive sides are perpendicular, having negative reciprocal slopes, so the given points are vertices of a square.

Hence, the points $(-2, 9)$, $(4, 6)$ and $(1, 0)$, $(-5, 3)$ are vertices of a square.

14. $a)|AB| = \sqrt{(}3+1)^2 + (11-3)^2 = \sqrt{8}0 = \sqrt{(}16 \times 5) = 4\sqrt{5}$

    $|BC| = \sqrt{(}5-3)^2 + (15-11)^2 = \sqrt{2}0 = \sqrt{(}4 \times 5) = 2\sqrt{5}$

    $|AC| = \sqrt{(}5+1)^2 + (15-3)^2 = \sqrt{1}80 = \sqrt{(}36 \times 5) = 6\sqrt{5}$

    $|AB| + |BC| = |AC|$

    $4\sqrt{5} + 2\sqrt{5} = 6\sqrt{5}$

    $6\sqrt{5} = 6\sqrt{5}$

    $b) slope AB = m_1 = \frac{11-3}{3+1} = 2$

    $slope BC = m_2 = \frac{15-11}{5-3} = 2$

    $slope AC = m_3 = \frac{15-3}{5+1} = 2$

    $m_1 = m_2 = m_3$

15. If ABCD is a Parallelogram, than:

    1. AB // CD
    2. AD // BC
    3. d(AB) = d(CD)
    4. d(AD) = d(BC)

    1.

    AB // CD if m(AB) = m(CD)

    m(AB) = $\frac{y_b - y_a}{x_b - x_a}$

    m(AB) = $\frac{3}{6}$

    m(CD) = $\frac{y_d - y_c}{x_d - x_c}$

    m(CD) = $\frac{3}{6}$

    m(AB) = m(CD)

    2.

    AD // BC if m(AD) = m(BC)

    m(AD) = $\frac{y_d - y_a}{x_d - x_a}$

    m(AD) = $\frac{-2}{6}$

    m(BC) = $\frac{y_c - y_b}{x_c - x_b}$

    m(BC) = $\frac{-2}{6}$

    m(AD) = m(BC)

    3.

    d(AB) = d(CD)

    d(AB) = $\sqrt{(x_b - x_a)^2 + (y_b - y_a)^2}$

    $\sqrt{36+9}$= $\sqrt{45}$

    d(CD) = $\sqrt{(x_d - x_c)^2 + (y_d - y_c)^2}$

    $\sqrt{36+9}$= $\sqrt{45}$

    d(AB) = d(CD)

    4.

    d(AD) = d(BC)

    d(AD) = $\sqrt{(x_d - x_a)^2 + (y_d - y_a)^2}$

    $\sqrt{36+4}$= $\sqrt{40}$

    d(BC) = $\sqrt{(x_c - x_b)^2 + (y_c - y_b)^2}$

    $\sqrt{36+4}$= $\sqrt{40}$

    d(AD) = d(BC)

    Thus, all conditions are met and ABCD is proved of being a parallelogram.

16. 1. d(AB) = d(CD)

2. d(DA) = d(BC)

3. AB $\perp$ DA

1.

d(AB) = d(CD)

d(AB) $= \sqrt{(x_b - x_a) + (y_b - y_b)}$

d(AB) $= \sqrt{10^2 + 2^2}$

d(AB) $= \sqrt{104}$

d(CD) $= \sqrt{(x_d - x_c) + (y_d - y_c)}$

d(CD) $= \sqrt{(-10)^2 + (-2)^2}$

d(CD) $= \sqrt{104}$

d(AB) = d(CD)

2.

d(DA) = d(BC)

d(DA) $= \sqrt{(x_d - x_a) + (y_d - y_a)}$

d(DA) $= \sqrt{1^2 + (-5)^2}$

d(DA) $= \sqrt{26}$

d(BC) $= \sqrt{(x_c - x_b) + (y_c - y_b)}$

d(BC) $= \sqrt{1^2 + (-5)^2}$

d(BC) $= \sqrt{26}$

d(DA) = d(BC)

3.

AB $\perp$ DA

$m_{AB} * m_{DA} =$ -1

$m_{AB} = \frac{y_b - y_a}{x_b - x_a}$

$m_{AB} = \frac{2}{10}$

$m_{DA} = \frac{y_d - y_a}{x_d - x_a}$

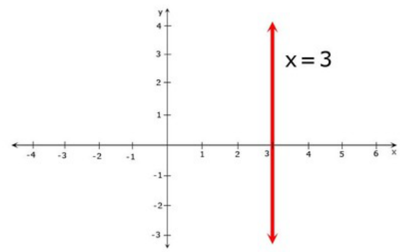
$m_{DA} = \frac{-5}{1}$

$\frac{2}{10} * \frac{-5}{1} =$ -1

Thus, hence all the conditions are proven, ABCD is a rectangle

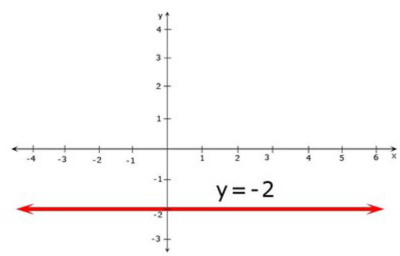
17. The graph of the equation $x = 3$ is a vertical line that passes through the point $(3, 0)$

We can see the graph of the equation $x = 3$ below.



18. The graph of the equation $y = -2$ is a horizontal line that passes through the point $(0, -2)$

We can see the graph of the equation $y = -2$ below.

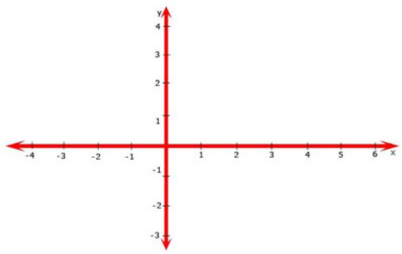


19. If $xy = 0$, then $x = 0$ or $y = 0$

The graph of the equation $xy = 0$ is a horizontal line that passes through the point $(0, 0)$ and a vertical line that passes through the point $(0, 0)$

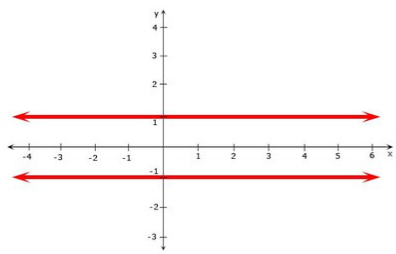
Note that a horizontal line that passes through the point $(0, 0)$ is the graph of the equation $y = 0$ and a vertical line that passes through the point $(0, 0)$ is the graph of the equation $x = 0$

We can see the graph of the equation $xy = 0$ below.



20. The graph of the equation $|y| = 1$ is a horizontal line that passes through the point $(0, -1)$ and another horizontal line that passes through the point $(0, 1)$

    We can see the graph of the equation $|y| = 1$ below.



21. To find an equation of a line with slope $m = 6$ that passes through $(2, -3)$,

    we use the formula $y = mx + b$.

    We plug in the values of m, y, and x to solve for b: $-3 = 6(2) + b$.

    Solve for b: $b = -15$.

    Plug values of m and b into equation to get the equation of line: $y = 6x - 15$.

22. To find an equation of a line with slope $m = -3$ that passes through $(-1, 4)$,

    we use the formula $y = mx + b$.

    We plug in the values of m, y, and x to solve for b: $4 = -3(-1) + b$.

    Solve for b:

    $b + 3 = 4$

    $b = 1$.

    Plug the values of m and b into the formula to get the equation of line: $y = -3x + 1$.

23. To find an equation of a line with slope $m = \frac{2}{3}$ that passes through $(1, 7)$,

    we use the formula $y = mx + b$.

    We plug in the values of m, y, and x to solve for b: $7 = \frac{2}{3}(1) + b$.

    Solve for b:

    $b + \frac{2}{3} = 7$

    $b = \frac{19}{3}$.

    Plug values of m and b into the formula to get the equation of the line: $y = \frac{2}{3}x + \frac{19}{3}$.

    Multiply by 3 on both sides to simplify: $3y = 2x + 19$.

24. To find an equation of a line with slope $m = -\frac{7}{2}$ that passes through $(-3, -5)$,

    we use formula $y = mx + b$.

    We plug in the values of m, y, and x to solve for b: $-5 = -\frac{7}{2}(-3) + b$.

    Solve for b:

    $b + \frac{21}{2} = -5$

    $b = -\frac{31}{2}$.

    Plug the values of m and b into the formula to get the equation of the line: $y = -\frac{7}{2}x - \frac{31}{2}$.

    Multiply by 2 on both sides to simplify: $2y = -7x - 31$.

25. First, we find the slope.

    To find the slope of the line passing through $(2, 1)$ and $(1, 6)$,

    we use the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

    Plug in the given values of x and y: $m = \frac{6-1}{1-2}$.

    Simplify to arrive at the slope: $m = -5$.

    We plug in the slope and the values of x and y into the equation $y = mx + b$:

    $1 = -5(2) + b$.

Solve for b:

$1 = b - 10$

$b = 11.$

Plug in the values of m and b into the formula to find the equation of the line:

$y = -5x + 11.$

26. First, we find the slope.

    To find the slope of the line passing through $(-1, -2)$ and $(4, 3)$,

    we use the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

    Plug in the given values of x and y: $m = \frac{3-(-2)}{4-(-1)}$.

    Simplify to arrive at the slope: $m = 1$.

    We plug in the slope and the values of x and y into the equation $y = mx + b$:

    $3 = 1(4) + b.$

    Solve for b: $3 = b + 4$

    $b = -1.$

    Plug in the values of m and b into the formula to find the equation of the line:

    $y = x - 1.$

27. Since we are given values for the slope (m) and y-intercept (b), we can plug these values into the equation $y = mx + b$.

    $m = 3, b = -2$. So

    $y = 3x - 2$

28. Since we are given values for the slope (m) and y-intercept (b), we can plug these values into the equation $y = mx + b$.

    $m = \frac{2}{5}, b = 4$. So

    $y = \frac{2}{5}x + 4$

29. We are told to find the equation of the line that satisfies the following conditions:

    The x-intercept is 1 and the y-intecept is -3

    To find the equation of the line we must find the slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

    and then use point slope form: $y`y_1 = m(x`x_1)$

    It may not be obvious at first, but the intercepts given are actually the points: $(1, 0)$ and $(0, -3)$

    $m = \frac{-3-0}{0-1} = \frac{-3}{-1} = 3$

    $y - (-3) = 3(x)$

    $y = -5x + 10 + 1$

    $y = 3x - 3$

    or

    $y = mx + b$

    $m = 3$

    $b = -3$

    $y = 3x - 3$

30. We are told to find the equation of the line that satisfies the following conditions:

    The x-intercept is -8 and the y-intecept is 6

    To find the equation of the line we must find the slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

    and then use point slope form: $y`y_1 = m(x`x_1)$

    It may not be obvious at first, but the intercepts given are actually the points: $(-8, 0)$ and $(0, 6)$

    $m = \frac{6-0}{0-(-8)} = \frac{6}{8} = \frac{3}{4}$

    $y - 6 = \frac{3}{4}x$

    $y = \frac{3}{4}x + 6$

    or

    $y = mx + b$

$m = \frac{3}{4}$

$b = 6$

$y = \frac{3}{4}x + 6$

31. Since it is parallel to the x axis, the line has a slope of 0.

    The line is a horizontal line with y-intercept 5, so we can represent it with the equation $y = 5$.

32. Since it is parallel to the y-axis, the line has a constant x value of 4. Therefore, we can represent the function by the equation $x = 4$.

33. We are told to find the equation of the line that satisfies the following conditions:

    Passes through the point $(1, -6)$, and parallel to the line $x + 2y = 6$

    To find the equation of the line we must find the slope and then use point slope form: $y \grave{} y_1 = m(x \grave{} x_1)$

    First solve the given equation for y:

    $x + 2y = 6$

    $2y = -x + 6$

    $y = -\frac{1}{2}x + 3$

    Since the lines are parallel they have the same slope, so the slope of our equation is $m = -\frac{1}{2}$

    $y \grave{} (-6) = -\frac{1}{2}(x \grave{} 1)$

    $y = -\frac{1}{2}x + \frac{1}{2} - 6$

    $y = -\frac{1}{2}x - \frac{11}{2}$

34. We are told to find the equation of the line that satisfies the following conditions:

    y-intercept is 6, and parallel to the line $2x + 3y + 4 = 0$

    To find the equation of the line we must find the slope

    then write it in the form:$y = mx + b$

    First solve the given equation for y:

    $2x + 3y + 4 = 0$

    $3y = -2x - 4$

    $y = -\frac{2}{3}x - \frac{4}{3}$

    Since the lines are parallel they have the same slope, so the slope of our equation is $m = -\frac{2}{3}$

    $b = 6$

    $y = -\frac{2}{3}x + 6$

35. We are told to find the equation of the line that satisfies the following conditions:

    Passes through the point (-1,-2), and perpendicular to the line $2x + 5y + 8 = 0$

    To find the equation of the line we must find the slope

    then write it in the form:$y = mx + b$

    First solve the given equation for y:

    $2x + 5y + 8 = 0$

    $5y = -2x - 8$

    $y = -\frac{2}{5}x - \frac{8}{5}$

    Since the lines are perpendicular the slope is the negative reciprocal, so the slope of our equation is $m = \frac{5}{2}$

    $y - (-2) = \frac{5}{2}(x - (-1)$

    $y = \frac{5}{2}x + \frac{5}{2} - 2$

    $y = \frac{5}{2}x + \frac{1}{2}$

36. We are told to find the equation of the line that satisfies the following conditions:

    Passes through the point $(\frac{1}{2}, -\frac{2}{3})$, and perpendicular to the line $4x - 8y = 1$

    To find the equation of the line we must find the slope

    then write it in the form:$y = mx + b$

    First solve the given equation for y:

$4x - 8y = 1$

$-8y = -4x + 1$

$y = \frac{1}{2}x - \frac{1}{8}$

Since the lines are perpendicular the slope is the negative reciprocal, so the slope of our equation is $m = -2$

$y - (-\frac{2}{3}) = -2(x - \frac{1}{2})$

$y = -2x + 1 - \frac{2}{3}$

$y = -2x + \frac{1}{3}$

37. The slope-intercept form of the equation of a line is $y = mx + b$ where m is the slope and b is the y-intercept.

$x + 3y = 0$

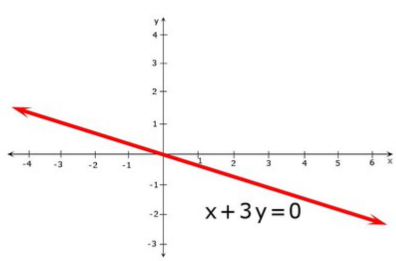
$3y = -x$

$y = -\frac{1}{3}x + 0$

The slope is $-\frac{1}{3}$

The y-intercept is 0

We can see the graph of the line $x + 3y = 0$:



38. The slope-intercept form of the equation of a line is $y = mx + b$ where m is the slope and b is the y-intercept.

$2x - 5y = 0$

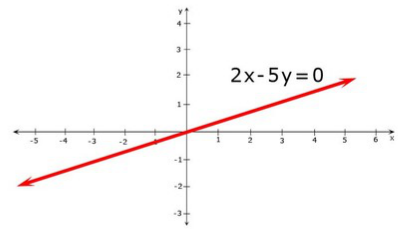
$-5y = -2x$

$y = \frac{2}{5}x + 0$

The slope is $\frac{2}{5}$

The y-intercept is 0

We can see the graph of the line $2x - 5y = 0$:



39. Rewrite it in the slope-intercept form:

$y = mx + b$

The given becomes :

$y = -2$

Answer:

$m = 0$

$b = -2$

40. Rewrite it in the slope-intercept form:

$y = mx + b$

$y = 2/3x + 2$

Answer:

$m = 2/3$

$b = 2$

41. Rewrite it in the slope-intercept form:

$y = mx + b$

$y = 3/4x - 3$

Answer:

$m = 3/4$

$b = -3$

42. Rewrite it in the slope-intercept form:

$y = mx + b$

$y = -4/5x + 2$

$m = -4/5$

$b = 2$

43. Compare the inequality with the equation

    x=0

    This is a vertical line through the point (0,0).

    Because we have a ”less than” symbol, the shaded region will be to the left of the line.

44. Compare the inequality with the equation

    y=0

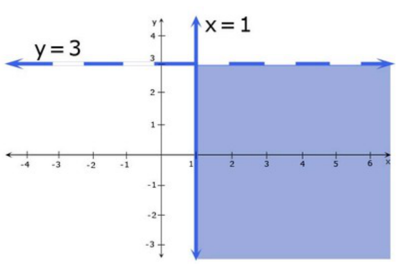
    This is a horizontal line through (0,0).

    Because we have a $>$ symbol, the shaded region will be above the line.

45. The product of x and y is negative. This implies that x is negative and y is positive, or vice versa. Thus the shaded region will be in quadrants II and IV.

46. $\{(x, y)|\ x \geq 1\ and\ y < 3\ \}$

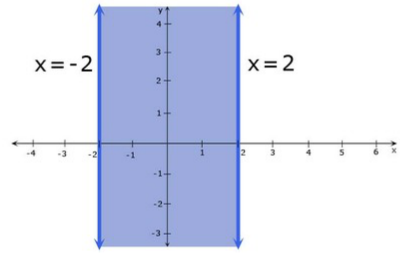
    The region includes the points which are to the right of the vertical line $x = 1$, (as well as the points on this line), and below the horizontal line $y = 3$, (but not including the points on this horizontal line).



47. $\{(x, y)|\ |x| \leq 2\ \}$

    The region includes all the points where $-2 \leq x \leq 2$

    Thus, the region includes all the points between the vertical lines $x = -2$ and $x = 2$, as well as the points on these vertical lines.

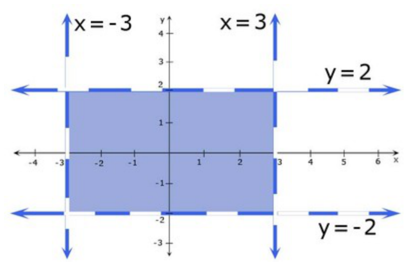


48. $\{(x, y)|\ |x| < 3\ and\ |y| < 2\ \}$

    The region includes all the points where $-3 < x < 3$ and $-2 < y < 2$

    Thus, the region includes all the points which are between the vertical lines $x = -3$ and $x = 3$, and between the horizontal lines $y = -2$ and $y = 2$.

    Note that the points on these four lines are not included in the region.

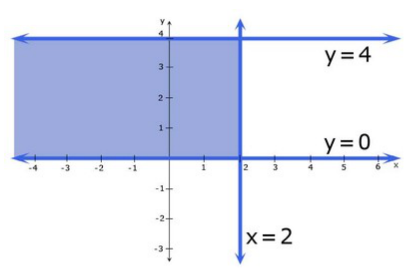


49. $\{(x, y)|\ 0 \leq y \leq 4\ and\ x \leq 2\ \}$

    The region includes all the points where $0 \leq y \leq 4$ and $x \leq 2$

    Thus, the region includes all the points which are between the horizontal lines $y = 0$ and $y = 4$, and to the left of the vertical line $x = 2$

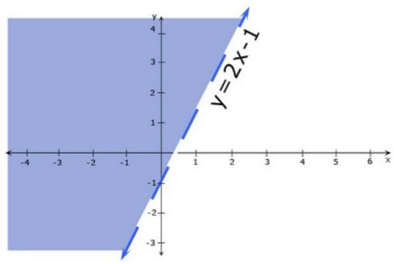
    Note that the points on these three lines are also included in the region.



50. $\{(x, y)|\ y > 2x - 1\ \}$

The region includes all the points that are above the line $y = 2x - 1$

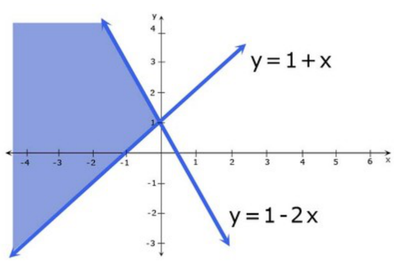
Note that the points on this line are not included in the region.



51. $\{(x, y)|\ 1 + x \leq y \leq 1 - 2x\ \}$

The region includes all the points that are above the line $y = 1 + x$ and below the line $y = 1 - 2x$

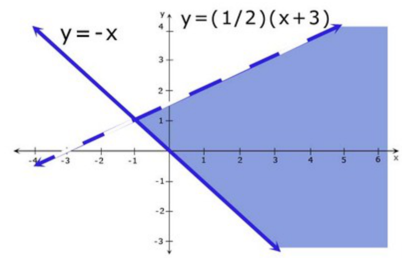
Note that the points on these two lines are also included in the region.



52. $\{(x, y)|\ -x \leq y < \frac{1}{2}(x + 3)\ \}$

The region includes all the points that are above the line $y = -x$ and below the line $y = \frac{1}{2}(x + 3)$

Note that the points on the line $y = -x$ are also included in the region, but the points on the line $y = \frac{1}{2}(x + 3)$ are not included in the region.



53. Let the point on the y-axis be $(0, y)$

We can find an expression for the distance from $(0, y)$ to the point $(1, 1)$:

$d_1 = \sqrt{(0-1)^2 + (y-1)^2}$

$d_1 = \sqrt{1 + (y^2 - 2y + 1)}$

$d_1 = \sqrt{y^2 - 2y + 2}$

We can find an expression for the distance from $(0, y)$ to the point $(5, -5)$:

$d_2 = \sqrt{(0-5)^2 + (y-(-5))^2}$

$d_2 = \sqrt{25 + (y^2 + 10y + 25)}$

$d_2 = \sqrt{y^2 + 10y + 50}$

We can equate the two distances to find y:

$d_1 = d_2$

$\sqrt{y^2 - 2y + 2} = \sqrt{y^2 + 10y + 50}$

$y^2 - 2y + 2 = y^2 + 10y + 50$

$-2y + 2 = 10y + 50$

$12y = -48$

$y = -4$

The point on the y-axis is $(0, -4)$

54. The midpoint formula states that for points $P_1(x_1; y_1)$ and $P_2(x_2; y_2)$, the midpoint of a segment XY has the coordinates: $(\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2})$

55. (a) We can find the midpoint of the line segment joining the two points $(1, 3)$ and $(7, 15)$:

$x = \frac{1+7}{2} = 4$

$y = \frac{3+15}{2} = 9$

The midpoint is $(4, 9)$

(b) We can find the midpoint of the line segment joining the two points $(-1, 6)$ and $(8, -12)$:

$x = \frac{-1+8}{2} = \frac{7}{2}$

$y = \frac{6+(-12)}{2} = -3$

The midpoint is $(\frac{7}{2}, -3)$

56. M(AB) = $(\frac{3+1}{2}; \frac{6+0}{2})$ = (2;3) = E

$$
\begin{aligned}
d(EC) &= \sqrt{(8-2)^2 + (2-3)^2} \\
&= \sqrt{6^2 + 1^2} \\
&= \sqrt{37}
\end{aligned}
$$

M(BC) = $(\frac{3+8}{2}; \frac{2+6}{2}) = (\frac{11}{2}; 4)$ = F

$$
\begin{aligned}
d(FA) &= \sqrt{(1 - \frac{11}{2})^2 + (0-4)^2} \\
&= \sqrt{(-\frac{9}{2})^2 + (-4)^2} \\
&= \frac{\sqrt{145}}{2}
\end{aligned}
$$

M(AC) = $(\frac{1+8}{2}; \frac{0+2}{2}) = (\frac{9}{2}; 1)$ = G

$$
\begin{aligned}
d(GB) &= \sqrt{(3 - \frac{9}{2})^2 + (6-1)^2} \\
&= \sqrt{(-\frac{3}{2})^2 + (5)^2} \\
&= \frac{\sqrt{109}}{2}
\end{aligned}
$$

57. The slope-intercept form of the equation of a line is $y = mx + b$ where m is the slope and b is the y-intercept.

We can find the slope of the line $2x - y = 4$:

$2x - y = 4$

$y = 2x - 4$

The slope of the line is 2

We can find the slope of the line $6x - 2y = 10$:

$6x - 2y = 10$

$2y = 6x - 10$

$y = 3x - 5$

The slope of the line is 3

Parallel lines have equal slopes. Since these two lines do not have equal slopes, they are not parallel.

We can find the x-coordinate of the point of intersection:

$2x - 4 = 3x - 5$

$x = 1$

We can find the y-coordinate of the point of intersection:

$y = 2x - 4$

$y = 2(1) - 4$

$y = -2$

The point of intersection is $(1, -2)$

58. The slope-intercept form of the equation of a line is $y = mx + b$ where m is the slope and b is the y-intercept.

We can find the slope of the line $3x - 5y + 19 = 0$:

$3x - 5y + 19 = 0$

$5y = 3x + 19$

$y = \frac{3}{5}x + \frac{19}{5}$

The slope of the line is $\frac{3}{5}$

We can find the slope of the line $10x + 6y - 50 = 0$:

$10x + 6y - 50 = 0$

$6y = -10x + 50$

$y = -\frac{5}{3}x + \frac{25}{3}$

The slope of the line is $-\frac{5}{3}$

We can find the product of the slopes of the two lines:

$(\frac{3}{5})(-\frac{5}{3}) = -1$

If the product of the slopes of two lines is -1, then the lines are perpendicular. Therefore, these two lines are perpendicular.

We can find the x-coordinate of the point of intersection:

$\frac{3}{5}x + \frac{19}{5} = -\frac{5}{3}x + \frac{25}{3}$

$9x + 57 = -25x + 125$

$34x = 68$

$x = \frac{68}{34}$

$x = 2$

We can find the y-coordinate of the point of intersection:

$y = \frac{3}{5}x + \frac{19}{5}$

$y = (\frac{3}{5})(2) + \frac{19}{5}$

$y = \frac{6}{5} + \frac{19}{5}$

$y = \frac{25}{5}$

$y = 5$

The point of intersection is $(2, 5)$

59. We can find the midpoint of the line segment joining the two points:

$x = \frac{1+7}{2} = 4$

$y = \frac{4+(-2)}{2} = 1$

The midpoint is $(4, 1)$

We can find the slope of the line segment joining the two points:

$m = \frac{4-(-2)}{1-7} = \frac{6}{-6} = -1$

The slope of the perpendicular bisector is $(-\frac{1}{m}) = 1$

Note that the perpendicular bisector passes through the point $(4, 1)$

We can find an equation of the perpendicular bisector:

$y - 1 = (1)(x - 4)$

$y = x - 4 + 1$

$y = x - 3$

60. This answer hasn't been written yet!

61. (a) Let the equation of a straight line be $y = mx + b$

Suppose the x-intercept is a.

$y = mx + b$

$0 = m(a) + b$

$ma = -b$

$m = -\frac{b}{a}$

Then:

$y = mx + b$

$y = (-\frac{b}{a})x + b$

$y - b = -\frac{bx}{a}$

$-\frac{y}{b} + 1 = \frac{x}{a}$

$\frac{x}{a} + \frac{y}{b} = 1$

(b) $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{6} + \frac{y}{(-8)} = 1$

$4x - 3y = 24$

$4x - 3y - 24 = 0$

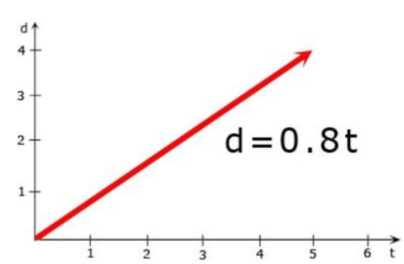
62. (a) $\frac{distance}{time} = \frac{40\ mi}{50\ min} = 0.8\ mi/min$

We can write an equation for the distance d in terms of the time t:

$d = 0.8\ t$

(b) We can see the graph below.

(c) The slope of the line is $0.8\ mi/min$ and this represents the speed of the car.



第三章 Appendix C: Graphs of Second-Degree Equations

3.1 Answer

1. $(x-3)^2+(y+1)^2=25$

2. The equation for the circle that has Center (-2,-8) and radius 10 is:

   $(x+2)^2+(y+8)^2=100$

3. $x^2+y^2=65$

4. $(x+1)^2+(y-5)^2=130$

5. center $(h,k)=(2,-5)$ and $r=4$

6. center $(h,k)=(0,-3)$ and $r=\sqrt{7}$

7. $(h,k)=(-\frac{1}{2},0)$ and $r=\frac{1}{2}$

8. center,$(h,k)=(-\frac{1}{4},-1)$ and $r=1$

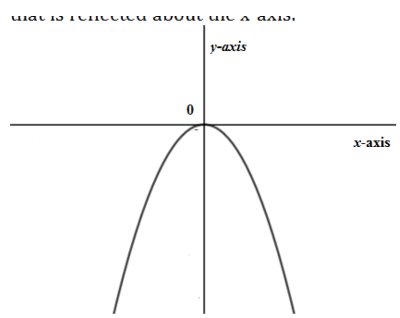
9. center,$(h,k)=(\frac{1}{4},-\frac{1}{4})$ and $r=(\frac{\sqrt{10}}{4})$

10. As long as $(\frac{a^2}{4}+\frac{b^2}{4}-c)>0$, then this is the equation of a circle.

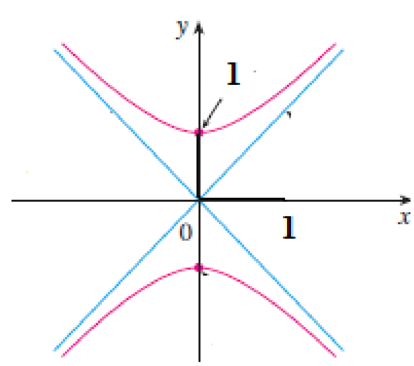
    The center of the circle is $(-\frac{a}{2},-\frac{b}{2})$

    The radius is $\sqrt{\frac{a^2}{4}+\frac{b^2}{4}-c}$

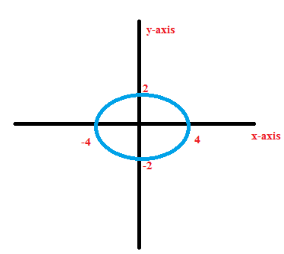
11. Parabola



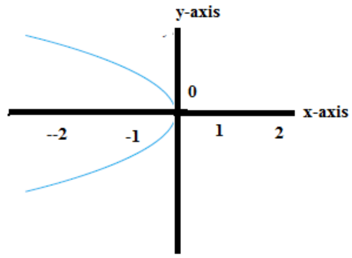
12. Hyperbola



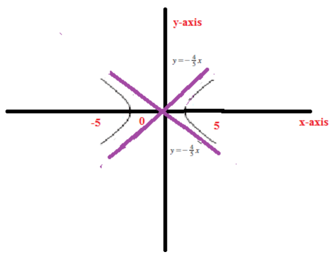
13. Ellipse



14. Parabola

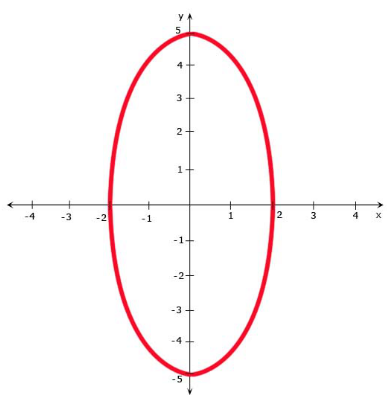


15. Hyperbola

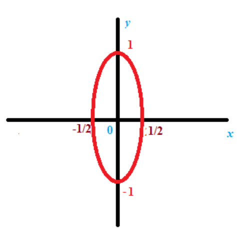


16. $\frac{x^2}{4} + \frac{y^2}{25} = 1$

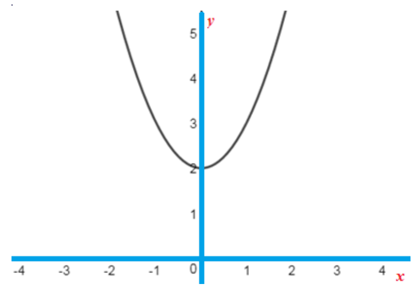
This is an equation of an ellipse.



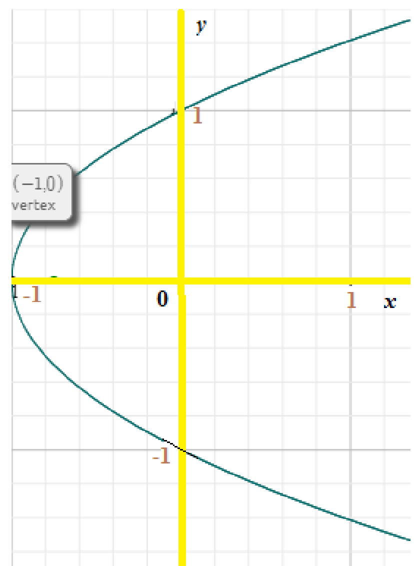
17. Ellipse



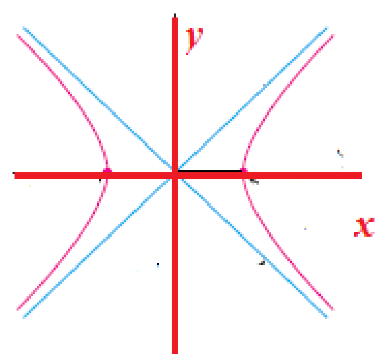
18. Parabola



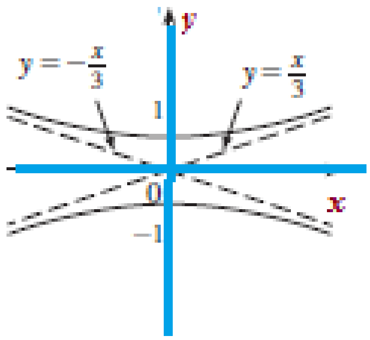
19. parabola



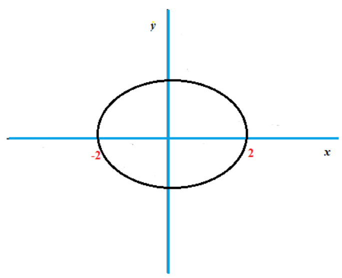
20. Hyperbola



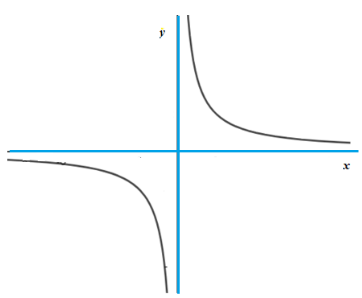
21. Hyperbola



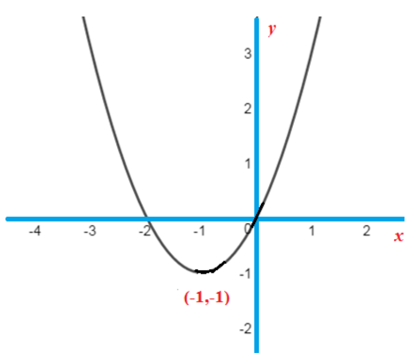
22. Ellipse



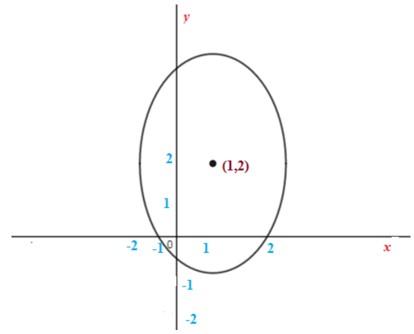
23. Hyperbola



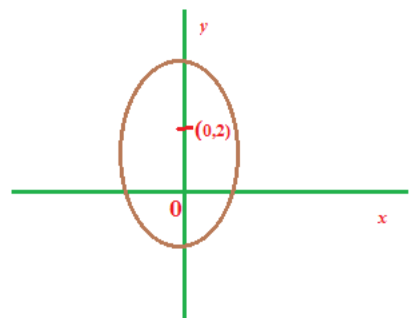
24. parabola



25. Ellipse



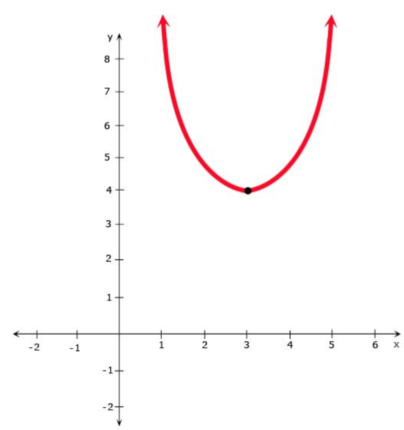
26. Ellipse



27. $y = x^2 - 6x + 13$

$y = (x-3)^2 + 4$

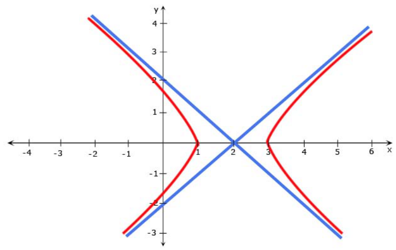
This is an equation of a parabola with vertex $(3, 4)$



28. $x^2 - y^2 - 4x + 3 = 0$

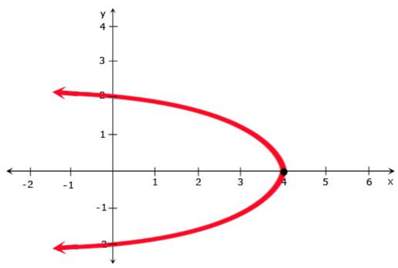
$(x-2)^2 - y^2 = 1$

This is the equation of a hyperbola, where $a = 1, b = 1, h = 2$, and $k = 0$



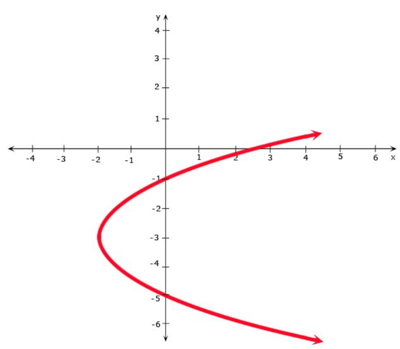
29. $x = 4 - y^2$

    This is an equation of a parabola with vertex $(4, 0)$

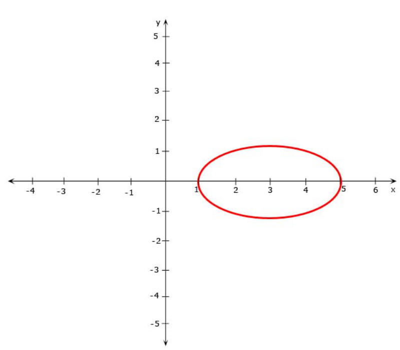


30. $x = \frac{1}{2}(y+3)^2 - 2$

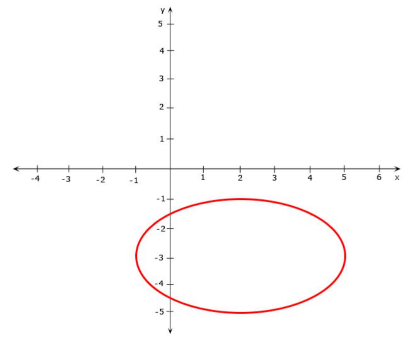
    This is the equation of a parabola with vertex $(-2, -3)$



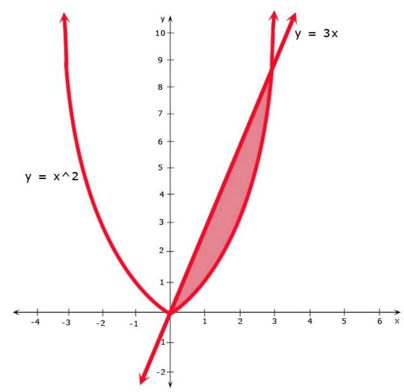
31. $\frac{(x-3)^2}{4} + y^2 = 1$



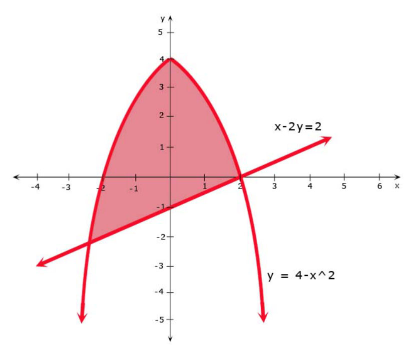
32. $\frac{(x-2)^2}{9} + \frac{(y+3)^2}{4} = 1$



33. We can sketch the region bounded by the line $y = 3x$ and the parabola $y = x^2$



34. We can sketch the region bounded by the line $x - 2y = 2$ and the parabola $y = 4 - x^2$

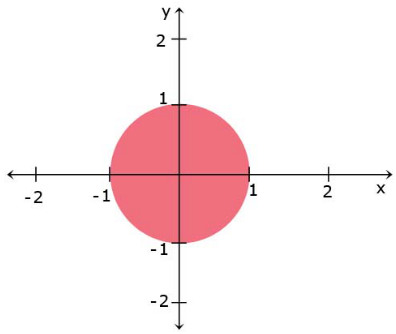


35. The equation of the parabola is $y = (x-1)^2 - 1$

36. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

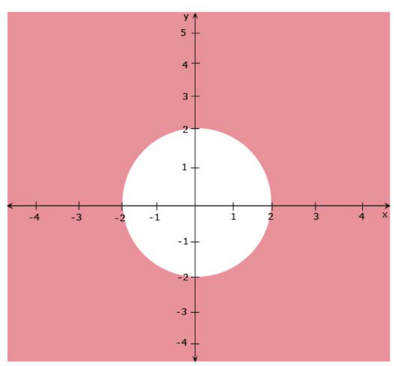
37. The set includes all the points inside the circle $x^2 + y^2 = 1$

Note that the points on the circle are also included in the set.



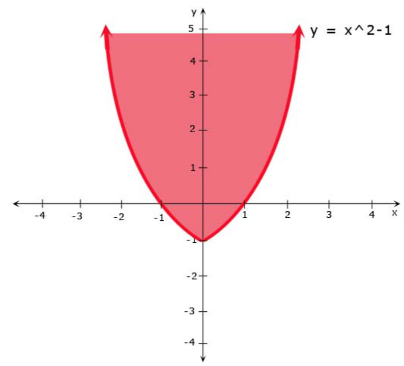
38. The set includes all the points outside the circle $x^2 + y^2 = 4$

    Note that the points on the circle are not included in the set.



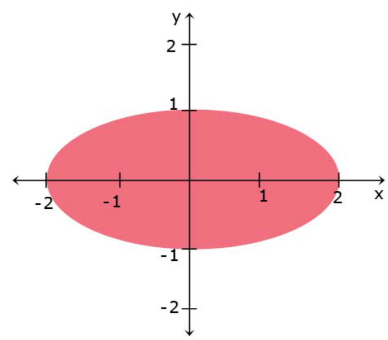
39. The set includes all the points above the parabola $y = x^2 - 1$

    Note that the points on the parabola are also included in the set.



40. The set includes all the points inside the ellipse $\frac{x^2}{4} + y^2 = 1$

    Note that the points on the ellipse are also included in the set.



3.2 Step by Step

1. knowing the equation of the circle:

   $(x-h)^2 + (y-k)^2 = r^2$

   we say that:

   $h = 3$

   $k = -1$

   $r = 5$

   then

   $(x-3)^2 + (y+1)^2 = 5^2$

   $(x-3)^2 + (y+1)^2 = 25$

2. Having the general equation of the circle we have:

   $(x-h)^2 + (y-k)^2 = r^2$

   We know that h is the displacement in the x axis and k is the displacement in the y axis. Then the equation for the circle that has Center (-2,-8) and radius 10 is:

   $(x+2)^2 + (y+8)^2 = 100$

3. An equation of the circle with center (h, k) and

radius r is given as:

$(x-h)^2+(y-k)^2=r^2$

Given: center $(h,k)=(0,0)$ and $(x,y)=(4,7)$

Then

$(4-0)^2+(7-0)^2=r^2$

$r^2=65$

Hence, $x^2+y^2=65$

4. An equation of the circle with center (h,k) and

   radius r is given as:

   $(x-h)^2+(y-k)^2=r^2$

   Given: center $(h,k)=(-1,5)$ and $(x,y)=(-4,-6)$

   Then

   $(-4+1)^2+(-6-5)^2=r^2$

   $r^2=130$

   Hence, $(x+1)^2+(y-5)^2=130$

5. An equation of the circle with center (h,k) and

   radius r is given as:

   $(x-h)^2+(y-k)^2=r^2$ ...(1)

   Given: $x^2+y^2-4x+10y+13=0$

   The above equation can be written in the standard equation of the circle as follows:

   $(x-2)^2+(y-(-5))^2=4^2$

   Compare it with equation (1) to obtain:

   $(h,k)=(2,-5)$ and $r=4$

6. An equation of the circle with center (h,k) and

   radius r is given as:

   $(x-h)^2+(y-k)^2=r^2$ ...(1)

   Given: $x^2+y^2+6y+2=0$

   The above equation can be written in the standard equation of the circle as follows:

   $(x-0)^2+(y-(-3))^2=(\sqrt{7})^2$

   Compare it with equation (1) to have

   Hence $(h,k)=(0,-3)$ and $r=\sqrt{7}$

7. An equation of the circle with center (h,k) and

   radius r is given as:

   $(x-h)^2+(y-k)^2=r^2$ ...(1)

   Given: $x^2+y^2+x=0$

   The above equation can be written in the standard equation of the circle as follows:

   $(x-(-\frac{1}{2}))^2+(y-0)^2=(\frac{1}{2})^2$

   Hence, $(h,k)=(-\frac{1}{2},0)$ and $r=\frac{1}{2}$.

8. An equation of the circle with center (h,k) and

   radius r is given as:

   $(x-h)^2+(y-k)^2=r^2$ ...(1)

   Given: $16x^2+16y^2+8x+32y+1=0$

   The above equation can be written in the standard equation of the circle as follows:

   $(x-(-\frac{1}{4}))^2+(y-(-1))^2=1^2$

   Hence, $(h,k)=(-\frac{1}{4},-1)$ and $r=1$

9. An equation of the circle with center (h,k) and

   radius r is given as:

   $(x-h)^2+(y-k)^2=r^2$ ...(1)

   Given: $2x^2+2y^2-x+y=0$

   The above equation can be written in the standard equation of the circle as follows:

   $(x-\frac{1}{4})^2+(y-(-\frac{1}{4}))^2=(\frac{\sqrt{10}}{4})^2$

   Hence $(h,k)=(\frac{1}{4},-\frac{1}{4})$ and $r=(\frac{\sqrt{10}}{4})$

10. We can write the general equation for a circle:

    $(x-u)^2 + (y-v)^2 = r^2$

    where (u, v) is the center of the circle and r is the radius

    We can rearrange the equation given in the question:

    $x^2 + y^2 + ax + by + c = 0$

    $(x^2 + ax) + (y^2 + by) + c = 0$

    $(x + \frac{a}{2})^2 - \frac{a^2}{4} + (y + \frac{b}{2})^2 - \frac{b^2}{4} + c = 0$

    $(x + \frac{a}{2})^2 + (y + \frac{b}{2})^2 = \frac{a^2}{4} + \frac{b^2}{4} - c$

    As long as $(\frac{a^2}{4} + \frac{b^2}{4} - c) > 0$, then this is the equation of a circle.

    The center of the circle is $(-\frac{a}{2}, -\frac{b}{2})$

    The radius is $\sqrt{\frac{a^2}{4} + \frac{b^2}{4} - c}$

11. The vertex, the point where the parabola changes direction, is the origin. We see that the parabola $y = ax^2$opens upward if $a > 0$ and downward if $a < 0$.

    Given: $y = -x^2$

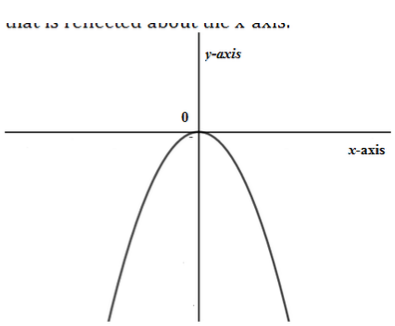
    Here, $a = -1$

    This represents the equation of a parabola which has the standard form as follows:

    $y = a(x-h)^2 + k$

    vertex, $(h, k) = (0, 0)$

    The graph for $y = -x^2$ will be a parabola acting downwards.

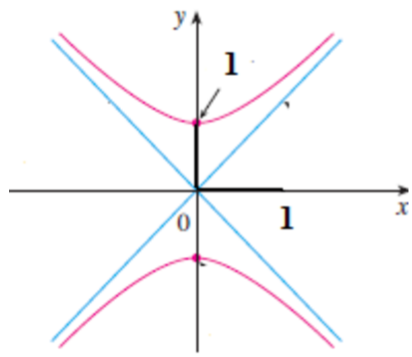


12. Given: $y^2 - x^2 = 1$

    Divide both sides by 1

    $\frac{y^2}{1} - \frac{x^2}{1} = 1$

    Above is the standard form of the equation of a hyperbola.

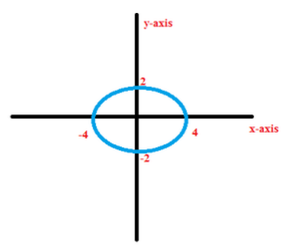


13. Given: $x^2 + 4y^2 = 16$

    Divide both sides by 16: $\frac{x^2}{16} + \frac{y^2}{16} = 1$

    $\frac{x^2}{4^2} + \frac{y^2}{2^2} = 1$

    which is the standard form of the equation of an ellipse.



14. The vertex, the point where the parabola changes direction, is the origin. We see that the parabola $x = ay^2$opens to the right side if $a > 0$ and opens to the left side if $a < 0$.

    Given: $x = -2y^2$

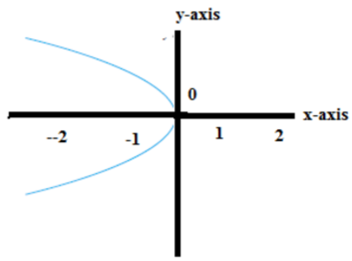
    Here, $a = -2$

    This represents the equation of a parabola which has the standard form as follows:

    $y = a(x-h)^2 + k$

vertex, $(h, k) = (0, 0)$

The graph will be a parabola that opens to the left side as depicted below:



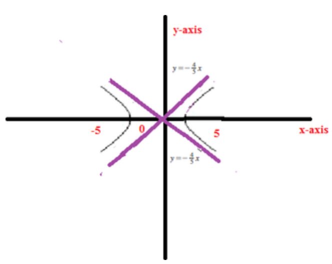
15. Given: $16x^2 - 25y^2 = 400$

    Divide both sides by 400

    $\frac{16x^2}{400} - \frac{25y^2}{400} = 1$

    $\frac{x^2}{5^2} - \frac{y^2}{4^2} = 1$

    which is the standard form of the equation of a hyperbola.



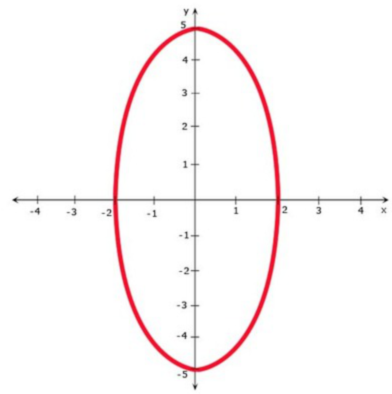
16. $25x^2 + 4y^2 = 100$

    $\frac{x^2}{4} + \frac{y^2}{25} = 1$

    We can write the general form of an ellipse:

    $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

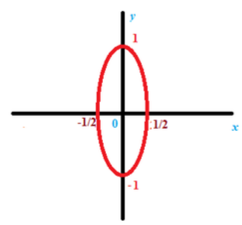
    The equation in the question is an equation of an ellipse, where $a = 2, b = 5, h = 0$, and $k = 0$



17. Given: $4x^2 + y^2 = 1$

    $\frac{x^2}{(\frac{1}{2})^2} - \frac{y^2}{1^2} = 1$

    which is the standard form of the equation of an ellipse.



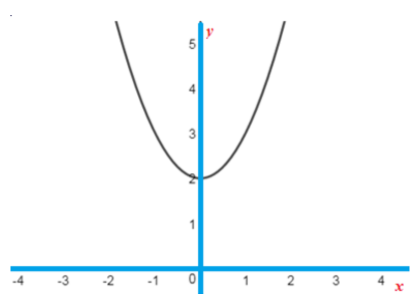
18. Given: $y = x^2 + 2$

    $y = a(x - h)^2 + k$

    Here, $(h, k) = (0, 2)$

    which is the standard form of the equation of a parabola.

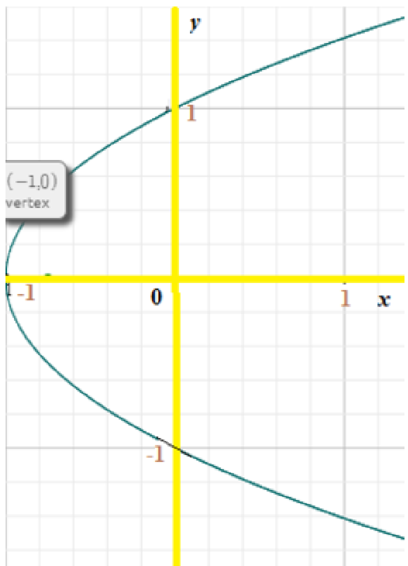
    The graph will be just like a graph of $y = x^2$ that is shifted 2 units upward.



19. Given: $x = y^2 - 1$

    This is a parabola in the standard form as:

4.$\frac{1}{4}(x-(-1)=(y-0)^2$

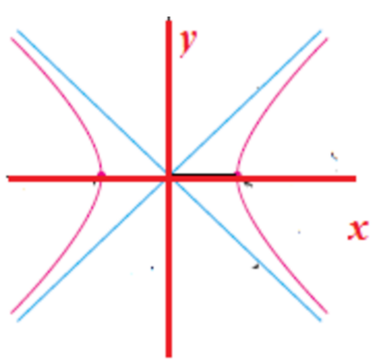


20. Given: $9x^2 - 25y^2 = 225$

Divide by 225.

$\frac{x^2}{(5)^2} - \frac{y^2}{(3)^2} = 1$

which is the standard form of the equation of a Hyperbola.

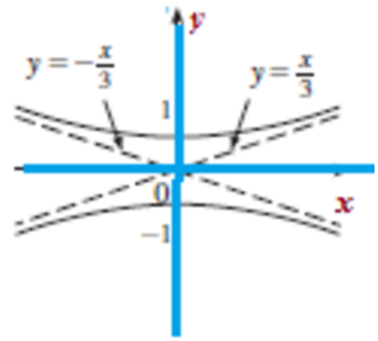


21. Given: $9y^2 - x^2 = 9$

Divide by 9.

$\frac{y^2}{(1)^2} - \frac{x^2}{(3)^2} = 1$

which is the standard form of the equation of a Hyperbola.

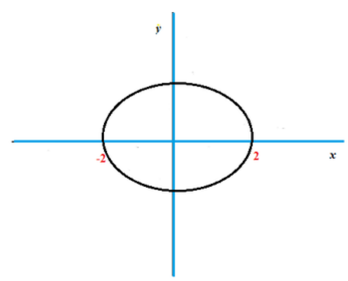


22. Given: $2x^2 + 5y^2 = 10$

Divide by 10.

$\frac{x^2}{(\sqrt{5})^2} + \frac{y^2}{(\sqrt{2})^2} = 1$

which is the standard form of the equation of an Ellipse.

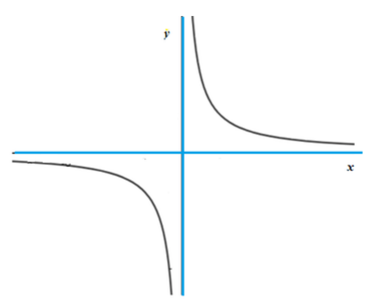


23. Given: $xy = 4$

Dividing by x

$y = \frac{4}{x}$

This represents a rotated Hyperbola similar to $y = \frac{1}{x}$ that is stretched vertically by a factor of 4.



24. Given: $y = x^2 + 2x$

By completing the square, we get

$y + 1 = x^2 + 2x + 1$

$y + 1 = (x + 1)^2$

$y = (x + 1)^2 - 1$

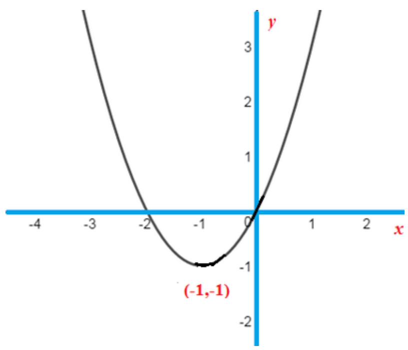
This is a parabola in the standard form as:

$y = a(x - h)^2 + k$

Here, $(h, k) = (-1, -1)$

which is the standard form of the equation of a parabola.

The graph will be just like a graph of $y = x^2$ that is shifted 1 unit to the left and 1 unit downward.



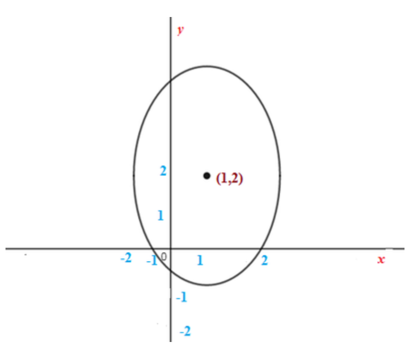
25. Given: $9(x - 1)^2 + 4(y - 2)^2 = 36$

Divide by 36.

$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{9} = 1$

which is a standard form of an Ellipse.

Here, $(h, k) = (1, 2)$



26. Given: $16x^2 + 9y^2 - 36y = 108$

By completing the square, we get

$16x^2 + 9(y^2 - 4y + 4) = 108 + 9(4)$

$16x^2 + 9(y - 2)^2 = 144$

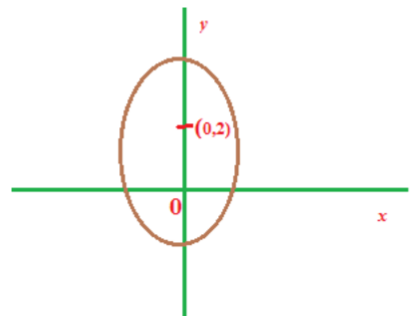
Divide by 144.

$\frac{x^2}{9} + \frac{(y-2)^2}{16} = 1$

which is the standard form of the equation of an Ellipse.

$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

Here, $(h, k) = (0, 2)$

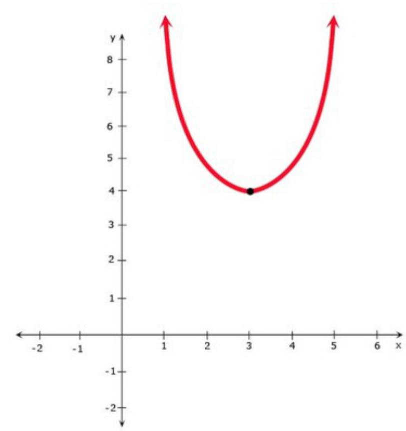


27. $y = x^2 - 6x + 13$

$y = x^2 - 6x + 9 + 4$

$y = (x - 3)^2 + 4$

This is an equation of a parabola with vertex $(3, 4)$



28. $x^2 - y^2 - 4x + 3 = 0$

$x^2 - 4x - y^2 + 3 = 0$

$x^2 - 4x + 4 - 4 - y^2 + 3 = 0$

$(x - 2)^2 - y^2 = 1$

We can write the general form of a hyperbola:

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

The equation in this question is the equation of a hyperbola, where $a = 1, b = 1, h = 2$, and $k = 0$

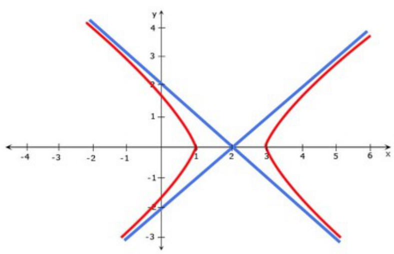
The equations of the asymptotes are $y = x - 2$ and $y = -(x-2) = 2 - x$

Note that these two lines intersect at the point $(2, 0)$

The x-intercepts are 1 and 3

The y-intercepts are $\pm\sqrt{3}$

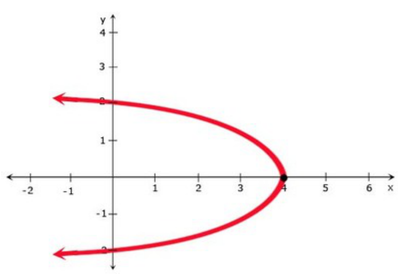
We can sketch a graph of this hyperbola:



29. $x = 4 - y^2$

    $x = -y^2 + 4$

    This is an equation of a parabola with vertex $(4, 0)$



30. $y^2 - 2x + 6y + 5 = 0$

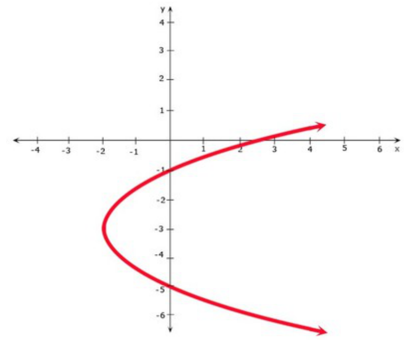
    $2x = y^2 + 6y + 5$

    $2x = y^2 + 6y + 9 - 9 + 5$

    $2x = (y+3)^2 - 4$

    $x = \frac{1}{2}(y+3)^2 - 2$

    This is the equation of a parabola with vertex $(-2, -3)$



31. $x^2 + 4y^2 - 6x + 5 = 0$

    $x^2 - 6x + 9 - 9 + 4y^2 + 5 = 0$

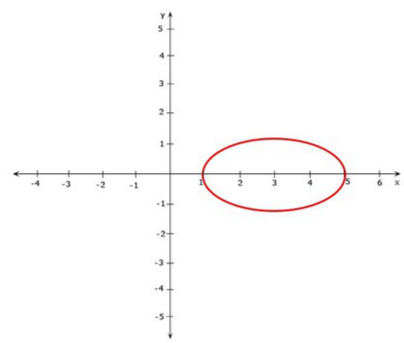
    $(x-3)^2 + 4y^2 = 4$

    $\frac{(x-3)^2}{4} + y^2 = 1$

    We can write the general form of an ellipse:

    $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

    The equation in the question is an equation of an ellipse, where $a = 2, b = 1, h = 3$, and $k = 0$



32. $4x^2 + 9y^2 - 16x + 54y + 61 = 0$

    $4x^2 - 16x + 9y^2 + 54y + 61 = 0$

    $4(x^2 - 4x) + 9(y^2 + 6y) + 61 = 0$

    $4(x^2-4x+4) - 16 + 9(y^2+6y+9) - 81 + 61 = 0$

    $4(x-2)^2 + 9(y+3)^2 = 16 + 81 - 61$

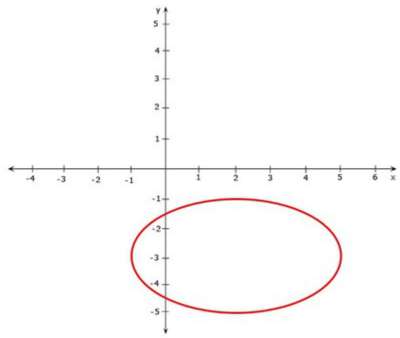
    $4(x-2)^2 + 9(y+3)^2 = 36$

    $\frac{(x-2)^2}{9} + \frac{(y+3)^2}{4} = 1$

We can write the general form of an ellipse:

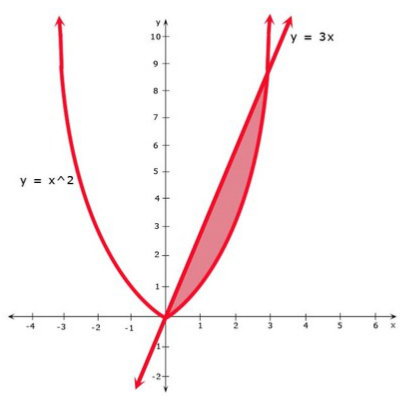
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

The equation in the question is an equation of an ellipse, where $a = 3, b = 2, h = 2$, and $k = -3$



33. $y = 3x$ is a straight line and and $y = x^2$ is a parabola.

    We can sketch the region bounded by the line $y = 3x$ and the parabola $y = x^2$



34. $x - 2y = 2$ is a straight line and and $y = 4 - x^2$ is a parabola.

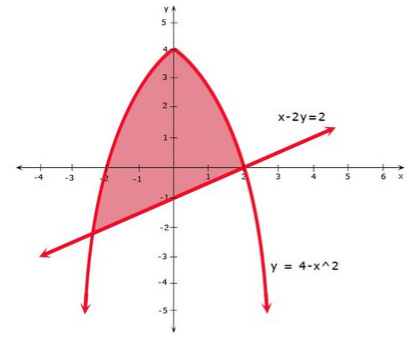
    We can rearrange the equation of the straight line:

    $x - 2y = 2$

    $2y = x - 2$

    $y = \frac{1}{2}x - 1$

    We can sketch the region bounded by the line $x - 2y = 2$ and the parabola $y = 4 - x^2$



35. The vertex is $(1, -1)$ and the parabola passes through the points $(-1, 3)$ and $(3, 3)$

    The equation of the parabola is $y = a(x-1)^2 - 1$

    We can find a:

    $y = a(x-1)^2 - 1$

    $3 = a(3-1)^2 - 1$

    $3 = 4a - 1$

    $a = 1$

    The equation of the parabola is $y = (x-1)^2 - 1$

36. We can write the general form of an ellipse:

    $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

    Since the center is the origin, we can rewrite the equation of the ellipse as follows:

    $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

    We can use the two data pairs to make two equations:

    equation 1:

    $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

    $\frac{(1)^2}{a^2} + \frac{(-10\sqrt{2}/3)^2}{b^2} = 1$

    $\frac{1}{a^2} + \frac{200}{9b^2} = 1$

    $\frac{-4}{a^2} + \frac{-800}{9b^2} = -4$

equation 2:

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{(-2)^2}{a^2} + \frac{(5\sqrt{5}/3)^2}{b^2} = 1$

$\frac{4}{a^2} + \frac{125}{9b^2} = 1$

We can add both sides of equation 1 and equation 2:

$\frac{-800}{9b^2} + \frac{125}{9b^2} = -4 + 1$

$\frac{-675}{9b^2} = -3$

$b^2 = \frac{675}{27}$

$b^2 = \frac{225}{9}$

$b^2 = \frac{75}{3}$

$b^2 = 25$

$b = 5$

We can use equation 1 to find a:

$\frac{1}{a^2} + \frac{200}{9b^2} = 1$

$\frac{1}{a^2} + \frac{200}{9(5)^2} = 1$

$\frac{1}{a^2} + \frac{8}{9} = 1$

$\frac{1}{a^2} = \frac{1}{9}$

$a^2 = 9$

$a = 3$

We can write the equation of the ellipse:

$\frac{x^2}{9} + \frac{y^2}{25} = 1$

37. $\{(x, y)|\ x^2 + y^2 \leq 1\ \}$

We can write the general equation for a circle:

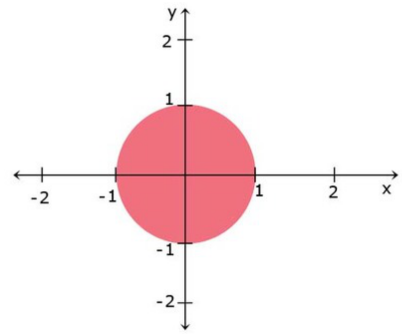
$(x - a)^2 + (y - b)^2 = r^2$

where (a, b) is the center of the circle and r is the radius

The set includes all the points inside the circle $x^2 + y^2 = 1$

The center of the circle is $(0, 0)$ and the radius of the circle is 1

Note that the points on the circle are also included in the set.



38. $\{(x, y)|\ x^2 + y^2 > 4\ \}$

We can write the general equation for a circle:

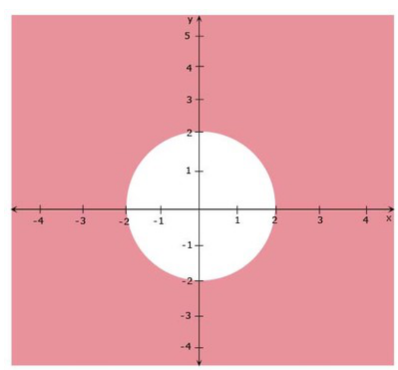
$(x - a)^2 + (y - b)^2 = r^2$

where (a, b) is the center of the circle and r is the radius

The set includes all the points outside the circle $x^2 + y^2 = 4$

The center of the circle is $(0, 0)$ and the radius of the circle is 2

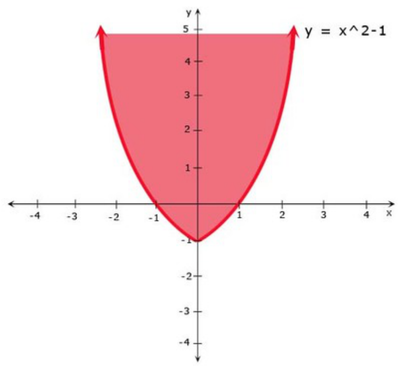
Note that the points on the circle are not included in the set.



39. $\{(x, y)|\ y \geq x^2 - 1\ \}$

The set includes all the points above the parabola $y = x^2 - 1$

Note that the points on the parabola are also included in the set.



40. $\{(x, y)|\ x^2 + 4y^2 \leq 4\ \}$

$x^2 + 4y^2 = 4$

$\frac{x^2}{4} + y^2 = 1$

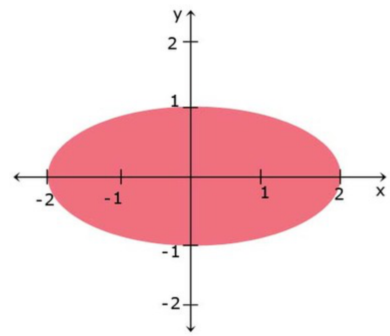
We can write the general form of an ellipse:

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

The equation in the question is an equation of an ellipse, where $a = 2, b = 1, h = 0$, and $k = 0$

The set includes all the points inside the ellipse $\frac{x^2}{4} + y^2 = 1$

Note that the points on the ellipse are also included in the set.



第四章　Appendix D: Trigonometry

4.1 Answer

1. This answer hasn't been written yet!
2. This answer hasn't been written yet!
3. This answer hasn't been written yet!
4. This answer hasn't been written yet!
5. This answer hasn't been written yet!
6. This answer hasn't been written yet!
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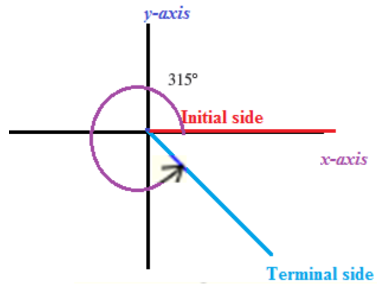
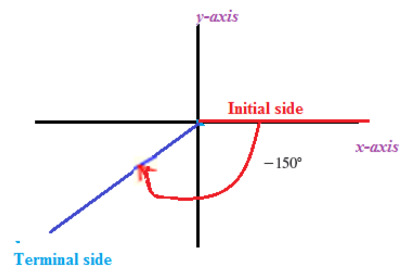
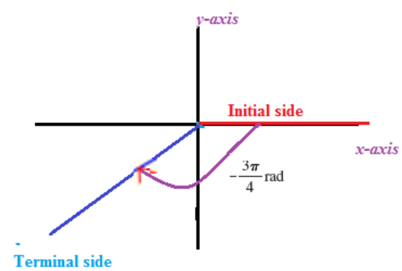
9. This answer hasn't been written yet!
10. This answer hasn't been written yet!
11. This answer hasn't been written yet!
12. This answer hasn't been written yet!
13. This answer hasn't been written yet!
14. This answer hasn't been written yet!
15. $\frac{2}{3}$ radians
16. $\frac{8}{\pi}$ cm
17. The given angle is positive therefore, we will move counterclockwise from the positive x-axis as depicted in the figure:



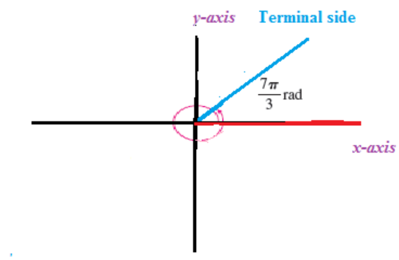
18. The answer is below.



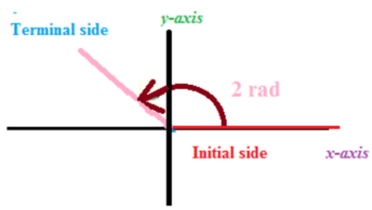
19. The answer is below.



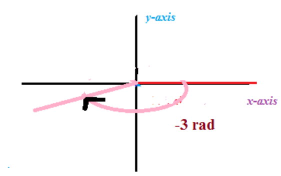
20. The answer is below.



21. The given angle is positive; therefore, we will move counterclockwise from the x-axis as depicted in the figure:



22. The answer is below.



23. $sin\frac{3\pi}{4} = sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

    $csc\frac{3\pi}{4} = \frac{1}{sin\frac{3\pi}{4}} = \sqrt{2}$

    $cos\frac{3\pi}{4} = -cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

    $sec\frac{3\pi}{4} = \frac{1}{cos\frac{3\pi}{4}} = -\sqrt{2}$

    $tan\frac{3\pi}{4} = -tan\frac{\pi}{4} = -1$

    $cot\frac{3\pi}{4} = \frac{1}{tan\frac{3\pi}{4}} = -1$

24. $sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

    $cos\frac{4\pi}{3} = -\frac{1}{2}$

    $tan\frac{4\pi}{3} = \sqrt{3},$

    $csc\frac{4\pi}{3} = -\frac{2\sqrt{3}}{3},$

    $sec\frac{4\pi}{3} = -2,$

    $cot\frac{4\pi}{3} = \frac{\sqrt{3}}{3}$

25. $sin\frac{9\pi}{2} = 1$

    $cos\frac{9\pi}{2} = 0$

    $tan\frac{9\pi}{2} = undefined$

    $csc\frac{9\pi}{2} = 1$

    $sec\frac{9\pi}{2} = undefined$

    $cot\frac{9\pi}{2} = 0$

26. $sin(-5\pi) = 0$

    $cos(-5\pi) = -1$

    $tan(-5\pi) = 0$

    $csc(-5\pi) = undefined$

    $sec(-5\pi) = -1$

    $cot(-5\pi) = undefined$

27. $sin\frac{5\pi}{6} = \frac{1}{2}$

    $cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

    $tan\frac{5\pi}{6} = -\frac{\sqrt{3}}{3}$

    $csc\frac{5\pi}{6} = 2$

    $sec\frac{5\pi}{6} = -\frac{2\sqrt{3}}{3}$

    $cot\frac{5\pi}{6} = -\sqrt{3}$

28. $sin\frac{11\pi}{4} = \frac{\sqrt{2}}{2}$

    $cos\frac{11\pi}{4} = -\frac{\sqrt{2}}{2}$

    $tan\frac{11\pi}{4} = -1$

    $csc\frac{11\pi}{4} = \sqrt{2}$

    $sec\frac{11\pi}{4} = -\sqrt{2}$

    $cot\frac{11\pi}{4} = -1$

29. $cos\theta = \frac{4}{5}$

    $tan\theta = \frac{3}{4}$

    $csc\theta = \frac{5}{3}$

    $sec\theta = \frac{5}{4}$

    $cot\theta = \frac{4}{3}$

30. $sin\alpha = \frac{2\sqrt{5}}{5}$

    $cos\alpha = \frac{\sqrt{5}}{5}$

$csc\alpha = \frac{\sqrt{5}}{2}$

$sec\alpha = \sqrt{5}$

$cot\alpha = \frac{1}{2}$

31. $sin\phi = \frac{\sqrt{5}}{3}$

    $cos\phi = -\frac{2}{3}$

    $tan\phi = -\frac{\sqrt{5}}{2}$

    $csc\phi = \frac{3\sqrt{5}}{5}$

    $cot\phi = -\frac{2\sqrt{5}}{5}$

32. $sinx = -\frac{2\sqrt{2}}{3}$

    $tanx = 2\sqrt{2}$

    $cscx = -\frac{3}{2\sqrt{2}} = -\frac{3\sqrt{2}}{4}$

    $secx = -3$

    $cotx = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

33. $sin\beta = -\frac{1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$

    $cos\beta = -\frac{3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$

    $tan\beta = \frac{1}{3}$

    $csc\beta = -\sqrt{10}$

    $sec\beta = -\frac{\sqrt{10}}{3}$

34. $sin\theta = -\frac{3}{4}$

    $cos\theta = \frac{\sqrt{7}}{4}$

    $tan\theta = -\frac{3}{\sqrt{7}} = -\frac{3\sqrt{7}}{7}$

    $sec\theta = \frac{4}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$

    $cot\theta = -\frac{\sqrt{7}}{3}$

35. 5.73576 cm

36. 19.15111 cm

37. 24.62147 cm

38. 57.48877 cm

39. (a) $sin(-\theta) = -sin\theta$

    (b) $cos(-\theta) = cos\theta$

40. (a) $tan(x+y) = \frac{tanx+tany}{1-tanxtany}$

    (b)$tan(x-y) = \frac{tanx-tany}{1+tanxtany}$

41. (a) $sinx\ cosy = \frac{1}{2}[sin(x+y)+sin(x-y)]$

    (b) $cosx\ cosy = \frac{1}{2}[cos(x+y)+cos(x-y)]$

    (c) $sinx\ siny = \frac{1}{2}[cos(x-y)-cos(x+y)]$

42. $cos(\frac{\pi}{2}-x) = sinx$

43. $sin(\frac{\pi}{2}+x) = cosx$

44. $sin(\pi-x) = sinx$

45. $sin\theta\ cot\theta = cos\theta$

46. $(sinx+cosx)^2 = 1+sin2x$

47. $(secy-cosy) = tany\ siny$

48. $tan^2\alpha - sin^2\alpha = tan^2\alpha\ sin^2\alpha$

49. $cot^2\theta + sec^2\theta = tan^2\theta + csc^2\theta$

50. $2\ csc\ 2t = sec\ t\ csc\ t$

51. $tan2\theta = \frac{2tan\theta}{1-tan^2\theta}$

52. $\frac{1}{1-sin\theta} + \frac{1}{1+sin\theta} = 2sec^2\theta$

53. $sinx$*$sin2x$ +$cosxcos2x = cosx$

54. $sin(x+y)\ sin(x-y) = sin^2x - sin^2y$

55. $csc\phi$ +$cot\phi = \frac{sin\phi}{1-cos\phi}$

56. $tanx + tany = \frac{sin(x+y)}{cosxcosy}$

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4.2 Step by Step

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15. We use the formula $s = r$.

    We solve the formula for : $= \frac{s}{r}$.

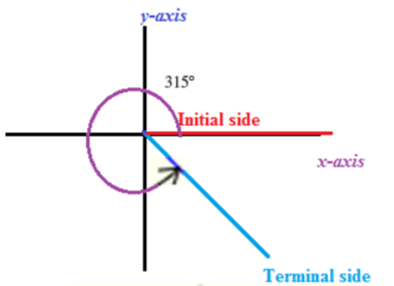
    We plug in the given values and simplify: $= \frac{1}{1.5} = \frac{2}{3}$ radians.
16. We use the formula $s = r$.

    We solve the formula for r: $r = \frac{s}{}$.

    We plug in the given values and simplify: $r = \frac{6}{\frac{3\pi}{4}} = \frac{4\times 6}{3\pi} = \frac{8}{\pi}$ cm.
17. Given: $315°$

    The standard position of an angle occurs when we place its vertex at the origin of a coordinate system and its initial side on the positive x-axis.

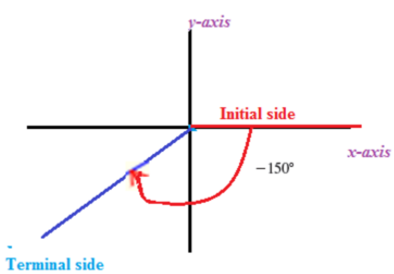
    A positive angle is obtained by rotating the initial side counterclockwise until it coincides with the terminal side. Likewise, negative angles are obtained by clockwise rotation.



18. Given: $-150°$

The standard position of an angle occurs when we place its vertex at the origin of a coordinate system and its initial side on the positive x-axis.

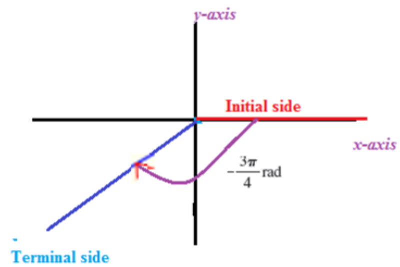
A positive angle is obtained by rotating the initial side counterclockwise until it coincides with the terminal side. Likewise, negative angles are obtained by clockwise rotation.



19. Given: $-\frac{3\pi}{4}$ rad $= -135°$

The standard position of an angle occurs when we place its vertex at the origin of a coordinate system and its initial side on the positive x-axis.

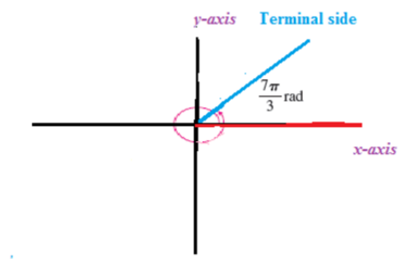
A positive angle is obtained by rotating the initial side counterclockwise until it coincides with the terminal side. Likewise, negative angles are obtained by clockwise rotation.



20. Given: $\frac{7\pi}{3}$ rad $= 420°$

The standard position of an angle occurs when we place its vertex at the origin of a coordinate system and its initial side on the positive x-axis.

A positive angle is obtained by rotating the initial side counterclockwise until it coincides with the terminal side. Likewise, negative angles are obtained by clockwise rotation.

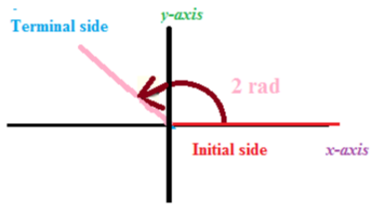


21. Given: 2 rad $= 114.6°$

The standard position of an angle occurs when we place its vertex at the origin of a coordinate system and its initial side on the positive x-axis.

A positive angle is obtained by rotating the initial side counterclockwise until it coincides with the terminal side. Likewise, negative angles are obtained by clockwise rotation.

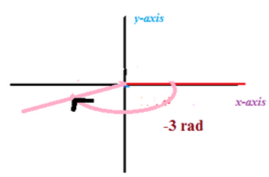
The given angle is positive, so we will move counterclockwise from the x-axis as depicted in the figure:



22. Given: $-3rad = -171.9°$

The standard position of an angle occurs when we place its vertex at the origin of a coordinate system and its initial side on the positive x-axis.

A positive angle is obtained by rotating the initial side counterclockwise until it coincides with the terminal side. Likewise, negative angles are obtained by clockwise rotation.



23. Since $\frac{3\pi}{4}$ lies in the second quadrant, its reference angle will be $\pi - \frac{3\pi}{4} = \frac{\pi}{4}$, which can be considered as a common angle to all trigonometric ratios.

The trigonometric ratios and their inverse trigonometric ratios are given as follows:

$sin\frac{3\pi}{4} = sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$csc\frac{3\pi}{4} = \frac{1}{sin\frac{3\pi}{4}} = \sqrt{2}$

$cos\frac{3\pi}{4} = -cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$sec\frac{3\pi}{4} = \frac{1}{cos\frac{3\pi}{4}} = -\sqrt{2}$

$tan\frac{3\pi}{4} = -tan\frac{\pi}{4} = -1$

$cot\frac{3\pi}{4} = \frac{1}{tan\frac{3\pi}{4}} = -1$

24. The trigonometric ratios and their inverse trigonometric ratios are given as follows:

$sin\frac{4\pi}{3} = -sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

$csc\frac{4\pi}{3} = \frac{1}{sin\frac{4\pi}{3}} = -\frac{2\sqrt{3}}{3}$

$cos\frac{4\pi}{3} = -cos\frac{\pi}{3} = -\frac{1}{2}$

$sec\frac{4\pi}{3} = \frac{1}{cos\frac{4\pi}{3}} = -2$

$tan\frac{4\pi}{3} = tan\frac{\pi}{3} = \sqrt{3}$

$cot\frac{4\pi}{3} = \frac{1}{tan\frac{4\pi}{3}} = \frac{\sqrt{3}}{3}$

25. The trigonometric ratios and their inverse trigonometric ratios are given as follows:

$sin\frac{9\pi}{2} = sin\frac{\pi}{2} = 1$

$cos\frac{9\pi}{2} = cos\frac{\pi}{2} = 0$

$tan\frac{9\pi}{2} = tan\frac{\pi}{2} = undefined$

$csc\frac{9\pi}{2} = \frac{1}{sin\frac{9\pi}{2}} = 1$

$sec\frac{9\pi}{2} = \frac{1}{cos\frac{9\pi}{2}} = undefined$

$cot\frac{9\pi}{2} = \frac{1}{tan\frac{9\pi}{2}} = 0$

26. The trigonometric ratios and their inverse trigonometric ratios are given as follows:

$sin(-5\pi) = sin\pi = 0$

$cos(-5\pi) = cos\pi = -1$

$tan(-5\pi) = tan\pi = 0$

$csc(-5\pi) = \frac{1}{sin(-5\pi)} = undefined$

$sec(-5\pi) = \frac{1}{cos(-5\pi)} = -1$

$cot(-5\pi) = \frac{1}{tan(-5\pi)} = undefined$

27. The trigonometric ratios and their inverse trigonometric ratios are given as follows:

$sin\frac{5\pi}{6} = sin\frac{\pi}{6} = \frac{1}{2}$

$cos\frac{5\pi}{6} = -cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

$tan\frac{5\pi}{6} = -tan\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$

$csc\frac{5\pi}{6} = \frac{1}{sin\frac{5\pi}{6}} = 2$

$sec\frac{5\pi}{6} = \frac{1}{cos\frac{5\pi}{6}} = -\frac{2\sqrt{3}}{3}$

$cot\frac{5\pi}{6} = \frac{1}{tan\frac{5\pi}{6}} = -\sqrt{3}$

28. The trigonometric ratios and their inverse trigonometric ratios are given as follows:

$sin\frac{11\pi}{4} = sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$cos\frac{11\pi}{4} = -cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$tan\frac{11\pi}{4} = -tan\frac{\pi}{4} = -1$

$csc\frac{11\pi}{4} = \frac{11}{sin\frac{3\pi}{4}} = \sqrt{2}$

$sec\frac{11\pi}{4} = \frac{1}{cos\frac{11\pi}{4}} = -\sqrt{2}$

$cot\frac{11\pi}{4} = \frac{1}{tan\frac{11\pi}{4}} = -1$

29. Since $sin\theta = \frac{3}{5}$, we can label the opposite side as having length 3 and the hypotenuse as having length 5.

    The Pythagorean Theorem gives

    adjacent side $= \sqrt{5^2 - 3^2} = 4$

    The other five trigonometric functions are given as follows:

    $cos\theta = \frac{4}{5}$

    $tan\theta = \frac{3}{4}$

    $csc\theta = \frac{5}{3}$

    $sec\theta = \frac{5}{4}$

    $cot\theta = \frac{4}{3}$

30. Since $tan\alpha = 2$, we can label the opposite side as having length 2 and the adjacent side as having length 1.

    The Pythagorean Theorem gives

    hypotenuse side $= \sqrt{2^2 + 1^2} = \sqrt{5}$

    The other five trigonometric functions are given as follows:

    $sin\alpha = \frac{2\sqrt{5}}{5}$

    $cos\alpha = \frac{\sqrt{5}}{5}$

    $csc\alpha = \frac{\sqrt{5}}{2}$

    $sec\alpha = \sqrt{5}$

    $cot\alpha = \frac{1}{2}$

31. Since $sec\phi = -1.5 = -\frac{3}{2}$, we can label the hypotenuse side as having length 3 and the adjacent side as having length 2.

    The Pythagorean Theorem gives

    opposite side $= \sqrt{3^2 - 2^2} = \sqrt{5}$

    The other five trigonometric functions are given as follows:

    $sin\phi = \frac{\sqrt{5}}{3}$

    $cos\phi = -\frac{2}{3}$

    $tan\phi = -\frac{\sqrt{5}}{2}$

    $csc\phi = \frac{3\sqrt{5}}{5}$

    $cot\phi = -\frac{2\sqrt{5}}{5}$

32. Since $cosx = -\frac{1}{3}$, we can label the adjacent side as having length 1 and the hypotenuse as having length 3.

    Then Pythagorean Theorem gives

    opposite side $= \sqrt{3^2 - 1^2} = 2\sqrt{2}$

    The other five trigonometric functions are given as follows:

    $sinx = -\frac{2\sqrt{2}}{3}$

    $tanx = 2\sqrt{2}$

    $cscx = -\frac{3}{2\sqrt{2}} = -\frac{3\sqrt{2}}{4}$

    $secx = -3$

    $cotx = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

33. Since $cot\beta = 3$, we can label the adjacent side as having length 3 and the opposite side as having length 1.

    The Pythagorean Theorem gives

    hypotenuse side$= \sqrt{3^2 + 1^2} = \sqrt{10}$

    The other five trigonometric functions are given as follows:

    $sin\beta = -\frac{1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$

    $cos\beta = -\frac{3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$

    $tan\beta = \frac{1}{3}$

    $csc\beta = -\sqrt{10}$

    $sec\beta = -\frac{\sqrt{10}}{3}$

34. Since $csc\theta = -\frac{4}{3}$, we can label the hypotenuse side as having length 4 and the opposite side as having length 3.

The Pythagorean Theorem gives

adjacent side $= \sqrt{4^2 - 3^2} = \sqrt{7}$

The other five trigonometric functions are given as follows:

$sin\theta = -\frac{3}{4}$

$cos\theta = \frac{\sqrt{7}}{4}$

$tan\theta = -\frac{3}{\sqrt{7}} = -\frac{3\sqrt{7}}{7}$

$sec\theta = \frac{4}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$

$cot\theta = -\frac{\sqrt{7}}{3}$

35. $sin35° = \frac{x}{10}$

Need to solve for x.

$x = sin35° \times 10$

Hence, $x = 5.73576$ cm

36. $cos\theta = \frac{adj}{hyp}$

$cos40° = \frac{x}{25}$

Need to solve for x.

$x = cos40° \times 25$

Hence, $x = 19.15111$ cm

37. $tan\theta = \frac{opp}{adj}$

$tan(\frac{2\pi}{5}) = \frac{x}{8}$

Need to solve for x.

$x = tan(\frac{2\pi}{5}) \times 8$

Hence, $x = 24.62147$ cm

38. $cos\theta = \frac{adj}{hyp}$

$cos(\frac{3\pi}{8}) = \frac{22}{x}$

Need to solve for x.

$x = \frac{22}{cos(\frac{3\pi}{8})}$

Hence, $x = 57.48877$ cm

39. (a) Need to prove $sin(-\theta) = -sin\theta$

For a right triangle as depicted below

$sin\theta = \frac{y}{\sqrt{x^2+y^2}}$

If we interchange the sign for θ, we will have

$sin(-\theta) = -\frac{y}{\sqrt{x^2+y^2}}$

Hence, $sin(-\theta) = -sin\theta$

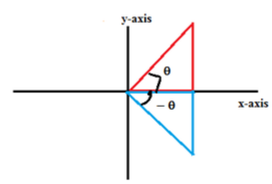
(b) (a) Need to prove $cos(-\theta) = cos\theta$

For a right triangle , $cos\theta = \frac{x}{\sqrt{x^2+y^2}}$

If we interchange the sign for θ, we will have

$cos(-\theta) = \frac{x}{\sqrt{x^2+y^2}}$

Hence, $cos(-\theta) = cos\theta$



40. (a) $tan(x+y) = \frac{sin(x+y)}{cos(x+y)}$

$sin(x+y) = sinxcosy + cosxsiny$

and

$cos(x+y) = cosxcosy - sinxsiny$

Thus,

$tan(x+y) = \frac{sinxcosy+cosxsiny}{cosxcosy-sinxsiny}$

Divide the numerator and denominator by $cosx\ cosy$:

$tan(x+y) = \frac{\frac{sinx}{cosx}+\frac{siny}{cosy}}{1-\frac{sinxsiny}{cosxcosy}}$

Hence, $tan(x+y) = \frac{tanx+tany}{1-tanxtany}$

(b) $tan(x-y) = \frac{sin(x-y)}{cos(x-y)}$

$sin(x-y) = sinxcosy - cosxsiny$

and

$cos(x-y) = cosxcosy + sinxsiny$

Thus,

$tan(x-y) = \frac{sinxcosy-cosxsiny}{cosxcosy+sinxsiny}$

Divide the numerator and denominator by $cosx\ cosy$:

$tan(x-y) = \frac{\frac{sinx}{cosx} - \frac{siny}{cosy}}{1+\frac{sinxsiny}{cosxcosy}}$

Hence, $tan(x-y) = \frac{tanx - tany}{1+tanxtany}$

41. (a) $sin(x+y) = sinxcosy + cosxsiny$

and

$sin(x-y) = sinxcosy - cosxsiny$

Thus,

$sin(x+y) + sin(x-y) = (sinxcosy + cosxsiny) + (cosxcosy - sinxsiny)$

$sin(x+y) + sin(x-y) = (sinxcosy + cosxsiny) + (sinxcosy - cosxsiny)$

$sin(x+y) + sin(x-y) = (2sinxcosy)$

Hence, $sinx\ cosy = \frac{1}{2}[sin(x+y) + sin(x-y)]$

(b) $cos(x+y) = cosxcosy - sinxsiny$

and

$cos(x-y) = cosxcosy + sinxsiny$

Thus,

$cos(x+y) + cos(x-y) = (cosxcosy - sinxsiny) + (cosxcosy + sinxsiny)$

$cos(x+y) + cos(x-y) = (2cosxcosy)$

Hence, $cosx\ cosy = \frac{1}{2}[cos(x+y) + cos(x-y)]$

(c) $cos(x+y) = cosxcosy - sinxsiny$

and

$cos(x-y) = cosxcosy + sinxsiny$

Thus,

$cos(x-y) - cos(x+y) = (cosxcosy + sinxsiny) - (cosxcosy - sinxsiny)$

$cos(x-y) - cos(x+y) = (2sinxsiny)$

Hence, $sinx\ siny = \frac{1}{2}[cos(x-y) - cos(x+y)]$

42. Need to prove $cos(\frac{\pi}{2} - x) = sinx$

$cos(A-B) = cosAcosB + sinAsinB$

Thus,

$cos(\frac{\pi}{2} - x) = cos\frac{\pi}{2}cosx + sin\frac{\pi}{2}sinx$

$cos(\frac{\pi}{2} - x) = 0 * cosx + 1 * sinx$

Hence, $cos(\frac{\pi}{2} - x) = sinx$

43. Need to prove $sin(\frac{\pi}{2} + x) = cosx$

$sin(A+B) = sinAcosB + cosAsinB$

Thus,

$sin(\frac{\pi}{2} + x) = sin\frac{\pi}{2}cosx + cos\frac{\pi}{2}sinx$

$sin(\frac{\pi}{2} + x) = 1 * cosx + 0 * sinx$

Hence, $sin(\frac{\pi}{2} + x) = cosx$

44. Need to prove $sin(\pi - x) = sinx$

$sin(A-B) = sinAcosB - cosAsinB$

Thus,

$sin(\pi - x) = sin\pi cosx - cos\pi sinx$

$sin(\pi - x) = 0 * cosx - (-1)sinx$

Hence, $sin(\pi - x) = sinx$

45. Need to prove $sin\theta\ cot\theta = cos\theta$

$cot\theta = \frac{cos\theta}{sin\theta}$ (quotient property)

Thus,

$sin\theta cot\theta = sin\theta \times \frac{cos\theta}{sin\theta}$

Hence, $sin\theta\ cot\theta = cos\theta$

46. We need to prove the identity $(sinx + cosx)^2 = 1 + sin2x$

We have:

$(sinx+cosx)^2 = sin^2x+cos^2x+2sinxcosx$

Also, $2sinxcosx = sin2x$, and $sin^2x + cos^2x = 1$

Hence, $(sinx + cosx)^2 = 1 + sin2x$

47. We need to prove the identity $secy - cosy = tany\ siny$

In order to prove this, take the left side and isolate.

$secy - cosy = \frac{1}{cosy} - cosy$

$= \frac{1-cos^2y}{cosy}$

$1 - cos^2y = sin^2y,$

Thus, $secy - cosy = \frac{sin^2y}{cosy}$

Now, $secy - cosy = \frac{siny}{cosy} \times siny$

Hence, $(secy - cosy) = tany\ siny$

48. We need to prove the identity $tan^2\alpha - sin^2\alpha = tan^2\alpha\ sin^2\alpha$

$tan^2\alpha - sin^2\alpha = \frac{sin^2\alpha}{cos^2\alpha} - sin^2\alpha$

$tan^2\alpha - sin^2\alpha = \frac{sin^2\alpha - sin^2\alpha cos^2\alpha}{cos^2\alpha}$

Thus, $tan^2\alpha - sin^2\alpha = \frac{1-cos^2}{cos^2\alpha} \times sin^2\alpha$

$1 - cos^2\alpha = sin^2\alpha$

Now, $tan^2\alpha - sin^2\alpha = \frac{sin^2\alpha}{cos^2\alpha} \times sin^2\alpha$

Hence, $tan^2\alpha - sin^2\alpha = tan^2\alpha\ sin^2\alpha$

49. We can prove the identity:

$cot^2\ \theta + sec^2\ \theta$

$= \frac{cos^2\ \theta}{sin^2\ \theta} + \frac{1}{cos^2\ \theta}$

$= \frac{1-sin^2\ \theta}{sin^2\ \theta} + \frac{1}{cos^2\ \theta}$

$= \frac{1}{sin^2\ \theta} - 1 + \frac{1}{cos^2\ \theta}$

$= \frac{1}{sin^2\ \theta} - \frac{cos^2\ \theta}{cos^2\ \theta} + \frac{1}{cos^2\ \theta}$

$= \frac{1}{sin^2\ \theta} + \frac{1-cos^2\ \theta}{cos^2\ \theta}$

$= \frac{1}{sin^2\ \theta} + \frac{sin^2\ \theta}{cos^2\ \theta}$

$= \frac{sin^2\ \theta}{cos^2\ \theta} + \frac{1}{sin^2\ \theta}$

$= tan^2\ \theta + csc^2\ \theta$

50. We prove the identity $2\ csc\ 2t = sec\ t\ csc\ t$

$2\ csc\ 2t = \frac{2}{sin2t}$

Since $sin2t = 2sint\ cost$

$2\ csc\ 2t = \frac{2}{2sintcost}$

$2\ csc\ 2t = \frac{1}{sint} \times \frac{1}{cost}$

As $\frac{1}{sint} = csct$ and $\frac{1}{cost} = sect$

Therefore, $2\ csc\ 2t = csct\ sect$

Hence, $2\ csc\ 2t = sec\ t\ csc\ t$

51. We need to prove the identity $tan2\theta = \frac{2tan\theta}{1-tan^2\theta}$

$tan2\theta = tan(\theta + \theta)$

$tan(\theta+\theta) = \frac{tan\theta + tan\theta}{1-tan\theta \times tan\theta}$ [sum identity for tangent]

Thus,

$tan(\theta+\theta) = \frac{2tan\theta}{1-tan^2\theta}$

Hence, $tan2\theta = \frac{2tan\theta}{1-tan^2\theta}$

52. We need to prove the identity $\frac{1}{1-sin\theta} + \frac{1}{1+sin\theta} = 2sec^2\theta$

$\frac{1}{1-sin\theta} + \frac{1}{1+sin\theta} = \frac{1+sin\theta}{(1-sin\theta)(1+sin\theta)} + \frac{1-sin\theta}{(1-sin\theta)(1+sin\theta)}$

$\frac{1}{1-sin\theta} + \frac{1}{1+sin\theta} = \frac{2}{1-sin^2\theta}$

Since $sin^2\theta + cos^2\theta = 1$:

$\frac{1}{1-sin\theta} + \frac{1}{1+sin\theta} = \frac{2}{cos^2\theta}$

Also, $\frac{1}{cos\theta} = sec\theta$

Hence, $\frac{1}{1-sin\theta} + \frac{1}{1+sin\theta} = 2sec^2\theta$

53. We need to prove the identity $sinx{*}sin2x + cosxcos2x = cosx$

$sin2x = 2sinxcosx$

and

$cos2x = cos^2x - sin^2x$

Thus,

$sinx{*}sin2x + cosxcos2x$

$= sinx(2sinxcosx) + cosx(cos^2x - sin^2x)$

$sinx{*}sin2x + cosxcos2x = 2sin^2xcosx + cos^3x - cosx \times sin^2x$

$sinx{*}sin2x + cosxcos2x = sin^2xcosx + cos^3x$

$sinx{*}sin2x + cosxcos2x = cosx(sin^2x + cos^2x)$

$sinx{*}sin2x + cosxcos2x = cosx(1)$

Hence, $sinx{*}sin2x + cosxcos2x = cosx$

54. We need to prove the identity

    $sin(x+y)\ sin(x-y) = sin^2x - sin^2y$

    Let us solve the left side of the given identity.

    $sin(x+y)\ sin(x-y) = (sinxcosy + cosxsiny) \times (sinxcosy - cosxsiny)$

    $sin(x+y)\ sin(x-y) = sin^2xcos^2y - cos^2xsin^2y$

    $sin(x+y)\ sin(x-y) = sin^2x(1-sin^2y) - (1-sin^2x)sin^2y$

    $sin(x+y)\ sin(x-y) = (sin^2x - sin^2xsin^2y) - (sin^2y - sin^2xsin^2y)$

    $sin(x+y)\ sin(x-y) = sin^2x - sin^2xsin^2y - sin^2y + sin^2xsin^2y$

    Hence, $sin(x+y)\ sin(x-y) = sin^2x - sin^2y$

55. We need to prove the identity

    $csc\phi + cot\phi = \frac{sin\phi}{1-cos\phi}$

    Let us solve the left side of the given identity.

    $csc\phi + cot\phi = \frac{1}{sin\phi} + \frac{cos\phi}{sin\phi}$

    $= \frac{1+cos\phi}{sin\phi}$

    $= \frac{1+cos\phi}{sin\phi} \times \frac{1-cos\phi}{1-cos\phi}$

    $= \frac{1-cos^2\phi}{sin\phi(1-cos\phi)}$

    $= \frac{sin^2\phi}{sin\phi(1-cos\phi)}$

    Hence, $csc\phi + cot\phi = \frac{sin\phi}{1-cos\phi}$

56. We need to prove the identity

    $tanx + tany = \frac{sin(x+y)}{cosxcosy}$

    Let us solve the right side of the given identity.

    $\frac{sin(x+y)}{cosxcosy} = \frac{sinxcosy+cosxsiny}{cosxcosy}$

    $\frac{sin(x+y)}{cosxcosy} = \frac{sinx}{cosx} + \frac{siny}{cosy}$

    Hence, $tanx + tany = \frac{sin(x+y)}{cosxcosy}$

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第五章 Appendix E: Sigma Notation

5.1 Answer

1. $\sqrt{1}+\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{5}$
2. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}$
3. $3^4+3^5+3^6$
4. $4^3+5^3+6^3$
5. $-1+\frac{1}{3}+\frac{3}{5}+\frac{5}{7}+\frac{7}{9}$
6. $x^5+x^6+x^7+x^8$
7. $1^{10}+2^{10}+3^{10}+4^{10}+\cdots+n^{10}$
8. $n^2+(n+1)^2+(n+2)^2+(n+3)^2$
9. $1-1+1-1+\cdots+(-1)^{n-1}$
10. $f(x_1)\triangle x_1 + f(x_2)\triangle x_2 + f(x_3)\triangle x_3 + f(x_4)\triangle x_4+\cdots+f(x_n)\triangle x_n$
11. $\sum_{i=1}^{10} i$
12. $\sum_{i=3}^{7} \sqrt{i}$
13. $\sum_{i=1}^{19} \frac{i}{i+1}$
14. $\sum_{i=3}^{23} \frac{i}{i+4}$
15. $\sum_{i=1}^{n} 2i$
16. $\sum_{i=1}^{n}(2i-1)$
17. $\displaystyle\sum_{i=0}^{5} 2^i$
18. $\displaystyle\sum_{i=1}^{6} \frac{1}{i^2}$
19. $\displaystyle\sum_{i=1}^{n} x^i$
20. $\sum_{i=0}^{n}(-1)^i x^i$
21. 80
22. 122
23. 3276
24. 1
25. 0
26. 400
27. 61
28. 63.5 or $\frac{127}{2}$
29. $n(n+1)$
30. $-\frac{n(5n+1)}{2}$
31. $\frac{1}{3}n\left(n^2+6n+17\right)$
32. $\frac{1}{3}n(4n^2+24n+47)$
33. $\sum\limits_{i=1}^{n}(i+1)(i+2)=\frac{1}{3}n(n^2+6n+11)$
34. $\sum\limits_{i=1}^{n} i(i+1)(i+2)=\frac{1}{4}n(n+1)[n^2+5n+6]$
35. $\sum\limits_{i=1}^{n}(i^3-i-2)=\frac{1}{4}n(n^3+2n^2-n-10)$
36. 12
37. $\sum\limits_{i=1}^{n} c = nc$

38. $\sum\limits_{i=1}^{n} i^3 = [\frac{n(n+1)}{2}]^2$

39. $\sum_{i=1}^{n} i^3 = [\frac{n(n+1)}{2}]^2$

40. The area for $G_i = i^3$

    The area of ABCD is $\sum_{i=1}^{n} i^3 = [\frac{n(n+1)}{2}]^2$

41. (a) n^4

    (b) $5^{100} - 1$

    (c) $\frac{97}{300}$

    (d) $a_n - a_0$

42. $|\sum\limits_{i=1}^{n} a_i| \leq \sum\limits_{i=1}^{n} |a_i|$

43. $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{1}{n}(\frac{i}{n})^2 = \frac{1}{3}$

44. $\frac{5}{4}$

45. 14

46. $\frac{195}{4}$

47. $\sum\limits_{i=1}^{n} ar^{i-1} = a + ar + ar^2 + ... + ar^{n-1} = \frac{a(r^n-1)}{r-1}$

48. $6 - 3(\frac{1}{2})^{n-1}$

49. $2^{n+1} + n^2 + n - 2$

50. $\sum\limits_{i=1}^{m}[\sum\limits_{j=1}^{n}(i+j)] = \frac{1}{2}mn(m+n+2)$

5.2 Step by Step

1. We expand the sigma notation by writing $\sqrt{i}$ as i increases from 1 to 5:

$$\sum_{i=1}^{5} \sqrt{i} = \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5}$$

2. We expand the sigma notation by writing $\frac{1}{i+1}$ as i increases from 1 to 6:

$$\sum_{i=1}^{6} \frac{1}{i+1} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$$

3. We expand the sigma notation by writing 3^i as i increases from 4 to 6:

$$\sum_{i=4}^{6} 3^i = 3^4 + 3^5 + 3^6$$

4. We expand the sigma notation by writing i^3 as i increases from 4 to 6:

$$\sum_{i=4}^{6} i^3 = 4^3 + 5^3 + 6^3$$

5. We expand the sigma notation by writing $\frac{2k-1}{2k+1}$ as k increases from 0 to 4:

$$\sum_{k=0}^{4} \frac{2k-1}{2k+1}$$

$$= \frac{2*0-1}{2*0+1} + \frac{2*1-1}{2*1+1} + \frac{2*2-1}{2*2+1} + \frac{2*3-1}{2*3+1} + \frac{2*4-1}{2*4+1}$$

$$= -1 + \frac{1}{3} + \frac{3}{5} + \frac{5}{7} + \frac{7}{9}$$

6. We expand the sigma notation by writing x^k as k increases from 5 to 8:

$$\sum_{k=5}^{8} x^k = x^5 + x^6 + x^7 + x^8$$

7. We expand the sigma notation by writing i^{10} as i increases from 1 to n:

$$\sum_{i=1}^{n} i^{10} = 1^{10} + 2^{10} + 3^{10} + 4^{10} + \cdots + n^{10}$$

8. We expand the sigma notation by writing j^2 as j increases from n to n+3:

$$\sum_{j=n}^{n+3} j^2 = n^2 + (n+1)^2 + (n+2)^2 + (n+3)^2$$

   (We could expand this, but this is not required of the problem.)

9. We expand the sigma notation by writing $(-1)^j$ as j increases from 0 to n-1:

$\sum_{j=0}^{n-1}(-1)^j$

$= (-1)^0+(-1)^1+(-1)^2+(-1)^3+(-1)^4+ ... + (-1)^{n-1}$

$= 1-1+1-1+\cdots+(-1)^{n-1}$

(We see that most terms will cancel, but we don't know what the final answer will be without knowing whether n is even or odd.)

10. We expand the sigma notation by writing $f(x_i)\triangle x_i$ as i increases from 1 to n:

$\sum_{i=1}^{n} f(x_i)\triangle x_i = f(x_1)\triangle x_1 + f(x_2)\triangle x_2 + f(x_3)\triangle x_3 + f(x_4)\triangle x_4 + \cdots + f(x_n)\triangle x_n$

11. We contract the sum into sigma notation by noting that it follows the pattern of a sum of integers (e.g. i) from 1 to 10:

$$1+2+3+4+\cdots+10 = \sum_{i=1}^{10} i$$

12. We contract the sum into sigma notation by noting that it follows the pattern of a sum of square roots (e.g. $\sqrt{i}$) from 3 to 7:

$$\sqrt{3}+\sqrt{4}+\sqrt{5}+\sqrt{6}+\sqrt{7} = \sum_{i=3}^{7}\sqrt{i}$$

13. We contract the sum into sigma notation by noting that it follows the pattern of a sum of fractions with integers 1-19 on top and a denominator increased by 1 (e.g. $\frac{i}{i+1}$):

$$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\cdots+\frac{19}{20} = \sum_{i=1}^{19}\frac{i}{i+1}$$

14. We contract the sum into sigma notation by noting that it follows the pattern of a sum of fractions with integers 3-23 on top and a denominator increased by 4 (e.g. $\frac{i}{i+4}$):

$$\frac{3}{7}+\frac{4}{8}+\frac{5}{9}+\frac{6}{10}+\cdots+\frac{23}{27} = \sum_{i=3}^{23}\frac{i}{i+4}$$

15. We contract the sum into sigma notation by noting that it follows the pattern of a sum of even integers (e.g. $2i$), starting with 2 (e.g. $i=1$), and ending with $2n$ (e.g. $i=n$).

$$2+4+6+8+\cdots+2n = \sum_{i=1}^{n} 2i$$

16. We contract the sum into sigma notation by noting that it follows the pattern of a sum of odd integers (e.g. $2i-1$), starting with 1 (e.g. $i=1$), and ending with $2n-1$ (e.g. $i=n$).

$$1+3+5+7+\cdots+(2n-1) = \sum_{i=1}^{n}(2i-1)$$

17. We contract the sum into sigma notation by noting that it follows the pattern of a sum of powers of 2 (e.g. 2^i), starting with 1 (e.g. $i=0$), and ending with 32 (e.g. $i=5$).

$$1+2+4+8+16+32 = \sum_{i=0}^{5} 2^i$$

18. We contract the sum into sigma notation by noting that it follows the pattern of a sum of fractions with 1 on top and squares of integers on the bottom (e.g. $\frac{1}{i^2}$), starting with $\frac{1}{1}$ (e.g. $i=1$), and ending with $\frac{1}{36}$ (e.g. $i=6$).

$$\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\frac{1}{36} = \sum_{i=1}^{6}\frac{1}{i^2}$$

19. We contract the sum into sigma notation by noting that it follows the pattern of a sum of powers of x (e.g. x^i), starting with x (e.g. $i=1$) and ending with x^n (e.g. $i=n$):

$$x+x^2+x^3+\cdots+x^n = \sum_{i=1}^{n} x^i$$

20. We contract the sum into sigma notation by noting that it follows the pattern of a sum of powers of x (e.g. x^i), starting with $1 = x^0$ (e.g. $i = 0$) and ending with x^n (e.g. $i = n$). In addition, the sign of the terms alternates, so we include $(-1)^n$:
$1-x+x^2-x^3+\cdots+(-1)^n x^n = \sum_{i=0}^{n}(-1)^i x^i$

21. We expand the sum:
$\sum_{i=4}^{8}(3i-2)$
$= [3(4)-2]+[3(5)-2]+[3(6)-2]+[3(7)-2]+[3(8)-2]$
$= (12-2)+(15-2)+(18-2)+(21-2)+(24-2)$
$= 10+13+16+19+22$
$= 80$

22. We expand and simplify the sum:
$\sum_{i=3}^{6} i(i+2)$
$3(3+2)+4(4+2)+5(5+2)+6(6+2)$
$= 3\cdot 5+4\cdot 6+5\cdot 7+6\cdot 8$
$= 15+24+35+48$
$= 122$

23. We expand and simplify the sum:
$\sum_{j=1}^{6} 3^{j+1}$
$= 3^{1+1}+3^{2+1}+3^{3+1}+3^{4+1}+3^{5+1}+3^{6+1}$
$= 3^2+3^3+3^4+3^5+3^6+3^7$
$= 9+27+81+243+729+2187$
$= 3276$

24. We expand and simplify the sum:
$\sum_{k=0}^{8}\cos k\pi = \cos 0\pi+\cos 1\pi+\cos 2\pi+\cos 3\pi+\cos 4\pi+\cos 5\pi+\cos 6\pi+\cos 7\pi+\cos 8\pi$
$= 1-1+1-1+1-1+1-1+1$
$= 1$

25. We expand and simplify the sum. We note that the terms alternate from -1 to 1. Since there are an even number of terms, the sum is 0.
$\sum_{n=1}^{20}(-1)^n$
$= -1+1-1+1-1+1-1+1-1+1-1+1-1+1-1+1-1+1-1+1$
$= 0$

26. We expand and simplify the sum:
$\sum_{i=1}^{100} 4$
$= 4+4+4+\cdots+4$
Since there are exactly 100 terms:
$= 100 * 4$
$= 400$

27. We expand and simplify the sum:
$\sum_{i=0}^{4}(2^i+i^2)$
$= (2^0+0^2)+(2^1+1^2)+(2^2+2^2)+(2^3+3^2)+(2^4+4^2)$
$= (1+0)+(2+1)+(4+4)+(8+9)+(16+16)$
$= 61$

28. We expand and simplify the sum:
$\sum_{i=-2}^{4} 2^{3-i}$
$= 2^5+2^4+2^3+2^2+2^1+2^0+2^{-1}$
$= 32+16+8+4+2+1+0.5$
$= 63.5$ or $\frac{127}{2}$

29. We simplify:
$\sum_{i=1}^{n} 2i$
We can take out 2 because it is a constant ($\sum cx = c\sum x$):

$= 2\sum_{i=1}^{n} i$

We know that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, so:

$= 2 * \frac{n(n+1)}{2}$

$= n(n+1)$

30. We simplify:

$$\sum_{i=1}^{n}(2-5i)$$

We know that we can split up the sums ($\sum(x+y) = \sum x + \sum y$):

$= \sum_{i=1}^{n} 2 - \sum_{i=1}^{n} 5i$

We can take out 5 because it is a constant ($\sum cx = c\sum x$):

$= 2n - 5\sum_{i=1}^{n} i$

We know that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, so:

$= 2n - \frac{5n(n+1)}{2}$

$= \frac{4n}{2} - \frac{5n^2+5n}{2}$

$= -\frac{4n-5n^2-5n}{2}$

$= -\frac{-n-5n^2}{2}$

$= -\frac{n(5n+1)}{2}$

31. First, we will use the fact that the sum passes through terms individually, as well as the fact that the constant can be put in front of the sum:

$$\begin{aligned} A &= \sum_{i=1}^{n}(i^2+3i+4) \\ &= \sum_{i=1}^{n} i^2 + 3\sum_{i=1}^{n} i + \sum_{i=1}^{n} 4 \end{aligned}$$

Now, using equalities from Theorem 3 we get

$$\begin{aligned} A &= \frac{n(n+1)(2n+1)}{6} + 3\frac{n(n+1)}{2} + 4n \\ &= n\left(\frac{(n+1)(2n+1)}{6} + \frac{3(n+1)}{2} + 4\right) \\ &= n\left((n+1)\left(\frac{2n+1}{6} + \frac{3}{2}\right) + 4\right) \\ &= n\left((n+1)\left(\frac{2n+1}{6} + \frac{3}{2}\right) + 4\right) \\ &= n\left((n+1)\frac{2n+1+9}{6} + 4\right) \\ &= n\left(\frac{(n+1)(n+5)}{3} + 4\right) \\ &= \frac{1}{3}n\left(n^2+6n+17\right) \end{aligned}$$

32. Expanding the expression under the sum and then summing term by term, with formulas from theorem 3, we get, step by step:

$$\begin{aligned} A &= \sum_{i=1}^{n}(3+2i)^2 = \sum_{i=1}^{n}(9+12i+4i^2) \\ &= \sum_{i=1}^{n} 9 + 12\sum_{i=1}^{n} i + 4\sum_{i=1}^{n} i^2 \\ &= 9n + 12\frac{n(n+1)}{2} + 4\frac{n(n+1)(2n+1)}{6} \\ &= 9n + 6n(n+1) + \frac{2}{3}n(n+1)(2n+1) \\ &= n\left(9 + 6(n+1) + \frac{2}{3}(n+1)(2n+1)\right) \\ &= n\left(9 + (n+1)\left(6 + \frac{2}{3}(2n+1)\right)\right) \\ &= n\left(9 + (n+1)\left(\frac{4n+20}{3}\right)\right) \\ &= n\left(9 + \frac{4(n+1)(n+5)}{3}\right) \\ &= \frac{1}{3}n(4n^2+24n+47) \end{aligned}$$

33. Find the value of the sum $\sum\limits_{i=1}^{n}(i+1)(i+2)$

After expanding the terms, we have

$\sum\limits_{i=1}^{n}(i+1)(i+2) = \sum\limits_{i=1}^{n}(i^2+3i+2)$

$= \sum_{i=1}^{n} i^2 + 3\sum_{i=1}^{n} i + \sum_{i=1}^{n} 2$

$= [\frac{n(n+1)(2n+1)}{6}] + 3[\frac{n(n+1)}{2}] + 2n$

$= \frac{1}{3}n^3 + 2n^2 + \frac{11}{3}n$

Hence, $\sum_{i=1}^{n}(i+1)(i+2) = \frac{1}{3}n(n^2+6n+11)$

34. Find the value of the sum $\sum_{i=1}^{n} i(i+1)(i+2)$

After expanding the terms, we have

$\sum_{i=1}^{n} i(i+1)(i+2) = \sum_{i=1}^{n}(i^3+3i^2+2i)$

$= \sum_{i=1}^{n} i^3 + 3\sum_{i=1}^{n} i^2 + 2\sum_{i=1}^{n} i$

$= [\frac{n(n+1)}{2}]^2 + 3[\frac{n(n+1)(2n+1)}{6}] + 2[\frac{n(n+1)}{2}]$

$= \frac{1}{4}n^2(n+1)^2 + \frac{1}{2}n(n+1)(2n+1) + n(n+1)$

$= \frac{1}{4}n(n+1)[n^2+n+4n+2+4]$

$= \frac{1}{4}n(n+1)[n^2+5n+6]$

Hence, $\sum_{i=1}^{n} i(i+1)(i+2) = \frac{1}{4}n(n+1)[n^2+5n+6] = \frac{1}{4}n(n+1)(n+2)(n+3)$

35. Find the value of the sum $\sum_{i=1}^{n}(i^3-i-2)$

After expanding the terms, we have

$\sum_{i=1}^{n}(i^3-i-2) = \sum_{i=1}^{n} i^3 - \sum_{i=1}^{n} i - 2\sum_{i=1}^{n} 1$

$= [\frac{n(n+1)}{2}]^2 - [\frac{n(n+1)}{2}] - 2n$

$= \frac{1}{4}n^2(n+1)^2 + \frac{1}{2}n(n+1) - 2n$

$= \frac{1}{4}n[n(n+1)^2 - 2(n+1) - 8]$

Hence, $\sum_{i=1}^{n}(i^3-i-2) = \frac{1}{4}n(n^3+2n^2-n-10)$

36. Find the value of n for the sum $\sum_{i=1}^{n} i = 78$

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Thus,

$\frac{n(n+1)}{2} = 78$

$n^2 + n = 78 \times 2$

$n^2 + n = 156$

$n^2 + n - 156 = 0$

$(n+13)(n-12) = 0$

$n = -13, 12$

Through neglecting the negative value of n, we have

$n = 12$

Hence, $n = 12$

37. Theorem 3(b) defines as

$\sum_{i=1}^{n} c = nc$

The left side can be written as

$\sum_{i=1}^{n} c = c + c + c + c + c.... + c$ (n- times)

Since c has been added n-times, it can be generally multiplied by n as nc.

Hence, $\sum_{i=1}^{n} c = nc$

38. We need to prove $\sum_{i=1}^{n} i^3 = [\frac{n(n+1)}{2}]^2$

Consider $n = 1, k, k+1$

$\sum_{i=1}^{1} i^3 = [\frac{1(1+1)}{2}]^2 = 1$

$\sum_{i=1}^{k} i^3 = [\frac{k(k+1)}{2}]^2$

$\sum_{i=1}^{k+1} (i)^3 = [\frac{k(k+1)}{2}]^2 + (k+1)^3$

$\sum_{i=1}^{k+1} (i)^3 = \frac{1}{4}(k+1)^2(k+2)^2$

$\sum_{i=1}^{k+1} i^3 = [\frac{(k+1)((k+1)+1)}{2}]^2$

Hence, $\sum_{i=1}^{n} i^3 = [\frac{n(n+1)}{2}]^2$

39. $\sum_{i=1}^{n}(1+i)^4 - i^4$

$= (2^4 - 1^4) + (3^4 - 2^4) + (4^4 - 3^4) + ... + [(n+1)^4 - n^4]$

$= (n+1)^4 - 1^4$

$= n^4 + 4n^3 + 6n^2 + 4n$

Also:

$\sum_{i=1}^{n}(1+i)^4 - i^4$

$= \sum_{i=1}^{n}(4i^3 + 6i^2 + 4i + 1)$

$= 4\sum_{i=1}^{n} i^3 + 6\sum_{i=1}^{n} i^2 + 4\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1$

$= 4S + 6(\frac{2n^2+3n^2+n}{6}) + 4(\frac{n^2+n}{2}) + n$

$= 4S + (2n^3 + 3n^2 + n) + (2n^2 + 2n) + (n)$

$= 4S + 2n^3 + 5n^2 + 4n$

We can equate these two expressions:

$4S + 2n^3 + 5n^2 + 4n = n^4 + 4n^3 + 6n^2 + 4n$

$4S = n^4 + 2n^3 + n^2$

$S = \frac{n^2(n^2+2n+1)}{4}$

$S = \frac{n^2(n+1)^2}{4}$

$S = [\frac{n(n+1)}{2}]^2$

Therefore:

$\sum_{i=1}^{n} i^3 = [\frac{n(n+1)}{2}]^2$

40. Area for $G_i = [\frac{i(i+1)}{2}]^2 - [\frac{i(i-1)}{2}]^2$

$= \frac{i^2(i+1)^2}{4} - \frac{i^2(i-1)^2}{4}$

$= \frac{1}{4}i^2[(i+1)^2 - (i-1)^2]$

$= \frac{1}{4}i^2[4i]$

Hence, the area for $G_i = i^3$

So the area of ABCD is $\sum_{i=1}^{n} i^3 = [\frac{n(n+1)}{2}]^2$

41. (a) $\sum_{i=1}^{n}[i^4 - (i-1)^4]$

$= (1^4 - 0^4) + (2^4 - 1^4) + (3^4 - 2^4) + ... + [n^4 - (n-1)^4]$

$= n^4 - 0^4$

$= n^4$

(b) $\sum_{i=1}^{100}(5^i - 5^{i-1})$

$= (5^1 - 5^0) + (5^2 - 5^1) + (5^3 - 5^2) + ... + (5^{100} - 5^{99})$

$= 5^{100} - 5^0$

$= 5^{100} - 1$

(c) $\sum_{i=3}^{99}(\frac{1}{i} - \frac{1}{i+1})$

$= (\frac{1}{3} - \frac{1}{4}) + (\frac{1}{4} - \frac{1}{5}) + (\frac{1}{5} - \frac{1}{6}) + ... + (\frac{1}{99} - \frac{1}{100})$

$= (\frac{1}{3} - \frac{1}{100})$

$= (\frac{100}{300} - \frac{3}{300})$

$= \frac{97}{300}$

(d) $\sum_{i=1}^{n}(a_i - a_{i-1})$

$= (a_1 - a_0) + (a_2 - a_1) + (a_3 - a_2) + ... + (a_n - a_{n-1})$

$= a_n - a_0$

42. $|\sum_{i=1}^{n} a_i| \leq \sum_{i=1}^{n} |a_i|$

Use the triangular inequality $|a + b| \leq |a| + |b|$

Expand the summation on both sides.

$a_1 + a_2 + ... + a_n = a_1 + a_2 + ... + a_n$

Take the absolute values.

$|a_1 + a_2 + ... + a_n| = |a_1 + a_2 + ... + a_n|$

Thus,

$|a_1 + a_2 + ... + a_n| \leq |a_1| + |a_2| + ... + |a_n|$

(Triangular inequality $|a + b| \leq |a| + |b|$)

Hence, $|\sum_{i=1}^{n} a_i| \leq \sum_{i=1}^{n} |a_i|$

43. Find the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}(\frac{i}{n})^2$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}(\frac{i}{n})^2 = \lim_{n\to\infty} \frac{1}{n^3} \sum_{i=1}^{n}(i^2)$

Since, $\sum_{i=1}^{n} i^2 = [\frac{n(n+1)(2n+1)}{6}]$

Thus,

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}(\frac{i}{n})^2 = \lim_{n\to\infty} [\frac{1}{n^3}(\frac{n(n+1)(2n+1)}{6})]$

$= \lim_{n\to\infty} [\frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}]$

$= \frac{1}{3} + 0 + 0$

Hence, $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}(\frac{i}{n})^2 = \frac{1}{3}$

44. Find the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}[(\frac{i}{n})^3 + 1]$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}[(\frac{i}{n})^3 + 1] = \lim_{n\to\infty} (\frac{1}{n^4} \sum_{i=1}^{n} i^3 + \frac{1}{n} \sum_{i=1}^{n} 1)$

$\sum_{i=1}^{n} i^3 = [\frac{n(n+1)}{2}]^2$

Thus,

$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}[(\frac{i}{n})^3+1] = \lim_{n\to\infty}(\frac{1}{n^4}[\frac{n(n+1)}{2}]^2 + \frac{1}{n}(n)$

$= \lim_{n\to\infty}[\frac{1}{4}+\frac{1}{2n}+\frac{1}{4n^2}+1]$

$= \frac{1}{4}+0+0+1$

Hence, $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}[(\frac{i}{n})^3+1] = \frac{5}{4}$

45. Find the limit $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}[(\frac{2i}{n})^3+5(\frac{2i}{n})]$

$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}[(\frac{2i}{n})^3+5(\frac{2i}{n})] = \lim_{n\to\infty}(\frac{16}{n^4}\sum_{i=1}^{n}i^3 + \frac{20}{n^2}\sum_{i=1}^{n}i)$

$\sum_{i=1}^{n}i^3 = [\frac{n(n+1)}{2}]^2$

Thus,

$= \lim_{n\to\infty}[\frac{16}{n^4}(\frac{n(n+1)}{2})^2 + \frac{20}{n^2}(\frac{n(n+1)}{2})]$

$= \lim_{n\to\infty}[4+\frac{8}{n}+\frac{4}{n^2}+10+\frac{10}{n}]$

$= 4+0+0+10+0$

Hence, $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}[(\frac{2i}{n})^3+5(\frac{2i}{n})] = 14$

46. Find the limit $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}[(1+\frac{3i}{n})^3-2(1+\frac{3i}{n})]$

$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}[(1+\frac{3i}{n})^3-2(1+\frac{3i}{n})] = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}[(-1+\frac{3i}{n}+\frac{27i^2}{n^2}+\frac{27i^3}{n^3}]$

$\lim_{n\to\infty}[-\frac{3}{n}\sum_{i=1}^{n}1+\frac{9}{n^2}\sum_{i=1}^{n}i+\frac{81}{n^3}\sum_{i=1}^{n}i^2+\frac{81}{n^4}\sum_{i=1}^{n}i^3$

$\sum_{i=1}^{n}i^2 = [\frac{n(n+1)(2n+1)}{6}], \sum_{i=1}^{n}i^3 = [\frac{n(n+1)}{2}]^2$

Thus,

$= \lim_{n\to\infty}[-3+\frac{9}{2}+\frac{9}{2n}+27+\frac{81}{2n}+\frac{27}{2n^2}+\frac{81}{4}+\frac{81}{2n}+\frac{81}{4n^2}]$

$= \lim_{n\to\infty}[-3+\frac{9}{2}+0+27+0+0+\frac{81}{4}+0+0]$

Hence, $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}[(1+\frac{3i}{n})^3-2(1+\frac{3i}{n})] = \frac{195}{4}$

47. Consider $\sum_{i=1}^{n}ar^{i-1} = a+ar+ar^2+....+ar^{n-1}$

$r\times\sum_{i=1}^{n}ar^{i-1} = r\times(a+ar+ar^2+....+ar^{n-1})$

$r\times\sum_{i=1}^{n}ar^{i-1} = ar+ar^2+....+ar^{n-1}+ar^n$

$r\sum_{i=1}^{n}ar^{i-1} - \sum_{i=1}^{n}ar^{i-1} = ar^n - a$

$(r-1)\sum_{i=1}^{n}ar^{i-1} = a(r^n-1)$

$\sum_{i=1}^{n}ar^{i-1} = \frac{a(r^n-1)}{r-1}$

Hence, $\sum_{i=1}^{n}ar^{i-1} = a+ar+ar^2+....+ar^{n-1} = \frac{a(r^n-1)}{r-1}$

48. Evaluate $\sum_{i=1}^{n}\frac{3}{2^{i-1}}$

$\sum_{i=1}^{n}\frac{3}{2^{i-1}} = \frac{3}{2^0}+\frac{3}{2^1}+\frac{3}{2^2}+....+\frac{3}{2^n}$

$\sum_{i=1}^{n}\frac{3}{2^{i-1}} = \frac{3}{2^0}+\sum_{i=1}^{n-1}\frac{3}{2^i}$

Here, $\sum_{i=1}^{n-1}\frac{3}{2^i}$ shows a geometric series with first term $a=\frac{1}{2}$ and common ratio $r=\frac{1}{2}$.

Therefore,

$\sum_{i=1}^{n}\frac{3}{2^{i-1}} = 3+\frac{\frac{1}{2}(\frac{1}{2}^n-1)}{\frac{1}{2}-1}$

$= 3+3(1-\frac{1}{2}^{(n-1)})$

$= 3+3-3(\frac{1}{2}^{(n-1)})$

Hence, $\sum_{i=1}^{n}\frac{3}{2^{i-1}} = 6-3(\frac{1}{2})^{n-1}$

49. Evaluate $\sum_{i=1}^{n}(2i+2^i)$

$\sum_{i=1}^{n}(2i+2^i) = \sum_{i=1}^{n}2i+\sum_{i=1}^{n}2^i$

Here, $\sum_{i=1}^{n}2^i$ shows a geometric series with first term $a=1$ and common ratio $r=2$.

Therefore,

$\sum_{i=1}^{n}\frac{3}{2^{i-1}} = 2[\frac{n(n+1)}{2}]+\frac{(2^n-1)}{2-1}$

$= n(n+1)+\frac{(2^n-1)}{2-1}$

$= n^2 + n + 2^n - 1$

50. Evaluate $\sum_{i=1}^{m}[\sum_{j=1}^{n}(i+j)]$

$\sum_{i=1}^{m}[\sum_{j=1}^{n}(i+j)] = i\sum_{j=1}^{n}1 + \sum_{j=1}^{n}j$

$= n[\frac{m(m+1)}{2}] + m[\frac{n(n+1)}{2}]$

$= \frac{1}{2}mn[(m+1)+(n+1)]$

Hence, $\sum_{i=1}^{m}[\sum_{j=1}^{n}(i+j)] = \frac{1}{2}mn(m+n+2)$

第六章　Appendix G: The Logarithm Defined as an Integral

6.1 Answer

1. This answer hasn't been written yet!
2. This answer hasn't been written yet!
3. This answer hasn't been written yet!
4. This answer hasn't been written yet!
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6.2 Step by Step

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