Online Appendices: Balancing Policy Objectives in Social Security Reform

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Online Appendix A: Complete Optimization Model Specification

A.1 Sets

$$C = \{1, 2, \ldots, 142\} \quad \text{Set of Courses of Action (COAs)} \tag{1}$$

$$K = \{ICB, T, E, S, AF, PV\} \quad \text{Set of policy constructs} \tag{2}$$

$$L = \{1, 2, 3, 4\} \quad \text{Set of Large Language Models} \tag{3}$$

A.2 Parameters and Data

$$R_{i,c,m} \in [-5, 5] \quad \text{Rating of COA } i \text{ by model } m \text{ on construct } c \tag{4}$$

$$A_i \in \mathbb{R} \quad \text{Actuarial impact of COA } i \text{ (\% of payroll)} \tag{5}$$

$$T_i \in \mathbb{R}^+ \quad \text{Target performance for COA } i \tag{6}$$

A.3 Stochastic Components

Construct-level weights:

$$\mathbf{w} = (w_{ICB}, w_T, w_E, w_S, w_{AF}, w_{PV}) \sim \text{Dirichlet}(\alpha, \alpha, \alpha, \alpha, \alpha, \alpha) \tag{7}$$

where $\alpha = 2$ for all constructs, ensuring $\sum_{c \in K} w_c = 1$ and $w_c \geq 0 \; \forall c \in K$.

Model-level weights within constructs:

$$\mathbf{b}_c = (b_{c,1}, b_{c,2}, b_{c,3}, b_{c,4}) \sim \text{Dirichlet}(\beta, \beta, \beta, \beta) \quad \forall c \in K \tag{8}$$

where $\beta = 2$ for all models, ensuring $\sum_{m \in L} b_{c,m} = 1$ and $b_{c,m} \geq 0 \; \forall m \in L, \forall c \in K$.

Combined goal weights:

$$y_{c,m} = w_c \cdot b_{c,m} \quad \forall c \in K, \forall m \in L \tag{9}$$

Composite COA scores:

$$S_i = \sum_{c \in K} \sum_{m \in L} y_{c,m} \cdot R_{i,c,m} = \sum_{c \in K} w_c \left(\sum_{m \in L} b_{c,m} \cdot R_{i,c,m} \right) \quad \forall i \in C \tag{10}$$

Penalty parameter:

$$\lambda \sim \text{Uniform}(0.2, 0.7) \tag{11}$$

Maximum portfolio size:

$$N_{\text{max}} \sim \text{Discrete Uniform}\{1, 2, 3, 4, 5, 6\} \tag{12}$$

A.4 Decision Variables

$x_i \in \{0, 1\}$	Binary selection variable for COA i	(13)
	$x_i = 1$ if COA i is selected, 0 otherwise	(14)
$d_i^- \geq 0$	Negative deviation from target for COA i	(15)

A.5 Objective Function

$$\max Z = \sum_{i \in C} S_i \cdot x_i - \lambda \sum_{i \in C} d_i^- \tag{16}$$

The objective maximizes the total weighted contribution of selected COAs minus penalties for negative deviations from aspirational targets.

A.6 Constraints

Actuarial balance requirement:

$$\sum_{i \in C} A_i \cdot x_i \geq 3.5 \tag{17}$$

This hard constraint ensures the selected portfolio achieves at least 3.5% of taxable payroll in 75-year actuarial balance, satisfying the statutory solvency requirement.

Privatization exclusivity:

$$\sum_{i=141}^{142} x_i \leq 1 \tag{18}$$

At most one privatization proposal (COAs 141 or 142) may be selected, as these represent fundamentally incompatible approaches.

Portfolio cardinality:

$$\sum_{i \in C} x_i \leq N_{\max} \tag{19}$$

The total number of selected COAs cannot exceed the stochastically determined maximum portfolio size.

Goal deviation tracking:

$$S_i \cdot x_i - d_i^- \leq T_i \quad \forall i \in C \tag{20}$$

These constraints capture the extent to which selected COAs fall short of aspirational performance targets. Only negative deviations are penalized; overachievement is not discouraged.

Non-negativity:

$$d_i^- \geq 0 \quad \forall i \in C \tag{21}$$

A.7 Complete Model Formulation

The complete Mixed Integer Goal Programming model is:

$$\max \quad Z = \sum_{i=1}^{142} S_i \cdot x_i - \lambda \sum_{i=1}^{142} d_i^- \tag{22}$$

$$\text{subject to} \quad \sum_{i=1}^{142} A_i \cdot x_i \geq 3.5 \tag{23}$$

$$\sum_{i=141}^{142} x_i \leq 1 \tag{24}$$

$$\sum_{i=1}^{142} x_i \leq N_{\max} \tag{25}$$

$$S_i \cdot x_i - d_i^- \leq T_i \quad \forall i \in \{1, \ldots, 142\} \tag{26}$$

$$x_i \in \{0, 1\} \quad \forall i \in \{1, \ldots, 142\} \tag{27}$$

$$d_i^- \geq 0 \quad \forall i \in \{1, \ldots, 142\} \tag{28}$$

where:

- S_i is computed via Equation (10) using stochastically sampled weights
- λ is sampled from Uniform(0.2, 0.7) for each simulation run
- $N_{\max}$ is sampled from Discrete Uniform$\{1, 2, 3, 4, 5, 6\}$ for each run
- All other parameters are data inputs or derived quantities

Online Appendix B: Simulation Algorithm

B.1 Monte Carlo Simulation Procedure

For each simulation run $r = 1, 2, \ldots, 100$:

Step 1: Sample Construct Weights

$$\mathbf{w}^{(r)} = (w_{ICB}^{(r)}, w_T^{(r)}, w_E^{(r)}, w_S^{(r)}, w_{AF}^{(r)}, w_{PV}^{(r)}) \sim \text{Dirichlet}(2, 2, 2, 2, 2, 2) \tag{29}$$

Step 2: Sample Model Weights for Each Construct

For each construct $c \in \{ICB, T, E, S, AF, PV\}$:

$$\mathbf{b}_c^{(r)} = (b_{c,1}^{(r)}, b_{c,2}^{(r)}, b_{c,3}^{(r)}, b_{c,4}^{(r)}) \sim \text{Dirichlet}(2, 2, 2, 2) \tag{30}$$

Step 3: Compute Combined Weights

For each construct c and model m:

$$y_{c,m}^{(r)} = w_c^{(r)} \cdot b_{c,m}^{(r)} \tag{31}$$

Step 4: Compute Composite COA Scores

For each COA i:

$$S_i^{(r)} = \sum_{c \in K} \sum_{m \in L} y_{c,m}^{(r)} \cdot R_{i,c,m} \tag{32}$$

Step 5: Sample Penalty Parameter

$$\lambda^{(r)} \sim \text{Uniform}(0.2, 0.7) \tag{33}$$

Step 6: Sample Maximum Portfolio Size

$$N_{\max}^{(r)} \sim \text{Discrete Uniform}\{1, 2, 3, 4, 5, 6\} \tag{34}$$

Step 7: Solve Optimization Model

Solve the MIGP formulation from Appendix A using lpSolveAPI with parameters $S_i^{(r)}$, $\lambda^{(r)}$, and $N_{\max}^{(r)}$ to obtain optimal solution $\mathbf{x}^{(r)}$.

Step 8: Record Results

Store:

- Selected COA set: $\{i : x_i^{(r)} = 1\}$
- Objective value: $Z^{(r)}$
- Actuarial balance achieved: $\sum_i A_i \cdot x_i^{(r)}$
- Deviation variables: $\mathbf{d}^{-(r)}$
- Weight vectors: $\mathbf{w}^{(r)}$ and $\{\mathbf{b}_c^{(r)}\}_{c \in K}$

B.2 Post-Processing and Analysis

After completing all 100 runs:

Selection Frequency Analysis:

$$f_i = \frac{1}{100} \sum_{r=1}^{100} x_i^{(r)} \quad \forall i \in C \tag{35}$$

where f_i represents the proportion of runs in which COA i was selected.

Jaccard Similarity for Solution Stability:

For runs r and s with the same $N_{\max}$ value:

$$J(r, s) = \frac{|\{i : x_i^{(r)} = 1\} \cap \{i : x_i^{(s)} = 1\}|}{|\{i : x_i^{(r)} = 1\} \cup \{i : x_i^{(s)} = 1\}|} \tag{36}$$

Co-Selection Correlation:

For COAs i and j:

$$\rho_{ij} = \mathrm{Corr}\left(\{x_i^{(r)}\}_{r=1}^{100}, \{x_j^{(r)}\}_{r=1}^{100}\right) \tag{37}$$

Positive ρ_{ij} indicates complementary policies frequently selected together; negative ρ_{ij} indicates substitutes.

B.3 Convergence Assessment

To verify that 100 runs provide sufficient sampling:

1. Compute average Jaccard similarity within each $N_{\max}$ category
2. Assess coefficient of variation for selection frequencies of top 20 COAs
3. Compare selection patterns between first 50 and last 50 runs

Results showing high Jaccard similarity (> 0.85) and low coefficient of variation (< 0.15) confirm adequate sampling.

Online Appendix C: Inter-Rater Reliability Statistics

C.1 Spearman Rank Correlation

For models m and m' within construct c:

$$\rho_s(c; m, m') = 1 - \frac{6\sum_{i=1}^{142}(r_{i,c,m} - r_{i,c,m'})^2}{142(142^2 - 1)} \tag{38}$$

where $r_{i,c,m}$ denotes the rank of COA i by model m on construct c.

C.2 Weighted Cohen's Kappa

For models m and m' within construct c, with rating scale $\{-5, -4, \ldots, 4, 5\}$:

$$\kappa_w = \frac{p_o - p_e}{1 - p_e} \tag{39}$$

where:

$$p_o = \frac{1}{N}\sum_{i=1}^{N} w(R_{i,c,m}, R_{i,c,m'}) \quad \text{(observed weighted agreement)} \tag{40}$$

$$p_e = \sum_{k=-5}^{5}\sum_{l=-5}^{5} w(k,l) \cdot p_k^{(m)} \cdot p_l^{(m')} \quad \text{(expected weighted agreement)} \tag{41}$$

The weight function for ordinal ratings:

$$w(k,l) = 1 - \frac{|k-l|}{10} \tag{42}$$

This assigns partial credit for near-agreement: ratings differing by 1 point receive weight 0.9, differing by 2 points receive 0.8, etc.

C.3 Interpretation Guidelines

Following standard conventions:

- $\kappa < 0.20$: Slight agreement
- $0.20 \leq \kappa < 0.40$: Fair agreement
- $0.40 \leq \kappa < 0.60$: Moderate agreement
- $0.60 \leq \kappa < 0.80$: Substantial agreement
- $\kappa \geq 0.80$: Almost perfect agreement