Problem Set 1

Your Name
Phil 411: Type Theory

For each of the following strings of symbols, state whether it is or is not a term. Use our syntactic convenience of writing $(\lambda x.A)$ without the outermost parentheses in A, but do not omit any other parentheses.

Examples. $(\lambda v.v'v)$

Term

$\lambda v.v'v$

Not a term

1. (vv)

Term / Not a term

2. $(v'v)v$

Term / Not a term

3. $(\lambda v.v)$

Term / Not a term

4. $(\lambda v.v'v')$

Term / Not a term

5. $(\lambda v.vv'v')$

Term / Not a term

6. $(\lambda v.v(v'v'))$

Term / Not a term

In the following strings of symbols, identify the free variables by putting a * in front of them (which makes them turn green). You can edit the terms in place; you don't need to copy them or anything.

Example. $(v(\lambda v.vv'))$

Answer: $(v(\lambda v.vv'))$

7. $(\lambda v'.v'v)$

8. $((vv')(\lambda v.v'v))$

9. $((\lambda v.vv')v)$

Complete the following substitutions. I now omit outermost parentheses.

Example. $v(\lambda v.vv')[(v'v')/v]$

$(v'v')(\lambda v.(v'v')v')$

10. $v'v[v'/v]$

11. $v'v[(\lambda v.vv)/v']$

12. $(v'v)(\lambda v'.v'v)[(\lambda v.vv)/v']$

β-reduce the following terms until you cannot perform any more β-conversions. You don't need to provide the intermediate steps, but I include them in the example for illustration.

Example. $(\lambda v'.(\lambda v.v'v)(v''v''))(v''')$

$$v'''(v''v''')$$

There are two chains of β-conversions that can reach this term:

$$\begin{array}{lllll}(\lambda v'.(\lambda v.v'v)(v''v'))(v''') & \to_\beta & (\lambda v.v'''v)(v''v''') & \to_\beta & v'''(v''v''') \\ (\lambda v'.(\lambda v.v'v)(v''v'))(v''') & \to_\beta & (\lambda v'.v'(v''v'))(v''') & \to_\beta & v'''(v''v''')\end{array}$$