193. Let $X \sim \text{gamma}(\nu, \beta)$, where $\nu > 0$ is the shape parameter and $\beta > 0$ is the scale parameter.

(a) Prove the moment generating function of X is

$$M_x(t) = \left(\frac{1}{1-\beta t}\right)^{\nu}, \quad t < \frac{1}{\beta}.$$

(b) Now suppose $X_1, ..., X_n$ are independently distributed according to a $\text{gamma}(\nu_i, \beta)$ distribution; that is, each X_i has a different shape parameter ν_i, but all have equal scale parameter β. Use the moment generating function to find the distribution of $Y = \sum_{i=1}^{n} X_i$.