

43. Let  $X_1, \dots, X_n$  be independent with probability density functions (PDFs)

$$f_{X_i}(x|\theta) = \exp\{i\theta - x\}I_{[i\theta, \infty]}(x).$$

(a) Prove that  $T(\mathbf{X}) = \min_i\{X_i/i\}$  is a sufficient statistic for  $\theta$ .

(b) Based on  $T$ , find the  $1 - \alpha$  confidence interval for  $\theta$  of the form  $[T + a, T + b]$  which is of minimum length.