

81. If X and Y have means μ_X and μ_Y and finite variances σ_X^2 and σ_Y^2 , use the Cauchy-Schwarz Inequality (given below) to prove that the correlation ρ_{XY} between X and Y is less than one in absolute value.

Cauchy-Schwarz Inequality: For any two random variables X and Y

$$|E[XY]| \leq E[|XY|] \leq (E|X|^2)^{1/2}(E|Y|^2)^{1/2}$$

Note: The Cauchy-Schwarz Inequality is a special case of Hölder's inequality which states

Hölder's inequality: For any two random variables X and Y

$$|E[XY]| \leq E[|XY|] \leq (E|X|^p)^{1/p}(E|Y|^q)^{1/q}$$

where p and q are positive numbers satisfying $p^{-1} + q^{-1} = 1$.