49. Let X $\sim$ geometric(p) distribution.

(a) Prove the mean $\mu = p^{-1}$.

(b) Prove the moment generating function $M_X(t) = pe^t/\{1-(1-p)e^t\}$.

(c) Use $M_X(t)$ to find the second moment of the distribution, and use that to prove $Var(X) = (1-p)/p^2$.

(d) Prove the geometric distribution is memoryless.