32. Let μ_n denote the n-th central moment of a random variable X. Two quantities of interest, in addition to the mean and variance are

$$\alpha_3 = \frac{\mu_3}{(\mu_2)^{3/2}} \quad \text{and} \quad \alpha_4 = \frac{\mu_4}{\mu_2^2}.$$

The value α_3 is called the skewness and α_4 is called the kurtosis. The skewness measures the lack of symmetry in the PDF. The kurtosis, although harder to interpret, measures the peakedness or flatness of the PDF. If a PDF is symmetric, then $\alpha_3 = 0$.

(a) Calculate α_3 for $f(x) = e^{-x}, x \geq 0$, a PDF that is skewed to the right. Hint: Use the definition of the gamma function found in equations (3.3.2) through (3.3.4) on page 99 in Chapter 3 of your text.

(b) Calculate α_4 for each of the following PDFs and comment on the peakedness of each. Use R to draw a plot for each of the PDFs. If you recognize a distribution as having a specific form (family) and know the mean and variance for that distribution, you may use that result without proof as long as you state the name of the distribution's family.

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \quad -\infty < x < \infty$$
$$f(x) = \frac{1}{2}, \quad -1 < x < 1$$
$$f(x) = \frac{1}{2} e^{-|x|}, \quad -\infty < x < \infty$$