All Questions for Prelim Preparation

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Homework 1

1. The five words in the list below are names given to different types of results in mathematics, statistics, and other sciences. Look up and write the definitions of the words. Explain any hierarchy of the terms. Be sure to cite any references you use

   (a) Axiom

   (b) Theorem

   (c) Proposition (the mathematical term)

   (d) Lemma

   (e) Corollary

2. DeMorgan's Laws are stated below.
   DeMorgan's Laws. Consider a sample space $\mathcal{S}$, and any Borel field $\mathcal{B}$ of $\mathcal{S}$. Then for any two events A and B in $\mathcal{B}$,

   $$(A \cup B)^c = A^c \cap B^c$$
   $$(A \cap B)^c = A^c \cup B^c$$

   (a) Draw clearly labeled Venn diagrams to illustrate both parts of DeMorgan's Laws, one diagram for each law. Do not put both laws on the same diagram.

   (b) Prove DeMorgan's laws. Note: The Venn diagram is not a proof. It is a tool to help you visulize how you would prove the results

3. The notation $B \backslash AB$ is notation for subtracting (or removing) the set AB from the set B. Draw a Venn diagram illustrating

   $$BA^c = B \backslash AB,$$

   and prove the relation.

4. For a finite sample space $\mathcal{S}$, prove the power set $\mathcal{P}_{\mathcal{S}}$ is a σ-algebra.

5. Let $\mathcal{A}$ be a Borel field on a sample space $\mathcal{S}$, and let $A \in \mathcal{A}$. Define

   $$\mathcal{A}_B = \{C : C \in B \cap A\}.$$

   Prove $\mathcal{A}_{\mathcal{B}}$ is a Borel field.

6. Consider rolling two fair dice one at a time. The sample space of this experiment was given during the lectures on set theory. Let $\mathcal{P}_{\mathcal{S}}$ represent the power set on $\mathcal{S}$. Finally, define the function P by

   $$P(A) = \frac{C(A)}{36} \quad \text{for } A \in \mathcal{P}_{\mathcal{S}}.$$

   Prove that $P(A)$ satisfies Kolmogorov's Axioms.

7. Use Proof by Induction to prove Boole's inequality for a finite numbr m of sets.