27. Let $X_1, ..., X_n$ be a random sample from a normal population with mean μ and known variance σ^2. Consider testing

$$H_0 : \mu = \mu_0 \quad \text{versus} \quad H_1 : \mu \neq \mu_0.$$

(a) Find the maximum likelihood estimator $\hat{\mu}$ of μ.

(b) Derive the likelihood ratio test (LRT). Use R to create a graph of the LRT illustrating the rejection region.

(c) Show that the LRT rejection region can be simplified to rejecting H_0 for all samples $\{\boldsymbol{x} : |\bar{x} - \mu_0| \geq K\}$, and find an expression for K in terms of c in the LRT.

(d) Use the result in part (c) to derive the power function. Use R to plot the power function with $\alpha = 0.05$ for $\mu_0 = 0$ and $n = 1$, 4, 16, 64, and 100. Include your plot with your work, and your annotated code at the end of your work.