126. Let $X_1, X_2, ..., X_n$ be IID normal(μ, σ^2), where $\mu \in \mathbb{R}$ is the mean of the normal distribution and $\sigma^2 > 0$ is the variance.

(a) Show the normal distribution is an exponential family.

(b) Find a complete minimal sufficient statistic for (μ, σ^2).

(c) Let $\bar{X}$ and S^2 denote the sample mean and sample variance, respectively, defined by

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \text{ and } S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2.$$

Suppose the parameter σ^2 is known. Prove $\bar{X}$ and S^2 are independent.

(d) Use the Derived Distribution approach to prove

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

has a Student's T distribution with $n-1$ degrees of fredom.