105. A natural ancillary statistic in most problems is the sample size. For example, let N be a random variable having values 1, 2, ... with known probabilities $p_1, p_2, ...$ where $\sum p_i = 1$. Having observed $N = n$, perform n Bernoulli trials with success probability θ, getting X successes.

(a) Prove that the pair (X, N) is minimal sufficient and N is ancillary for θ.

(b) Prove that the estimator X/N is unbiased for θ and has variance $\theta(1-\theta)\mathrm{E}[N^{-1}]$.