34. Let X have a negative binomial distribution with PDF

$$f(x) = \binom{r+x-1}{x} p^r (1-p)^x, \quad x = 0, 1, 2...$$

where $0 < p < 1$ and $r > 0$ is an integer.

(a) Calculate the MGF of X.

(b) Define a new random variable $Y = 2pX$. Show that as $p \to 0$, the MGF of Y converges to a chi-square random variable with $2r$ degrees of freedom by showing that

$$\lim_{p \to 0} M_Y(t) = \left(\frac{1}{1-2t} \right)^r, \quad |t| < \frac{1}{2}$$