

124. Prove the following theorem. Sufficiency for Exponential Families. Let X_1, X_2, \dots, X_n be independent and identically distributed (IID) random variables from a probability mass function (PMF) or probability density function (PDF) $f(x | \boldsymbol{\theta})$, where $\boldsymbol{\theta} = (\theta_1, \dots, \theta_k)$ that belongs to an exponential family given by

$$f(x | \boldsymbol{\theta}) = h(x)c(\boldsymbol{\theta}) \exp \left\{ \sum_{j=1}^k \omega_j(\boldsymbol{\theta}) t_j(x) \right\}$$

Then

$$T(\mathbf{X}) = \left(\sum_{i=1}^n t_1(X_i), \dots, \sum_{i=1}^n t_k(X_i) \right)$$

is a sufficient statistic for $\boldsymbol{\theta}$.