154. Let $X_1, ..., X_n$ be IID normal(θ, 1), and let θ_0 be a specified value of θ.

(a) Find the UMP size α test of $H_0 : \theta \geq \theta_0$ versus $H_1 : \theta < \theta_0$. Write the rejection region in terms of the sample mean.

(b) Prove (or disprove) that the test in part (a) is an unbiased test.

(c) Show that the test in part (a) can be derived as a likelihood ratio test. Use R to draw the likelihood ratio test statistic for $\alpha = 0.05$, and $n = 2$, 5, and 10. Include your annotated code at the end of your solutions.

(d) Show that there does not exist a UMP size α test of $H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$.