$$f(x) = \frac{1}{2}, \quad -1 < x < 1$$
$$f(x) = \frac{1}{2}e^{-|x|}, \quad -\infty < x < \infty$$

33. Find the MGF and use it to find E[X] and Var(X) if

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, \quad -\infty < x < \infty; -\infty < \mu < \infty, \sigma > 0.$$

34. Let X have a negative binomial distribution with PDF

$$f(x) = \binom{r+x-1}{x} p^r (1-p)^x, \quad x = 0, 1, 2...$$

where $0 < p < 1$ and $r > 0$ is an integer.

(a) Calculate the MGF of X.

(b) Define a new random variable $Y = 2pX$. Show that as $p \to 0$, the MGF of Y converges to a chi-square random variable with $2r$ degrees of freedom by showing that

$$\lim_{p \to 0} M_Y(t) = \left(\frac{1}{1-2t}\right)^r, \quad |t| < \frac{1}{2}$$

Homework 5

35. Let U have a discrete uniform distribution on the integers a and b where $a < b$. Prove the variance is

$$\text{Var}(U) = \frac{(b-a+1)^2 - 1}{12}$$

36. Prove the binomial(n, p) probability mass function (PMF) sums to one.

37. Let Y have a binomial(n, p) distribution. Using Var(Y) = E[Y^2] - μ^2, prove Var(Y) = $np(1-p)$. You may use $\mu = np$ without proof.

38. Let $X_1, X_2, ..., X_n$ be independent Bernoulli(p), and define Y = $\Sigma_{i=1}^n X_i$. Use these facts to prove the following

(a) Var(Y) = $np(1-p)$. Hint: For independent random variables, the variance of a sum is the sum of the variances.

(b) $M_Y(t) = (1-p+pe^t)^n$. Hint: If W_1 and W_2 are independent random variables E[W_1W_2] = E[W_1]E[W_2]. This is generalizable to n mutually exclusive random variables.

39. For a specific professor in the Physics department, 30% of students withdraw from his elementary physics course after the first test. The day of the first exam, there are 50 students in his class. You can assume that students decide whether or not to withdraw from the course independently of one another.

(a) What is the random variable, and what is the distribution? Specifically, what is the name of the distribution and the values of the distribution's parameter?

(b) What is the probability that 10 students withdraw after the first exam?

(c) How many students does he expect will withdraw after the first exam?

(d) What is the probability that at least 40 students remain in the class after the first exam?

40. A standard drug is known to be effective in 80% of the cases in which it is used. A new drug is tested on 100 patients and found to be effective in 85 cases. Is the new drug superior?

41. A manufacturer receives a lot of 100 parts from a vendor. The lot will be unacceptable if more than five of the parts are defective. The manufacturer will randomly select K parts from the lot for inspection and the lot will be accepted if no defective parts are found in the sample. Use R for all computations.