83. Let X and Y be independent, identically distributed zero-mean normal random variables with variance σ^2.

(a) Find the joint distribution of U and V where

$$U = X^2 + Y^2, \quad \text{and} \quad V = \frac{X}{\sqrt{X^2 + Y^2}}$$

(b) Show U and V are independent

(c) Create the following graph. It is permissible to do this by-hand as long as you're careful and neat.

i. Create the Euclidean plane, and draw some (x, y) as a point on the plane.

ii. Illustrate what $u = x^2 = y^2$ represents with respect to (x, y).

iii. Do likewise for $v = w\sqrt{u}$.