- 110. One advantage of using a minimal sufficient statistic is that unbiased estimators will have smaller variance, as the following will show. Suppose that  $T_1$  is sufficient and  $T_2$  is minimal sufficient, U is an unbiased estimator of  $\theta$  and define  $U_1 = \mathrm{E}[U|T_1]$  and  $U_2 = \mathrm{E}[U|T_2]$ .
  - (a) Show that  $U_2 = E[U_1|T_2]$ .
  - (b) Now use the conditional variance formula to show that  $Var(U_2) \leq Var(U_1)$ .