50. Suppose that $X_1, ..., X_n$ are IID Poisson(λ). Find the uniformly minimum variance unbiased estimator (UMVUE) and find the maximum likelihood estimator (MLE) of

(a) $P(X = 0)$.

(b) $P(X = 1)$.

(c) For parts (a) and (b), calculate the asymptotic relative efficiency (ARE) for the UMVUEs with respect to the MLE.

(d) A preliminary test of a possible carcinogenic compound can be performed by measuring the mutation rate of microorganisms exposed to the compound. An experimenter places the compound in 15 petri dishes and records the following number of mutant colonies.

$$10, 7, 8, 13, 8, 9, 5, 7, 6, 8, 3, 6, 6, 3, 5.$$

Estimate the probability that no mutant colonies emerge and the probability that one mutant colony will emerge using both the MLE and the UMVUE. In this experiment, which would you prefer? Why?