71. Let $\boldsymbol{X} = (X_1, ..., X_n)$ be IID gamma(ν, θ), where the shape parameter $\nu > 0$ is known and the scale parameter $\theta > 0$ is unknown. Consider testing

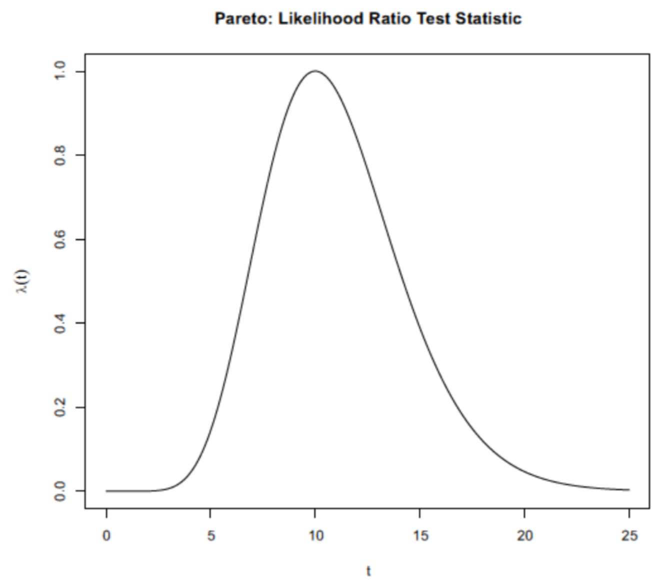
$$H_0 : \theta \leq \theta_0 \quad \text{and} \quad H_1 : \theta > \theta_0.$$

You may use without proof $T(\boldsymbol{X}) = \sum_{i=1}^{n} X_i$ is a complete minimal sufficient statistic for θ and that maximum likelihood estimator of θ is $\hat{\theta} = \bar{X}/\nu$.

(a) Find the size α likelihood ratio test (LRT).

(b) Using the axes in the figure below, sketch the LRT statistic and illustrate the rejection region. Label the axes. Show which regions are the null and alternative parameter spaces. The more relevant information, the better!

Figure 3: Likelihood ratio test statistic $\lambda(t)$ as a function of the sufficient statistic $T(\boldsymbol{x})$ given in equation (1).



(c) Using the Karlin-Rubin Theorem, find the size α uniformly most powerful (UMP) test.

(d) Use the axes in the figure below to sketch the power function of the UMP test. Label the axes. Show which regions correspond to the null and alternative parameter space. Also illustrate the type I error probability.

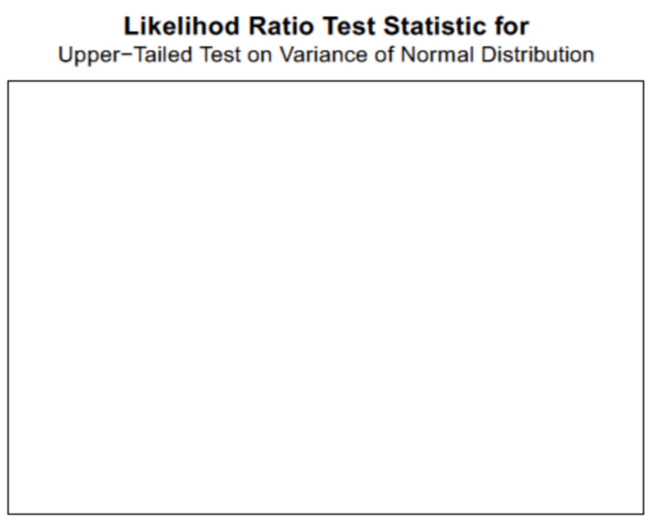


Figure 1: Axes for graph of likelihood ratio test statistic.

(e) For the size α UMP test, find a valid p-value.

(f) Find the size $(1 - \alpha)$ uniformly most accurate (UMA) confidence bound for θ.

(g) Derive the approximate size α Wald test.

(h) Using the Wald test, find an approximate $(1 - \alpha)$ confidence bound for θ.