

60. For any sets of constants $\mathbf{a} = (a_1, \dots, a_k)$ and $\mathbf{b} = (b_1, \dots, b_k)$, show that under the oneway ANOVA assumptions

$$\text{Cov} \left(\sum_{i=1}^k a_i \bar{Y}_i, \sum_{i=1}^k b_i \bar{Y}_i \right) = \sigma^2 \sum_{i=1}^k \frac{a_i b_i}{n_i}$$

Therefore, in the oneway ANOVA cell means model, contrasts are uncorrelated (orthogonal) if $\sum_{i=1}^k a_i b_i / n_i = 0$.