

104. Prove the following theorem. Sufficiency for Exponential Families. Let X_1, X_2, \dots, X_n be iid observations from a PDF or PMF $f(x|\boldsymbol{\theta})$ that belongs to an exponential family given by

$$f(x|\boldsymbol{\theta}) = h(x)c(\boldsymbol{\theta}) \exp \left\{ \sum_{i=1}^k \omega_i(\boldsymbol{\theta}) t_i(x) \right\},$$

where $\boldsymbol{\theta} = (\theta_1, \theta_2, \dots, \theta_k), d \leq k$. Then

$$T(\mathbf{X}) = \left(\sum_{j=1}^n t_1(X_j), \dots, \sum_{j=1}^n t_k(X_j) \right)$$

is a sufficient statistic for $\boldsymbol{\theta}$.