

158. Let X_1, \dots, X_n be IID Bernoulli(p). Derive a $1 - \alpha$ upper confidence bound for p by inverting a test based on $T(\mathbf{X}) = \sum_{i=1}^n X_i$ for

$$H_0 : p = p_0 \text{ versus } H_1 : p < p_0$$

Specifically, if $\mathbf{X} = (X_1, \dots, X_n)$ find an expression for $U(\mathbf{X})$ in an interval of the form $[0, U(\mathbf{X})]$, where $P_p(p \in (0, U(\mathbf{X}))) \geq 1 - \alpha$.