101. Let $X_i, i = 1, 2, ...$, be independent Bernoulli(p) random variables and let $Y_n = n^{-1} \sum_{i=1}^{n} X_i$.

(a) Show that $\sqrt{n}(Y_n - p) \xrightarrow{\mathcal{D}} N(0, p(1 - p))$

(b) Show that for $p \neq 0.5$, the estimate of variance $Y_n(1 - Y_n)$ satisfies

$$\sqrt{n}\{Y_n(1 - Y_n) - p(1 - p)\} \xrightarrow{\mathcal{D}} N(0, (1 - 2p)^2 p(1 - p)).$$

(c) Show that for $p = 0.5$, $n\{Y_n(1 - Y_n) - 0.25\} \xrightarrow{\mathcal{D}} -0.25\chi_1^2$. This may seem strange, but it's not. Examine values on the left-hand side, and briefly explain why it's not.