132. If S^2 is the sample variance based on a sample of size n from a normal population, we know that $(n-1)S^2/\sigma^2$ has a χ^2_{n-1} distribution. The conjugate prior for σ^2 is the inverted gamma PDF, $\text{IG}(\alpha, \beta)$ given by

$$\pi(\sigma^2) = \frac{1}{\Gamma(\alpha)\beta^\alpha} \frac{1}{(\sigma^2)^{\alpha+1}} \exp\left\{-\frac{1}{\beta\sigma^2}\right\}, \quad 0 < \sigma^2 < \infty$$

where $\alpha > 0$ and $\beta > 0$.

(a) Show that the posterior distribution of σ^2 is $\text{IG}(\alpha + (n-1)/2, \{(n-1)S^2/2 + (1/\beta)\}^{-1})$.

(b) Find the Bayes estimator of σ^2.