

80. For the following hierarchical settings,

(a) For the hierarchical model

$$Y|\Lambda \sim \text{Poisson}(\Lambda) \quad \text{and} \quad \Lambda \sim \text{gamma}(\alpha, \beta),$$

find the marginal distribution, mean, and variance of Y . Show that the marginal distribution of Y is a negative binomial if α is a positive integer.

(b) Show that the three-stage model

$$Y \mid N \sim \text{binomial}(N, p), \quad N \mid \Lambda \sim \text{Poisson}(\Lambda) \quad \text{and} \quad \Lambda \sim \text{gamma}(\alpha, \beta)$$

leads to the same marginal (unconditional) distribution of Y .