166. Let $X_1, ..., X_n$ be independent with probability density functions (PDFs)

$$f_{X_i}(x|\theta) = \exp\{i\theta - x\} I_{[i\theta,\infty]}(x).$$

(a) Prove that $T(\mathbf{X}) = \min_i\{X_i/i\}$ is a sufficient statistic for θ.

(b) Based on T, find the $1-\alpha$ confidence interval for θ of the form $[T+a, T+b]$ which is of minimum length.