1. Prove the following theorem. Sufficiency for Exponential Families. Let $X_1, X_2, ..., X_n$ be independent and identically distributed (IID) random variables from a probability mass function (PMF) or probability density function (PDF) f(x| $\boldsymbol{\theta}$), where $\boldsymbol{\theta} = (\theta_1, ..., \theta_k)$ that belongs to an exponential family given by

$$f(x \mid \boldsymbol{\theta}) = h(x)c(\boldsymbol{\theta}) \exp \left\{ \sum_{j=1}^{k} \omega_j(\boldsymbol{\theta}) t_j(x) \right\}$$

Then

$$T(\mathbf{X}) = \left(\sum_{i=1}^{n} t_1(X_i), ..., \sum_{i=1}^{n} t_k(X_i) \right)$$

is a sufficient statistic for $\boldsymbol{\theta}$.