51. In this exercise, the asymptotic relative efficiency (ARE) of the sample mean $\bar{X}_n$ and sample median M_n will be investigated for a number of situations

(a) Find a general expression for the ARE $(M_n, \bar{X}_n)$.

(b) Find the ARE $(M_n, \bar{X}_n)$ for the following distributions. For each distribution, determine which estimator is asymptotically more efficient for estimating the center of $f(x|\theta)$.

i. Normal with location parameter μ and scale parameter σ^2.

ii. Logistic with location parameter μ and scale parameter β.

iii. LaPlace (double exponential) with location parameter μ and scale parameter σ^2.

iv. Student's t distribution with degrees of freedom $\nu = 3, 5, 10, 25, 50$ and ∞.

(c) Show that the ARE $(M_n, \bar{X}_n)$ is unaffected by changes in scale.