176. For testing

$$H_0 : p = p_0 \quad \text{versus} \quad H_1 : p \neq p_0,$$

suppose we observe $X_1, ..., X_n \sim$ IID Bernoulli(p)

(a) Derive an expression for -2log$\{\lambda(\mathbf{x})\}$ where $\lambda(\mathbf{x})$ is the likelihood ratio test statistic.

(b) Use R to simulate -2log$\{\lambda(\mathbf{x})\}$. Simulate 5000 replications of samples of sizes $n = 10, 30, 50, 100$ and 500, each for three values of $p_0 = 0.25, 0.5$ and 0.9. Compare the histogram of the test statistic with the χ^2_1 PDF. Create QQ plots of the empirical statistics vs the χ^2_1 distribution. Compute the empirical 25th, 50th, 75th, 90th, 95th, and 99th percentiles and compare them to the χ^2_1 percentiles and put these in a table. How does increasing the sample size affect the fit of the χ^2_1 distribution the empirical distribution of -2log$\{\lambda(\mathbf{x})\}$? Does the value of p_0 affect the approximation? Explain.