68. Let $X_1, ..., X_n$ be independent and identically distributed (IID) according to a geometric(p) distribution.
Hint: The geometric probability mass function is

$$f(x|p) = p(1-p)^{x-1}, \quad x = 1, 2, ..., \quad 0 \leq p \leq 1.$$

(a) Find the method of moments estimator $\tilde{p}$ of p.

(b) Find the maximum likelihood estimator $\hat{p}$ of p.

(c) Find the maximum likelihood estimator of $P(X = 3)$.

(d) Find a complete sufficient statistic for p.

(e) Determine if a minimal sufficient statistic exists, and if so, find it.

(f) Find the Cramér-Rao Lower Bound (CRLB) for the variance of any estimator of a continuous function $\tau(p)$ of p.

(g) Use the Rao-Blackwell approach to find the uniformly minimum variance unbiased estimator (UMVUE) for p.
Hint: $T = \sum_{i=1}^{n} X_i$ has a negative binomial distribution with parameters n and p, with PMF

$$f(t|n,p) = f(x) = \binom{t-1}{n-1} p^n (1-p)^{t-n}, \quad t = n, n+1, ...$$