189. Let $Y_{ij}, i = 1, ..., k, j = 1, ..., n_i$ represent random samples of sizes $n_1, ..., n_k$.

(a) Write the cell means model, using the most general assumptions.

(b) If the errors in the cell means model in part (a) are normally distributed and are homoscedastic, in what other ways are the assumptions affected?

(c) What are the classic analysis of variance (ANOVA) hypotheses?

(d) Without using Cochran's Theorem, prove the expected value of mean square between (MSB) is

$$\text{E[MSB]} = \sigma_\epsilon^2 + \frac{1}{k-1} \sum_{i=1}^{k} n_i(\theta_i - \theta)^2,$$

where σ_ϵ^2 is the common variance of the errors in the cell means model, θ_i is the mean of population i, and θ is equal to the average of the θ_i; that is $\theta = n^{-1} \sum_{i=1}^{k} n_i \theta_i$.