115. Consider the χ^2 distribution with ν degrees of freedom, χ^2_ν.

(a) Use the definition of the moment generating function (MGF) to prove the MGF of χ^2_ν is

$$M_{\chi^2_\nu}(t) = \left(\frac{1}{1-2t}\right)^{\nu/2}, \quad t < \frac{1}{2}$$

(b) Use the MGF to find the mean of the χ^2_ν distribution.

(c) Let $Y_1, ..., Y_n$ be independent χ^2 random variables with degrees of freedom $\nu_i, i = 1, ..., n$, respectively; that is, for $i = 1, ..., n, Y_i \sim \chi^2_{\nu_i}$. Prove that $W = \sum_{i=1}^n Y_i$ has a χ^2_ν distribution where $\nu = \sum_{i=1}^n v_i$.