28. For a random sample $Y_1, ..., Y_m$ from a Bernoulli(θ) population, it is desired to test

$$H_0 : \theta = 0.49 \quad \text{versus} \quad H_1 : \theta = 0.51.$$

(a) Use the Neyman-Pearson Lemma to find a UMP level α test. Rewrite the rejection region in terms of $W = \sum_{i=1}^{m} Y_i$.

(b) For $m = 10$, create a table that contains values of k from the Neyman-Pearson Lemma, corresponding critical values for the rejection region in terms of $\sum_{i=1}^{m} Y_i$, and the values of α (the size of the test). You may use R to compute these values without providing the code.

(c) Use the Central Limit Theorem to determine, approximately, the sample size (m) needed so that the two error probabilities are both about 0.01. Use a test function that rejects H_0 if $\sum_{i=1}^{m} Y_i$ is large.

(d) Now consider testing

$$H_0 : \theta \leq \frac{1}{2} \quad \text{versus} \quad H_1 : \theta > \frac{1}{2}.$$

Use the Karlin-Rubin Theorem to find a level α UMP test, where $\alpha = P_{\theta_0}(\boldsymbol{Y} \in R)$, and R is the rejection region.

(e) For the test in part (d), for $m = 10$, create a table of sizes for $w_0 = 0, 1, ..., m$. Which values of w_0 would be used for levels $\alpha = 0.01$, 0.05, and 0.10 if $m = 10$? For those critical values, what is the size of the test? (You may use R without including your code).

(f) For the test in part (d), find an expression for a valid p-value and calculate the p-value of the test if of the 10 Bernoulli trials seven are successes.