147. Let $\boldsymbol{X} = (X_1, ..., X_n)$ be an independent and identically distributed (IID) collection of Poisson(λ) random variables. Consider estimating the function

$$\tau(\lambda) = P(X = 1) = \lambda e^{-\lambda}.$$

Let $W(\boldsymbol{X})$ be defined as

$$W(\boldsymbol{X}) = \begin{cases} 1 & \text{if } X_1 = 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Prove $W(\boldsymbol{X})$ is an unbiased estimator of $P(X = 1)$.

(b) Find the MLE of $P(X = 1)$. You may use without proof that the MLE of λ is $\hat{\lambda} = \bar{X}$, the sample mean.

(c) Find a best unbiased estimator of $\tau(\lambda)$. You may use without proof that $T(\boldsymbol{X}) = \sum_{i=1}^{n} X_i$ is a sufficient statistic for λ.