

191. Let  $X_1, \dots, X_n$  be independent and identically distributed (IID) according to a geometric( $p$ ) distribution.  
Hint: The geometric probability mass function is

$$f(x|p) = p(1-p)^{x-1}, \quad x = 1, 2, \dots, \quad 0 \leq p \leq 1.$$

- (a) Find the method of moments estimator  $\tilde{p}$  of  $p$ .
- (b) Find the maximum likelihood estimator  $\hat{p}$  of  $p$ .
- (c) Find the maximum likelihood estimator of  $P(X = 3)$ .
- (d) Find a complete sufficient statistic for  $p$ .
- (e) Determine if a minimal sufficient statistic exists, and if so, find it.
- (f) Find the Cramér-Rao Lower Bound (CRLB) for the variance of any estimator of a continuous function  $\tau(p)$  of  $p$ .
- (g) Use the Rao-Blackwell approach to find the uniformly minimum variance unbiased estimator (UMVUE) for  $p$ .  
Hint:  $T = \sum_{i=1}^n X_i$  has a negative binomial distribution with parameters  $n$  and  $p$ , with PMF

$$f(t|n, p) = f(x) = \binom{t-1}{n-1} p^n (1-p)^{t-n}, \quad t = n, n+1, \dots$$