

35. Consider a finite sample space  $\mathcal{S}$  consisting of 4 equally likely outcomes; that is,  $\mathcal{S} = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ .
- (a) What is the smallest Borel field on  $\mathcal{S}$  (call it  $\mathcal{A}$ ), also called the "trivial  $\sigma$ -algebra"? There is no need to prove this is a Borel field.
  - (b) Describe, in one sentence, the power set,  $\mathcal{P}_{\mathcal{S}}$  on  $\mathcal{S}$ . How many elements are in the power set  $\mathcal{P}_{\mathcal{S}}$  of  $\mathcal{S}$ ? Provide a listing of the sets in  $\mathcal{P}_{\mathcal{S}}$ .
  - (c) Prove that  $\mathcal{P}_{\mathcal{S}}$  is a Borel field.