

20. Let X_1, \dots, X_n be a random sample from a normal population with mean θ and variance σ^2 . Consider testing

$$H_0 : \theta \leq \theta_0 \text{ versus } H_1 : \theta > \theta_0$$

If σ^2 is known, find the likelihood ratio test of the hypotheses. You may use $\hat{\theta} = \bar{X}$ is the maximum likelihood estimator of θ and that \bar{X} is sufficient for θ without proof. For the LRT, show that it reduces to a test that rejects H_0 if

$$\bar{X} > \theta_0 + K$$

Write an R function to illustrate the LRT statistic as a function of \bar{x} . Include a figure and code.