8. A random sample without replacement of size $n = 5$ is selected from a finite population with $N = 12$ individuals. The sample is selected in such a way that all samples of size five are equally likely. Samples are unordered - in other words, a sample with the same five individuals are selected in a different order are still considered the same sample.

   (a) Describe the sample space $\mathcal{S}$. Note: There are 792 outcomes in the sample space.

   (b) Describe the power set $\mathcal{P}_{\mathcal{S}}$ of $\mathcal{S}$. Give an expression for $\mathcal{C}(\mathcal{P}_{\mathcal{S}})$.

9. Consider the probability space $(\mathcal{S}, \mathcal{B}, P)$, and suppose A and B are two sets in $\mathcal{B}$ such that A and B are exhaustive for $\mathcal{S}$. Moreover, assume $P(A) = 0.8$ and $P(B) = 0.5$. Find $P(AB)$. Explain your steps.

Homework 2

10. A simple random sample of size n is a sample selected in such a way that all possible samples of size n are equally likely. If $N = 20$ and $n = 7$, find the following if (1) sampling is with replacement, and then if (2) sampling is without replacement.

    (a) Describe the sample space. Specificially, what is the job? What is the task? What does an outcome in the sample space "look like"? Are repeats allowed? Are outcomes ordered or unordered?

    (b) Find $\mathcal{C}(\mathcal{S})$. Explain your reasoning.

    (c) What is $\mathcal{B} = \mathcal{P}_{\mathcal{S}}$, the power set of S? How many elements are in $\mathcal{B}$? (Use R if necessary, and exponential notation).

11. Justify your answer. How many different sets of initials can be formed if every person has one surname and

    (a) exactly two given names?

    (b) either one or two given names?

12. (Note: We skipped this question because the wording was difficult to understand). Justify your answer. If m balls are placed at random into m cells, find the probability that exactly one cell remains empty. For values of $m = 2, 3, ..., 10$, compute the probability. You can use a calculator, or R. As m increases, what do the numbers suggest is the limit of the probability?

13. A bowl contains 16 chips, of which six are Doritos, seven are Ruffles, and three are Fritos. Jane likes Doritos, Ruffles are her favorite, and she does not like Fritos. If four chips are taken at random and without replacement (because replacement would be gross and rude), find the following.

    (a) Describe the sample space. Specificially, what is the job? What is the task? What does an outcome in the sample space "look like"? Are repeats allowed? Are outcomes ordered or unordered?

    (b) Find the probability that all four chips are Ruffles.

    (c) Find the probability that none of the four chips are Ruffles.

    (d) Find the probability that there is at least one chip of each brand. (Hint: Use DeMorgan's Laws).

14. In a lot of 50 light bulbs, there are 4 bad bulbs. From the 50 bulbs, an inspector randomly selects 5 without replacement.

    (a) Find the probability of at least one defective bulb among the five. (Be sure to justify using the method you choose).

    (b) How many bulbs should be examined so that the probability of finding at least one defective bulb exceeds 0.5?

15. Suppose there are $n \leq 365$ people in the same room. What is the probability that at least two of them have the same birthday? At what value of n does the probability reach 99%? Note: This is called the birthday paradox, and is a famous problem.