

58. Under the oneway ANOVA assumptions for the cell means model

$$Y_{ij} = \theta_i + \epsilon_{ij}, \quad i = 1, \dots, k, \quad j = 1, \dots, n_i$$

where ϵ_{ij} are zero-mean homoscedastic independent normal random variables with variance $\sigma^2 < \infty$

- (a) Show that the statistic $(\bar{Y}_1, \bar{Y}_2, \dots, \bar{Y}_k, \text{MSW})$ is sufficient for $(\theta_1, \theta_2, \dots, \theta_k, \sigma^2)$.
- (b) Show that $\text{MSW} = (N - k)^{-1} \sum_{i=1}^k (n_i - 1) S_i^2$ is independent of each \bar{Y}_i . In this, for each $i = 1, \dots, k$, $S_i^2 = (n_i - 1)^{-1} \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_i)^2$ and $N = \sum_{i=1}^k n_i$.