197. Consider the oneway analysis of variance (ANOVA) cell means model

$$Y_{ij} = \theta_i + \epsilon_{ij}, \quad i = 1, ..., k \quad j = 1, ..., n_i.$$

where ϵ_{ij} are IID as zero-mean normal random variables with common variance $\sigma^2 < \infty$.

(a) Without using Cochran's Theorem, prove the expected value of mean square within (MSW) is σ^2.

(b) The classic ANOVA hypotheses are

$$H_0 : \theta_1 = ... = \theta_k \quad \text{versus} \quad H_1 : \theta_i \neq \theta_j \text{for some} i \neq j.$$

Use the Derived Distributions approach to show the sampling distribution of F is

$$F = \frac{\text{MSB}}{\text{MSW}} \sim F_{k-1,n-k},$$

under H_0, where $n = \sum_{i=1}^{k} n_i$ and $F_{k-1,n-k}$ is an F distribution with $k - 1$ numerator degrees of freedom and $n - k$ denominator degrees of freedom. You may use without proof that under H_0, $\text{E[MSB]} = \sigma^2$. You may use Cochran's Theorem for any independence result you may need.