

56. For $X_1, \dots, X_n \sim \text{IID Poisson}(\lambda)$

(a) Find the limiting distribution of

$$\sqrt{n} \left(\frac{\bar{X} - \lambda}{\sqrt{\lambda}} \right).$$

(b) Show that this approximation is optimal according to Wilks; that is, show that

$$\sqrt{n} \left(\frac{\bar{X} - \lambda}{\sqrt{\lambda}} \right) = \frac{\frac{\partial}{\partial \lambda} \log L(\lambda | \mathbf{x})}{\sqrt{-E_{\lambda} \left[\frac{\partial^2}{\partial \lambda^2} \log L(\lambda | \mathbf{X}) \right]}}.$$

Let us find the log-likelihood first

(c) Use this to find an approximate $1 - \alpha$ confidence interval estimator for λ .