

66. Let  $Y_{ij}, i = 1, \dots, k, j = 1, \dots, n_i$  represent random samples of sizes  $n_1, \dots, n_k$ .

- (a) Write the cell means model, using the most general assumptions.
- (b) If the errors in the cell means model in part (a) are normally distributed and are homoscedastic, in what other ways are the assumptions affected?
- (c) What are the classic analysis of variance (ANOVA) hypotheses?
- (d) Without using Cochran's Theorem, prove the expected value of mean square between (MSB) is

$$E[\text{MSB}] = \sigma_\epsilon^2 + \frac{1}{k-1} \sum_{i=1}^k n_i (\theta_i - \theta)^2,$$

where  $\sigma_\epsilon^2$  is the common variance of the errors in the cell means model,  $\theta_i$  is the mean of population  $i$ , and  $\theta$  is equal to the average of the  $\theta_i$ ; that is  $\theta = n^{-1} \sum_{i=1}^k n_i \theta_i$ .