

84. Prove that if $(X, Y) \sim \text{bivariate normal}(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho)$ then the following are true.

- (a) Using the definition of marginal distribution, prove the marginal distribution of X is normal(μ_X, σ_X^2).
- (b) Using the definition of conditional distribution, prove the conditional distribution of Y given $X = x$ is normal with mean $\mu_Y + \rho(\frac{\sigma_Y}{\sigma_X})(x - \mu_X)$ and variance $\sigma^2 = \sigma_Y^2(1 - \rho^2)$.
- (c) For any constants a and b , use transformation methods to prove the distribution of $aX + bY$ is normal with mean $a\mu_X + b\mu_Y$ and variance $a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\rho\sigma_X\sigma_Y$.