

151. For a random sample Y_1, \dots, Y_m from a Bernoulli(θ) population, it is desired to test

$$H_0 : \theta = 0.49 \quad \text{versus} \quad H_1 : \theta = 0.51.$$

- (a) Use the Neyman-Pearson Lemma to find a UMP level α test. Rewrite the rejection region in terms of $W = \sum_{i=1}^m Y_i$.
- (b) For $m = 10$, create a table that contains values of k from the Neyman-Pearson Lemma, corresponding critical values for the rejection region in terms of $\sum_{i=1}^m Y_i$, and the values of α (the size of the test). You may use R to compute these values without providing the code.
- (c) Use the Central Limit Theorem to determine, approximately, the sample size (m) needed so that the two error probabilities are both about 0.01. Use a test function that rejects H_0 if $\sum_{i=1}^m Y_i$ is large.
- (d) Now consider testing

$$H_0 : \theta \leq \frac{1}{2} \quad \text{versus} \quad H_1 : \theta > \frac{1}{2}.$$

Use the Karlin-Rubin Theorem to find a level α UMP test, where $\alpha = P_{\theta_0}(\mathbf{Y} \in R)$, and R is the rejection region.

- (e) For the test in part (d), for $m = 10$, create a table of sizes for $w_0 = 0, 1, \dots, m$. Which values of w_0 would be used for levels $\alpha = 0.01, 0.05$, and 0.10 if $m = 10$? For those critical values, what is the size of the test? (You may use R without including your code).
- (f) For the test in part (d), find an expression for a valid p -value and calculate the p -value of the test if of the 10 Bernoulli trials seven are successes.