140. Suppose that we observe m independent and identically distributed (IID) Bernoulli(θ) random variables $Y_1, Y_2, ..., Y_m$. Consider testing

$$H_0 : \theta \leq \theta_0 \text{ versus } H_1 : \theta > \theta_0$$

(a) Find the maximum likelihood estimator (MLE) $\hat{\theta}$ of θ.

(b) Under $H_0 : \theta \leq \theta_0$, find the restricted MLE $\hat{\theta}_0$ of θ.

(c) Use R to draw the likelihood function $L(\theta)$, and illustrate both $\hat{\theta}$ and $\hat{\theta}_0$ on the graph.

(d) Find the likelihood ratio test (LRT) of

$$H_0 : \theta \leq \theta_0 \text{ versus } H_1 : \theta > \theta_0$$

(e) Use R to graph the likelihood ratio (LR) test statistic as a function of $\sum_{i=1}^{n} Y_i$. On the graph, illustrate the rejection region

(f) Prove the likelihood ratio test statistic $\lambda(\mathbf{y}) \leq c$ if and only if $\bar{y} \geq k$ (or equivalently $\sum_{i=1}^{m} y_i > mk$)