

31. Let  $X_1, \dots, X_n$  be IID  $\text{normal}(\theta, 1)$ , and let  $\theta_0$  be a specified value of  $\theta$ .
- (a) Find the UMP size  $\alpha$  test of  $H_0 : \theta \geq \theta_0$  versus  $H_1 : \theta < \theta_0$ . Write the rejection region in terms of the sample mean.
  - (b) Prove (or disprove) that the test in part (a) is an unbiased test.
  - (c) Show that the test in part (a) can be derived as a likelihood ratio test. Use *R* to draw the likelihood ratio test statistic for  $\alpha = 0.05$ , and  $n = 2, 5$ , and  $10$ . Include your annotated code at the end of your solutions.
  - (d) Show that there does not exist a UMP size  $\alpha$  test of  $H_0 : \theta = \theta_0$  versus  $H_1 : \theta \neq \theta_0$ .