

174. In this exercise, the asymptotic relative efficiency (ARE) of the sample mean  $\bar{X}_n$  and sample median  $M_n$  will be investigated for a number of situations
- (a) Find a general expression for the ARE  $(M_n, \bar{X}_n)$ .
  - (b) Find the ARE  $(M_n, \bar{X}_n)$  for the following distributions. For each distribution, determine which estimator is asymptotically more efficient for estimating the center of  $f(x|\theta)$ .
    - i. Normal with location parameter  $\mu$  and scale parameter  $\sigma^2$ .
    - ii. Logistic with location parameter  $\mu$  and scale parameter  $\beta$ .
    - iii. LaPlace (double exponential) with location parameter  $\mu$  and scale parameter  $\sigma^2$ .
    - iv. Student's  $t$  distribution with degrees of freedom  $\nu = 3, 5, 10, 25, 50$  and  $\infty$ .
  - (c) Show that the ARE  $(M_n, \bar{X}_n)$  is unaffected by changes in scale.