

45. Let  $X_1, X_2, \dots, X_n$  be independent Bernoulli( $p$ ), and define  $Y = \sum_{i=1}^n X_i$ . Use these facts to prove the following
- (a)  $\text{Var}(Y) = np(1 - p)$ . Hint: For independent random variables, the variance of a sum is the sum of the variances.
  - (b)  $M_Y(t) = (1 - p + pe^t)^n$ . Hint: If  $W_1$  and  $W_2$  are independent random variables  $E[W_1 W_2] = E[W_1]E[W_2]$ . This is generalizable to  $n$  mutually exclusive random variables.