20. Suppose we have a finite sample space $\mathcal{S} = \{\omega_1, \omega_2, ..., \omega_n\}$ with a probability function $\mathcal{P}(\mathcal{S}) \rightarrow [0, 1]$. Define a random variable X with range being some subset $\mathcal{X}$ of the real numbers and that maps $\mathcal{S}$ to the subset; that is, $X(\omega_i) = x_j$ for some $x_j \in \mathcal{X} \in\subset \mathbb{R}$, so that $X(\mathcal{S}) \rightarrow \mathcal{X}$. We say that we observe $X = x_j$ if and only if the outcome of the random experiment is an $\omega_j \in \mathcal{S}$ such that $X(\omega_j) = x_i$. Then the induced probability function on X is

$$P_X(X = x_i) = P(\{\omega_j \in \mathcal{S} : X(\omega_j) = x_i\}).$$

Prove $P_X(\cdot)$ satisfies Kolmogorov's Axioms.