38. In location, scale, and location-scale families, there are many pivotal quantities. If $X_1, ..., X_n$ is a random sample from the indicated probability density function (PDF) and let $\bar{X}$ and S^2 be the (traditional) sample mean and sample variance; specifically

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{and} \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$$

Show that each of the quantities provided is a pivotal quantity.

Form of PDF	Type of PDF	Pivotal Quantity
$f(x - \mu)$	Location	$Q(\bar{X}, \mu) = \bar{X} - \mu$
$\frac{1}{\sigma}f(x/\sigma)$	Scale	$Q(\bar{X}, \sigma) = \frac{\bar{X}}{\sigma}$
$\frac{1}{\sigma}f\left(\frac{x-\mu}{\sigma}\right)$	Location-scale	$Q(\bar{X}, S, \mu, \sigma) = \frac{\bar{X}-\mu}{S}$