181. Under the oneway ANOVA assumptions for the cell means model

$$Y_{ij} = \theta_i + \epsilon_{ij}, \quad i = 1, ..., k, \quad j = 1, ..., n_i$$

where ϵ_{ij} are zero-mean homoscedastic independent normal random variables with variance $\sigma^2 < \infty$

(a) Show that the statistic $(\bar{Y_1}, \bar{Y_2}, ..., \bar{Y_k}, \text{MSW})$ is sufficient for $(\theta_1, \theta_2, ..., \theta_k, \sigma^2)$.

(b) Show that $\text{MSW} = (N-k)^{-1}\sum_{i=1}^{k}(n_i - 1)S_i^2$ is independent of each $\bar{Y_i}$. In this, for each $i = 1, ..., k, S_i^2 = (n_i - 1)^{-1}\sum_{i=1}(Y_{ij} - \bar{Y_i})^2$ and $N = \sum_{i=1}^{k} n_i$.