

54. Let  $X_1, \dots, X_n$  be a random sample from a normal distribution with mean  $\mu$  and variance  $\sigma^2$ .

(a) If  $\mu$  is unknown and  $\sigma^2$  is known, show that

$$Z = \sqrt{n} \left( \frac{\bar{X} - \mu_0}{\sigma} \right)$$

is a Wald statistic for testing the hypothesis  $H_0 : \mu = \mu_0$ .

(b) If  $\sigma^2$  is unknown and  $\mu$  is known, find a Wald statistic for testing  $H_0 : \sigma = \sigma_0$ .