CS4300 Artificial Intelligence

Tom Henderson

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What is AI?

I → is automated - in machine or constructed artifact

- From biological cells? (slime mold)
- By mimicking biology? (see Edelman)
- From mathematics? (see Turing)

Are (other) animals intelligent?

What is AI?

- Is AI abstract (i.e., a collection of algorithms)
- Or must it be embodied?
- Is Eagle Eye an AI system?

Eagle Eye as AI?

Book's list of capabilities:

- NLP √
- Knowledge Rep √
- Automated Reasoning √
- Machine Learning √
- Computer Vision √
- Robotics √

Can't distinguish system from human

Some Issues

Thinking:

- Like human (Cognitive Science)
- Rationally (Logic)

Acting:

- Like human (Turing Test)
- Rationally (Best Expected Outcome)

Some Other Issues

Intelligence:

- Abstract (Descartes)
- Physical (Skinner)

Computation:

- Discrete (Turing Machines)
- Continuous (Real Numbers)

Abstract Smarts

- Language (e.g., English)
- Math (algebra, logic, geometry)
- Games (chess, cards, backgammon)
- Science (Physics)
- Problem Solving (word problems)
- Planning (moving, route optimization)

Physical Smarts

- Sports (soccer)
- Dexterity (surgery, manipulation)
- Social skills (humanoid robots)
- Service (health, disability)
- Security (homes, business)
- Military

Is AI Math or Science?

- Math: axiomatic method
- Science:
  - Observe Phenomenon
  - Develop Model (math, process, concept)
  - Predict Phenomena
  - Validate predictions via experiment

Math

- Development of abstractions that idealize aspects of the world
  - E.g., "straight line"
- Axiomatic method
  - Assume axioms true
  - Choose inference procedures
  - Theory is what follows
- Validity: syntax or semantics

Requires mapping to specific domain

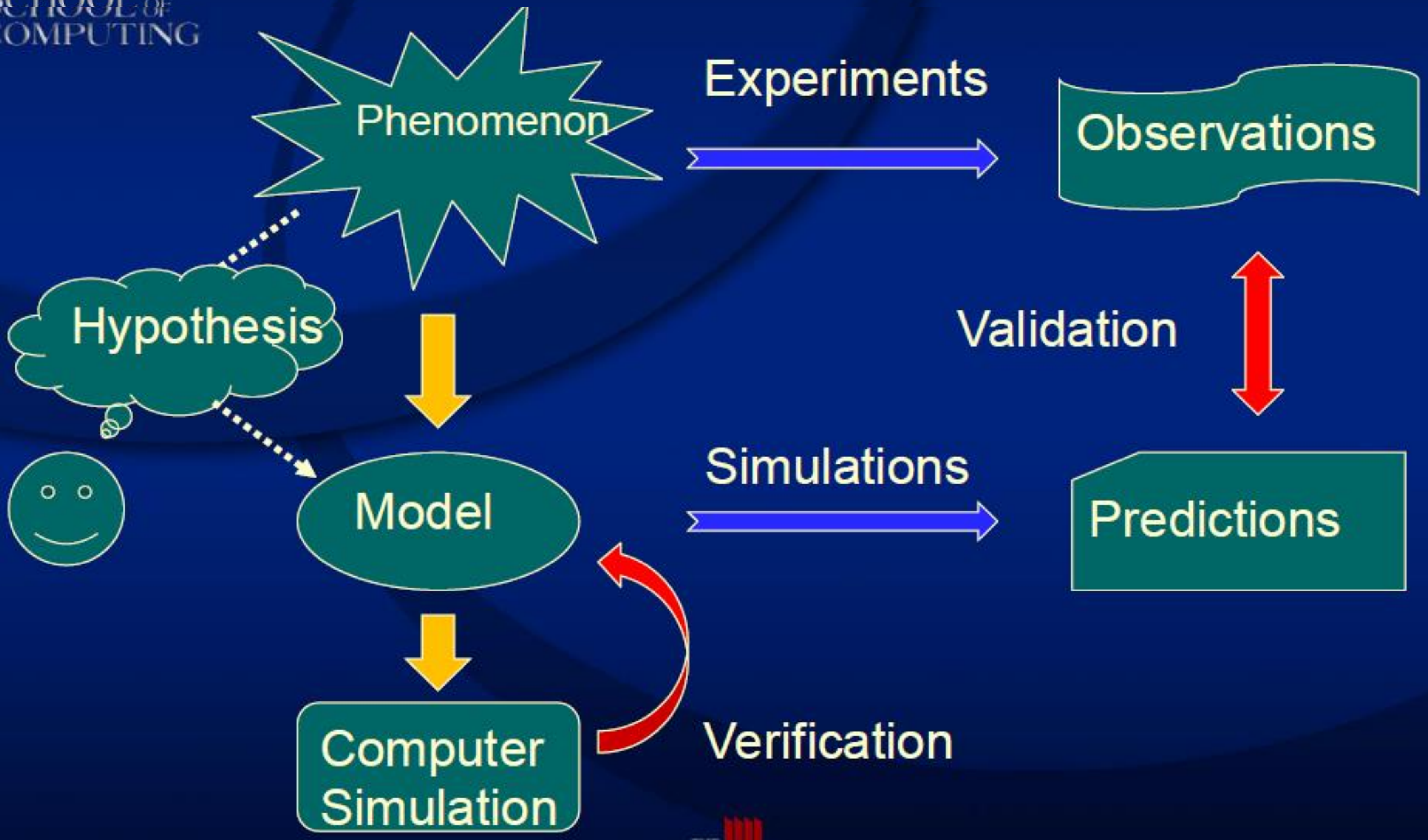
Logic Example

1. Socrates is a man
2. All men are mortal

? Conclusion

➔ Socrates better think of something fast if he doesn't want to die!

Scientific Method



Scientific Method

- ➔Theory must be falsifiable
  - ➔I.e., there exist experiments to distinguish from other theories
  - ➔E.g., "Intelligence arises from an undetectable force from a parallel universe"

Outlook

- Why optimistic?

  Living creatures offer proof of existence

- Why pessimistic?

  Success does not come easily!

Misleading Jargon

"agent" → program

"percept" → sensor data

"knowledge" → data/structures

"learning" → modify data structures

"artificial life" → e.g., Brooks is working on primordial soup

Mapping as a Pardigm

- Discrete: if – then –
- Continuous: dx/dt = f(x,…) + del x

Allows for moving around in a space

Can roll downhill to solution

Have attractors and repellors

Need to know some math (chemistry?)

Environment Modeling

- If performance is important (!!), then need model of environment and an evaluation function

Why not simulate to determine best action?

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Environment Modeling

- If performance in real world is important, how can we hope abstraction will succeed (we leave out too much)?

Use real world to train/learn etc.

Foundations

- Philosophy
- Mathematics
- Engineering
- Psychology
- Economics
- Neuroscience
- Linguistics

Foundations

AI, taking humans as the primary object of study, involves the attempt to understand all aspects of what is means to be human.

Why Study AI?

- Cultural imperative:

  “So God created man in his own image”
- Androids to take menial tasks
- Mental prosthetic
- …

Fundamental Issues

- Knowledge
- Inference
- Goals
- Language

→ Do these exist? Or are they linguistic structures?

Is AI Possible?

Weak AI Hypothesis: Machines can act intelligently

Strong AI Hypothesis: Machines can be intelligent

Physical Symbol Hypothesis

- Computational processes operate on symbol structures
- Cognition is symbol manipulation

(Proposed by Newell and Simon)

PSH: A necessary and sufficient condition for general intelligent action is the ability to manipulate symbol structures

View from the Other Side

- Are humans just machines (e.g., meat machines as Marvin Minsky says)?
- Will reductionism work? I.e., figure out the physics and chemistry in a bottom-up approach.

Meaning?

- Functionalism: “a mental state is any intermediate causal condition between input and output … any two systems with isomorphic causal processes would have the same mental state.” (p. 954)

➔ Can this ever occur?

The Chinese Room

This teaser gets at the heart of the matter:

A program is running, and the computer (human or machine) does not really understand anything that happens or why.

Is this the way a human body functions?

Searle's Axioms

1. Computer programs are formal, syntactic entities.
2. Minds have mental contents, or semantics.
3. Syntax by itself is not sufficient for semantics.
4. Brains cause minds.

Thus, programs are not sufficient for mind.

(Can you prove this using propositional logic?)

Do you accept axiom 3?

Major AI Contexts (1)

"Mind can be modeled through the adequate combination of interacting functional machines (modules)… the mind is a multitude of interacting components … and a hack … only a model incorporating a rich spectrum of (animal or human) mental functioning will give us a correct picture of the broad principles underlying intelligence." from: "Contructionist Design Methodology for Interactive Intelligences," AI Magazine, Winter 2004, Vol. 25, No 4, pp.77-90

Major AI Contexts (2)

"A biologically oriented definition of intelligence is the starting point of the investigation. Intelligence is defined with respect to the capability of an autonomous system to maintain itself. This gives an objective criterion, as opposed to subjective criteria based on judgment of performance or the ascription of knowledge or reasoning. This definition is refined by considering functionalities used to increase the chances of survival: representation, specialization, cooperation, communication, reflection, etc.

Major Ai Context (2) cont'd

"A theory of intelligence must be compatible with the basic laws of physics and biology and it must be a universal theory, I.e., independent of a particular embodiment (wetware or silicon) or system level (brain component, individual agent, society). This universality can be achieved by using complex dynamical systems theory as a foundation. Intelligence then is seen as the result of a set of nonlinear processes which exhibit properties also found in other physical systems. Phenomena like behavioral coherence, cooperation, or the emergence of diversity between agents can be explained using bifurcation theory, chaos, self-organization, etc." from: "Intelligence – Dynamics and Representation," Luc Steels, in <u>The Biology and Technology of Intelligent Autonomous Agents</u>, Springer, 1995.

Patterns from Dynamical Systems

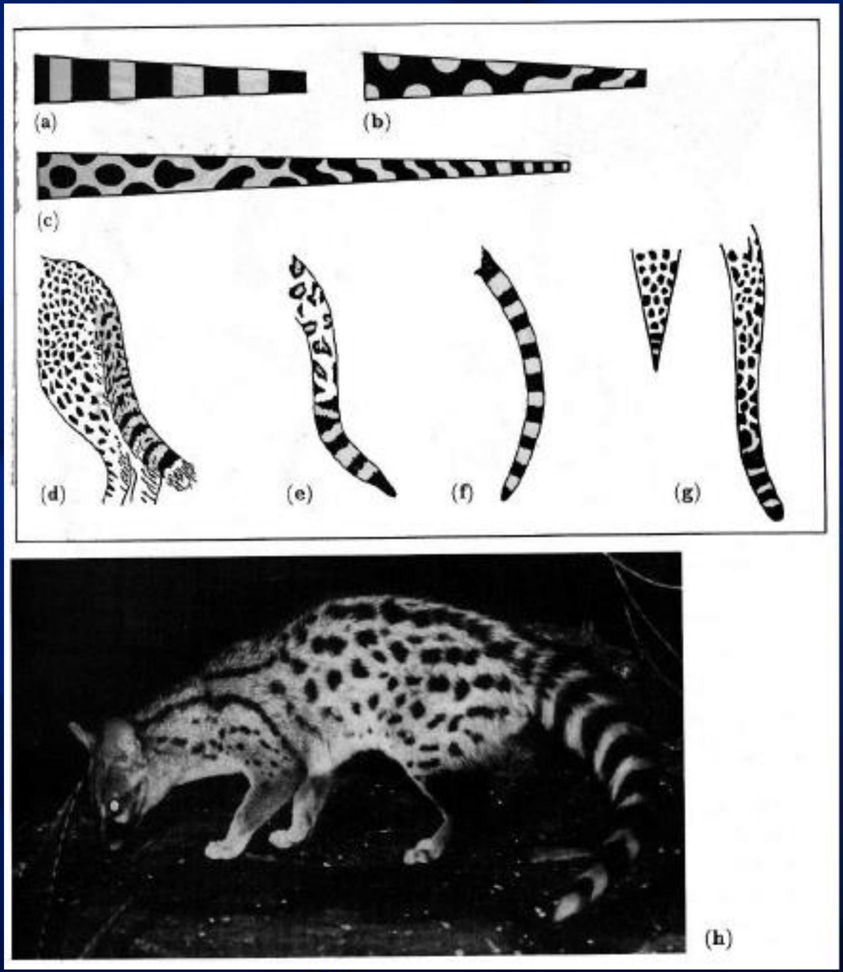
E.g., Stripe Pattern

Figure 3.7. Stripe Pattern

Biosystems Observations

- Pattern Formation in Nature
- Role of Developmental Cycle
- Environmental Niche
- Context, Signals and Algorithms
- Dissipative Information Systems

Patterns and Coloration



Role of Development

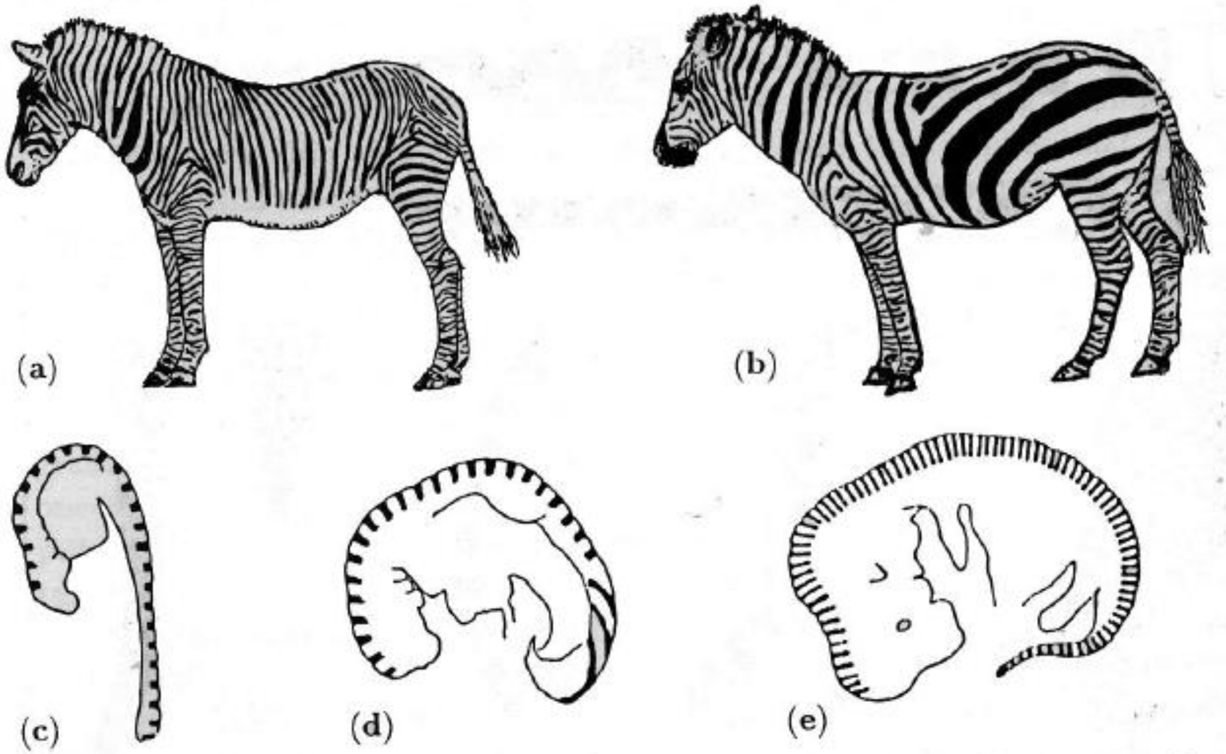


Fig. 15.3a-e. Typical zebra patterns: (a) *Equus grevyi*; (b) *Equus burchelli*. Proposed stripe pattern 0.4 mm apart superimposed on two zebra embryos: (c) 21 day embryo; (d) The effect of 3–4 days of the pattern in (c). (e) A similar stripe pattern laid down on a 5 week old embryo. ((c)-(e) redrawn after Bard 1977)

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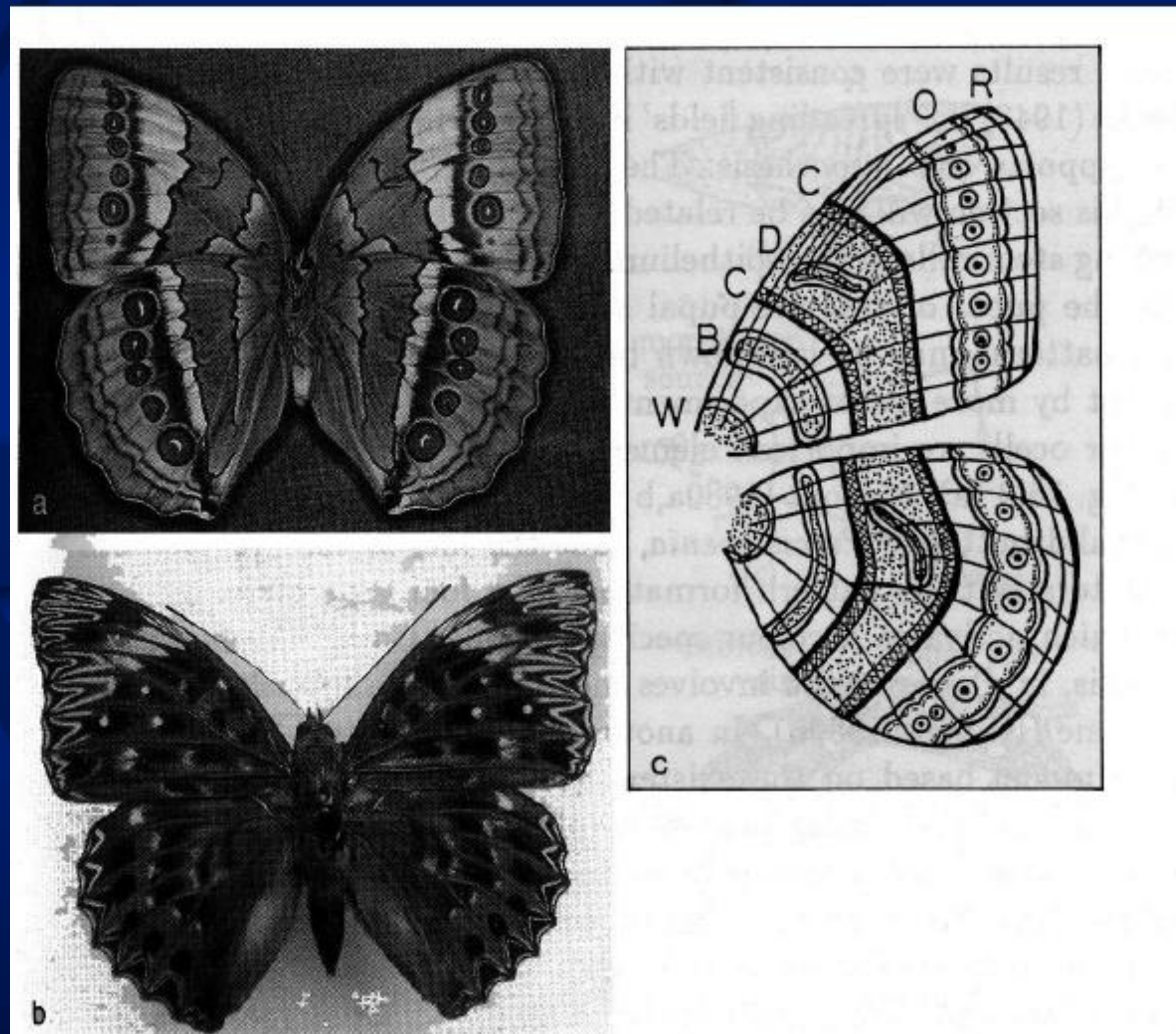
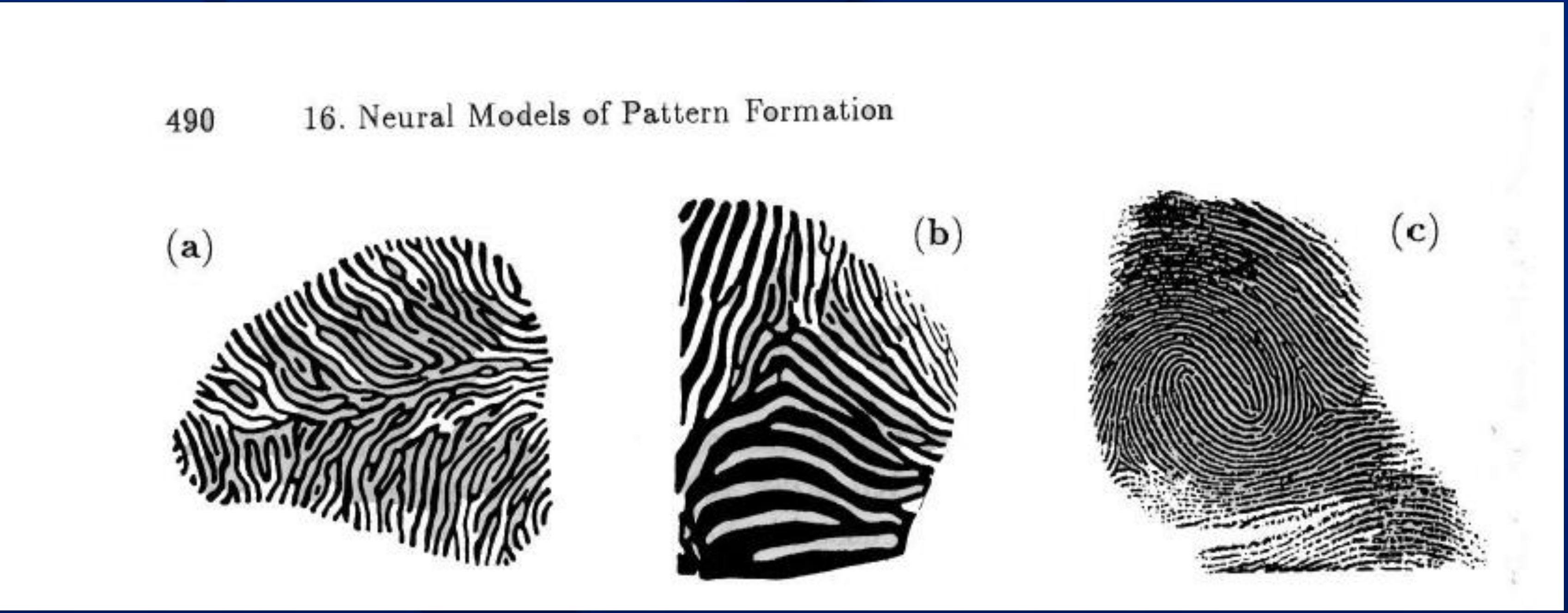
Evolutionary Niche

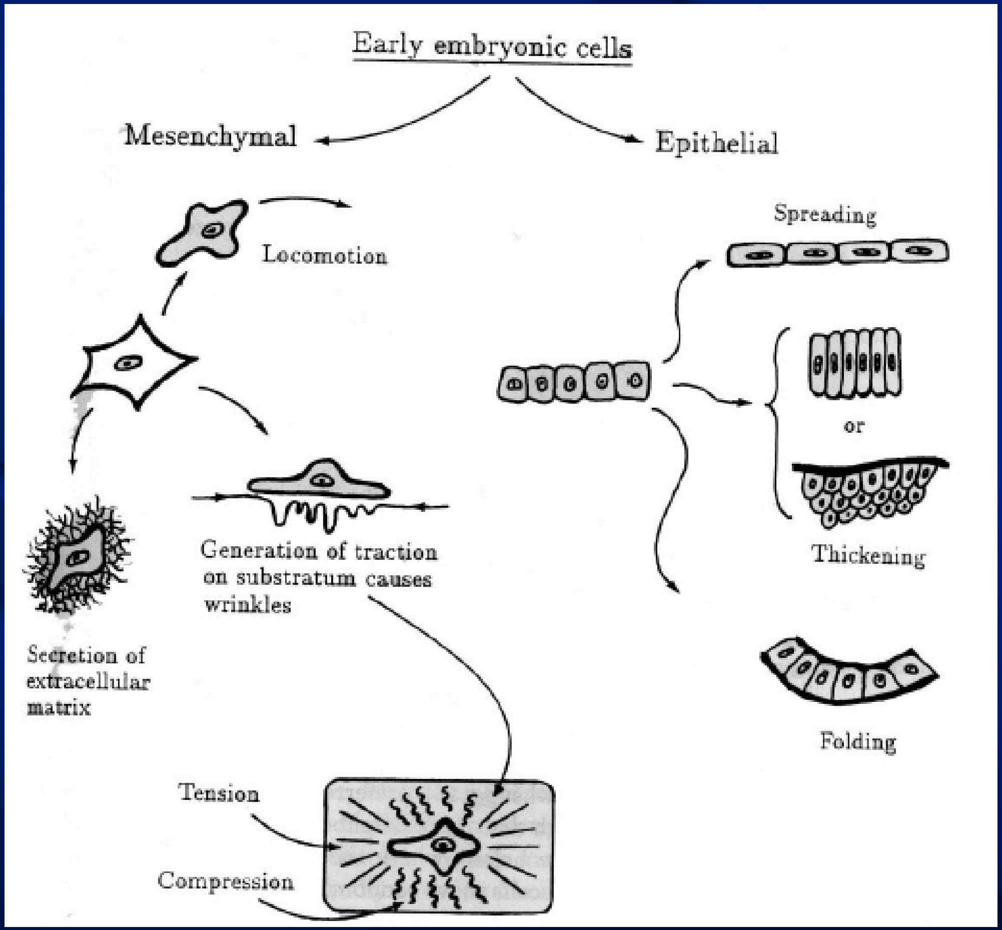


Similarity of Patterns


490 16. Neural Models of Pattern Formation

(a) (b) (c)

Context and Signals



Turing's Morphogenesis Idea

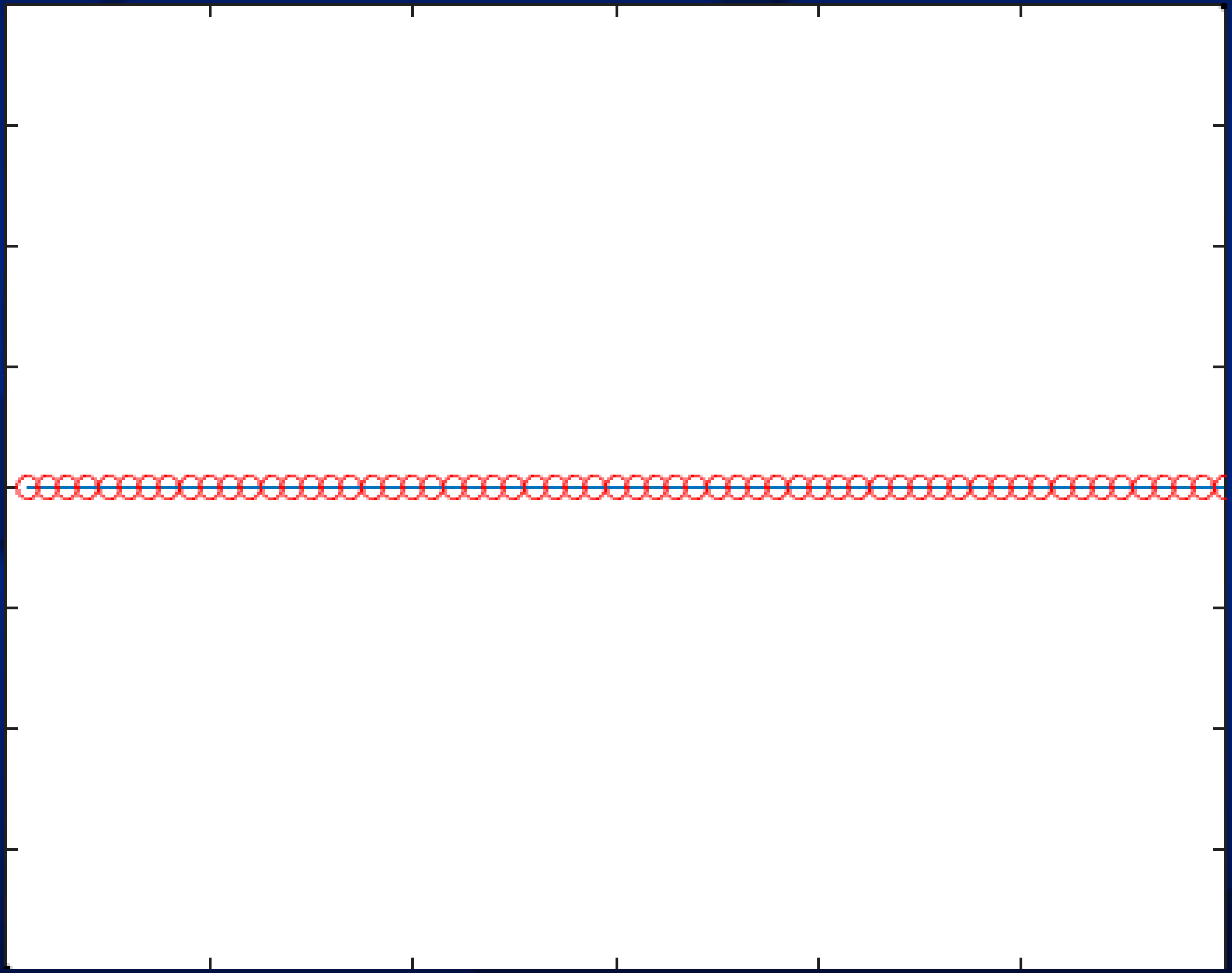
- "The Chemical Basis of Morphogenesis"
- http://www.dna.caltech.edu/courses/cs191/paperscs191/turing.pdf

Reaction-Diffusion Patterns

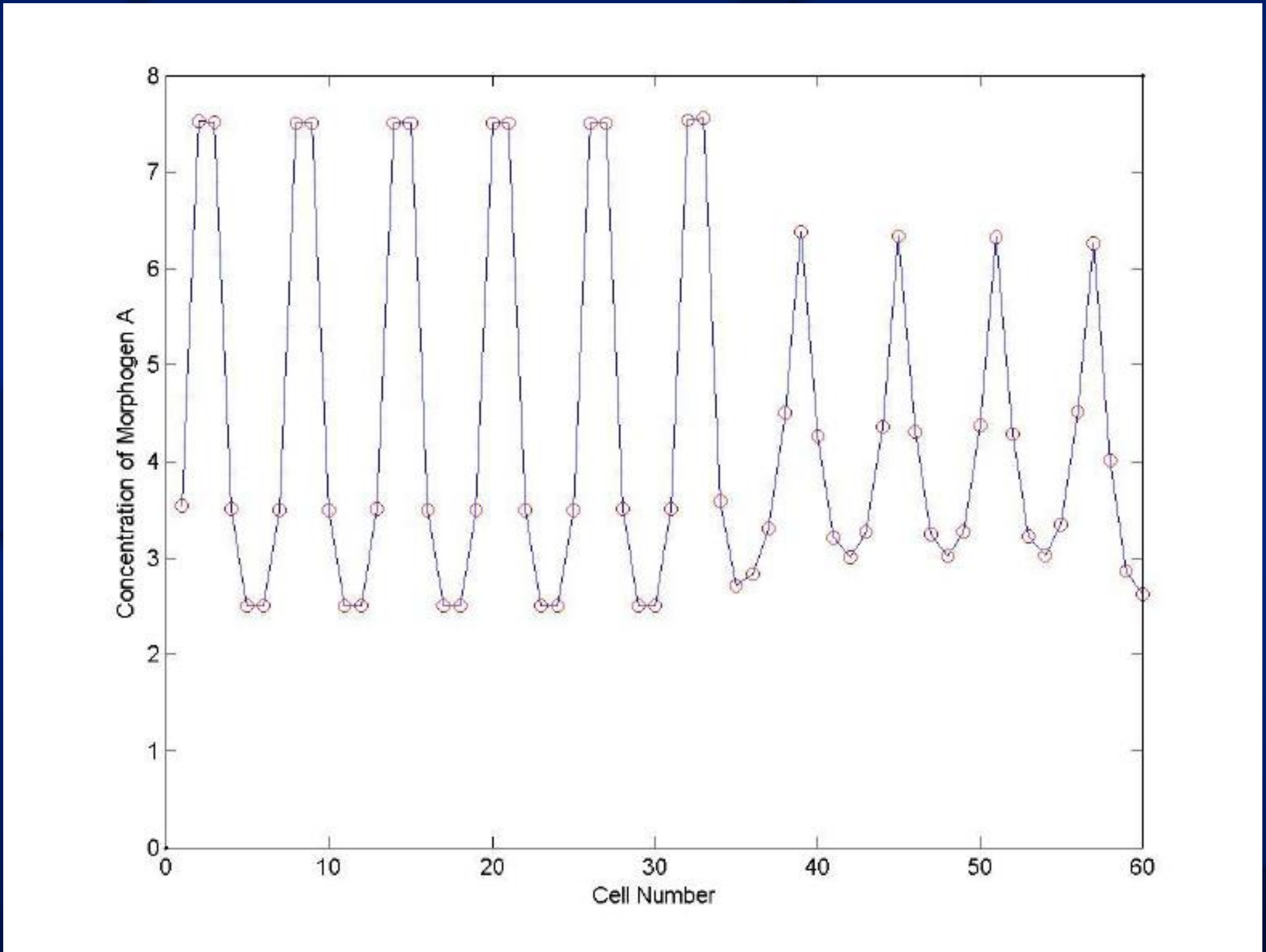
Formula:

$$\frac{\partial c}{\partial t} = \underbrace{f(c)}_{\text{reaction}} + \underbrace{D\nabla^2 c}_{\text{diffusion}}$$

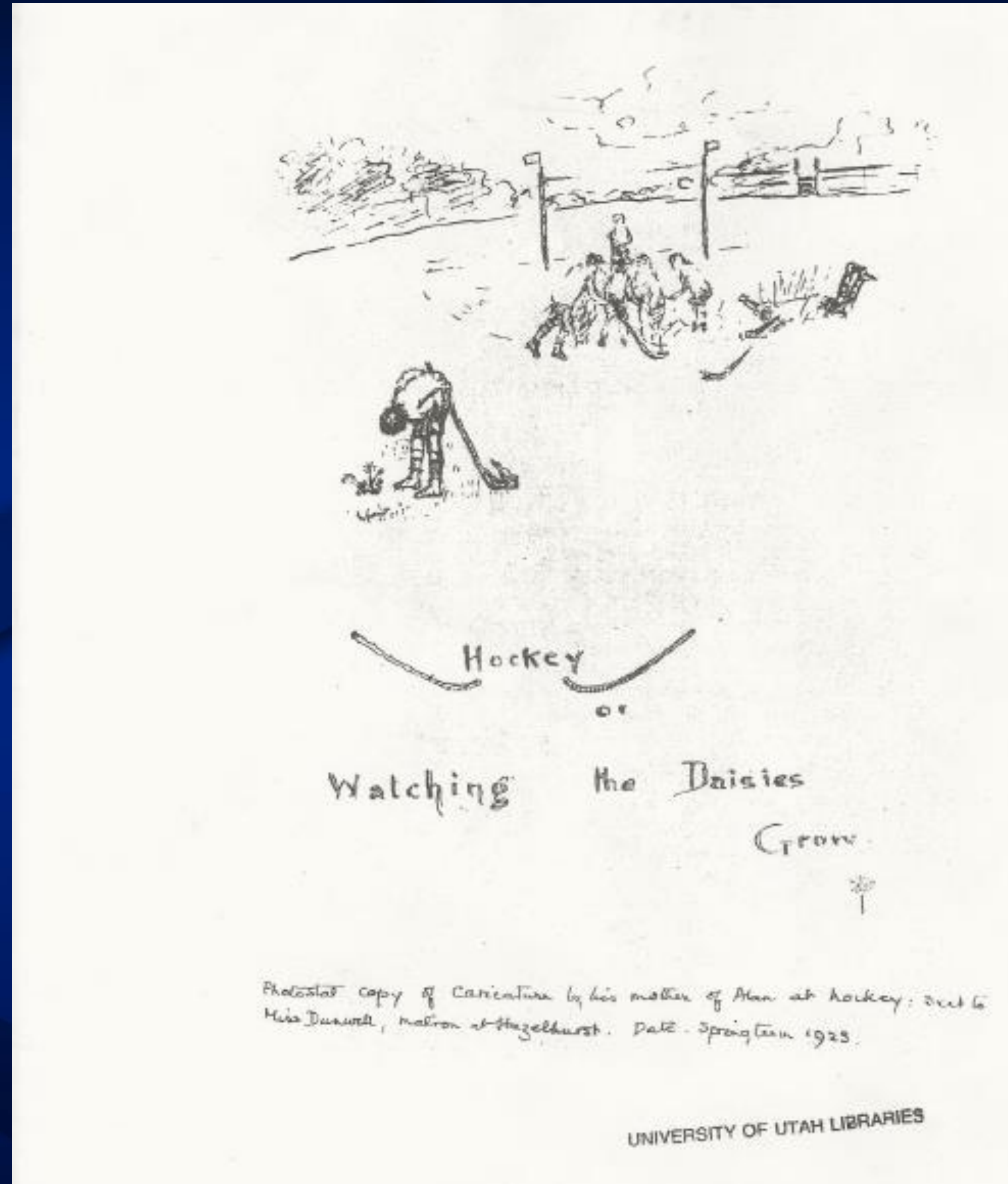
Example of Morphogenesis



Example of Morphogenesis



Caricature of Alan Turing Playing hockey (by his mom)



Major AI Contexts (3)

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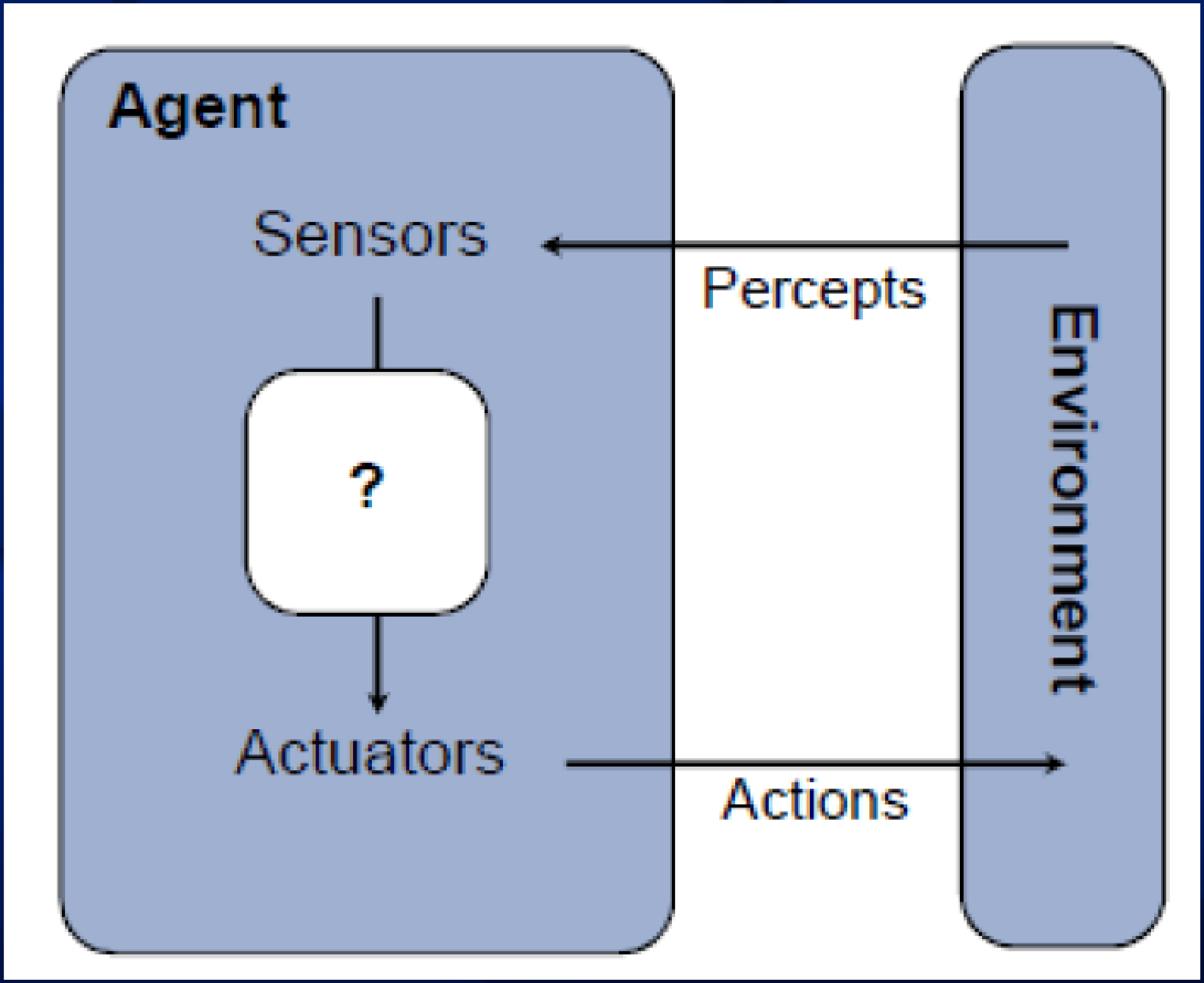
Gerald Edelman: An adequate theory of consciousness must contain an explanation of the properties of conscious experience. It should account both for intentionality and for the discriminability of qualia or phenomenal experiences. To qualify as a scientific account, a theoretical analysis of consciousness must achieve four goals: (1) propose explicit *neural* models that explain how consciousness can arise; (2) relate these models to the emergence of consciousness during evolution and development; (3) relate these models to concept formation, memory, and language; and (4) describe stringent tests for the models in terms of known neurobiological facts.

Questions?

What is an Agent Function?



Agent Function

- Abstract
- External View
- Complete Map

$$f: \underbrace{\{P\}^*}_{\text{percept}} \rightarrow \underbrace{A}_{\text{action}}$$

What is a Percept?

A Percept

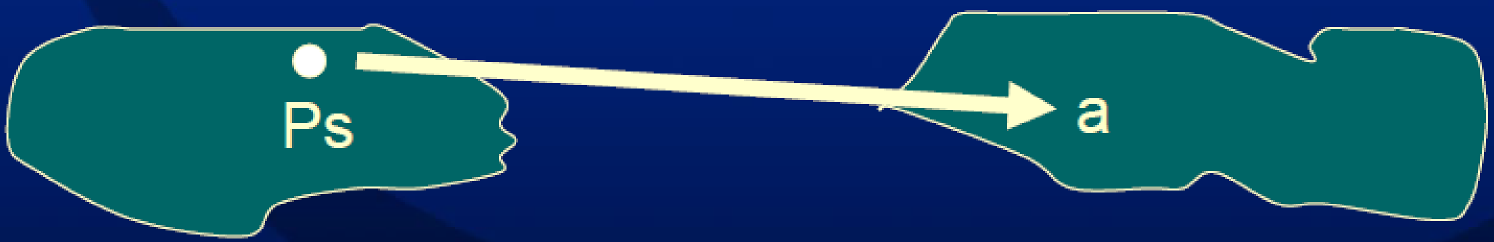
A vector:

$$P = P_1 \times P_2 \times \ldots \times P_n$$

Must a percept vector be finite?

Percepts

- Enumerable single percepts
- Enumerable sequences
- Experimentally find where percept sequence goes



How to Organize Ps Selection

- One Percept?

E.g., [<room>,<dirt status>]

[A,Clean], [A,Dirty],[B,Clean],[B,Dirty]

#rooms * #status_states

What if there're an infinite #???

Two Percepts

Number possibilities is |S1|*|S2|

Are all perception sequences possible?

Can we arrange to try them all?

If so, why?

→ Build Behavior Model (Agent Function)

What is an **Agent Program**?

Agent Program

- Concrete
- Internal
- Implementation
- Just Current Percept
- Maybe persistent state

How Should We Design an Agent Function?

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Do the right thing

Do the most successful thing

Need measures of success (performance measures)

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What is a Rational Agent?

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Rational Agent

For every perception sequence

Select an action that is

Expected to maximize a

Performance measure

Given evidence of P*

and knowledge

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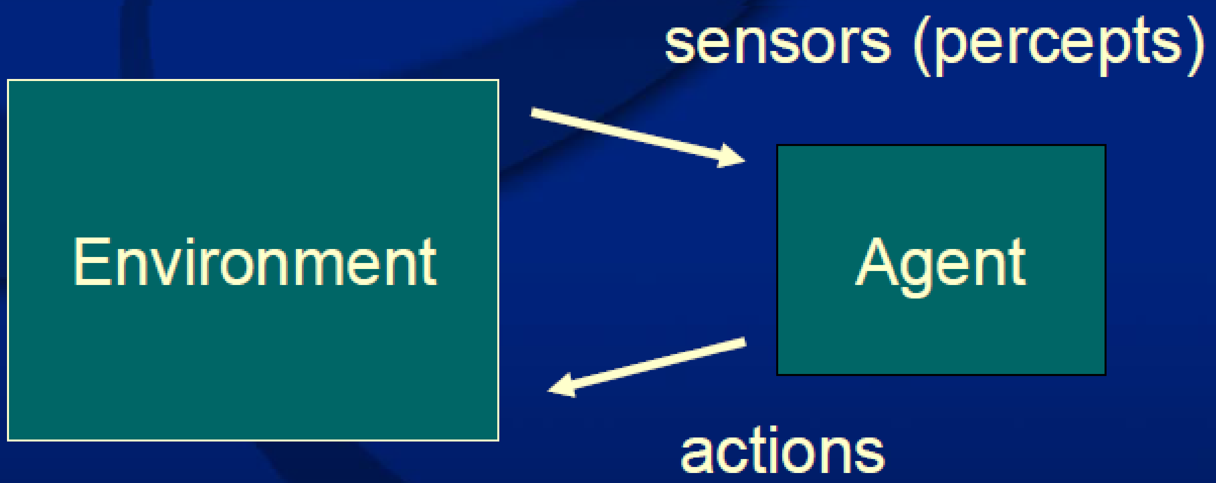
Example

Given an agent that always vacuums, and the performance criterion p. 38 and one more: -1 for vacuuming,

Show it is not a rational agent.

Agents



Current Approach

Programmer does most of work!

- Skeleton agent
- Table-driven agent
- Simple reflex agent
- Reflex agent with state
- Utility agent

Autonomy

What is it?

- Freedom to choose action?
- Behavior based on experience?
- Behavior not programmed in?

Autonomy

Book argues:

- Compensate for partial or incorrect knowledge
- With experience, becomes effectively independent of prior knowledge

Do you buy that?

Autonomy

Last assertion too strong:

Choose appropriate action based on context.

Agent Structure

Map percepts (and state!) to actions

Percepts: Bump, Dirt, Home

Actions: Forward, Right, Left, Suck, Stop

Agent Structure

- Map percept (and state!) to action
- Percepts: Stench, Breeze, Glitter, Bump, Scream
- Actions: FORWARD, RIGHT, LEFT, GRAB, SHOOT, CLIMB

Agent Functions & Programs

- An agent is completely specified by the agent function mapping percepts to actions
- One agent function is rational
- Aim: Find a way to implement concisely
- An agent program takes a single percept as input, keeps internal state

Enumeration vs. ???

Is there an approach to capture the infinite percept and behavior issue?

Consider π

What is it?

Geometrically?

Numerically?

Skeleton Agent

function SKELETON-AGENT(*percept*)
  returns action

 static: *memory*, the agent's memory of the world

 memory ← Update-Memory(*memory*,*percept*)
 action ← Choose-Best-Action(*memory*)
 memory ← Update-Memory(*memory*,*action*)

 return *action*

Matlab Agent Template

```
function action = CS4300_X(percept)

persistent state

if isempty(state)
    state = 0;
end
…  % e.g.,
if state == 0
   action = FORWARD;
end
```

Matlab Agent (cont'd)

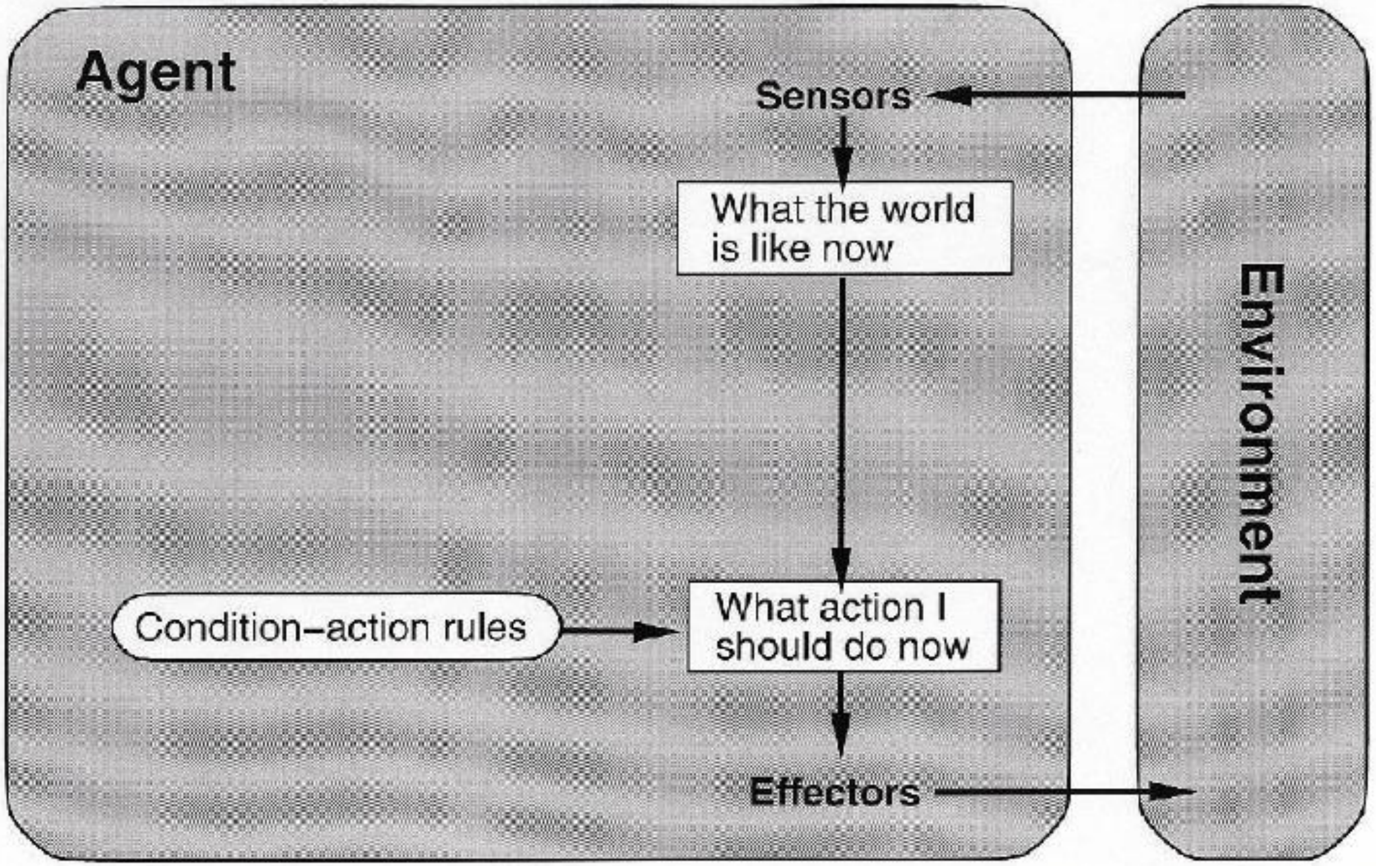
```
switch state
   case 0
      action = FORWARD;
      state = 1;
   case 1
       action = LEFT;
       state = 2;
   case 2
       action = LEFT;
       state = 3;
   case 3
    …
```

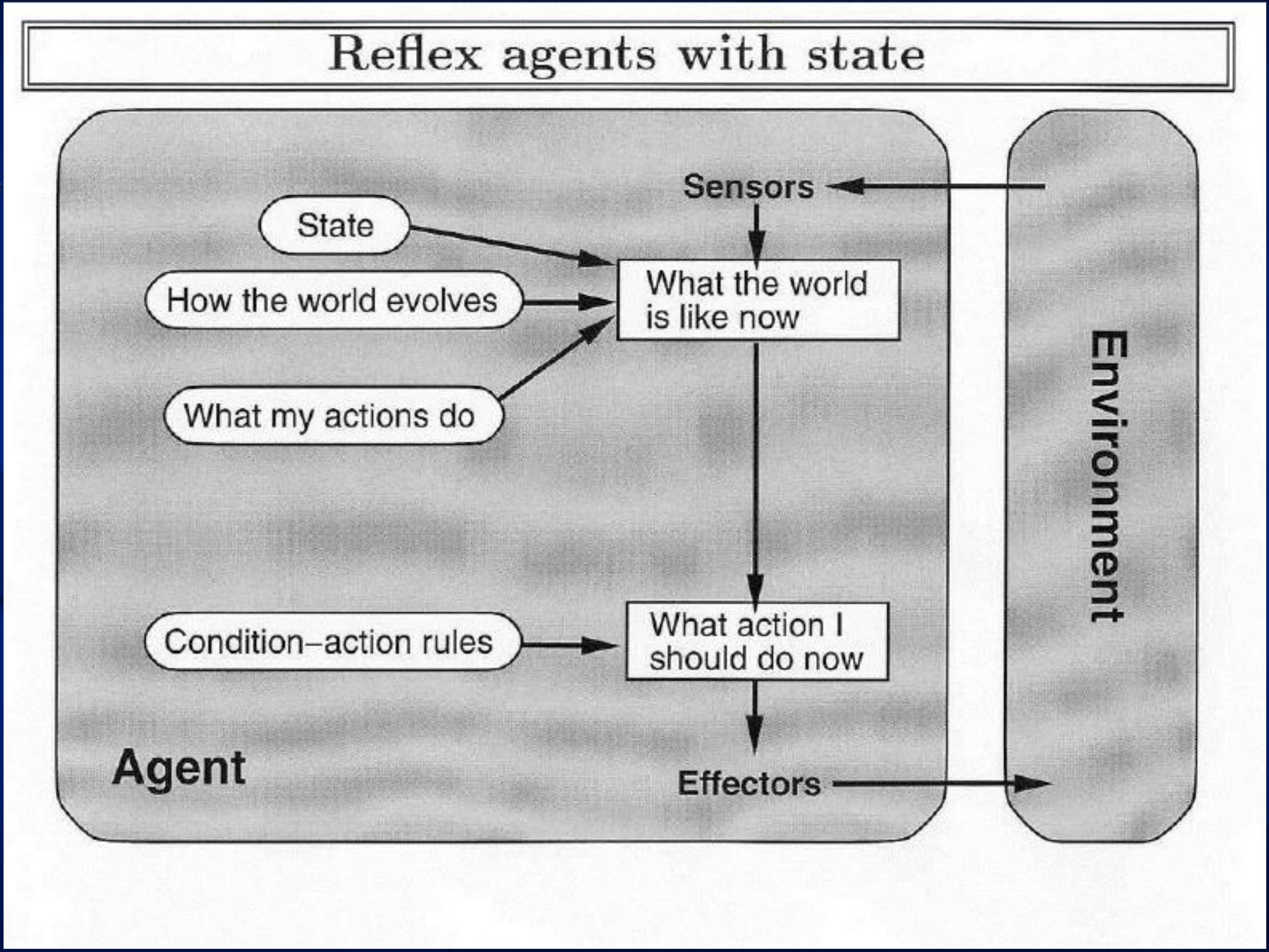
Agent Types

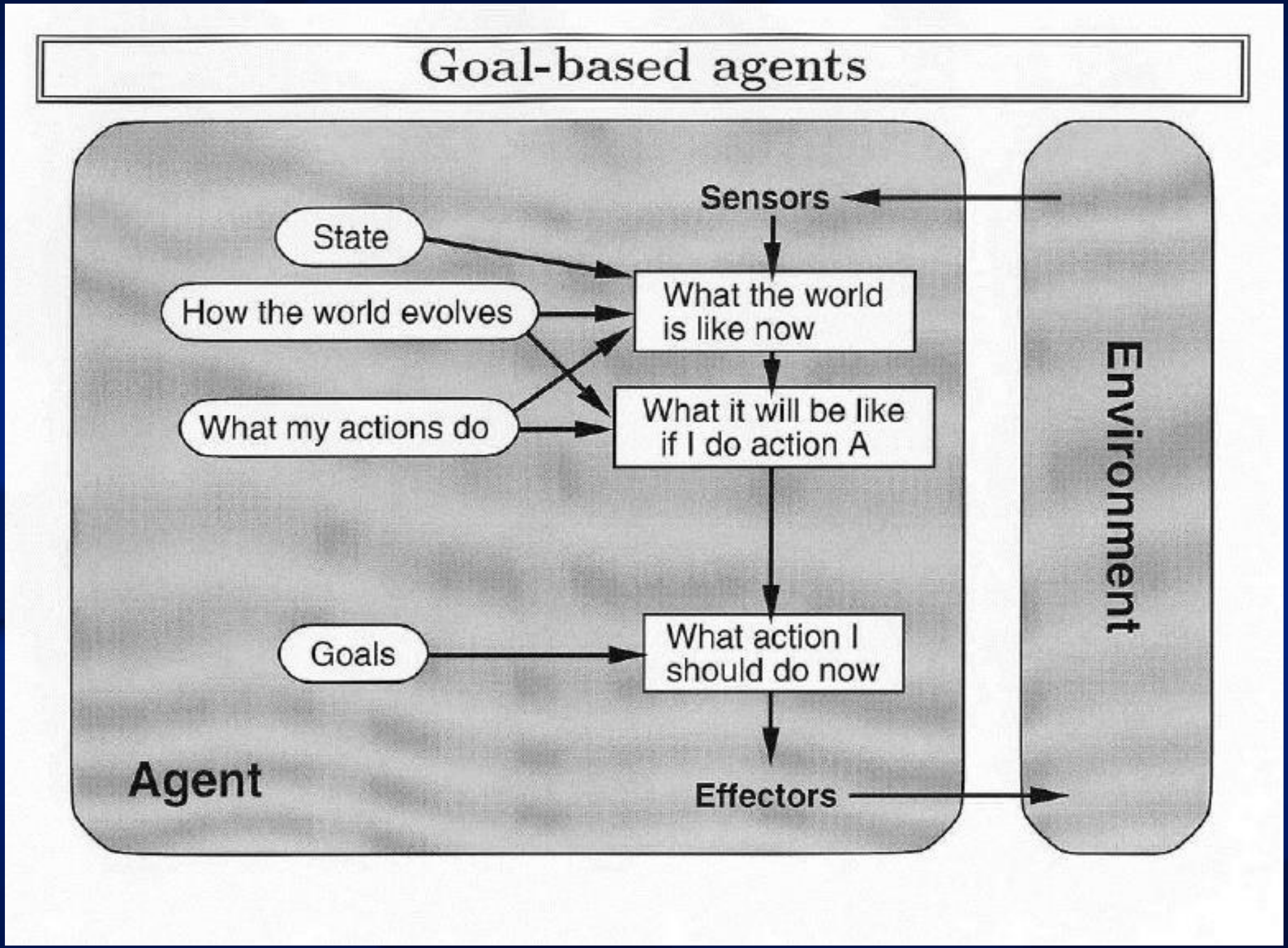
Four basic types in order of increasing generality:

- Simple reflex agents
- Reflex agents with state
- Goal-based agents
- Utility-based agents

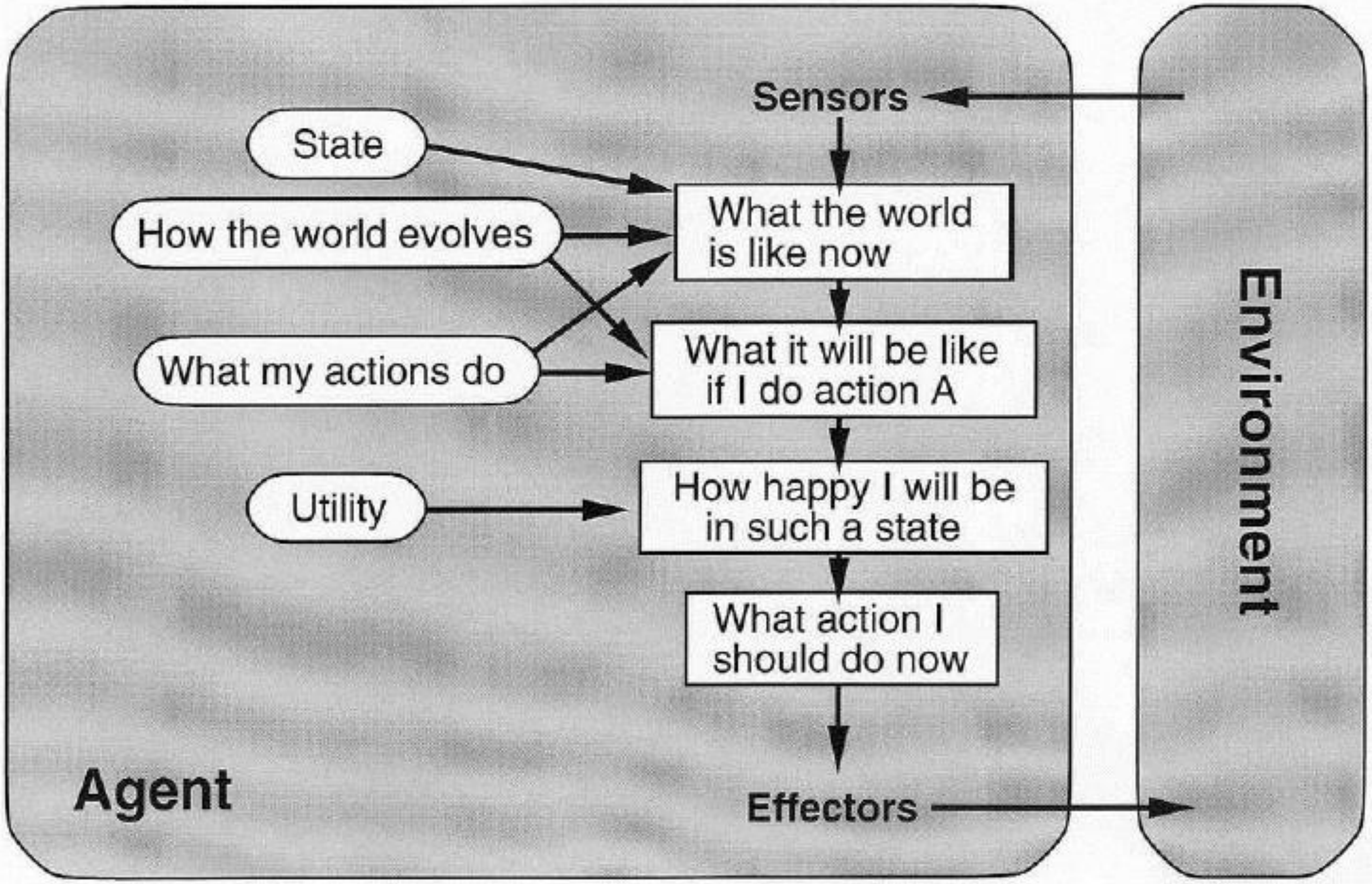
Simple reflex agents




Reflex agents with state
Sensors
State
How the world evolves
What the world is like now
What my actions do
Environment
Condition–action rules
What action I should do now
Agent
Effectors


Goal-based agents
Sensors
State
What the world
is like now
How the world evolves
What my actions do
What it will be like
if I do action A
Goals
What action I
should do now
Agent
Effectors
Environment

Utility-based agents



The Wumpus World

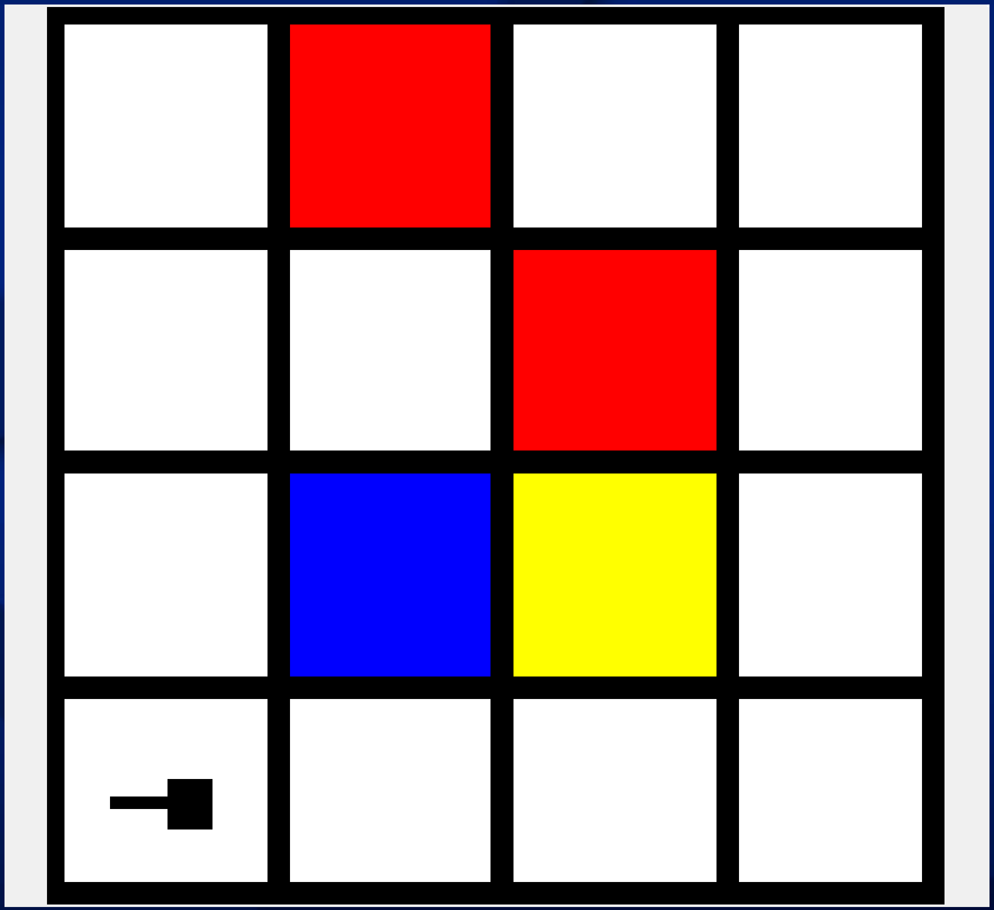
- Percepts: [Stench,Breeze,Glitter,Bump,Scream]
- Actions: [Forward,Turn_Right,Turn_Left,Grab,Shoot, Climb]
- Costs:
  - -1: each action
  - -50: shoot arrow
  - -1000: die
  - 1000: get gold and exit

Some Issues

- hidden goals in reflex agent; determine from:

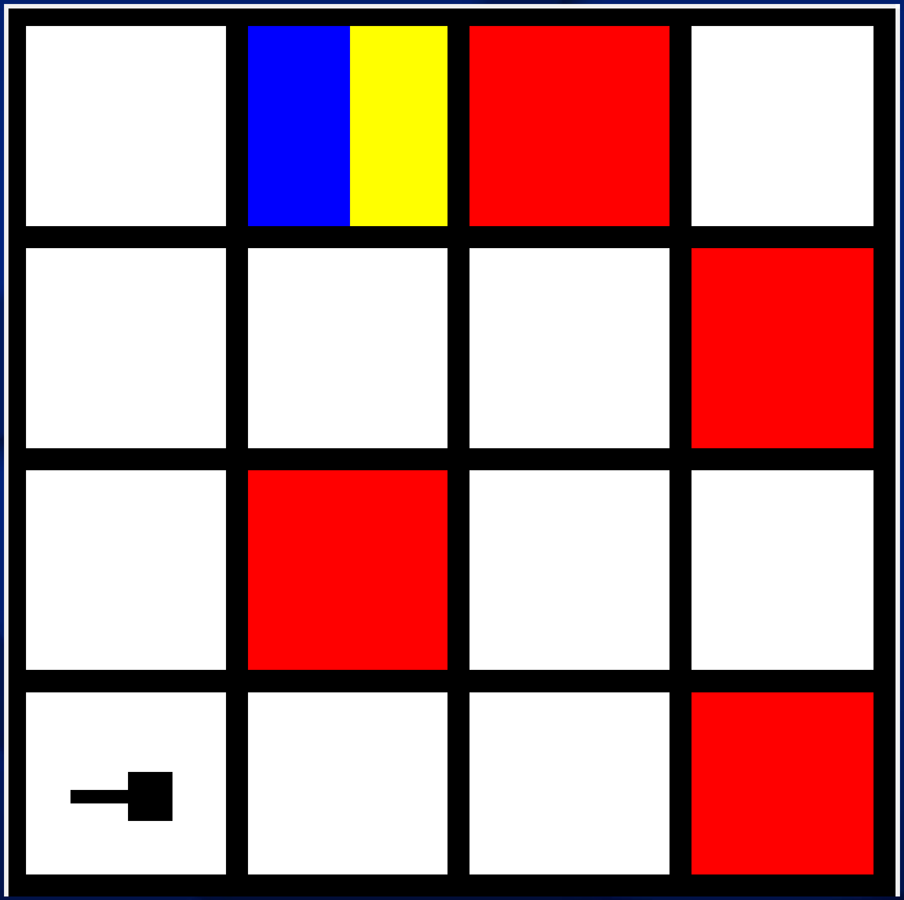
  Behavior? Observables?
- learned vs. prior knowledge
  - Nature/nurture,   learned/innate
- agent = architecture+program: does this cover everything? How about people? Adaptation?
- model how world evolves and how actions affect world: simulation?

Example 1



```
function action = CS4300_Example1(percept)
% CS4300_Example1 - simple agent example
%     It moves right, then left, then climbs out
% On input:
%      percept (1x5 Boolean vector): percept values
%       (1): Stench
%       (2): Pit
%       (3): Glitters
%       (4): Bumped
%       (5): Screamed
% On output:
%      action (int): action selected by agent
%        FORWARD = 1;
%        ROTATE_RIGHT = 2;
%        ROTATE_LEFT = 3;
%        GRAB = 4;
%        SHOOT = 5;
%        CLIMB = 6;
% Call:
%      a = CS4300_Example1([0,1,0,0,0]);
% Author:
%      T. Henderson
%      UU
%      Summer 2015
%

persistent state

FORWARD = 1;
ROTATE_RIGHT = 2;
ROTATE_LEFT = 3;
GRAB = 4;
SHOOT = 5;
CLIMB = 6;

if isempty(state)
    state = 0;
end

switch state
    case 0
        action = FORWARD;
        state = 1;
    case 1
        action = ROTATE_LEFT;
        state = 2;
    case 2
        action = ROTATE_LEFT;
        state = 3;
    case 3
        action = FORWARD;
        state = 4;
    case 4
        action = CLIMB;
end
```

Example 2 (A1)



From: Simulation, Modeling and Analysis
A.M. Law

4.5
CONFIDENCE INTERVALS AND HYPOTHESIS TESTS FOR THE MEAN

Let $X_1, X_2, \ldots, X_n$ be IID random variables with finite mean μ and finite variance σ^2. (Also assume that $\sigma^2 > 0$, so that the X_i's are not degenerate random variables.) In this section we discuss how to construct a confidence interval for μ and also the complementary problem of testing the hypothesis that $\mu = \mu_0$.

We begin with a statement of the most important result in probability theory, the classical central limit theorem. Let Z_n be the random variable $[\bar{X}(n) - \mu]/\sqrt{\sigma^2/n}$, and let $F_n(z)$ be the distribution function of Z_n for a sample size of n; that is, $F_n(z) = P(Z_n \le z)$. [Note that μ and σ^2/n are the mean and variance of $\bar{X}(n)$, respectively.] Then the *central limit theorem* is as follows [see Chung (1974, p. 169) for a proof].

THEOREM 4.1. $F_n(z) \to \Phi(z)$ as $n \to \infty$, where $\Phi(z)$, the distribution function of a normal random variable with $\mu = 0$ and $\sigma^2 = 1$ (henceforth called a *standard normal random variable*; see Sec. 6.2.2), is given by

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-y^2/2}\, dy \qquad \text{for } -\infty < z < \infty$$

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The theorem says, in effect, that if n is "sufficiently large," the random variable Z_n will be approximately distributed as a standard normal random variable, regardless of the underlying distribution of the X_i's. It can also be shown for large n that the sample mean $\overline{X}(n)$ is approximately distributed as a normal random variable with mean μ and variance σ^2/n.

The difficulty with using the above results in practice is that the variance σ^2 is generally unknown. However, since the sample variance $S^2(n)$ converges to σ^2 as n gets large, it can be shown that Theorem 4.1 remains true if we replace σ^2 by $S^2(n)$ in the expression for Z_n. With this change the theorem says that if n is sufficiently large, the random variable $t_n = [\overline{X}(n) - \mu]/\sqrt{S^2(n)/n}$ is approximately distributed as a standard normal random variable. It follows for large n that

$$P\left(-z_{1-\alpha/2} \le \frac{\overline{X}(n)-\mu}{\sqrt{S^2(n)/n}} \le z_{1-\alpha/2}\right)$$

$$= P\left[\overline{X}(n) - z_{1-\alpha/2}\sqrt{\frac{S^2(n)}{n}} \le \mu \le \overline{X}(n) + z_{1-\alpha/2}\sqrt{\frac{S^2(n)}{n}}\right]$$

$$\approx 1-\alpha \qquad (4.10)$$

where the symbol $\approx$ means "approximately equal" and $z_{1-\alpha/2}$ (for $0 < \alpha < 1$) is the upper $1-\alpha/2$ critical point for a standard normal random variable (see Fig. 4.15 and the last line of Table T.1 of the Appendix at the back of the book). Therefore, if n is sufficiently large, an approximate $100(1-\alpha)$ percent confidence interval for μ is given by

$$\overline{X}(n) \pm z_{1-\alpha/2}\sqrt{\frac{S^2(n)}{n}} \qquad (4.11)$$

For a given set of data $X_1, X_2, \ldots, X_n$, the lower confidence-interval endpoint $l(n, \alpha) = \overline{X}(n) - z_{1-\alpha/2}\sqrt{S^2(n)/n}$ and the upper confidence-interval endpoint $u(n, \alpha) = \overline{X}(n) + z_{1-\alpha/2}\sqrt{S^2(n)/n}$ are just numbers (actually, specific realizations of random variables) and the confidence interval $[l(n, \alpha), u(n, \alpha)]$ either contains μ

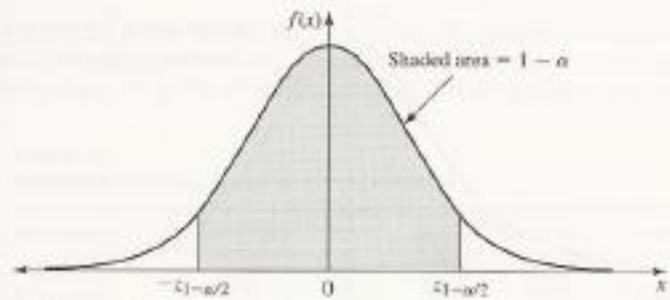

FIGURE 4.15
Density function for the standard normal distribution.

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or does not contain μ. Thus, there is nothing probabilistic about the single confidence interval $[l(n, \alpha), u(n, \alpha)]$ *after* the data have been obtained and the interval's endpoints have been given numerical values. The correct interpretation to give to the confidence interval (4.11) is as follows [see (4.10)]: If one constructs a very large number of independent $100(1 - \alpha)$ percent confidence intervals, each based on n observations, where n is sufficiently large, the proportion of these confidence intervals that contain (cover) μ should be $1 - \alpha$. We call this proportion the *coverage* for the confidence interval.

The difficulty in using (4.11) to construct a confidence interval for μ is in knowing what "n sufficiently large" means. It turns out that the more skewed (i.e., nonsymmetric) the underlying distribution of the X_i's, the larger the value of n needed for the distribution of t_n to be closely approximated by $\Phi(z)$. (See the discussion later in this section.) If n is chosen too small, the actual coverage of a desired $100(1 - \alpha)$ percent confidence interval will generally be less than $1 - \alpha$. This is why the confidence interval given by (4.11) is stated to be only approximate.

In light of the above discussion, we now develop an alternative confidence-interval expression. If the X_i's are *normal* random variables, the random variable $t_n = [\bar{X}(n) - \mu]/\sqrt{S^2(n)/n}$ has a t distribution with $n - 1$ degrees of freedom (df) [see, for example, Hogg and Craig (1995, pp. 181–182)], and an *exact* (for any $n \geq 2$) $100(1 - \alpha)$ perce ... ven by

$$\boxed{\bar{X}(n) \pm t_{n-1,1-\alpha/2}\sqrt{\frac{S^2(n)}{n}}} \tag{4.12}$$

where $t_{n-1,1-\alpha/2}$ is the upper $1 - \alpha/2$ critical point for the t distribution with $n - 1$ df. These critical points are given in Table T.1 of the Appendix at the back of the book. Plots of the density functions for the t distribution with 4 df and for the standard normal distribution are given in Fig. 4.16. Note that the t distribution is less peaked and has longer tails than the normal distribution, so, for any finite n, $t_{n-1,1-\alpha/2} > z_{1-\alpha/2}$. We call (4.12) the *t confidence interval*.

The quantity that we add to and subtract from $\bar{X}(n)$ in (4.12) to construct the confidence interval is called the *half-length* of the confidence interval. It is a measure of how precisely we know μ. It can be shown that if we increase the sample

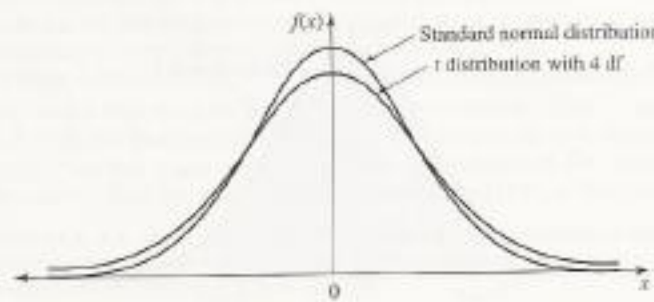

FIGURE 4.16
Density functions for the t distribution with 4 df and for the standard normal distribution.

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size from n to $4n$ in (4.12), then the half-length is decreased by a factor of approximately 2 (see Prob. 4.20).

In practice, the distribution of the X_i's will rarely be normal, and the confidence interval given by (4.12) will also be approximate in terms of coverage. Since $t_{n-1,1-\alpha/2} > z_{1-\alpha/2}$, the confidence interval given by (4.12) will be larger than the one given by (4.11) and will generally have coverage closer to the desired level $1 - \alpha$. For this reason, we recommend using (4.12) to construct a confidence interval for μ. Note that $t_{n-1,1-\alpha/2} \to z_{1-\alpha/2}$ as $n \to \infty$; in particular, $t_{40,0.95}$ differs from $z_{0.95}$ by less than 3 percent. However, in most of our applications of (4.12) in Chaps. 9, 10, and 12, n will be small enough for the difference between (4.11) and (4.12) to be appreciable.

EXAMPLE 4.26. Suppose that the 10 observations 1.20, 1.50, 1.68, 1.89, 0.95, 1.49, 1.58, 1.55, 0.50, and 1.09 are from a normal distribution with unknown mean μ and that our objective is to construct a 90 percent confidence interval for μ. From these data we get

$$\bar{X}(10) = 1.34 \qquad \text{and} \qquad S^2(10) = 0.17$$

which results in the following confidence interval for μ:

$$\bar{X}(10) \pm t_{9,0.95}\sqrt{\frac{S^2(10)}{10}} = 1.34 \pm 1.83\sqrt{\frac{0.17}{10}} = 1.34 \pm 0.24$$

Note that (4.12) was used to construct the confidence interval and that $t_{9,0.95}$ was taken from Table T.1. Therefore, subject to the interpretation stated above, we claim with 90 percent confidence that μ is in the interval [1.10, 1.58].

We now discuss how the coverage of the confidence interval given by (4.12) is affected by the distribution of the X_i's. In Table 4.1 we give estimated coverages for 90 percent confidence intervals based on 500 independent experiments for each of the sample sizes $n = 5$, 10, 20, and 40 and each of the distributions normal, exponential, chi square with 1 df (a standard normal random variable squared; see the discussion of the gamma distribution in Sec. 6.2.2), lognormal (e^Y, where Y is a standard normal random variable; see Sec. 6.2.2), and hyperexponential whose distribution function is given by

$$F(x) = 0.9F_1(x) + 0.1\,F_2(x)$$

where $F_1(x)$ and $F_2(x)$ are the distribution functions of exponential random variables with means 0.5 and 5.5, respectively. For example, the table entry for the exponential distribution and $n = 10$ was obtained as follows. Ten observations were generated

TABLE 4.1
Estimated coverages based on 500 experiments

Distribution	Skewness ν	$n = 5$	$n = 10$	$n = 20$	$n = 40$
Normal	0.00	0.910	0.902	0.898	0.900
Exponential	2.00	0.854	0.878	0.870	0.890
Chi square	2.83	0.810	0.830	0.848	0.890
Lognormal	6.18	0.758	0.768	0.842	0.852
Hyperexponential	6.43	0.584	0.586	0.682	0.774

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$\gamma = P(T_\nu \le t_{\nu,\gamma})$, where T_ν is a random variable …
satisfying $\gamma = P(Z \le z_\gamma)$, where Z is a standard normal random variable

r	γ: 0.6000	0.7000	0.8000	0.9000	0.9333	0.9500	0.9600	0.9667	0.9750	0.9800	0.9833	0.9875	0.9900	0.9917	0.
1	0.325	0.727	1.376	3.078	4.702	6.314	7.916	9.524	12.706	15.895	19.043	25.452	31.821	38.342	5
2	0.289	0.617	1.061	1.886	2.456	2.920	3.320	3.679	4.303	4.849	5.334	6.205	6.965	7.665	
3	0.277	0.584	0.978	1.638	2.045	2.353	2.605	2.823	3.182	3.482	3.738	4.177	4.541	4.864	
4	0.271	0.569	0.941	1.533	1.879	2.132	2.333	2.502	2.776	2.999	3.184	3.495	3.747	3.966	
5	0.267	0.559	0.920	1.476	1.790	2.015	2.191	2.337	2.571	2.757	2.910	3.163	3.365	3.538	
6	0.265	0.553	0.906	1.440	1.735	1.943	2.104	2.237	2.447	2.612	2.748	2.969	3.143	3.291	
7	0.263	0.549	0.896	1.415	1.698	1.895	2.046	2.170	2.365	2.517	2.640	2.841	2.998	3.130	
8	0.262	0.546	0.889	1.397	1.670	1.860	2.004	2.122	2.306	2.449	2.565	2.752	2.896	3.018	
9	0.261	0.543	0.883	1.383	1.650	1.833	1.973	2.086	2.262	2.398	2.508	2.685	2.821	2.936	
10	0.260	0.542	0.879	1.372	1.634	1.812	1.948	2.058	2.228	2.359	2.465	2.634	2.764	2.872	
11	0.260	0.540	0.876	1.363	1.621	1.796	1.928	2.036	2.201	2.328	2.430	2.593	2.718	2.822	
12	0.259	0.539	0.873	1.356	1.610	1.782	1.912	2.017	2.179	2.303	2.402	2.560	2.681	2.782	
13	0.259	0.538	0.870	1.350	1.601	1.771	1.899	2.002	2.160	2.282	2.379	2.533	2.650	2.748	
14	0.258	0.537	0.868	1.345	1.593	1.761	1.887	1.989	2.145	2.264	2.359	2.510	2.624	2.720	
15	0.258	0.536	0.866	1.341	1.587	1.753	1.878	1.978	2.131	2.249	2.342	2.490	2.602	2.696	
16	0.258	0.535	0.865	1.337	1.581	1.746	1.869	1.968	2.120	2.235	2.327	2.473	2.583	2.675	
17	0.257	0.534	0.863	1.333	1.576	1.740	1.862	1.960	2.110	2.224	2.315	2.458	2.567	2.657	
18	0.257	0.534	0.862	1.330	1.572	1.734	1.855	1.953	2.101	2.214	2.303	2.445	2.552	2.641	
19	0.257	0.533	0.861	1.328	1.568	1.729	1.850	1.946	2.093	2.205	2.293	2.433	2.539	2.627	
20	0.257	0.533	0.860	1.325	1.564	1.725	1.844	1.940	2.086	2.197	2.285	2.423	2.528	2.614	
21	0.257	0.532	0.859	1.323	1.561	1.721	1.840	1.935	2.080	2.189	2.277	2.414	2.518	2.603	
22	0.256	0.532	0.858	1.321	1.558	1.717	1.835	1.930	2.074	2.183	2.269	2.405	2.508	2.593	
23	0.256	0.532	0.858	1.319	1.556	1.714	1.832	1.926	2.069	2.177	2.263	2.398	2.500	2.584	
24	0.256	0.531	0.857	1.318	1.553	1.711	1.828	1.922	2.064	2.172	2.257	2.391	2.492	2.575	
25	0.256	0.531	0.856	1.316	1.551	1.708	1.825	1.918	2.060	2.167	2.251	2.385	2.485	2.568	
26	0.256	0.531	0.856	1.315	1.549	1.706	1.822	1.915	2.056	2.162	2.246	2.379	2.479	2.561	
27	0.256	0.531	0.855	1.314	1.547	1.703	1.819	1.912	2.052	2.158	2.242	2.373	2.473	2.554	
28	0.256	0.530	0.855	1.313	1.546	1.701	1.817	1.909	2.048	2.154	2.237	2.368	2.467	2.548	
29	0.256	0.530	0.854	1.311	1.544	1.699	1.814	1.906	2.045	2.150	2.233	2.364	2.462	2.543	
30	0.256	0.530	0.854	1.310	1.543	1.697	1.812	1.904	2.042	2.147	2.230	2.360	2.457	2.537	
40	0.255	0.529	0.851	1.303	1.532	1.684	1.796	1.886	2.021	2.123	2.203	2.329	2.423	2.501	
50	0.255	0.528	0.849	1.299	1.526	1.676	1.787	1.875	2.009	2.109	2.188	2.311	2.403	2.479	
75	0.254	0.527	0.846	1.293	1.517	1.665	1.775	1.861	1.992	2.090	2.167	2.287	2.377	2.450	
100	0.254	0.526	0.845	1.290	1.513	1.660	1.769	1.855	1.984	2.081	2.157	2.276	2.364	2.436	
∞	0.253	0.524	0.842	1.282	1.501	1.645	1.751	1.834	1.960	2.054	2.127	2.241	2.326	2.395	

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Questions?