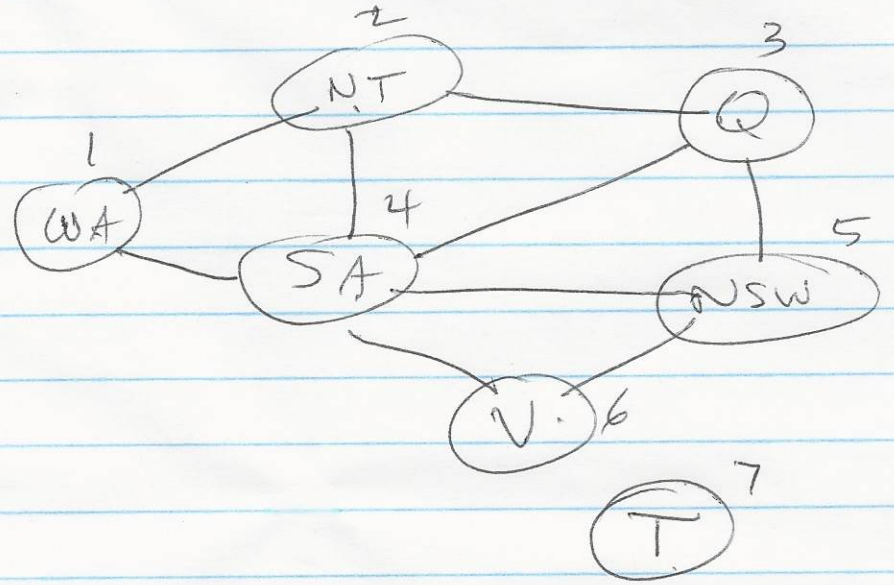
Chapter 6

Problem from book:



Find color assignment so no 2 neighbors have same color.
Colors: $\{R, G, B\}$

Enumerate all possible assignments

3 colors: 0, 1, 2

7 variables : 7 digit base 3 number

0000000
0000001
0000002
0000010
⋮
2222222

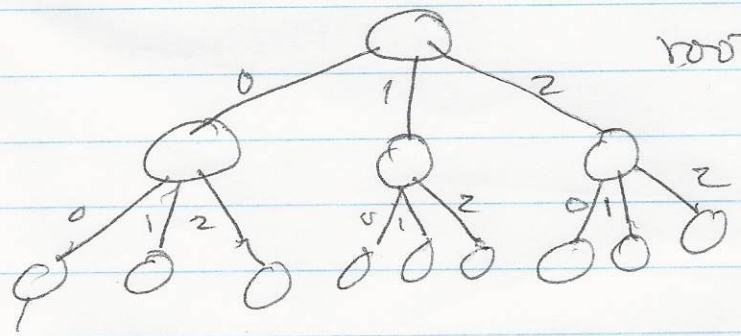
check

$d_1 \neq d_2$
$d_1 \neq d_4$
$d_2 \neq d_3$
$d_2 \neq d_4$
$d_3 \neq d_4$
$d_3 \neq d_5$
$d_4 \neq d_5$
$d_4 \neq d_6$
$d_5 \neq d_6$

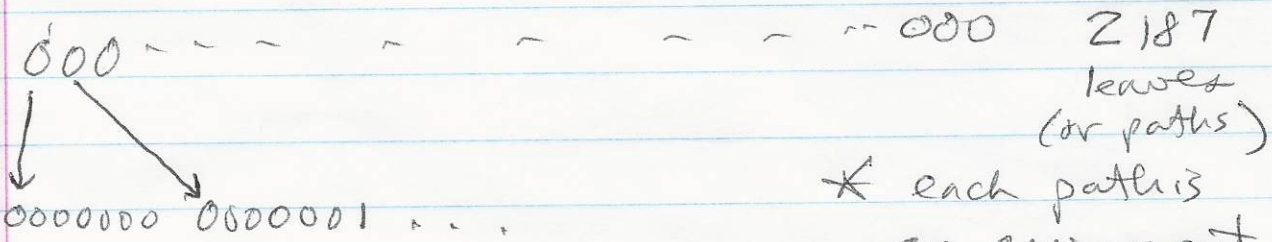
$3^7 = 2187$

CS4310-crypt
CS4310_F2015/EC2

View it as search



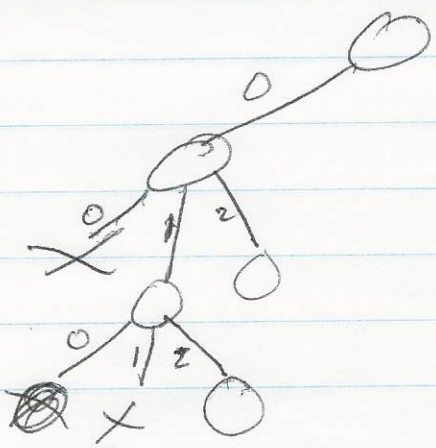
root: no assignment
V1 possibilities
V2 possibilities



2187 leaves (or paths)

* each path is an assignment

But: can prune wherever path fails



number of nodes expanded in search is less.

How can this be done in systematic way?

6. Constraint Satisfaction Problems (CSP)

set of 3 components X, D, C

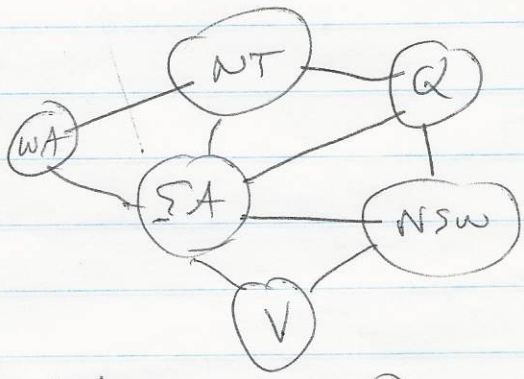
$X = \{X_1, \ldots X_n\}$ set of variables
$D = \{D_1, \ldots D_n\}$ set of domains
$C = \{C_1, \ldots C_k\}$ constraints that specify allowable sets of values on variable combinations

$D_i = \{v_1, \ldots v_k\}$ values X_i can take on

C_i : pair $\langle$scope, rel$\rangle$
scope is a tuple of variables
rel is a relation on those tuples
generally, given as $R(X_1, X_2 \ldots X_p)$

a state : assignment of values to some of the variables
consistent : does not violate constraints
complete : every variable assigned
solution : complete, consistent

E.g., map coloring
use 3 colors
no neighbors have same color

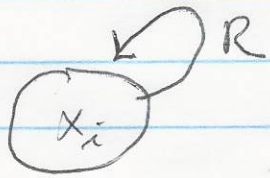


Once a color is assigned,
many assignments are eliminated
e.g., assign SA ← blue ⇒ $3^5 = 243$
using constraint $2^5 = 32$

once partial assignment fails, don't pursue others

Constraints

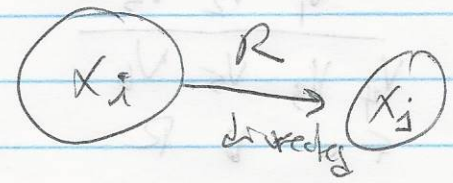
unary $R(X_i)$



binary $R(X_i, X_j)$

$(V_1, V_2) \in R$

does not necessarily imply $(V_2, V_1) \in R$



if symmetric relation use non-directed edge

e.g., different color

global constraint $R(X_{i_1} \dots X_{i_p})$

e.g., in cryptarithmetic, the variables are all different

~~T-438 345 8305 Anna~~

```
  TWO
+TWO
-----
 FOUR
```

example in book

$V_1\ V_2\ V_3$

$V_1\ V_2\ V_3$

$V_4\ V_3\ V_5\ V_6$

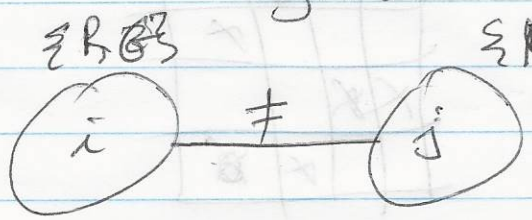
How many solutions? ~~15~~ 19 e.g., 765130 $C^{10}\ C^{100}\ C^{1000}$ 1 1 1

How many combinations? $10^6 * 2^3 = 8M$

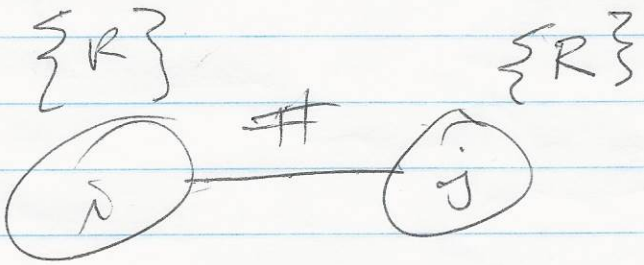
CS4300 - crypt

constraint propagation : reduce possible # of labels

(arc) consistency : every label at a node has at least one support label at each neighbor



consistent
G at i supports R at j



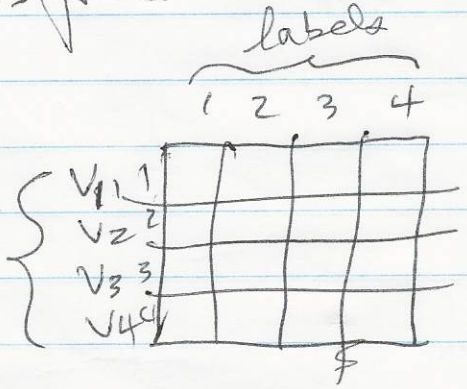
not consistent

node consistency

unary relations satisfied

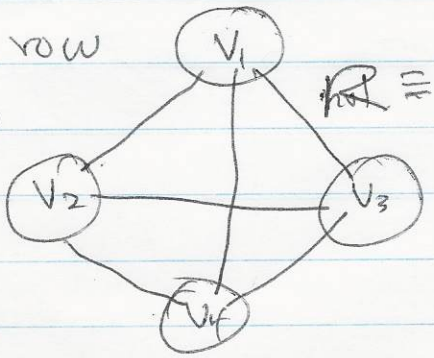
e.g., N-queens

N variables
K-graph



each variable represents a row
each label " a col.

How many combinations?
N^N



≡ not attack

Arc Consistent variable

X_i ac wrt X_j if $\forall d_i \in D_i \; \exists e \in D_j \Rightarrow R(d,e)$

AC-3 creates arc consistent set of domains

① REMOVE function

returns true if some label deleted

② AC-3

if labels deleted from D_i
check all neighbors that
depend on labels in D_i

N-queens 3×3 ; 4×4

Path Consistency

$\{X_i, X_j\}$ pc wrt X_m

if $\forall X_i = a \; \& \; X_j = b \Rightarrow R(a,b)$

$\exists X_j = c \Rightarrow R(a,c) \; \& \; R(c,b)$

OK for pruning, but must be embedded in search.

AC-4 !

Backtracking

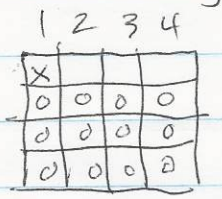
When constraints are applied, does not result in a solution.

4 Queens

$D_i = \{1, 2, 3, 4\}$

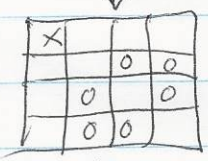
and AC3 does not eliminate any labels!

Apply search: consider single variable at each node
[may link them to level]

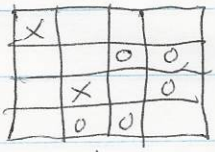


← Set Q1 ← 1

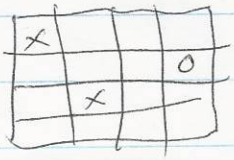
↓ Apply constraints



↓ Set Q2 ← 3



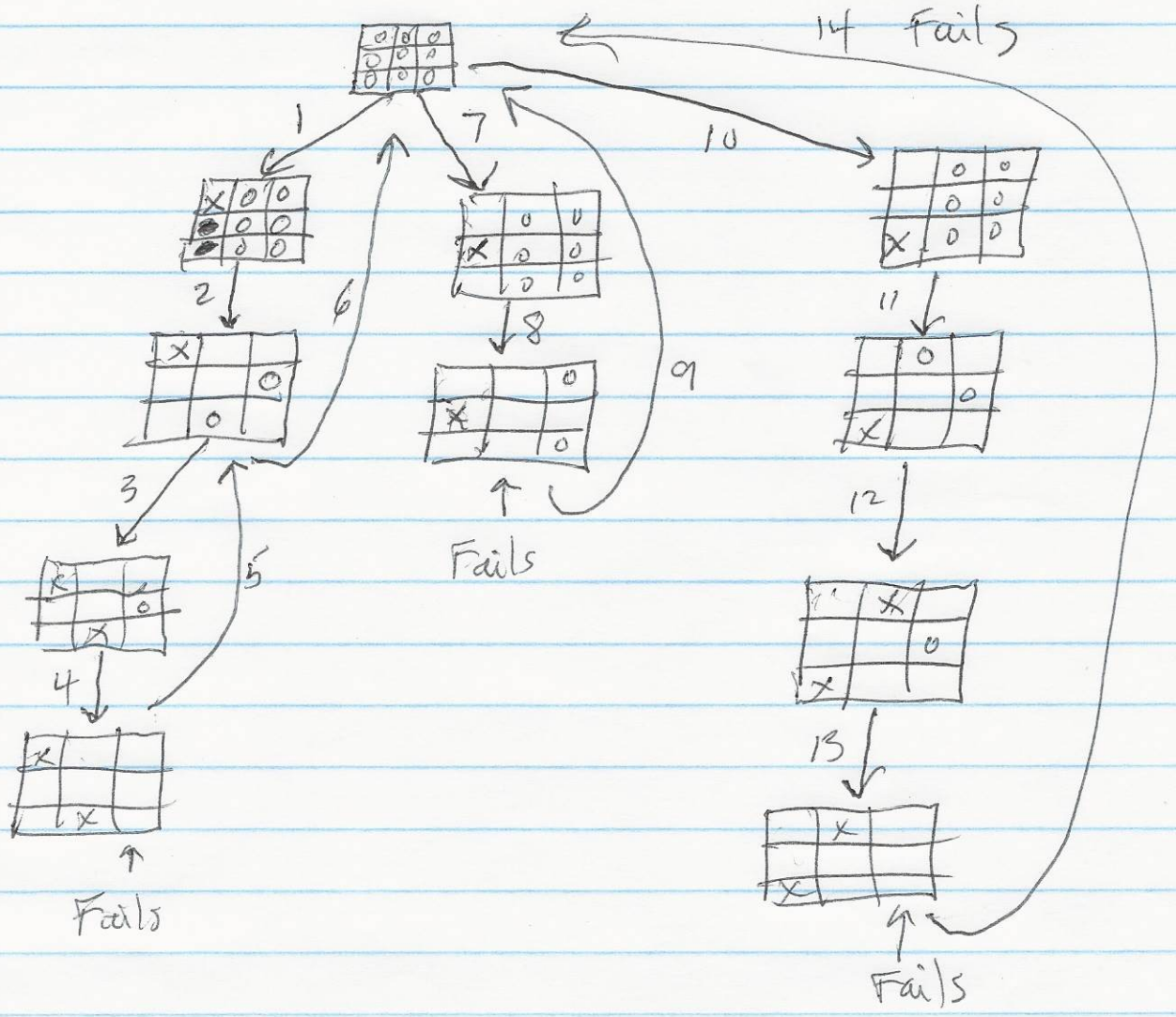
↓ Apply constraints



← Failure since Q3 domain is $\emptyset$

backtrack search

- depth-first
- one variable assigned a value at a time
- backs up when no values left



6/9

Issues: * Which variable to assign value
* What value to assign
* What inferences to perform (constraints)
* How to avoid thrashing (repeated failure)

E.g., choose variable with smallest # of values
choose variable with largest degree
:

Apply arc consistency