

HERIOT-WATT UNIVERSITY

MASTERS THESIS

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# Formal Verification of Neural Networks in Go

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for the degree of MSc. Robotics*

*in the*

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&

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# Declaration of Authorship

I, Arran DINSMORE, declare that this thesis titled, 'Formal Verification of Neural Networks in Go' and the work presented in it is my own. I confirm that this work submitted for assessment is my own and is expressed in my own words. Any uses made within it of the works of other authors in any form (e.g., ideas, equations, figures, text, tables, programs) are properly acknowledged at any point of their use. A list of the references employed is included.

Signed: Arran Dinsmore

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Date: April 2021

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*“Program testing can be used to show the **presence** of bugs, but never to show their **absence**!”*

Edsger W. Dijkstra

# *Abstract*

As machine learning for safety critical applications such as autonomous vehicles are starting to be developed beyond proof of concepts, and enter into production within society, there is a need to ensure these systems do not fail.

Traditional rigorous testing methods are not a viable approach for such black box systems, and thus a need for formal verification methods that can prove the robustness of a system are required.

Additionally, the choice of programming language used for these tasks has grown with new machine learning extensions being developed on existing languages.

This project will investigate how robust programming infrastructures can be used to enhance formal verification approaches for machine learning tasks, with the main objective of developing a formal methods framework for verifying neural networks in the Go programming language.

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# List of Abbreviations

AI	Artificial Intelligence. <a href="#">3</a> , <a href="#">4</a>
AIV	Artificial Intelligence Verification. <a href="#">3</a> , <a href="#">4</a> , <a href="#">6</a> , <a href="#">11</a>
BGD	Batch Gradient Descent. <a href="#">10</a>
CNN	Convolutional Neural Network. <a href="#">9</a>
CV	Computer Vision. <a href="#">7–9</a>
DNN	Deep Neural Network. <a href="#">9</a>
FP	Functional Programming. <a href="#">4</a>
GD	Gradient Descent. <a href="#">10</a>
MAS	Multi-Agent System. <a href="#">3</a>
ML	Machine Learning. <a href="#">1–4</a> , <a href="#">9</a>
MLP	Multi-Layer Perceptron. <a href="#">9</a>
NN	Neural Network. <a href="#">1–11</a>
ReLU	Rectified Linear Unit. <a href="#">7</a> , <a href="#">8</a>
RNN	Recurrent Neural Network. <a href="#">9</a>
SGD	Stochastic Gradient Descent. <a href="#">10</a>
SMT	Satisfiability Modulo Theories. <a href="#">4</a>

# Chapter 1

## Introduction

### 1.1 Context

[Machine Learning \(ML\)](#) algorithms are becoming increasingly present in systems that operate within shared environments with humans, or involve direct interaction with humans themselves [[Pereira and Thomas, 2020](#)]. These systems are often defined as safety-critical, such that their failures lead to unintended and potentially harmful behaviours [[Amodei et al., 2016](#)]. Examples of these systems include autonomous automotive systems, traffic control systems, medical devices, aviation software, industrial robotics, and many more cyber-physical systems that interact with our environment. Many of these systems have so far only existed as proof of concepts, but are steadily approaching commercial use within our society.

Additionally, recent research has exposed broad vulnerabilities to adversarial attacks within data driven [ML](#) algorithms, including [Neural Networks \(NNs\)](#); where applying small but intentional perturbations to an input which are not noticeable to humans, can lead to a model outputting an incorrect classification with high confidence [[Goodfellow et al., 2014](#)]. An example of such an attack can be seen in *Fig. 1.1*. Consequently, the testing and verification of [ML](#) for the use of controlling safety-critical systems has become a focused area of research in recent years.

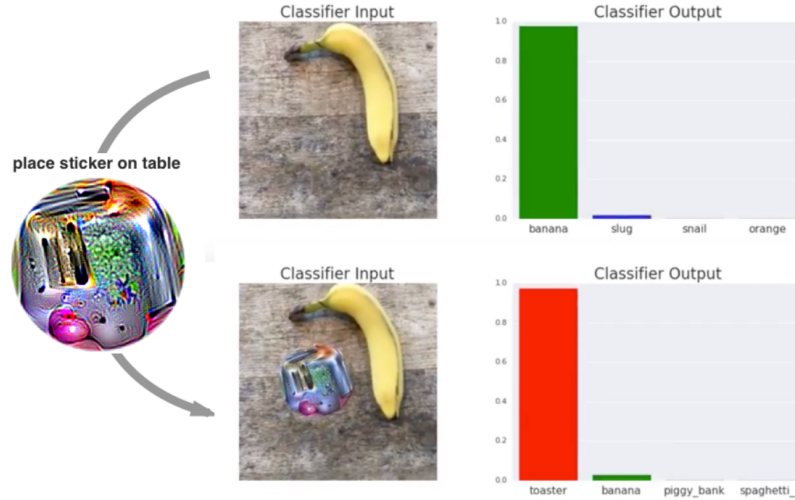


FIGURE 1.1: **Google’s Adversarial Patch** – An example of a method to create targeted adversarial attacks on NNs by adding carefully designed noise via a physical patch [Brown et al., 2018].

This thesis will use the following definitions for software testing and verification. Software testing, or validation, is defined as the evaluation of a system under various conditions and observing its behaviour while looking for defects [Pereira and Thomas, 2020]. In the context of ML development, testing is used to ensure that a trained model generalises accurately to some previously unseen test data.

Verification is defined as the process of determining whether the products of a phase of the software development process fulfill the requirements established during the previous phase [Ammann and Offutt, 2008]. Formal verification in other words, formulates logical arguments that a system will not act abnormally under a wide range of circumstances, and can be used to determine not only generality, but also the robustness and correctness of a system.

The challenges regarding verification of ML models stem from the typically less deterministic and more statistically-oriented nature of their algorithms, which lead to a lower degree of understanding than software that is explicitly programmed to perform a specific task [Bishop, 2006]. These types of systems are commonly referred to as *black box* systems, where the internal mechanisms are not revealed; in other words, it is impossible to understand a model just by looking at its parameters [Molnar, 2019].

## 1.2 Motivation

Public calls for *sensible* or *verifiable* *Artificial Intelligence (AI)* have been raised in recent years due to ever increasing development of complex and pervasive systems that are entering into our everyday lives [Russell et al., 2016].

Formal verification of deterministic software systems has seen significant progress since the early verification systems. These early systems [Boyer and Moore, 1990, Guaspari et al., 1993, Polak, 1979] often struggled to be widely adopted into industry applications. However, due to the ever increasing complexity of deployed software, new verification tools have been developed with the intent of being accessible to a wide range of industry software engineers [Fisher et al., 2017].

On the other hand, verification of non-deterministic systems has seen relatively little progress, with the exception of *Multi-Agent Systems (MASs)* [Kouvaros and Lomuscio, 2016, Lomuscio et al., 2017]. Indeed, due to the nature of *Artificial Intelligence Verification (AIV)* research, there are limited resources with regard to the programming tools available for researchers in this area. This is especially true for work within *ML*, as the programming languages and tools commonly used for *traditional* verification are often disparate from those widely adopted by the *ML* communities.

Popular programming languages used for *ML* such as Python or Matlab currently have comparatively less formal verification tools available than those concerned with system infrastructure or embedded applications. Additionally, *AIV* toolkits for *ML* tasks in these languages are still in early stages of development, and mainly focused on the verification of *NNs* [Kokke, 2020].

Furthermore, the landscape of *ML* programming itself is forever shifting, and while there is yet a programming language dedicated for *ML* tasks, huge efforts from programming language designers have been made in developing *ML* libraries for existing languages. This is necessary in order to handle the extremely high computational demands, and to simplify model languages to make them easier to add domain-specific optimisations and features [Innes et al., 2017].

A prime example of such development can be seen in the Go programming language, or *GoLang*. A relatively new language, originally developed by Google in 2009 with the intention of creating a modern general-purpose language similar to C. GoLang has seen a surge in popularity within the *ML* community since the release of its first extensive *ML* package, *Gorgonia*, in 2016, which heavily relies on

the use of expression graphs [Chew, 2016]. This package allows GoLang developers to take advantage of automatic and symbolic differentiation, gradient descent optimisations, numerical stabilisation, added support for CUDA/GPGPU computation, and comparatively quick speeds than its Python counterparts (Theano and TensorFlow) [GoLang, 2020].

Another example of a programming paradigm shift towards dedicated ML languages, is Microsoft’s efforts in developing an efficient differentiable version of the Functional Programming (FP) language F [Shaikhha et al., 2019].

Consequently, as programming languages continue to develop ML capabilities, there is a need for exploring new and scalable approaches for developing AIV tools in these languages. This is especially important for programming languages which are being adopted by industry to implement ML models for the use within safety-critical or pervasive systems.

### 1.3 Aims & Objectives

The aim of this project is to investigate the current programming paradigms within ML development, and to explore the suitability of current formal verification toolkits available to them. Subsequently, this thesis will aim to design and implement a GoLang formal methods framework for Gorgonia NNs, providing GoLang ML developers with a set of tools which will allow them to produce safe and fair AI applications.

This framework will extend upon the work made by [Kokke, 2020], and the Sapphire library implemented in Python which successfully translates TensorFlow feed-forward NN models to the Z3 Satisfiability Modulo Theories (SMT) solver created by Microsoft Research [De Moura and Bjørner, 2008].

To achieve this project’s aims, the following objectives should be met:

- *Objective 1* - Conduct a feasibility study with regards to developing a formal methods framework for NNs in Go.
- *Objective 2* - Implement bindings that map the parameters of a Gorgonia NN model to Z3 variables.
- *Objective 3* - Select data in order to train and verify NN models using this project’s formal methods framework.

- 
- *Objective 4* - Implement a series of NN models in Gorgonia using the data sets mentioned in *Objective 3*.
  - *Objective 5* - Verify the correctness of Gorgonia NNs using the bindings mentioned in *Objective 2*.
  - *Objective 6* - Make conclusions about the developed framework's benefits and limitations, and discuss future improvements to the methodology as described in *Objective 1*.

## Chapter 2

# Background & Literature Review

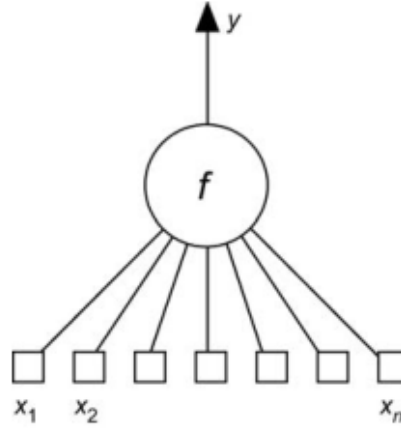
This chapter will provide a background understanding to the important concepts that are required by this thesis, and explore the current trends within [AIV](#) research. This includes an introduction to formal verification, both within deterministic and non-deterministic systems; an overview of the current state of [NN](#) and deep learning research, and the programming paradigms used for their development; and finally an investigation into the Go programming language infrastructure and the feasibility of using it for verifying [NNs](#).

### 2.1 Neural Networks & Deep Learning

This section aims to clarify the concepts of [NNs](#) and deep learning, as well as to show the successes and failures of the field, and to demonstrate the need for formal approaches for developing such algorithms.

#### 2.1.1 Overview

[NNs](#) are learning algorithms based on a loose analogy of how the human brain functions. They consist of nodes, or neurons (*see Fig. 2.1*), which act as functions that output a nonlinear combination of weighted inputs and a bias [[Dreyfus, 2005](#)]. Learning is achieved by adjusting the weights on the connections between nodes, which are analogous to synapses and neurons in nature [[Sammut and Webb, 2010](#)].




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FIGURE 2.1: **Artificial Neuron** – a nonlinear bounded function  $y = f(x_1, x_2, \dots, x_n; w_1, \dots, w_n)$  where the  $x_i$  are the input values and the  $w_i$  are the weights of the neuron [Dreyfus, 2005].

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A weight is assigned to each of a neuron's inputs. They are the co-efficients of a neuron's equation and therefore reflect the importance of individual inputs. A bias is a constant value assigned to each neuron. They are used to shift a neuron's activation function output in a positive or negative direction [Malik, 2019b].

A **NN** is made up of a series of layers; an input layer, a number of hidden layers, and an output layer. Each layer is made up of a set of neurons, where each neuron is fully connected to all neurons in the previous layer. Each neuron within a single layer does not share connections with, and operates completely independently from one another [Stanford Vision and Learning Lab, 2012].

Using the case of **Computer Vision (CV)** as an example, the input layer of a **NN** consists of neurons encoding the values of image pixels (RGB or greyscale intensities). The encoding is typically achieved by passing the raw input value through an activation function which outputs a normalised value. Often, activation functions in modern **NNs** output non-linearities, an example is to use a Sigmoid Function which maps an input to a value between 0 and 1 (*see Fig. 2.2 left*) [Nielsen, 2015].

However a more common activation function found in current **NN** models for **CV** is the **Rectified Linear Unit (ReLU)**. It also adds non-linearity to the output, however it maps the input to a value within the range of 0 and  $\infty$  (*see Fig. 2.2 right*) [Malik, 2019a].



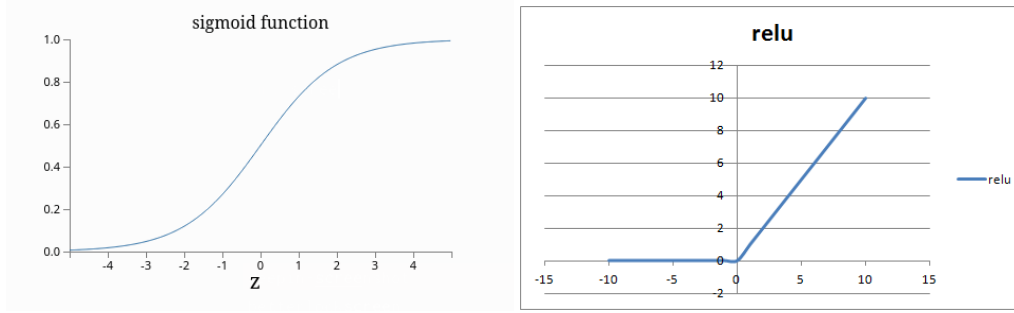


FIGURE 2.2: *Left*: The Sigmoid Function is one type of activation function. "A bounded, differentiable, real function that is defined for all real input values and has a non negative derivative at each point" [Han and Moraga, 1995]. *Right*: An example of a ReLU activation function transforming  $x$  to a value between 0 and  $\infty$  [Malik, 2019a].

The output layer of a CV classification network contains neurons representing the class scores of the task (see Fig. 2.3). For example, in a NN attempting to classify handwritten digits, the output layer would contain 10 neurons, representing the digits 0 - 9. If the first neuron fires, i.e. has an output  $\approx l$ , this will indicate that the network is confident the handwritten digit is 0, and so on [Nielsen, 2015].

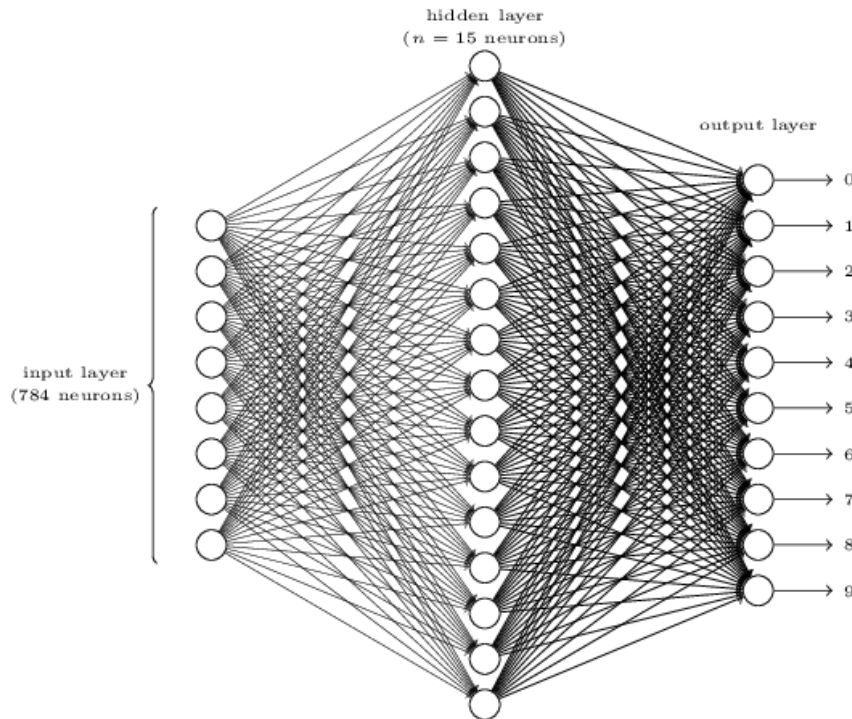


FIGURE 2.3: Neural Network. Example of a NN to classify handwritten digits. The input is a single vector of 28x28 pixels, i.e. 784 neurons, and outputs 10 neurons representing digits 0-9 [Nielsen, 2015].

[NNs](#) with a single hidden layer are able to approximate functions that contain any continuous mapping from one finite space to another, whereas with no hidden layers a [NN](#) model would only be able to represent linear functions or decision boundaries [[Hornik, 1991](#)].

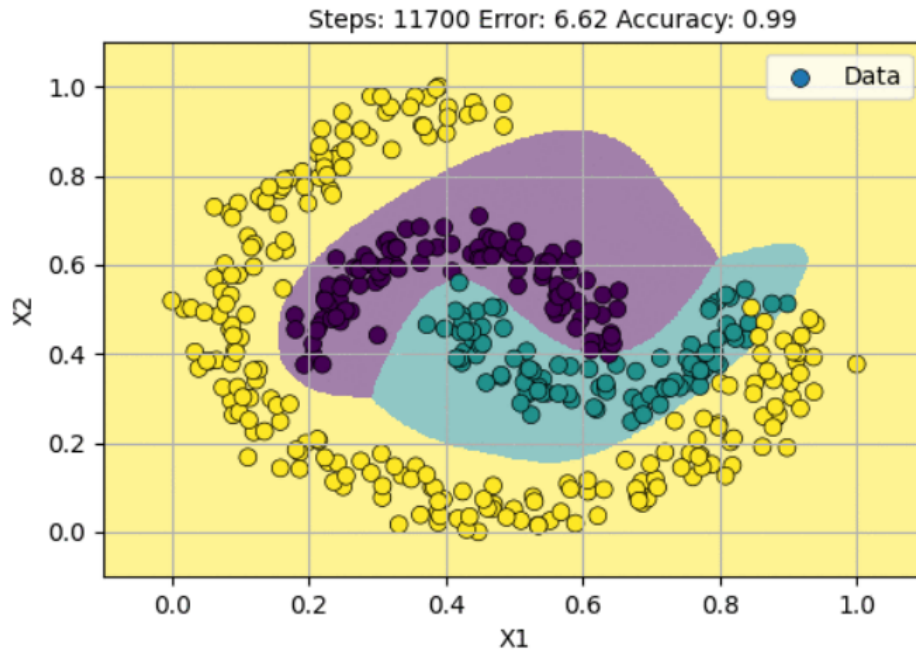


FIGURE 2.4: **Complex Decision Boundary** – Example of a decision boundary made capable by deep learning [[Sapkota, 2020](#)]

[NNs](#) are especially powerful when additional hidden layers are added to a network's architecture. By doing so, a model can not only approximate continuous functions to a high accuracy with less computational cost, but it can also represent complex composite functions [[Sapkota, 2020](#)]. An example of the complex decision boundaries that are possible from [NNs](#) with more than one hidden layer can be seen in *Fig. 2.4*.

[NNs](#) with two or more hidden layers fall under the category of deep learning, and are often referred to as [Deep Neural Networks \(DNNs\)](#) or [Multi-Layer Perceptrons \(MLPs\)](#). This subset of [ML](#) has become increasingly powerful with the rise of powerful variations of [DNNs](#), namely [Convolutional Neural Networks \(CNNs\)](#) and [Recurrent Neural Networks \(RNNs\)](#) in recent years due to their successes within the field of [CV](#).

### 2.1.2 Learning & Backpropagation

Training a [NN](#) consists of iteratively adjusting the values of weights at each neuron in order to minimise the model's output error. Although there are many algorithms available for determining the optimum values of weights, a common approach is by using some flavour of [Gradient Descent \(GD\)](#) together with a technique for efficiently computing partial derivatives within a directed graph called *backpropagation*.

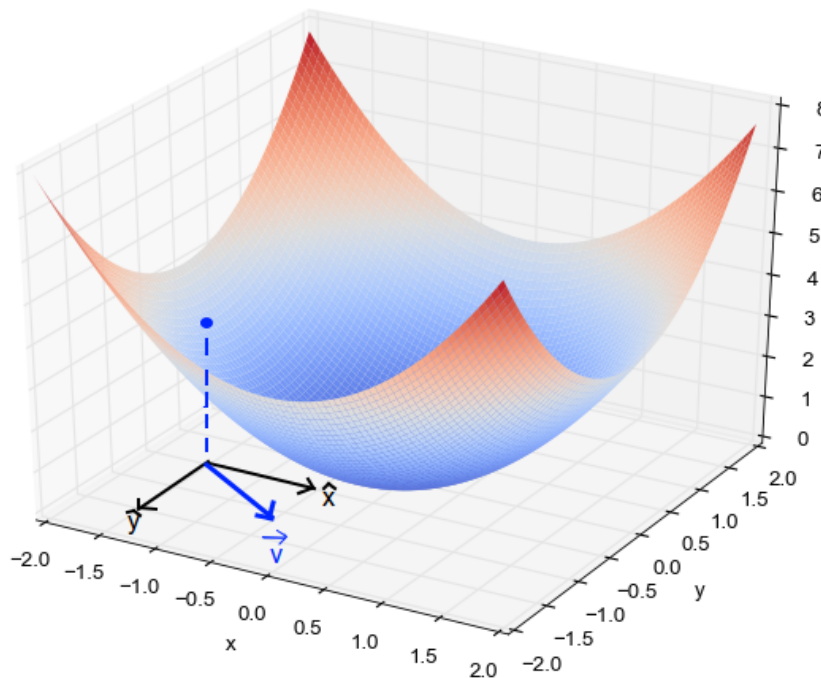


FIGURE 2.5: **Visualisation of GD Search Space** – An example of an *ideal* search space, where the vertical  $z$  axis shows the loss function  $f(x, y)$ , and  $\vec{v}$  represents the resulting vector applied to the parameter in question [[Bendersky, 2016](#)].

[GD](#), comes in many forms. The most popular [Stochastic Gradient Descent \(SGD\)](#) and [Batch Gradient Descent \(BGD\)](#) ...

backpropagation is a computational technique for calculating partial derivatives used for [GD](#) algorithms in linear time. This is an important element of training neural networks in reasonable times.

### **2.1.3 Vulnerabilities to Adversarial Attacks**

### **2.1.4 Discrimination and Neural Networks**

## **2.2 Formal Verification**

Formal verification is an extensive field which has seen development in many areas of software engineering. As such, this section will attempt to provide a succinct overview of the ideas behind formal verification while keeping the focus on areas related to this thesis.

### **2.2.1 Background**

### **2.2.2 Current Frameworks**

## **2.3 Formal Verification of AI**

This section will provide a more detailed investigation into the current research undertaken within [AIV](#), with a focus on [NNs](#) and deep learning tasks.

### 2.3.1 Overview

### 2.3.2 Sapphire

## 2.4 Programming Paradigms for Machine Learning

### 2.4.1 Computational Graphs

### 2.4.2 Auto Differentiability

## 2.5 The Go Programming Language

### 2.5.1 Brief History

### 2.5.2 Go for ML

### 2.5.3 Go for Formal Verification

## 2.6 Conclusions

## Chapter 3

# Methodology

## Chapter 4

# Implementation

## Chapter 5

# Analysis



## Chapter 6

## Conclusions

## Appendix A

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