3.8 Buoyancy, Flotation and Stability of Floating Bodies

In the preceding experiment we have seen that the upthrust, or buoyancy force, is equal to the weight of water displaced. It should be noted that this depends only on the immersed volume of the body and not on its density or weight. However, the question of whether a body will float or sink does depend on these factors. The maximum buoyancy force is produced when the body is fully immersed and if this is less that its total weight it will sink. Conversely, if it is greater, the body will find an equilibrium position with only parts of its volume immersed, such that the buoyancy force just balances the weight of the body, i.e. the body will float. The criterion for floating, therefore, is that the average density must be less than that of water.

Simple Demonstrations

Various simple demonstrations can be carried out using one of the plastic beakers and the weights supplied with the apparatus.

a) Attempt to float the beaker in the upper tank by placing it in the water, bottom downwards. It is unstable and tips over, and will float or sink depending on whether water gets into it or not. The beaker is 'top heavy' and will not float in the upright position.

b) Place a few weights in the centre of the bottom of the beaker and again place it in the water. It is now more stable and will float in a more upright position, although it is lower in the water due to the extra weight. The extra stability is produced by lowering the centre of gravity.

c) If further weights are added, the stability will further improve and the beaker will float lower down in the water.

d) If sufficient weights are added, the beaker will sink. The volume is approximately 450 ml, so approximately 450 g should be required to make it sink. Note that at this point, the average density of the beaker is equal to that of water, even though much of the volume is filled with air.

Stability of a Floating Body

Note: SECTION 4.0 includes alternative theory for this experiment.

The type of behaviour demonstrated above can be quantified and analysed using the Stability of a Floating Body apparatus (15) supplied with the bench. The stability will be found to depend on the position of the centre of gravity and, in particular, its position in relation to the centre of buoyancy. This leads to a definition of the metacentric height as a measure of stability.

The question of the stability of a body, such as a ship, which floats in the surface of a liquid, is one of obvious importance. Whether the equilibrium is stable, neutral or unstable is determined by the height of its centre of gravity, and, in this experiment, the stability of a pontoon may be determined with its centre of gravity at various heights. A comparison with calculated stability may also be made.

The arrangement of the apparatus is shown in Figure 12. A pontoon of rectangular form floats in water and carries a plastic sail, with five rows of V-slots at equally-spaced heights on the sail. The slots' centres are spaced at 7.5 mm intervals, equally disposed about the centre sail line. An adjustable weight, consisting of two machined cylinders which can be screwed together, fits into the V-slots on the sail; this can be used to change the height of the centre of gravity and the angle of list of the pontoon. A plum bob is suspended from the top centre of the sail and is used in conjunction with the scale fitted below the base of the sail to measure the angle of list.

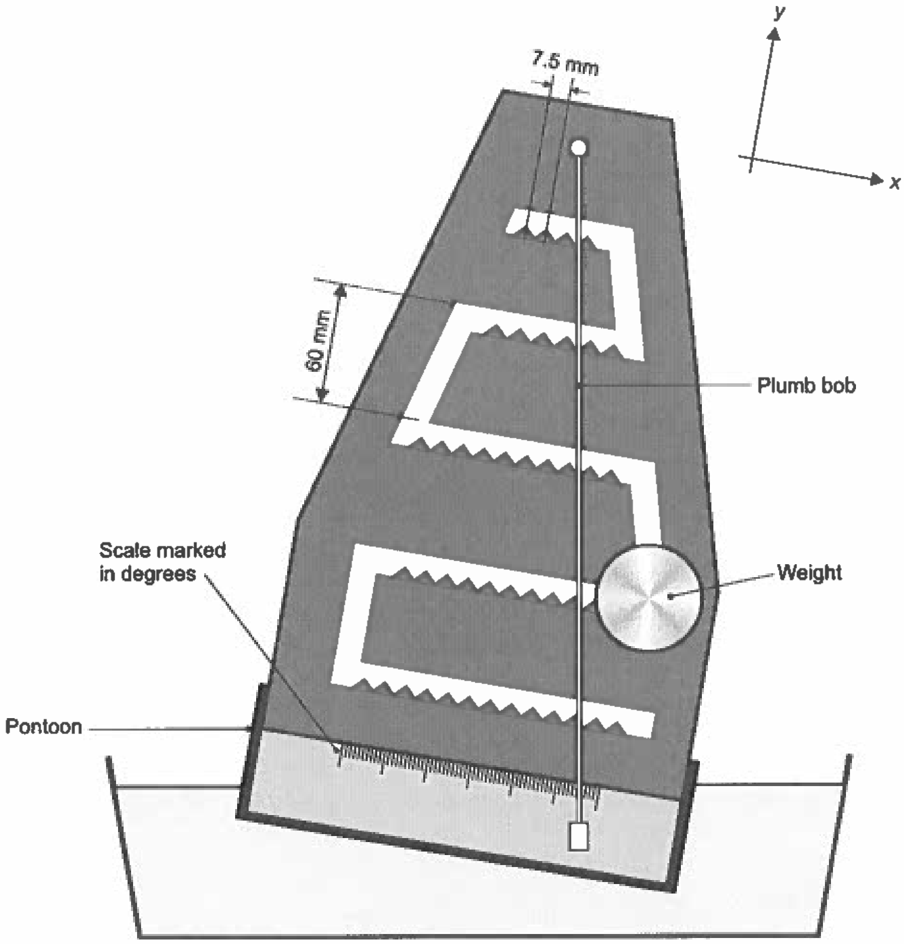


Figure 12 Arrangement of the Floating Pontoon

Consider the rectangular pontoon shown floating in equilibrium on an even keel, as shown in the cross-section of Figure 13(a). The weight of the floating body acts vertically downwards through its centre of gravity G and this is balanced by an equal and opposite buoyancy force acting upwards through the centre of buoyancy B, which lies at the centre of gravity of the liquid displaced by the pontoon.

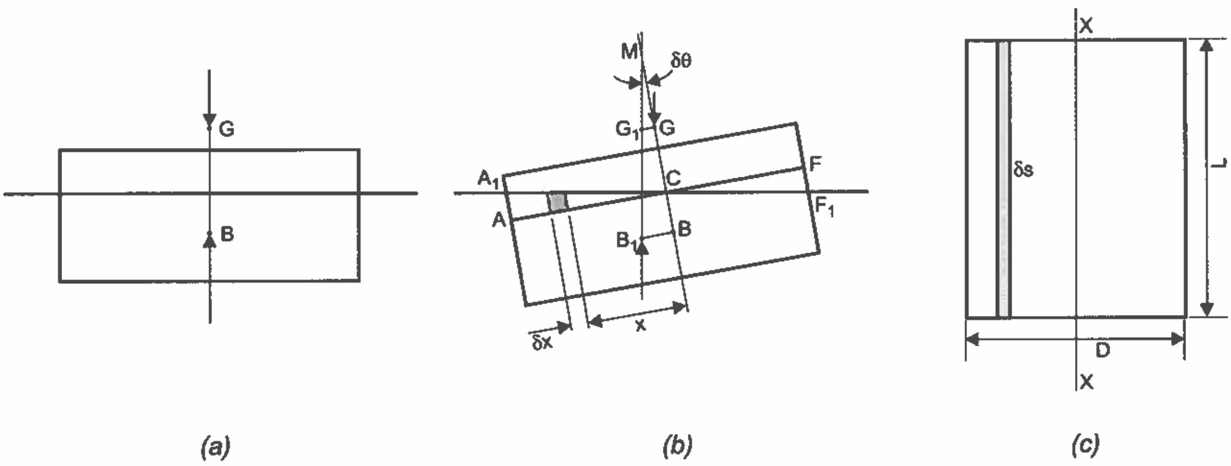


Figure 13 Derivation of the Stability of a Floating Pontoon

To investigate the stability of the system, consider a small angular displacement $\delta\theta$ from the equilibrium position as shown in Figure 13(b). The centre of gravity of the liquid displaced by the pontoon shifts from B to B_1. The vertical line action of the buoyancy force is shown in the diagram and intersects the extension of line BG at M, the metacentre.

The equal and opposite forces through G and B_1 exert a couple on the pontoon, and provided that M lies above G, as shown in Figure 13(b), this couple acts in the sense of restoring the pontoon to even keel, i.e. the pontoon is stable. If, however, the metacentre M lies below the centre of gravity G, the sense of the couple is to increase the angular displacement and the pontoon is unstable. The special case of the neutral stability occurs when M and G coincide.

Figure 13(b) shows clearly how the metacentric height GM may be established experimentally using the adjustable weight (of mass ω) to displace the centre of gravity sideways from G.

Suppose the adjustable weight is moved a distance δx_1 from its central position. If the weight of the whole floating assembly is W, then the corresponding movement of the centre of gravity of the whole, in a direction parallel to the base of the pontoon, is $\frac{\omega}{W}\delta x_1$. If this movement produces a new equilibrium position at an angle of list $\delta\theta$, then in Figure 13(b), G_1 is the position of the centre of gravity of the whole, i.e.

$$GG_1 = \frac{\omega}{W}\delta x_1 \qquad (11)$$

Now, from the geometry of the figure:

$$GG_1 = GM \cdot \delta\theta \qquad (12)$$

Eliminating GG_1, between these equations we derive:

$$GM = \frac{\omega}{W} \cdot \frac{\delta x_1}{\delta\theta} \qquad (13)$$

or in the limit:

$$GM = \frac{\omega}{W}\left(\frac{dx_1}{d\theta}\right) \qquad (14)$$

The metacentric height may thus be determined by measuring $\left(\frac{dx_1}{d\theta}\right)$ knowing ω and W.

Quite apart from experimental determinations, BM may be calculated from the mensuration of the pontoon and the volume of liquid which it displaces. Referring again to Figure 13(b), it may be noted that the restoring moment about B, due to shift of the centre of buoyancy to B_1, is produced by additional buoyancy represented by triangle AA_1C to one side of the centre line, and reduced buoyancy represented by triangle FF_1C to the other. The element shaded in Figure 13(b) and Figure 13(c) has an area δs in plan view and a height $x\delta\theta$ in vertical section, so that its volume is $x\delta s\delta\theta$. The weight of liquid displaced by this element is $wx\delta s\delta\theta$, where w is the specific weight of the liquid, and this is the additional buoyancy due to the element. The moment of this elementary buoyancy force about B is $wx^2\delta s\delta\theta$, so that the total restoring moment about B is given by the expression:

$$w\delta\theta\int x^2 ds$$

where the integral extends over the whole area s of the pontoon at the plane of the water surface. The integral may be referred to as I, where:

$$I = \int x^2 ds \qquad (15)$$

the second moment of area of s about the axis XX.

The total restoring moment about B may also be written as the total buoyancy force, wV, in which V is the volume of liquid displaced by the pontoon, multiplied by the lever arm BB_1. Equating this product to the expression for total retiring moment derived previously:

$$wV \cdot BB_1 = w\delta\theta\int x^2 ds$$

Substituting from Equation (15) for the integral and using the expression:

$$BB_1 = BM \cdot \delta\theta \qquad (16)$$

which follows from the geometry of Figure 13(b), leads to:

$$BM = \frac{I}{V} \qquad (17)$$

This result, which depends only on the mensuration of the pontoon and the volume of liquid which it displaces, will be used to check the accuracy of the experiment. It applies to a floating body of any shape, provided that I is taken about an axis through the centroid of the area of the body at the plane of the water surface, the axis being perpendicular to the plane in which angular displacement takes place. For a rectangular pontoon, B lies at a depth below the water surface equal to half the total depth of immersion, and I may readily be evaluated in terms of the dimensions of the pontoon as:

$$I = \int x^2 ds = \int_{-D/2}^{D/2} x^2 L dx = \frac{1}{12} LD^3 \qquad (18)$$

Experimental Procedure

The total mass of the apparatus (including the two magnetic weights, but not the jockey weight) is written on a label affixed to the sail housing.

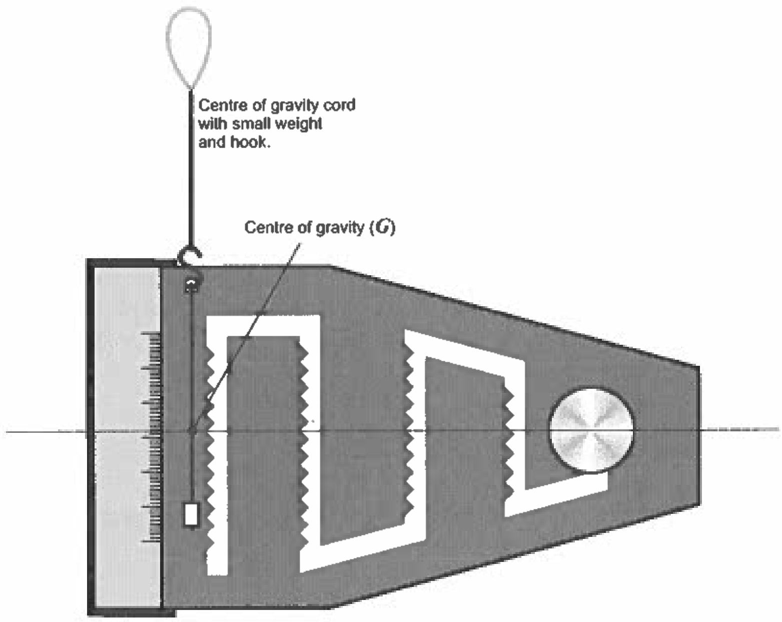


Figure 14 Method of Finding the Centre of Gravity

With reference to Figure 14, the height of the centre of gravity may be found as follows:

a) Fit the two magnetic weights to the base of the pontoon. Refer to 'Magnets' on page 8.

b) Fit the hook of the centre of gravity cord, through the hole in the sail, ensuring that the plumb weight is free to hang down on the side of the sail which has the scored centre line.

c) Clamp the adjustable weight into the V-slot on the centre line of the lowest row and suspend the pontoon from the free end of the thick cord. Mark the point where the plumb line crosses the sail centre line (for example, with typist's correcting fluid or a similar marking fluid).

d) Repeat step c) for the other four rows.

With the adjustable weight situated in the centre of one of the rows, allow the pontoon to float in water and position the two magnetic weights on the base of the pontoon to trim the vessel. When the vessel has been trimmed correctly, the adjustable weight may be moved to positions either side of the centre line for each of the five rows. At each position, the displacement can be determined by the angle the plumb line from the top of the sail makes with the scale on the sail housing.

Results and Calculations

Total weight of floating assembly (W)	=	N
Adjustable weight (ω)	=	N
Breadth of pontoon (D)	=	mm
Length of pontoon (L)	=	mm
Second moment of area $I = \frac{LD^3}{12} \times 10^{-12}$	=	m^4
Volume of water displaced $V = \frac{W}{10^3}\rho$	=	m^3
Height of metacentre above centre of buoyancy $BM = \frac{I}{V}$	=	m
Depth of immersion of pontoon $= \frac{V \times 10^6}{LD}$	=	m
Since L and D are in mm depth of centre of buoyancy $CB = \frac{V \times 10^6}{2LD}$	=	m

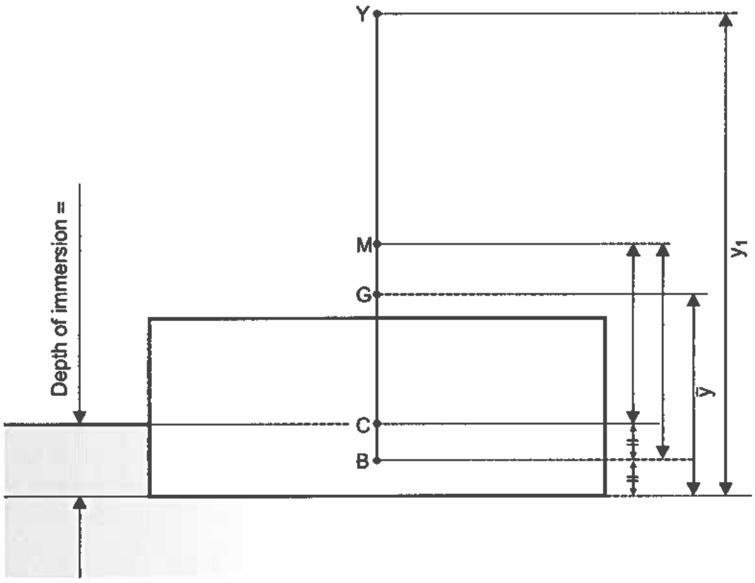


Figure 15 Standard Dimensions of the Pontoon

It is recommended that Figure 15 is marked up appropriately and referred to each time the apparatus is used. Note that when measuring the heights $\bar{y}$ and y_1, as it is only is convenient to measure from the inside floor of the pontoon, the thickness of the sheet metal bottom should be added to $\bar{y}$ and y_1 measurements. The position of G (and hence the value of $\bar{y}$) and a corresponding value of y was marked earlier in the experiment when the assembly was balanced.

The height of $\bar{y}$ of G above the base will vary with the height y of the adjustable weight above the base, according to the equation:

$$\bar{y} = y_1\frac{\omega}{W} + A \qquad (19)$$

where A is a constant, which pertains to the centre of gravity of the pontoon and the height of the adjustable weight.

Using one set of results for the centre of gravity of the pontoon and the height of the adjustable weight, $\bar{y}$ and y_1 can be measured and the constant A calculated. This can then be used in calculations for subsequent heights of $\bar{y}$ and y_1 which can be checked against the markings.

Values of angles of list produced by lateral movement of the adjustable weight height y_1 should be recorded in the form of Table 4. A graph (Figure 16) for each height y_1, of lateral position of adjustable weight against angle of list, can then be plotted.

Note: Decide which side of the sail centre line is to be termed negative and then term list angles on that side negative.

Height of jockey weight y_1 mm (l)	**Angles of list for adjustable weight lateral displacement from sail centre line x_1 mm**														
	-52.5	-45	-37.5	-30	-22.5	-15	-7.5	0	7.5	15	22.5	30	37.5	45	52.5

Table 4 Values of List Angles for Height and Position of Adjustable Weight

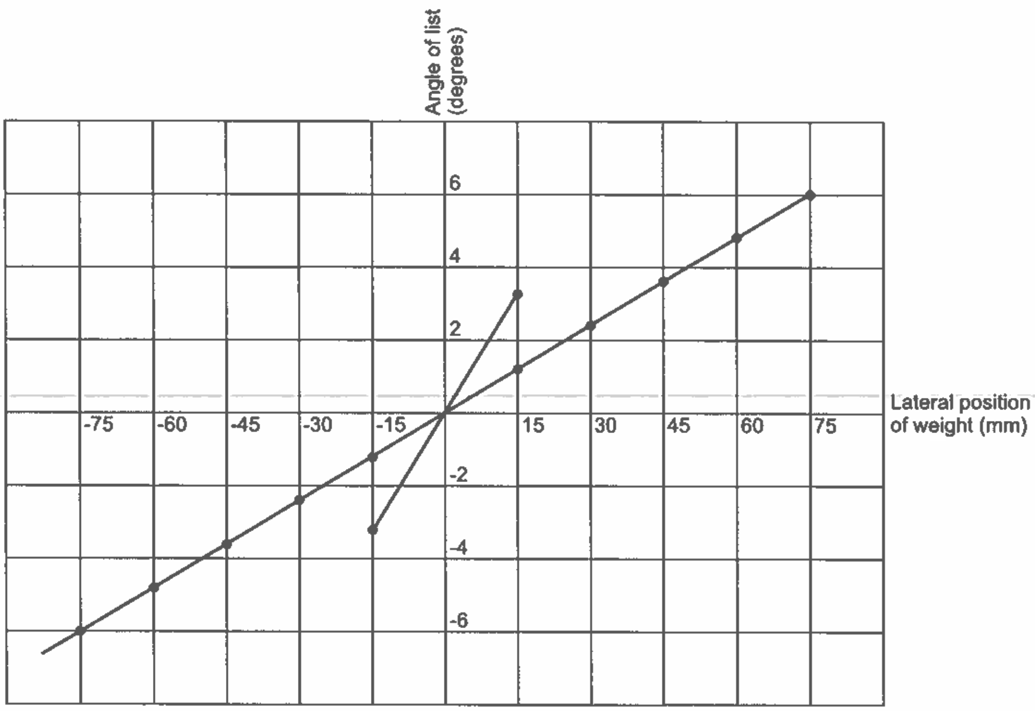


Figure 16 Variation of Angle List with Lateral Position of Weight

From Figure 16, for the five values of y_1, the corresponding values of $\frac{dx_1}{d\theta}$ can be extracted. Using Equation (14), values of GM can be obtained. Using Equation (19) and knowing the immersion depth, values of CG can be derived. Also, since CM = CG + GM, values of CM can be calculated. These values should be calculated and arranged in tabular form as shown in Table 5.

Height of adjustable weight y_1 (mm)	Height of G above water surface CG (mm)	$\frac{dx_1}{d\theta}$ (mm/°)	Metacentric height CM (mm)	Height of M above water surface CM (mm)

Table 5 Derivation of Metacentric Height from Experimental Results

The values of $\frac{dx_1}{d\theta}$ can now be plotted against CG, the height of G above the water line. Extrapolation of this plot will indicate the limiting value of CG above which the pontoon will be unstable (i.e. when $\frac{dx_1}{d\theta}$ is zero and CG = M).

3.9 Forces on Plane Surfaces: Centre of Pressure

The centre of pressure may be defined as: *"The point in a plane at which the total fluid thrust can be said to be acting normal to that plane."*

The apparatus permits the moment due to the total fluid thrust on a wholly or partially submerged plane surface to be measured directly and compared with theoretical analysis.

Water is contained in a quadrant tank assembly (16) as part of a balance. The cylindrical sides of the quadrant have their axes coincident with the centre of rotation of the tank assembly, and therefore the total fluid pressure acting on these surfaces exerts no moment about that centre. The only moment present is that due to the fluid pressure acting on the plane surface. This moment is measured experimentally by applying weights (17) to a weight hanger mounted on the opposite side to the quadrant tank.

A second tank, situated on the same side of the assembly as the weight hanger, provides a trimming facility. A scale on the quadrant tank measures the level of the water below the pivot (h).

TecQuipment supply coloured dye with the apparatus, which can be added to the water to help see its level during the experiments. Only a few drops of dye should be needed.

Before each experiment, make sure both tanks are empty and trim the assembly to bring the submerged plane to the vertical (0° position). To do this, gently pour water into the trim tank until the balance reaches the 0° position. You may need to add one of the weight hangers supplied and a few masses to help (hook it to the bar next to the trim tank).

TecQuipment supply a pipette (29) to help remove excess water. To use it, dip it into the tank and put a finger on the top of the pipette to hold the water then lift the pipette and transfer the water to the reservoir.

Now add the second weight hanger and additional weights. Pour water into the quadrant tank until a 0° balance is restored. Note the additional weight (not the trim weight) and the level of the water (h). Repeat the procedure for the full range of weights.

Now you can investigate the readings for $\theta = 0°$.

Readings should be tabulated in the form outlined in Table 6 and the results calculated in line with the theory given.

ω (g)	$M = \dfrac{W \times 9.81 \times R_3}{10^3}$ (Nm)	h (mm)	h (m)	h^3 (m^3)	$M + \dfrac{\omega B R_2^2 h}{2}$ (Nm)

Table 6 Format of Results Table – Centre of Pressure for the Particular Case of θ = 0°

The following analysis is applied to the general condition of a plane surface at various angles when it is wholly or partially submerged in a fluid.

Note: SECTION 4.0 includes alternative theory for this experiment.

Let breadth of quadrant = B
and weight per unit volume = ω

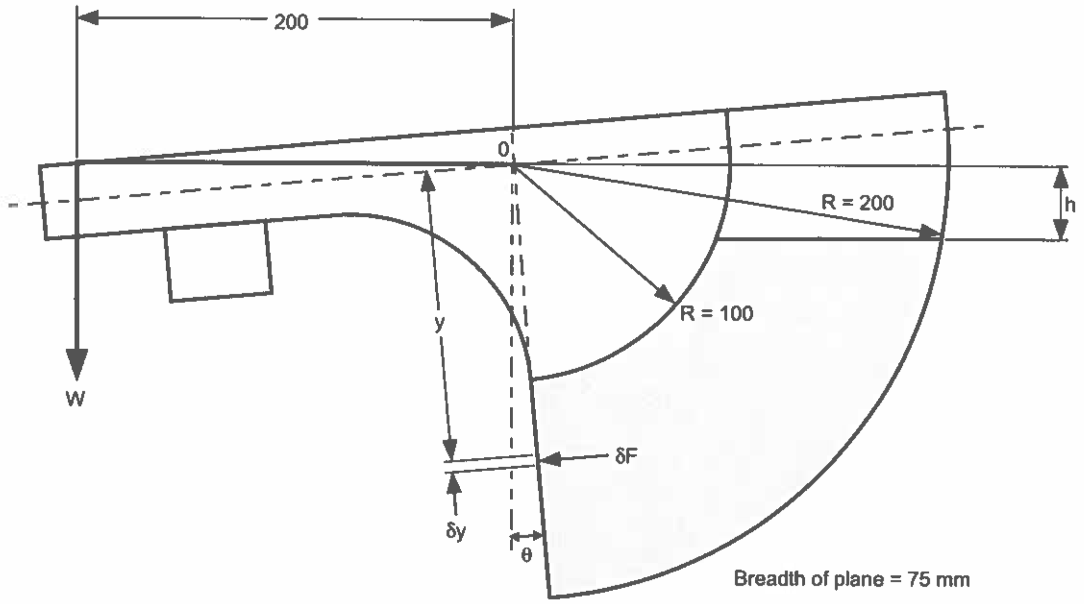


Figure 17 Pressure Forces on a Plane Surface

Referring to Figure 17, consider an element at start depth y, width δy.

Therefore, force on element $\delta F = \omega(y\cos\theta - h)B\delta y$ and moment of force on element about $0 = \omega B(y\cos\theta - h)y\delta y$. Therefore:

$$\text{Total moment about } 0 = M = \omega B \int (\cos\theta y^2 - hy)dy$$

Case 1: Plane Fully Submerged

Limits R_1 and R_2

$$M = \omega B \int_{R_1}^{R_2} (\cos\theta y^2 - hy)dy$$

$$M = \omega B\left[\frac{\cos\theta^3}{3} - \frac{hy^2}{2} + c\right]_{R_1}^{R_2}$$

$$M = \frac{\omega B\cos\theta}{3}(R_2^3 - R_1^3) - \frac{\omega B}{2}(R_2^2 - R_1^2)h \qquad (20)$$

This equation is of the form of y = mx + c

A plot of M against h will yield a straight line graph of gradient $-\frac{\omega B}{2}(R_2^2 - R_1^2)$. The value of ω can now be calculated.

Case 2: Plane Partially Submerged

Limits R_2 and $h\mathrm{Sec}\theta$

Hence:

$$M = \omega B \int_{h\sec\theta}^{R_2} (\cos\theta y^2 - hy)dy$$

$$M = \left[\frac{\omega B \cos\theta y^3}{3} - \frac{hy^2}{2} + c\right]_{h\sec\theta}^{R_2}$$

$$M = \frac{\omega B \cos\theta}{3}(R_2^3 - h^3\sec^3\theta) - \frac{\omega B h}{2}(R_2^2 - h^2\sec^2\theta)$$

$$M = \frac{\omega B \cos\theta R_2^3}{3} - \frac{\omega B R_2^2 h}{2} + \frac{\omega B \sec^2\theta h^3}{6}$$

Rearranging:

$$M + \frac{\omega B R_2^2 h}{2} = \frac{\omega B \sec 2\theta h^3}{6} + \frac{\omega B \cos\theta R_2^3}{3}$$

Obtain ω from Case 1 and plot h^3 against $M + \dfrac{\omega B R_2^2 h}{2}$

Figure 18 and Figure 19 show the general form of the graphs expected from this experiment.

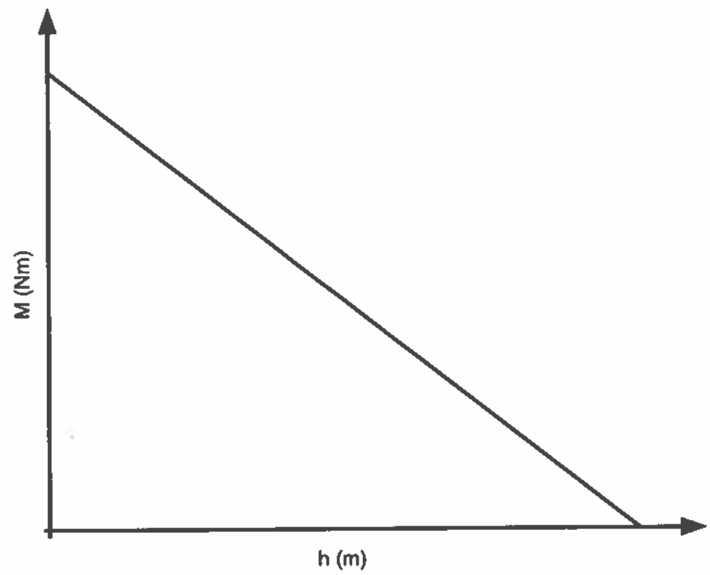


Figure 18 Centre of Pressure Graph: Plane Fully Submerged

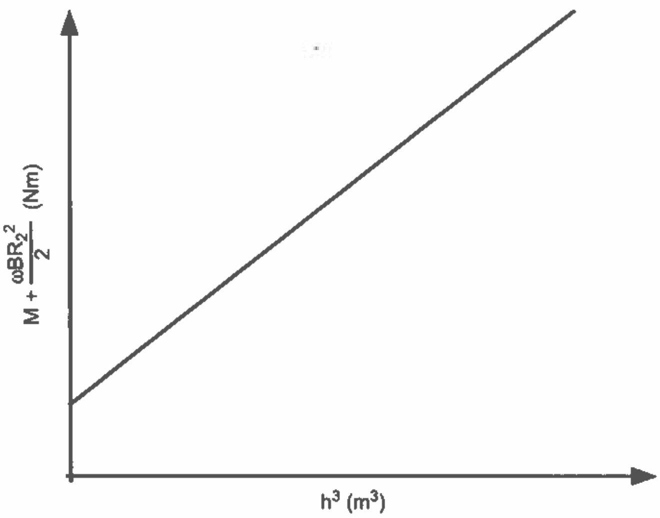


Figure 19 Centre of Pressure Graph: Plane Partially Submerged

Two different kinds of error may normally be expected in a gauge of this type. Firstly, there is the possibility of hysteresis, friction and backlash which will yield smaller gauge readings when the pressure is increasing than when it is decreasing. Typically, the gauge tested here will have an error in the range of 1 kN/m^2 of the entire range, which is acceptably small. Secondly, there is error due to the scale being marked off incorrectly. It will be found that this error increases to a maximum of around 2.5% of the full-scale reading. This is acceptably small for many engineering purposes, although gauges with an error of only 0.5% of the full-scale reading are commercially obtainable.

Liquid Column Manometers

Columns of liquid can be used in a wide range of configurations for measuring pressures in both static and moving fluids. A barometer represents a special case in which absolute pressure is measured but, in general, liquid columns are used to measure differential pressures; that is to say the difference in pressure between two points in a fluid system. Strictly speaking, the term manometer relates to all methods of measuring pressure but, in normal usage, it is taken to refer to liquid columns and particularly those in the form of U-tubes.

Figure 21 shows the general case of a U-tube manometer measuring the differential pressure between two points in a system containing fluid (liquid or gas) of density ρ_1. The U-tube is filled with a heavier fluid (liquid) of density ρ_2 and the differential pressure is measured in terms of the difference in height $\Delta h = (h_4 - h_3)$ of the two columns.

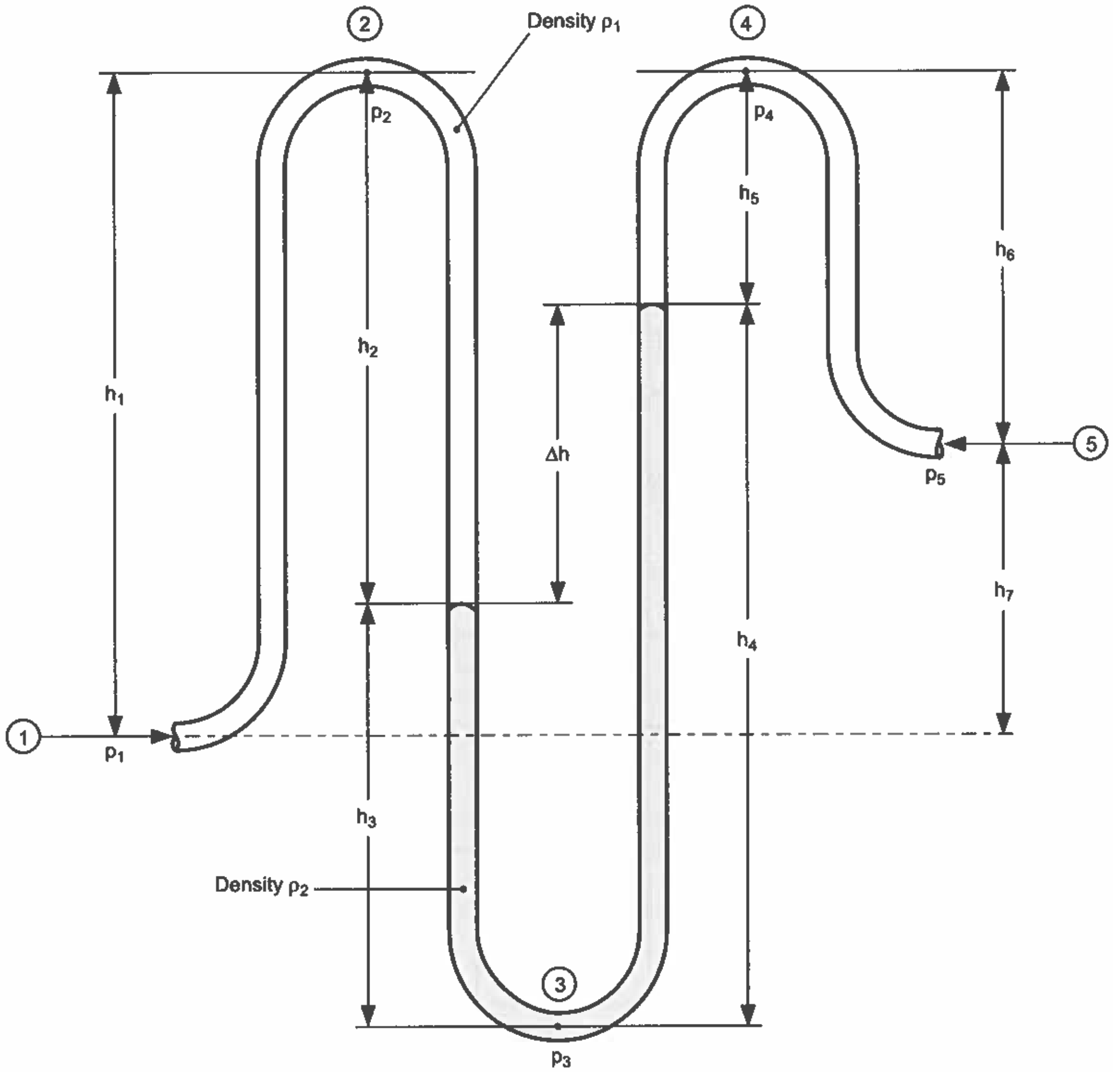


Figure 21 Pressure in a U-tube Manometer

The reminder of the U-tube and the connecting tubes are filled with the working fluid of density ρ_1. First consider the pressure at point 1 due to the column h of working fluid:

$$p_1 = p_2 + \rho_1 g h_1$$

or

$$p_2 = p_1 - \rho_1 g h_1$$

Now consider the pressure at the bottom of the U-tube due to fluid in the left-hand column:

$$p_3 = p_2 + \rho_1 g h_2 + \rho_2 g h_3 \qquad (21)$$

and substituting for p_2 we obtain:

$$p_3 = p_1 - \rho_1 g(h_1 - h_2) + \rho_2 g h_3$$

Similarly, for the right-hand column and connecting tube:

$$p_3 = p_5 - \rho_1 g(h_6 - h_5) + \rho_2 g h_4 \qquad (22)$$

Then, on equating the right-hand sides of Equations (21) and (22) and rearranging we obtain:

$$(p_1 - p_5) = \rho_2 g(h_4 - h_3) + \rho_1 g(h_1 - h_6) - \rho_1 g(h_2 - h_5) \qquad (23)$$

Finally, substituting $h_4 - h_3 = h_2 - h_5 = \Delta h$ and $h_1 - h_6 = h_7$ we obtain:

$$(p_1 - p_5) = (\rho_2 - \rho_1)g\Delta h + \rho_1 g h_7 \qquad (24)$$

This represents the general case where the pressure tappings are at different heights and the density of the working fluid is significant compared to that of the manometer fluid. It can be seen that the difference in height Δh of the manometer columns gives a measure of $(p_1 - p_5)$, but a correction has to be made for the different heights of the pressure tappings. This is most important and must be remembered whenever a manometer is used with liquid rather than gas as the working fluid in the connecting tubes.

Notice also that the apparent density of the manometer fluid is reduced by ρ_1, the density of the working fluid. A common case is a mercury under water manometer where the apparent density is $13600 - 1000 = 12600$ kg/m^3 instead of 13600 kg/m^3 for a mercury/air manometer. There are two important cases which lead to a simplification of Equation (24).

Tappings at the same height

If the tappings are at the same height, the last term in Equation (24) becomes zero giving:

$$(p_1 - p_5) = (\rho_2 - \rho_1)g\Delta h \qquad (25)$$

Thus, for this case, the pressure difference is proportional to Δh multiplied by the apparent density $(\rho_2 - \rho_1)$.

Gas as the working fluid

If a gas is the working fluid, its density can usually be taken as negligible compared to that of the manometer fluid and Equations (24) and (25) reduce to:

$$(p_1 - p_5) = \rho_2 g h\,\Delta h \qquad (26)$$

In this case, the pressure difference depends directly on Δh and the actual density of the manometer fluid.

Equations (25) and (26) are quite general. The following two cases are worthy of further discussion.

Manometers open to atmospheric pressure

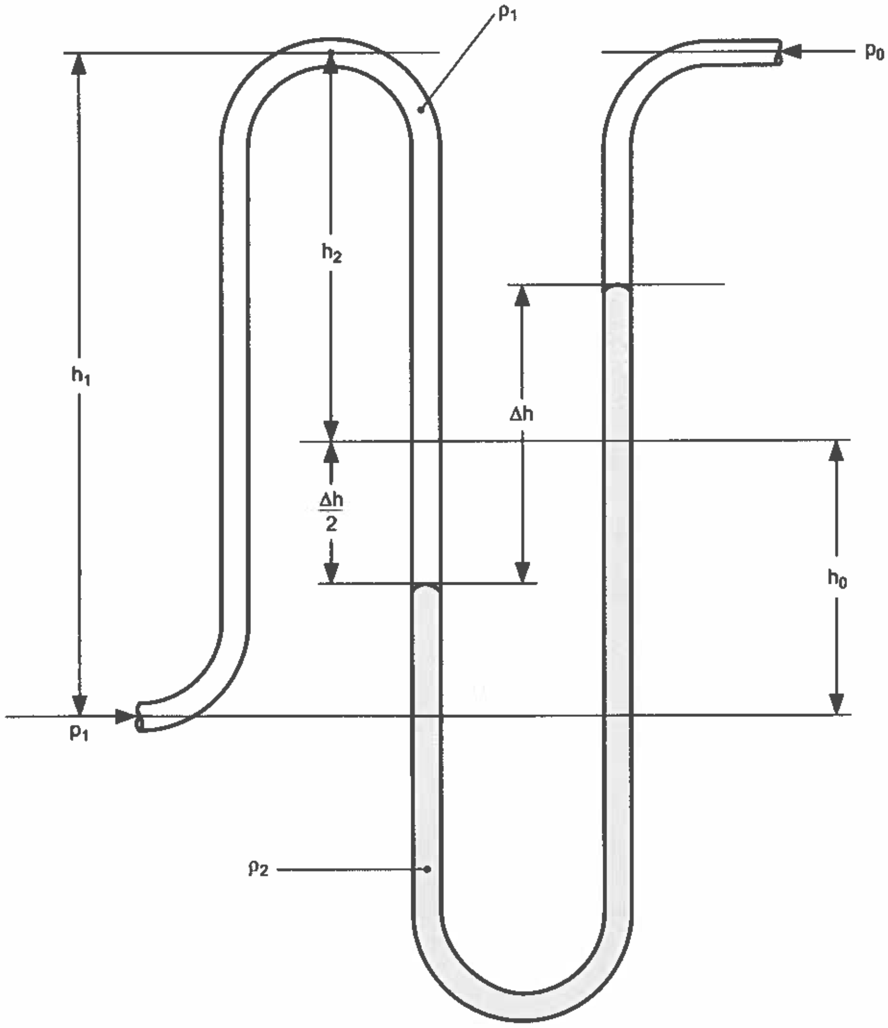


Figure 22 Manometer Open to Atmospheric Pressure

The manometers fitted to the H314 apparatus all have one column which is open to atmospheric pressure and this provides a common datum pressure. In the case of a liquid/air manometer, Equation (26) applies except that the pressure difference is measured relative to a fixed datum of atmospheric pressure p_0, rather than a varying pressure p_5. If the right-hand column is open to atmospheric pressure p_0 and there is no water in the connecting pipe as shown in Figure 22, Equation (23) becomes:

$$(p_1 - p_0) = \rho_2 g\Delta h + \rho_1 g(h_1 - h_2) \qquad (27)$$

It may be noted that the second term on the right-hand side is a correction for the varying column of water in the connecting tube. This variable correction can be eliminated by substituting $h_1 - h_2 = h_0 - \frac{\Delta h}{2}$, where h_0 is the height of the columns above the pressure tapping when the pressure difference is zero (see Figure 22). Equation (27) can then be written:

$$(p_1 - p_0) = \left[\rho_2 - \frac{\rho_2}{2}\right]g\Delta h + \rho_1 g h_0 \qquad (28)$$

where the second term on the right-hand side is now a constant.

It should be noted that this is based on the assumption that each column moves the same distance from level h_0 when displaced and this means that Equation (28) only applies if the tubes are of the same cross-sectional area. The significance of this is explained in the next case.

Manometer with limbs of different cross-sectional areas

In the preceding cases, the pressure difference is measured in terms of the height Δh, which is usually obtained as the difference between two measured heights. This is often laborious and it would be easier if the pressure difference could be obtained in terms of a single direct reading. One possible solution is to take the initial level of the columns at zero pressure difference as a datum and then measure the deflection of one column relative to this. If the limbs were of equal cross-sectional area, each column would move by $\Delta h/2$ and the reading could be doubled to obtain Δh. This is not often done in practice due to the loss of accuracy which results, but this can be overcome by using limbs of different cross-sectional areas.

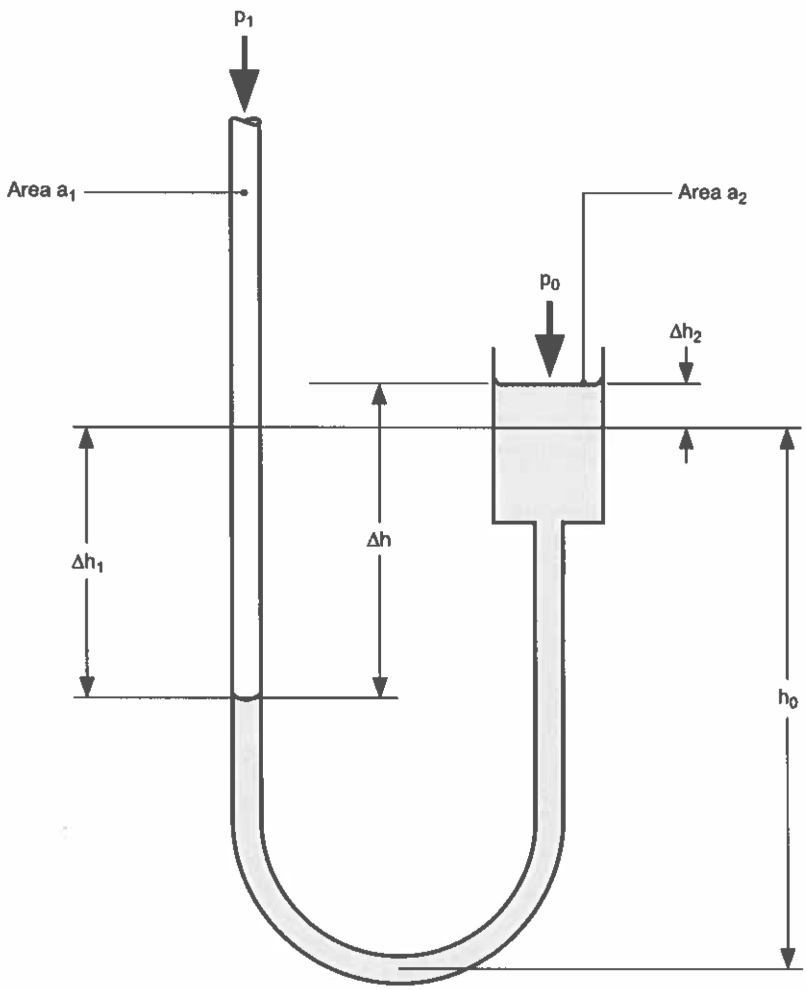


Figure 23 Manometer with Unequal Limbs

Consider the manometer shown in Figure 23 which has limbs of different cross-sectional areas. From the results of previous experiments (see page 12, Measuring Beaker) it will be appreciated that the differing areas do not affect the pressures in the columns, so for the case shown (a liquid/air manometer), Equation (26) still applies. The significant point is that the columns move different distances from the datum level h_0. The volume of liquid displaced is the same for each column so we can write:

$$\Delta h_1 a_1 = \Delta h_2 a_2$$

or alternatively:

$$\Delta h_2 = \Delta h_1 \frac{a_1}{a_2} \tag{29}$$

If we make a_2 large compared to a_1 then nearly all of the change in height will occur in column 1 and the deflection can be measured to a greater accuracy. Since $\Delta h = \Delta h_1 + \Delta h_2$ we can obtain h by substituting using Equation (29):

$$\Delta h = \Delta h_1\left(1 + \frac{a_1}{a_2}\right) \tag{30}$$

This arrangement is the basis of a common form of direct-reading, 'single-limb' manometer. The second limb is in the form of a reservoir tank of large cross-sectional area, such that the change in level in it is small compared to that in the narrow limb. Equation (30) can be used to define a scale for the narrow limb which is calibrated to read Δh directly from the level in that limb. Alternatively, Δh_1 can be measured and a small correction applied using Equation (30) to obtain Δh, or if a_2 is made sufficiently large, the measurement could be taken as a direct reading of h without introducing significant errors.

Manometer Experiments

The equations and underlying principles described previously can be demonstrated using the manometers fitted to the rear panel of the H314 apparatus. The manometers should be filled and all air bubbles removed from the water lines.

Comparison of different fluids

a) With the isolating valve on pipe (V3) to the air/water manometer open, attach the air pump to the Schrader valve and gently pump until a pressure is registered. This will be shown on both the fluid and water manometers.

b) Compare the readings of each manometer until the water manometer approaches the limit of its range.

c) Isolate the water manometer and increase the pressure until the fluid manometer is at full capacity.

d) To vent both manometers, insert the top of the Schrader valve cap into the centre of the valve and vent the fluid manometer. Open the isolating valve for the water manometer and again vent the air via the Schrader valve.

3.11 Hare's Tube Apparatus (Optional Ancillary H314b)

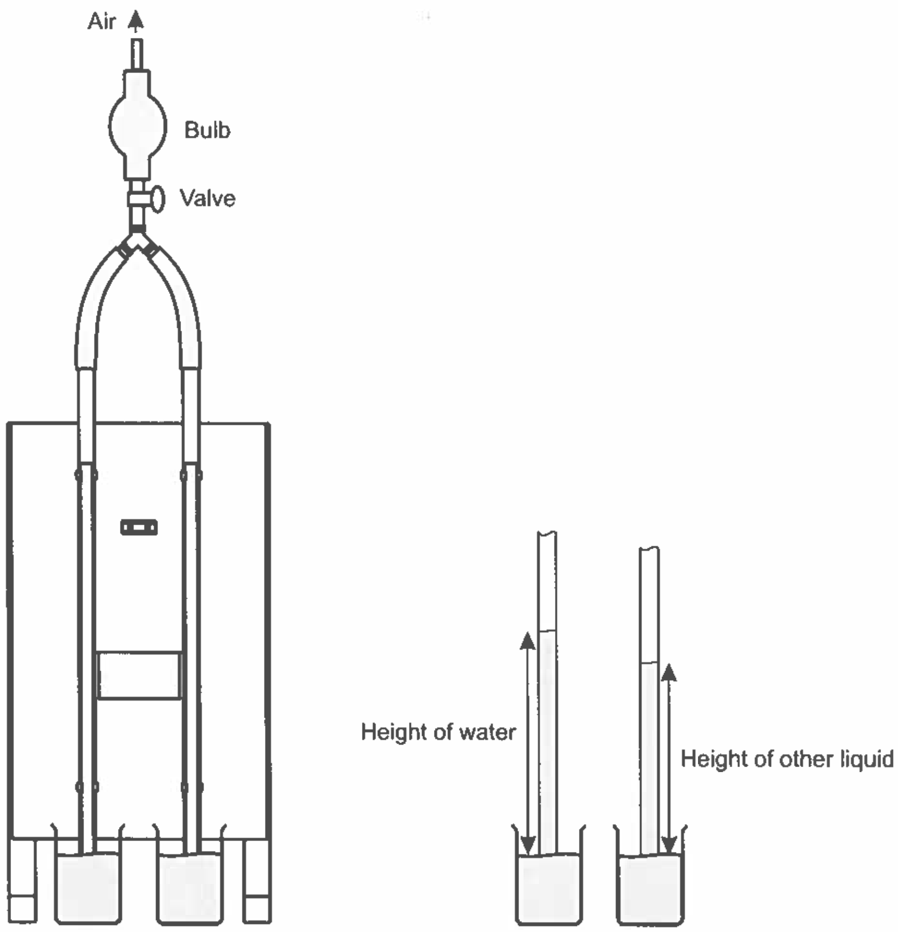


Figure 24 Using Hare's Tube

This apparatus enables the specific gravity of a liquid to be determined when compared with water. This is done as follows:

a) Fill one 100 ml beaker with water and another 100 ml beaker with the other liquid. Ensure the two liquids are at the same level.

b) Place the filled beakers under the vertical glass tubes, ensuring the tubes are at the same height, so the tube goes into the liquid.

c) The suction bulb includes a built-in one way valve, so that it can only 'suck' air from the tubes, but the equipment also includes a small hand-operated valve under the suction bulb to retain the suction when you release the bulb.

d) Fully open the small valve underneath the bulb, squeeze the bulb and slowly release it to draw the two liquids up the glass tubes. Now shut the small valve underneath to keep the fluids in the tube.

e) Record the liquid levels and then fully open the small valve again to allow the fluids to return to their containers (you may need to squeeze the bulb a few times to help).

f) Use the two liquid levels to calculate the unknown liquid's specific gravity as shown:

$$\text{Specific gravity (relative density)} = \frac{\text{Height of water}}{\text{Height of other liquid}}$$

3.12 Depth Gauge (also known as a 'Hook' Gauge)

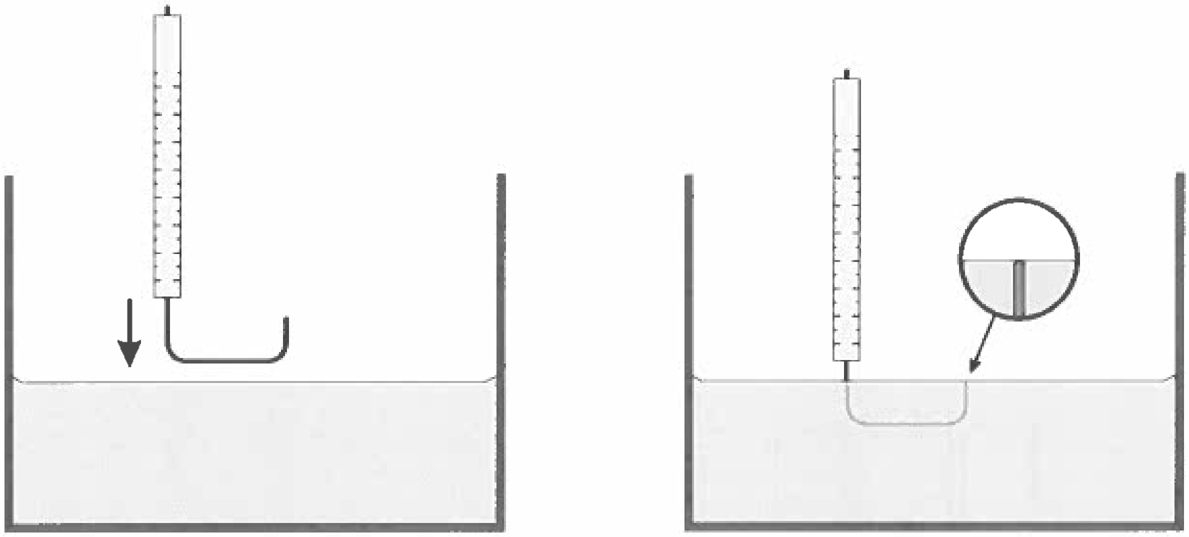


Figure 25 Using the Depth Gauge

The Depth Gauge in the header tank accurately measures changes in liquid level. It uses a pointed hook to 'touch' the surface of the liquid and a scale to help you measure differences in liquid height. To use it you must choose one of two methods and be consistent. If you are not consistent, the small meniscus of liquid that forms around the tip will affect your results.

Method 1: Gently lower the hook into the liquid and note when the tip just passes under the surface of the liquid. Note the reading on the scale. Repeat for different liquid levels and subtract the differences in scale readings to find the change in liquid level.

Method 2: Gently lower the hook into the liquid until it is completely submerged, then slowly raise it until the tip just breaks the liquid surface from underneath. Repeat for different liquid levels and subtract the differences in scale readings to find the change in liquid level.

SECTION 4.0 ALTERNATIVE THEORY

This section includes alternative but relevant theory for some parts of the product.

4.1 Alternative Theory for Buoyancy, Flotation and Stability of Floating Bodies on page 25

on page 25

Introduction

When designing a vessel such as a ship, which is to float on water, it is clearly necessary to be able to establish beforehand that it will float upright in stable equilibrium.

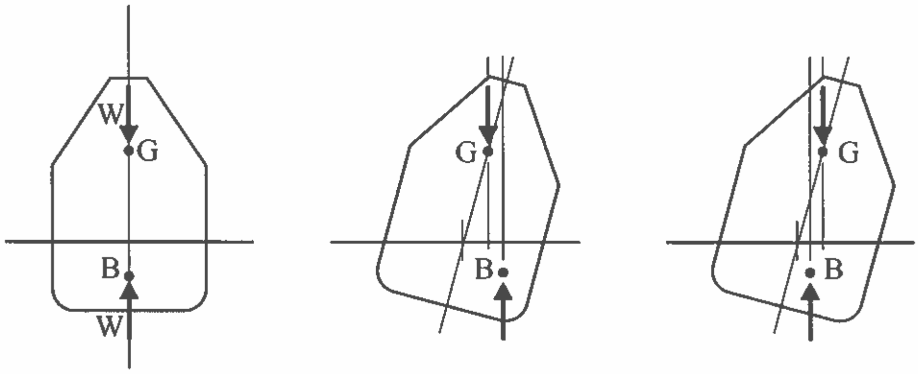


Figure 26 Forces Acting on a Floating Body

Figure 26 (a) shows such a floating body, which is in equilibrium under the action of two equal and opposite forces, namely, its weight W acting vertically downwards through its centre of gravity G, and the buoyancy force, of equal magnitude W, acting vertically upwards at the centre of buoyancy B. This centre of buoyancy is located at the centre of gravity of the fluid displaced by the vessel. When in equilibrium, the points G and B lie in the same vertical line. At first sight, it may appear that the condition for stable equilibrium would be that G should lie below B. However, this is not so.

To establish the true condition for stability, consider a small angular displacement from the equilibrium position, as shown in Figure 26 (b) and Figure 26 (c). As the vessel tilts, the centre of buoyancy moves sideways, remaining always at the centre of gravity of the displaced liquid. If, as shown on Figure 26 (b), **the weight and the buoyancy forces together produce a couple which acts to restore the vessel to its initial position, the equilibrium is stable. If however, the couple acts to move the vessel even further from its initial position, as in Figure 26 (c), then the equilibrium is unstable.** The special case when the resulting couple is zero represents the condition of neutral stability. It will be seen from Figure 26 (b) that it is perfectly possible to obtain stable equilibrium when the centre of gravity G is located above the centre of buoyancy B.

In the following text, we shall show how the stability may be investigated experimentally, and then how a theoretical calculation can be used to predict the results.

Experimental Determination of Stability

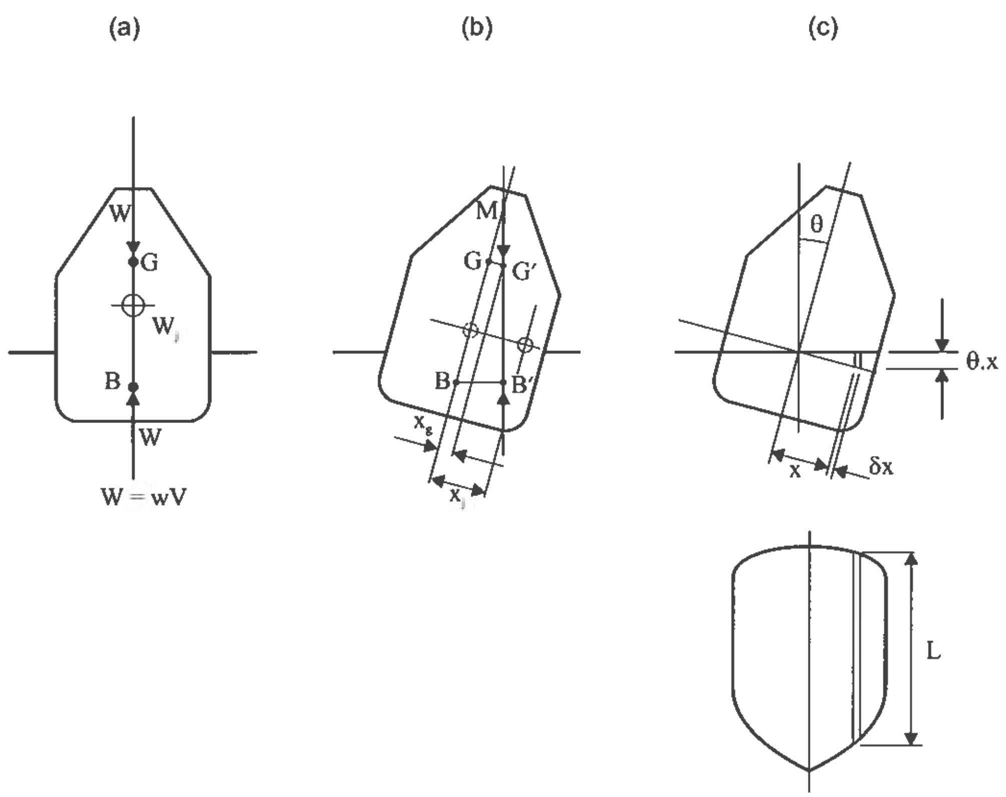


Figure 27 Derivation of Conditions for Stability

Figure 27(a) shows a body of total weight W floating on even keel. The centre of gravity G may be shifted sideways by moving a jockey of weight W_j across the width of the body. When the jockey is moved a distance x_j, as shown in Figure 27(b), the centre of gravity of the whole assembly moves to G'. The distance GG', denoted by x_g, is given from elementary statics as:

$$x_g = \frac{W_j x_j}{W} \qquad (31)$$

The shift of the centre of gravity causes the body to tilt to a new equilibrium position, at a small angle θ to the vertical, as shown in Figure 27(b), with an associated movement of the centre of buoyancy from B to B'. The point B' must lie vertically below G', since the body is in equilibrium in the tilted position. Let the vertical line of the upthrust through B' intersect the original line of upthrust BG at the point M, called the metacentre. We may now regard the jockey movement as having caused the floating body to swing about the point M. Accordingly, the equilibrium is stable if the metacentre lies above G. Provided that θ is small, the distance GM is given by:

$$GM = \frac{x_g}{\theta} \qquad (32)$$

where θ is in circular measure. Substituting for x_g from Equation (31) gives the result:

$$GM = \frac{W_j}{W} \times \frac{x_j}{\theta} \qquad (33)$$

The dimension GM is called the metacentric height. In the experiment described below, it is measured directly from the slope of a graph of x_j against θ, obtained by moving a jockey across a pontoon.

Analytical Determination of BM

A quite separate theoretical calculation of the position of the metacentre can be made as follows.

The movement of the centre of buoyancy to B' produces a moment of the buoyancy force about the original centre of buoyancy B. To establish the magnitude of this moment, first consider the element of moment exerted by a small element of **change** in displaced volume, as indicated on Figure 27(c). An element of width δx, lying at distance x from B, has an **additional** depth $\theta.x$ due to the tilt of the body. Its length, as shown in the plan view on Figure 27(c), is L. So the volume δV of the element is:

$$\delta V = \theta.x.L.\delta x = \theta L x \delta x$$

and the element of **additional** buoyancy force δF is:

$$\delta F = w.\delta x = w\theta L x \delta x$$

where w is the specific weight of water. The element of moment about B produced by the element of force is δM, where

$$\delta M = \delta F.x = w\theta L x^2 \delta x$$

The total moment about B is obtained by integration over the whole of the plan area of the body, in the plane of the water surface:

$$M = w\theta \int L x^2 dx = w\theta I \qquad (34)$$

In this, 'I' represents the second moment, about the axis of symmetry, of the water plane area of the body.

Now this moment represents the movement of the upthrust wV from B to B', namely, wV.BB'. Equating this to the expression for M in Equation 34.

$$wV.BB' = w\theta I$$

From the geometry of the figure, we see that

$$BB' = \theta.BM$$

and eliminating BB' between these last two equations gives BM as:

$$BM = I/V \qquad (35)$$

For the particular case of a body with a rectangular planform of width D and length L, the second moment I is readily found as:

$$I = \int_{-D/2}^{D/2} L x^2 dx = L \int_{-D/2}^{D/2} x^2 dx = L\left[\frac{x^3}{3}\right]_{-D/2}^{D/2} = \frac{LD^3}{12}$$

Now the distance BG may be found from the computed or measured positions of B and of G, so the metacentric height GM follows from Equation (35) and the geometrical relationship:

$$GM = BM - BG \qquad (36)$$

This gives an independent check on the result obtained experimentally by traversing a jockey weight across the floating body.

4.2 *Alternative Theory for Forces on Plane Surfaces: Centre of Pressure on page 32*

Introduction

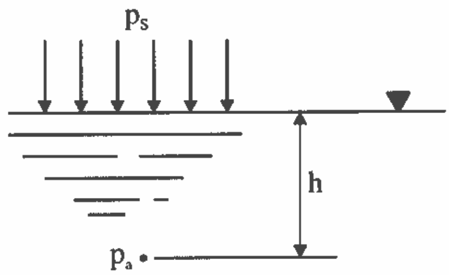


Figure 28 Hydrostatic Pressure at Depth

The hydrostatic pressure exerted by a liquid of density ρ or specific weight w, at depth h below the surface, is:

$$p = \rho g h = wh \tag{37}$$

This is the gauge pressure, due solely to the liquid column of height h. To obtain the absolute pressure p_a at depth h, we must add whatever pressure p_s is applied at the liquid's surface, giving:

$$p_a = p_s + p$$
or
$$p_a = p^s + wh$$

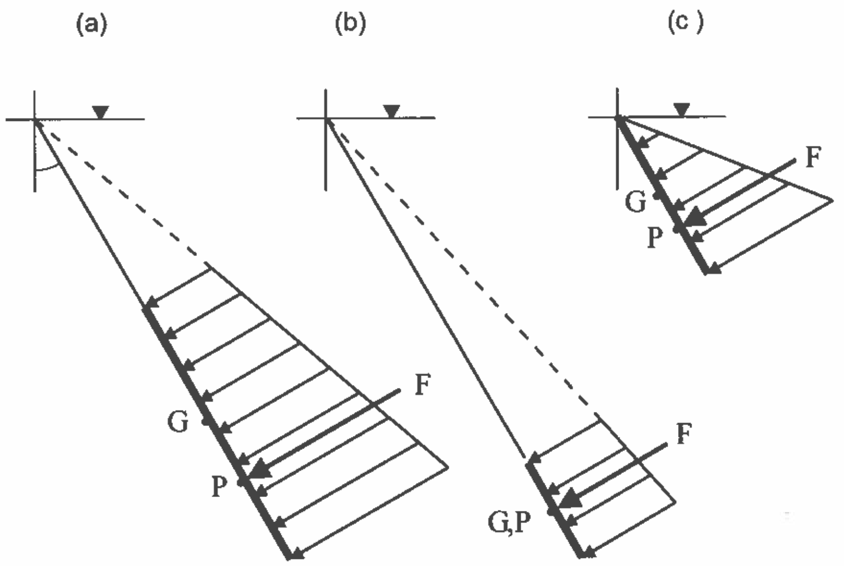


Figure 29 Hydrostatic Thrust on Plane Surfaces

Hydrostatic pressure generates thrust on any surface over which it acts. Figure 29 (a) indicates the distribution of gauge pressure over a plane surface in the liquid. The pressure acts normal to the surface, increasing linearly with depth below the water surface. The result of the pressure forces is a single force F, which of course is also normal to the surface. If the pressure were uniform, F would act through the centroid G of the plane surface. However, as the pressure increases with depth, the line of action of F is through some lower point P, called the **centre of pressure**. If the extent of the plane surface is small compared with its depth, as shown in Figure 29(b), the hydrostatic pressure is very nearly constant over it, so the centre of pressure lies nearly at the centroid. If however, the upper edge of the plane area lies in the water surface the pressure distribution is triangular, as shown in Figure 29(c), and P will lie at a significant distance below G.

The hydrostatic force on a submerged surface, such as the face of a dam or on a lock gate, can be extremely large, so it is necessary to be able to calculate such a force with certainty.

Analytical Determination of Position of Centre of Pressure

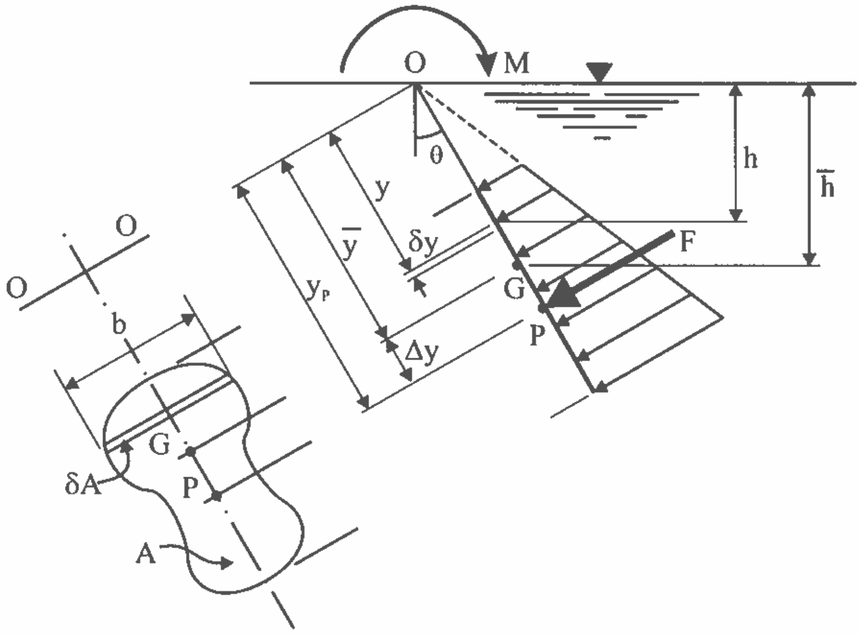


Figure 30 Determination of position of centre of pressure

Consider a plane surface A, inclined at angle θ to the vertical, as shown in edge and front view in Figure 30. For simplicity, the surface is taken to be symmetrical in front view. The area δA of the element shown at depth h, is:

$$\delta A = b\delta y$$

in which b is its width and δy its length, measured down the slope. The hydrostatic pressure on the element is

$$p = wh$$

so the element of hydrostatic force δF is

$$\delta F = p\delta A = wh\delta A$$

The resultant force F is obtained by integrating over the whole area A, and is therefore

$$F = w\int h\delta A$$

Now the depth $\bar{h}$ of the centroid G is, by definition, given by:

$$\bar{h}A = \int h\delta A$$

From these last two equations we see that

$$F = w\bar{h}A \qquad (38)$$

Writing $\bar{p}$ as the hydrostatic pressure at the centroid, then

$$\bar{p} = w\bar{h}$$

so the force F may be written as

$$F = \bar{p}A \tag{39}$$

The **magnitude** of F is therefore simply the product of the hydrostatic pressure at the centroid, and the area of the surface. To obtain the **position** of F, we take moments about the axis through O, lying in the water surface.

The element of moment δM about O produced by the element of force δF is

$$\delta M = y\delta F$$

Integrating over the surface, the resultant moment is

$$M = w\sec\theta\int y^2\delta A$$

Now the second moment of area of A about the axis at O is, by definition,

$$I_o = \int y^2\delta A$$

so, substituting this into the previous equation,

$$M = w\sec\theta I_o \tag{40}$$

This moment may also be expressed as

$$M = Fy_p$$

where y_p is the depth, measured down the slope, of the centre of pressure P below the water surface. Equation 38 gives F as

$$F = w\bar{h}A = w\bar{y}\sec\theta A$$

Substituting from this for F in the previous equation, and then equating the result to M as given by Equation 40 leads to

$$y_p = \frac{I_o}{A\bar{y}} \tag{41}$$

This result may be simplified by using the "theorem of parallel axes", which relates the second moment of area I_o about the axis at O to the second moment I_g about the parallel axis through G by

$$I_o = A\bar{y}^2 + I_g = A\bar{y}^2 + Ak_g^2 \tag{42}$$

In this,

I_o is the second moment of area A about the axis at O;

I_g is the second moment of area A about the axis at G;

k_g is the radius of gyration of A about the axis at G.

Substituting into Equation 41, we derive the result

$$y_p = \bar{y} + \frac{k_g^2}{\bar{y}} \qquad (43)$$

Finally, the slant distance, $\Delta y = y_p - \bar{y}$, between P and G is

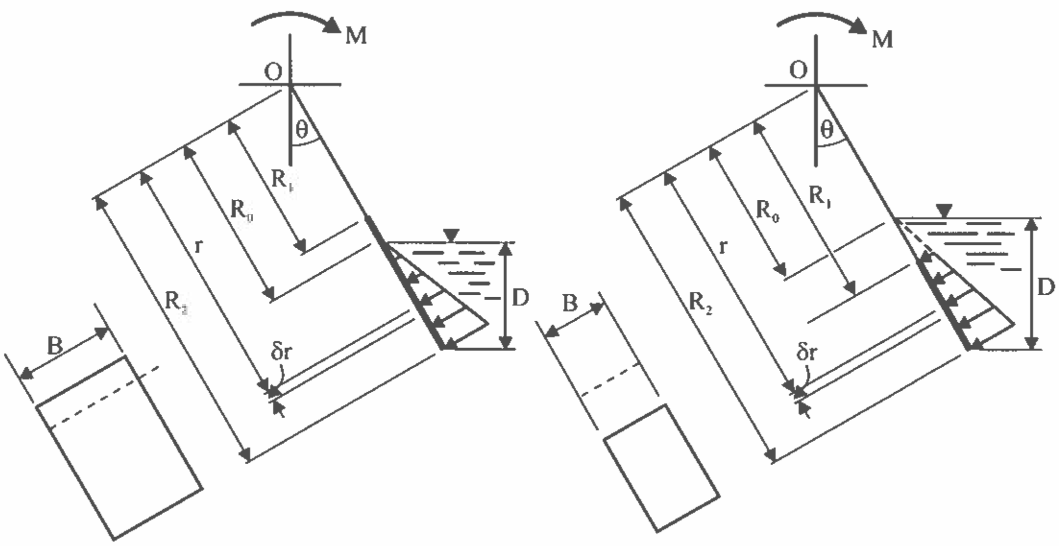
$$\Delta y = \frac{k_g^2}{\bar{y}} \qquad (44)$$

In summary, **the resultant force F is $\bar{p}A$, the product of the hydrostatic pressure $\bar{p}$ at the centroid G and the area A of the surface. It acts at the centre of pressure P, which lies at the slant distance Δy below G given by:**

$$\Delta y = \frac{k_g^2}{\bar{y}}$$

In the experiment described below, a plane surface is subjected to hydrostatic pressure, and measurement is made of the resultant moment about a fixed axis above the water surface. This moment is compared with a value derived from an analysis similar to that presented above.

Analytical Determination of Moment about an Axis above the Water Surface



(a) Partial Submergence

(B) Full Submergence

Figure 31 Determination of Moment about an Axis above the Water Surface

Consider a rectangular plate, inclined at angle θ to the vertical, which is subjected to the action of an increasing depth of water. Figure 31 (a) shows the case where the plate is only partially submerged, and Figure 31(b) shows the case when the plate is completely submerged below the level of the water surface. We now derive expressions for the moment produced about a fixed axis at O above the water surface.

Let D be the depth of water above the lower edge of the rectangle. Let R_1 be the slant distance from the axis O to the upper edge, and R_2 be the corresponding slant distance to the lower edge. It is convenient also to define R_0 as the slant distance to the water surface. To find the moment about the axis at O, consider first the moment produced by the action of hydrostatic pressure on an element, lying at slant distance r from the axis, and of slant length δr. The width of the rectangular plate is B, so the area of the element δA is

$$\delta A = B\delta r$$

The depth of the element below the water surface is $(r-R_0)\cos\theta$, so the hydrostatic pressure p on it is:

$$p = w(r-R_0)\cos\theta$$

The hydrostatic force δF on the element is:

$$\delta F = p\delta A = wB(r-Ro)\cos\theta\delta r$$

This force acts at radius r from the axis at O, so the moment δM produced about O is:

$$\delta M = wB(r-Ro)\cos\theta\delta r$$

The total moment M, obtained by integration over the submerged area, is:

$$M = wB\cos\theta\int r(r-R_0)dr \qquad (45)$$

Now the limits of integration for the two cases shown in Figure 31 are different. For the case of Figure 31(a), where the rectangular surface is only partially submerged:

$R_0 > R_1$ and

$$M = wB\cos\theta\int_{R_0}^{R_2} r(r-R_0)dr = wB\cos\theta\int_{R_0}^{R_2}(r^2-R_0r)dr$$

which leads to:

$$M = wB\cos\theta\left[\frac{r^3}{3}-\frac{R_0r^2}{2}\right]_{R_0}^{R_2}$$

or

$$M = wB\cos\theta\left[\frac{R_2^3-R_0^{\;3}}{3}-\frac{R_0(R_2^2-R_0^2)}{2}\right] \qquad (46)$$

For the case where the whole of the rectangle is submerged, as illustrated in Figure 31(b),

$R_0 > R_1$ and

$$M = wB\cos\theta\int_{R_1}^{R_2} r(r-R_0)dr = wB\cos\theta\int_{R_1}^{R_2}(r^2-R_0r)dr$$

or

$$M = wB\cos\theta\left[\frac{R_2^3-R_1^{\;3}}{3}-\frac{R_0(R_2^2-R_1^2)}{2}\right] \qquad (47)$$

SECTION 5.0 MAINTENANCE, SPARE PARTS AND CUSTOMER CARE

5.1 Maintenance

Regularly check all parts of the apparatus for damage, renew if necessary.

When not in use, store the apparatus in a dry, dust-free area, covered with a plastic sheet. If the apparatus becomes dirty, wipe the surfaces with a damp, clean cloth. Do not use abrasive cleaners.

Regularly check all fixings and fastenings for tightness, adjust where necessary.

NOTE *Renew faulty or damaged parts with an equivalent item of the same type or rating.*

Pressure Measurement Apparatus

Always smear the whole piston surface lightly with oil (supplied) after use. Do not attempt to polish the piston rod with emery cloth, or any harsh abrasive. Use only a mixture of powdered chalk and oil to remove discoloration.

5.2 Spare Parts

Check the Packing Contents List to see what spare parts we send with the apparatus.

If you need technical help or spares, please contact your local TecQuipment agent, or contact TecQuipment direct.

When you ask for spares, please tell us:
- Your name
- The full name and address of your college, company or institution
- Your email address
- The TecQuipment product name and product reference
- The TecQuipment part number (if you know it)
- The serial number
- The year it was bought (if you know it)

Please give us as much detail as possible about the parts you need and check the details carefully before you contact us.

If the product is out of warranty, TecQuipment will let you know the price of the spare parts.

5.3 Customer Care

We hope you like our products and manuals. If you have any questions, please contact our Customer Care department:

Telephone: +44 115 954 0155

Fax: +44 115 973 1520

Email: **customercare@tecquipment.com**

For information about all TecQuipment products visit: **www.tecquipment.com**

Air Valves

TecQuipment's

Fluid Mechanics Products

Instruction Sheets

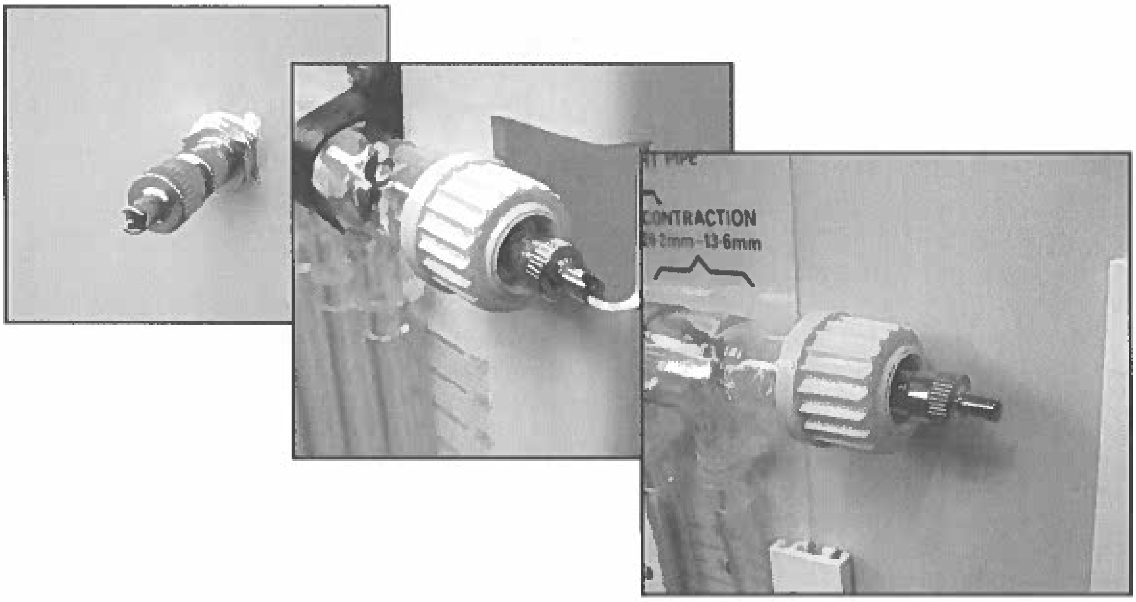


Figure 32 Typical Air Valves on Some of TecQuipment's Products

Many of the products in TecQuipment's Fluid Mechanics range use air valves at the tops of manometers or piezometers. The valves keep the air in the manometer tubes to allow you to offset the pressure range of the manometer or piezometer.

The valves are similar to valves used in vehicle tyres and include a special cap. The hand pump supplied with the equipment is similar to those used for bicycle tyres, except that TecQuipment remove the cross-shape part of the flexible pipe.

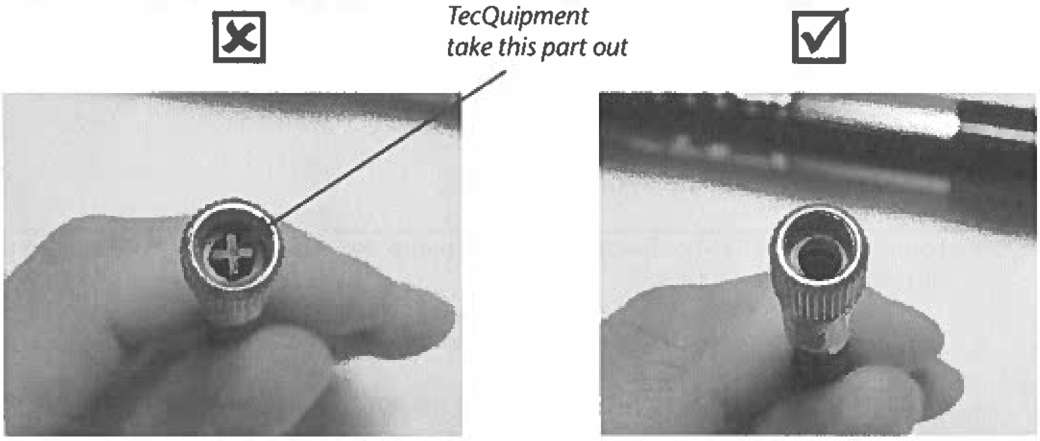


Figure 33 TecQuipment Remove the Cross-shape Part of the Flexible Pipe

Normally, when you connect the flexible pipe to an air valve, the cross-shape piece in the flexible pipe pushes open the valve as you pump air with the hand pump. With TecQuipment fluid mechanics products, this could allow water back out through the valve. For this reason TecQuipment remove the cross-shape piece. Without the cross-shape piece, only pressurised air can go through the valve in one direction, and no water can come back out.

Figure 34 The Hand Pump and Flexible Pipe

When you first use the hand pump with the air valve, you may find it hard to push air through the valve. This is because the valve is new and you do not have the cross-shape piece to help push it open. The valve will open more easily after you have pumped air through it a few times.

You may need some practice to use the air valve. To do it correctly:

1. Unscrew the cap from the valve.

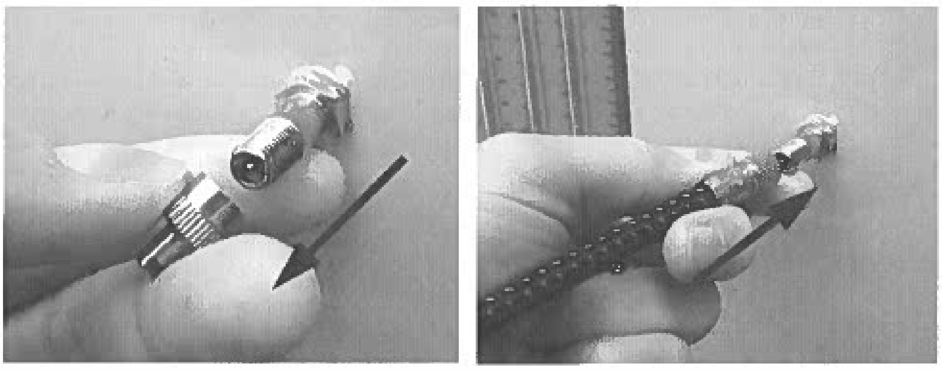


Figure 35 Unscrew the Cap and Fit the Pipe

2. Connect the flexible pipe to the valve.
3. Connect the hand pump to the flexible pipe.
4. Using complete strokes, ***slowly and firmly*** pump the hand pump to force air into the manometer or piezometer.
5. Unscrew the hand pump and flexible pipe and refit the valve cover.
6. To let air back out through the air valve, use the end of the special cap to press on the inner part of the valve (see Figure 36).

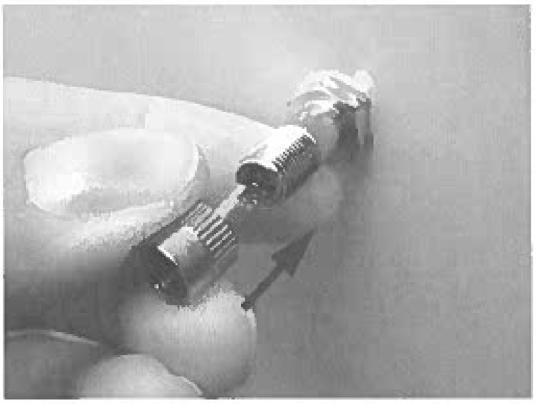


Figure 36 To Let Air Out - Use the End of the Special Cap to Press the Inner Part of the Valve

WARNING

Take care when you let air back out from the air valve. Water may come out!

Clean up any water spills immediately.

If using the hand pump is too difficult, the valve may be stuck. If you need to check the valve is working, use the special cap to unscrew the valve, then gently press the end of the valve. It should move easily and return back to its original position (see Figure 37).

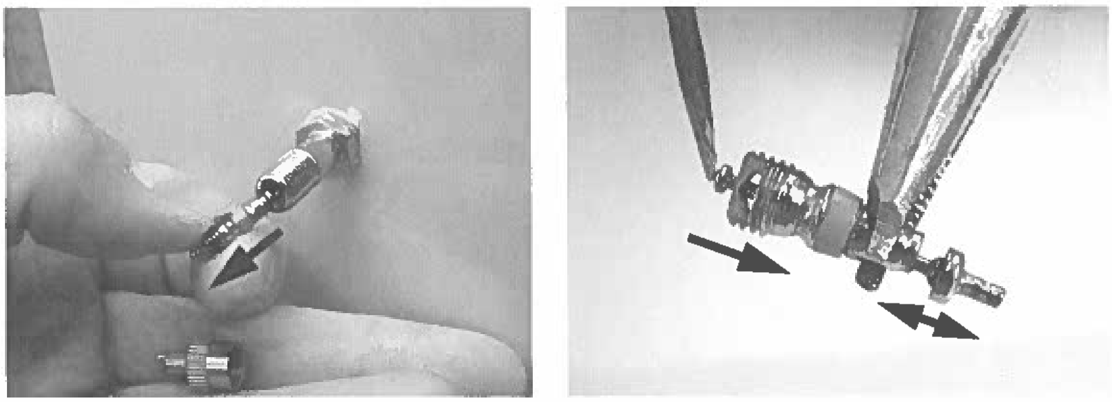


Figure 37 Unscrew the Valve and Check it

If the valve does not move easily, then contact TecQuipment Customer Services for help.

Telephone: +44 115 9722611

Fax: +44 115 973 1520

Email: **customer.care@tecquipment.com**

TecQuipment 0809 DB