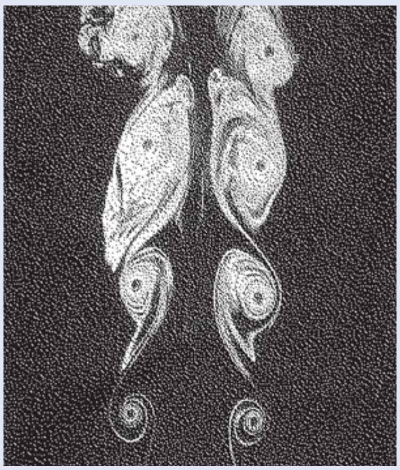
C H A P T E R 13

Flow Measurements



The pattern of flow that occurs when an air jet is discharged into quiescent air is revealed in this photo. (Courtesy Cecilia D. Richards.)

SIGNIFICANT LEARNING OUTCOMES

Contextual Knowledge

- Sketch common measuring instruments.
- Explain the operating principles of common measuring instruments.
- List the advantages and disadvantages of common measuring instruments (common measurment instruments include the hot-wire anemometer, orifice meter, laser-Doppler anemometer venturi meter, rotameter, and the weir).

Procedural Knowledge

- Calculate flow rate for an orifice meter, a venturi meter, or a weir.
- Calculate flow rate by integrating velocity distribution data.
- For a weir or an orifice meter, estimate uncertainty using the RMS method.

Measurement techniques are important because fluid mechanics relies heavily on experiments. Thus, Chapter 3 described instruments for measuring pressure including the piezometer, the manometer, the Bourdon-tube gage, and the pressure transducer. Chapter 4 describes the Pitot-static tube and the Pitot tube. This chapter builds on this knowledge by introducing additional ways to measure flow rate, pressure, and velocity. Also, this chapter describes how to estimate the uncertainty of a measurement.

13.1 Measuring Velocity and Pressure

Stagnation (Pitot) Tube

The stagnation tube, also called the Pitot tube, is shown in Fig. 13.1*a*. A Pitot tube measures stagnation pressure with an open tube that is aligned parallel with the velocity direction and then senses pressure in the tube using a pressure gage or transducer.

When the stagnation tube was introduced in Chapter 4, viscous effects were not discussed. Viscous effects are noteable because they can influence the accuracy of a measurement. The effects of viscosity, from reference (1), are shown in Fig. 13.2. This shows the pressure coefficient C_p plotted as a function of the Reynolds number. Viscous effects are important when $C_p > 1.0$.

Figure 13.1

Section view of (a) Pitot tube, (b) Static tube, (c) the Pitot-static tube.

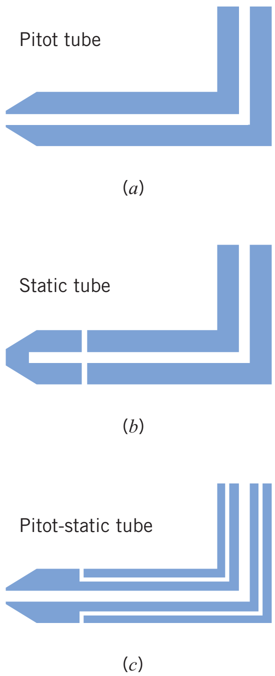


Figure 13.2

Viscous effects on C_p. [After Hurd, Chesky, and Shapiro (1). Used with permission from ASME.]

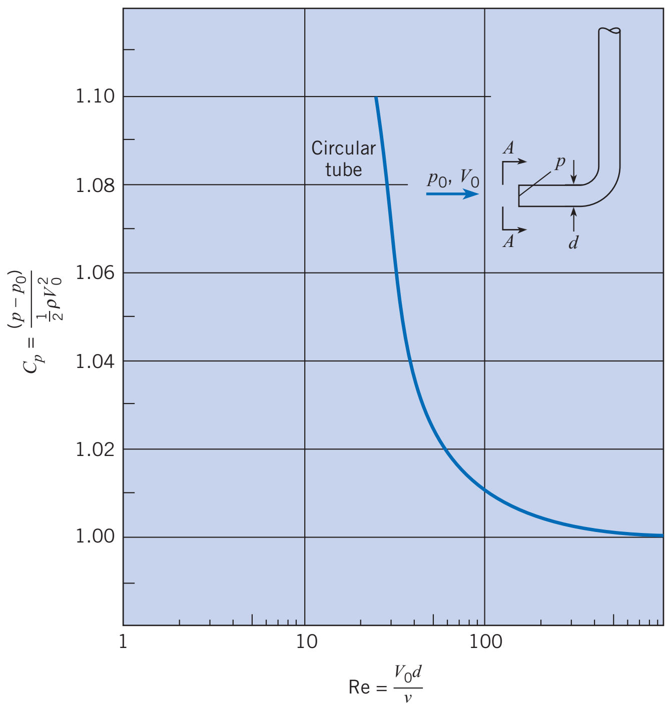
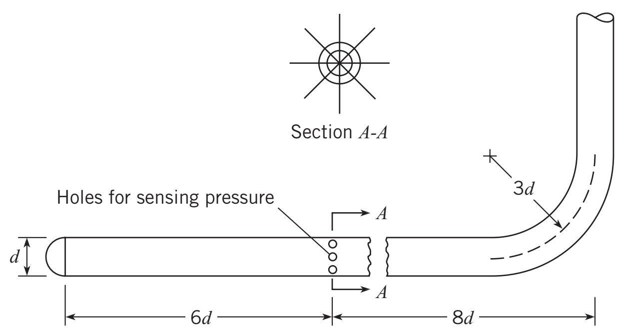


Figure 13.3

Static tube.



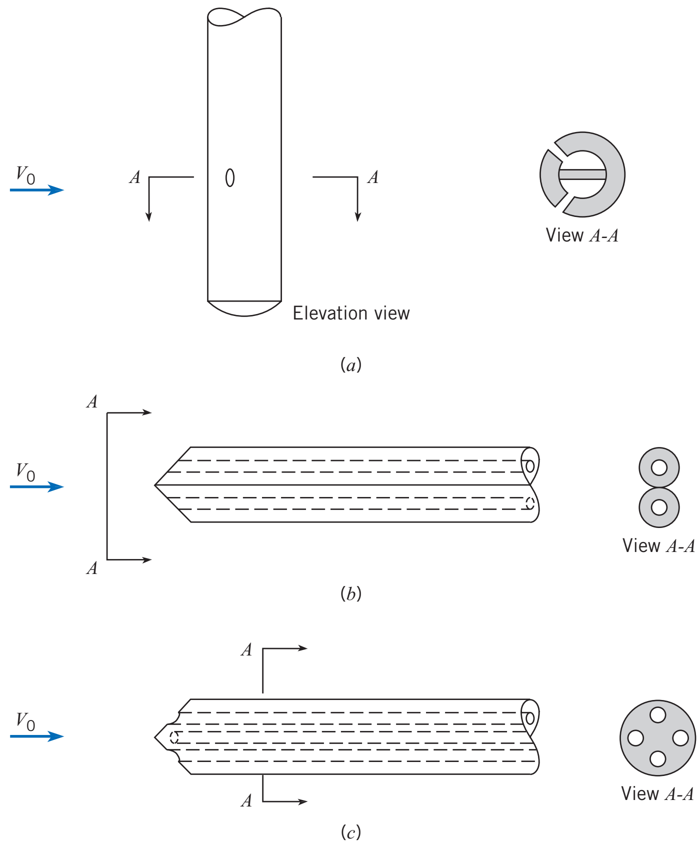
In Fig. 13.2 it is seen that when the Reynolds number for the circular stagnation tube is greater than 60, the error in measured velocity is less than 1%. For boundary-layer measurements a stagnation tube with a flattened end can be used. By flattening the end of the tube, the velocity measurement can be taken nearer the boundary than if a circular tube were used. For these flattened tubes, the pressure coefficient remains near unity for a Reynolds number as low as 30. See reference (15) for more information on flattened-end stagnation tubes.

Static Tube

A *static tube*, as shown in Fig. 13.1*b*, is an instrument for measuring static pressure. Static pressure is the pressure in a fluid that is stationary or in a fluid that is flowing. When the fluid is flowing, the static pressure must be measured in a way that does not disturb the pressure. Thus, in the design of the static tube, as shown in Fig. 13.3, the placement of the holes along the probe is critical because the rounded nose on the tube causes some decrease of pressure along the tube and the downstream stem causes an increase in pressure in front of it. Hence the location for sensing the static pressure must be at the point where these two effects cancel each other. Experiments reveal that the optimum location is at a point approximately six diameters downstream of the front of the tube and eight diameters upstream from the stem.

Figure 13.4

Various types of yaw meters.
(a) Cylindrical-tube yaw meter.
(b) Two-tube yaw meter.
(c) Three-dimensional yaw meter.



Pitot-Static Tube

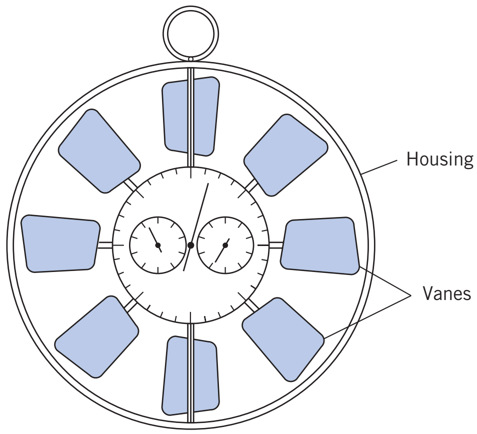
The Pitot-static tube, Fig. 13.1c, measures velocity by using concentric tubes to measure static pressure and dynamic pressure. Application of the Pitot-static tube is presented in Chapter 4.

Yaw Meters

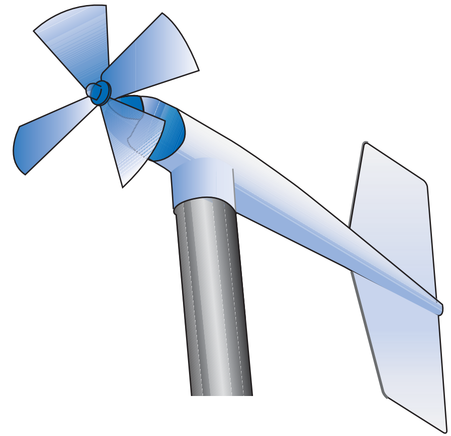
A *yaw meter*, Fig. 13.4, is an instrument for measuring velocity by using multiple pressure ports to determine the magnitude and direction of fluid velocity. The first two yaw meters in Fig. 13.4 can be used for two-dimensional flow, where flow direction in only one plane needs to be found. The third yaw meter in Fig. 13.4 is used for determining flow direction in three dimensions. In all these devices, the tube is turned until the pressure on symmetrically opposite openings is equal. This pressure is sensed by a differential pressure gage or manometer connected to the openings in the yaw meter. The flow direction is sensed when a null reading is indicated on the differential gage. The velocity magnitude is found by using equations that depend on the type of yaw meter that is used.

The Vane or Propeller Anemometer

The term *anemometer* originally meant an instrument that was used to measure the velocity of the wind. However, anemometer now means an instrument that is used to measure fluid velocity, because anemometers are used in water, air, nitrogen, blood, and many other fluids. See (18) for an overview of the many types of anemometers.



(*a*)



(*b*)

Figure 13.5

(a) Vane anemometer, (b) Propeller anemometer.

The *vane anemometer* (Fig. 13.5*a*) and the *propeller anemometer* (Fig. 13.5*b*) measure velocity by using vanes typical of a fan or propeller, respectively. These blades rotate with a speed of rotation that depends on the wind speed. Typically, an electronic circuit converts the rotational speed into a velocity reading. On some older instruments the rotor drives a low-friction gear train that, in turn, drives a pointer that indicates feet on a dial. Thus if the anemometer is held in an airstream for 1 min and the pointer indicates a 300 ft change on the scale, the average airspeed is 300 ft/min.

Cup Anemometer

Instead of using vanes, the cup anemometer, in Fig. 13.6, is a device that uses the drag on cup-shaped objects to spin a rotor around a central axis. Since the rotational speed of the rotor is related to drag force, the frequency of rotation is related to the fluid velocity by appropriate calibration data. A typical rotor comprises three to five hemispherical or conical cups. In addition to applications in air, engineers use a cup anemometer to measure the velocity in streams and rivers.

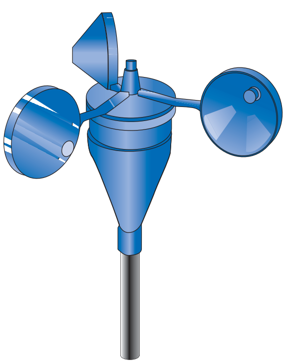


Figure 13.6

Cup anemometer.

Hot-wire and Hot-film Anemometers

The *hot-wire anemometer* (HWA), Fig. 13.7, is an instrument for measuring velocity by sensing the heat transfer from a heated wire. As velocity increases, more energy is needed to keep the wire hot and the corresponding changes in electrical characteristics can be used to determine the velocity of the fluid that is passing by the wire.

The HWA has advantages over other instruments. The HWA is well suited for measuring velocity fluctuations that occur in turbulent flow, whereas instruments such as the Pitot-static tube are only suitable for measuring velocity that either is steady or changes slowly with time. The sensing element of the HWA is quite small, allowing the HWA to be used in locations such as the boundary layer, where the velocity is varying in a region that is small in size. Many other instruments are too large for recording velocity in a region that is geometrically small. Another advantage of the HWA is that it is sensitive to low-velocity flows, a characteristic lacking in the Pitot tube and other instruments. The main disadvantages of the HWA are its delicate nature (the sensor wire is easily broken), its relatively high cost, and its need for an experienced user.

The basic principle of the hot-wire anemometer is described as follows: A wire of very small diameter—the sensing element of the hot-wire anemometer—is welded to supports as

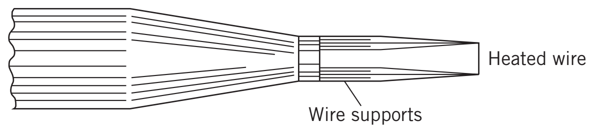


Figure 13.7

Probe for hot-wire anemometer (enlarged).

shown in Fig. 13.7. In operation the wire either is heated by a fixed flow of electric current (the constant-current anemometer) or is maintained at a constant temperature by adjusting the current (the constant-temperature anemometer).

A flow of fluid past the hot wire causes the wire to cool because of convective heat transfer. In the constant-current anemometer, the cooling of the wire causes its resistance to change, and a corresponding voltage change occurs across the wire. Because the rate of cooling is a function of the speed of flow past the heated wire, the voltage across the wire is correlated with the flow velocity. The more popular type of anemometer, the constant-temperature anemometer, operates by varying the current in such a manner as to keep the resistance (and temperature) constant. The flow of current is correlated with the speed of the flow: the higher the speed, the greater the current needed to maintain a constant temperature. Typically, the wires are 1 mm to 2 mm in length and heated to 150°C. The wires may be 10 μm or less in diameter; the time response improves with the smaller wire. The lag of the wire's response to a change in velocity (thermal inertia) can be compensated for more easily, using modern electronic circuitry, in constant-temperature anemometers than in constant-current anemometers. The signal from the hot wire is processed electronically to give the desired information, such as mean velocity or the root-mean-square of the velocity fluctuation.

To illustrate the versatility of these instruments, note that the hot-wire anemometer can measure accurately gas flow velocities from 30 cm/s to 150 m/s; it can measure fluctuating velocities with frequencies up to 100,000 Hz; and it has been used satisfactorily for both gases and liquids.

The single hot wire mounted normal to the mean flow direction measures the fluctuating component of velocity in the mean flow direction. Other probe configurations and electronic circuitry can be used to measure other components of velocity.

For velocity measurements in liquids or dusty gases, where wire breakage is a problem, the hot-film anemometer is more suitable. This anemometer consists of a thin conducting metal film (less than 0.1 μm thick) mounted on a ceramic support, which may be 50 μm in diameter. The hot film operates in the same fashion as the hot wire. Recently, the split film has been introduced. It consists of two semicylindrical films mounted on the same cylindrical support and electrically insulated from each other. The split film provides both speed and directional information.

For more detailed information on the hot-wire and hot-film anemometers, see King and Brater (2) and Lomas (3).

Laser-Doppler Anemometer

The *laser-Doppler anemometer* (LDA) is an instrument for measuring velocity by using the Doppler shift that occurs when a particle in a flow scatters light from crossed laser beams. Advantages of the LDA are (a) the flow field is not disturbed by the presence of a probe, and (b) it provides excellent spatial resolution. Disadvantages of the LDA include cost, complexity, the need for a transparent fluid, and requirement for particle seeding.

There are several different configurations for the LDA. The dual-beam mode, Fig. 13.8, splits a laser beam into two parallel beams and then uses a converging lens to cause the two beams to cross. The point where beams cross is called the measuring volume, which might best be described as an ellipsoid that is typically 0.3 mm in diameter and 2 mm long, illustrating the excellent spatial resolution achievable. The interference of the two beams generates a series of light and dark fringes in the measuring volume perpendicular to the plane of the two

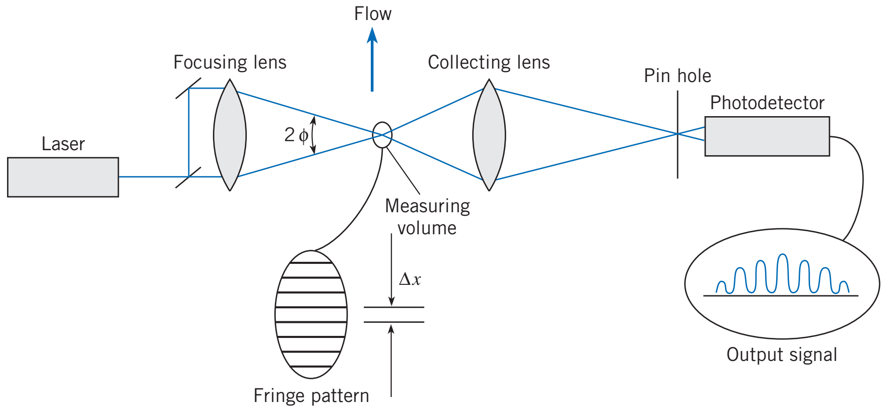


Figure 13.8

Dual-beam laser-Doppler anemometer.

beams. As a particle passes through the fringe pattern, light is scattered and a portion of the scattered light passes through the collecting lens toward the photodetector. A typical signal obtained from the photodetector is shown in the figure.

It can be shown from optics theory that the spacing between the fringes is given by

$$\Delta x = \frac{\lambda}{2\sin\phi} \tag{13.1}$$

where λ is the wavelength of the laser beam and ϕ is the half-angle between the crossing beams. By suitable electronic circuitry, the frequency of the signal (f) is measured, so the velocity is given by

$$U = \frac{\Delta x}{\Delta t} = \frac{\lambda f}{2\sin\phi} \tag{13.2}$$

The operation of the laser-Doppler anemometer depends on the presence of particles to scatter the light. These particles need to move at the same velocity as the fluid. Thus the particles need to be small relative to the size of flow patterns, and they need to have a density near that of the ambient fluid. In liquid flows, impurities of the fluid can serve as scattering centers. In water flows, adding a few drops of milk is common. In gaseous flows, it is common to "seed" the flow with small particles. Smoke is often used for this seeding.

Laser-Doppler anemometers that provide two or three velocity components of a particle traveling through the measuring volume are now available. This is accomplished by using laser-beam pairs of different colors (wavelengths). The measuring volumes for each color are positioned at the same physical location but oriented differently to measure a different component. The signal-processing system can discriminate the signals from each color and thereby provide component velocities.

Another recent technological advance in laser-Doppler anemometry is the use of fiber optics. The fiber optics transmit the laser beams from the laser to a probe that contains optical elements to cross the beams and generate a measuring volume. Thus measurements at different locations can be made by moving the probe and without moving the laser. For more applications of the laser-Doppler technique see Durst (4).

Marker Methods

The marker method for determining velocity involves particles that are placed in the stream. By analyzing the motion of these particles, one can deduce the velocity of the flow itself. Of course, this requires that the markers follow virtually the same path as the surrounding fluid elements. It means, then, that the marker must have nearly the same density as the fluid or that it must be so small that its motion relative to the fluid is negligible. Thus for water flow it is common to use colored droplets from a liquid mixture that has nearly the same density as the water. For example, Macagno (6) used a mixture of *n*-butyl phthalate and xylene with a bit of white paint to yield a mixture that had the same density as water and could be photographed effectively. Solid particles, such as plastic beads, that have densities near that of the liquid being studied can also be used as markers.

Hydrogen bubbles have also been used for markers in water flow. Here an electrode placed in flowing water causes small bubbles to be formed and swept downstream, thus revealing the motion of the fluid. The wire must be very small so that the resulting bubbles do not have a significant rise velocity with respect to the water. By pulsing the current through the electrode, it is possible to add a time frame to the visualization technique, thus making it a useful tool for velocity measurements. Figure 13.9 shows patches of tiny hydrogen bubbles that were released with a pulsing action from noninsulated segments of a wire located to the left of the picture. Flow is from left to right, and the necked-down section of the flow passage has higher water velocity. Therefore, the patches are longer in that region. Next to the walls the patches of bubbles are shorter, indicating less distance traveled per unit of time. Other details concerning the marker methods of flow visualization are described by Macagno (6).

A relatively new marker method is particle image velocimetry (PIV), which provides a measurement of the velocity field. In PIV, the marker or seeding particles may be minuscule spheres of aluminum, glass, or polystyrene. Or they may be oil droplets, oxygen bubbles (liquids only), or smoke particles (gases only). The seeding particles are illuminated in order to produce a photographic record of their motion. In particular, a sheet of light passing through a cross section of the flow is pulsed on twice, and the scattered light from the particles is recorded by a camera. The first pulse of light records the position of each particle at time t, and the second pulse of light records the position at time $t + \Delta t$. Thus, the displacement $\Delta \mathbf{r}$ of each particle is recorded on the photograph. Dividing $\Delta \mathbf{r}$ by Δt yields the velocity of each particle. Because PIV uses a sheet of light, the method provides a simultaneous measurement of velocity at locations throughout a cross section of the flow. Hence, PIV is identified as a whole-field technique. Other velocity measurements, the LDA method, for example, are limited to measurements at one location.

PIV measurement of the velocity field for flow over a backward-facing step is shown in Fig. 13.10. This experiment was carried out in water using 15 μm–diameter, silver-coated hollow spheres as seeding particles. Notice that the PIV method provided data over the cross section of the flow. Although the data shown in Fig. 13.10 are qualitative, numerical values of the velocity at each location are also available.

The PIV method is typically performed using digital hardware and computers. For example, images may be recorded with a digital camera. Each resulting digital image is evaluated with software that calculates the velocity at points throughout the image. This evaluation

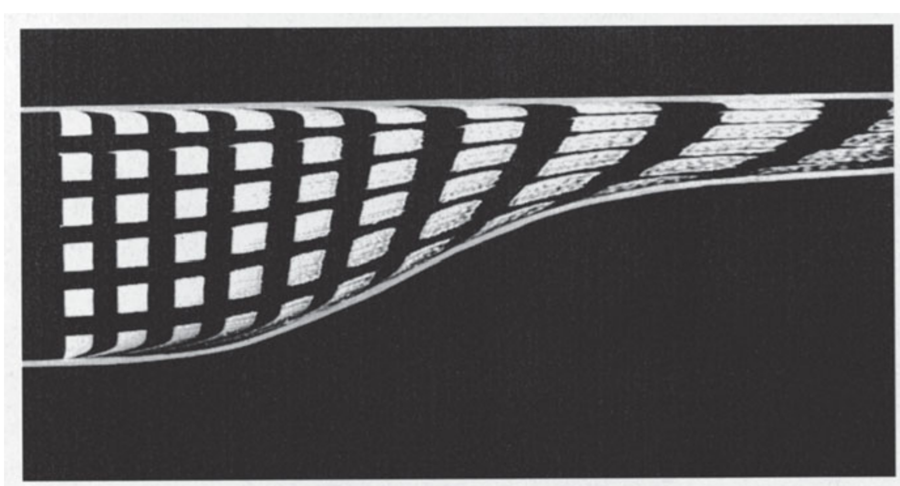


Figure 13.9

Combined time-streak markers (hydrogen bubbles); flow is from left to right. [After Kline (5) Courtesy of Education Development Center, Inc., Newton, MA.]

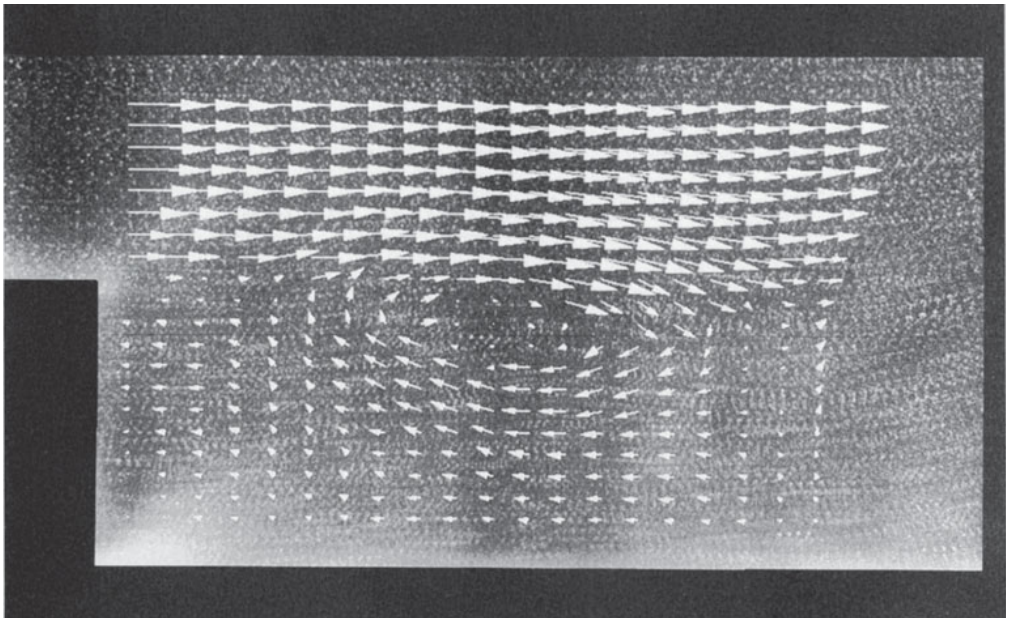


Figure 13.10

Velocity vectors from PIV measurements. (Courtesy TSI, Inc., and Florida State University.)

proceeds by dividing the image into small sub-areas called "interrogation areas." Within a given interrogation area, the displacement vector ($\mathbf{\Delta r}$) of each particle is found by using statistical techniques (auto- and cross-correlation). After processing, the PIV data are typically available on a computer screen. Additional information on PIV systems is provided by Raffel et al. (7).

Smoke is often used as a marker in flow measurement. One technique is to suspend a wire vertically across the flow field and allow oil to flow down the wire. The oil tends to accumulate in droplets along the wire. Applying a voltage to the wire vaporizes the oil, creating streaks from the droplets. Figure 13.11 is an example of a flow pattern revealed by such a method. Smoke generators that provide smoke by heating oils are also commercially available. It is also possible to position a thin sheet of laser light through the smoke field to obtain an improved spatial definition of the flow field indicated by the smoke. Another technique is to introduce titanium tetrachloride ($TiC1_4$) in a dried-air flow, which reacts with the water vapor in the ambient air to produce micron-sized titanium oxide particles, which serve as tracers. The flow pattern obtained for an upward-flowing air jet using this technique in conjunction with a laser light sheet

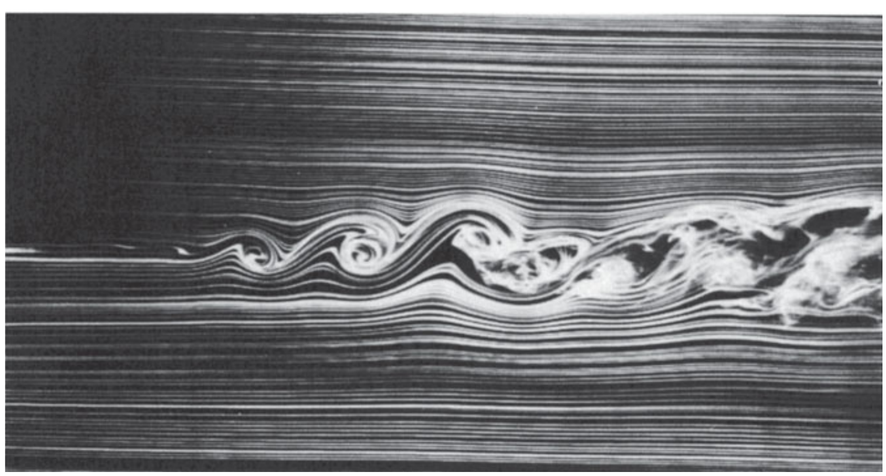


Figure 13.11

Flow pattern in the wake of a flat plate.

is shown in the photograph at the beginning of this chapter (p. 435). This jet is subjected to an acoustic field, which enhances the vortex shedding pattern observed in the jet.

13.2 Measuring Flow Rate (Discharge)

Measuring flow rate is important in research, design, testing, and in many commercial applications.

Direct Measurement of Volume or Weight

For liquids, a simple and accurate method is to collect a sample of the flowing fluid over a given period of time Δt. Then the sample is weighed, and the average weight rate of flow is $\Delta W/\Delta t$, where ΔW is the weight of the sample. The volume of a sample can also be measured (usually in a calibrated tank), and from this the average volume rate of flow is calculated as $\Delta \forall/\Delta t$, where $\Delta \forall$ is the volume of the sample. This method has several disadvantages: It cannot be used for an unsteady flow, and it is not always possible to collect a sample.

Integrating a Measured Velocity Distribution

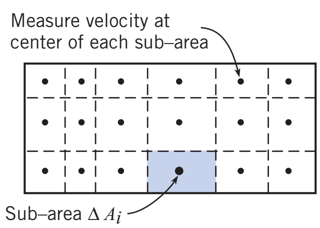


Figure 13.12

Dividing a rectangular conduit into sub-areas for approximating discharge.

Flow rate can be found by measuring a velocity distribution and then integrating using the volume flow rate equation (5.8):

$$Q = \int_A V dA$$

For example, one can divide a rectangular conduit into sub-areas and then measure velocity at the center of each sub-area as shown in Fig. 13.12. Then flow rate is determined by

$$Q = \int_A V\,dA \approx \sum_{i=1}^{N} V_i(\Delta A)_i \tag{13.3}$$

where N is the number of sub-areas. When the flow area occurs in a round pipe, then the sub-area is a ring as shown by Example 13.1.

EXAMPLE 13.1 DISCHARGE FROM VELOCITY DATA

The data given in the table are for a velocity traverse of air flow in a pipe 100 cm in diameter. What is the volume rate of flow in cubic meters per second?

r (cm)	V (m/s)
0.00	50.0
5.00	49.5
10.00	49.0
15.00	48.0
20.00	46.5
25.00	45.0
30.00	43.0
35.00	40.5
40.00	37.5
45.00	34.0
47.50	25.0
50.00	0.0

EXAMPLE 13.2

Situation:

1. Air is flowing in a round pipe ($D = 1.0$ m).
2. Velocity in m/s is known as a function of radius (see table).

Find: Volume flow rate (in m^3/s) in the pipe.

Assumptions: The velocity distribution is symmetric around the centerline of the pipe.

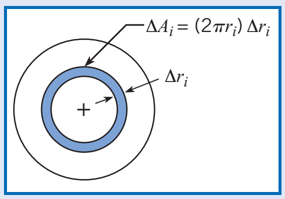
Plan

1. Develop an equation for a round pipe by applying Eq. (13.3).
2. Find discharge by using a spreadsheet program.

Solution

The flow rate is given by

$$Q = \sum_{i=1}^{N} V_i (\Delta A)_i$$



The area ΔA_i is shown in the sketch above. Visualize this area as a strip of length $2\pi r_i$ and width Δr_i. Then $\Delta A_i \approx (2\pi r_i)\Delta r_i$. The flow rate equation becomes

$$Q = \sum_{i=1}^{N} V_i (\Delta A)_i = \sum_{i=1}^{N} V_i (2\pi r_i)\Delta r_i$$

To perform the sum, use a spreadsheet as shown. To see how the table is set up, consider the row $i = 2$. The area is

$$\Delta A_2 = (2\pi r_2)\Delta r_2 = (2\pi(0.05 \text{ m}))(0.05 \text{ m}) = 0.0157 \text{ m}^2$$

which is given in the sixth column. The last column gives

$$V_2(\Delta A)_2 = (49.5 \text{ m/s})(0.0157 \text{ m}^2) = 0.778 \text{ m}^3/\text{s}$$

i	r_i (cm)	V_i (m/s)	$2*\pi*r_i$ (m)	Δr_i (m)	ΔA_i (m^2)	$V_i*\Delta A_i$ (m^3/s)
1	0.0	50.0	0.0000	0.0250	0.0000	0.000
2	5.0	49.5	0.3142	0.0500	0.0157	0.778
3	10.0	49.0	0.6283	0.0500	0.0314	1.539
4	15.0	48.0	0.9425	0.0500	0.0471	2.262
5	20.0	46.5	1.2566	0.0500	0.0628	2.922
6	25.0	45.0	1.5708	0.0500	0.0785	3.534
7	30.0	43.0	1.8850	0.0500	0.0942	4.053
8	35.0	40.5	2.1991	0.0500	0.1100	4.453
9	40.0	37.5	2.5133	0.0500	0.1257	4.712
10	45.0	34.0	2.8274	0.0375	0.1060	3.605
11	47.5	25.0	2.9845	0.0250	0.0746	1.865
12	50.0	0.0	3.1416	0.0125	0.0393	0.000
			SUM==>	0.50	0.79	29.72

Discharge is found by summing the last column. As shown

$$Q = \sum_{i=1}^{12} V_i (\Delta A)_i = \boxed{29.7 \frac{\text{m}^3}{\text{s}}}$$

To check the validity of the calculation, sum the column labeled Δr_i and check to ensure that this value equals the radius of the pipe. As shown, this sum is 0.5 m. Similarly, the pipe area of

$$A = \pi r^2 = \pi(0.5 \text{ m})^2 = 0.785 \text{m}^2$$

should be produced by summing the column labeled ΔA_i. As shown, this is the case.

Calibrated Orifice Meter

An *orifice meter* is an instrument for measuring flow rate by using a carefully designed plate with a round opening and situating this device in a pipe, as shown in Fig. 13.13. Flow rate is found by measuring the pressure drop across the orifice and then using an equation to calculate the appropriate flow rate. One common application of the orifice meter is metering of natural gas in pipelines. Because large quantities of natural gas are measured and the associated costs are high, accuracy is very important. This section describes the main ideas associated with orifice meters. Details about using orifice meters are presented in standards such as reference (10).

Flow through a sharp-edged orifice is shown in Fig. 13.13. Note that the streamlines continue to converge a short distance downstream of the plane of the orifice. Hence the minimum-flow area is actually smaller than the area of the orifice. To relate the minimum-

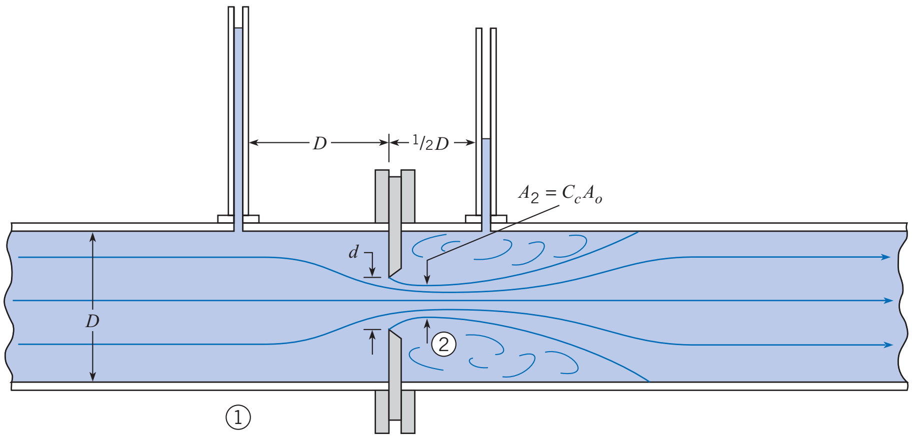


Figure 13.13

Flow through a sharp-edged pipe orifice.

flow area, often called the contracted area of the jet, or *vena contracta*, to the area of the orifice A_o, one uses the contraction coefficient, which is defined as

$$A_j = C_c A_o$$

$$C_c = \frac{A_j}{A_o}$$

Then, for a circular orifice,

$$C_c = \frac{(\pi/4)d_j^2}{(\pi/4)d^2} = \left(\frac{d_j}{d}\right)^2$$

Because d_j and d_2 are identical, $C_c = (d_2/d)^2$. At low values of the Reynolds number, C_c is a function of the Reynolds number. However, at high values of the Reynolds number, C_c is only a function of the geometry of the orifice. For d/D ratios less than 0.3, C_c has a value of approximately 0.62. However, as d/D is increased to 0.8, C_c increases to a value of 0.72.

To derive the orifice equation, consider the situation shown in Fig. 13.13. Apply the Bernoulli equation between section 1 and section 2:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

V_1 is eliminated by means of the continuity equation $V_1A_1 = V_2A_2$. Then solving for V_2 gives

$$V_2 = \left\{\frac{2g[(p_1/\gamma + z_1) - (p_2/\gamma + z_2)]}{1 - (A_2/A_1)^2}\right\}^{1/2} \tag{13.4a}$$

However, $A_2 = C_c A_o$ and $h = p/\gamma + z$, so Eq. (13.4a) reduces to

$$V_2 = \sqrt{\frac{2g(h_1 - h_2)}{1 - C_c^2 A_o^2/A_1^2}} \tag{13.4b}$$

Our primary objective is to obtain an expression for discharge in terms of h_1, h_2, and the geometric characteristics of the orifice. The discharge is given by $V_2 A_2$. Hence, multiply both sides of Eq. (13.4b) by $A_2 = C_c A_o$, to give the desired result:

$$Q = \frac{C_c A_o}{\sqrt{1 - C_c^2 A_o^2 / A_1^2}} \sqrt{2g(h_1 - h_2)} \tag{13.5}$$

Equation (13.5) is the discharge equation for the flow of an incompressible inviscid fluid through an orifice. However, it is valid only at relatively high Reynolds numbers. For low and moderate values of the Reynolds number, viscous effects are significant, and an additional coefficient called the *coefficient of velocity,* C_v, must be applied to the discharge equation to relate the ideal to the actual flow.* Thus for viscous flow through an orifice, we have the following discharge equation:

$$Q = \frac{C_v C_c A_o}{\sqrt{1 - C_c^2 A_o^2 / A_1^2}} \sqrt{2g(h_1 - h_2)}$$

The product $C_v C_c$ is called the *discharge coefficient,* C_d, and the combination $C_v C_c / (1 - C_c^2 A_o^2 / A_1^2)^{1/2}$ is called the *flow coefficient,* K. Thus, $Q = K A_o \sqrt{2g(h_1 - h_2)}$, where

$$K = \frac{C_d}{\sqrt{1 - C_c^2 A_o^2 / A_1^2}} \tag{13.6}$$

If Δh is defined as $h_1 - h_2$, then the final form of the orifice equation reduces to

$$Q = K A_o \sqrt{2g \Delta h} \tag{13.7a}$$

If a differential pressure transducer is connected across the orifice, it will sense a piezometric pressure change that is equivalent to $\gamma \Delta h$, so the orifice equation becomes

$$Q = K A_o \sqrt{2 \frac{\Delta p_z}{\rho}} \tag{13.7b}$$

Experimentally determined values of K as a function of d/D and Reynolds number based on orifice size are given in Fig. 13.14. If Q is given, Re_d is equal to $4Q/\pi d\nu$. Then K is obtained from Fig. 13.14 (using the vertical lines and the bottom scale), and Δh is computed from Eq. (13.7a), or Δp_z can be computed from Eq. (13.7b). However, one is often confronted with the problem of determining the discharge Q when a certain value of Δh or a certain value of Δp_z is given. When Q is to be determined, there is no direct way to obtain K by entering Fig. 13.14 with Re, because Re is a function of the flow rate, which is still unknown. Hence another scale, which does not involve Q, is constructed on the graph of Fig. 13.14. The variables for this scale are obtained in the following manner: Because $\mathrm{Re}_d = 4Q/\pi d\nu$ and $Q = K(\pi d^2/4)\sqrt{2g\Delta h}$, write Re_d in terms of Δh:

$$\mathrm{Re}_d = K \frac{d}{\nu} \sqrt{2g\Delta h}$$

* At low Reynolds numbers the coefficient of velocity may be quite small; however, at Reynolds numbers above 10^5, C_v typically has a value close to 0.98. See Lienhard (8) for C_v analyses.

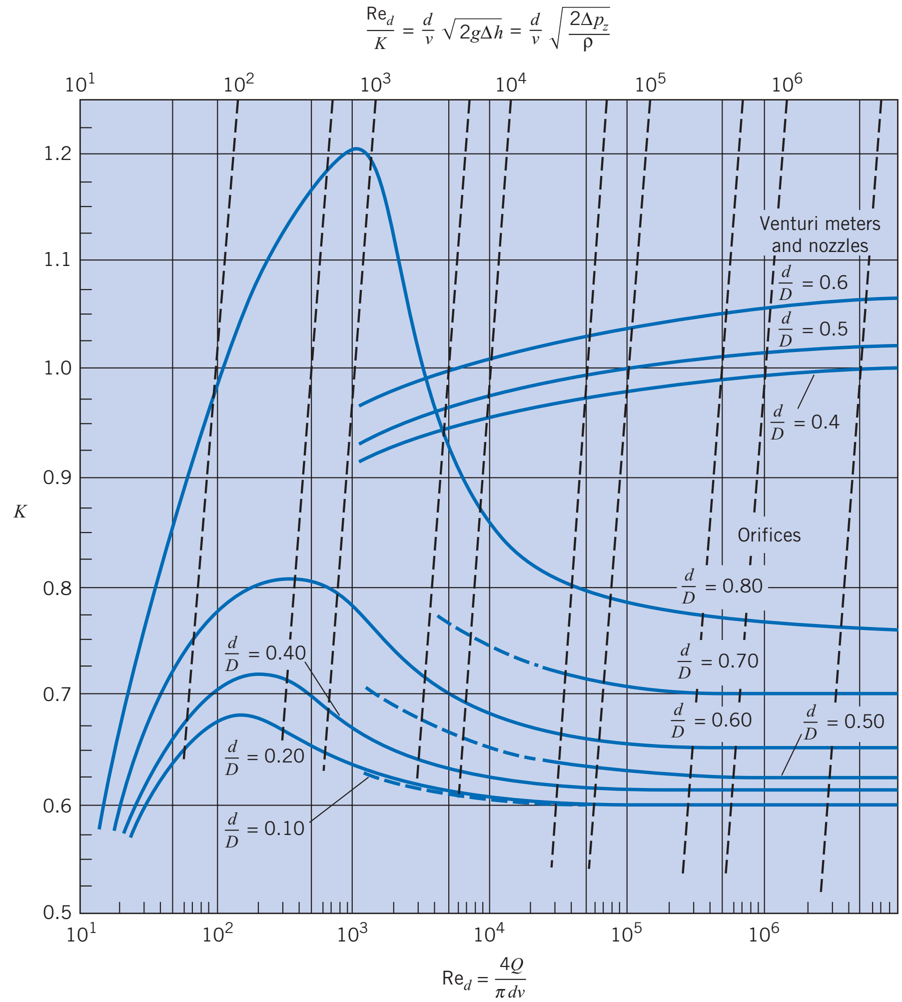


Figure 13.14

Flow coefficient K and Re$_d$/*K versus the Reynolds number for orifices, nozzles, and venturi meters. [After Tuve and Sprenkle (9) and ASME (10). Permission to use Tuve granted by* Instrumentation & Control Systems *magazine, formerly* Instruments *magazine.]*

or

$$\frac{\text{Re}_d}{K} = \frac{d}{v}\sqrt{2g\Delta h} = \frac{d}{v}\sqrt{\frac{2\Delta p_z}{\rho}}$$

Thus the slanted dashed lines and the top scale are used in Fig. 13.14 when Δh is known and the flow rate is to be determined. If a certain value of Δp is given, one can apply Fig. 13.14 by using $\Delta p_z/\rho$ in place of $g\Delta h$ in the parameter at the top of Fig. 13.14.

The literature on orifice flow contains numerous discussions concerning the optimum placement of pressure taps on both the upstream side and the downstream side of an orifice. The data given in Fig. 13.14 are for "corner taps." That is, on the upstream side the pressure readings were taken immediately upstream of the orifice plate (at the corner of the orifice plate and the pipe wall), and the downstream tap was at a similar downstream location. However, pressure data from flange taps (1 in. upstream and 1 in. downstream) and from the taps shown in Fig. 13.13 all yield virtually the same values for K—the differences are no greater than the deviations involved in reading Fig. 13.14. For more precise values of K with specific types of taps, see the ASME report on fluid meters (10).

Head Loss for Orifices

Some head loss occurs between the upstream side of the orifice and the vena contracta. However, this head loss is very small compared with the head loss that occurs downstream of the vena contracta. This downstream portion of the head loss is like that for an abrupt expansion. Neglecting all head loss except that due to the expansion of the flow, gives

$$h_L = \frac{(V_2 - V_1)^2}{2g} \tag{13.8}$$

where V_2 is the velocity at the vena contracta and V_1 is the velocity in the pipe. It can be shown that the ratio of this expansion loss, h_L, to the change in head across the orifice, Δh, is given as

$$\frac{h_L}{\Delta h} = \frac{\dfrac{V_2}{V_1} - 1}{\dfrac{V_2}{V_1} + 1} \tag{13.9}$$

Table 13.1 shows how the ratio increases with increasing values of V_2/V_1. It is obvious that an orifice is very inefficient from the standpoint of energy conservation. Examples 13.2 and 13.3 illustrate how to make calculations when orifice meters are used.

TABLE 13.1 RELATIVE HEAD LOSS FOR ORIFICES

$V_2/V_1 \rightarrow$	1	2	4	6	8	10
$h_L/\Delta h \rightarrow$	0	0.33	0.60	0.71	0.78	0.82

EXAMPLE 13.3 ANALYSIS OF AN ORIFICE METER

A 15 cm orifice is located in a horizontal 24 cm water pipe, and a water-mercury manometer is connected to either side of the orifice. When the deflection on the manometer is 25 cm, what is the discharge in the system, and what head loss is produced by the orifice? Assume the water temperature is 20°C.

Problem Definition

Situation:

1. Water flows through an orifice $(d = 0.15\text{ m})$ in a pipe $(D = 0.24\text{ m})$.
2. A mercury-water manometer is used to measure pressure drop.

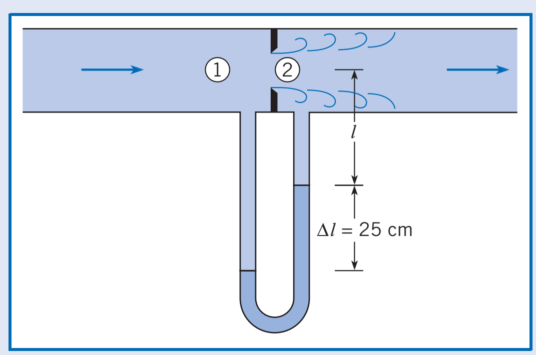
Find:

1. Discharge (in m^3/s) in pipe.
2. Head loss (in meters) produced by the orifice.

Properties:

1. Water (20°C), Table A.5: $\nu = 1 \times 10^{-6}\text{ m}^2/\text{s}$.
2. Mercury (20°C), Table A.4: $S = 13.6$.

Sketch:



Plan

1. Calculate $\Delta h = h_1 - h_2$ using the manometer equation (3.18).
2. Find the flow coefficient K using Fig. 13.14.
3. Find discharge Q using Eq. (13.7a).
4. Calculate the coefficient of contraction C_c using Eq. (13.6).

5. Solve for the velocity V_2 at the vena contracta.
6. Calculate head loss using Eq. (13.8).

Solution

1. Change in piezometric head
   - Apply manometer equation from 1 to 2.
$$p_1 + \gamma_w(l + \Delta l) - \gamma_{Hg}\Delta l - \gamma_w l = p_2$$
   - Solve for Δh.
$$\Delta h = \frac{p_1 - p_2}{\gamma_w} = \Delta l \frac{\gamma_{Hg} - \gamma_w}{\gamma_w} = \Delta l\left(\frac{\gamma_{Hg}}{\gamma_w} - 1\right)$$
$$\Delta h = (0.25 \text{ m})(13.6 - 1) = 3.15 \text{ m of water}$$
2. Flow coefficient
   - Calculate (Re_d/K).
$$\frac{\text{Re}_d}{K} = \frac{d\sqrt{2g\Delta h}}{\nu} = \frac{0.15 \text{ m}\sqrt{2(9.81 \text{ m/s}^2)(3.15 \text{ m})}}{1.0 \times 10^{-6} \text{ m}^2\text{/s}}$$
$$= 1.2 \times 10^6$$
   - From Fig. 13.14 with $d/D = 0.625$, $K = 0.66$ (interpolated).
3. Discharge
$$Q = 0.66 A_o \sqrt{2g\Delta h}$$
$$= 0.66\frac{\pi}{4}d^2\sqrt{2(9.81 \text{ m/s}^2)(3.15 \text{ m})}$$
$$= 0.66(0.785)(0.15^2 \text{ m}^2)(7.86 \text{ m/s}) = \boxed{0.092 \text{ m}^3\text{/s}}$$
4. Coefficient of contraction C_c
$$K = \frac{C_d}{1 - C_c^2 A_o^2/A_1^2}$$
Let $K = 0.66$. The ratio $(A_o/A_1)^2 = (0.625)^4 = 0.1526$ and $C_d = C_v C_c$. Assuming $C_v = 0.98$ (see the footnote on page 446) and solving for C_c, gives $C_c = 0.633$.
5. Velocity at the vena contracta
$$V_2 = Q/(C_c A_o)$$
$$(0.092 \text{ m}^3\text{/s})/[(0.633)(\pi/4)(0.15^2 \text{ m}^2)] = 8.23 \text{ m/s}$$
$$V_1 = Q/A_{\text{pipe}}$$
$$(0.092 \text{ m}^3\text{/s})/[(\pi/4)(0.24^2 \text{ m}^2)] = 2.03 \text{ m/s}$$
6. Head loss
$$h_L = (V_2 - V_1)^2/2g = (8.23 - 2.03)^2/(2 \times 9.81)$$
$$= \boxed{1.96 \text{ m}}$$

EXAMPLE 13.4 MANOMETER DEFLECTION FOR AN ORIFICE METER

An air-water manometer is connected to either side of an 8 in. orifice in a 12 in. water pipe. If the maximum flow rate is 2 cfs, what is the deflection on the manometer? The water temperature is 60°F.

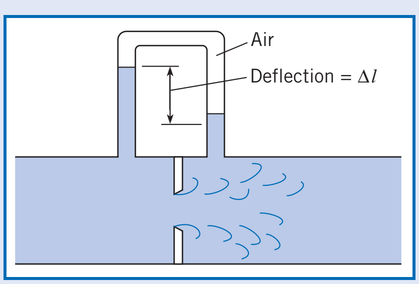
Problem Definition

Situation:

1. Water flows ($Q = 2$ cfs) through an orifice ($d = 8$ in.) in a pipe ($D = 12$ in.).
2. An air-water manometer is used to measure pressure drop.

Find: Deflection (in ft) of water in the manometer.

Sketch:



Properties: Water (60°F), Table A.5: $\nu = 1.22 \times 10^{-5}$ ft^2/s.

Plan

1. Calculate Reynolds number.
2. Find the flow coefficient K from Fig. 13.14.
3. Solve for Δh by using Eq. (13.7a).
4. Solve for Δl by using the manometer equation (3.19)

Solution

1. Reynolds number.
$$\text{Re} = \frac{4Q}{\pi d\nu} = \frac{(4)(2 \text{ ft}^3\text{/s})}{\pi((8/12) \text{ ft})(1.22 \times 10^{-5} \text{ ft}^2\text{/s})} = \boxed{3.1 \times 10^5}$$
2. Flow coefficient.
   - Use Fig. 13.14. Interpolate for $d/D = 8/12 = 0.667$ to find $K \approx 0.68$.

3. Change in piezometric head

- From $Q = KA_o\sqrt{2g\Delta h}$, solve for Δh:

$$\Delta h = \frac{Q^2}{2gK^2A_o^2} = \frac{4}{64.4(0.68^2)[((\pi)/4)(8/12)^2]^2} = 1.1 \text{ ft}$$

4. Manometer deflection

- The deflection is related to Δh by

$$\Delta h = \Delta l\left(\frac{\gamma_w - \gamma_{air}}{\gamma_w}\right)$$

- Since $\gamma_w \gg \gamma_{air}$, $\Delta l = \Delta h = 1.1$ ft. $\boxed{\Delta l = 1.1 \text{ ft}}$

The sharp-edged orifice can also be used to measure the mass flow rate of gases. The discharge equation [Eq. (13.7b)] is multiplied by the upstream gas density and an empirical factor to account for compressibility effects (10). The resulting equation is

$$\dot{m} = YA_oK\sqrt{2\rho_1(p_1 - p_2)} \tag{13.10}$$

where K, the flow coefficient, is found using Fig. 13.14 and Y is the compressibility factor given by the empirical equation

$$Y = 1 - \left\{\frac{1}{k}\left(1 - \frac{p_2}{p_1}\right)\left[0.41 + 0.35\left(\frac{A_o}{A_1}\right)^2\right]\right\} \tag{13.11}$$

In this case both the pressure difference across the orifice and the absolute pressure of the gas are needed. One must remember when using the equation for the compressibility factor that the absolute pressure must be used.

EXAMPLE 13.5 MASS FLOW RATE OF NATURAL GAS

The mass flow rate of natural gas is to be measured using a sharp-edged orifice. The upstream pressure of the gas is 101 kPa absolute, and the pressure difference across the orifice is 10 kPa. The upstream temperature of the methane is 15°C. The pipe diameter is 10 cm, and the orifice diameter is 7 cm. What is the mass flow rate?

Problem Definition

Situation:

1. Natural gas (methane) is flowing through a sharp-edged orifice.
2. Pipe diameter is $D = 0.1$ m. Orifice diameter is $d = 0.07$ m.
3. Pressure difference across orifice is 10 kPa.

Find: Mass flow rate (in kg/s).

Properties: Natural gas (15°C, 1 atm), Table A.2: $\rho = 0.678 \text{ kg/m}^3$, $\nu = 1.59 \times 10^{-5} \text{ m}^2/\text{s}$, $K = 1.31$.

Plan

1. Find the flow coefficient K from Fig. 13.14.
2. Calculate the compressibility factor Y using Eq. (13.11).
3. Calculate the mass flow rate using Eq. (13.10).

Solution

1. Flow coefficient
   - Calculate (Re_d/K):

$$\frac{\mathrm{Re}_d}{K} = \frac{d}{\nu}\sqrt{2\frac{\Delta p}{\rho_1}} = \frac{0.07}{1.59(10^{-5})}\sqrt{2\frac{10^4}{0.678}} = 7.56 \times 10^5$$

   - Using Fig. 13.14, $K = 0.7$.

2. Compressibility factor

$$Y = 1 - \left\{\frac{1}{1.31}\left(1 - \frac{91}{101}\right)(0.41 + 0.35 \times 0.7^4)\right\} = 0.962$$

3. Mass flow rate of methane

$$\dot{m} = YA_oK\sqrt{2\rho_1(p_1 - p_2)}$$

$$= 0.962\left(\frac{\pi}{4}0.07^2\right)(0.7)\sqrt{2(0.678)(10^4)}$$

$$= \boxed{0.302 \text{ kg/s}}$$

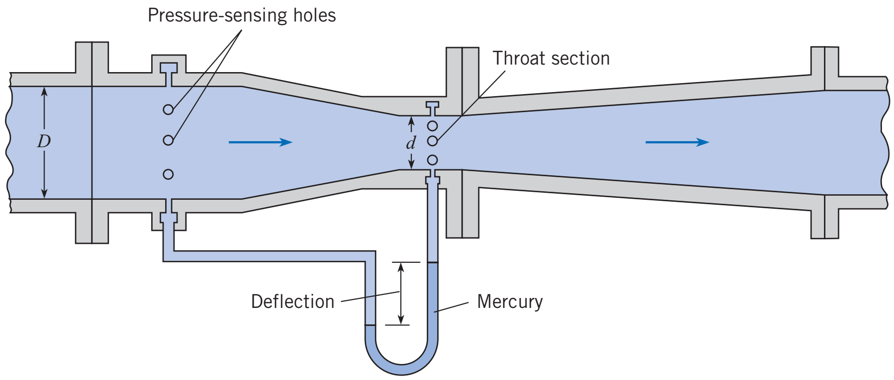


Figure 13.15

Typical venturi meter.

The foregoing examples involved the determination of either Q or Δh for a given size of orifice. Another type of problem is determination of the diameter of the orifice for a given Q and Δh. For this type of problem a trial-and-error procedure is required. Because one knows the approximate value of K, that is guessed first. Then the diameter is solved for, after which a better value of K can be determined, and so on.

Venturi Meter

The *venturi meter*, Fig. 13.15, is an instrument for measuring flow rate by using measurements of pressure across a converging-diverging flow passage. The main advantage of the venturi meter as compared to the orifice meter is that the head loss for a venturi meter is much smaller. The lower head loss results from streamlining the flow passage, as shown in Fig. 13.15. Such streamlining eliminates any jet contraction beyond the smallest flow section. Consequently, the coefficient of contraction has a value of unity, and the venturi equation is

$$Q = \frac{A_t C_d}{\sqrt{1-(A_t/A_p)^2}}\sqrt{2g(h_p - h_t)} \tag{13.12}$$

$$Q = KA_t\sqrt{2g\Delta h} \tag{13.13}$$

where A_t is the throat area and Δh is the difference in piezometric head between the venturi entrance (pipe) and the throat. Note that the venturi equation is the same as the orifice equation. However, K for the venturi meter approaches unity at high values of the Reynolds number and small d/D ratios. This trend can be seen in Fig. 13.14, where values of K for the venturi meter are plotted along with similar data for the orifice.

Flow Nozzles

The *flow nozzle,* Fig. 13.16, is an instrument for measuring flow rate by using the pressure drop across a nozzle that is typically placed inside a conduit. Similar to an orifice meter, design and application of the flow nozzle is described by engineering standards (10). As compared to an orifice meter, the flow nozzle is better in flows that cause wear (e.g., particle-laden flow). The reason is that erosion of an orifice will produce more change in the pressure-drop versus flow-rate relationship. Both the flow nozzle and orifice meter will produce about the same overall head loss.

EXAMPLE 13.6 FLOW RATE USING A VENTURI METER

The pressure difference between the taps of a horizontal venturi meter carrying water is 35 kPa. If $d = 20$ cm and $D = 40$ cm, what is the discharge of water at 10°C?

Problem Definition

Situation:

1. Water flows through a horizontal venturi meter.
2. Pipe diameter is $D = 0.40$ m. Venturi throat diameter is $d = 0.2$ m.

Find: Discharge (in $\mathrm{m^3/s}$).

Properties: Water (10°C), Table A.5: $\nu = 1.31 \times 10^{-6}\ \mathrm{m^2/s}$, and $\gamma = 9810\ \mathrm{N/m^3}$.

Plan

1. Compute $\Delta h = h_1 - h_2$.
2. Find the flow coefficient K from Fig. 13.14.
3. Find discharge Q using Eq. (13.7a).

Solution

1. Change in piezometric head

$$\Delta h = \frac{\Delta p}{\gamma} + \Delta z = \frac{\Delta p}{\gamma} + 0 = \frac{35{,}000\ \mathrm{N/m^2}}{9810\ \mathrm{N/m^3}} = 3.57\ \text{m of water}$$

2. Flow coefficient
   - Calculate (Re_d/K):

$$\frac{\mathrm{Re}_d}{K} = \frac{d\sqrt{2g\Delta h}}{\nu} = \frac{0.20\sqrt{2(9.81)(3.57)}}{1.31(10^{-6})} = 1.28 \times 10^6$$

   - From Fig. 13.14, find that $K = 1.02$.

3. Discharge

$$Q = 1.02 A_2 \sqrt{2g\Delta h}$$
$$= 1.02(0.785)(0.20^2)\sqrt{2(9.81)(3.57)} = \boxed{0.268\ \mathrm{m^3/s}}$$

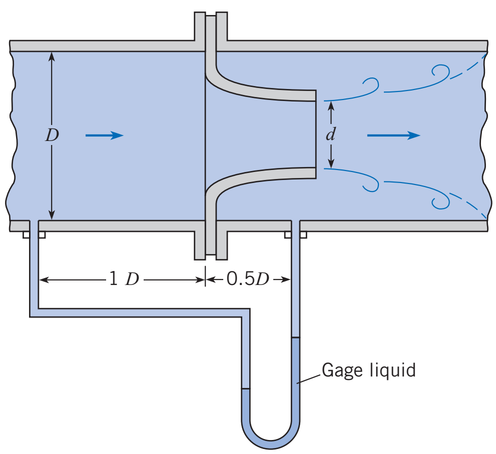


Figure 13.16

Typical flow nozzle.

Electromagnetic Flow Meter

All of the flow meters described so far require that some sort of obstruction be placed in the flow. The obstruction may be the rotor of a vane anemometer or the reduced cross-section of an orifice or venturi meter. A meter that neither obstructs the flow nor requires pressure taps, which are subject to clogging, is the *electromagnetic flow meter*. Its basic principle is that a conductor that moves in a magnetic field produces an electromotive force. Hence liquids having a degree of conductivity generate a voltage between the electrodes, as in Fig. 13.17, and this voltage is proportional to the velocity of flow in the conduit. It is interesting to note that the basic principle of the electromagnetic flow meter was investigated by Faraday in 1832. However, practical application of the principle was not made until approximately a century later, when it was used to measure blood flow. Recently, with the need for a meter to

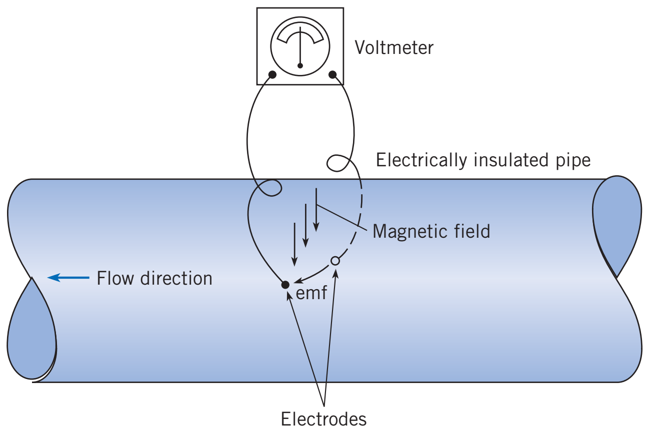


Figure 13.17

Electromagnetic flow meter.

measure the flow of liquid metal in nuclear reactors and with the advent of sophisticated electronic signal detection, this type of meter has found extensive commercial use.

The main advantages of the electromagnetic flow meter are that the output signal varies linearly with the flow rate and that the meter causes no resistance to the flow. The major disadvantages are its high cost and its unsuitability for measuring gas flow.

For a summary of the theory and application of the electromagnetic flow meter, the reader is referred to Shercliff (11). This reference also includes a comprehensive bibliography on the subject.

Ultrasonic Flow Meter

Another form of nonintrusive flow meter that is used in diverse applications ranging from blood flow measurement to open-channel flow is the *ultrasonic flow meter*. Basically, there are two different modes of operation for ultrasonic flow meters. One mode involves measuring the difference in travel time for a sound wave traveling upstream and downstream between two measuring stations. The difference in travel time is proportional to flow velocity. The second mode of operation is based on the Doppler effect. When an ultrasonic beam is projected into an inhomogeneous fluid, some acoustic energy is scattered back to the transmitter at a different frequency (Doppler shift). The measured frequency difference is related directly to the flow velocity.

Turbine Flow Meter

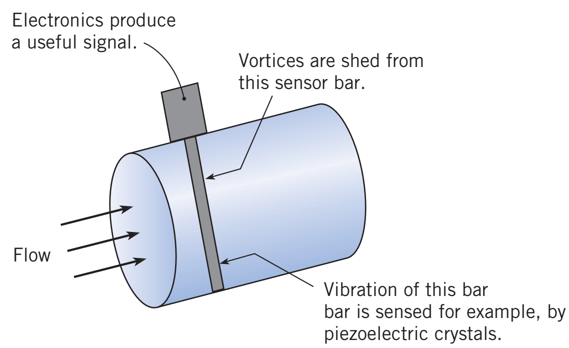
The *turbine flow meter* consists of a wheel with a set of curved vanes (blades) mounted inside a duct. The volume rate of flow through the meter is related to the rotational speed of the wheel. This rotational rate is generally measured by a blade passing an electromagnetic pickup mounted in the casing. The meter must be calibrated for the given flow conditions. The turbine meter is versatile in that it can be used for either liquids or gases. It has an accuracy of better than 1% over a wide range of flow rates, and it operates with small head loss. The turbine flow meter is used extensively in monitoring flow rates in fuel-supply systems.

Vortex Flow Meter

The *vortex flow meter*, shown in Fig. 13.18, measures flow rate by relating vortex shedding frequency to flow rate. The vortices are shed from a sensor tube that is situated in the center of a pipe. These vortices cause vibrations, which are sensed by piezoelectric crystals that are located inside the sensor tube, and are converted to an electronic signal that is directly proportional to flow rate.

Figure 13.18

Vortex flow meter.



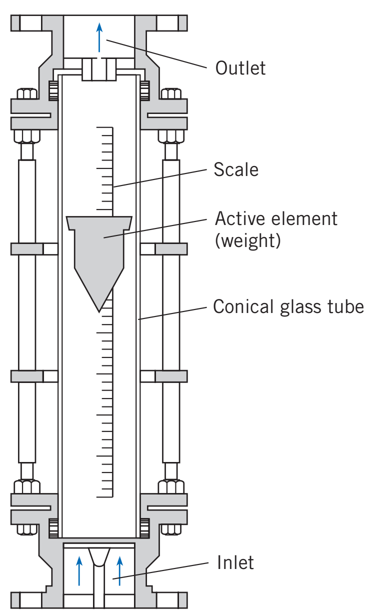
This vortex meter gives accurate and repeatable measurements with no moving parts. However, the corresponding head loss is comparable to that from other obstruction-type meters.

Rotameter

The *rotameter*, Fig. 13.19, is an instrument for measuring flow rate by sensing the position of an active element (weight) that is situated in a tapered tube. The equilibrium position of the weight is related to the flow rate. Because the velocity is lower at the top of the tube (greater flow section there) than at the bottom, the rotor seeks a neutral position where the drag on it just balances its weight. Thus the rotor "rides" higher or lower in the tube depending on the rate of flow. The weight is designed so that it spins, thus it stays in the center of the tube. A calibrated scale on the side of the tube indicates the rate of flow. Although venturi and orifice meters have better accuracy (approximately 1% of full scale) than the rotameter (approximately 5% of full scale), the rotameter offers other advantages, such as ease of use and low cost.

Figure 13.19

Rotameter.



Rectangular Weir

A *weir*, shown in Fig. 13.20, is an instrument for determining flow rate in liquids by measuring the height of water relative to an obstruction in an open channel. The discharge over the weir is a function of the weir geometry and of the head on the weir. Consider flow over the weir in a rectangular channel, shown in Fig. 13.20. The head H on the weir is defined as the vertical distance between the weir crest and the liquid surface taken far enough upstream of the weir to avoid local free-surface curvature (see Fig. 13.20).

The discharge equation for the weir is derived by integrating $V\,dA = VL\,dh$ over the total head on the weir. Here L is the length of the weir and V is the velocity at any given distance h below the free surface. Neglecting streamline curvature and assuming negligible velocity of approach upstream of the weir, one obtains an expression for V by writing the Bernoulli equation between a point upstream of the weir and a point in the plane of the weir (see Fig. 13.21). Assuming the pressure in the plane of the weir is atmospheric, this equation is

$$\frac{p_1}{\gamma} + H = (H - h) + \frac{V^2}{2g} \tag{13.14}$$

Figure 13.20

Definition sketch for sharp-crested weir. (a) Plan view. (b) Elevation view.

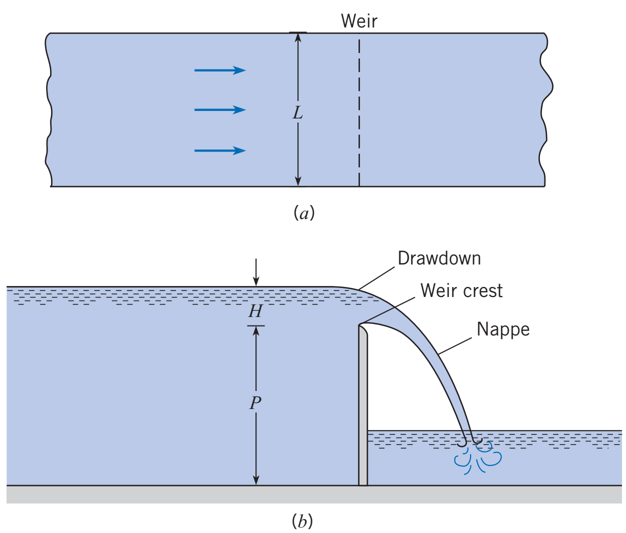
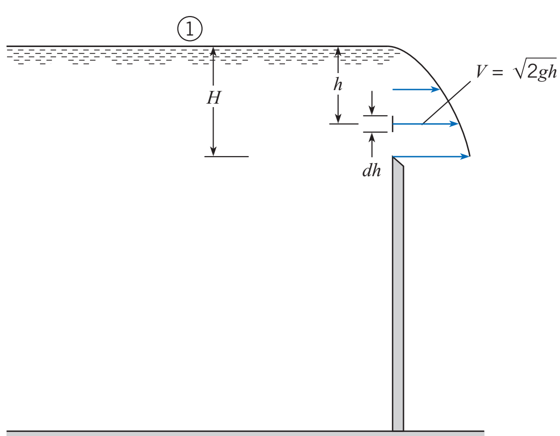


Figure 13.21

Theoretical velocity distribution over a weir.



Here the reference elevation is the elevation of the crest of the weir, and the reference pressure is atmospheric pressure. Therefore $p_1 = 0$, and Eq. (13.14) reduces to

$$V = \sqrt{2gh}$$

Then $dQ = \sqrt{2gh}\, Ldh$, and the discharge equation becomes

$$\begin{aligned} Q &= \int_0^H \sqrt{2gh}\, Ldh \\ &= \tfrac{2}{3} L\sqrt{2g} H^{3/2} \end{aligned} \tag{13.15}$$

In the case of actual flow over a weir, the streamlines converge downstream of the plane of the weir, and viscous effects are not entirely absent. Consequently, a discharge coefficient C_d must be applied to the basic expression on the right-hand side of Eq. (13.15) to bring the theory in line with the actual flow rate. Thus the rectangular weir equation is

$$\begin{aligned} Q &= \tfrac{2}{3} C_d \sqrt{2g} L H^{3/2} \\ &= K\sqrt{2g} L H^{3/2} \end{aligned} \tag{13.16}$$

For low-viscosity liquids, the flow coefficient K is primarily a function of the relative head on the weir, H/P. An empirically determined equation for K adapted from Kindsvater and Carter (12) is

$$K = 0.40 + 0.05\frac{H}{P} \tag{13.17}$$

This is valid up to an H/P value of 10 as long as the weir is well ventilated so that atmospheric pressure prevails on both the top and the bottom of the weir nappe.

When the rectangular weir does not extend the entire distance across the channel, as in Fig. 13.22, additional end contractions occur. Therefore, K will be smaller than for the weir without end contractions. The reader is referred to King (13) for additional information on flow coefficients for weirs.

EXAMPLE 13.7 FLOW RATE FOR A RECTANGULAR WEIR

The head on a rectangular weir that is 60 cm high in a rectangular channel that is 1.3 m wide is measured to be 21 cm. What is the discharge of water over the weir?

Problem Definition

Situation:

1. Water flows over a rectangular weir.
2. The weir has a height of $P = 0.6$ m and a width of $L = 1.3$ m.
3. Head on the weir is $H = 0.21$ m.

Find: Discharge (in m^3/s).

Plan

1. Calculate the flow coefficient K using Eq. (13.17).
2. Calculate flow rate using the rectangular weir equation (13.16).

Solution

1. Flow coefficient

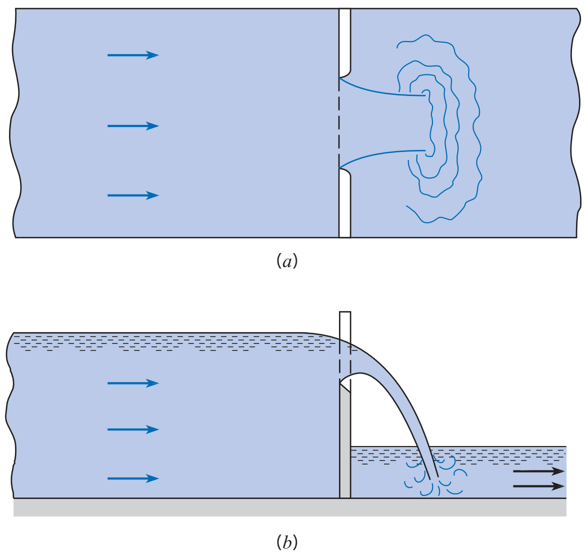
$$K = 0.40 + 0.05\frac{H}{P} = 0.40 + 0.05\left(\frac{21}{60}\right) = 0.417$$

2. Discharge

$$\begin{aligned} Q &= K\sqrt{2g} L H^{3/2} = 0.417\sqrt{2(9.81)}(1.3)(0.21^{3/2}) \\ &= 0.23 \text{ m}^3/\text{s} \end{aligned}$$

Figure 13.22

Rectangular weir with end contractions.
(a) Plan view.
(b) Elevation view.



Triangular Weir

A definition sketch for the triangular weir is shown in Fig. 13.23. The primary advantage of the triangular weir is that it has a higher degree of accuracy over a much wider range of flow than does the rectangular weir, because the average width of the flow section increases as the head increases.

The discharge equation for the triangular weir is derived in the same manner as that for the rectangular weir. The differential discharge $dQ = V\,dA = VL\,dh$ is integrated over the total head on the weir to give

$$Q = \int_0^H \sqrt{2gh}\,(H-h)2\tan\left(\frac{\theta}{2}\right)dh$$

which integrates to

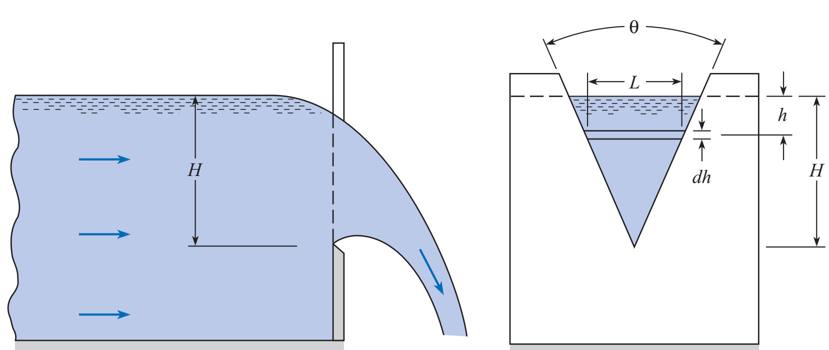
$$Q = \frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{5/2}$$

However, a coefficient of discharge must still be used with the basic equation. Hence

$$Q = \frac{8}{15}C_d\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{5/2} \tag{13.18}$$

Figure 13.23

Definition sketch for the triangular weir.



Experimental results with water flow over weirs with $\theta = 60°$ and $H > 2$ cm indicate that C_d has a value of 0.58. Hence the triangular weir equation with these limitations is

$$Q = 0.179\sqrt{2g}H^{5/2} \tag{13.19}$$

More details about flow-measuring devices for incompressible flow can be found in references (14) and (15).

EXAMPLE 13.8 FLOW RATE FOR A TRIANGULAR WEIR

The head on a 60° triangular weir is measured to be 43 cm. What is the flow of water over the weir?

Problem Definition

Situation:

1. Water flows over a 60° triangular weir.
2. Head on the weir is $H = 0.43$ m.

Find: Discharge (in m^3/s).

Plan

Apply the triangular weir equation Eq. (13.19).

Solution

$$Q = 0.179\sqrt{2g}H^{5/2} = 0.179 \times \sqrt{2 \times 9.81} \times (0.43)^{5/2}$$
$$= \boxed{0.096 \text{ m}^3/\text{s}}$$

13.3

Measurement in Compressible Flow

This section describes how to measure velocity, pressure, and flow rate in compressible flows. Since fluid density is changing in these flows, the Bernoulli equation is invalid. Thus, compressible flow theory from Chapter 12 will be applied to develop valid measurement techniques.

Pressure Measurements

Static-pressure measurements can be made using the conventional static-pressure taps of a probe. However, if the boundary layer is disturbed by the presence of a shock wave in the vicinity of the pressure tap, the reading may not give the correct static pressure. The effect of the shock wave on the boundary layer is smaller if the boundary layer is turbulent. Therefore an effort is sometimes made to trip the boundary layer and ensure a turbulent boundary layer in the region of the pressure tap.

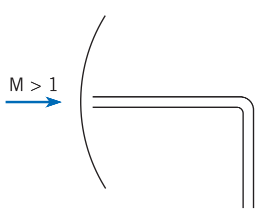


Figure 13.24

Stagnation tube in supersonic flow.

The stagnation pressure can be measured with a stagnation tube aligned with the local velocity vector. If the flow is supersonic, however, a shock wave forms around the tip of the probe, as shown in Fig. 13.24, and the stagnation pressure measured is that downstream of the shock wave and not that of the free stream. The stagnation pressure in the free stream can be calculated using the normal shock relationships, provided the free-stream Mach number is known. See Chapter 12 for more details about normal shock waves.

Mach Number and Velocity Measurements

A Pitot-static tube can be used to measure Mach numbers in compressible flows. Taking the measured stagnation pressure as the total pressure, one can calculate the Mach number in subsonic flows from the total-to-static-pressure ratio according to Eq. (12.31):

$$\mathrm{M} = \left\{ \frac{2}{k-1} \left[\left(\frac{p_t}{p} \right)^{(k-1)/k} - 1 \right] \right\}^{1/2}$$

It is interesting to note here that one must measure the stagnation and static pressures separately to determine the pressure ratio, whereas one needs only the pressure difference to calculate the velocity of a flow.

If the flow is supersonic, then the indicated stagnation pressure is the pressure behind the shock wave standing off the tip of the tube. By taking this pressure as the total pressure downstream of a normal shock wave and the measured static pressure as the static pressure upstream of the shock wave, one can determine the Mach number of the free stream (M_1) from the static-to-total-pressure ratio (p_1/p_{t_2}) according to the expression

$$\frac{p_1}{p_{t_2}} = \frac{\{[2k/(k+1)]\mathrm{M}_1^2 - [(k-1)/(k+1)]\}^{1/(k-1)}}{\{[(k+1)/2]\mathrm{M}_1^2\}^{k/(k-1)}} \tag{13.20}$$

which is called the Rayleigh supersonic Pitot formula. Note, however, that M_1 is an implicit function of the pressure ratio and must be determined graphically or by some numerical procedure. Many normal-shock tables, such as those in reference (16), have p_1/p_{t_2} tabulated versus M_1, which enables one to find M_1 quite easily by interpolation.

Once the Mach number is determined, more information is needed to evaluate the velocity—namely, the local speed of sound. This can be done by inserting a probe into the flow to measure total temperature and then calculating the static temperature using Eq. (12.22):

$$T = \frac{T_t}{1 + [(k-1)/2]\mathrm{M}_1^2}$$

The local speed of sound is then determined by Eq. (12.11):

$$c = \sqrt{kRT}$$

and the velocity is calculated from

$$V = \mathrm{M}_1 c$$

The hot-wire anemometer can also be used to measure velocity in compressible flows, provided it is calibrated to account for Mach-number effects.

Mass Flow Measurement

Measuring the flow rate of a compressible fluid using a truncated nozzle was discussed in some detail in Chapter 12. Basically, the flow nozzle is a truncated nozzle located in a pipe, so the equations developed in Chapter 12 can be used to determine the flow rate through the flow nozzle. Strictly speaking, the flow rate so calculated should be multiplied by the discharge coefficient. For the high Reynolds numbers characteristic of compressible flows, however, the discharge coefficient can be taken as unity. If the flow at the throat of the flow nozzle is sonic (i.e., Mach number at the throat is 1.0), it is conceivable that the complex flow field existing downstream of the nozzle will make the reading from the downstream pressure tap difficult to interpret. That is, there can be no assurance that the measured pressure is the true back pressure. In such a case, it is advisable to use a venturi meter because the pressure is measured directly at the throat.

The mass flow rate of a compressible fluid through a venturi meter can easily be analyzed using the equations developed in Chapter 12. Consider the venturi meter shown in Fig. 13.25. Writing the energy equation, Eq. (12.15), for the flow of an ideal gas between stations 1 and 2 gives

$$\frac{V_1^2}{2} + \frac{kRT_1}{k-1} = \frac{V_2^2}{2} + \frac{kRT_2}{k-1} \tag{13.21}$$

By conservation of mass, the velocity V_1 can be expressed as

$$V_1 = \frac{\rho_2 A_2 V_2}{\rho_1 A_1}$$

Substituting this result into Eq. (13.21), using the ideal-gas law to eliminate temperature, and solving for V_2 gives

$$V_2 = \left\{ \frac{[2k/(k-1)][(p_1/\rho_1) - (p_2/\rho_2)]}{1 - (\rho_2 A_2/\rho_1 A_1)^2} \right\}^{1/2} \tag{13.22}$$

Assuming that the flow is isentropic,

$$\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^k$$

the equation for the velocity at the throat can be rewritten as

$$V_2 = \left\{ \frac{[2k/(k-1)](p_1/\rho_1)[1 - (p_2/p_1)^{(k-1)/k}]}{1 - (p_2/p_1)^{2/k}(D_2/D_1)^4} \right\}^{1/2} \tag{13.23}$$

The mass flow is obtained by multiplying V_2 by $\rho_2 A_2$. This analysis, however, has been based on a one-dimensional flow, and two-dimensional effects can be accounted for by the discharge coefficient C_d. The final result is

$$\dot{m} = C_d \rho_2 A_2 V_2 = C_d A_2 \left(\frac{p_2}{p_1}\right)^{1/k} \left\{ \frac{[2k/(k-1)]p_1 \rho_1 [1 - (p_2/p_1)^{(k-1)/k}]}{1 - (p_2/p_1)^{2/k}(D_2/D_1)^4} \right\}^{1/2} \tag{13.24}$$

This equation is valid for all flow conditions, subsonic or supersonic, provided no shock waves occur between station 1 and station 2. It is good design practice to avoid supersonic flows in the venturi meter in order to prevent the formation of shock waves and the attendant total pressure losses. Also, the discharge coefficient can generally be taken as unity if no shock waves occur between 1 and 2.

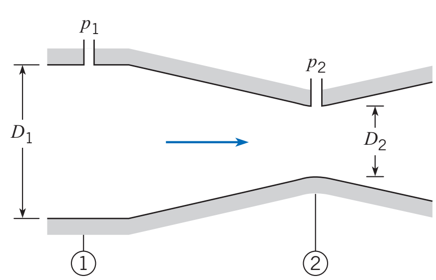


Figure 13.25

Venturi meter.

EXAMPLE 13.9 FLOW RATE FOR AIR THROUGH A VENTURI METER (COMPRESSIBLE FLOW)

Calculate the mass flow rate of air (inlet static temperature = 27°C) flowing through a venturi meter. The venturi throat is 1 cm in diameter (D_2), and the pipe is 3 cm in diameter (D_1). Upstream static pressure is 150 kPa, and throat pressure is 100 kPa.

Problem Definition

Situation:

1. Air flows through a venturi meter.
2. Pipe diameter is $D = 0.03$ m. Venturi throat diameter is $d = 0.01$ m.
3. Upstream conditions: Static temperature is 27°C; static pressure is 150 kPa.
4. Pressure in throat = 100 kPa.

Find: Mass flow rate (in kg/s).

Properties: Air (27°C), Table A.2: $k = 1.4$, and $R = 287\ \text{J/kg}\cdot\text{K}$.

Plan

1. Calculate density of air in the pipe (upstream) using the ideal gas law.
2. Calculate mass flow rate using Eq. (13.24).

Solution

1. Ideal gas law

$$\rho_1 = \frac{p_1}{RT_1} = \frac{150\times 10^3\ \text{N/m}^2}{(287\ \text{J/kg K})(300\ \text{K})} = 1.74\ \text{kg/m}^3$$

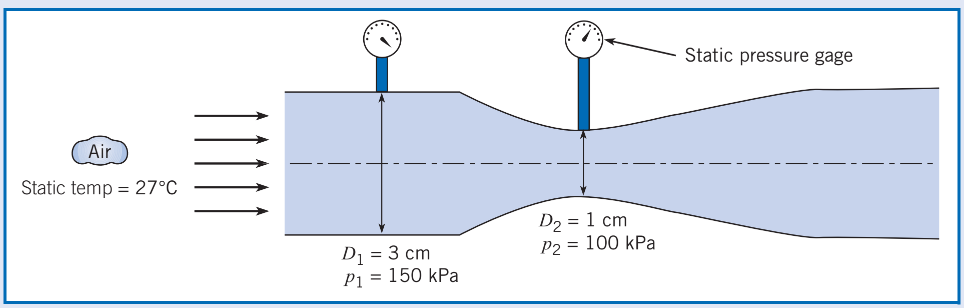
2. Mass flow rate

$$\dot{m} = 1\times 0.785\times 10^{-4}\ \text{m}^2\left(\frac{1}{1.5}\right)^{0.714}$$

$$\times\left\{\frac{7\times 150\times 10^3\ \text{N/m}^2\times 1.74\ \text{kg/m}^3[1-(1/1.5)^{0.286}]}{[1-(1/1.5)^{1.43}(1/3)^4]}\right\}^{1/2}$$

$$= \boxed{0.0264\ \text{kg/s}}$$

Sketch:



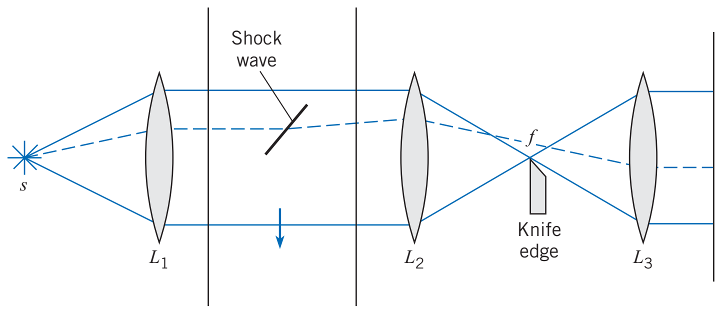
Shock Wave Visualization

When studying supersonic flow in a wind tunnel, it is important to be able to locate and identify the shock wave pattern. Unfortunately, shock waves cannot be seen with the naked eye, so the application of some type of optical technique is necessary. There are three techniques by which shock waves can be seen: the shadowgraph, the interferometer, and the schlieren system. Each technique has its special application related to the type of information on density variation that is desired. The schlieren technique, however, finds frequent use in shock wave visualization.

An illustration of the essential features of the schlieren system is given in Fig. 13.26. Light from the source s is collimated by lens L_1 to produce a parallel light beam. The light then passes through a second lens L_2 and produces an image of the source at plane f. A third lens L_3 focuses the image on the display screen. A sharp edge, usually called the knife edge, is positioned at plane f so as to block out a portion of the light.

Figure 13.26

Schlieren system.



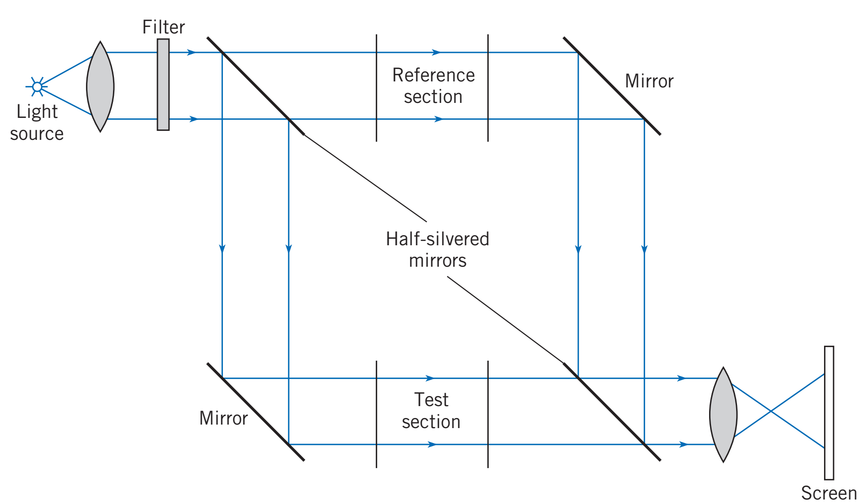
If a shock wave occurs in the test section, the light is refracted by the density change across the wave. As illustrated by the dashed line in Fig. 13.26, the refracted ray escapes the blocking effect of the knife edge, and the shock wave appears as a lighter region on the screen. Of course, if the beam is refracted in the other direction, the knife edge blocks out more light, and the shock wave appears as a darker region. The contrast can be increased by intercepting more light with the knife edge.

Interferometry

The interferometer allows one to map contours of constant density and to measure the density changes in the flow field. The underlying principle is the phase shift of a light beam on passing through media of different densities. The system now employed almost universally is the Mach-Zender interferometer, shown in Fig. 13.27. Light from a common source is split into two beams as it passes through the first half-silvered mirror. One beam passes through the test section, the other through the reference section. The two beams are then recombined and projected onto a screen or photographic plate. If the density in the test section and that in the reference section are the same, there is no phase shift between the two beams, and the screen is uniformly bright. However, a change of density in the test section changes the light speed of the test section beam, and a phase shift is generated between the two beams. Upon recombination of the beams, this phase shift gives rise to a series of dark and light bands on the screen. Each band represents a uniform-density contour, and the change in density across each band can be determined for a given system.

Figure 13.27

Schematic diagram of a Mach-Zender interferometer.



13.4

Accuracy of Measurements

When a parameter is measured, it is important to assess the accuracy of the measurement. The resulting analysis, called an *uncertainty analysis,* provides an estimate of the upper and lower bounds of the parameter. For example, if Q is a measured value of discharge, uncertainty analysis provides an estimate of the uncertainty U_Q in this measurement. The measurement would then be reported as $Q \pm U_Q$.

Commonly, a parameter of interest is not directly measured but is calculated from other variables. For example, discharge for an orifice meter is calculated using Eq. (13.7a). Such an equation is called a data reduction equation. Consider a data reduction equation of the form

$$x = f(y_1, y_2, \ldots, y_n)$$

where x is the parameter of interest and y_1 through y_n are the independent variables. Then, the uncertainty in x, which is written as U_x, is given by

$$U_x = \left[\left(\frac{\partial x}{\partial y_1}U_{y_1}\right)^2 + \left(\frac{\partial x}{\partial y_2}U_{y_2}\right)^2 + \cdots + \left(\frac{\partial x}{\partial y_n}U_{y_n}\right)^2\right]^{0.5} \tag{13.25}$$

where U_{y_i} is the uncertainty in variable y_i. Equation (13.25), known as the uncertainty equation, is very useful for quantifying the accuracy of an experimental measurement, and for planning experiments. Additional information about uncertainty analysis is provided by Coleman and Steele (17).

EXAMPLE 13.10 UNCERTAINTY ESTIMATE FOR AN ORIFICE METER

For the orifice meter described in Example 13.2, estimate the uncertainty of the calculated discharge. Assume that uncertainty in K is 0.03, the uncertainty in diameter is 0.15 mm, and the uncertainty in measured head is 10 mm-Hg.

Problem Definition

Situation:

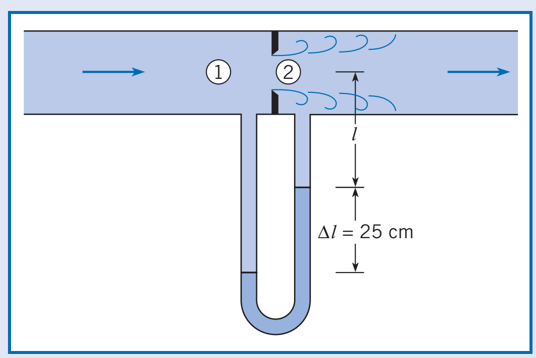
1. Water flows through an orifice ($d = 0.15$ m) in a pipe ($D = 0.24$ m).
2. A mercury-water manometer is used to measure pressure drop.

Find: Uncertainty (in m^3/s) for the calculated discharge Q.

Plan

1. Identify the data reduction equation (DRE).
2. Within the DRE, identify each variable that contributes to uncertainty.
3. Develop an equation for uncertainty by applying Eq. (13.25).
4. Calculate uncertainty by using the equation developed in step 3.

Sketch:



Solution

1. The data reduction equation is the orifice equation, Eq. (13.7a).

$$Q = K(\pi d^2/4)\sqrt{2g\Delta h}$$

2. Variables that cause uncertainty are K, d, g, and h. Neglect the influence of g.
3. Derive an equation for the uncertainty

$$U_Q^2 = \left(\frac{\partial Q}{\partial K}U_K\right)^2 + \left(\frac{\partial Q}{\partial d}U_d\right)^2 + \left(\frac{\partial Q}{\partial h}U_h\right)^2$$

Evaluate each partial derivative and then divide both sides of this equation by Q^2:

$$\left(\frac{U_Q}{Q}\right)^2 = \left(\frac{U_K}{K}\right)^2 + \left(\frac{2U_d}{d}\right)^2 + \left(\frac{U_h}{2h}\right)^2$$

4. Substitute values from Example 13.2:

$$\left(\frac{U_Q}{Q}\right)^2 = \left(\frac{0.03}{0.66}\right)^2 + \left(\frac{2 \times 0.15}{150}\right)^2 + \left(\frac{10}{2 \times 250}\right)^2$$

$$\left(\frac{U_Q}{Q}\right)^2 = (20.7 \times 10^{-4}) + (0.04 \times 10^{-4}) + (4 \times 10^{-4})$$

$$U_Q = 0.0497Q = 0.0497 \times (0.092 \text{ m}^3/\text{s})$$

$$= 0.0046 \text{ m}^3/\text{s}$$

Thus

$$\boxed{Q = (0.092 \pm 0.046) \text{ m}^3/\text{s}}$$

Review

The primary source of uncertainty in the discharge is due to U_K. The term U_h has a small effect, and U_d has a negligible effect.

13.5

Summary

There are many methods and instruments for measuring velocity, pressure, and flow rate:

- For velocity measurement: stagnation tube, Pitot tube, yaw meter, vane and cup anemometers, hot-wire and hot-film anemometers, laser-Doppler anemometer, and particle image velocimetry
- For pressure measurement: static tube, piezometer, differential manometer, Bourdon-tube gage, and several types of pressure transducers
- For flow rate measurement: direct volume or weight measurement, velocity-area integration, orifice meter, flow nozzle, venturi meter, electromagnetic flow meter, ultrasonic flow meter, turbine flow meter, vortex flow meter, rotameter, and weir

Flow rate or discharge for a flow meter that uses a restricted opening (i.e., an orifice, flow nozzle, or venturi) is calculated using

$$Q = KA_o\sqrt{2g\Delta h} = KA_o\sqrt{2\Delta p_z/\rho}$$

where K is a flow coefficient that depends on Reynolds number and the type of flow meter, A_o is the area of the opening, Δh is the change in piezometric head across the flowmeter, and Δp_z is drop in piezometric pressure across the flowmeter.

Discharge for a rectangular weir of length L is given by

$$Q = K\sqrt{2g}LH^{3/2}$$

where K is the flow coefficient that depends on H/P. The term H is the height of the water above the crest of the weir, as measured upstream of the weir, and P is the height of the weir. Discharge for a 60° triangular weir with $H > 2$ cm is given by

$$Q = 0.179\sqrt{2g}H^{5/2}$$

When flow is compressible, instruments such as the stagnation tube, hot-wire anemometer, Pitot tube, and flow nozzle may be used. However, equations correlating velocity and discharge need to be altered to account for the effects of compressibility. To observe shock waves in compressible flow, a schlieren technique or an interferometer may be used.

Uncertainty analysis provides a way to quantify the accuracy of a measurement. When a parameter of interest x is evaluated using an equation of the form $x = f(y_1, y_2, \ldots, y_n)$, where y_1 through y_n are the independent variables, the uncertainty in x is given by

$$U_x = \left[\left(\frac{\partial x}{\partial y_1}U_{y_1}\right)^2 + \left(\frac{\partial x}{\partial y_2}U_{y_2}\right)^2 + \cdots + \left(\frac{\partial x}{\partial y_n}U_{y_n}\right)^2\right]^{0.5}$$

where U_{y_i} is the uncertainty in variable y_i. This equation, known as the uncertainty equation, is very useful for estimating uncertainty and for planning experiments.

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Problems

Overview

13.1 ℙℚ◀ List five different instruments or approaches that engineers use to measure fluid velocity. For each instrument or approach, list two advantages and two disadvantages.

13.2 ℙℚ◀ List five different instruments or approaches that engineers use to measure flow rate (discharge). For each instrument or approach, list two advantages and two disadvantages.

Stagnation Tubes

13.3 ℙℚ◀ Consider measuring the speed of automobile by building a stagnation tube from a drinking straw and then using this device with a water-filled *U*-tube manometer.

a. Make a sketch that illustrates how you would propose making this measurement.

b. Determine the lowest velocity that could be measured. Assume that the lower limit is based on the resolution of the manometer.

13.4 Without exceeding an error of 2.5%, what is the minimum air velocity that can be obtained using a 1 mm circular stagnation tube if the formula.

$$V = \sqrt{2\Delta p_{stag}/\rho} = \sqrt{2gh_{stag}}$$

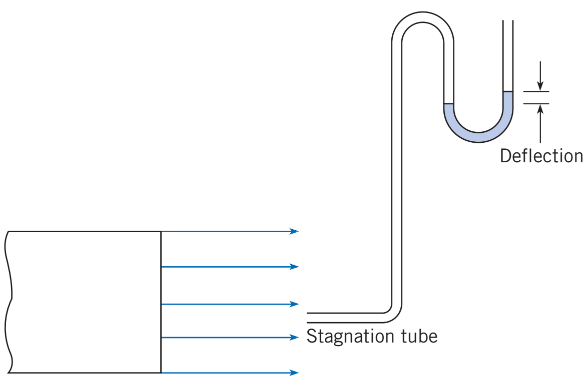
is used for computing the velocity? Assume standard atmospheric conditions.

13.5 Without exceeding an error of 1%, what is the minimum water velocity that can be obtained using a 1.5 mm circular stagnation tube if the formula

$$V = \sqrt{2\Delta p_{stag}/\rho} = \sqrt{2gh_{stag}}$$

is used for computing the velocity? Assume the water temperature is 20°C.

13.6 A stagnation tube 2 mm in diameter is used to measure the velocity in a stream of air as shown. What is the air velocity if the deflection on the air-water manometer is 1.0 mm? Air temperature = 10°C, and p = 1 atm.

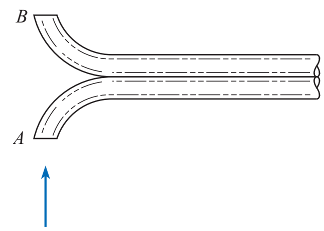


PROBLEMS 13.6, 13.7

13.7 If the velocity in an airstream (p_a = 98 kPa; T = 10°C) is 12 m/s, what deflection will be produced in an air-water manometer if the stagnation tube is 2 mm in diameter?

13.8 What would be the error in velocity determination if one used a C_p value of 1.00 for a circular stagnation tube instead of the true value? Assume the measurement is made with a stagnation tube 2 mm in diameter that is measuring air (T = 25°C, p = 1 atm) velocity for which the stagnation pressure reading is 5.00 Pa.

13.9 A velocity-measuring probe used frequently for measuring stack gas velocities is shown. The probe consists of two tubes bent away from and toward the flow direction and cut off on a plane normal to the flow direction, as shown. Assume the pressure coefficient is 1.0 at A and –0.4 at B. The probe is inserted in a stack where the temperature is 300°C and the pressure is 100 kPa absolute. The gas constant of the stack gases is 410 J/kg K. The probe is connected to a water manometer, and a 1.0 cm deflection is measured. Calculate the stack gas velocity.



PROBLEM 13.9

Volume Flow Rate (Discharge)

13.10 Water from a pipe is diverted into a tank for 4 min. If the weight of diverted water is measured to be 10 kN, what is the discharge in cubic meters per second? Assume the water temperature is 20°C.

13.11 Water from a test apparatus is diverted into a calibrated volumetric tank for 5 min. If the volume of diverted water is measured to be 80 m^3, what is the discharge in cubic meters per second, gallons per minute, and cubic feet per second?

13.12 A velocity traverse in a 24 cm oil pipe yields the data in the table. What are the discharge, mean velocity, and ratio of maximum to mean velocity? Does the flow appear to be laminar or turbulent?

r (cm)	*V* (m/s)	*r* (cm)	*V* (m/s)
0	8.7	7	5.8
1	8.6	8	4.9
2	8.4	9	3.8
3	8.2	10	2.5
4	7.7	10.5	1.9
5	7.2	11.0	1.4
6	6.5	11.5	0.7

13.13 A velocity traverse inside a 16 in. circular air duct yields the data in the table. What is the rate of flow in cubic feet per second and cubic feet per minute? What is the ratio of V_{max} to V_{mean}? Does it appear that the flow is laminar or turbulent? If p = 14.3 psia and T = 70°F, what is the mass flow rate?

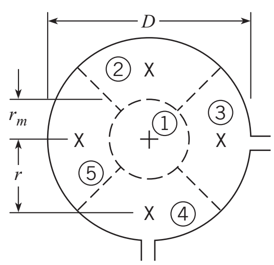
*y**	*V* (ft/s)	*y**	*V* (ft/s)
0.0	0	2.0	110
0.1	72	3.0	117
0.2	79	4.0	122
0.4	88	5.0	126
0.6	93	6.0	129
1.0	100	7.0	132
1.5	106	8.0	135

*Distance from pipe wall, in.

13.14 The asymmetry of the flow in stacks means that flow velocity must be measured at several locations on the cross-flow plane. Consider the cross section of the cylindrical stack shown. The two access holes through which probes can be inserted are separated by 90°. Velocities can be measured at the five points shown (five-point method).

a. Determine the ratio r_m/D such that the areas of the five measuring segments are equal.
b. Determine the ratio r/D (probe location) that corresponds to the centroid of the segment.
c. The data in the table are taken for a stack 2 m in diameter in which the gas temperature is 300°C, the pressure is 110 kPa absolute, and the gas constant is 400 J/kg K. The data represent the deflection on a water manometer connected to a Pitot-static tube located at the measuring stations. Calculate the mass flow rate.

Station	Δh (cm)
1	1.2
2	1.1
3	1.1
4	0.9
5	1.05

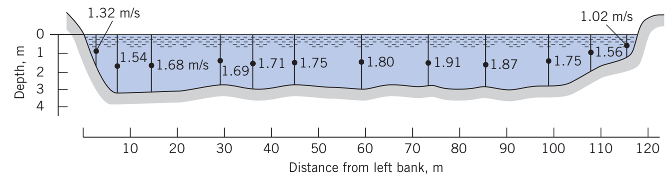


PROBLEM 13.14

13.15 Repeat Prob. 13.14 for the case in which three access holes are separated by 60° and seven measuring points are used. The diameter of the stack is 1.5 m, the gas temperature is 250°C, the pressure is 115 kPa absolute, and the gas constant is 420 J/kg K. The data in the following table represent the deflection of a water manometer connected to a Pitot-static tube at the measuring stations. Calculate the mass flow rate.

Station	Δh (mm)
1	8.2
2	8.6
3	8.2
4	8.9
5	8.0
6	8.5
7	8.4

13.16 Theory and experimental verification indicate that the mean velocity along a vertical line in a wide stream is closely approximated by the velocity at 0.6 depth. If the indicated velocities at 0.6 depth in a river cross section are measured, what is the discharge in the river?



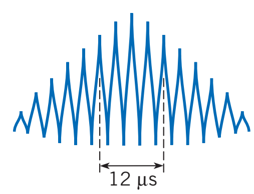
PROBLEM 13.16

Laser-Doppler Anemometers

13.17 PQ◀ Literature Review. On the Internet, locate quality resources relevant to the LDA. Skim these resources, and then

a. Write down five findings that are relevant to engineering practice and interesting to you.
b. Write down two questions about LDAs that are interesting and insightful.

13.18 A laser-Doppler anemometer (LDA) system is being used to measure the velocity of air in a tube. The laser is an argon-ion laser with a wavelength of 4880 angstroms. The angle between the laser beams is 20°. The time interval is determined by measuring the time between five spikes, as shown, on the signal from the photodetector. The time interval between the five spikes is 12 microseconds. Find the velocity.



PROBLEM 13.18

Orifice Meters

13.19 PQ◀ On the Internet, locate quality knowledge resources relevant to orifice meters. Skim these resources, and then

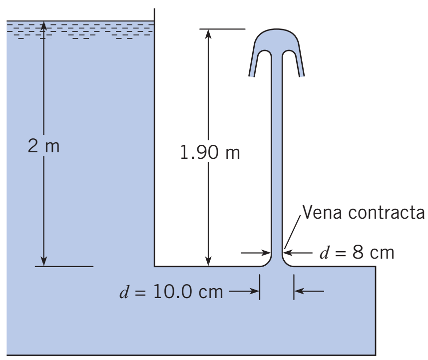
a. Write down five findings that are relevant to engineering practice and interesting to you.
b. Write down two questions that are interesting, insightful, and relevant to orifice meters.

13.20 For the jet and orifice shown, determine C_v, C_c, and C_d.

13.21 A fluid jet discharging from a 3 cm orifice has a diameter of 2.7 cm at its vena contracta. What is the coefficient of contraction?

13.22 Figure 13.13 is of a sharp-edged orifice. Note that the metal surface immediately downstream of the leading edge makes an acute angle with the metal of the upstream face of the orifice. Do you think the orifice would operate the same (have the same flow coefficient, K) if that angle were 90°? Explain how you came to your conclusion.

13.23 New orifices such as that shown in Fig. 13.13 will have definite flow coefficients as given in Fig. 13.14. With age,

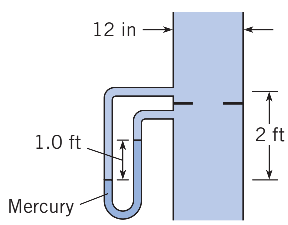


PROBLEM 13.20

however, physical changes could occur to the orifice. Explain what changes these might be and how (if at all) these physical changes might affect the flow coefficients.

13.24 A 6 in. orifice is placed in a 10 in. pipe, and a mercury manometer is connected to either side of the orifice. If the flow rate of water (60°F) through this orifice is 4.5 cfs, what will be the manometer deflection?

13.25 Determine the discharge of water through this 6 in. orifice that is installed in a 12 in. pipe.



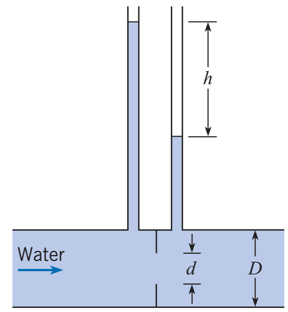
PROBLEM 13.25

13.26 The flow coefficient values for orifices given in Fig. 13.14 were obtained by testing orifices in relatively smooth pipes. If an orifice were used in a pipe that was very rough, do you think you would get a valid indication of discharge by using the flow coefficient of Fig. 13.14? Justify your conclusion.

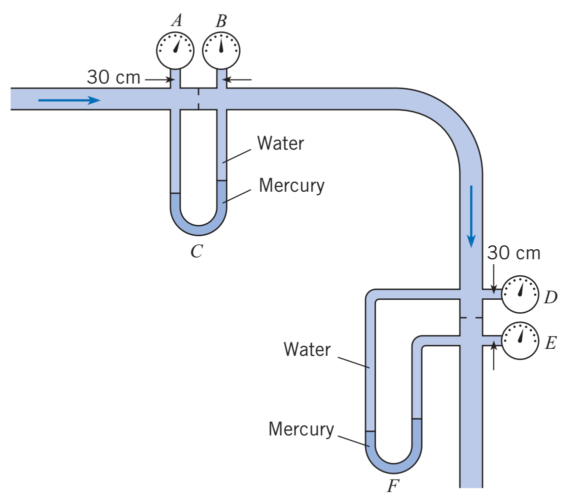
13.27 Determine the discharge of water ($T = 60°F$) through the orifice shown if $h = 5$ ft, $D = 6$ in., and $d = 3$ in.

13.28 A pressure transducer is connected across an orifice to measure the flow rate of kerosene at 20°C. The pipe diameter is 2 cm, and the ratio of orifice diameter to pipe diameter is 0.6. The pressure differential as indicated by the transducer is 10 kPa. What is the mean velocity of the kerosene in the pipe?

13.29 The 10 cm orifice in the horizontal 30 cm pipe shown is the same size as the orifice in the vertical pipe. The manometers are mercury-water manometers, and water ($T = 20°C$) is flowing in the system. The gages are Bourdon-tube gages. The flow, at a rate of 0.1 m^3/s, is to the right in the horizontal pipe and therefore downward in the vertical pipe. Is Δp as indicated by gages A and B the same as Δp as indicated by gages D and E? Determine their values. Is the deflection on manometer C the same as the deflection on manometer F? Determine the deflections.



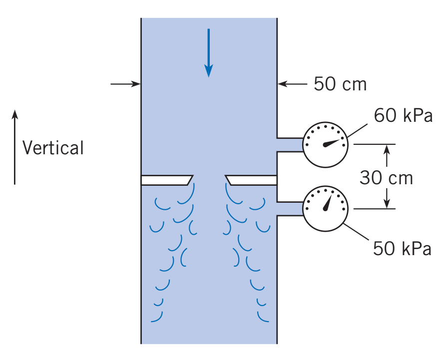
PROBLEM 13.27



PROBLEM 13.29

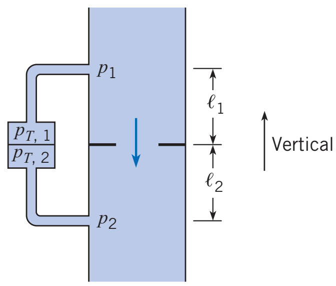
13.30 A 15 cm plate orifice at the end of a 30 cm pipe is enlarged to 20 cm. With the same pressure drop across the orifice (approximately 50 kPa), what will be the percentage of increase in discharge?

13.31 If water (20°C) is flowing through this 5 cm orifice, estimate the rate of flow.



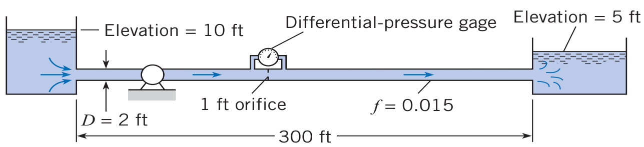
PROBLEM 13.31

13.32 A pressure transducer is connected across an orifice as shown. The pressure at the upstream pressure tap is p_1, and the pressure at the downstream tap is p_2. The pressure at the transducer connected to the upstream tap is $p_{T,1}$ and to the downstream pressure tap, $p_{T,2}$. Show that the difference in piezometric pressure defined as $(p_1 + \gamma z_1) - (p_2 + \gamma z_2)$ is equal to the pressure difference across the transducer, $p_{T,1} - p_{T,2}$.



PROBLEM 13.32

13.33 Water ($T = 50°F$) is pumped at a rate of 20 cfs through the system shown in the figure. What differential pressure will occur across the orifice? What power must the pump supply to the flow for the given conditions? Also, draw the HGL and the EGL for the system. Assume $f = 0.015$ for the pipe.



PROBLEM 13.33

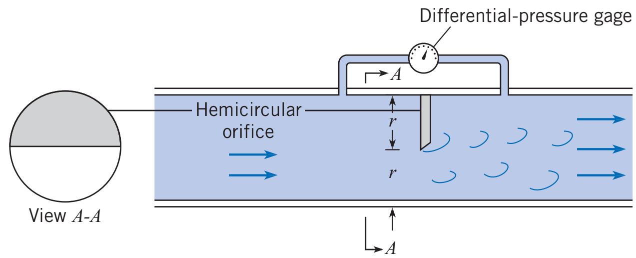
13.34 Determine the size of orifice required in a 15 cm pipe to measure 0.03 m^3/s of water with a deflection of 1 m on a mercury-water manometer.

13.35 What is the discharge of gasoline (S = 0.68) in a 10 cm horizontal pipe if the differential pressure across a 6 cm orifice in the pipe is 50 kPa?

13.36 What size orifice is required to produce a change in head of 6 m for a discharge of 2 m^3/s of water in a pipe 1 m in diameter?

13.37 An orifice is to be designed to have a change in pressure of 50 kPa across it (measured with a differential-pressure transducer) for a discharge of 3.0 m^3/s of water in a pipe 1.2 m in diameter. What diameter should the orifice have to yield the desired results?

13.38 Hemicircular orifices such as the one shown are sometimes used to measure the flow rate of liquids that also transport sediments. The opening at the bottom of the pipe allows free passage of the sediment. Derive a formula for Q as a function of Δp, D, and other relevant variables associated with the problem. Then, using that formula and guessing any unknown data, estimate the water discharge through such an orifice when Δp is read as 80 kPa and flow is in a 30 cm pipe.



PROBLEM 13.38

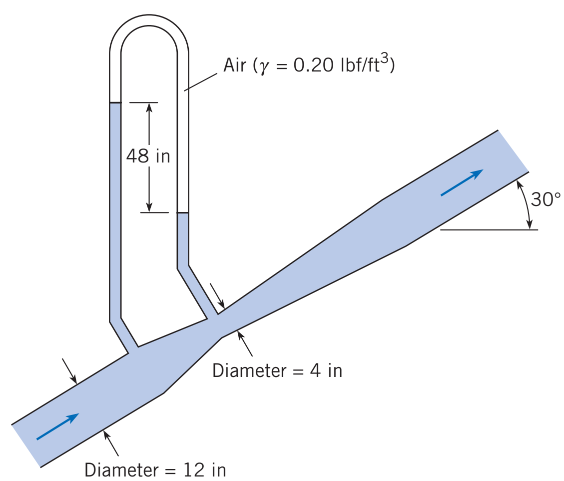
Venturi Meters

13.39 PQ◀ What is the main advantage of a venturi meter versus an orifice meter? The main disadvantage?

13.40 Water flows through a venturi meter that has a 30 cm throat. The venturi meter is in a 60 cm pipe. What deflection will occur on a mercury-water manometer connected between the upstream and throat sections if the discharge is 0.75 m^3/s? Assume $T = 20°C$.

13.41 What is the throat diameter required for a venturi meter in a 200 cm horizontal pipe carrying water with a discharge of 10 m^3/s if the differential pressure between the throat and the upstream section is to be limited to 200 kPa at this discharge?

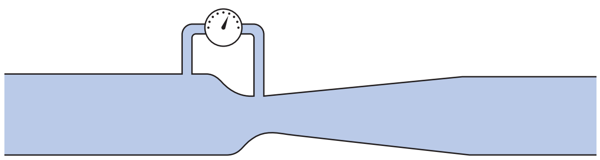
13.42 Estimate the rate of flow of water through the venturi meter shown.



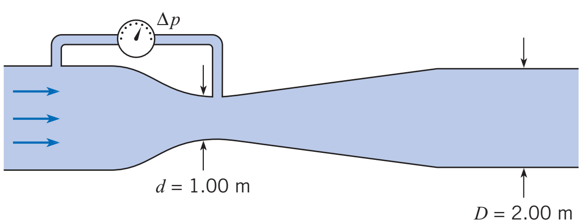
PROBLEM 13.42

13.43 When no flow occurs through the venturi meter, the indicator on the differential-pressure gage is straight up and indicates a Δp of zero. When 5 cfs of water flows to the right, the differential-pressure gage indicates $\Delta p = +10$ psi. If the flow is now reversed and 5 cfs flow to the left through the venturi meter, in which range would Δp fall? (a) $\Delta p < -10$ psi, (b) -10 psi $< \Delta p < 0$, (c) $0 < \Delta p < 10$ psi, or (d) $\Delta p = 10$ psi.

13.44 The pressure differential across this venturi meter is 100 kPa. What is the discharge of water through it?



PROBLEM 13,43



PROBLEM 13.44

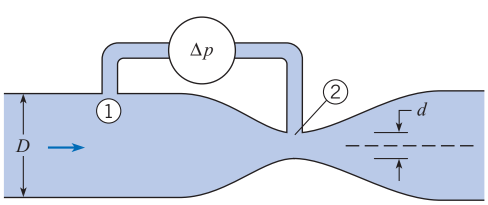
13.45 Engineers are calibrating a poorly designed venturi meter for the flow of an incompressible liquid by relating the pressure difference between taps 1 and 2 to the discharge. By applying the Bernoulli equation and assuming a quasi–one-dimensional flow (velocity uniform across every cross section), the engineers find that

$$Q_0 = A_2[2(p_1 - p_2)/\rho]^{0.5}[1 - (d/D)^4]^{-0.5}$$

where D and d are the duct diameters at stations 1 and 2. However, they realize that the flow is not quasi–one-dimensional and that the pressure at tap 2 is not equal to the average pressure in the throat because of streamline curvature. Thus the engineers introduce a correction factor K into the foregoing equation to yield

$$Q = KQ_0$$

Use your knowledge of pressure variation across curved streamlines to decide whether K is larger or smaller than unity, and support your conclusion by presenting a rational argument.

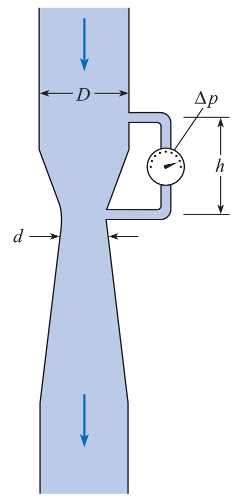


PROBLEM 13.45

13.46 The differential-pressure gage on the venturi meter shown reads 6.2 psi, $h = 25$ in., $d = 6$ in., and $D = 12$ in. What is the discharge of water in the system? Assume $T = 50°\text{F}$.

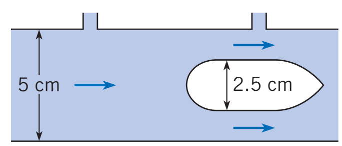
13.47 The differential-pressure gage on the venturi meter reads 45 kPa, $d = 20$ cm, $D = 40$ cm, and $h = 80$ cm. What is the discharge of gasoline (S $= 0.69$; $\mu = 3 \times 10^{-4}$ N $\cdot$ s/m^2) in the system?

13.48 A flow nozzle has a throat diameter of 2 cm and a beta ratio (d/D) of 0.5. Water flows through the nozzle, creating a pressure difference across the nozzle of 8 kPa. The viscosity of the water is 10^{-6} m^2/s, and the density is 1000 kg/m^3. Find the discharge.



PROBLEM 13.46

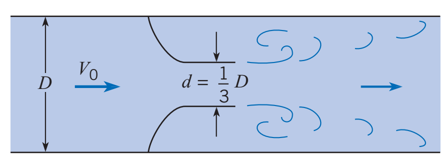
13.49 Water flows through an annular venturi consisting of a body of revolution mounted inside a pipe. The pressure is measured at the minimum area and upstream of the body. The pipe is 5 cm in diameter, and the body of revolution is 2.5 cm in diameter. A head difference of 1 m is measured across the pressure taps. Find the discharge in cubic meters per second.



PROBLEM 13.49

Miscellaneous Measurement Techniques

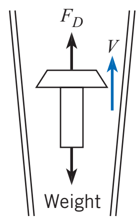
13.50 What is the head loss in terms of $V_0^2/2g$ for the flow nozzle shown?



PROBLEM 13.50

13.51 A vortex flow meter is used to measure the discharge in a duct 5 cm in diameter. The diameter of the shedding element is 1 cm. The Strouhal number based on the shedding frequency from one side of the element is 0.2. A signal frequency of 50 Hz is measured by a pressure transducer mounted downstream of the element. What is the discharge in the duct?

13.52 A rotameter operates by aerodynamic suspension of a weight in a tapered tube. The scale on the side of the rotameter is calibrated in scfm of air—that is, cubic feet per minute at standard conditions ($p = 1$ atm and $T = 68°F$). By considering the balance of weight and aerodynamic force on the weight inside the tube, determine how the readings would be corrected for nonstandard conditions. In other words, how would the actual cubic feet per minute be calculated from the reading on the scale, given the pressure, temperature, and gas constant of the gas entering the rotameter?



PROBLEM 13.52

13.53 A rotameter is used to measure the flow rate of a gas with a density of 1.0 kg/m^3. The scale on the rotameter indicates 5 liters/s. However, the rotameter is calibrated for a gas with a density of 1.2 kg/m^3. What is the actual flow rate of the gas (in liters per second)?

13.54 One mode of operation of ultrasonic flow meters is to measure the travel times between two stations for a sound wave traveling upstream and then downstream with the flow. The downstream propagation speed with respect to the measuring stations is $c + V$, where c is the sound speed and V is the flow velocity. Correspondingly, the upstream propagation speed is $c - V$.

a. Derive an expression for the flow velocity in terms of the distance between the two stations, L; the difference in travel times, Δt; and the sound speed.
b. The sound speed is typically much larger than V ($c \gg V$). With this approximation, express V in terms of L, c, and Δt.
c. A 10 ms time difference is measured for waves traveling 20 m in a gas where the speed of sound is 300 m/s. Calculate the flow velocity.

Weirs

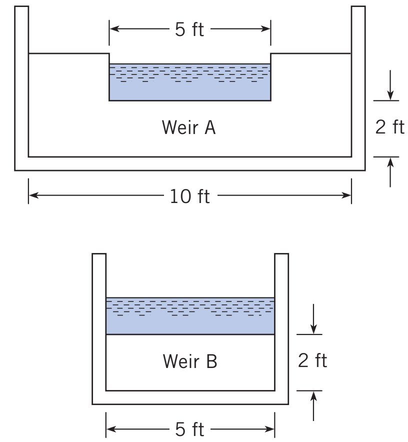
13.55 ℙℚ◀What variables influence flow rate through a rectangular weir?

13.56 ℙℚ◀ On the Internet locate quality resources relevant to weirs, skim these resources, and write down five important findings.

13.57 Water flows over a rectangular weir that is 4 m wide and 30 cm high. If the head on the weir is 20 cm, what is the discharge in cubic meters per second?

13.58 The head on a 60° triangular weir is 25 cm. What is the discharge over the weir in cubic meters per second?

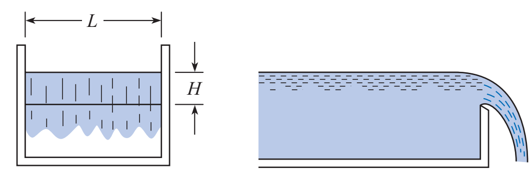
13.59 Water flows over two rectangular weirs. Weir A is 5 ft long in a channel 10 ft wide; weir B is 5 ft long in a channel 5 ft wide. Both weirs are 2 ft high. If the head on both weirs is 1.00 ft, then one can conclude that (a) $Q_A = Q_B$, (b) $Q_A > Q_B$, or (c) $Q_A < Q_B$.



PROBLEM 13.59

13.60 A 1 ft–high rectangular weir (weir 1) is installed in a 2 ft–wide rectangular channel, and the head on the weir is observed for a discharge of 10 cfs. Then the 1 ft weir is replaced by a 2 ft–high rectangular weir (weir 2), and the head on the weir is observed for a discharge of 10 cfs. The ratio H_1/H_2 should be (a) equal to 1.00, (b) less than 1.00, or (c) greater than 1.00.

13.61 A 3 m–long rectangular weir is to be constructed in a 3 m–wide rectangular channel, as shown (*a*). The maximum flow in the channel will be 4 m^3/s. What should be the height P of the weir to yield a depth of water of 2 m in the channel upstream of the weir?



(*a*) Rectangular weir (end view) (*b*) Elevation view

PROBLEMS 13.61, 13.62, 13.63, 13.64

13.62 Consider the rectangular weir described in Prob. 13.61. When the head is doubled, the discharge is (a) doubled, (b) less than doubled, or (c) more than doubled.

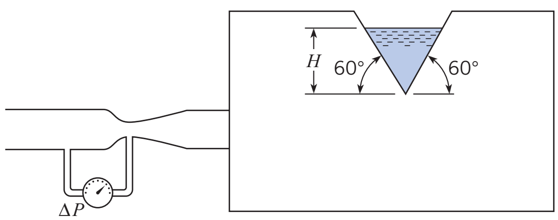
13.63 A basin is 50 ft long, 2 ft wide, and 4 ft deep. A sharp-crested rectangular weir is located at one end of the basin, and it spans the width of the basin (the weir is 2 ft long). The crest of the weir is 2 ft above the bottom of the basin. At a given instant water in the basin is 3 ft deep; thus water is flowing over the weir and out of the basin. Estimate the time it will take for the water in the basin to go from the 3 ft depth to a depth of 2 ft 2 in.

13.64 Water at 50°F is piped from a reservoir to a channel like that shown. The pipe from the reservoir to the channel is a 4 in. steel pipe 100 ft in total length. There are two 90° bends, $r/D = 1$, in the line, and the entrance and exit are sharp-edged. The weir is 2 ft long. The elevation of the water surface in the reservoir is 100 ft, and the elevation of the bottom of the channel is 70 ft. The crest of the weir is 3 ft above the bottom of the channel. For steady flow conditions determine the water surface elevation in the channel and the discharge in the system.

13.65 At one end of a rectangular tank 1 m wide is a sharp-crested rectangular weir 1 m high. In the bottom of the tank is a 10 cm sharp-edged orifice. If 0.10 m^3/s of water flows into the tank and leaves the tank both through the orifice and over the weir, what depth will the water in the tank attain?

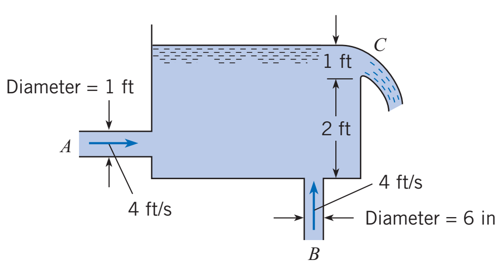
13.66 What is the water discharge over a rectangular weir 3 ft high and 10 ft long in a rectangular channel 10 ft wide if the head on the weir is 1.5 ft?

13.67 A reservoir is supplied with water at 60°F by a pipe with a venturi meter as shown. The water leaves the reservoir through a triangular weir with an included angle of 60°. The flow coefficient of the venturi is unity, the area of the venturi throat is 12 in.^{2}, and the measured Δp is 10 psi. Find the head, H, of the triangular weir.



PROBLEM 13.67

13.68 At a particular instant water flows into the tank shown through pipes A and B, and it flows out of the tank over the rectangular weir at C. The tank width and weir length (dimensions normal to page) are 2 ft. Then, for the given conditions, is the water level in the tank rising or falling?

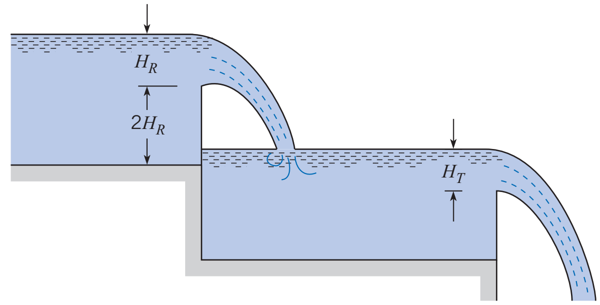


PROBLEM 13.68

13.69 Water flows from the first reservoir to the second over a rectangular weir with a width-to-head ratio of 3. The height P of the weir is twice the head. The water from the second reservoir flows over a 60° triangular weir to a third reservoir. The discharge across both weirs is the same. Find the ratio of the head on the rectangular weir to the head on the triangular weir.

13.70 Given the initial conditions of Prob. 13.69, tell, qualitatively and quantitatively, what will happen if the flow entering the first reservoir is increased 50%.

13.71 A rectangular irrigation canal 3 m wide carries water with a discharge of 6 m^3/s. What height of rectangular weir installed across the canal will raise the water surface to a level 2 m above the canal floor?

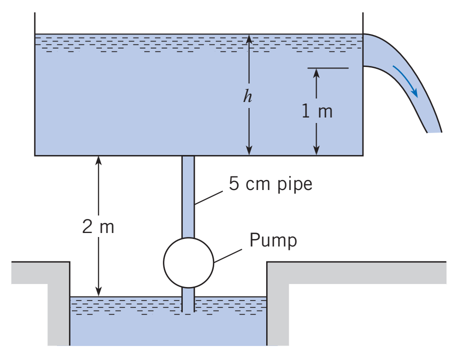


PROBLEM 13.69

13.72 The head on a 60° triangular weir is 1.5 ft. What is the discharge of water over the weir?

13.73 An engineer is designing a triangular weir for measuring the flow rate of a stream of water that has a discharge of 10 cfm. The weir has an included angle of 45° and a coefficient of discharge of 0.6. Find the head on the weir.

13.74 A pump is used to deliver water at 10°C from a well to a tank. The bottom of the tank is 2 m above the water surface in the well. The pipe is commercial steel 2.5 m long with a diameter of 5 cm. The pump develops a head of 20 m. A triangular weir with an included angle of 60° is located in a wall of the tank with the bottom of the weir 1 m above the tank floor. Find the level of the water in the tank above the floor of the tank.



PROBLEM 13.74

Measurements in Compressible Flow

13.75 A Pitot-static tube is used to measure the Mach number in a compressible subsonic flow of air. The stagnation pressure is 140 kPa, and the static pressure is 100 kPa. The total temperature of the flow is 300 K. Determine the Mach number and the flow velocity.

13.76 Use the normal shock wave relationships developed in Chapter 12 to derive the Rayleigh supersonic Pitot formula.

13.77 The static and stagnation pressures measured by a Pitot-static tube in a supersonic air flow are 54 kPa and 200 kPa, respectively.

The total temperature is 350 K. Determine the Mach number and the velocity of the free stream.

13.78 A venturi meter is used to measure the flow of helium in a pipe. The pipe is 1 cm in diameter, and the throat diameter is 0.5 cm. The measured upstream and throat pressures are 120 kPa and 80 kPa, respectively. The static temperature of the helium in the pipe is 17°C. Determine the mass flow rate.

13.79 Hydrogen at atmospheric pressure and 15°C flows through a sharp-edged orifice with a beta ratio, d/D, of 0.5 in a 2 cm pipe. The pipe is horizontal, and the pressure change across the orifice is 1 kPa. The flow coefficient is 0.62. Find the mass flow (in kilograms per second) through the orifice.

13.80 A hole 0.2 in. in diameter is accidentally punctured in a line carrying natural gas (methane). The pressure in the pipe is 50 psig, and the atmospheric pressure is 14 psia. The temperature in the line is 70°F. What is the rate at which the methane leaks through the hole (in lbm/s)? The hole can be treated as a truncated nozzle.

Uncertainty Analysis

13.81 Consider the stagnation tube of Prob. 13.6. If the uncertainty in the manometer measurement is 0.1 mm, calculate the velocity and the uncertainty in the velocity. Assume that $C_p = 1.00$, $\rho_{air} = 1.25 \text{ kg/m}^3$, and the only uncertainty is due to the manometer measurement.

13.82 Consider the orifice meter in Prob. 13.25. Calculate the flow rate and the uncertainty in the flow rate. Assume the following values of uncertainty: 0.03 in flow coefficient, 0.05 in. in orifice diameter, and 0.5 in. in height of mercury.

13.83 Consider the weir in Prob. 13.66. Calculate the discharge and the uncertainty in the discharge. Assume the uncertainty in K is 5%, in H is 3 in., and in L is 1 in.

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