Name: SOLUTIONS

CE 3305 Engineering Fluid Mechanics
Exercise Set 12
Spring 2014

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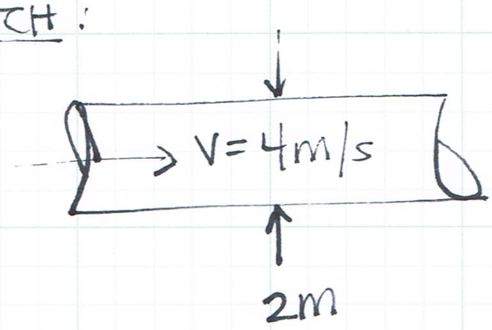
3. Problem 5.23, pg 197
4. Problem 5.26, pg 198

5.9) A pipe with a 2 m diameter carries water having a velocity of 4m/s. What is the discharge in cubic meters per second and in cubic feet per second?

SKETCH:



KNOWN

Dia = 2m
V=4m/s

GOVERNING EQN.

$Q = VA$

UNKNOWN

$Q = ?$ (ft^3/s) & (m^3/s)

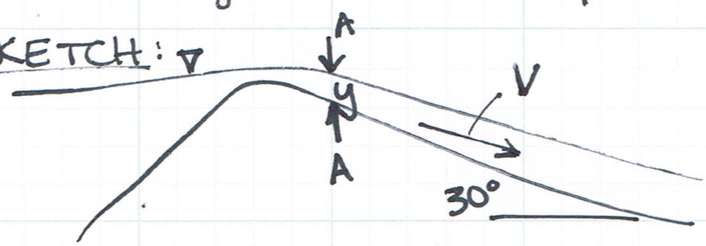
SOLUTION

$$Q = VA = 4m/s\left(\frac{\pi(2m)^2}{4}\right) = \boxed{12.56 m^3/s = Q}$$

$$\frac{12.56\, m^3}{s} * \left(\frac{3.28\, ft}{1m}\right)^3 = \boxed{443.2 ft^3/s = Q}$$

5.19) The velocity at section A-A is 15 ft/s, and the vertical depth y at the same section is 4 ft. If the width of the channel is 28 ft, what is the discharge in cubic feet per second?

SKETCH:



KNOWN:

$V = 15\ ft/s$ $\quad X = 28\ ft$
$y = 4\ ft$

UNKNOWN:

Q in ft^3/s

GOVERNING EQN

$Q = VA$

SOLUTION:

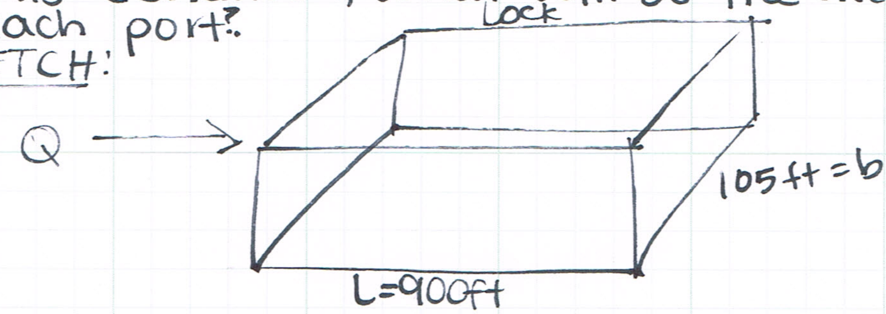
$$Q = V \times A$$

$$= \left(\frac{15\ ft}{s}\right)\left(4\ ft \cos(30°)\right)(28\ ft) = 1455\ ft^3/s$$

$$\boxed{Q = 1455\ ft^3/s}$$

5.23) Water enters the lock of a ship canal through 180 ports, each port having a 2ft by 2ft cross section. The lock is 900ft long and 105ft wide. The lock is designed so that the water surface in it will rise at a maximum rate of 6ft/min. For this condition, what will be the mean velocity in each port?

SKETCH:



KNOWN

180 ports
$A_p = 2ft \times 2ft = 4ft^2$
$A_{rise} = 105ft \times 900ft = 94,500ft^2$
$V_{rise} = 6ft/min$

UNKNOWN:

mean velocity, $\bar{V}$

GOVERNING EQNS

$$\sum V_p A_p = V_{rise} \times A_{rise}$$

SOLUTION:

$$180 \times V_p \times (A_p) = V_{rise} * A_{rise}$$

$$180 * V_p * 4ft^2 = \frac{6ft}{min} * \frac{1min}{60sec} * 94500ft^2$$

$$\boxed{V_p = 13.12ft/s}$$

5.26) The velocity of flow in a circular pipe varies according to the equation $V/V_c = (1 - r^2/r_o^2)^n$, where V_c is the centerline velocity, r_o is the pipe radius, and r is the radial distance from the centerline. The exponent n is general and is chosen to fit a given profile ($n=1$ for laminar flow). Determine the mean velocity as a function of V_c and n.

GOVERNING EQN

$$V = V_c(1-(r/r_o)^2)^n$$

FIND:

$$\bar{V} = \bar{V}(V_c, n)$$

SOLUTION:

Flowrate equation

$$Q = \int_A V\,dA$$

$$= \int_0^{r_o} V_c\left[1-\left(\frac{r}{r_o}\right)^2\right]^n 2\pi r\,dr$$

$$= -\pi r_o^2 V_c \int_0^{r_o}\left(1-\left(\frac{r}{r_o}\right)^2\right)^n\left(\frac{-2r}{r_o^2}\right)dr$$

use u substitution because integral is in the following form

$$\int_0^u u^n\,du = \frac{u^{n+1}}{n+1}$$

$$Q = -\pi r_o^2 V_c\left(\frac{\left(1-\left(\frac{r}{r_o}\right)^2\right)^{n+1}}{n+1}\right)\Bigg|_0^{r_o}$$

$$= \left(\frac{1}{n+1}\right)V_c \pi r_o^2$$

$$\bar{V} = \frac{Q}{A}$$

$$\Rightarrow \boxed{\bar{V} = \left(\frac{1}{n+1}\right)V_c}$$