CE 3305 – Fluid Mechanics Exam 3

Purpose

Demonstrate ability to apply fluid mechanics and problem solving principles covering topics such as: Dimensional analysis and similitude; turbulent flow in closed conduits; pump system performance.

Instructions

1. Choose any **four (4)** of the six (6) problems. You do **not** need to complete all six problems.
2. Put your name on each sheet you submit.
3. Use additional sheets as needed.
4. Begin each problem on a separate page. Ok to disassemble the exam to keep pages in order.
5. Do not write on the back of sheets (I don't even look)
6. Use the **problem solving protocol** in the class notes. The discussion section can simply be the word "discussion"
7. Label and/or underline answers, be sure to include units.

Allowed Resources

1. Your notes
2. Your textbook
3. The mighty Internet with following proviso:

   You may not communicate with other people during the exam

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1. A smooth pipe designed to carry crude oil is to be modeled with a smooth pipe 0.75 inches in diameter carrying water (T =60^{o}F). The protoype properties are:

   - $D = 47$ inches
   - $\rho = 1.75$ slugs/ft^3
   - $\mu = 4 \times 10^{-4}$ $\frac{lbf\ s}{ft^2}$

   Determine:

   (a) The mean velocity of the water in the model to ensure dynamically similar conditions, if the mean velocity in the prototype is to be 2 ft/s,?

SKETCH +10

$\rho_p = 1.75$ slug/ft^3

$\mu = 4 \cdot 10^{-4} \frac{lb \cdot s}{ft^2}$

$\rho_m = 1.94$ slugs/ft^3

$\mu = 2.36 \cdot 10^{-5} \frac{lb \cdot s}{ft^2}$

KNOWN +1

VALUES IN SKETCH

UNKNOWN +1

V_m

GOVERNING PRINCIPLES +1

$Re_m = Re_p$ (Dynamic similitude)

SOLUTION

$$Re_p = \frac{\rho_p V_p D_p}{\mu_p} \quad +1 \qquad Re_m = \frac{\rho_m V_m D_m}{\mu_m} \quad +1$$

SET EQUAL; SOLVE FOR UNKNOWN

$$\frac{\rho_p V_p D_p}{\mu_p} = \frac{\rho_m V_m D_m}{\mu_m}$$

$$\frac{\mu_m \rho_p V_p D_p}{\mu_p \rho_m D_m} = V_m \quad +1$$

SUBSTITUTE NUMERICAL VALUES

$$\frac{(2.36 \cdot 10^{-5})(1.75)(2.0)\left(\frac{47}{12}\right)}{(4 \cdot 10^{-4})(1.94)\left(\frac{0.75}{12}\right)} = \underbrace{6.7 \ ft/s}_{+2}$$

+1

DISCUSSION +1

DIRECT APPLICATION OF Re SIMILARITY & ARITHMETIC

2. Flow around a bridge pier is studied using a $\frac{1}{12}$ scale model. The approach velocity in the model is 0.9 $\frac{m}{s}$ and at this speed the standing wave at the bridge pier nose is measured to be 2.5 cm in height (above the undisturbed water surface).

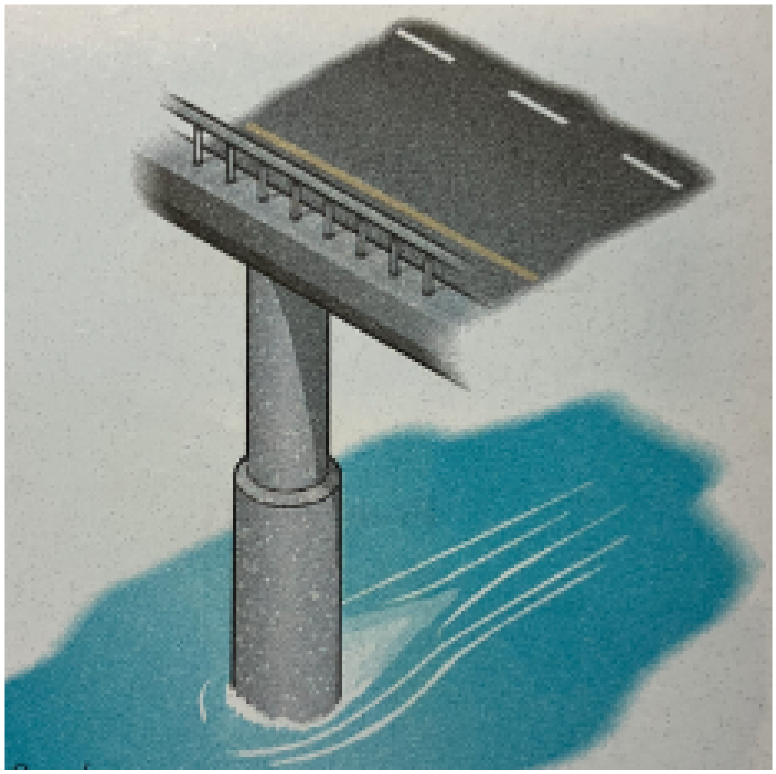


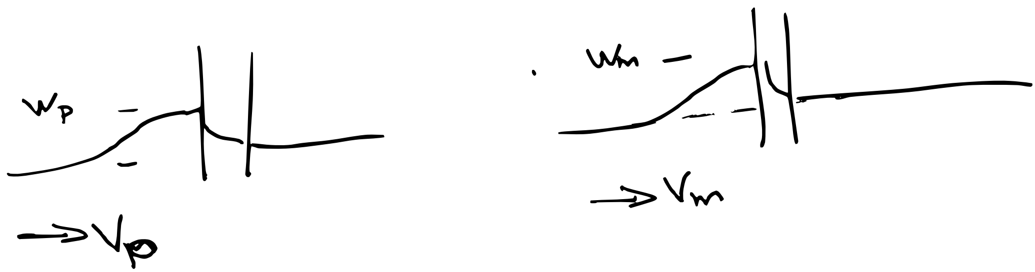
Figure 1:

Determine:

(a) The approach velocity in the prototype using Froude number matching ($Fr = \frac{V}{\sqrt{gL}}$).

(b) The wave height in the prototype.

SKETCH



KNOWN

1:12 GEOMETRIC SCALE

$V_m = 0.9 \text{ m/s}$

$W_m = 0.025 \text{ m}$

UNKNOWN

V_p, W_p

GOVERNING PRINCIPLES

Fr SIMILARITY

SOLUTION

$$\frac{V_p}{\sqrt{g L_p}} = \frac{V_m}{\sqrt{g L_m}} \qquad \frac{L_p}{L_m} = \frac{12}{1} \text{ (GIVEN)}$$

$$V_p = \frac{V_m \sqrt{L_p}}{\sqrt{L_m}} = \frac{V_m \sqrt{12}}{\sqrt{1}} = \frac{0.9 \sqrt{12}}{\sqrt{1}} = \underline{\underline{3.1 \text{ m/s}}}$$

$$W_p = 12\, W_m = \underline{\underline{0.3 \text{ m}}} \text{ (30 cm)}$$

DISCUSSION

DIRECT APPLICATION DEFN. Fr MATCHING &

GEOMETRIC SCALING

3. A prototype littoral frigate-class vessel has a length of 421 ft and is designed to travel on water at 75 ft/s[1]. A 4.21-ft-long model is tested in oil to maintain the same Froude number ($Fr = \frac{V}{\sqrt{gL}}$) and Reynolds number ($Re = \frac{\rho V D}{\mu}$) as the prototype. Determine:

   (a) The geometric scaling factor

   (b) The speed of model (V_m)

   (c) The required kinematic viscosity of oil $\cancel{(\mu_m)}$. $\nu_m = \frac{\mu}{\rho}$

SKETCH

KNOWN

PHYSICAL DIMENSIONS

PROTOTYPE VELOCITY

PROTOTYPE LIQUID

UNKNOWN

GEOMETRIC SCALE

V_{model}

ν_{model}

[1]Roughly the specifications of the USS Independence (LCS-2) Littoral Combat Ship

GOVERNING PRINCIPLES

Fr scaling desired

Re scaling desired

Experimental controls of ν_{oil} & V_{model}

SOLUTION

GEOMETRIC SCALING:

$$\frac{L_{model}}{L_{PROTOTYPE}} = \frac{4.21}{421} = \frac{1}{100}$$

SO MODEL is 1:100 GEOMETRIC SCALE

$$Re_p = \frac{V_p L_p}{\nu_p} \qquad Re_m = \frac{V_m L_m}{\nu_m}$$

WANT $Re_p = Re_m$ (PROBLEM STATEMENT)

$$\frac{V_p L_p}{\nu_p} = \frac{V_m L_m}{\nu_m} \quad (1)$$

$$Fr_p = \frac{V_p}{\sqrt{g L_p}} \qquad F_m = \frac{V_m}{\sqrt{g L_m}}$$

WE WANT

$$Fr_p = Fr_m \Rightarrow \frac{V_p}{\sqrt{g L_p}} = \frac{V_m}{\sqrt{g L_m}}$$

OR

$$\frac{V_p}{\sqrt{L_p}} = \frac{V_m}{\sqrt{L_m}} \qquad (2)$$

$$\frac{V_p L_p}{\nu_p} = \frac{V_m L_m}{\nu_m} \qquad (1) \text{ (FROM PRIOR PAGE)}$$

2 EQNS. (1) & (2)   2 UNKS. V_m & ν_m - SOLVABLE

(2) $$V_m = \frac{\sqrt{L_m}\, V_p}{\sqrt{L_p}}$$

SUBSTITUTE NUMBERS

(1) $$\nu_m = \frac{V_m L_m \nu_p}{V_p L_p}$$

$$V_m = \frac{\sqrt{4.21}\ 75\ ft/s}{\sqrt{421}} = 7.5\ ft/s$$

$$\nu_{oil} = \frac{(7.5\ ft/s)(4.21\ ft)(1.66 \cdot 10^{-5}\ ft^2/s)}{(75\ ft/s)(421\ ft)}$$

$$= \boxed{1.66 \cdot 10^{-8}\ ft^2/s}$$

(No such liquid substance on Earth)

DISCUSSION

READ PP. 432 – 433 FOR SUGGESTED METHOD METHOD FOR SUCH EXPERIMENT.

4. In the design of a lift station, a bypass line is often installed parallel to the pump so some liquid recirculates as shown on Figure 2. The bypass valve then controls the flow rate in the system.

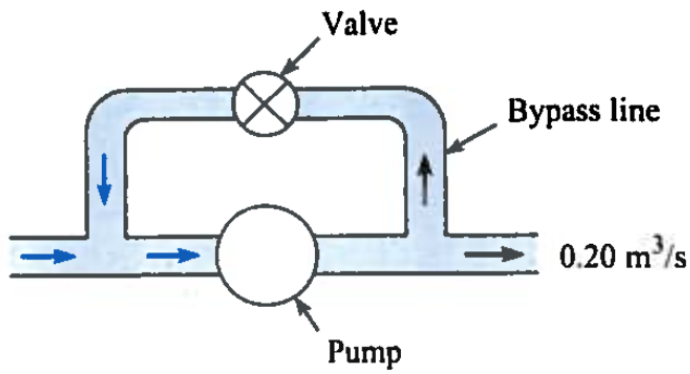


Figure 2:

The pump performance function is

$$h_p = 100 - 100Q$$

where h_p is in meters, and Q is in $\frac{m^3}{sec}$. The bypass line is 10 cm in diameter. The valve setting produces a fitting loss coefficient of $K = 0.2$ and this valve loss is the only meaningful head loss at the lift station. For a discharge leaving the lift station of $0.2\ \frac{m^3}{sec}$

Determine:

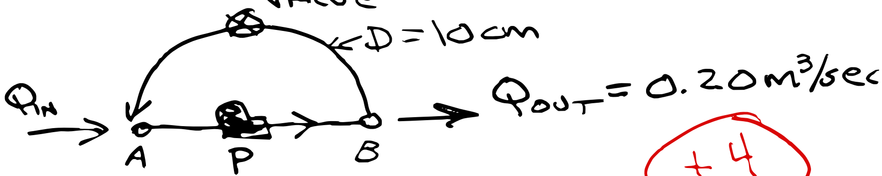
(a) The discharge through the pump

(b) The discharge through the bypass line

Sketch
(Next sheet)

SKETCH

$K_v = 0.2$



+4

$$h_p = 100 - 100Q$$

Q in m^3/sec

KNOWN:

+1

$Q_{OUT} = 0.2\ m^3/s$

$D_{BYPASS} = 0.10$ m

$A_{BYPASS} = \dfrac{\pi (0.10)^2}{4} = 0.0078\ m^2$

$h_p = 100 - 100Q$

NOTE LINEAR IN "Q"!

$K_V = 0.2$

UNKNOWN

+2

Q_{BYPASS}

Q_{PUMP}

GOVERNING PRINCIPLES

+3

CONTINUITY:

$$Q_{OUT} = Q_{IN} = Q_P - Q_{BY}$$

ENERGY

$$h_B - h_A = K \frac{V_{BY}^2}{2g} \quad \text{(head loss in bypass)}$$

$$h_A + h_p = h_B \quad \text{(added head in pump)}$$

SOLUTION

SUBSTITUTE PUMP CURVE INTO ; SUBSTITUTE ②

INTO ①

$$\cancel{h_A} + (100 - 100Q_P) - \cancel{h_A} = \frac{K V_{BY}^2}{2g}$$

+4

$$V_{BY} = \frac{Q_{OUT} - Q_P}{A_{BY}} = \frac{0.2 - Q_P}{0.0078}$$

$$100 - 100Q_P = \frac{K}{2g}\left(\frac{0.2 - Q_P}{0.0078}\right)^2$$

$$100 - 100Q_P = \frac{0.2}{2(9.8)}\left(\frac{0.2 - Q_P}{0.0078}\right)^2$$

$$100 - 100Q_P = 0.0102\left(\frac{0.2 - Q_P}{0.0078}\right)^2$$

$$\frac{100 - 100Q_P}{100} = \frac{167.65}{100}(0.2 - Q_P)^2$$

$$1 - Q_P = 1.6765(0.2 - Q_P)^2$$

$$-Q_P = 1.6765(0.2 - Q_P)^2 - 1$$

+2 (ALGEBRA + ARITHMETIC)

$$Q_p = 1 - 1.6765\left(0.2 - Q_p^*\right)^2$$

+3

QUADRATIC EQN OR BRUTE FORCE OR SOLVER

① Q_p^*	② $1-1.6765(0.2-Q_p^*)^2$	ERROR ①-②
0.2	1.0	0.8
0.3	0.983	0.683
0.9	0.178	−0.721
0.6	0.731	0.131
0.731	0.527	−0.203
0.665	0.636	−0.028
0.66	0.645	−0.014
0.657	0.649	−0.007
0.6545	0.6536	−0.0008

$\underbrace{}_{1 \text{ part}/10{,}000}$

$Q_p = 0.6545\ m^3/sec$ +2

$$Q_{BY} = \underbrace{0.6545}_{Q_P} - \underbrace{0.2}_{Q_{OUT}} = 0.4545\ m^3/sec$$

+2

DISCUSSION

CHECK HEAD LOSS IN BYPASS +1

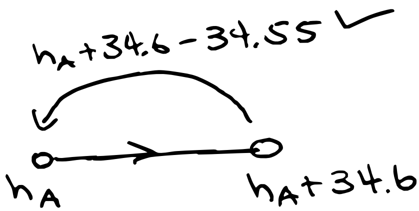
$$h_{Loss} = \frac{0.2}{2(9.8)}\left(\frac{0.4545}{0.0078}\right)^2 = 34.6m$$

CHECK ADDED HEAD

+1

$$h_p = 100 - 100\,Q$$

$$= 100 - 100\,(0.6545) = 34.55\,m$$



HEADS OK!

5. The figure below is a schematic of a pumped-storage system. Water is pumped from the lower reservoir in a pipeline with the following characteristics: D = 300 mm, L = 150 m, f = 0.029, ΣK = 5.0. The radial-flow pump characteristic curve for a single-stage pump is $H_p = 22.9 + 10.7Q - 109Q^2$ where H_p is in meters and Q is in $\frac{m^3}{sec}$.

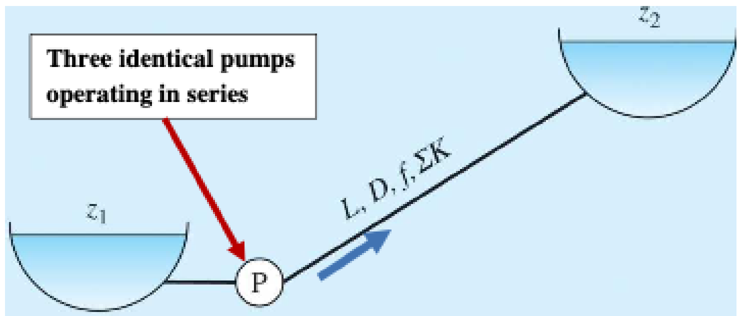


Figure 3:

Determine:

(a) Plot the lift station composite pump curve, and the system curve on the same graph.

(b) The discharge Q_D and pump added head H_D if the lift $(z_2 - z_1)$ is 40 m using a three-stage pump (treat as three identical pumps operating in series).

SKETCH: +4

SAME AS ABOVE

KNOWN +1

D = 0.3 m

L = 150 m

f = 0.029

ΣK = 5.0

$H_{P①} = 22.9 + 10.7Q - 109Q^2$

$H_{P(COMPOSITE)} = 3 * H_{P①}$

$= 68.7 + 32.1Q - 327Q^2$

UNKNOWN +3

PUMP CURVE (PLOT)
SYSTEM CURVE (PLOT)

Q_0, H_0 for $z_2 - z_1 = 40\,m$

PRINCIPLES +3

CONTINUITY
ENERGY; PIPE LOSS + MINOR LOSS
PUMPS IN SERIES

SOLUTION

$$\underbrace{\cancel{\frac{P_1}{\gamma}}}_{0\ \text{gage}} + z_1 + \underbrace{\cancel{\frac{V_1^2}{2g}}}_{\text{small at reservoir}} + H_p = \underbrace{\cancel{\frac{P_2}{\gamma}}}_{0\ \text{gage}} + z_2 + \underbrace{\cancel{\frac{V_2^2}{2g}}}_{\text{small at reservoir}} + H_L \quad +1$$

$$z_1 + H_p = z_2 + H_L$$

$$H_p = \underbrace{(z_2 - z_1)}_{\text{GIVEN AS } 40\,m} + H_L \quad +1$$

$$\frac{16\,Q^2}{\pi^2 D^4}$$

$$H_p = 40 + H_L \quad +1$$

NOW LOSSES

$$H_L = \frac{8 f L Q^2}{\pi^2 g D^5} + \sum K \frac{8 Q^2}{\pi^2 g D^4} \quad +1$$

$$H_L = \frac{8 f L Q^2}{\pi^2 g D^5} + \sum K \frac{8 Q^2}{\pi^2 g D^4} \quad +1$$

$$H_p = 68.7 + 32.1Q - 327Q^2$$

$$H_p = 40 + H_L \quad +1$$

BRUTE FORCE OR SOLVE QUADRATIC OK

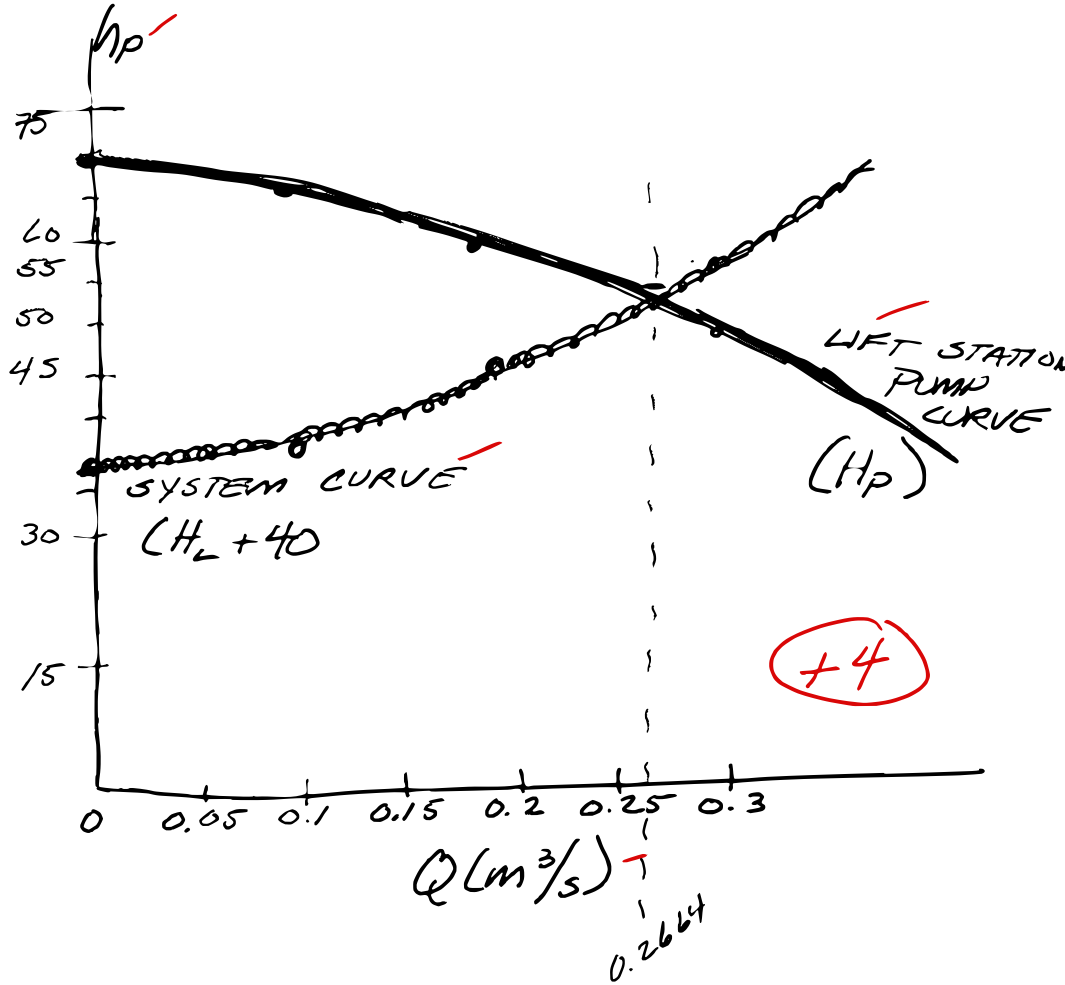
① ② ③

Q ①	40+H_L ②	H_p ③	② − ③
0	40	68.7	−28.7 make Q↑
0.1	41.99	68.6	−26.6
0.2	47.96	62.04	−14.07 sign change
0.3	57.9	48.9	9.02

IN here

0.22	49.63	59.9	−10.29
0.24	51.4	57.5	−6.09
0.26	53.46	54.94	−1.48
0.27	54.51	53.52	0.987
0.267	54.19	53.95	0.23
0.2644	54.13 +1	54.04	0.09 close enough

$$Q_p = 0.2644\ m^3/sec \quad +2$$



IF HAVE PLOTTING SOFTWARE
MAKE PRETTIER.

DISCUSSION

BRUTE FORCE GIVES INFO FOR
PLOTS. +1

6. The figure below is a schematic of a parallel pipe system. Flow occurs from A to B as shown. To augment the flow a pump is located between C and C'. The network is on a plane (flat) surface; all the junction elevations are the same. The pipes are commercial steel.

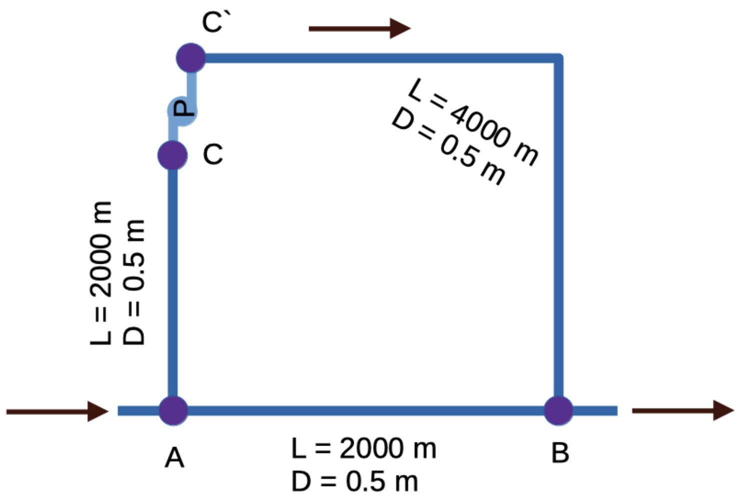


Figure 4:

The pump characteristic curve is shown below:

Total discharge is 0.60 $\frac{m^3}{sec}$

Determine:

(a) The division of flow between pipes A-B and A-C-B

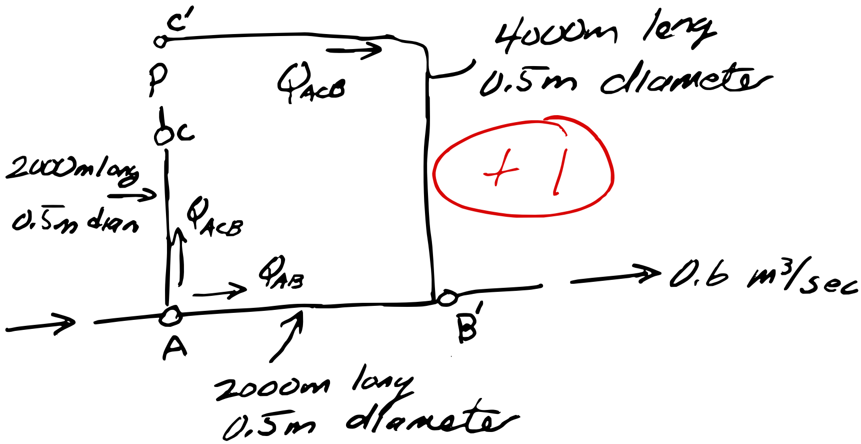
(b) The head loss in pipe A-B

(c) The head loss in pipe A-C

(d) The head loss in pipe C'-B

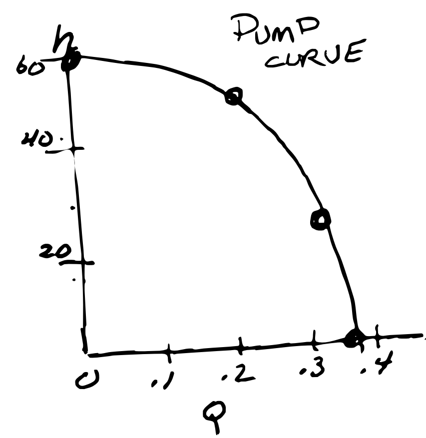
(e) The pump operating conditions.

SKETCH



KNOWN +1

PIPE LENGTHS, DIAMETER, MATERIAL
PUMP CURVE (GIVEN)
TOTAL Q



UNKNOWN +1

Q_{AB}, Q_{ACB}, Δh_{AB}, Δh_{AC}, $\Delta h_{C'B}$

PRINCIPLES +1

CONTINUNITY
ENERGY
HEAD LOSS MODEL

SOLUTION

$$\Delta h_{AB} = \frac{8 f L Q_{AB}^2}{\pi^2 g D^5}$$ (head loss AB) +1

$$\Delta h_{C'B} = \frac{8 f L Q_{ACB}^2}{\pi^2 g D^5}$$ (head loss C'B) +1

$$\Delta h_{AC} = \frac{8 f L Q_{ACB}^2}{\pi^2 g D^5}$$ (head loss AC) +1

$$Q_{AB} + Q_{ACB} = Q_{TOTAL}$$ (continuity whole network) +1

FROM PUMP CURVE

$$\Delta h_{AB} = \Delta h_{AC} - h_p + \Delta h_{c'B}$$ (Expressed as loss) +1

Will need f from Re & Moody Chart

$\varepsilon_{steel} = 0.045$

OR use tabulated friction factor for SCH 40 Steel Pipe $f = 0.012$ +1

Use & check Re later on.

+2

Q_{AB}	Q_{ACB}	Δh_{AB}	h_p: $\Delta h_{AC} + \Delta h_{c'B} - \Delta h_{AB}$	
0.1	0.5	0.6	46.95	Then plot on Pump Curve
0.2	0.4	2.53	27.921	
0.3	0.3	5.71	11.42	
0.4	0.2	10.15	-2.5	

$\dot{Q}_{AB}$	Q_{ACB}	Δh_{AB}	$\Delta h_{AC} + \overline{\Delta h_{C'B}}^{D} - \Delta h_{AB}$	
0.1	0.5	0.6	46.95	Then
0.2	0.4	2.53	27.921	plot
0.3	0.3	5.71	11.42	on
0.4	0.2	10.15	-2.5	Pump Curve

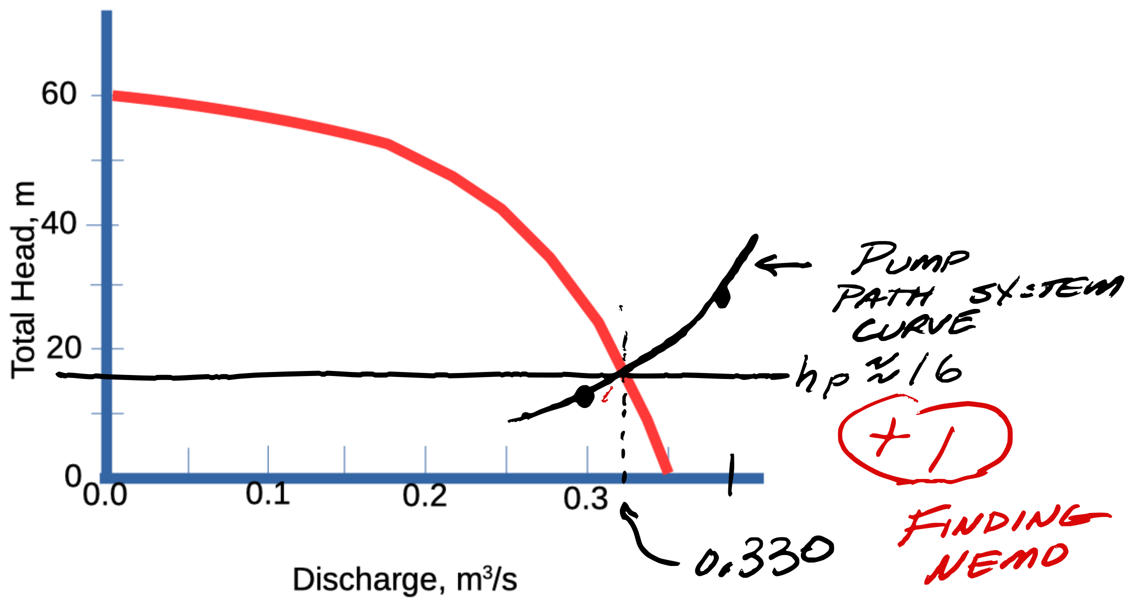


Figure 5:

$$Q_{ACB} = 0.330 \ m^3/s$$

$$Q_{AB} = 0.27 \ m^3/s$$

SCORING NEXT PAGE

Q_{AB}	Q_{ACB}	Δh_{AB}	h_p: $\Delta h_{AC} + \Delta h_{C'B} - \Delta h_{AB}$	
0.1	0.5	0.6	46.95	Then plot on Pump Curve
0.2	0.4	2.53	27.921	
0.3	0.3	5.71	11.42	
0.4	0.2	10.15	-2.5	
0.27	0.33	4.626	16.1055	Check

Check hydraulics

Path $A \to B$, let $h_A = 100$

$$h_A - \Delta h_{A\cdot} = h_B$$

$$100 - 4.626 = 95.374$$

check Optional

Path $A \to C \to C' - B$

$$h_A - \Delta h_{AC} + h_p - \Delta h_{CB} = h_B$$

$$100 - 6.9105 + 16.1055 - 13.821 = 95.37\cancel{4}$$

$\therefore$ $Q_{AB} = 0.27\ m^3/s$

$Q_{ACB} = 0.33\ m^3/s$ $\quad h_p = 16.1055$

$h_{AB} = 4.626\ m$

$h_{AC} = 6.9105\ m$

$h_{C'B} = 13.821\ m$

+12

ANSWERS; UNITS; LABELS.

DISCUSSION

+1

DIRECT APPLICATION PIPE HYDRAULICS; BUT HAVE TO LOOK UP h_p FROM CURVE, PLOT LOSS EQN ONTO CHART TO GET CLOSE

* CHECK f

$$V = \frac{Q}{\frac{\pi D^2}{4}} = \frac{4(0.33)}{\pi (0.5)^2} = 1.68\ m/sec$$

$$Re = \frac{\rho V D}{\mu} = \frac{(998\ kg/m^3)(1.68)(0.5)}{0.001\ (N\text{-}s/m^2)}$$

$$\approx 838320$$

$$\varepsilon_{STEEL} = 0.045\ mm$$

$$\frac{\varepsilon}{D} = \frac{0.045 \cdot 10^{-3}}{0.5} = 9 \cdot 10^{-5}$$

$f = 0.0135$

So guess using 0.012 is good