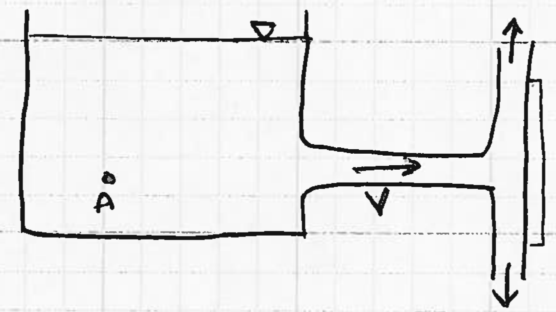
CE 3305 Engineering Fluid Mechanics
Exercise Set 15
Spring 2014

1. Problem 6.13, pg 239
2. Problem 6.64, pg 246
3. Problem 6.82, pg 248

6.13) A horizontal water jet at 70°F issues from a circular orifice in a large tank. The jet strikes a vertical plate that is normal to the axis of the jet. A force of 600 lbf is needed to hold the plate in place against the action of the jet. If the pressure in the tank is 25 psig at point A, what is the diameter of the jet just downstream of the orifice?

Sketch:



Known:

$P_A = 25$ psig
$F_x = 600$ lbf
$T = 70°F$
$\rho = 1.94$ slugs/ft^3
$\gamma = 62.4$ lbf/ft^3

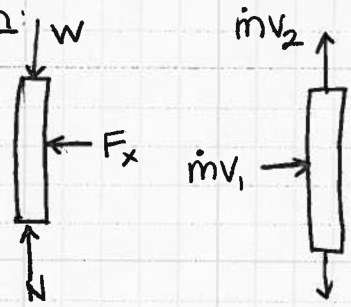
Unknown:

$D_{jet} = ?$

Governing Equation

Bernoulli & momentum equation.

Solution:



From the inside of tank to nozzle exit Bernoulli's Eqn.

$$\frac{P_A}{\rho} = \frac{V_1^2}{2}$$

$$V_1 = \sqrt{\frac{2P_A}{\rho}} = \sqrt{\frac{2(25\ \text{lbf/in}^2)(144\ \text{in}^2/\text{ft}^2)}{62.4\ \text{lbf/ft}^3\left(\frac{\text{slug}}{32.2\ \text{ft/s}^2}\right)}} = 60.9\ \text{ft/s}$$

6.13 continued)

$$\Sigma F_x = -\dot{m} V_{1x}$$

$$F_x = -(\dot{m} V_1) = \rho A V_1^2$$

$$A = \frac{F_x}{\rho V_1^2} = \frac{600\,lbf}{62.4\,lbf/ft^3 * slug/32.2ft/s^2 (60.9ft/s)^2} = \frac{600\,lbf}{7187.2 \frac{lbf \cdot ft^2}{ft^4 \cdot s^2}}$$

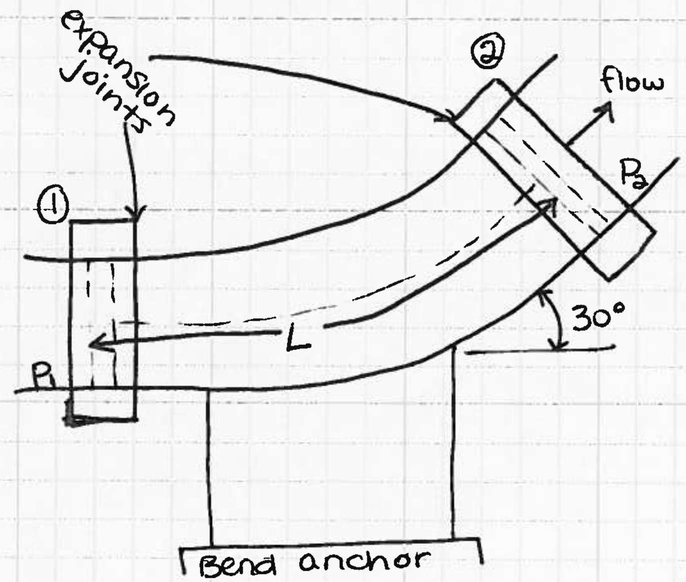
$$A = 0.083ft^2$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.083ft^2)}{\pi}} = 0.326ft$$

$$\boxed{A = 0.326ft}$$

6.64) This 30° vertical bend in a pipe with a 2ft diameter carries water ($\rho = 62.4\ lbm/ft^3$) at a rate of 31.4 cfs. If the pressure P_1 is 10 psi at the lower end of the bend, where the elevation is 100ft, and P_2 is 8.5 psi at the upper end, where the elevation is 103ft, what will be the vertical component of force that must be exerted by the "anchor" on the bend to hold it in position? The bend itself weighs 300lb and the length L is 4ft.

Sketch:



Known

$D = 2ft$

30° bend

$\rho = 62.4\ lbm/ft^3$

$Q = 31.4\ cfs$

$P_1 = 10psi$ $\quad z_1 = 100ft$

$P_2 = 8.5psi$ $\quad z_2 = 103ft$

$W_{Bend} = 300lb$

$L = 4ft$

Unknown:

$F_{anchor} = ?$

Governing Equation:

Momentum Equation

Solution:

$$\Sigma F_y = \rho Q (V_{2y} - V_{1y})$$

$$F_a - W_{water} - W_{bend} - P_2 A_2 \sin(30°) = \rho Q (V\sin(30°) - V\sin(0°))$$

$$V = \frac{Q}{A} = \frac{31.4\ cfs}{\frac{\pi}{4}(2ft)^2} = 10.0\ ft/s$$

6.64 continued

$$F_a = \frac{\pi}{4}(2ft)^2(4ft)(62.4\,lbm/ft^3) + 300lb +$$

$$\left(\frac{144in^2}{ft^2}\right)8.5\frac{lb}{in^2}\left(\frac{\pi}{4}(2ft)^2\right)\sin(30^\circ) + 1.94\frac{lbf}{ft^4 s^2}(31.4cfs)(10ft/s(0.5) - 0)$$

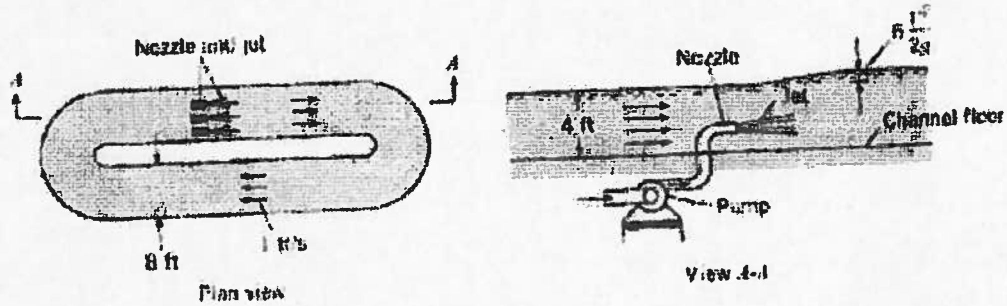
$$F_a = 784\,lbf + 300lb + 1922.1bf + 304.6\,lbf = 3310.\,lbf$$

$$\boxed{F_a = 3310\,lbf}$$

Name: ______________________________

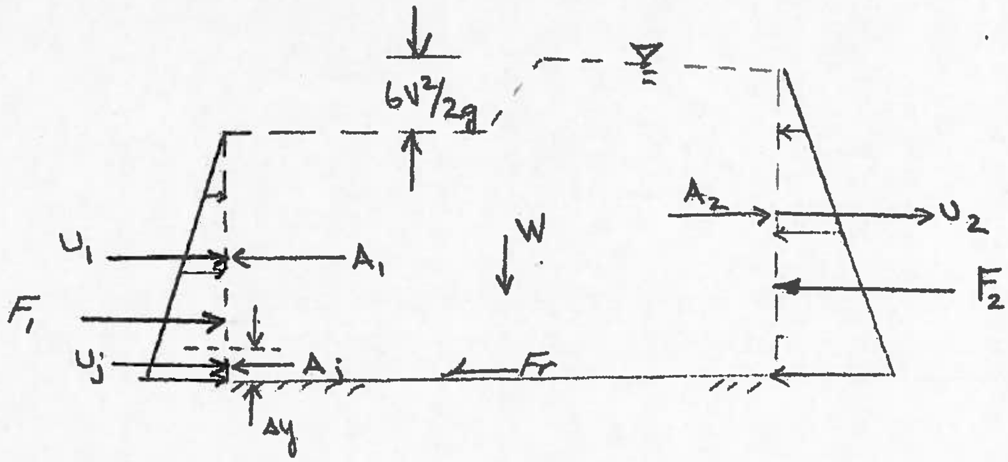
Problem 5 (*Design* Problem)

Jet pumps are sometimes used for special purposes such as pumping slurries or to circulate flow in basins where contact with mechanical impellers is not feasible, such as a fish-race in a hatchery. The figure below depicts the basic concept for this type of application. For the indicated fish-race, the jets would have to increase the water surface elevation downstream of the jets by an amount equal to $6V^2/2g$, where V is the average velocity in the basin (1 ft/sec in the drawing). ***Design*** a jet pump system (speed, nozzle diameter, and number of nozzles) to make such a re-circulating system work for a channel 8-feet wide and 4-feet deep. Your ***design*** should specify nominal diameters (i.e nozzles made from pipes commonly available 1-in, 2-in, etc.)



Hints: This problem involves "analysis" to determine the required momentum added by the jet(s), and the total jet area and speed. The design should be based on this analysis with the understanding that the total jet area should be small as compared to the total flow area (otherwise the fish will get stuck!).

MIXING ZONE



NEGLECT FRICTION ($F_r \sim 0$)

F_1 = PRESSURE FORCE APPROACH

F_2 = PRESSURE FORCE EXIT

U_1, A_1 = FREE STREAM SPEED & AREA

U_j, A_j = JET STREAM SPEED & AREA

U_2, A_2 = EXIT STREAM SPEED & AREA

MOMENTUM

$$\Sigma F_x = \frac{d}{dt}\int_{CV} V\rho\, d\forall + \int_{CS} V\rho (V \cdot dA)$$

$$A_1 = 8(y_1 - \Delta y)$$
$$A_2 = 8y_2$$
$$A_j = 8\Delta y$$

$$F_1 - F_2 = -U_j \rho U_j A_j - U_1 \rho U_1 A_1 + U_2 \rho U_2 A_2$$

$$F_1 = \rho g \frac{y_1}{2} y_1 8$$

$$F_2 = \rho g \frac{y_2}{2} y_2 8$$

$$\therefore \cancel{\rho} g \cancel{8}\left(\frac{y_1^2}{2} - \frac{y_2^2}{2}\right) = -\cancel{\rho} U_j^2 \cancel{8} \Delta y - \cancel{\rho} U_1^2 \cancel{8}(y_1 - \Delta y) + \cancel{\rho} U_2^2 \cancel{8} y_2$$

$$g\left(\frac{y_1^2 - y_2^2}{2}\right) = -U_j^2 \Delta y - U_1^2 (y_1 - \Delta y) + U_2^2 y_2$$

NOTE: y_2 IS KNOWN!

$$0 = \frac{d}{dt}\int_{cv} \rho V d\forall + \int_{cs} \rho (V \cdot dA)$$

$$0 = u_2 A_2 - u_1 A_1 - u_j A_j$$

$$0 = u_2 \cancel{b} y^2 - u_1 \cancel{b}(y_1 - \Delta y) - u_j \cancel{b} \Delta y$$

$$= u_2 y_2 - u_1 (y_1 - \Delta y) - u_j \Delta y$$

$\therefore$

$$u_2 y_2 = u_1 (y_1 - \Delta y) - u_j \Delta y$$

$$u_2 = \frac{u_1 (y_1 - \Delta y) - u_j \Delta y}{y_2}$$

NOW ARRANGE MOMENTUM IN TERMS u_j

$$u_j^2 \Delta y = -u_1^2 (y_1 - \Delta y) + u_2^2 y_2 - \frac{g}{2}(y_1^2 - y_2^2)$$

SUBSTITUTE u_2 FROM CONTINUITY

$$u_j^2 \Delta y = -u_1^2 (y_1 - \Delta y) + \left[\frac{u_1 (y_1 - \Delta y) - u_j \Delta y}{y_2}\right]^2 y_2 - \frac{g}{2}\left[y_1^2 - y_2^2\right]$$

IMPLICIT EQUATION IN $u_j \Delta y$

SOLVE BY

1. TRIAL & ERROR <u>OR</u>
2. NEWTON'S METHOD

VERIFY WHAT'S KNOWN

$u_1 = 1 \text{ ft/s}$ (GIVEN)

$y_1 = 4 \text{ ft}$ (GIVEN)

$$y_2 = 4 \text{ ft} + \frac{6(1 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = 4.0932 \text{ ft}$$

$u_2 = f(u_j, \Delta y)$

u_j = UNKNOWN

Δy = UNKNOWN

$g = 32.2 \text{ ft/s}^2$

RECALL $\Delta y \propto$ JET AREA.

JETS ARE SMALL! (IN SIZE)

$$u_j^2 \cancel{\Delta y} = \frac{-u_1^2(y_1-\Delta y)}{\Delta y} + \frac{u_1^2(y_1-\Delta y)^2}{y_2\,\Delta y} - \frac{2u_1(y_1-\Delta y)u_j\cancel{\Delta y}}{y_2\,\cancel{\Delta y}} + \frac{u_j^2\Delta y^{\cancel{2}}}{y_2\,\cancel{\Delta y}} - \frac{g}{2}\left[y_1^2-y_2^2\right]\frac{1}{\Delta y}$$

$$u_j^2 - \frac{u_j^2\Delta y}{y_2} + \frac{2u_1(y_1-\Delta y)u_j}{y_2} = \frac{-u_1^2(y_1-\Delta y)}{\Delta y} + \frac{u_1^2(y_1-\Delta y)^2}{y_2\,\Delta y} - \frac{g}{2\Delta y}\left[y_1^2-y_2^2\right]$$

NOW IF WE "PICK" Δy ONLY UNKNOWN IS U_j $\therefore$ SOLVABLE

SEE ATTACHED WORKSHEET

FOR ΔY'S FROM 0.1 TO 0.3

ΔY	U_j	A_j
0.1	10.1 ft/s	0.8 ft^2
0.2	7.0 ft/s	1.6 ft^2
0.3	5.6 ft/s	2.4 ft^2

<u>NOW</u> DETERMINE JET AREAS IN 8 FOOT CHANNEL

$A_j = \Delta Y \; 8 \text{ ft}$

FOR EACH A_j, DETERMINE NOZZLE SPECS

SEE SECOND WORKSHEET

# CIVE 3434 Exam#1, Problem #5 Jet Design					
# INPUT					
u1	1	Approach Speed			
y1	4	Approach Depth			
y2	4.0932	Exit Depth			
g	32.2	Gravitational Constant			
# DESIGN_VARIABLE(S)					
delta_y	0.1	Jet Depth (Area)			
uj	10.13858	Jet Speed			
# COMPUTED_VALUES					
y1-delta_y	3.9				
y1^2-y2^2	-0.75429				
y2*delat_y	0.40932				
u1^2	1				
uj^2	102.7905				
# COMPUTED_VALUES					
	Term_1	102.7905		Term_1	-39
	Term_2	-2.51125		Term_2	37.15919
	Term_3	19.32004		Term_3	121.4401
	LHS	119.5993		RHS	119.5993
	DIFF	9.65E-08			

① SELECT THIS VALUE

BY CHANGING THIS CELL

USE SOLVER TO MAKE THIS CELL = 0.0 ②

# INPUT					
u1	1	Approach Speed			
y1	4	Approach Depth			
y2	4.0932	Exit Depth			
g	32.2	Gravitational Constant			
# DESIGN_VARIABLE(S)					
delta_y	0.2	Jet Depth (Area)			
uj	6.983909	Jet Speed			
# COMPUTED_VALUES					
y1-delta_y	3.8				
y1^2-y2^2	-0.75429				
y2*delat_y	0.81864				
u1^2	1				
uj^2	48.77498				
# COMPUTED_VALUES					
	Term_1	48.77498		Term_1	-19
	Term_2	-2.38322		Term_2	17.63901
	Term_3	12.96729		Term_3	60.72004
	LHS	59.35905		RHS	59.35905
	DIFF	-2.6E-07			

# INPUT					
u1	1	Approach Speed			
y1	4	Approach Depth			
y2	4.0932	Exit Depth			
g	32.2	Gravitational Constant			
# DESIGN_VARIABLE(S)					
delta_y	0.3	Jet Depth (Area)			
uj	5.608987	Jet Speed			
# COMPUTED_VALUES					
y1-delta_y	3.7				
y1^2-y2^2	-0.75429				
y2*delat_y	1.22796				
u1^2	1				
uj^2	31.46074				
# COMPUTED_VALUES					
	Term_1	31.46074		Term_1	-12.3333
	Term_2	-2.30583		Term_2	11.14857
	Term_3	10.14036		Term_3	40.48003
	LHS	39.29527		RHS	39.29527
	DIFF	-4.3E-07			

#_CIVE 3434 Exam#					
#_INPUT					
u1	1	Approach Speed			
y1	4	Approach Depth			
y2	4.0932	Exit Depth			
g	32.2	Gravitational Constan			
#_DESIGN_VARIABL					
delta_y	0.1	Jet Depth (Area)			
uj	10.1385641236786	Jet Speed			
#_COMPUTED_VAL					
y1-delta_y	=B4-B8				
y1^2-y2^2	=B4^2-B5^2				
y2*delat_y	=B8*B5				
u1^2	=B3^2				
uj^2	=B9^2				
#_COMPUTED_VAL					
	Term_1	=B15		Term_1	=-B14*B11/B8
	Term_2	=-B15*B8/B5		Term_2	=B14*B11*B11/B13
	Term_3	=(2*B3*B11*B9)/B5		Term_3	=-0.5*(B6*B12)/B8
	LHS	=C19+C20+C21		RHS	=F19+F20+F21
	DIFF	=C23-F23			
#_INPUT					
u1	1	Approach Speed			
y1	4	Approach Depth			
y2	4.0932	Exit Depth			
g	32.2	Gravitational Constan			
#_DESIGN_VARIABL					
delta_y	0.2	Jet Depth (Area)			
uj	6.98390894304089	Jet Speed			
#_COMPUTED_VAL					
y1-delta_y	=B29-B33				
y1^2-y2^2	=B29^2-B30^2				
y2*delat_y	=B33*B30				
u1^2	=B28^2				
uj^2	=B34^2				
#_COMPUTED_VAL					
	Term_1	=B40		Term_1	=-B39*B36/B33
	Term_2	=-B40*B33/B30		Term_2	=B39*B36*B36/B38
	Term_3	=(2*B28*B36*B34)/B3		Term_3	=-0.5*(B31*B37)/B33
	LHS	=C44+C45+C46		RHS	=F44+F45+F46
	DIFF	=C48-F48			
#_INPUT					
u1	1	Approach Speed			
y1	4	Approach Depth			
y2	4.0932	Exit Depth			
g	32.2	Gravitational Constan			
#_DESIGN_VARIABL					
delta_y	0.8	Jet Depth (Area)			
uj	5.60898737282977	Jet Speed			
#_COMPUTED_VAL					
y1-delta_y	=B54-B58				
y1^2-y2^2	=B54^2-B55^2				
y2*delat_y	=B58*B55				
u1^2	=B53^2				
uj^2	=B59^2				
#_COMPUTED_VAL					
	Term_1	=B65		Term_1	=-B64*B61/B58
	Term_2	=-B65*B58/B55		Term_2	=B64*B61*B61/B63
	Term_3	=(2*B53*B61*B59)/B5		Term_3	=-0.5*(B56*B62)/B58
	LHS	=C69+C70+C71		RHS	=F69+F70+F71
	DIFF	=C73-F73			

Nozzle_Design

10 ft/sec design

Diameter(in)	Area(ft^2)	Number	Specify	Total_Actual_Jet_Area
1	0.005454	146.7	147	0.80
2	0.021817	36.7	37	0.81
3	0.049087	16.3	17	0.83
4	0.087266	9.2	10	0.87
6	0.19635	4.1	5	0.98
8	0.349066	2.3	3	1.05

7 ft/sec design

Diameter(in)	Area(ft^2)	Number		Total_Actual_Jet_Area
1	0.005454	293.4	293	1.60
2	0.021817	73.3	73	1.59
3	0.049087	32.6	34	1.67
4	0.087266	18.3	19	1.66
6	0.19635	8.1	9	1.77
8	0.349066	4.6	5	1.75

5.6 ft/sec design

Diameter(in)	Area(ft^2)	Number		Total_Actual_Jet_Area
1	0.005454	440.0	441	2.41
2	0.021817	110.0	111	2.42
3	0.049087	48.9	49	2.41
4	0.087266	27.5	28	2.44
6	0.19635	12.2	13	2.55
8	0.349066	6.9	7	2.44

All these specifications will work but:

Small diameters require a lot of nozzles, and these would be hard to maintain -- thus discard 1,2,3 in.

8-in. would require a lot of head to generate required jet speeds

Either 30 4-in nozzles or 15 6-in. would be good design. Using manifolds and valving one could select nozzle "banks" to produce desired fish race at variable flow speeds.