8.4 DETERMINE WHICH OF FOLLOWING ARE DIMENSIONALLY HOMOGENEOUS.

a) $Q = \frac{2}{3} C_L \sqrt{2g}\, H^{3/2}$

(C_L: DIMENSIONLESS; $\sqrt{2g}$: CONSTANT)

DIMENSIONAL HOMOGENEOUS MEANS

$$[\#] = [\#]$$

$$\left[\frac{L^3}{T}\right] = [L]\left[\frac{L}{T^2}\right]^{1/2}[L]^{3/2} = [L]\left[\frac{L^2}{T}\right] = \left[\frac{L^3}{T}\right]$$

HOMOGENEOUS

b) $V = \frac{1.49}{n} A R^{2/3} S^{1/2}$

($\frac{1.49}{n}$: CONSTANT)

$$\left[\frac{L}{T}\right] = [L]^{1/6}[L]^{2/3} = [L]^{-1/6}[L]^{4/6} = [L]^{3/6}$$

NOT-HOMOGENEOUS

c) $h_f = f \frac{L}{D} \frac{V^2}{2g}$

(f: DIMENSIONLESS; NO NEED TO CONSIDER; $\frac{1}{2g}$: CONSTANT)

$$[L] = \frac{[L]}{[L]} \frac{\left[\frac{L}{T}\right]^2}{\left[\frac{L}{T^2}\right]} = \frac{[L]^2}{[T]^2} \cdot \frac{[T]^2}{[L]} = [L]$$

HOMOGENEOUS

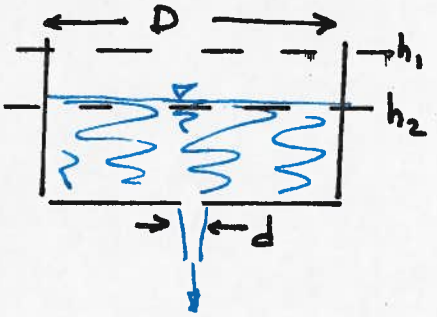
d) $D = \frac{0.074}{Re^{0.2}} \left(\frac{B \times \rho V^2}{2}\right)$

(Re: DIMENSIONLESS; $\frac{0.074}{Re^{0.2}}$: CONSTANT)

$$\left[\frac{ML}{T^2}\right] = [L][L][M]\frac{[L]^2}{[L^3][T]^2} = \frac{[M][L]}{[T^2]}$$

HOMOGENEOUS

8.6) TIME TO DROP $\Delta h = h_1 - h_2$ IN TANK OF DIAMETER D THROUGH HOLE d. FIND π-GROUPS



1) ID SIGNIFICANT VARIABLES

$\Delta h, d, D, \rho, g, t, h_1$

$\Delta h = h_1 - h_2$ (SO REALLY SURROGATE FOR h_2)

EXPRESS AS FUNCTIONAL RELATION

$$\frac{\Delta h}{d} = f(D, d, \rho, g, t, h_1)$$

2) π-GROUPS:

7 VARIABLES

3 FUNDAMENTAL DIMENSIONS L, M, T

V	[ ]	V	[ ]
Δh	L	$\frac{\Delta h}{d}$	$\frac{L}{L}=1$
h_1	L	$\frac{h_1}{d}$	$\frac{L}{L}=1$
d	L	$\frac{d}{d}$	$\frac{L}{L}=1$
D	L	D/d	$\frac{L}{L}=1$
ρ	M/L^3	ρd^3	M
t	T	t	T
g	L/T^2	$\rho g d^2$	$\frac{M}{T^2}$

VARIABLE DIVIDE BY ITSELF

$\frac{\rho d^3}{\rho d^3} = \frac{M}{M} = 1$

$\frac{\rho g d^2 t^2}{\rho d^3} \frac{MT^2}{MT^2} = 1$

$\frac{\rho g d^2 \cdot t^2}{\rho d^3} = \frac{g t^2}{d}$

⇐ 3) SET UP TABLE
COMBINE VARIABLE(S) TO ELIMINATE DIMENSION

$\frac{\Delta h}{d}, \frac{h_1}{d}, \frac{D}{d}$ EASY 3-π GROUPS.

GET FOURTH COMBINING ρ, t, g

4) EXPRESS RESULT AS CORRELATION FUNCTION

$$\frac{\Delta h}{d} = f\left(\frac{D}{d}, \frac{g t^2}{d}, \frac{h_1}{d}\right)$$

8.12) CENTRIFUGAL PUMP

Δp IS KNOWN TO BE FUNCTION OF IMPELLER DIAMETER D, ANGULAR SPEED n, DISCHARGE Q, AND DENSITY ρ. FIND π-GROUPS RELATING THESE VARIABLES.

1) IDENTIFY ALL SIGNIFICANT VARIABLES EXPRESS AS FUNCTIONAL RELATION

$$\Delta p = f(D, n, Q, \rho)$$

2) π-GROUPS

5 VARIABLES

3 DIMENSIONS $\quad 5-3 = 2\ \pi$-GROUPS

3) SET-UP TABLE

V	[ ]					
Δp	$\frac{M}{LT^2}$	$\Delta p D$	$\frac{M}{T^2}$	$\frac{\Delta p}{\rho D^3}$	$\frac{1}{T^2}$	$\frac{\Delta p D}{\rho D^3 n^2} \frac{T^2}{T^2} = 1$
D	L	$\frac{D}{D}$	$\frac{L}{L} = 1$			
n	$\frac{1}{T}$					
Q	$\frac{L^3}{T}$	$\frac{Q}{D^3}$	$\frac{1}{T}$	$\frac{nQ}{D^3}$	$\frac{T}{T} = 1$	
ρ	$\frac{M}{L^3}$	ρD^3	M			

COMBINE VARIABLES TO ELIMINATE DIMENSION

4) EXPRESS RESULT AS CORRELATION FUNCTION

$$\frac{\Delta p D}{\rho D^3 n^2} = f\left(\frac{nQ}{D^3}\right)$$ SIMPLIFY TO $$\frac{\Delta p}{\rho D^2 n^2} = f\left(\frac{nQ}{D^3}\right)$$