CE 3305 Fluid Mechanics; Exam 1 Name: SOLUTION

CE 3305 Fluid Mechanics
Exam 1
Spring 2014

1. A design team is developing a CO_2 cartridge for a rubber raft. The cartridge will allow a flight crew to rapidly inflate the raft to escape a downed and sinking aircraft. The raft is shown in Figure 1. The raft can be conceptualized as two parallel long tubes, and four parallel short tubes. The desired inflation pressure is 3 psig. Estimate the raft volume when inflated and the mass of CO_2 required, in grams, in the cartridge.

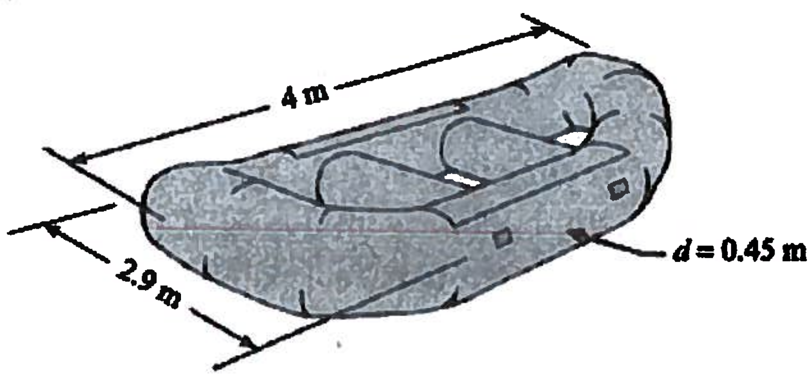
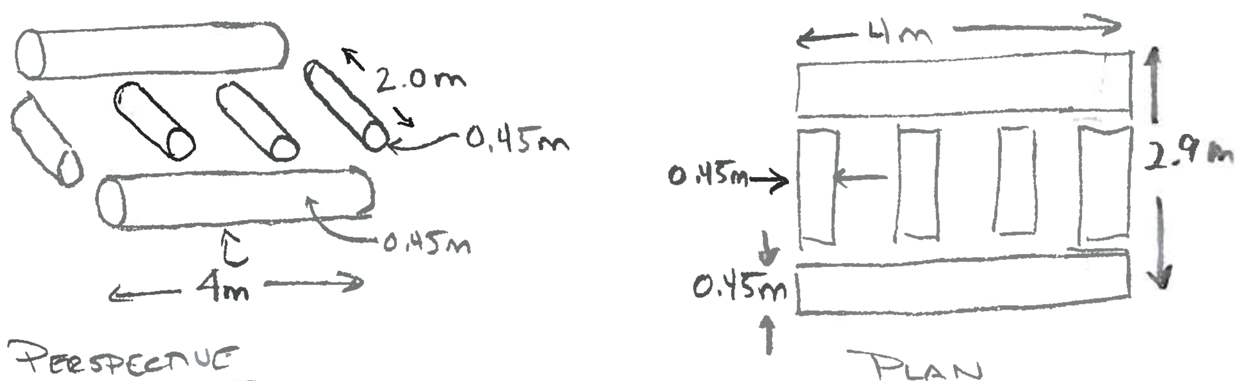


Figure 1: Aircrew escape raft, typical dimensions

SKETCH



Problem 1 (Continued)

GOVERNING EQUATIONS

$$p\forall = \frac{m}{M} RT$$

$$\forall_{CYL} = \frac{\pi d^2}{4} \cdot L$$

GIVEN

DIMENSIONS PRIOR PAGE.

$$p = 3\,psig = \frac{3}{14.75} = 0.203\,atm\text{-}gage$$
$$= 1.203\,atm\text{-}abs$$

$$M_{CO_2} = \underset{\overset{\uparrow}{C}}{12} + 2(\underset{\overset{\uparrow}{O}}{16}) = 44\,g/mol$$

from periodic chart, quiz 1

$T = ?$ ASSUME $20°C = 273 + 20 = 293K$

FIND

m_{CO_2} IN GRAMS

$\forall_{RAFT}$

SOLUTION

$$\forall_{RAFT} = \forall_{LONG\ TUBES} + \forall_{SHORT\ TUBES}$$

$$= 2\frac{\pi (0.45m)^2}{4}(4m) + \frac{4\pi (0.45m)^2}{4}(2m) = \underline{\underline{2.54m^3}} \longleftarrow \forall_{RAFT}$$

$$m_{CO_2} = \frac{p\forall}{RT} M_{CO_2} = \frac{(1.203atm)(2.54m^3 \frac{1000L}{m^3})(44g/mol)}{(0.0821 \frac{L\cdot atm}{K\cdot mol})(293K)}$$

$$= \underline{\underline{5{,}589\,g_{CO_2}}} \longleftarrow m_{CO_2}$$

REMARKS

1) 5.5 kg CO_2 WOULD BE A CARTRIDGE THE SIZE OF A FIRE EXTINGUISHER

2) NEED TO EXPRESS WORKING PRESSURE AS ABSOLUTE FOR IDEAL GAS LAW (AND IN ATMOSPHERE)

3) COULD USE GAS SPECIFIC R FROM BOOK AND GET SAME RESULT.

2. Emulsion with a specific gravity $SG = 0.85$ and viscosity $\mu = 2.15 \times 10^{-3}$ lbf-sec/ft^2 flows steadily down an inclined surface as depicted in Figure 2 in a film of thickness of h=0.125 inches. The velocity profile in the film is

$$V = \frac{\rho g}{\mu}(hy - \frac{y^2}{2})sin\theta \qquad (1)$$

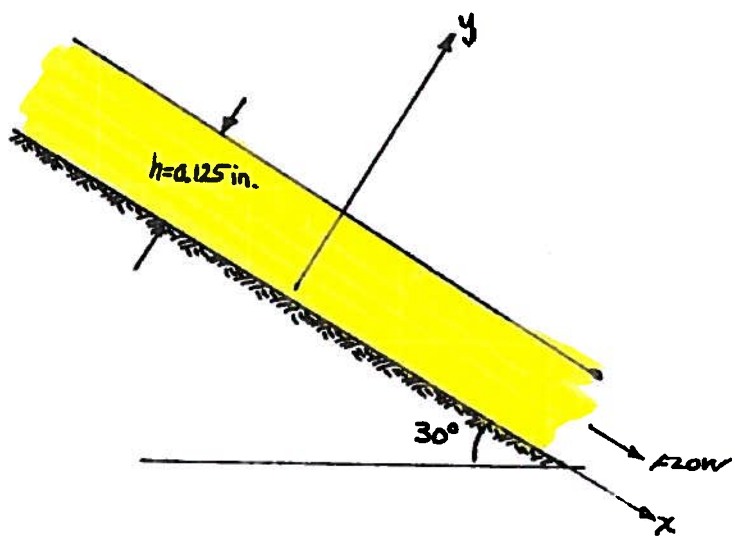


Figure 2: Emulsion flowing down inclined plane

Compute the velocity in the emulsion at y=0, y=h/4, y=h/2, and y=h. Plot the profile (V versus y, y axis is plotted up). Determine the magnitude and direction of shear stress that acts on the inclined surface.

given

$\mu = 2.15 \cdot 10^{-3}$ lbf-s/ft^2

$\rho g = 0.85(62.4 \text{ lbf/ft}^3) = 53.04 \text{ lbf/ft}^3$

$h = 0.125 \text{ in} = \frac{0.125}{12} = 0.01041 \text{ ft}$

FIND

PLOT PROFILE

FIND τ_{WALL}

Name: SOLUTION

Problem 2 (Continued)

	y (ft)	V (ft/s)
	0	0
$h/4 =$	0.0026	0.291 ft/s
$h/2 =$	0.0052	0.5005 ft/s
$h =$	0.0104	0.668 ft/s

ARITHMETIC

$$V(y) = \frac{53.04 \frac{lbf}{ft^3}}{\left(2.15\cdot 10^{-3} \frac{lbf\cdot s}{ft^2}\right)}\left[(0.0104)y - \frac{y^2}{2}\right]\sin 30$$

$$= 12334.8\left[0.0104y - \frac{y^2}{2}\right] \text{ ft/s}$$

$0.0104(0.0026) = 0.00002704$

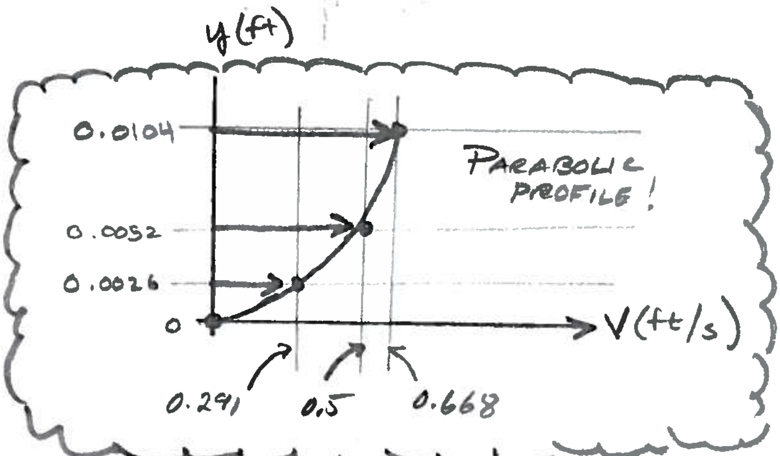
$0.0104(0.0052) = 0.00005408$

$0.0104(0.0104) = 0.00010816$

$\frac{0.0026^2}{2} = 0.00000338$

$\frac{0.0052^2}{2} = 0.0000135$

$\frac{0.0104^2}{2} = 0.000054$



PROFILE PLOT

$$\tau_{WALL} = \mu \frac{dV}{dy}\bigg|_{y=0}$$

$$\frac{dV}{dy} = \frac{d}{dy}\left[\left(\frac{\gamma g}{\mu}\sin\theta\right)\left(hy - \frac{y^2}{2}\right)\right]$$

$$= \frac{\gamma g \sin\theta}{\mu}(h-y)$$

$$\tau_{WALL} = \cancel{\mu}\frac{\gamma g\sin\theta}{\cancel{\mu}}(h-y)\bigg|_{y=0}$$

$$= \gamma g \sin\theta\, h$$

$$= \gamma g \sin 30^\circ h = \frac{\gamma g h}{2}$$

REVISION A

$$\tau_{WALL} = \frac{(53.04 \text{ lbf/ft}^3)(0.0104 \text{ ft})}{2} = \underline{0.276 \text{ lb/ft}^2} \leftarrow \tau_{WALL}$$

IN +X DIRECTION

3. A device for measuring the specific weight of a fluid consists of a U-tube manometer as shown. The manometer tube has an internal diameter of 0.5 cm and originally has water in it. Exactly 2 cm^3 of unknown liquid is poured into one leg of the manometer, and a displacement of 5 cm is observed between the surfaces as shown in Figure 3. What is the specific weight of the unknown liquid?

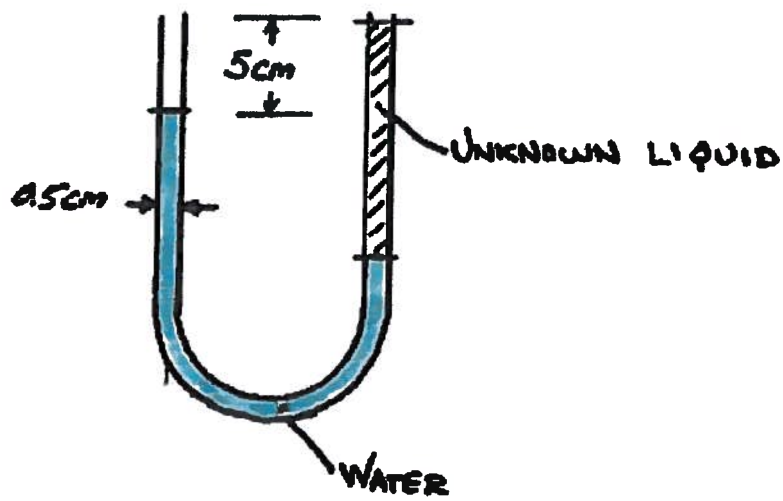


Figure 3: YouTube Manometer for Unknown Liquid

Ha Ha!

GIVEN

$\gamma_{H_2O} = 9800 N/m^3$

$d = 0.5 cm$

$\forall_{UNK} = 2 cm^3$

FIND

$\gamma_{UNKNOWN}$

EQUATION(S)

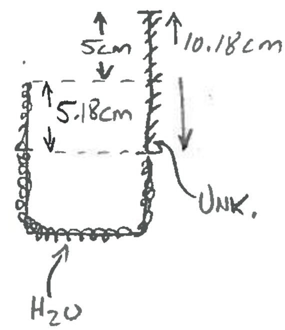
$\forall_{UNK} = \left(\frac{\pi d^2}{4}\right) L_{UNK}$

$h = \frac{p}{\gamma}$

Problem 3 (Continued)

SOLUTION

$$L_{UNK} = \frac{\forall_{UNK}}{\frac{\pi d^2}{4}} = \frac{2\,cm^3}{\frac{\pi (0.5cm)^2}{4}} = 10.18cm$$



$$\gamma_{H_2O}\, h_{H_2O} = \gamma_{UNK}\, h_{UNK}$$

$$\frac{\gamma_{H_2O}\, h_{H_2O}}{h_{UNK}} = \gamma_{UNK}$$

$$\frac{(9800 N/m^3)(0.0518m)}{(0.1018m)} = \gamma_{UNK} = \underline{\underline{4986.6\ N/m^3}} \longleftarrow \gamma_{UNKNOWN}$$

4. A rectangular gate is hinged as shown in Figure 4. The gate is 2 meters tall and 6 meters wide. Find the force required at the bottom of the gate to keep it closed.

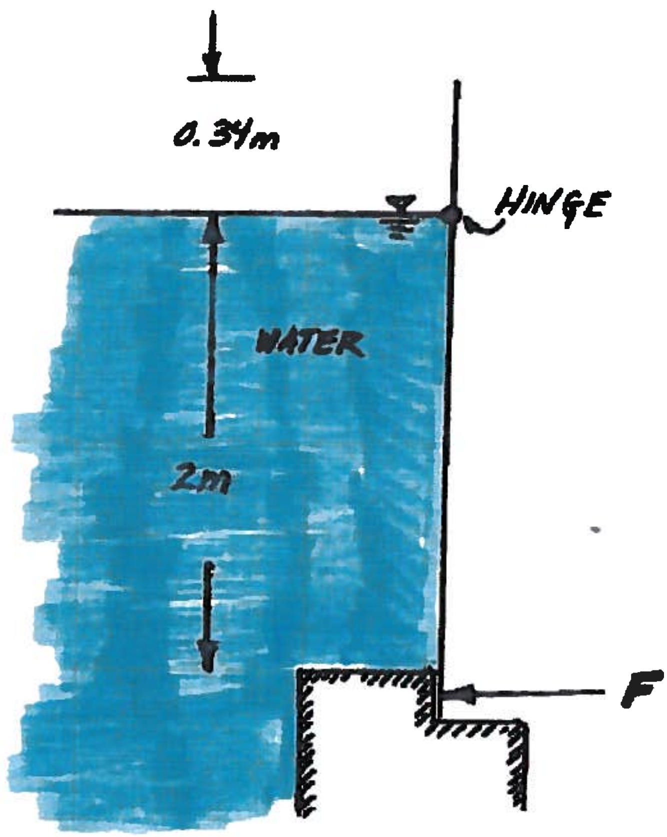


Figure 4: Rectangular gate

GIVEN

$\gamma_{H_2O} = 9800 N/m^3$

DEPTH = 2m

FIND

F

EQUATIONS

$F_p = \bar{p}A$

$\bar{p} = \gamma \bar{h}$

$+\circlearrowright \Sigma M_H = 0$

Problem 4 (Continued)

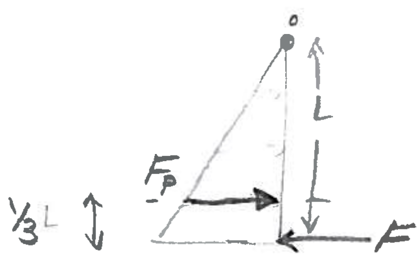
$\bar{p} = \rho g \bar{h}$ $\bar{h}$ = DEPTH TO CENTROID OF PLATE

$\bar{h} = 1m$

$\bar{p} = (9800 N/m^3)(1m) = 9800 N/m^2$

$A = (2m)(6m) = 12 m^2$

$F_p = (9800 N/m^2)(12 m^2) = 117600 N$



$\circlearrowleft \Sigma M_o = F_p\left(\frac{2}{3} 2m\right) - F(2m)$

$F = \dfrac{F_p\left(\frac{2}{3}\right)(2m)}{2m} = \frac{2}{3} F_p$

$= \frac{2}{3}(117600 N) = \underline{\underline{78,400 N}} \leftarrow F$

5. A floating platform as shown in Figure 4 is moored (tied to the earth) with 4 polypropylene lines. The platform itself weighs 30kN, the pontoons are cylinders 1 meter in diameter, and 16 meters long. The platform is to be moored with a freeboard of 1 meter. What is the tension in each mooring line?

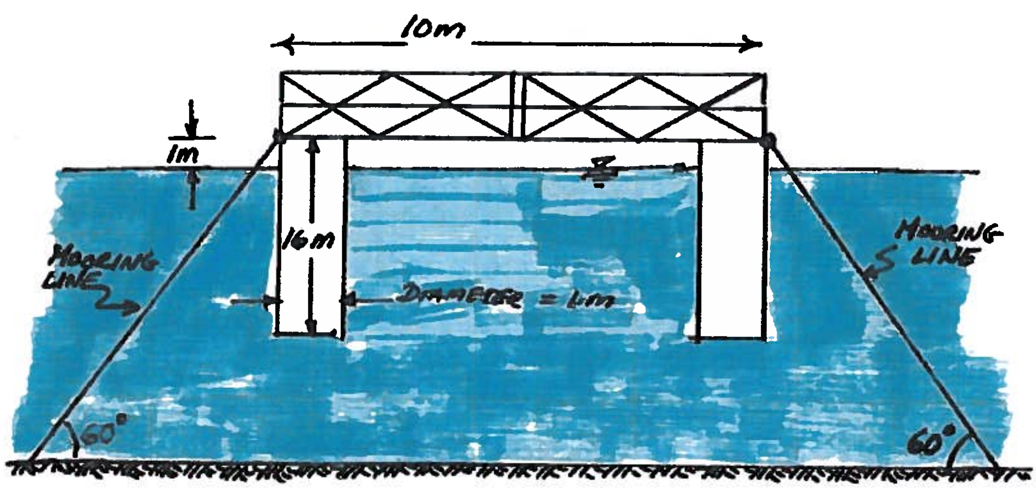
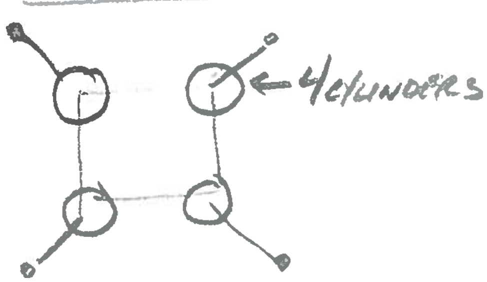


Figure 5: Floating platform (elevation view)

SKETCH



EQUATIONS

$$\Sigma F = m\cancel{a}^{0} = 0$$

$$F_B = \rho g V_{displaces}$$

FIND

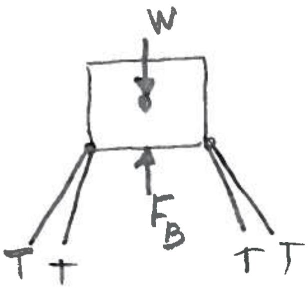
T IN MOORING LINE

KNOWN

$\rho g = 9800 N/m^3$

Problem 5 (Continued)

SOLUTION



HAS 4 PONTOONS; 4 MOORING LINES

$$\Sigma F_y = F_B - W - 4(T \sin 60) = 0$$

$$F_B = 4\,(9800\,N/m^3)(15m)\left(\frac{\pi 1^2}{4}\right) = 461{,}814\,N$$

$$W = 30{,}000\,N$$

$$\frac{461814 - 30{,}000}{4 \sin 60} = T = \underline{\underline{124653N}} \text{ per line}$$

** ALTERNATE (IF INTERPRET AS ONLY 2 PONTOONS) **

$$F_B = 2\left(9800 \frac{N}{m^3}\right)(15m)\left(\frac{\pi 1^2}{4}\right) = 230907\,N$$

$$W = 30{,}000\,N$$

$$\frac{230907 - 30000}{4 \sin 60} = T = \underline{\underline{57{,}996N}} \text{ per line}$$

REMARKS 1) THE PROBLEM DID NOT STATE 4 PONTOONS, SO WORKED BOTH WAYS.