

## PROBLEM 10-8

**Given** → As shown, water (15°C) flows from a tank through a tube and discharges to ambient. The tube has an ID of 8 mm, and  $L = 6\text{ m}$ . The resistance coefficient is  $f = 0.15$ . The water level is  $H = 3\text{ m}$ .

**Find** → The exit velocity in m/s. Sketch the HGL and EGL.

**Sketch**

**Assumptions** → The only head loss is in the tube  
Turbulent flow so  $\alpha_2 = 1.0$

**Solution** →  $T = 15^\circ\text{C} \Rightarrow \text{Table A.5} \Rightarrow \rho = 999\text{ kg/m}^3$   
 $\nu = 1.14 \times 10^{-6}\text{ m}^2/\text{s}$

$D = 0.008\text{ m}$   
 $L = 6\text{ m}$   
 $H = 3\text{ m}$   
 $f = 0.15$



Using the energy equation → location ① at free surface  
location ② at pipe exit

$$\frac{p_1}{\rho} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\rho} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$

$$\Rightarrow 0 + 0 + H + 0 = 0 + \alpha_2 \frac{V_2^2}{2g} + 0 + h_L$$

$$\Rightarrow H = \alpha_2 \frac{V_2^2}{2g} + h_L \quad \leftarrow \text{equation ①}$$

Using Darcy-Weisbach equation →

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \quad \leftarrow \text{equation ②}$$

Combining eq's ① and ② →

$$H = \frac{V_2^2}{2g} \left[ 1 + f \frac{L}{D} \right] \Rightarrow V_2 = \sqrt{\frac{2gH}{1 + f \frac{L}{D}}}$$

$$V_2 = \sqrt{\frac{2(9.81)(3\text{ m})}{1 + 0.15 \frac{(6\text{ m})}{(0.008\text{ m})}}} \Rightarrow \boxed{V_2 = 2.19\text{ m/s}}$$

Flow rate equation  $\rightarrow$

$$Q = VA = V \frac{\pi D^2}{4}$$

$$Q = \frac{2.192 \text{ m}}{s} \cdot \frac{\pi}{4} \cdot \frac{(0.008 \text{ m})^2}{4}$$

$$Q = .110 \text{ L/s}$$

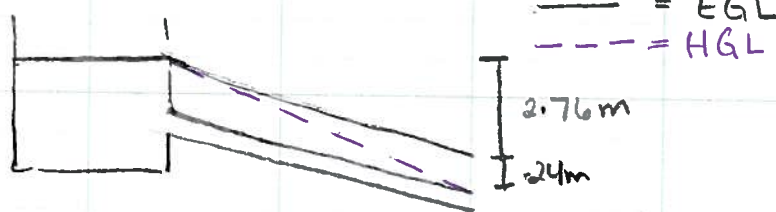
Sketch HGL and EGL  $\rightarrow$

locate HGL and EGL on the free surface of the tank

$$\text{Velocity head} \rightarrow \frac{V^2}{2g} = \frac{(2.192)^2}{2(9.81)} = .24 \text{ m}$$

$$\text{head loss} \rightarrow h_f = f \frac{L}{D} \frac{V^2}{2g} = \frac{.015(6 \text{ m})}{(0.008 \text{ m})} \cdot \frac{(2.192)^2}{2(9.81)}$$

$$h_f = 2.76 \text{ m}$$



Discussion  $\rightarrow$  For this problem to be correct, we worked with the assumption that the flow was turbulent. To check this assumption we use

$$Re = \frac{VD}{\nu} \Rightarrow \frac{2.192(0.008 \text{ m})}{1.14 \times 10^{-6} \text{ m}^2/\text{s}}$$

$$Re = 15400 > 3000$$

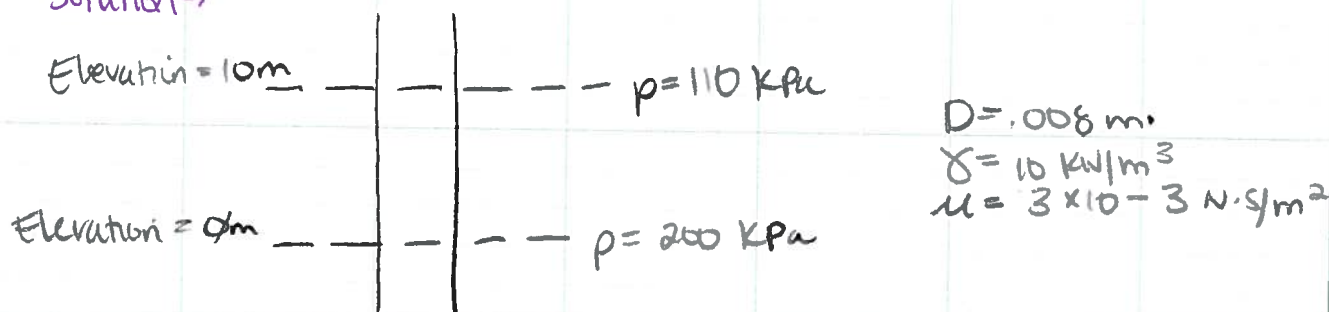
Therefore, our assumption that the flow is turbulent is correct.

## PROBLEM 10-15

Given  $\rightarrow$  Liquid ( $\gamma = 10 \text{ kN/m}^3$ ) is flowing in a pipe at steady rate, but the direction of the flow is unknown.  
The pipe diameter is 8mm and the liquid viscosity is  $3 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$ .

Find  $\rightarrow$  If the liquid is moving upward or downward.  
The magnitude of the mean velocity in the pipe

Solution  $\rightarrow$



Piezometric Head  $\rightarrow$

Location ① @  $z = 10 \text{ m}$

$$h_1 = \frac{p_1}{\gamma} + z_1 = \frac{110 \text{ kPa}}{(10 \text{ kN/m}^3)} + 10 \text{ m} = 21 \text{ m}$$

Location ② @  $z = 0 \text{ m}$

$$h_2 = \frac{p_2}{\gamma} + z_2 = \frac{200 \text{ kPa}}{(10 \text{ kN/m}^3)} + 0 \text{ m} = 20 \text{ m}$$

Since  $h_1 > h_2$ , the direction of flow is downward

Energy equation  $\rightarrow$

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

$$\Rightarrow \frac{p_1}{\gamma} + 0 + z_1 + 0 = \frac{p_2}{\gamma} + 0 + z_2 + h_f$$

$$21 \text{ m} = 20 \text{ m} + h_f$$

$$\Rightarrow h_f = 1 \text{ m}$$



Head loss (laminar flow)  $\rightarrow$

$$h_f = \frac{32\mu LV}{\gamma D^2} = V = \frac{h_f \gamma D^2}{32\mu L}$$

$$V = \frac{1\text{m}}{\text{m}^3} \cdot \frac{10000\text{ N}}{\text{m}^3} \cdot \frac{(0.06\text{m})^2}{32} \cdot \frac{\text{m}^2}{3 \times 10^{-3}\text{ N}\cdot\text{s}} \cdot \frac{1}{10\text{m}}$$

$$V = .667\text{ m/s}$$

**Discussion**  $\rightarrow$  To find the direction the flow moves, we compare the piezometric head values at two points. The direction of flow is always from a greater head to a lower head so since  $h_1 > h_2$ , the flow is downward.

To find the mean velocity, first the energy equation was used to solve for  $h_f$  in the pipe. Using this value, and the equation for head loss with laminar flow, I could plug in all known variables to solve for the velocity in the pipe.

## PROBLEM 10-71

**Given** → An engineer is making an estimate of hydroelectric power for a home owner. This owner has a small stream ( $Q = 2 \text{ cfs}$ ,  $T = 40^\circ\text{F}$ ) that is located at elevation  $H = 34 \text{ ft}$  above the owner's residence. The owner is proposing to divert the stream and operate a water turbine connected to an electric generator to supply electrical power to the residence. The max acceptable head loss in the penstock is 3 ft. The penstock has a length of 87 feet. If the penstock is going to be fabricated from commercial grade plastic pipe, find the minimum diameter that can be used. Neglect component head losses. Assume pipes are available in even sizes.

**Find** → Minimum diameter for the penstock pipe.

**Given data** → Stream →  $Q = 2 \text{ cfs}$   
 $T = 40^\circ\text{F}$   
 $H = 34 \text{ ft}$   
Penstock →  $h_L = 3 \text{ ft}$   
 $L = 87 \text{ ft}$

**Assumptions** → Neglect minor head losses.  
Assume smooth plastic pipe →  $K_s = 0$   
Assume turbulent flow.

**Solution** → In order to do this problem we must first find the Moody friction factor using Reynolds number and the Moody chart. The chart is shown on the following page.

$$f (\text{smooth pipe curve}) = .015$$

Guess and check method → For the following 3 equations, we need to guess a value for  $D$  and see which gets closest to the correct value for max head loss.

$$\text{Eq. ① } Q = VA \Rightarrow V = Q/A$$

$$\text{Eq. ② } Re = \frac{VD}{\nu} \quad V @ 40^\circ\text{F} = 1.664 \times 10^{-5} \text{ ft}^2/\text{s}$$

$$\text{Eq. ③ } h_f = f \frac{L}{D} \frac{V^2}{2g}$$

$h_f$  cannot exceed 3 ft





$D_{\text{guess}}$	$V$ (eq. 1)	$Re$ (eq. 2)	$n_f$ (eq. 3)	
20 in	.9172	91866	10.60	too high
30 in	.408	61244	1.396	too low
26 in	.592	70666	2.85	✓

$D$  should be 26 in

Discussion → The actual number we are trying to get for  $h_f$  is 3 ft max but since the pipes only come in even diameters, we choose 26 in for the final  $D$  because any smaller and the head loss would exceed 3 feet and any larger the cost would increase.

Material	$e$ (ft)	$e$ (mm)
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Cast iron	0.00085	0.25
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015

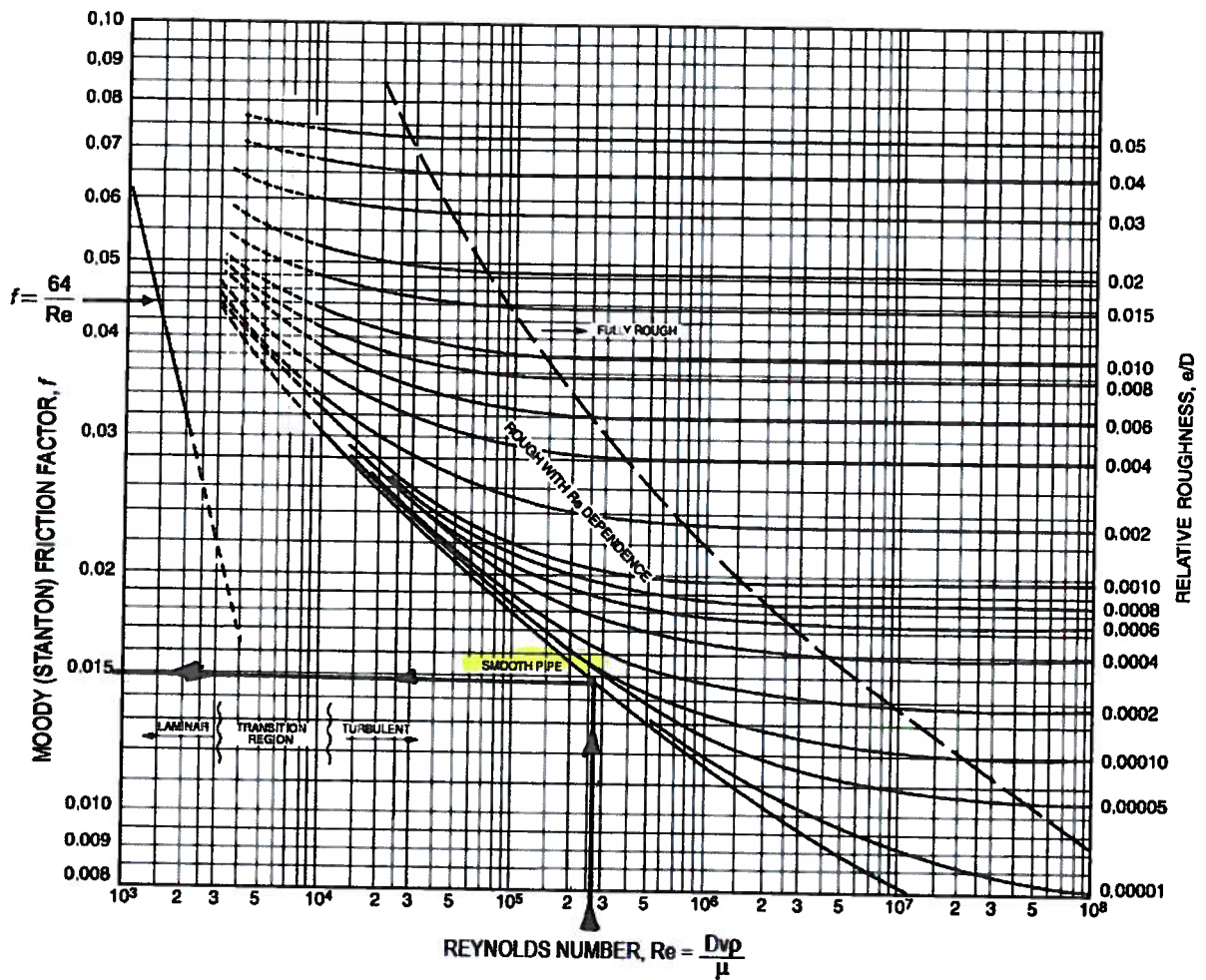


Figure 2: Moody-Stanton Diagram (from CITE NCEES).