Name: SOLUTION

CE 3305 Fluid Mechanics
Exam 2
Spring 2014

1. A 6-in. diameter cylinder falls at a rate of 4 ft/s in an 8-in. diameter tube containing an incompressible liquid. What is the mean velocity of the liquid (with respect to the tube) in the space between the falling cylinder and the tube wall?[1]

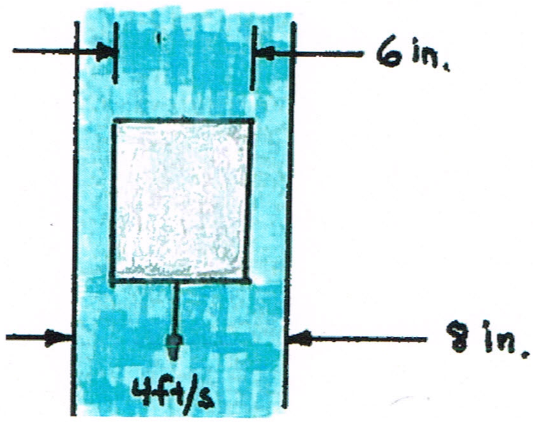
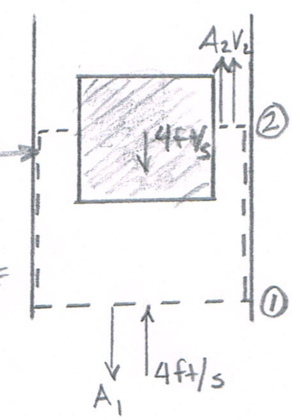


Figure 1: Falling cylinder in a tube.

C.V. ATTACHED TO AND MOVING WITH CYLINDER

C.V. MOVES DOWN AT 4ft/s; V_1 RELATIVE TO CV IS 4ft/s

SKETCH CV, AREAS, FLOWS/VELOCITY, STATE CV MOVES WITH CYLINDER +5



KNOWN

$$A_1 = \frac{\pi\left(\frac{8}{12}\right)^2}{4} = 0.349\ ft^2$$ +2 #, UNITS

$$A_2 = \frac{\pi\left(\frac{8}{12}\right)^2}{4} - \frac{\pi\left(\frac{6}{12}\right)^2}{4} = 0.153\ ft^2$$ +2 #, UNITS

[1] Continunity

CONTINUED ↓

20 POINTS TOTAL THIS PROBLEM

Problem 1 (Continued)

GOVERNING EQUATION(S)

CONTINUNITY (+1)

$$0 = \frac{d}{dt}\int_{CV} \rho \, d\forall + \int_{C.S.} \rho \bar{V} \cdot d\bar{A} \quad (+1)$$

$0; \ \frac{d\forall}{dt} = 0$

$$0 = -\rho A_1 V_1 + \rho A_2 V_2 \quad (+1)$$

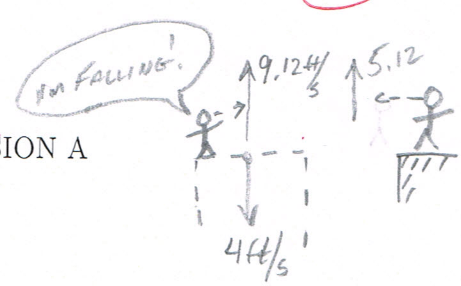
WATER, ρ = CONSTANT (+1)

$\therefore$

$$A_1 V_1 = A_2 V_2 \qquad V_2 = \frac{A_1 V_1}{A_2} \ (+1) = \frac{(0.349 ft^2)(4 ft/s)}{(0.153 ft^2)} = 9.12 ft/s \quad (+2) \text{ \#, UNITS}$$

HOWEVER, C.S. (2) IS MOVING DOWNWARD AT 4 ft/s

$$\therefore \ V = V_2 - V_{CYLINDER} \ (+2) = 9.12 ft/s - 4 ft/s = 5.12 ft/s \quad (+2) \text{ \#, UNITS}$$



2. How long will it take the water surface to drain from 9 ft. to 1 ft. for the 12 ft. diameter tank in Figure 2. Assume the outlet pipe is 2 in. in diameter and the pressure at the outlet is atmospheric.[2]

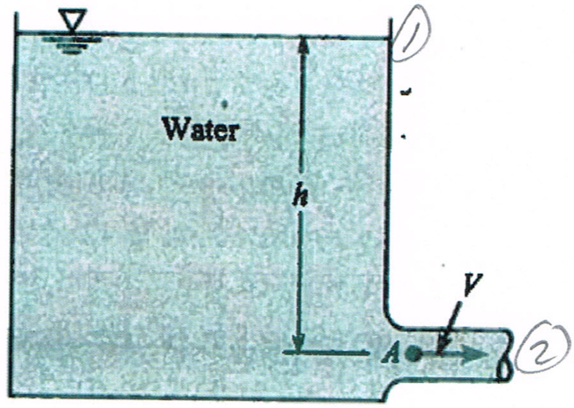
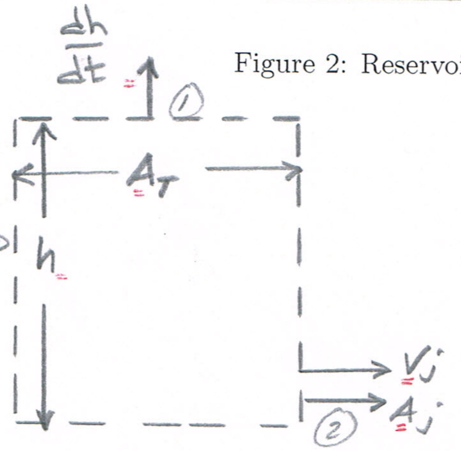


Figure 2: Reservoir draining through a pipe.

C.V. IS FREE SURFACE TO JET. TOP MOVES AT $\frac{dh}{dt}$



SKETCH OF CV WITH FLOWS, AREAS AND NOTE CV MOVES WITH FREE SURFACE (+5)

KNOWN

$$A_T = \frac{\pi(12)^2}{4} = 113.09\ ft^2 \quad (+2)\ \#,\ \text{UNITS}$$

$$A_j = \frac{\pi\left(\frac{2}{12}\right)^2}{4} = 0.022\ ft^2 \quad (+2)\ \#,\ \text{UNITS}$$

BERNOULLI FROM FREE SURFACE TO JET (+1)

$$\underbrace{\frac{p_1}{\gamma}}_{0\ gage} + \underbrace{\frac{V_1^2}{2g}}_{\text{Assume} \approx 0\ \left(\text{Actually } \frac{(dh/dt)^2}{2g}\right)} + \overbrace{z_1}^{h} = \underbrace{\frac{p_2}{\gamma}}_{0\ gage} + \frac{V_2^2}{2g} + \underbrace{z_2}_{0\ datum} \quad (+1)$$

$$\therefore\ h(t) = \frac{V^2}{2g}$$

$$V(t) = \sqrt{2gh(t)} \quad (+1)$$

[2] Bernoulli, Continunity, Related Rates

20 POINTS TOTAL THIS PROBLEM

Problem 2 (Continued)

CONTINUITY (+1)

$$0 = \frac{d}{dt}\int_{CV} \rho \, dV + \int_{CS} \rho \vec{V} \cdot d\vec{A} \quad = (+1)$$

$$0 = \cancel{\rho} \cdot \frac{dh}{dt} \cdot A_T + \cancel{\rho} V_j A_j \quad = (+1)$$

WATER ρ = CONSTANT

$$\therefore \quad \frac{dh}{dt} = -\frac{V_j A_j}{A_T} = -\frac{A_j}{A_T}\sqrt{2gh}$$

SEPERATE & INTEGRATE

$$\frac{1}{\sqrt{h}} dh = -\frac{A_j}{A_T}\sqrt{2g} \, dt \quad (+1)$$

$$\int \frac{dh}{\sqrt{h}} = \frac{-A_j \sqrt{2g}}{A_T} \int dt$$

CONSTANT OF INTEGRATION

$$2\sqrt{h} = -\frac{A_j \sqrt{2g}}{A_T} t + C \quad (+1)$$

(DEFINITE INTEGRAL OK TOO!)

Problem 2 (Continued)

APPLY BOUNDARY (INITIAL) CONDITIONS

+1 (IF DEFINITE INTEGRAL THEN $\int_1^9 = -\int_9^1$) MAY SWITCH DEPENDING ON SIGN LOCATION

$t=0,\ h=9ft$

$2\sqrt{9ft} = c$

$\therefore$

$$2\sqrt{h} = -\frac{A_j\sqrt{2g}}{A_T}t + 2\sqrt{9ft}$$

SET $h = 1ft$, SOLVE FOR t

$$-(2\sqrt{1} - 2\sqrt{9})\frac{A_T}{A_j\sqrt{2g}} = t$$

$$-(2-6)\left(\frac{113.09}{0.022}\right)\left(\frac{1}{\sqrt{2(32.2)}}\right) = 2562.23 \text{ Seconds}$$

+2 #, UNITS

 Name: SOLUTION

3. A turbofan engine (Figure 3) on a commercial jet takes in air , a portion of which passes through the compressor, combustion chamber where fuel is added and ignited, and the turbine, the remainder of the air bypasses the compressor and is accelerated by the fans. The mass flow rate of bypass air to the mass flow rate through the compressor-combuster-turbine path is called the "bypass ratio." The total flow rate of air entering the turbofan engine is 300 kg/s with a velocity relative to the engine of 300 m/s. The engine has a bypass ratio of 2.5. The bypass air exits at 600 m/s, whereas the air through the compressor-combustor-turbine path exits at 1000 m/s. What is the thrust generated by the turbofan engine? Clearly show the control volume and application of the linear momentum equation.[3]

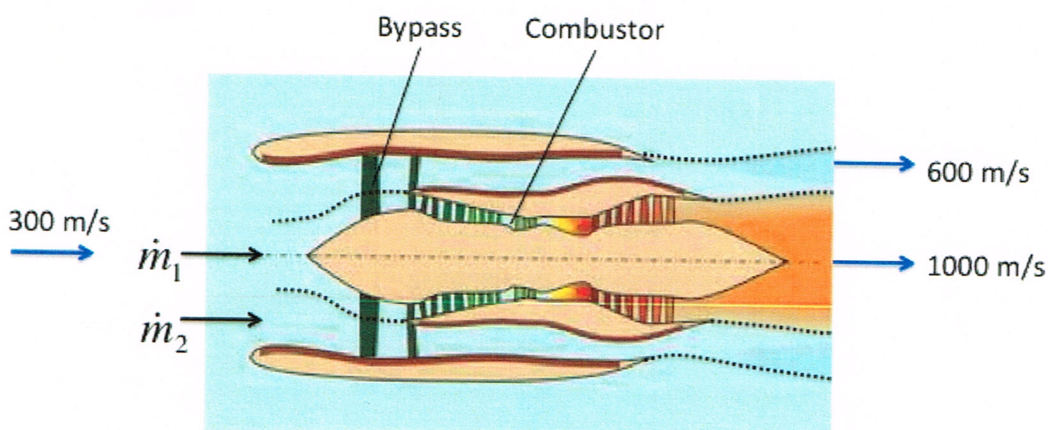
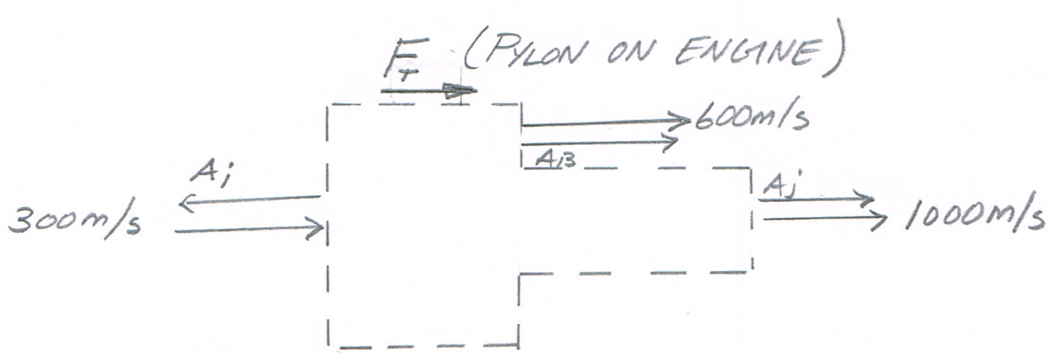


Figure 3: Typical turbofan engine



C.V. ENCLOSES JET ENGINE
THRUST IS EQUAL/OPPOSITE OF
PYLON FORCE ON ENGINE
SHOWN AS F_T.

[3]Continunity, Linear Momentum

REVISION A

C.V. SKETCH NEEDS TO SHOW
AREAS, FLOWS. OK IF $\dot{m}_1, \dot{m}_2$ +5 PTS
REPLACE INLET MASS FLOW.

Problem 3 (Continued)

MOMENTUM (+1)

$$\Sigma F = \frac{d}{dt}\int_{CV} \rho \cancel{V} d\forall + \int_{C.S.} \rho V (\bar{V} \cdot d\bar{A}) \quad (+1)$$

$0 \quad \frac{d\forall}{dt} = 0$

$$F_T = -\underbrace{\rho A_{INLET} \overset{+1}{V_{INLET}}}_{\overset{\bullet}{m}_1 + \overset{\bullet}{m}_2 \; +1} \cdot V_{INLET} + \underbrace{\rho A_j \overset{+1}{V_j}}_{\overset{\bullet}{m}_1 \; +1} V_j + \underbrace{\rho A_B \overset{+1}{V_B}}_{\overset{\bullet}{m}_2 \; +1} V_B$$

(+6) pts NEED TO RECOGNIZE THAT $\rho A V^2 = \overset{\bullet}{m} V$ OTHERWISE PROBLEM REALLY HARD TO SOLVE

KNOWN

$\overset{\bullet}{m}_1 + \overset{\bullet}{m}_2 = 300 \text{ kg/s}$ (GIVEN) (+1)

$\frac{\overset{\bullet}{m}_2}{\overset{\bullet}{m}_1} = 2.5$ (+1) (GIVEN BYPASS RATIO)

$V_{INLET} = 300 \text{ m/s}$

$V_j = 1000 \text{ m/s}$

$V_B = 600 \text{ m/s}$

REVISION A

Problem 3 (Continued)

$$F_T = -(\dot{m}_1 + \dot{m}_2) V_{INLET} + \dot{m}_1 V_j + \dot{m}_2 V_B$$

CONTINUNITY

$$0 = \underbrace{\frac{d}{dt}\int_{CV} \rho \, d\forall}_{0} + \int_{CS} \rho V \cdot dA \quad (+1)$$

$$0 = -(\dot{m}_1 + \dot{m}_2) + \dot{m}_1 + \dot{m}_2 = -(\dot{m}_1 + \dot{m}_2) + \dot{m}_1 + 2.5\dot{m}_1$$

SOLVE FOR $\dot{m}_1$

$$3.5\dot{m}_1 = \dot{m}_1 + \dot{m}_2 = 300 \text{ kg/s}$$

$$\dot{m}_1 = \frac{300 \text{ kg/s}}{3.5} = 85.71 \text{ kg/s} \quad (+1)$$

$$\dot{m}_2 = 300 \text{ kg/s} - 85.71 \text{ kg/s} = 214.28 \text{ kg/s} \quad (+1)$$

SUBSTITUTE INTO MOMENTUM AND SOLVE FOR F_T

$$F_T = -(300 \text{ kg/s})(300 \text{ m/s}) + (85.71 \text{ kg/s})(1000 \text{ m/s}) + (214.28 \text{ kg/s})(600 \text{ m/s})$$

$$= -90{,}000 \frac{\text{kg m}}{\text{s}^2} + 85{,}710 \frac{\text{kg m}}{\text{s}^2} + 128{,}568 \frac{\text{kg m}}{\text{s}^2}$$

$$= 124{,}278 \frac{\text{kg m}}{\text{s}^2} = 124.2 \text{ kN} \quad (+2)$$ #, UNITS

F_T IS FORCE OF PYLON ON ENGINE, HENCE ENGINE "PUSHES" ON PYLON $124.2\text{ kN} \leftarrow$

THRUST

4. Hydraulic mining uses high speed jets of water to demolish hills and the economically valuable material is then separated from the resulting slurry. Water flows from a reservoir through a pipe and discharges through a hydraulic monitor (a nozzle) as shown in Figure 4. The head loss in the pipe itself is $h_L = 0.025\frac{L}{D}\frac{V^2}{2g}$ where L and D are the length and diameter of the pipe and V is the velocity in the pipe. What is the discharge of the water. (Assume α=1.0) [4]

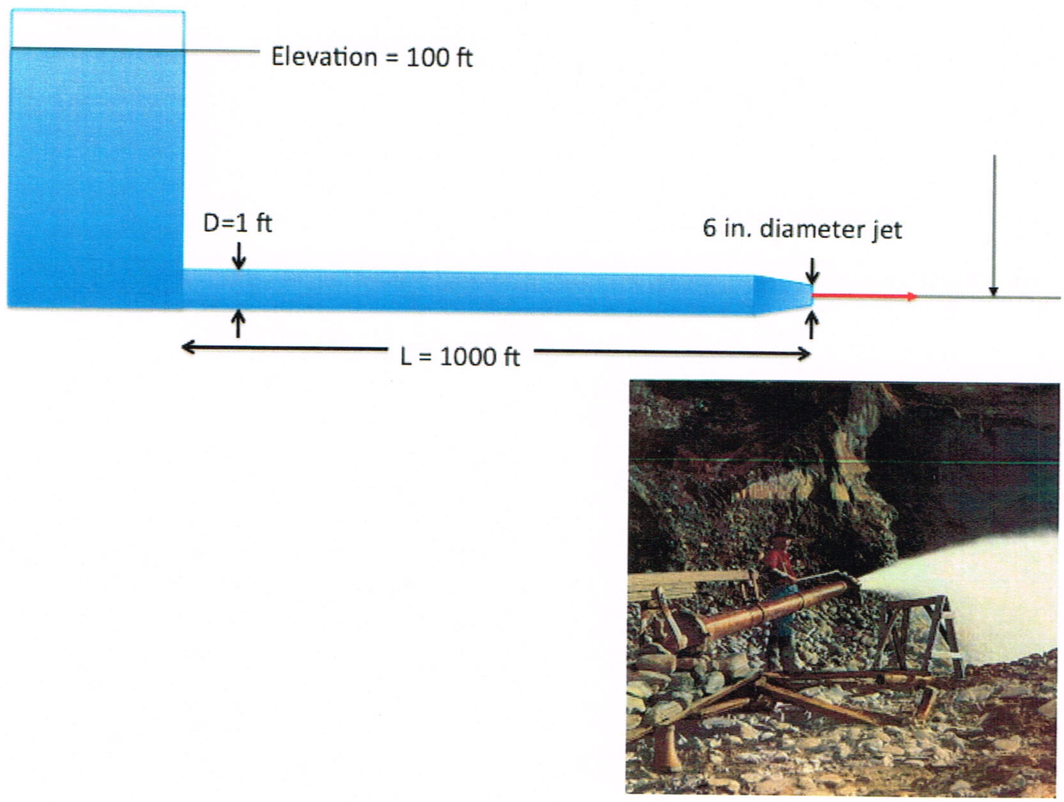
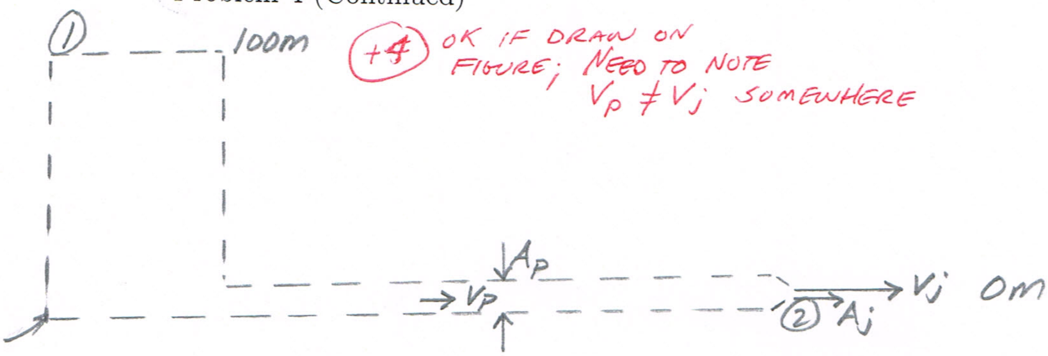


Figure 4: Schematic of monitor and supply line. Photograph of a monitor in use in California, circa late 1800s.

[4] Energy

20 POINTS TOTAL THIS PROBLEM

Problem 4 (Continued)



(+4) OK IF DRAW ON FIGURE; NEED TO NOTE $V_P \neq V_j$ SOMEWHERE

C.V. FIXED TO FREE SURFACE, ALL H_2O IN SYSTEM.

V_j = VELOCITY JET

V_P = VELOCITY IN PIPE

ENERGY (+1) FROM RESERVOIR TO JET

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \quad (+1)$$

0 gage; Reservoir surface ≈ 0; 0 gage; 0 datum

(+1) EVIDENCE OF CANCEL/ REASON CANCEL/ NEGLECT TERMS

V_j

V_P

$$z_1 = \frac{V_2^2}{2g} + 0.025 \frac{L}{D} \frac{V_P^2}{2g}$$

THIS IS AT JET $V_2 = V_j$

(+2) NEED TO NOTE V_j & V_P ARE DIFFERENT

EVALUATE CONSTANTS

$$0.025 \frac{L}{D} = \frac{0.025 (1000 ft)}{1 ft}$$

$$= 25$$

CONTINUITY (+1)

$$V_P A_P = V_j A_j$$

$$\frac{V_P A_P}{A_j} = V_j \quad (+1)$$

$$A_P = \frac{\pi (1 ft)^2}{4} = 0.785 ft^2 \quad (+2 \text{ \#, UNIT})$$

$$A_j = \frac{\pi (0.5 ft)^2}{4} = 0.196 ft^2 \quad (+2 \text{ \#, UNIT})$$

$$\therefore V_j = 3.99 V_P$$

$$V_j \sim 4 V_P \quad (+1)$$

Problem 4 (Continued)

EXPRESS EVERYTHING IN V_p

$$z_1 = \frac{(4V_p)^2}{2(32.2)} + \frac{25V_p^2}{2(32.2)}$$

$$z_1 = \frac{16V_p^2 + 25V_p^2}{64.4} = \frac{41V_p^2}{64.4}$$ (+1)

SUBSTITUTE IN NUMBERS AND SOLVE FOR V_p

$$100 = \frac{41V_p^2}{64.4} \qquad V_p^2 = \frac{64.4 \cdot 100}{41} = 157.07$$

$$V_p = \sqrt{157.07} = 12.5 \text{ ft/s}$$ (+1)

CONTINUNITY

$$Q = A_p V_p$$

$$= (0.785 \text{ ft}^2)(12.5 \text{ ft/s}) = 9.81 \text{ ft}^3/\text{s}$$ (+2) #, UNITS

5. A simple centrifugal pump consists of a 10 cm disk with radial ports as shown. Water is pumped from a reservoir through a central tube on the axis. The wheel spins at 3000 rev/min, and the liquid discharges to atmospheric pressure. To establish the maximum height for operation of the pump (called the shutoff height), assume the flow rate is nearly zero and the pressure at the pump intake is atmospheric pressure. Calculate the maximum operational height z for the pump.[5]

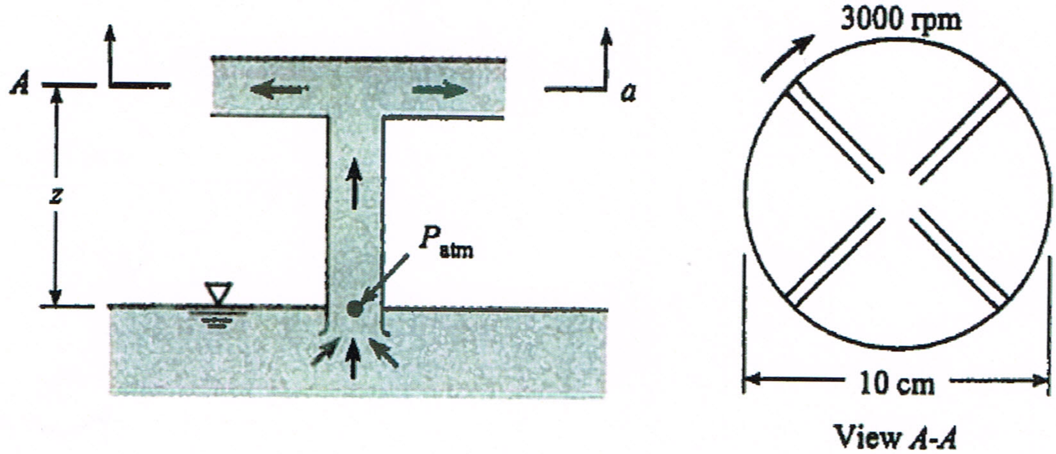
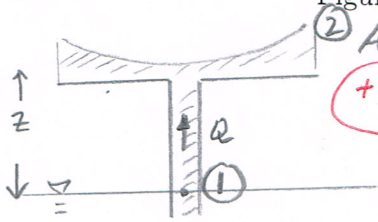


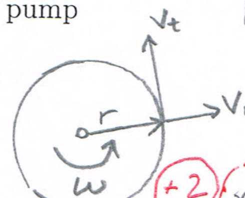
Figure 5: Sketch of simple centrifugal pump



AT SHUT OFF ($Q=0$)

+3

SKETCH (OK TO USE FIGURE) SHOW (1) & (2) AND z



+2

SKETCH TO SHOW LOGIC OF $V_t = r\omega$ (OK TO USE VIEW A-A)

IF $Q=0$, $V_r = 0$

ONLY $V_t = r\omega$ NON ZERO

+2 ±, UNIT

$r = \frac{0.1}{2} = 0.05\,m$ (GIVEN)

APPLY ROTATIONAL FORM EULER'S EQUATION

+1

+1 EQUATION

$$\cancel{\frac{p_1}{\gamma}} + \cancel{z_1} - \cancel{\frac{\omega^2 r_1^2}{2g}} = \cancel{\frac{p_2}{\gamma}} + z_2 - \frac{\omega^2 r_2^2}{2g}$$

0 gage 0 datum $=0$ r_1 IS AXIS OF ROTATION 0 gage

+4

EVIDENCE & LOGIC FOR "0" VALUES

[5]Euler and Bernoulli

TEXTBOOK FORM IS

$p_1 + \rho g z_1 - \frac{\rho \omega^2 r^2}{2} = C$ WAY EASIER IF DIVIDE EVERYTHING BY ρg

20 POINTS TOTAL THIS PROBLEM

13

Problem 5 (Continued)

$$0 = z_2 - \frac{\omega^2 r^2}{2g}$$ (+2)

z_2 IS SHUTOFF HEIGHT

$$z = \frac{\omega^2 r^2}{2g}$$

$$\omega = \frac{3000\ \text{rev}}{\text{min}} \cdot \frac{1\ \text{min}}{60\ \text{sec}} \cdot \frac{2\pi\ \text{rads}}{\text{rev}}$$ (+1)

$$= 314.15\ \text{rad/sec}$$ (+2)

$$z = \frac{\left(314.15\ \frac{\text{rad}}{\text{s}}\right)^2 (0.05\ \text{m})^2}{2(9.8\ \text{m/s}^2)} = 12.6\ \text{m}$$ (+2) #, UNITS

Problem 5 (Continued)

20
20
20
20
20
___
100