CE 3305 – Fluid Mechanics Exam 4

Purpose

Demonstrate ability to apply fluid mechanics and **problem solving principles** covering topics such as: Conservation of mass, continunity, conservation of linear momentum, and conservation of energy (modified bernoulli).

Instructions

1. Put your name on each sheet.
2. Use additional sheets as needed, if you add sheets put your name and the problem number on the added sheet.
3. Use the **problem solving protocol** in the class notes for the fully worked problems (Problems 7-9).
4. Label and/or underline answers, be sure to include units.

Allowed Resources

1. Your notes
2. Your textbook
3. The mighty Internet with following proviso
4. **You may not communicate with other people during the exam**

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1. Find the difference in pressure between the water and oil in Figure 1 if H = 25 cm.

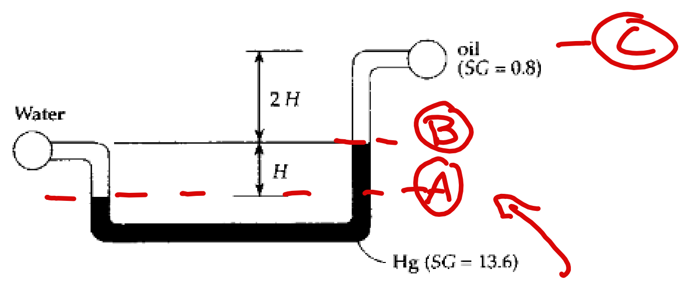


Figure 1:

SKETCH:

KNOWN: $H = 25\text{cm}$

$\rho_w = 1000 \text{ kg/m}^3$

$\rho_{Hg} = 13{,}600 \text{ kg/m}^3$

$\rho_{oil} = 800 \text{ kg/m}^3$

A) 42.3 kPa

B) 37.2 kPa

C) 34.8 kPa

D) 30.6 kPa

UNKNOWN

$p_w + \rho_w g H - \rho_{Hg} g H - \rho_{oil} g 2H = p_{oil}$ $\quad p_{oil} - p_{water}$

PRINCIPLES

Pascals Law

$p_w + (9800 - 13(9800) - 0.8(9800)(2))H = p_{oil}$

Solve For $p_{oil} - p_{water}$

$9800(0.25) - 13(9800)(0.25) - 0.8(2)(9800)(0.25) = p_{oil} - p_w$

$= -33{,}320 \frac{N}{m^2} = -33.3 \text{ kPa}$

$p_{water} - p_{oil} = 33.3 \text{ kPa}$

Choose C

2. The pressure drop across a valve, through which 0.04 m^3/s of water flows, is measured to be 100 kPa. Estimate the loss coefficient if the nominal diameter of the valve is 8 cm.

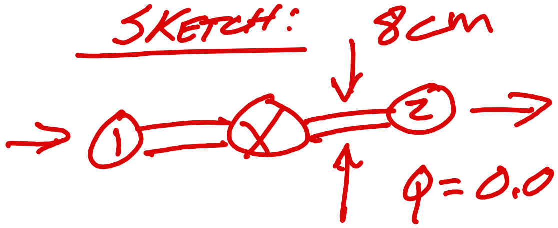
   A) 0.32

   B) 0.79

   C) 3.2

   D) 8.7

SKETCH:



$Q = 0.04\,m^3/s$

KNOWN:

$p_1 - p_2 = 100\,kPa$

UNKNOWN:

K_v

GOV. EQN

$h_L = K_v \frac{V^2}{2g}$

MOD. BERNOULLI

SOLUTION

$$\frac{p_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} = \frac{p_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2} + h_L$$

$z_1 = z_2$

$V_1 = V_2$

$p_1 - p_2 = \gamma h_L$

$100 \cdot 10^3 \frac{N}{m^2} = 9800 \frac{N}{m^3} h_L$

$100 \cdot 10^3 Pa = 9800 \frac{Pa}{m} \left(K_v \frac{V^2}{2g}\right)$

$V = \frac{0.04\,m^3/s}{\frac{\pi (0.08\,m)^2}{4}} = 7.95\,m/s$

$\frac{V^2}{2g} = \frac{63.3\,m^2/s^2}{2(9.8)} = 3.23\,m$

$\therefore K_v = \frac{100 \cdot 10^3 \frac{N}{m^2}}{(3.23\,m)(9800 \frac{N}{m^3})}$

$= 3.158$

Choose C

3. Find the expression for the force P needed to hold the gate of width w in the position shown.

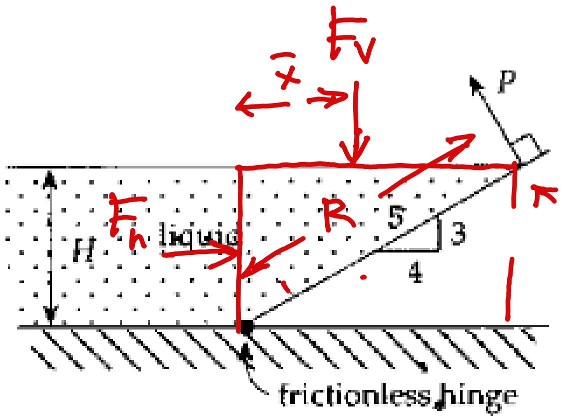


Figure 2:

A) $\frac{5}{18}\gamma w H^2$
B) $\frac{1}{6}\gamma w H^2$
C) $\frac{2}{9}\gamma w H^2$
D) $\frac{1}{2}\gamma w H^2$

SKETCH

KNOWN

H, w, γ_w, H

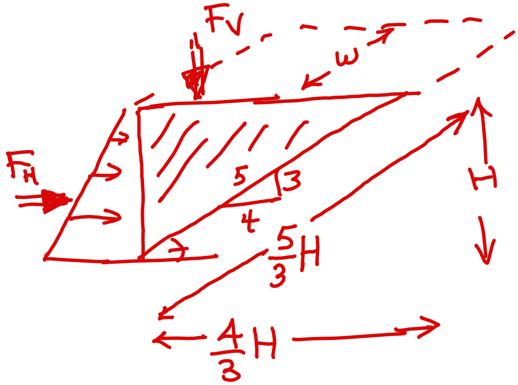
UNKNOWN

P

GOV. EQUN

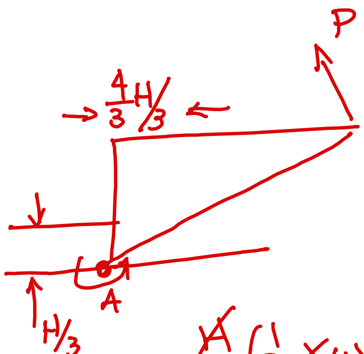
FLUID STATICS / GEOMETRY

$\Sigma M = 0$



$$F_V = \frac{1}{2}\gamma w \frac{4}{3}H^2 = \frac{4}{6}\gamma w H^2$$

$$F_H = \frac{1}{2}\gamma w H^2$$



$$\circlearrowright \Sigma M_A = 0 = \frac{5}{3}PH - \frac{H}{3}\left(\frac{1}{2}\gamma w H^2\right) - \frac{4H}{9}\left(\frac{4}{6}\gamma w H^2\right)$$

SOLVE FOR P

$$P = \frac{\frac{\cancel{H}}{\cancel{3}}\left(\frac{1}{2}\gamma w H^2\right) + \frac{4\cancel{H}}{\cancel{9}\,3}\left(\frac{4}{6}\gamma w H^2\right)}{\frac{5}{\cancel{3}}\cancel{H}}$$

$$P = \frac{\frac{1}{2}\gamma w H^2 + \frac{4}{3}\left(\frac{4}{6}\gamma w H^2\right)}{5}$$

$$= \frac{\left[\frac{3}{6} + \frac{4}{3}\left(\frac{4}{6}\right)\right]}{5}\gamma w H^2 = \frac{\frac{9}{18} + \frac{16}{18}}{5}\gamma w H^2$$

$$= \frac{\frac{25}{18}}{5}\gamma w H^2 = \frac{5}{18}\gamma w H^2$$

Choose A

4. The pressure drop over 15 m of 2-cm-diameter galvanized iron pipe is measured to be 60 kPa. If the pipe is horizontal, estimate the flow rate of water. ($\nu = 10^{-6} m^2/s$)

   A) 6.82 L/s

   B) 2.18 L/s

   C) 0.682 L/s

   D) 0.218 L/s

SOLUTION

$$\frac{p_1 - p_2}{\gamma} = \frac{8 f L Q^2}{\pi^2 g D^5}$$

$$\frac{60 \cdot 10^3 Pa}{9800 N/m^3} = 6.12 m = \frac{8 f L Q^2}{\pi^2 g D^5}$$

MOODY CHART

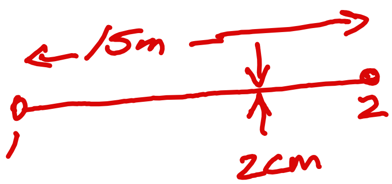
CONVERT SOLN.S TO VELOCITY

	V	Re	f
A	21.7 m/s	$4.34 \cdot 10^5$	0.041
B	6.9 m/s	$1.38 \cdot 10^5$	0.042
C	2.17 m/s	$4.34 \cdot 10^4$	0.043
D	0.6 m/s	$1.3 \cdot 10^4$	0.045

* 0.0427 is average

$$\frac{\varepsilon_s}{D} = \frac{0.26 mm}{20 mm} = 0.013$$

SKETCH



KNOWN

$p_1 - p_2 = 60 kPa$

iron pipe

UNKNOWN

Q

GOV. EQNS

$$Re_d = \frac{VD}{\nu}$$

$\varepsilon_s = 0.26 mm$ (look up)

$f(Re, \varepsilon_s)$

MOD. BERNOULLI

$$\frac{6.12(\pi^2 g D^5)}{8 f L} = Q^2$$

$$\sqrt{\frac{6.12(\pi^2(9.8)(0.02)^5}{8(0.0427)(15)}} = 0.000608 \ m^3/s$$

$\times 1000$

$= 0.608 \frac{L}{s}$

Choose C

5. Water flows through a converging fitting shown and discharges to the atmosphere as a free jet. Flow is incompressible, friction negligible.

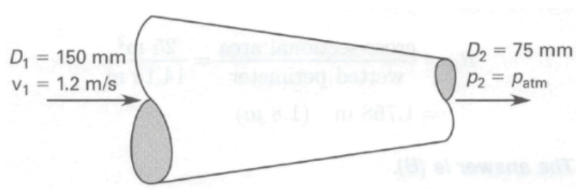


Figure 3:

The gage pressure at the inlet is

A) 10.2 kPa

B) 10.8 kPa

C) 11.3 kPa

D) 12.7 kPa

SKETCH

GIVEN

KNOWN

D_1, V_1, D_2, P_2

GOV. EQN

CONTINUITY

MOD. BERNOULI

CALCULATIONS

$$V_1 \frac{\cancel{\pi} D_1^2}{\cancel{4}} = V_2 \frac{\cancel{\pi} D_2^2}{\cancel{4}}$$

$$V_2 = \frac{V_1 D_1^2}{D_2^2} = \frac{(1.2 m/s)(150 mm)^2}{(75 mm)^2}$$

$$= 4.8 m/s$$

NOW BERNOULLIS

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} = \cancel{\frac{p_2}{\gamma}} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$z_1 = z_2$; $p_2 = 0$ gage

$$p_1 = \left(\frac{V_2^2 - V_1^2}{2g}\right)(\rho g)$$

$$\rho = 1000 \frac{kg}{m^3}$$

$$p_1 = 10800 \underbrace{\frac{m^2}{s^2} \cdot \frac{kg}{m^3}}$$

$$\frac{kg\, m}{s^2} \quad \frac{m}{m^3}$$

$$p_1 = 10800 \, N/m^2$$

or 10.8 kPa choose B

6. A model of a dam is constructed so the scale of prototype to model is 15:1. The similarity scaling is based on Froude numbers. At a certain point on the spillway of the model, the velocity is measured as 5 meters per second. At the corresponding point on the spillway of the actual (prototype) dam, the velocity is about

A) 6.7 $\frac{m}{s}$

B) 7.5 $\frac{m}{s}$

C) 15 $\frac{m}{s}$

D) 19 $\frac{m}{s}$

$$\frac{L_P}{L_m} = \frac{15}{1}$$

$$Fr_p = Fr_m$$

$$\frac{V_p}{\sqrt{gL_p}} = \frac{V_m}{\sqrt{gL_m}}$$

$$V_p = V_m \frac{\sqrt{gL_p}}{\sqrt{gL_m}}$$

$$V_p = V_m \sqrt{15}$$

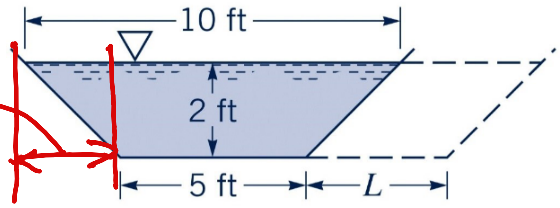
$$V_p = (5 m/s)\sqrt{15} = 19.36 \, m/s$$

choose D

7. The canal shown below is to be widened so that the water flow discharge can be tripled (i.e., flow discharge after widening is three times the initial flow discharge).

$\leftarrow 10 + L \rightarrow$

2.5



$\leftarrow 5 + L \rightarrow$

$\sqrt{2^2 + 2.5^2} = 3.2$

Figure 4: Cross section of trapezoidal channel

Determine:

(a) The additional width, L, required if all other parameters (i.e., flow depth, bottom slope, surface material, side slope) are to remain the same

SKETCH

KNOWN

GEOMETRY

UNKNOWN

L

GOVERNING EQN

MANNINGS

$Q = \frac{1}{n} A R^{2/3} S^{1/2}$

CALCS.

$Q_1 = \frac{1}{n} A R^{2/3} S^{1/2}$

$3Q_1 = \frac{1}{n} A R^{2/3} S^{1/2}$

$A_1 = (2)(5) + (2)(\frac{1}{2})(2.5)(2)$

$A_2 = (2)(5+L) + (2)(\frac{1}{2})(2.5)(2)$

$R_1 = \frac{A_1}{2(3.2) + 5}$

$R_2 = \frac{A_2}{2(3.2) + (5+L)}$

$$3\left(\cancel{\frac{1}{n}} A_1 R_1^{2/3} \cancel{S_0}\right) = \cancel{\frac{1}{n}}\left(A_2 R_2^{2/3} \cancel{S_0}\right)$$

$$3A_1 R_1^{2/3} = A_2 R_2^{2/3}$$

EXPRESS IN TERMS OF L; SOLVE FOR L

$$3(15)\left(\frac{15}{11.4}\right)^{2/3} = (15+2L)\left(\frac{15+2L}{11.4+L}\right)^{2/3}$$

$$54.034 = \frac{(15+2L)^{5/3}}{(11.4+L)^{2/3}}$$

① L	② $15+2L$	③ $11.4+L$	④ ②$^{5/3}$	⑤ ③$^{2/3}$	④/⑤
10	35	21.4	374.49	7.708	48.5
11	37	22.4	410.8	7.946	51.7
12	39	23.4	448.5	8.181	54.8
BETWEEN 11 & 12, CLOSER TO 12					
11.8	38.6	23.2	440.87	8.134	54.19
$\boxed{11.75}$	38.5	23.15	438.97	8.122	54.04
Close	enough				

$L = 11.75$ feet

SOLVE FOR L

$$3\left[\cancel{\frac{1}{n}}(2)(5)+(2)(2.5)\right]\left[\frac{(2)(5)+(2)(2.5)}{2(3.2)+5}\right]^{\cancel{5/0}\ 1/2}$$

$$= \left[\cancel{\frac{1}{n}}(2)(5+L)+(2)(2.5)\right]\left[\frac{2(5+L)+2(2.5)}{2(3.2)+(5+L)}\right]^{\cancel{5/0}\ 1/2}$$

$$3\left[15\right]\left[\frac{15}{11.4}\right]^{2/3} = \left[10+2L+5\right]\left[\frac{10+2L+5}{6.4+5+L}\right]^{2/3}$$

$$= \frac{(15+2L)(15+2L)}{6.4+5+L}$$

$$59.21 = \frac{(15+2L)^2}{11.4+L}$$

① L	② $(15+2L)^2$	③ 11.4+L	②/③	TARGET
0	225	11.4	19.73	59.21
1	289	12.4	23.3	59.21
2	361	13.4	26.9	59.21
5	625	16.4	38.1	59.21
10	1225	21.4	57.24	59.21
11	1369	22.4	61.11	59.21
10.2	1253.16	21.6	58.01	59.21
10.3	1267.36	21.7	58.4	59.41
10.4	1281.64	21.8	58.79	59.21
10.5	1296	21.9	59.17	59.21 ← Close Enough

$$\boxed{L = 10.5}$$

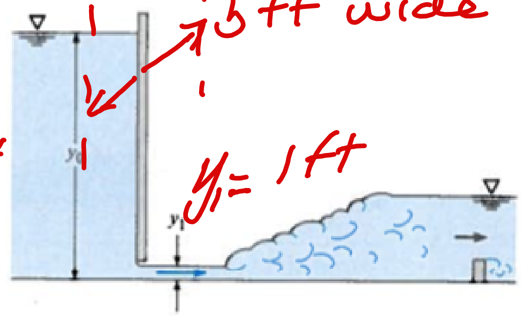
8. The figure below is a schematic of water flowing under a sluice gate in a horizontal channel 5 feet wide.

Sketch

5 ft wide

$y_0 = 65\text{ ft}$

$y_1 = 1\text{ ft}$



$y_2 = ?$

Figure 5: Supercritical flow under a sluice gate

Determine:

(a) Discharge through the sluice gate

(b) Power dissipated in the jump

(c) The alternate depth (depth of flow after the jump)

Known

$y_0 = 65\text{ ft}$ $\quad g = 32.2\text{ ft/s}^2$

$y_1 = 1\text{ ft}$ $\quad \gamma = 62.4\text{ lbf/ft}^3$

$b = 5\text{ ft}$

UNKNOWN

Q, P, y_2

GOV. PRINCIPLES

CONTINUITY $\quad V = \frac{Q}{A}$ $\qquad$ HYD. JUMP

POWER = $Q\gamma h$ $\qquad \frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2}-1\right)$

MOD. BERNOULLI

Bernoulli ⓪ → ①

$$y_0 = y_1 + \frac{V_1^2}{2g} + \cancel{z_1}^{=0}$$

$$\sqrt{(y_0 - y_1)2g} = V_1$$

$$\sqrt{(65-1)(2)(32.2)} = 64.2\ ft/s$$

Ⓐ

$$Q = VA = 64.2(b)(y_1)$$

$$= 64.2(5ft)(1ft) = \boxed{320.9\ ft^3/s} \leftarrow Q$$

$$\Delta E = E_2 - E_1$$

$$\Delta E = y_2 + \frac{V_2^2}{2g} + \cancel{z_2} - y_1 + \frac{V_1^2}{2g} + \cancel{z_1} \qquad z_2 = z_1$$

NEED y_2; SO USE HYD. JUMP EQN

$$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8(Fr_1)^2} - 1\right)$$

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{64.2}{\sqrt{32.2(1)}} = 11.314$$

Ⓒ

$$y_2 = \frac{1}{2}\left(\sqrt{1 + 8(11.314)^2} - 1\right) = \boxed{15.508\ ft} \leftarrow y_2$$

SOLVE FOR V_2

$$\frac{Q}{A_2} = V_2$$

$$\frac{320.9}{(5)(15.51)} = 4.14 \text{ ft/s}$$

$$\Delta E = y_2 + \frac{V_2^2}{2g} + \cancel{z_2} - \left(y_1 + \frac{V_1^2}{2g} + \cancel{z_1}\right)$$

$$= 15.51 + \frac{4.14^2}{2(32.2)} - 1.0 - \frac{64.2^2}{2(32.2)}$$

$$= 15.51 + 0.266 - 1.0 - 64$$

$$= -49.2 \text{ ft (loss)}$$

$$\text{Power} = Q\gamma h = -Q\gamma\Delta E$$

$$= (320.9 \text{ ft}^3/\text{s})(62.4 \text{ lb}/\text{ft}^3)(49.2 \text{ ft})$$

$$= 985.67 \cdot 10^3 \frac{\text{ft} \cdot \text{lb}}{\text{s}} * \frac{1 \text{HP}}{550 \frac{\text{ft-lb}}{\text{s}}}$$

B $= 1792$ HP dissipated in the jump

9. Figure 6 is a gravity-flow pipe network with water supplied from a fixed-grade reservoir (pool elevation 100 meters) connected to node N2. All pipes are ductile iron.

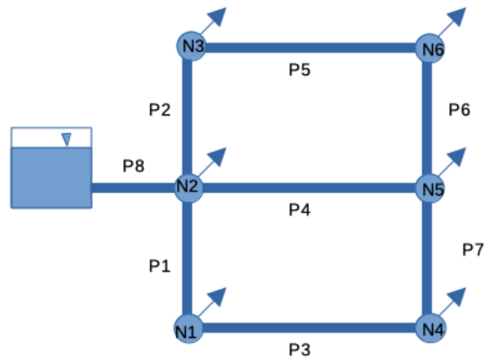
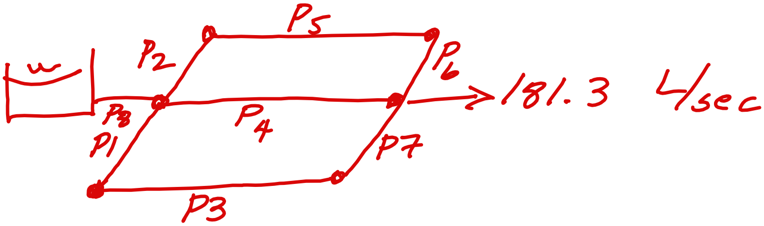


Figure 6: Gravity flow pipe network

The pipe dimensions and node demands are shown in the tables below.

Pipe ID	Length(m)	Diameter(mm)	Friction factor f
1	1,220	254	0.028
2	1,829	254	0.028
3	1,829	305	0.028
4	1,982	610	0.028
5	2,134	254	0.028
6	915	457	0.028
7	1,524	254	0.028
8	91	305	0.028

Node ID	Elevation(m)	Demand(liters/sec)
N1	51.8	0.0
N2	54.9	0.0
N3	50.3	0.0
N4	47.3	0.0
N5	45.7	181.3
N6	44.2	0.0



SKETCH

KNOWN

D, f, L each pipe

$h_0 = 100\,m$ Elev. each node

demand at N5

UNKNOWN

Q in each pipe

pressue each node

$\Delta h_{2 \to 6}$

Lowest p

GOVERNING EQN

$$h_L = \frac{8 f L Q^2}{\pi^2 g D^5} \quad \text{each pipe}$$

h_i any node is unique value

continuity at nodes

SOLUTION

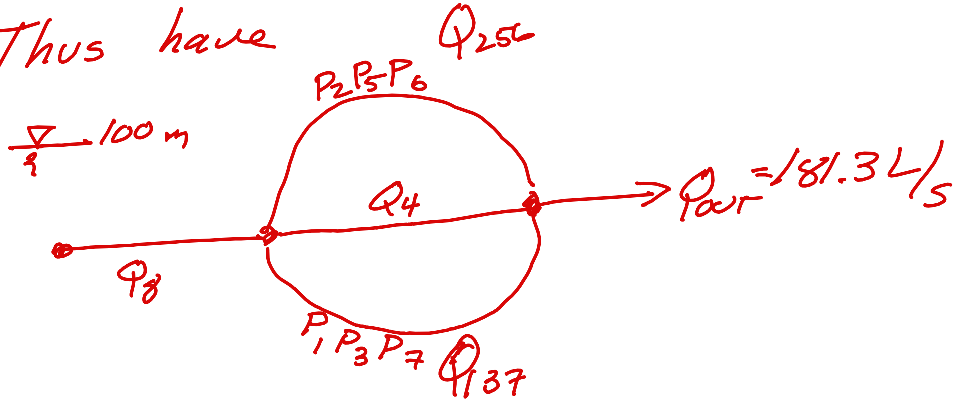
Observe continuity means

$$Q_{P_2} = Q_{P_5} = Q_{P_6}$$

AND

$$Q_{P_1} = Q_{P_3} = Q_{P_7}$$

Thus have



$$Q_8 = 181.3\ L/s = 0.181\ m^3/s$$

HEAD LOSS ALONG 3 PATHS MUST BE SAME

$$h_1 = \frac{8 f L_1 Q_{137}^2}{\pi^2 g D_1^5} = \frac{8(0.028)(1220)}{\pi^2 (9.8)(0.254)^5} Q^2$$

$$= 2672.47\, Q_{137}^2$$

$$h_2 = \frac{8f}{\pi^2 g} \frac{L_2}{D_2^5} Q_{256}^2 = 4006.52\, Q_{256}^2$$

$$h_3 = \frac{8f}{\pi^2 g} \frac{L_3}{D_3^5} Q_{137}^2 = 1604.85\, Q_{137}^2$$

⋮

2672.47
4006.52
1604.85

PIPE_ID	K
1	2672.47
2	4006.52
3	1604.85
4	54.34
5	4674
6	106.31
7	3338.4
8	79.84

$$h_L = KQ^2$$

FORM UP HEAD LOSS EQNS.

$$(4006.52 + 4674 + 106.31)Q_{256}^2 = h_L$$

$$54.34\, Q_4^2 = h_L$$

$$(2672.47 + 1604.85 + 3338.4)Q_{137}^2 = h_L$$

$$8786.83\, Q_{256}^2 = h_L$$

$$54.34\, Q_4^2 = h_L$$

$$7615.72\, Q_{137}^2 = h_L$$

		Q	h_L
Q_{256}	8786.83	60.4	32052734
Q_4	54.34	60.4	198459
Q_{137}	7615.72	60.4	27780738
	Σ		

make Q_4 bigger, Q_{256} smallest
Q_{137} slighty bigger than Q_{256}

		Q	h_L
Q_{256}	8787	0.01	0.878 ←
Q_4	54	0.1613	1.413
Q_{137}	7616	0.01	0.7616

NEAT TRICK: GUESS h_L VALUES
SOLVE FOR Q; STOP WHEN
FLOWS SUM UP TO 0.1813

$$8787Q_1^2 = \qquad Q_1 = 0.0125$$
$$54Q_2^2 = 1.39 \qquad Q_2 = 0.1604$$
$$7616Q_3^2 = \qquad Q_3 = 0.0135$$
$$\Sigma \; 0.1864$$

$$8787Q_1^2 = \qquad Q_1 = 0.0126$$
$$54Q_2^2 = 1.40 \qquad Q_2 = 0.1610$$
$$7616Q_3^2 = \qquad Q_3 = 0.0135$$
$$\Sigma \; 0.18715$$

$$8787Q_1^2 = \qquad Q_1 = 0.01244$$
$$54Q_2^2 = 1.36 \qquad Q_2 = 0.1587$$
$$7616Q_3^2 = \qquad Q_3 = 0.01336$$
$$\Sigma \; 0.1845$$

$$8787Q_1^2 = \qquad Q_1 = 0.01225$$
$$54Q_2^2 = 1.32 \qquad Q_2 = 0.15634$$
$$7616Q_3^2 = \qquad Q_3 = 0.01316$$
$$\Sigma \; 0.18175$$ ← Close

between these two

$$8787Q_1^2 = \qquad Q_1 = 0.01221$$
$$54Q_2^2 = 1.31 \qquad Q_2 = 0.1557538$$
$$7616Q_3^2 = \qquad Q_3 = 0.013151$$
$$\Sigma \; 0.181079$$

$8787 Q_1^2 =$

$54 Q_2^2 = 1.315$

$7616 Q_3^2 =$

$Q_1 = 0.0122333$

$Q_2 = 0.1560508$

$Q_3 = 0.0131401$

$\Sigma\ 0.18142$

Target is 0.1813

close enough to proceed

$Q_1 = 0.01314 = 13.14\ L/s$

$Q_2 = 0.01223 = 12.23\ L/s$

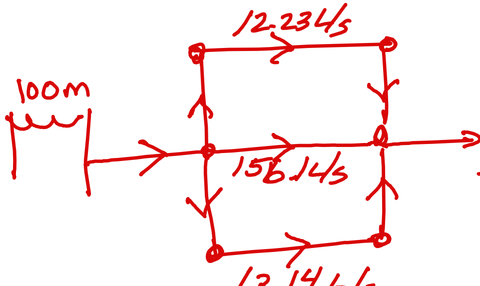
$Q_3 = 0.01314$

$Q_4 = 0.1561 = 156.1\ L/s$

$Q_5 = 0.01223$

$Q_6 = 0.01223$

$Q_7 = 0.01314$



$$\begin{array}{r} 156.1 \\ 12.23 \\ +\ 13.14 \\ \hline 181.47\ L/s \end{array}$$

(close)

NOW HEAD LOSSES FOR PRESSURE

- AT A NODE $p = \text{Head} - \text{Elev}$

NODE	HEAD	ELEV	PRESSURE
0	100m	100	0m
1	94.73	51.8	42.93m
2	97.37	54.9	42.47m ← LOWEST p (HIGHEST ELEV)
3	96.77	50.3	46.47
4	96.055	47.3	48.755
5	96.7	45.7	51.0
6	96.072	44.2	51.87

HEAD LOSSES (APPROX.)

PIPE	K	Q	Δh
1	2672	0.0314	2.634
2	4006.52	0.01223	0.599
3	1604	0.0314	1.5815
4	54	0.15605	1.315
5	4674	0.01223	0.699
6	106.31	0.01223	0.016
7	3338	0.0314	3.2911
8	79.84	0.18142	2.63m

Determine:

(a) The flow rate (and direction of flow) for each pipe in the network, for the case where the total head at the supply reservoir is 100 meters.

(b) The resultant pressure in SI units at each node.

(c) The Darcy-Weisbach friction factor for each ductile iron pipe of the network.

(d) The head loss from Node 2 to Node 6.

(e) The node with the lowest pressure.

0m
42.93m
42.47m ← LO
46.47 (HI
48.755
51.0
51.87

SOLUTION (DRAWING PRESSURES & FLOW)