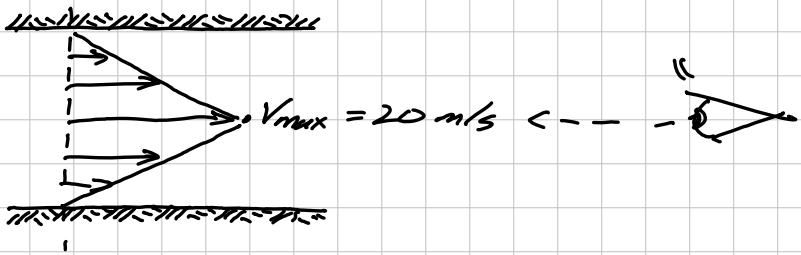
PROBLEM STATEMENT

$\gamma = 2700 N/m^3$

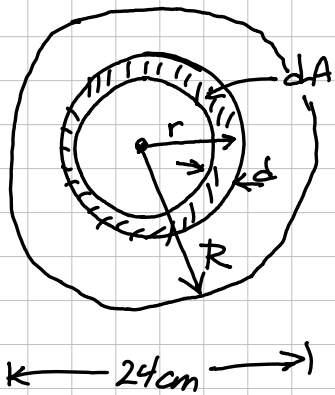
24cm diameter pipe, linear velocity profile.

Find Q if $V_{max} = 20 m/s$



SKETCH

OBSERVE THAT PIPE IS CIRCULAR



$dA = 2\pi r\, dr$

KNOWN

R, V_{max}, GEOMETRY

UNKNOWN

Q

GOVERNING PRINCIPLES

$$Q = VA; \quad Q = \int dQ$$

$$dQ = V(r)\,dA = V(r)\,2\pi r\,dr$$

$$V(r) = V_{max} - \frac{r}{R}V_{max} = V_{max}\left(1 - \frac{r}{R}\right)$$

or

$$V_{max}\left(\frac{R-r}{R}\right)$$

SOLUTION

$$Q = \int_0^R V_{max}\left(1 - \frac{r}{R}\right)2\pi r\,dr$$

$$= 2\pi V_{max}\int_0^R r\,dr - \frac{2\pi V_{max}}{R}\int_0^R r^2\,dr$$

$$= 2\pi(20\,m/s)\frac{R^2}{2} - \frac{2\pi(20\,m/s)}{R}\frac{R^3}{3}$$

$$= 2\pi(20\,m/s)\frac{(0.12)^2}{2} - 2\pi\frac{(20\,m/s)}{0.12}\frac{(0.12)^3}{3}$$

$$= 0.9047\,m^3/s - 0.6031\,m^3/s = 0.3016\,m^3/s$$

DISCUSSION

i) NEED TO RECOGNIZE CYLINDRICAL GEOMETRY

ii) VELOCITY AS FUNCTION OF r MEASURED FROM CENTERLINE.