PROBLEM 8.44

Given → A smooth pipe designed to carry crude oil ($D = 47$ in) $\rho = 1.75$ slug/ft³ and $\mu = 4\times10^{-4}$ lbf·s/ft² is to be modeled with a smooth pipe 4 in in diameter carrying water ($T = 60°F$). If the mean velocity in the prototype is 2 ft/s, what should be the mean velocity of the water in the model to ensure dynamically similar conditions?

Crude oil → $D_p = 47$ in $V_p = 2$ ft/s
$\rho_p = 1.75$ slugs/ft³
$\mu_p = 4\times10^{-4}$ lbf·s/ft²

water → $T = 60°F$
$\rho_m = 1.94$ slugs/ft³ $D_m = 4$ in.
$\mu_m = 2.73\times10^{-5}$ lbf·s/ft²

Find → mean velocity of the water in the model, V_m

Solution →

$Re_m = Re_p$ Dynamically similar model and prototype

$$Re = \frac{V d \rho}{\mu}$$ Reynolds number equation

$$Re_m = Re_p \Rightarrow \frac{V_m d_m \rho_m}{\mu_m} = \frac{V_p d_p \rho_p}{\mu_p}$$

$$\Rightarrow V_m = V_p \left(\frac{d_p}{d_m}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{\mu_m}{\mu_p}\right)$$

$$\Rightarrow \text{Units in ft} \Rightarrow D_p = \frac{47\text{ in}}{}\bigg|\frac{1\text{ ft}}{12\text{ in}} = 3.92\text{ ft}$$

$$D_m = \frac{4\text{ in}}{}\bigg|\frac{1\text{ ft}}{12\text{ in}} = .333\text{ ft}$$

$$\Rightarrow V_m = \frac{2\text{ ft}}{s}\bigg|\frac{3.92\text{ ft}}{.333\text{ ft}}\bigg|\frac{1.75\text{ slugs}}{\text{ft}^3}\bigg|\frac{\text{ft}^3}{1.94\text{ slugs}}\bigg|\frac{2.73e{-5}\text{ lbf s}}{\text{ft}^2}\bigg|\frac{\text{ft}^2}{4e{-4}\text{ lbf s}}$$

$$\Rightarrow \boxed{V_m = 1.45\ \frac{\text{ft}}{s}}$$

Discussion → The model and the prototype have different fluids so they have different fluid properties. To calculate the velocity in the model the Reynold's numbers have to match. Setting the Reynolds number for the model equal to the Reynolds number for the prototype lets us put the given fluid properties into the equation to solve for the velocity in the model.

PROBLEM 8.66

Given → Flow around a bridge pier is studied using a model 1/12 scale. When the velocity in the model is .9 m/s, the standing wave at the pier nose is observed to be 2.5 cm in height. What are the corresponding values of velocity and wave height in the prototype?

model → $V_m = .9\ m/s$
$L_m = 2.5\ cm$
$1/12$ scale

Find → - velocity of the prototype
- wave height of the prototype

Solution → Using Froude number matching ⇒ $Fr_p = Fr_m$

$$Fr = \frac{V}{\sqrt{L}} \Rightarrow \frac{V_p}{\sqrt{L_p}} = \frac{V_m}{\sqrt{L_m}}$$

$$\Rightarrow V_p = V_m \left[\sqrt{\frac{L_p}{L_m}}\right]$$

Since scale is $\frac{1}{12}$, $L_m = \frac{1}{12} L_p \Rightarrow 12 L_m = L_p$

put into equation → $V_p = V_m \left[\sqrt{\frac{12 L_m}{L_m}}\right] = V_m \sqrt{12}$

$$V_p = \frac{.9\ m}{s} \Big| \sqrt{12} \Rightarrow \boxed{V_p = 3.12\ m/s}$$

$$12 L_m = L_p \Rightarrow 12(2.5\ cm) = L_p$$

$$\Rightarrow \boxed{L_p = 30\ cm}$$

Discussion → Open channel flow almost always uses Froude number matching. By setting Fr_p equal to Fr_m, we can input the given model and scale factor data to solve for both the velocity and wave height of the prototype.

PROBLEM 8.77

Given → Experiments are performed to measure pressure drop in a pipe with water at 20°C and crude oil at the same temperature. Data was gathered with pipes of 2 diameters 5 cm and 10 cm. The data obtained is shown on the tables on the next page.
Pressure drop per unit length is assumed to be a function of pipe diameter, liquid density, viscosity, and velocity.

$$\frac{\Delta P}{L} = f(\rho, \mu, V, D)$$

Find → Dimensional analysis to obtain π groups, plot the results using the dimensionless parameters.

Solution → Dimensional analysis

$$\rho = \frac{kg}{m^3} \qquad \mu = \frac{N \cdot s}{m^2} = \frac{kg\,m}{s^2}\bigg|\frac{s}{m^2} = \frac{kg}{s \cdot m}$$

$$V = \frac{m}{s} \qquad D = m$$

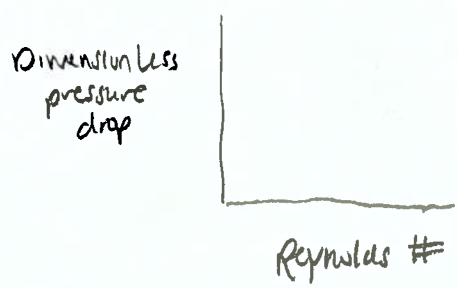
$$\frac{\Delta P}{L} = \frac{N}{m^3} = \frac{kg\,m}{s^2}\bigg|\frac{1}{m^3} = \frac{kg}{s^2 m^2}$$

$$\frac{\Delta P}{L} = \frac{kg}{s^2 m^2}\bigg|\ m\ \bigg|\frac{m^3}{kg}\bigg|\frac{s}{m}\bigg|\frac{s}{m} = \frac{\Delta P}{L}\bigg|\ D\ \bigg|\frac{1}{\rho}\bigg|\frac{1}{V}\bigg|\frac{1}{V}$$

$$\Rightarrow \frac{\Delta P\, D}{L \rho V^2} = \text{dimensionless}$$

$$\frac{\rho V D}{\mu} = \text{dimensionless} - \text{Reynolds number}$$

⇒ plot data using excel and data tables given. Using both dimensionless parameters



for both water and crude oil.
Excel tables of calculated and given values on next page.

Discussion → When the data is reduced to dimensionless parameters, the function f is obtained by passing a curve through the data. This curve eventually populates design publications

ES 20

Water Fluid Properties		
density=	998.2	kg/m3
viscosity=	1.00E-03	Pa s

Crude Oil Fluid Properties		
density=	900	kg/m3
viscosity=	2.00E-02	Pa s

Water

Given				Calculated	
Pipe Diameter	Pipe Diameter	Velocity	Pressure Drop	Dimensionless Pressure Drop	Reynolds Number
cm	m	$\frac{m}{s}$	$\frac{N}{m^3}$	$\frac{\Delta p D}{L \rho V^2}$	$\frac{\rho V D}{\mu}$
5	0.05	1	210	0.0105	4.98E+04
5	0.05	2	730	0.0091	9.96E+04
5	0.05	5	3750	0.0075	2.49E+05
10	0.1	1	86	0.0086	9.96E+04
10	0.1	2	320	0.0080	1.99E+05
10	0.1	5	1650	0.0066	4.98E+05

Crude Oil

Given				Calculated	
Pipe Diameter	Pipe Diameter	Velocity	Pressure Drop	Dimensionless Pressure Drop	Reynolds Number
cm	m	$\frac{m}{s}$	$\frac{N}{m^3}$	$\frac{\Delta p D}{L \rho V^2}$	$\frac{\rho V D}{\mu}$
5	0.05	1	310	0.0172	2.25E+03
5	0.05	2	1040	0.0144	4.50E+03
5	0.05	5	5300	0.0118	1.13E+04
10	0.1	1	130	0.0144	4.50E+03
10	0.1	2	450	0.0125	9.00E+03
10	0.1	5	2210	0.0098	2.25E+04