 Name: SOLUTIONS

CE 3305 Engineering Fluid Mechanics
Exercise Set 13
Spring 2014

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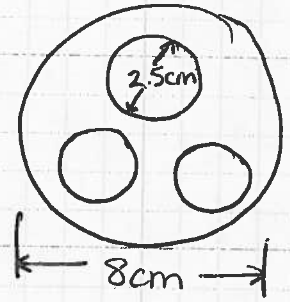
5.32) The cross section of a heat exchanger consists of three circular pipes inside a larger pipe. The internal diameter of the 3 smaller pipes is 2.5 cm, and the pipe wall thickness is 3 mm. The inside diameter of the larger 8 cm. If the velocity of the fluid in region between the smaller pipes and larger pipe is 10 m/s, what is the discharge in m³/s?

SKETCH:



KNOWN:

$V = 10\ m/s$
$D_{small} = 2.5cm$
$t_{wall} = 3mm = 0.3cm$
$D_{large} = 8cm$

UNKNOWN:

$Q_{large} = ?$

GOVERNING EQUATIONS:

$Q = AV$

SOLUTION:

$$A_{out} = \frac{\pi}{4}(2.5+0.6)^2 = 7.55cm^2$$

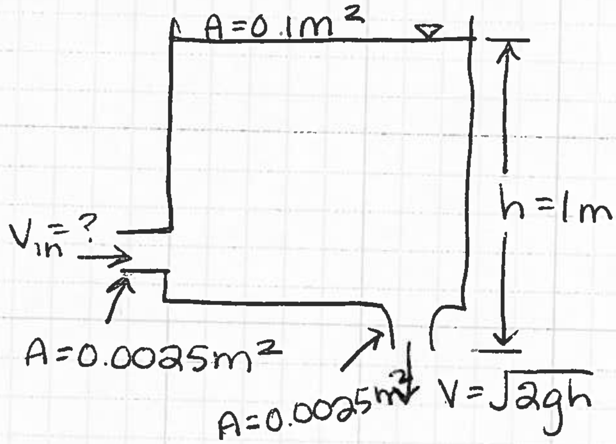
Area between large pipe and small pipes

$$A = \frac{\pi}{4}(8)^2 - 3(7.55cm^2) = 27.61cm^2 = 0.002761\ m^2$$

$$Q = AV = 0.002761\ m^2 \times 10m/s = \boxed{0.02761\ m^3/s = Q}$$

5.55) A tank has a hole in the bottom with a cross-sectional area of $0.0025 m^2$ and an inlet line on the side with a cross-sectional area of $0.0025 m^3$ as shown. The cross-sectional area of the tank is $0.1 m^2$. The velocity of the liquid flowing out the bottom hole is $V=\sqrt{2gh}$, where h is the height of the water surface in the tank above the outlet. At a certain time the surface level in the tank is 1m and rising at the rate of 0.1 cm/s. The liquid is incompressible. Find the velocity of the liquid through the inlet.

SKETCH:



KNOWN:

$A_{out} = 0.0025 m^2$
$A_{tank} = 0.1 m^2$
$h = 1m$
$\frac{dh}{dt} = 0.1m$
$V = \sqrt{2gh}$

Assumption: incompressible flow.

UNKNOWN:

$V_{in} = ?$

GOVERNING EQN:

$Q = AV$ continuity EQN.

SOLUTION:

$$\dot{m}_o - \dot{m}_i = -\frac{d}{dt}\int_{CV} \rho \, d\forall$$

$$-\rho V_{in} A_{in} + \rho V_{out} A_{out} = \frac{-d}{dt}(\rho A_{tank} h)$$

5.55 (continued)

$$-V_{in}A_{in} + V_{out}A_{out} = -A_{tank}\left(\frac{dh}{dt}\right)$$

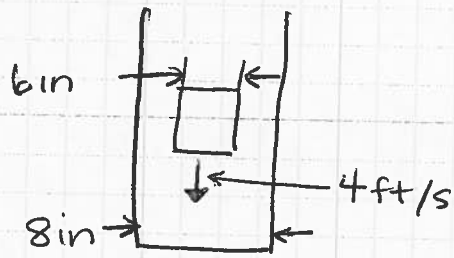
$$-V_{in}(0.0025) + \sqrt{2g(1)}(0.0025) = -0.1(0.1)\times 10^{-2}$$

$$V_{in} = \frac{\sqrt{19.62}(0.0025) + 10^{-4}}{0.0025}$$

$$\boxed{V_{in} = 4.47\ m/s}$$

5.57) A 6in. diameter cylinder falls at a rate of 4ft/s in an 8 in diameter tube containing an incompressible liquid. What is the mean velocity of the liquid (with respect to the tube) in the space between the cylinder and the tube wall?

SKETCH:



KNOWN:

$V_c = 4\,ft/s$
$D_{tube} = 8\,in$
$D_{cylinder} = 6\,in$

UNKNOWN:

mean velocity with respect of tube

GOVERNING EQN:

continuity equation.

SOLUTION:

$$0 = \frac{d}{dt}\int \rho\, dV + \dot{m}_o - \dot{m}_i$$

$$0 = \frac{d}{dt}(V) + V_T A_A$$

$$0 = V_c A_c + V_T (\pi/4)\left[(8in)^2 - (6in)^2\right]$$

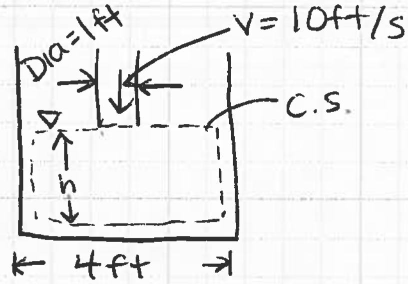
$$0 = -4ft/s\left(\frac{\pi}{4}\right)\left(\frac{6}{12}\right)^2 + V_T\left(\frac{\pi}{4}\right)\left[\left(\frac{8}{12}\right)^2 - \left(\frac{6}{12}\right)^2\right]$$

$$V_T = \frac{0.785\,ft^3/s}{0.1527\,ft^2} = \boxed{5.14\,ft/s = V_T}$$

velocity of liquid is upwards

5.58) The circular tank of water is being filled from a pipe as shown. The velocity of flow of water from the pipe is 10ft/s. What will be the rate of rise of the water surface in the tank?

SKETCH:



KNOWN:

$V_p = 10ft/s$
$D_T = 4ft$
$D_p = 1ft$

UNKNOWN:

rate of rise $V_{Rate\ of\ rise} = ?$

GOVERNING ~~SOE~~ EQN:

Continuity equation.

SOLUTION:

$$0 = \frac{d}{dt}\int_{cv} \rho\, dV + \dot{m}_o - \dot{m}_i$$

p = pipe
R = rise
T = Tank

$$0 = \frac{d}{dt}(hA_T) - ((10 + V_R)A_p)$$

$$0 = A_T\frac{dh}{dt} - 10A_p - V_R A_p$$

$$\frac{dh}{dt} = V_R$$

$$0 = A_T V_R - 10A_p - V_R A_p$$

$$V_R = \frac{10A_p}{A_T - A_p} = \frac{10ft/s(\pi/4)(1ft)^2}{\frac{\pi}{4}(4ft)^2 - \frac{\pi}{4}(1ft)^2} = \frac{7.85ft^3/s}{11.78ft^2} = 0.66\frac{ft}{s}$$

$$\boxed{V_R = 0.66ft/s}$$