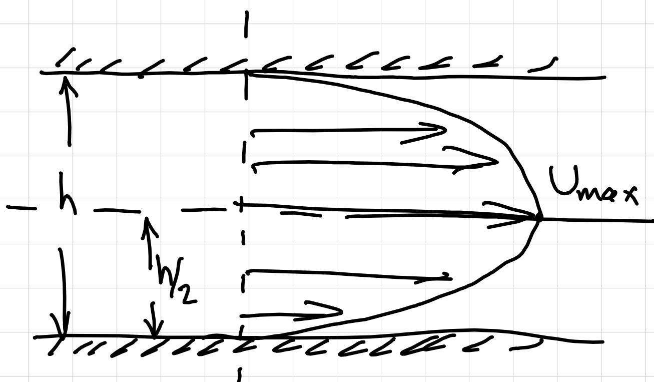
ES-8 SPRING 2024

P1. FLOW BETWEEN PARALLEL PLATES VELOCITY PROFILE:

$$u(y) = \frac{4U_{max}}{h^2}\left(hy - y^2\right)$$



KNOWN

GEOMETRY

$u(y)$

UNKNOWN

$\overline{U}$ (mean section velocity)

Q for depth W

GOVERNING PRINCIPLES

$$\overline{U} = \frac{\int_0^h u(y)\,dy}{\int_0^h dy} \qquad Q = \overline{U} A$$

SOLUTION

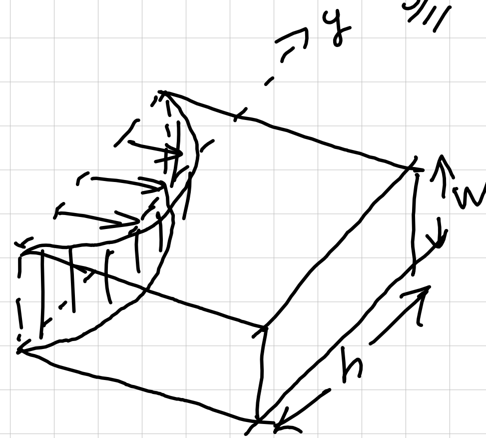
$$\bar{U} = \frac{\displaystyle\int_0^h \frac{4U_{max}}{h^2}\left(hy - y^2\right) dy}{\displaystyle\int_0^h dy}$$

$$= \frac{\dfrac{4U_{max}}{h^2}\left[\dfrac{hy^2}{2} - \dfrac{y^3}{3}\right]_0^h}{y\Big|_0^h}$$

$$= \frac{\dfrac{4U_{max}}{\cancel{h^2}}\left[\dfrac{\cancel{h^3}}{2} - \dfrac{\cancel{h^3}}{3}\right]}{\cancel{h}} = 4U_{max}\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= 4U_{max}\left[\frac{3}{6} - \frac{2}{6}\right] = 4U_{max}\left[\frac{1}{6}\right]$$

$$= \frac{2\cdot\cancel{2}\,U_{max}}{3\cdot\cancel{2}} = \underline{\underline{\frac{2}{3}U_{max}}} \leftarrow$$ MEAN SECTION VELOCITY



$$Q = \bar{U} A$$

$$\bar{U} = \frac{2}{3} U_{max}$$

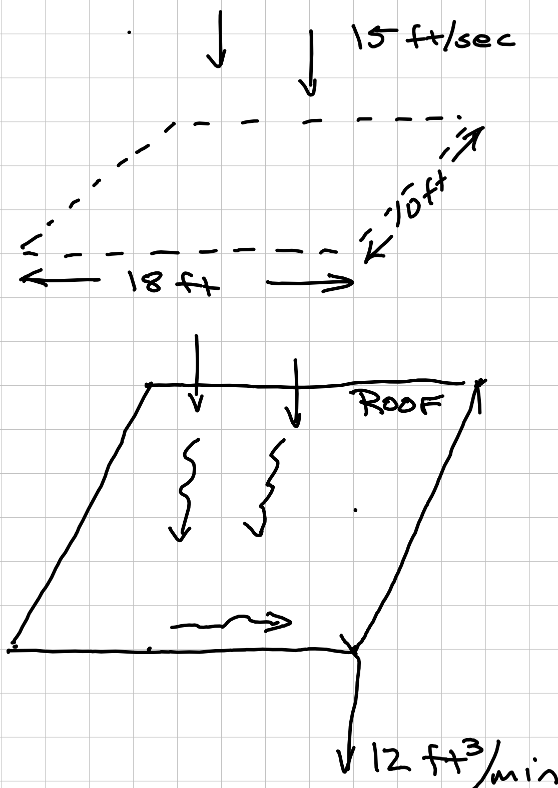
$$A = hw$$

$$\underline{\underline{Q = \frac{2}{3} U_{max} h w}}$$ ← DISCHARGE.

DISCUSSION

APPLICATION OF DEFN. MEAN SECTION VELOCITY (EQ. 4-3) IN DA BOOK!

PROBLEM 2



RAIN FALLS VERTICALLY ONTO ROOF WITH SPEED 15 ft/s. ROOF CATCH & ACCUMULATES 12 ft³/min

FIND

1) AMOUNT RAINWATER IN 1 ft³/AIR

2) DROPS PER ft³/air

KNOWN

DIMENSIONS ROOF, V_{fall}

$\forall_{drop}$

UNKNOWN

Q_{RAIN}/ft^3 AIR

$\#DROPS/ft^3$ AIR

GOVERNING EQN

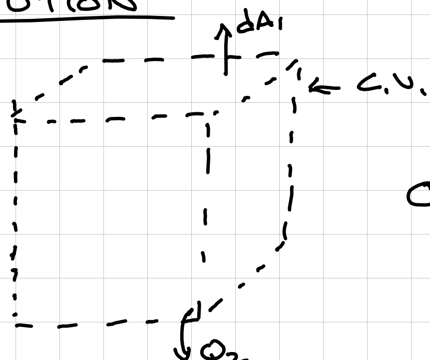
CONTINUITY

$Q = VA$

$\forall_{drop} = \frac{4}{3}\pi r^3 = \frac{\pi d^3}{6}$

SOLUTION

η = FRACTION OF AREA THAT IS DROPS.



CONTINUITY

$0 = \cancel{\frac{d}{dt}\int_{CV} \rho \, d\forall}^{\,0} + \int_{C.S.} \rho \, V \cdot dA$

$\eta \cancel{\rho} (15\,ft/s)(10\,ft)(18\,ft) - \cancel{\rho}\, 12\,ft^3/min = 0$

$\eta (15\,ft/s)(10\,ft)(18\,ft)\left(\frac{60\,s}{min}\right) = 12\,ft^3/min$

SOLVE FOR η

$\eta = 0.0000741$ ← FRACTION OF AREA THAT IS WATER

$\therefore$ $\forall_{WATER}$ per $1\ ft^3/air$ is

$0.0000741\ ft^3\ water/ft^3\ air$ ←

(SO MOSTLY AIR)

#DROPS

$$\forall_{drop} = \frac{\pi d^3}{6} = \frac{\pi\left(\frac{0.18}{12}\right)^3}{6}$$

$$= 0.0000018\ ft^3/drop$$

$\therefore$

$$\frac{0.0000741\ ft^3}{0.0000018\ \frac{ft^3}{drop}} = 41\ drops/ft^3$$

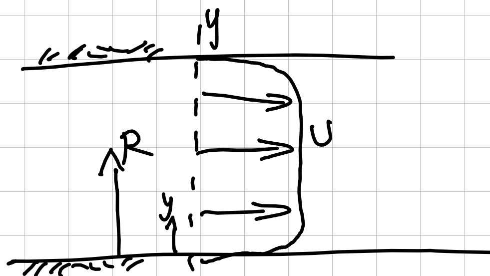
DISCUSSION

- REALLY JUST APPLICATION OF CONTINNUNITY. THE RELATIVELY SMALL NUMBERS COMPLICATES THINGS A LITTLE
- FIND $\forall/ft^3$; then how many drops make up volume

PROBLEM 3

CIRCULAR PIPE

$$u(y) = U\left(\frac{y}{R}\right)^{1/7}$$



FIND $\bar{U}$, Q

KNOWN

$u(y)$, GEOMETRY

UNKNOWN

$\bar{U}$, Q

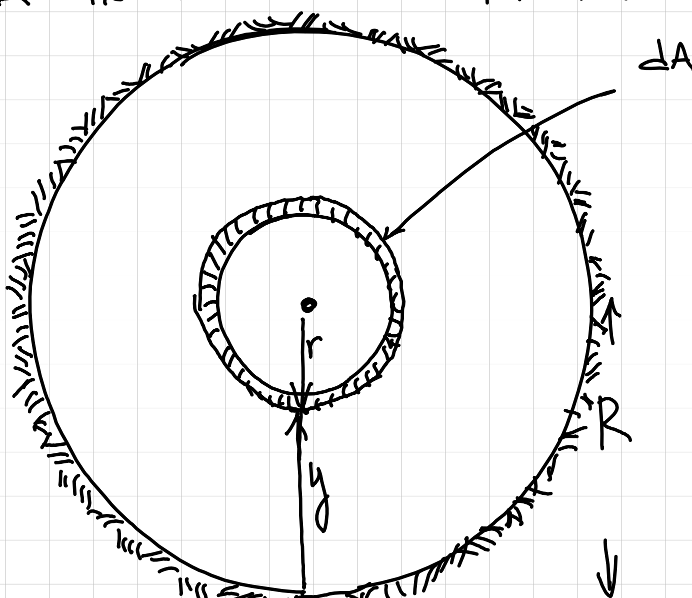
GOVERNING PRINCIPLES

$$\bar{U} = \frac{\int u \cdot dA}{\int dA}, \quad Q = \bar{U} A$$

SOLUTION

RECOGNIZE CIRCULAR CROSS-SECTION.

KEEP $U(y)$ FORM, BUT EXPRESS dA IN TERMS OF R AND y.



$$r = R - y; \quad dA = 2\pi (R-y)\, dy$$

$$\bar{U} = \frac{\int_0^R 2\pi (R-y)\, U \left(\frac{y}{R}\right)^{1/7} dy}{\int_0^R 2\pi (R-y)\, dy}$$

$$\bar{U} = \frac{\int_0^R 2\pi (R-y)\, U \left(\frac{y}{R}\right)^{1/7} dy}{\int_0^R 2\pi (R-y)\, dy}$$

$$= \frac{2\pi U \int_0^R \frac{R y^{1/7}}{R^{1/7}} - \frac{y^{8/7}}{R^{1/7}} \quad dy}{2\pi \int_0^R R - y \quad dy}$$

$$= \frac{\frac{2\pi U}{R^{1/7}} \left[\frac{R \; y^{8/7}}{8/7} - \frac{y^{15/7}}{15/7} \right]_0^R}{2\pi \; Ry - \frac{y^2}{2} \Big|_0^R}$$

$$= \frac{\frac{2\pi U}{R^{1/7}} \left(\frac{R \cdot R^{8/7}}{8/7} - \frac{R^{15/7}}{15/7} \right)}{2\pi R^2 - \frac{2\pi R^2}{2}}$$

$$= \frac{2\pi U \left(\frac{R \cdot R^{7/7}}{8/7} - \frac{R^{14/7}}{15/7} \right)}{2\pi R^2 - \pi R^2}$$

$$= \frac{2\cancel{\pi} U \left(\frac{R \cdot R^{7/7}}{8/7} - \frac{R^{14/7}}{15/7} \right)}{2\cancel{\pi} R^2 - \cancel{\pi} R^2}$$

$$= \frac{2U \left(\frac{R^2}{8/7} - \frac{R^2}{15/7} \right)}{R^2}$$

$$= \frac{2U\cancel{R^2} \left(7/8 - 7/15 \right)}{\cancel{R^2}}$$

$$= 2U \left(\frac{7(15) - 8(7)}{(8)(15)} \right)$$

$$= \cancel{2}U \left(\frac{49}{\cancel{120}_{60}} \right) = \underline{\underline{\frac{49}{60} U}} \leftarrow$$

TA DA!

VOLUMETRIC FLOW RATE

DEFN: $\bar{U} \cdot A$

$\overline{U} \cdot A \qquad A = \pi R^2$

$$\therefore \quad Q = \frac{49}{60} U \pi R^2$$

DISCUSSION

- DIRECT APPLICATION OF DEFN. $\overline{U}$.
- COMPLEXITY IN THAT WANT TO PERFORM INTEGRATION ACROSS PIPE IN r,y COORDINATES BECAUSE THE 1/7 POWER OTHERWISE COMPLICATES INTEGRAL