3.7) As shown, a mouse can use the mechanical advantage provided by a hydraulic machine to lift up an elephant.

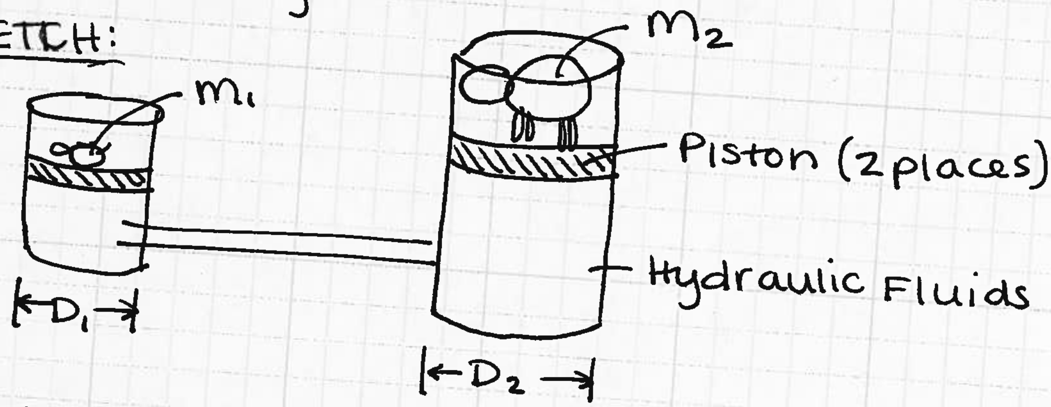
a) Derive an algebraic equation that gives the mechanical advantage of the hydraulic machine shown. Assume the pistons are frictionless and massless.

b) A mouse can have a mass of 25g and an elephant a mass of 7500kg. Determine a value of D_1 and D_2 so that the mouse can support the elephant.

KNOWN

$m_1 = 0.025kg$
$m_2 = 7500kg$

SKETCH:



UNKNOWN:

(a) Derive an algebraic equation for the mechanical advantage.

(b) Calculate D_1 and D_2 so the mouse can support the elephant.

Assumptions:

- Neglect the mass of the pistons
- Neglect the friction btn the piston & the cylinder wall.
- The pistons are @ the same elevation; thus, the pressure acting on the bottom of each piston is the same.
- A mouse can fit into a piston of diameter $D_1 = 70mm$.

3.7 cont'd)

Step a) mechanical advantage.

$$\left\{\begin{matrix}\text{mechanical}\\ \text{advantage}\end{matrix}\right\} = \frac{\text{weight "lifted" by the mouse}}{\text{weight of the mouse}}$$

$$\text{(eq. 1)} = \frac{W_2}{W_1} = \frac{\text{weight of elephant}}{\text{weight of mouse}}$$

Step b) Equilibrium (piston 1):

$$W_1 = p\left(\frac{\pi D_1^2}{4}\right)$$

$$P = W_1\left(\frac{4}{\pi D_1^2}\right) \quad \text{(eq. 2)}$$

Step c) Equilibrium (piston 2):

$$W_2 = p\left(\frac{\pi D_2^2}{4}\right)$$

$$P = W_2\left(\frac{4}{\pi D_2^2}\right) \quad \text{(eq. 3)}$$

Step D) Combine Eqs. (2) and (3)

$$P = W_1\left(\frac{4}{\pi D_1^2}\right) = W_2\left(\frac{4}{\pi D_2^2}\right) \quad \text{(eq. 5)}$$

Solve Eq. (5) for mechanical advantage

$$\boxed{\frac{W_2}{W_1} = \left(\frac{D_2}{D_1}\right)^2} \Rightarrow \text{Question 1a) algebraic eq.}$$

3.7 cont'd)

Step e) Calculate D_2

$$\frac{W_2}{W_1} = \left(\frac{D_2}{D_1}\right)^2$$

$$\frac{7500\,kg(9.8\,m/s^2)}{0.025\,kg(9.8\,m/s^2)} = 300{,}000 = \left(\frac{D_2}{0.07m}\right)^2$$

assumption

$$\Rightarrow D_2 = 38.3m$$

$$D_1 = 0.07m$$

The ratio of (D_2/D_1) needs to be $\sqrt{300{,}000}$.
If $D_1 = 70$ mm, then $D_2 = 38.3m$

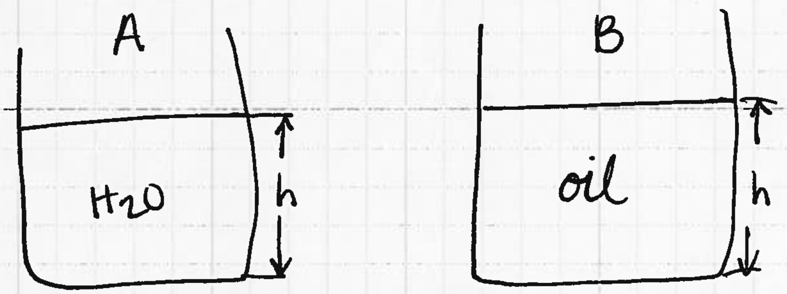
DISCUSSION:

① Notice. The mechanical advantage varies as the diameter ratio squared.

② The mouse needs a mechanical advantage of 300,000:1. This results in a piston that is impractical (dia. = 38.3m).

3.10) Imagine two tanks. Tank A is filled to depth h with water. Tank B is filled to depth h with oil. Which tank has the largest pressure? Why? Where in the tank does the pressure occur the largest?

SKETCH:



UNKNOWN

Which tank has the largest pressure?

Where in the tank does the largest pressure occur?

SOLUTION:

Pressure increases with depth, z, in both tanks.

$$P = -\gamma \Delta z$$

@ bottom of each tank: $P = \gamma h$

$\therefore$ Tank A, $P = \gamma_{H_2O} h$

Tank B, $P = \gamma_{oil} h$

Because $\gamma_{oil} < \gamma_{water}$, the pressure in tank A has the largest pressure because H_2O has a larger specific weight than oil.

The largest pressure occurs at the bottom of the tank.