 Name: Solutions

CE 3305 Engineering Fluid Mechanics
Exercise Set 14
Spring 2014

1. Problem 5.74, pg 203
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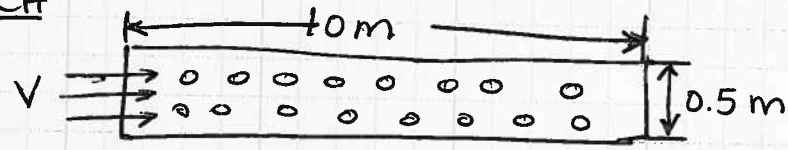
REVISION A

5.74) A pipe with a series of holes as shown in the figure is used in many engineering systems to distribute gas into a system.

$$Q = 0.67 A_o \left(\frac{2\Delta P}{\rho}\right)^{1/2}$$

A_o = area of the hole
ΔP = pressure difference across the hole.
ρ = density of gas

SKETCH



Known:

$$Q_{hole} = 0.67 A_o \left(\frac{2\Delta P}{\rho}\right)^{1/2}$$

$n_{hole} = 50/m = 500$ total holes
$L = 10m$
$D_{pipe} = 0.5m$
$D_{hole} = 2.5cm = 0.025m$
$T = 20°C$
$p_{pipe} = 100 Pa$ gage

Unknown:

$V_{air \text{ in pipe}} = ?$

Governing Equation:

$$Q = AV_{in} = NQ_{hole}$$

Solution:

$$N = 50 \times 10 = 500 \text{ holes}$$

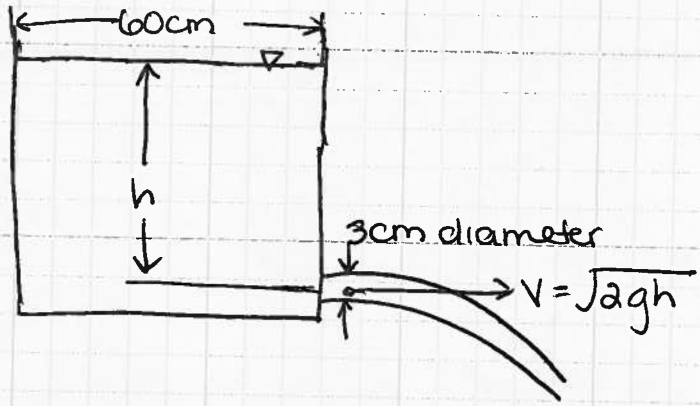
$$\rho = \frac{P}{RT} = \frac{100{,}000 + 100kPa}{287 \,\text{J/kgK}\,(273+20)K} = 1.19 kg/m^3$$

$$Q_{hole} = 0.67\left(\frac{\pi}{4}(0.025m)^2\right)\left(\frac{2 \times 100 Pa}{1.19 kg/m^3}\right)^{1/2} = 0.00426 m^3/s$$

$$V = \frac{NQ_{hole}}{A} = \frac{500(0.00426 m^3/s)}{\frac{\pi}{4}(0.5m)^2} = \boxed{10.8 m/s = V_{pipe}}$$

5.80) How long will it take the water surface in the tank shown to drop from h = 3m to h = 50cm?

SKETCH:



Known
$h_1 = 3m$
$h = 0.5m$
$D_T = 0.6m$
$D_2 = 3cm$

Unknown:

time = ?

Governing Equation:

$$t = \left(\frac{2A_T}{\sqrt{2g}\,A_2}\right)(h_1^{1/2} - h^{1/2})$$

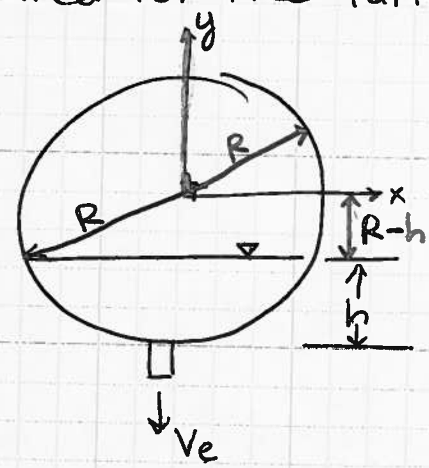
Solution:

$$t = \frac{2\left(\left(\frac{\pi}{4}\right)(0.6m)^2\right)(\sqrt{3} - \sqrt{0.5})m^{1/2}}{\sqrt{2\times 9.81 m/s^2}\left(\pi/4\,(0.03m)^2\right)} = \frac{0.579 m^{2.5}}{0.0031 m^{2.5}/s} = 184.9 s$$

$\boxed{t = 185 s}$

5.84) A spherical tank with a diameter of 1m is half filled with water. A port at the bottom of the tank is opened to drain the tank. The hole is 1 cm, and the velocity of the water draining from the hole is $V_e=\sqrt{2gh}$, where h is the elevation of the water surface above a hole. Find the time required for the tank to empty.

Sketch



Known:

$V_e=\sqrt{2gh}$

$R=0.5m$

$d_e= 1cm$

Unknown:

Time to empty tank

Governing Equation:

continuity equation.

Solution:

$$\rho\frac{d\forall}{dt} = -\rho A_e V_e$$

$$\frac{d\forall}{dt} = A\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-A_e}{A}\sqrt{2gh}$$

$$A_c = \pi\left[R^2-(R-h)^2\right] = \pi(2Rh-h^2)$$

$$\frac{\pi(-2Rh+h^2)}{A_e\sqrt{2gh}}\,dh = dt$$

$$\frac{\pi}{\sqrt{2g}A_e}\left(-2Rh^{1/2}+h^{3/2}\right)dh = dt$$

5.84 continued)

Integrating eqn.

$$\frac{\pi}{\sqrt{2g}\,A_e}\left(-\frac{4}{3}Rh^{3/2}+\frac{2}{5}h^{5/2}\right)\Big|_R^0=\Delta t$$

$$\frac{\pi}{\sqrt{2g}\,A_e}\,\frac{14}{15}\,R^{5/2}=\Delta t$$

for:

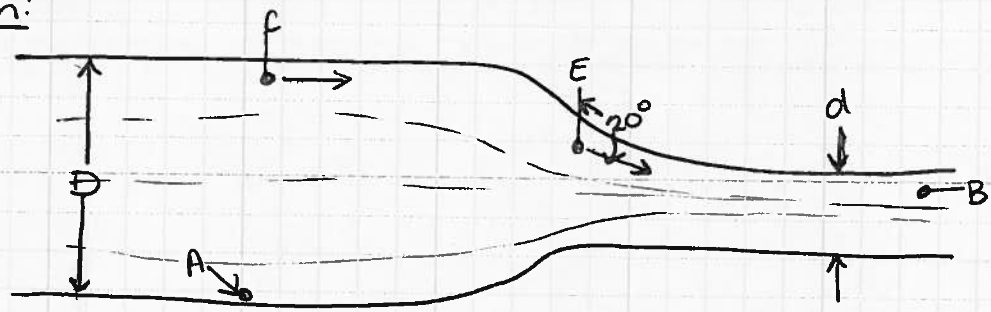
$R = 0.5\text{ m}$

$A_e = \frac{\pi}{4}(0.01\text{ m})^2 = 7.85\times10^{-5}\text{ m}^2$

$$\boxed{\Delta t = 1491\text{s or } 24.8\text{ min}}$$

5.94) The flow pattern through the pipe contraction is as shown, and the Q of water is 60cfs. For d=2ft and D=6ft what is the pressure at point B if the pressure at point C is 3200 psf?

Sketch:



Known:

$Q = 60 ft/s$
$d = 2ft$
$D = 6ft$
$P_c = 3200 psf$

water (70°F) = 62.3 lbf/ft^3

unknown:

$P_B = ?$

Governing Equation

$$\frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B = \frac{P_c}{\gamma} + \frac{V_c^2}{2g} + z_c$$

$Q = VA$

Solution:

$$V_B = \frac{Q}{A_B} = \frac{60 ft^3/s}{\pi/4 (2ft)^2} = 19.1 ft/s$$

$$V_C = \frac{Q}{A_C} = \frac{60 ft^3/s}{\pi/4 (6ft)^2} = 2.12 ft/s$$

$$\frac{P_B}{\gamma} = \frac{3200 psf}{62.3 lbf/ft^3} + \frac{(2.12 ft/s)^2}{64.4 ft/s^2} - \frac{(19.1 ft/s)^2}{64.4 ft/s^2} - 4ft$$

$$\frac{P_B}{\gamma} = 51.36 ft + 0.0697 ft - 5.66 ft - 4ft = 41.8 ft$$

$$\boxed{P_B = 2602 \text{ lbf/ft}^2 \quad \text{or} \quad 18.1 \text{ lbf/in}^2}$$