

CE 3354 Engineering Hydrology
Exercise Set 1

Exercises

1. Using the internet, textbook(s), and the on-line reading collection define the following (in a sentence or two); please cite your references (URL is sufficient):
 - (a) Alluvium
 - (b) Bankfull Discharge
 - (c) Best Management Practice
 - (d) Drainage Divide
 - (e) Evaporation
 - (f) Evapotranspiration
 - (g) Precipitation
 - (h) Flow Duration Curve
 - (i) Flood Frequency Curve
 - (j) Watershed
 - (k) Catchment

Solution(s)

- (a) **Alluvium** (from Latin alluvius, from alluere 'to wash against') is loose clay, silt, sand, or gravel that has been deposited by running water in a stream bed, on a floodplain, in an alluvial fan or beach, or in similar settings <https://en.wikipedia.org/wiki/Alluvium>
- (b) **Bankfull Discharge** is the maximum discharge that the channel can convey without overflowing onto the floodplain. http://www.extranet.vdot.state.va.us/locdes/hydraulic_design/nchrp_rpt544/content/html/WorksCited/Copeland_2001.pdf
- (c) **Stormwater Best Management Practices** are devices, practices, or methods that are used to manage stormwater runoff by controlling peak runoff rate, improving water quality, and managing runoff volume. <https://spcwater.org/topics/stormwater-management/stormwater-best-management-practices-2/>
- (d) **Drainage Divide**, water divide, ridgeline, watershed boundary, water parting

or height of land is elevated terrain that separates neighboring drainage basins. https://en.wikipedia.org/wiki/Drainage_divide

- (e) **Evaporation** is the process that changes liquid water to gaseous water (water vapor). Water moves from the Earth's surface to the atmosphere via evaporation. <https://www.usgs.gov/special-topics/water-science-school/science/evaporation-and-water-cycle>
- (f) **Evapotranspiration** is the sum of all processes by which water moves from the land surface to the atmosphere via evaporation and transpiration. <https://www.usgs.gov/special-topics/water-science-school/science/evapotranspiration-and-water-cycle>
- (g) **Precipitation** is water released from clouds in the form of rain, freezing rain, sleet, snow, or hail. Precipitation is the main way atmospheric water returns to the surface of the Earth. Most precipitation falls as rain. <https://www.usgs.gov/special-topics/water-science-school/science/precipitation-and-water-cycle>
- (h) **Flow Duration Curve** is a cumulative frequency curve that shows the percent of time specified discharges were equaled or exceeded during a given period. It combines in one curve the flow characteristics of a stream throughout the range of discharge, without regard to the sequence of occurrence. <https://pubs.er.usgs.gov/publication/wsp1542A>
- (i) **Flood Frequency Curve** is used to relate flood discharge values to return periods to provide an estimate of the intensity of a flood event. The discharges are plotted against return periods using either a linear or a logarithmic scale. In order to provide an estimate of return period for a given discharge or vice versa, the observed data is fitted with a theoretical distribution using a cumulative density function (CDF). <https://serc.carleton.edu/hydromodules/steps/168500.html>
- (j) **Watershed** is the land area that channels rainfall and snowmelt to creeks, streams, and rivers, and eventually to outflow points such as reservoirs, bays, and the ocean. <https://oceanservice.noaa.gov/facts/watershed.html>
- (k) **Catchment** is an area where water is collected by the natural landscape. <https://www.watarnsw.com.au/water-quality/education/learn/catchment>

2. Assuming that all water in the oceans is involved in the hydrologic cycle, estimate the average residence time of ocean water. [Problem 1.1.1 in Chow, Maidment, and Mays]

Solution(s)

Figure 1 shows the estimated residence time of an arbitrary water molecule in the ocean using Table 1.1.1 in Chow, Maidment, and Mays.

Use Table 1.1.1 in CMM

$$V_{\text{ocean}} = 1.38 \cdot 10^9 \text{ km}^3$$

Use Example 1.1.1 in CMM for throughput in atmosphere (precipitation rate)

Example 1.1.1 in CMM is source of precip throughput

$$\dot{P}_{\text{global}} = 5.77 \cdot 10^5 \text{ km}^3/\text{yr}$$

$$\bar{T}_r = \frac{S}{Q} = \frac{V_{\text{ocean}}}{\dot{P}_{\text{global}}} = \frac{1.38 \cdot 10^9 \text{ km}^3}{5.77 \cdot 10^5 \text{ km}^3/\text{yr}}$$

$$= 2300 \text{ years}$$

Discussion:

Value seems kind of large; idea is that oceans are huge and on average a water molecule can spend a long time in the ocean.

Figure 1: Ocean Residence Time Estimate

3. Assuming that all surface runoff to the oceans comes from rivers, estimate the average residence time of water in rivers. [Problem 1.1.2 in Chow, Maidment, and Mays]

Solution(s)

Figure 2 shows the estimated residence time of an arbitrary water molecule in the ocean using Table 1.1.1, and Table 1.1.2 in Chow, Maidment, and Mays.

Use Table 1.1.1 in CMM

$$V_{\text{RIVER}} = 2120 \text{ km}^3$$

Use Table 1.1.2 in CMM

$$\dot{R}_{\text{RIVER}} = 44,700 \text{ km}^3/\text{yr}$$

$$T_r = \frac{S}{Q} = \frac{2120 \text{ km}^3}{44,700 \text{ km}^3/\text{yr}} = 0.047 \text{ years} \\ = 17 \text{ days}$$

Discussion:

Water spends about 2½ weeks in a river before getting to ocean. Seems about right.

Figure 2: River Residence Time Estimate

4. The equation $k \frac{dQ}{dt} + Q(t) = I(t)$ has been used to describe the response of streamflow to a constant rate of precipitation continuing indefinitely on a watershed. For this problem, let $I(t) = 1$ for $t > 0$ and $Q(t) = 0$ for $t = 0$. Plot values of $I(t)$ and $Q(t)$ over a 10-hour period if $k = 2$. [Problem 1.3.2 in Chow, Maidment, and Mays]¹

Solution(s)

Figure 3 shows the analysis steps to produce an equation for plotting.

Sketch

Given

$$k \left(\frac{dQ}{dt} \right) + Q = I$$

$I(t) = 1$ for all $t > 0$
 $Q(0) = 0$

find
 $Q(t) \neq I(t)$ for $t = 0$ to 10 hrs
 $k = 2$ hrs

solution

$$k \frac{dQ}{dt} = I - Q \quad \text{* rearrange}$$

$$k \frac{dQ}{I - Q} = dt \quad \text{* separate}$$

$$\frac{dQ}{I - Q} = \frac{1}{k} dt \quad \text{* divide by k}$$

$$\frac{dQ}{I - Q} = \frac{1}{k} dt \quad \text{* substitute}$$

Let $u = I - Q$; $du = -dQ$

$$\therefore \frac{-(-dQ)}{I - Q} = -\frac{du}{u} = \frac{1}{k} dt$$

I is the constant "one"

$$\ln|u| = -\frac{1}{k} t + C$$

* Evaluate C at $t = 0$
 $u(t=0) = 1$; $Q = 0$
 $\therefore \ln|u| = -\frac{1}{k} t$

$$e^{\ln|u|} = u = e^{-\frac{1}{k} t}$$

* replace original variables

$$1 - Q(t) = e^{-\frac{1}{k} t}$$

or

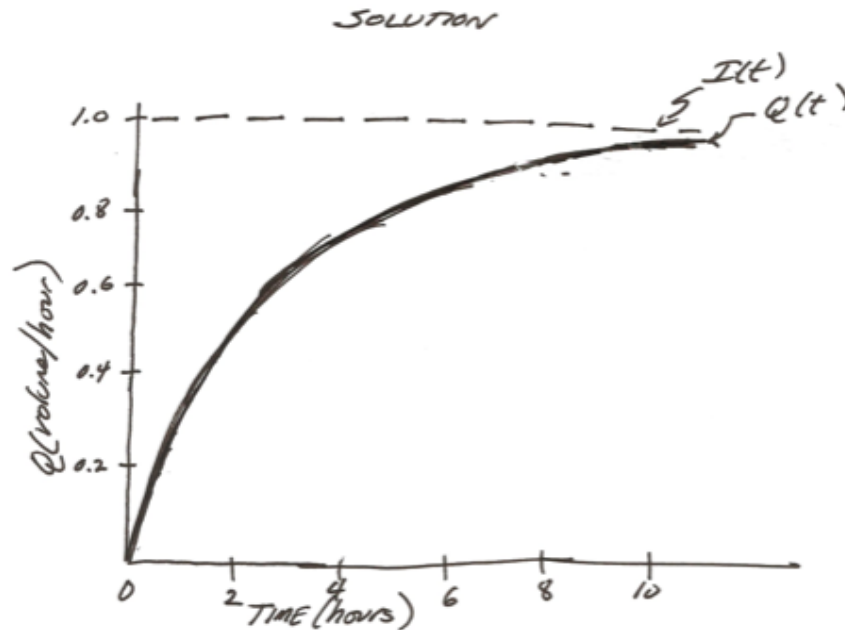
$$Q(t) = 1 - e^{-\frac{1}{k} t}$$

This is equation of discharge for the watershed

Figure 3: Analysis to produce $Q = f(I, T)$ equation.

¹You will need to solve the differential equation

Figure 4 shows a script to plot of the resulting relationship.



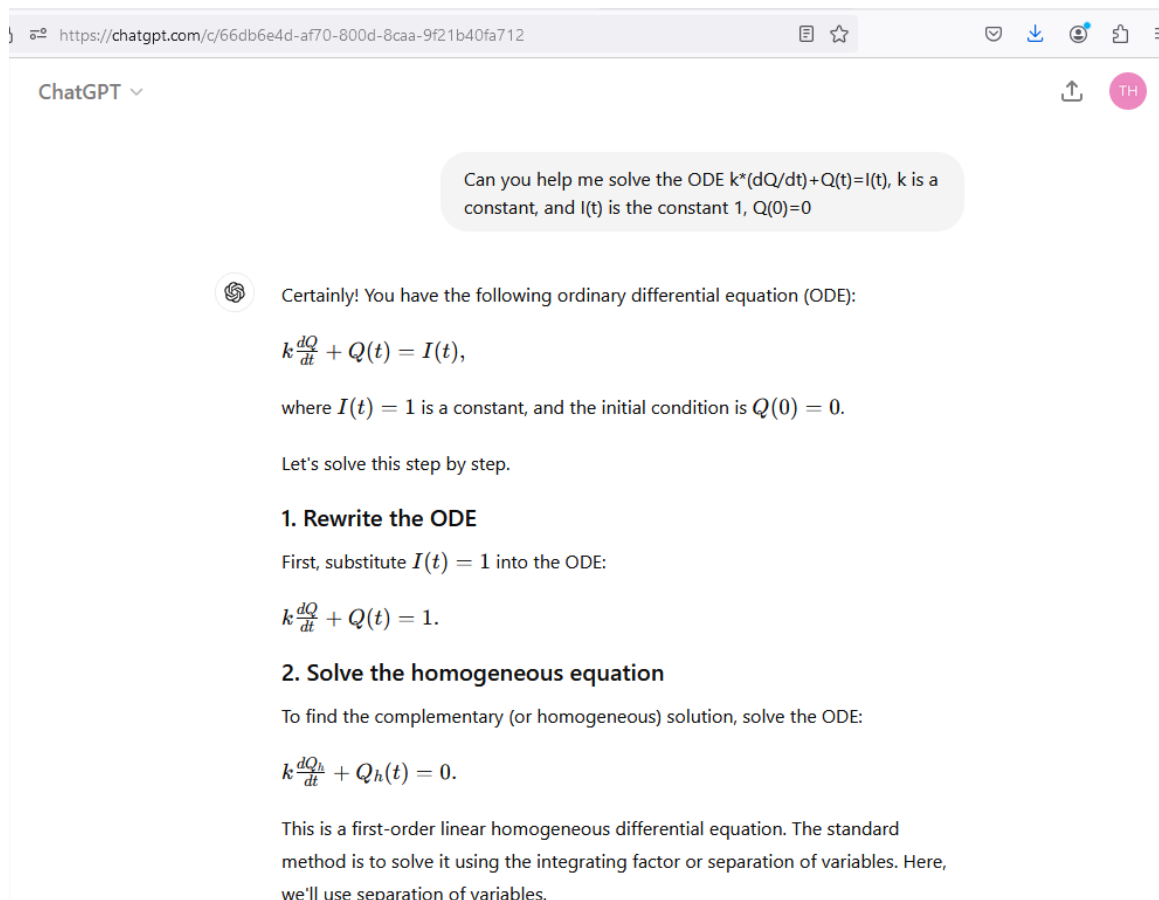
Discussion:

For constant rate input $I(t) = 1$, $Q(t)$ starts at 0 and exponentially approaches $I(t)$ — at large t
 $Q(t) = I(t)$ (as expected)

Figure 4: Plot values of $I(t)$ and $Q(t)$ over a 10-hour period if $k = 2$.

Chat GPT Solution

Figures 5 through 9 show how to use a large-language model to assist in the solution to the ODE. Note how the prompt is stated. In this case the GPT solution is correct for the problem.



The screenshot shows a web browser window with the URL <https://chatgpt.com/c/66db6e4d-af70-800d-8caa-9f21b40fa712>. The chat interface shows a user asking for help with an ODE. The response from ChatGPT identifies the equation as a first-order linear ODE and provides a step-by-step solution plan.

Can you help me solve the ODE $k \frac{dQ}{dt} + Q(t) = I(t)$, k is a constant, and $I(t)$ is the constant 1, $Q(0) = 0$

Certainly! You have the following ordinary differential equation (ODE):

$$k \frac{dQ}{dt} + Q(t) = I(t),$$

where $I(t) = 1$ is a constant, and the initial condition is $Q(0) = 0$.

Let's solve this step by step.

1. Rewrite the ODE

First, substitute $I(t) = 1$ into the ODE:

$$k \frac{dQ}{dt} + Q(t) = 1.$$


2. Solve the homogeneous equation



To find the complementary (or homogeneous) solution, solve the ODE:

$$k \frac{dQ_h}{dt} + Q_h(t) = 0.$$

This is a first-order linear homogeneous differential equation. The standard method is to solve it using the integrating factor or separation of variables. Here, we'll use separation of variables.

Figure 5: Chat GPT ODE solution (1 of 4)

ChatGPT 

Rearrange the terms:

$$\frac{dQ_h}{dt} = -\frac{Q_h(t)}{k}.$$

Separate variables:

$$\frac{dQ_h}{Q_h(t)} = -\frac{dt}{k}.$$

Integrate both sides:

$$\int \frac{1}{Q_h(t)} dQ_h = -\frac{1}{k} \int dt.$$

This gives:

$$\ln |Q_h(t)| = -\frac{t}{k} + C,$$

where C is the integration constant. Exponentiate both sides to solve for $Q_h(t)$:

$$Q_h(t) = C e^{-t/k}.$$

3. Find the particular solution

To find the particular solution for the non-homogeneous term 1, assume $Q_p(t)$ is a constant solution:

$$Q_p(t) = A.$$

Substitute $Q_p(t) = A$ into the original ODE:

Figure 6: Chat GPT ODE solution (2 of 4)

ChatGPT ▾



$$k \cdot 0 + A = 1.$$

So:

$$A = 1.$$

Thus, the particular solution is:

$$Q_p(t) = 1.$$

4. Combine solutions and apply initial condition

The general solution $Q(t)$ is the sum of the homogeneous solution and the particular solution:

$$Q(t) = Q_h(t) + Q_p(t) = Ce^{-t/k} + 1.$$

Apply the initial condition $Q(0) = 0$:

$$Q(0) = Ce^0 + 1 = C + 1.$$

Setting this equal to 0:

$$C + 1 = 0 \quad C = -1.$$

So the solution is:

$$Q(t) = -e^{-t/k} + 1.$$

Figure 7: Chat GPT ODE solution (3 of 4)

Final Solution

The solution to the ODE $k \frac{dQ}{dt} + Q(t) = 1$ with the initial condition $Q(0) = 0$ is:

$$Q(t) = 1 - e^{-t/k}.$$

Figure 8: Chat GPT ODE solution (4 of 4)

Final Solution

The solution to the ODE $k \frac{dQ}{dt} + Q(t) = 1$ with the initial condition $Q(0) = 0$ is:

$$Q(t) = 1 - e^{-t/k}.$$

Figure 9: Chat GPT ODE solution (4 of 4)

Figure 10 shows a script to plot of the resulting relationship.

```
[1]: # Plot the ODE Solution
import sys
# the flow function
def flow_for_constant_input(k,I,time):
    import math,sys
    try:
        qoft = I - math.exp(-time/k)
    except:
        print("Failed function call, Check if I or k are 0")
        sys.exit() # force program exit
    return(qoft)
# Set values
I = 1; k = 2
# Compute series
time_series = [float(i) for i in range(0,13)]
flow_series = [float(0) for i in range(0,13)] # initial flow vector all zero
rain_series = [float(I) for i in range(0,13)] # actual constant rain vector
for i in range(0,13):
    flow_series[i]=flow_for_constant_input(k,I,time_series[i])
# build and display plot
from matplotlib import pyplot as plt
plt.plot(time_series,flow_series,color='red', marker='.', linestyle='solid')
plt.plot(time_series,rain_series,color='blue', marker='.', linestyle='solid')
plt.title("Plot of I(t) and Q(t) for k=2, t=[0,12.0] \n")# add a title
plt.ylabel("I(t) or Q(t)")# add a label to the x and y-axes
plt.xlabel("Time, t ")
plt.legend(["Q(t)","I(t)"])
plt.show() # display the plot
```

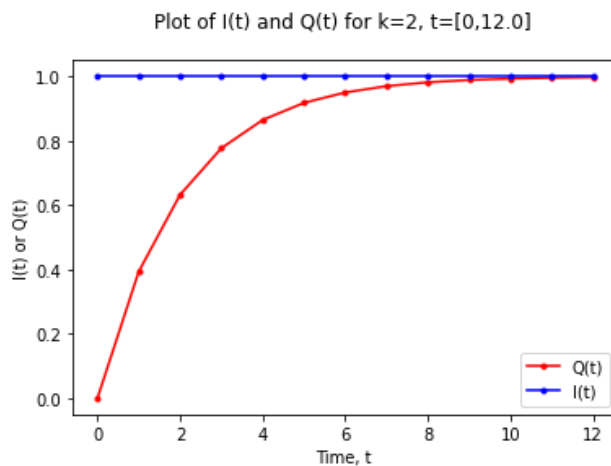


Figure 10: Plot values of $I(t)$ and $Q(t)$ over a 12-hour period if $k = 2$.

5. Figure 11 is a schematic of a 600-hectare farm; the land receives annual rainfall of 2500 mm. There is a river flowing through the farm land with inflow rate of $5 \text{ m}^3/\text{s}$ and outflow rate of $4 \text{ m}^3/\text{s}$. The annual water storage in the farm land increases by $2.5 \times 10^6 \text{ m}^3$. Using the water budget concept, estimate the annual evaporation amount in millimeters.²

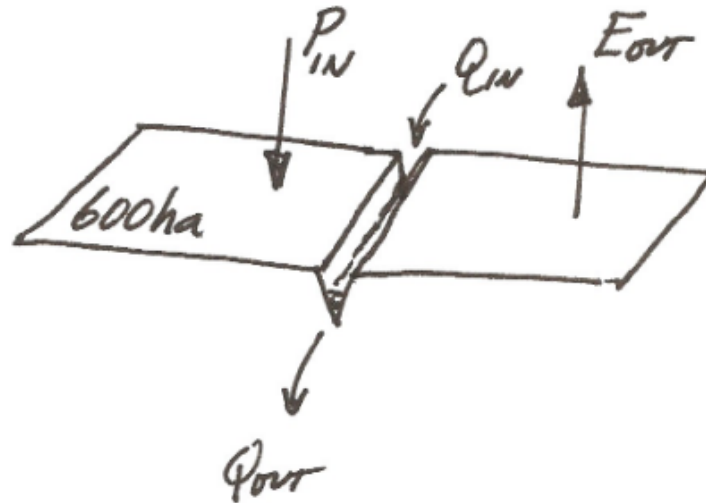
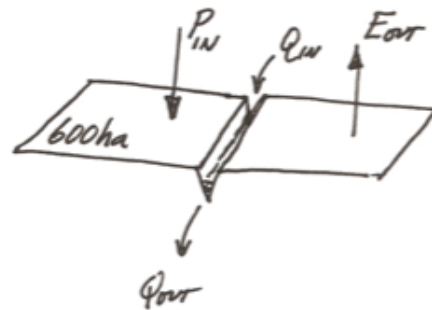


Figure 11: Schematic of Farmland

Solution(s)

Figure 12 shows the estimated annual evaporation for the 600-hectare farm in 11

²1 hectare = 10,000 m²



known

$$P_{IN} = 2500 \text{ mm/yr}$$

$$Q_{IN} = 5 \text{ m}^3/\text{s}$$

$$Q_{out} = 4 \text{ m}^3/\text{s}$$

$$\frac{\Delta S}{\Delta t} = 2.5 \cdot 10^6 \text{ m}^3/\text{yr}$$

UNKNOWN

E_{out}

SOLUTION

$$V_{IN_Q} = Q_{IN} \cdot 86400 \frac{\text{s}}{\text{day}} \cdot \frac{365 \text{ d}}{\text{yr}} = (5)(86400)(365) = 157,680,000 \text{ m}^3/\text{yr}$$

$$V_{out_Q} = Q_{out} \cdot 86400 \frac{\text{s}}{\text{day}} \cdot \frac{365 \text{ d}}{\text{yr}} = (4)(86400)(365) = 126,144,000 \text{ m}^3/\text{yr}$$

$$V_{IN_P} = P_{IN} (600 \text{ ha}) = (2.5 \frac{\text{m}}{\text{yr}})(600 \text{ ha}) \left(\frac{10000 \text{ m}^2}{\text{ha}} \right) = 15,000,000 \text{ m}^3/\text{yr}$$

$$\frac{\Delta S}{\Delta t} = (V_{IN_Q} + V_{IN_P}) - V_{out_Q} - E_{out}$$

$$E_{out} = (V_{IN_Q} + V_{IN_P}) - V_{out_Q} - \frac{\Delta S}{\Delta t} = 157,680,000 + 15,000,000 - 126,144,000 - 2,500,000$$

$$= 44,036,000 \text{ m}^3/\text{yr} \cdot \frac{1}{(600 \text{ ha}) \left(\frac{10000 \text{ m}^2}{\text{ha}} \right)}$$

$$= 7.339 \text{ m/yr}$$

$$= 7339 \text{ mm/yr.}$$

Discussion: Estimated evap. rate is nearly 3 times the rainfall rate in this region.

Figure 12: Farmland water balance estimates

6. A reservoir has a surface area of 690 acres. Figure 13 shows the monthly inflow of surface water, outflows as releases from the reservoir via the spillway, direct precipitation into the reservoir, and evaporation from the reservoir. The reservoir water surface elevation was 701.0 feet on January 1. Determine the reservoir water surface elevation at the end of each month (i.e. complete the table)

Lake Woodlands									
Average Surface Area = 690 acres									
Month	Inflow (acre-feet)	Outflow (acre-feet)	Precipitation (inches)	Precipitation (acre-feet)	Evaporation (inches)	Evaporation (acre-feet)	Storage Net Change (acre-feet)	Elevation Change (feet)	Water Surface Elevation (feet)
December									701.00
January	1732	175	2.75	158.13	1.05	60.38	1654.75	2.40	703.40
February	1755	190	3.05		1.55				
March	872	232	3.76		2.05				
April	955	375	4.11		2.80				
May	708	525	2.70		3.75				
June	312	955	1.05		4.25				
July	102	1720	0.75		5.15				
August	37	2250	1.25		5.76				
September	175	1575	1.55		4.92				
October	575	550	3.79		3.02				
November	1250	175	4.53		1.75				
December	1875	125	5.01		0.60				

Figure 13: Tabular Water Budget Values

Solution(s)

Figure 14 shows the estimated water budget for the 690-acre reservoir.

	A	B	C	D	E	F	G	H	I	J
1	Lake Woodlands									
2	Average Surface Area = 690 acres									
3	Month	Inflow (acre-feet)	Outflow (acre-feet)	Precipitation (inches)	Precipitation (acre-feet)	Evaporation (inches)	Evaporation (acre-feet)	Storage Net Change (acre-feet)	Elevation Change (feet)	Water Surface Elevation (feet)
4	December									701.00
5	January	1732	175	2.75	158.13	1.05	60.38	1654.75	2.40	703.40
6	February	1755	190	3.05	175.38	1.55	89.13	1651.25	2.39	705.79
7	March	872	232	3.76	216.20	2.05	117.88	738.33	1.07	706.86
8	April	955	375	4.11	236.33	2.80	161.00	655.33	0.95	707.81
9	May	708	525	2.70	155.25	3.75	215.63	122.63	0.18	707.99
10	June	312	955	1.05	60.38	4.25	244.38	-827.00	-1.20	706.79
11	July	102	1720	0.75	43.13	5.15	296.13	-1871.00	-2.71	704.08
12	August	37	2250	1.25	71.88	5.76	331.20	-2472.33	-3.58	700.50
13	September	175	1575	1.55	89.13	4.92	282.90	-1593.78	-2.31	698.19
14	October	575	550	3.79	217.93	3.02	173.65	69.28	0.10	698.29
15	November	1250	175	4.53	260.48	1.75	100.63	1234.85	1.79	700.08
16	December	1875	125	5.01	288.08	0.60	34.50	2003.58	2.90	702.98
21	Example Row showing formula entries for water balance									
22	January	1732	175	2.75	=D5*690/12	1.05	=F5*690/12	=B5+E5-C5-G5	=H5/690	=J4+I5
23										

Figure 1: Tabular Water Budget Values

see waterbudget spreadsheet

↑
Estimated pool
elevation for
each month

Figure 14: Tabular Water Budget (Complete - with annotations)