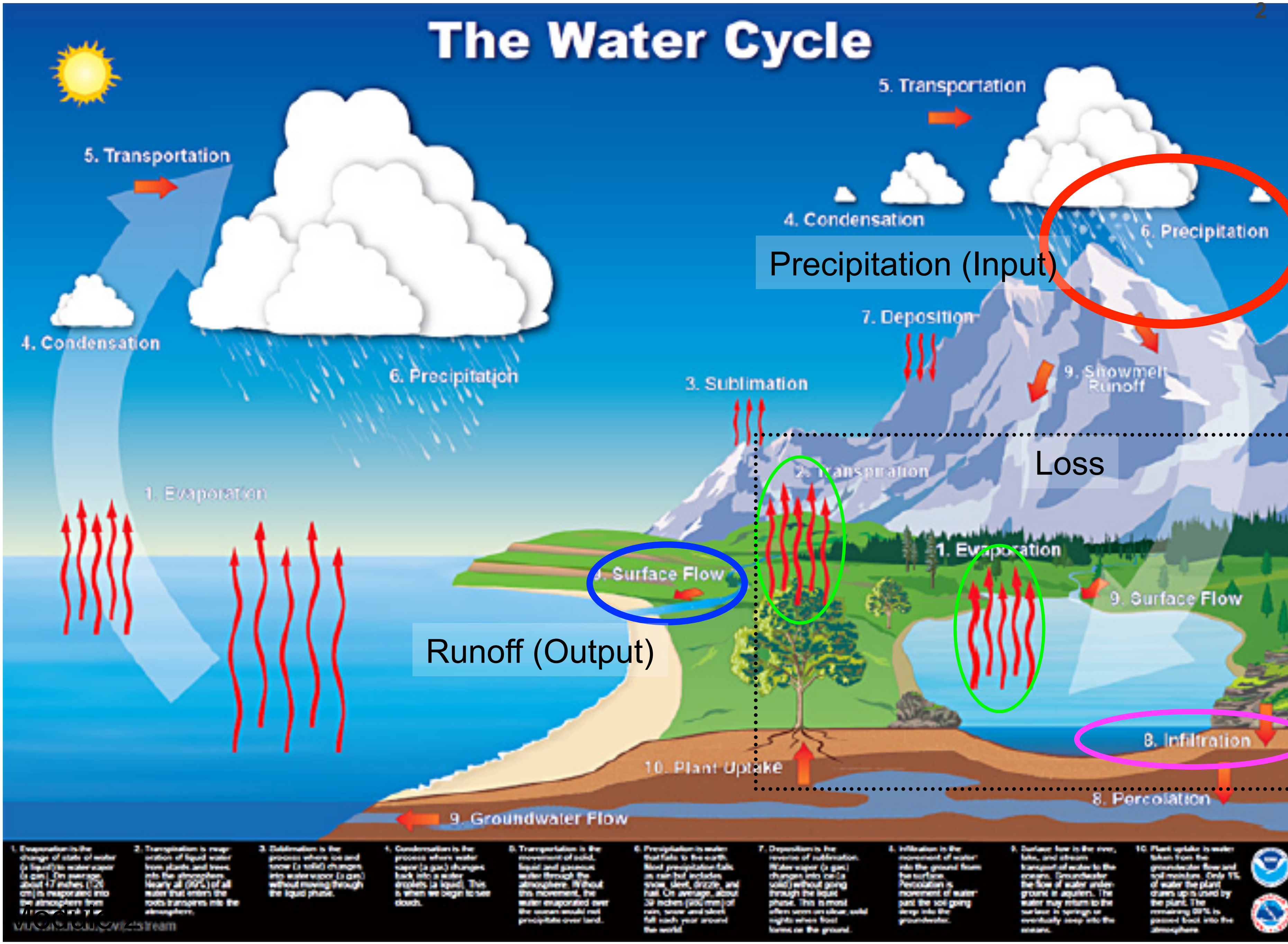
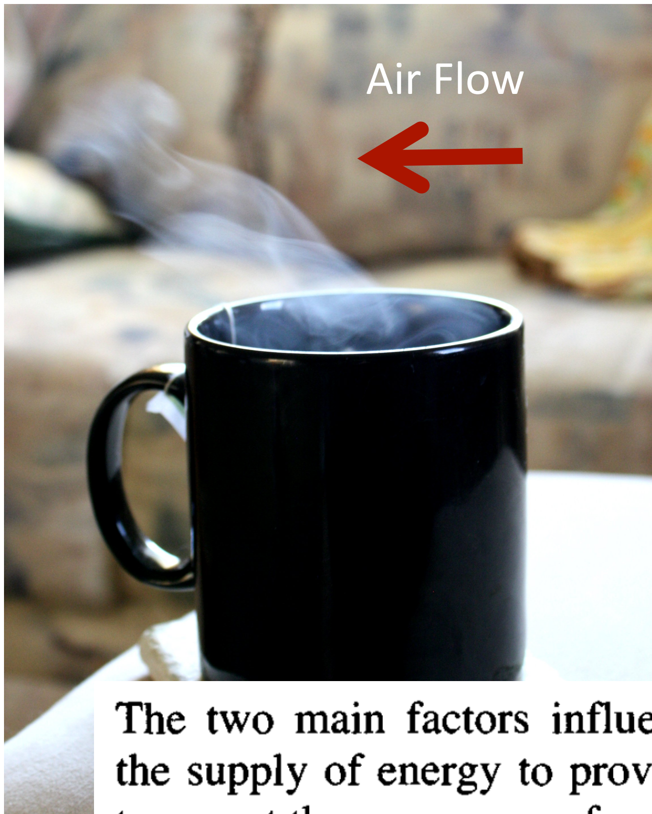
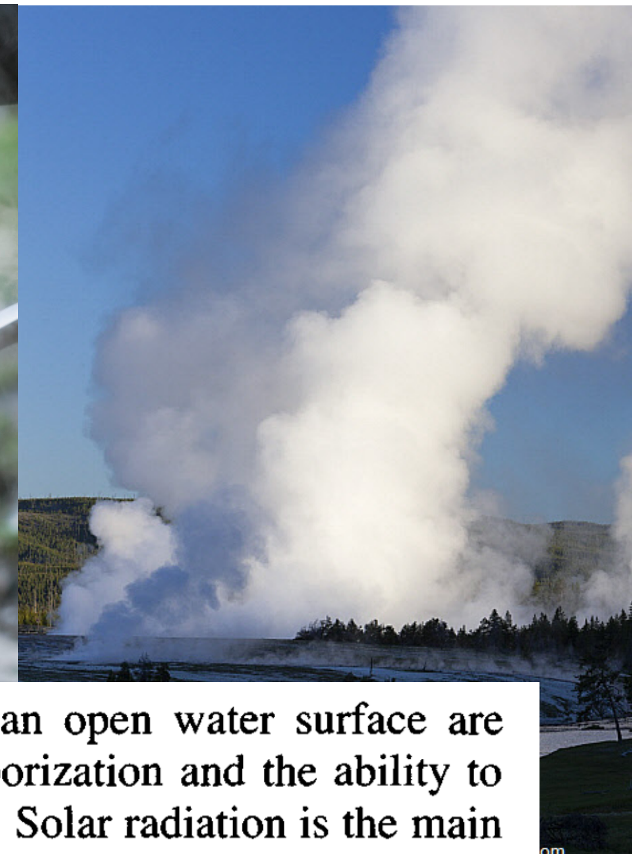
CE 5361 SURFACE WATER HYDROLOGY

WATERSHED PROCESS: EVAPOTRANSPIRITATON


The Water Cycle
5. Transportation
5. Transportation
4. Condensation
4. Condensation
6. Precipitation
6. Precipitation
Precipitation (Input)
7. Deposition
3. Sublimation
9. Snowmelt Runoff
1. Evaporation
2. Transpiration
Loss
1. Evaporation
9. Surface Flow
9. Surface Flow
Runoff (Output)
8. Infiltration
10. Plant Uptake
8. Percolation
9. Groundwater Flow

Evaporation Process

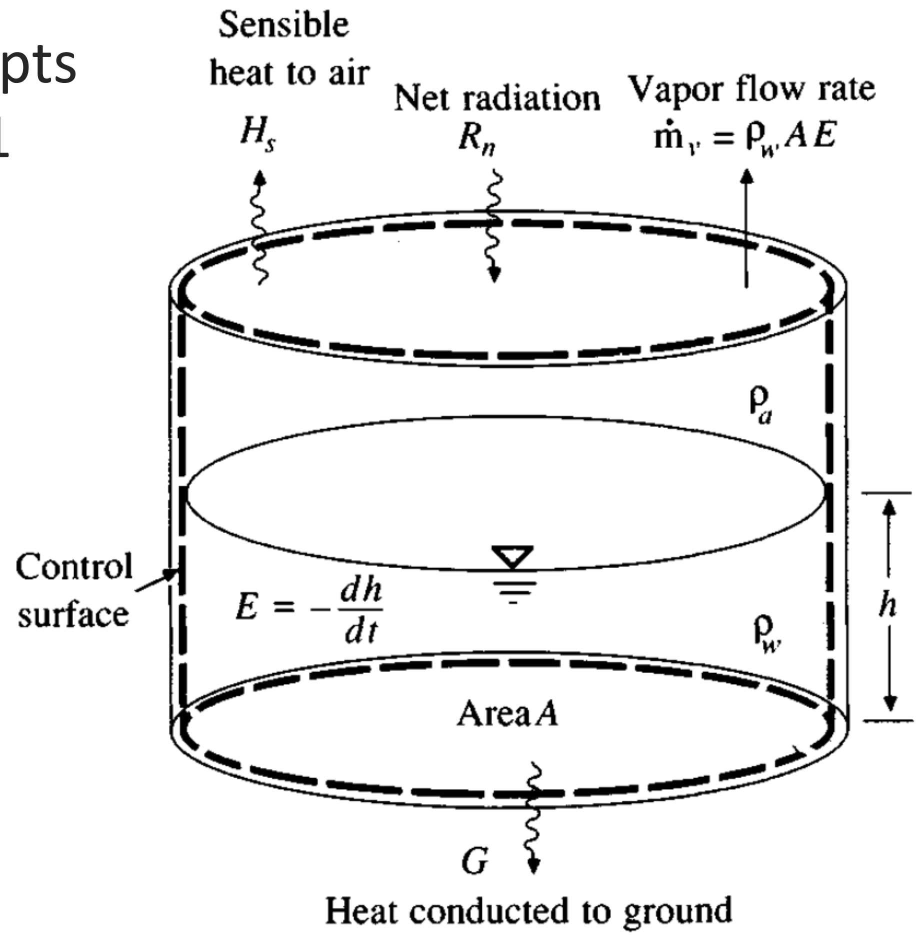




The two main factors influencing evaporation from an open water surface are the supply of energy to provide the latent heat of vaporization and the ability to transport the vapor away from the evaporative surface. Solar radiation is the main source of heat energy. The ability to transport vapor away from the evaporative surface depends on the wind velocity over the surface and the specific humidity gradient in the air above it.

Loss Processes – Evaporation

- Process Concepts CMM pp 80-91



Mass-Transfer Model

- Mass Transfer using linear driving force model.
  - Note the non-homogeneous units.
  - e_o and e_a are table look-up based on temperature of water/air for the study site.

$$E = N\,U\,(e_o - e_a)$$

where: E = evaporation, in inches per day;
N = mass-transfer coefficient;
U = wind speed, in miles per hour at 2 meters above the water surface;
e_o = saturation vapor pressure at the water-surface temperature, in millibars; and
e_a = vapor pressure of the air, in millibars.

Harbeck, G. E., Jr., 1962, A practical field technique for measuring reservoir evaporation utilizing mass-transfer theory: U.S. Geological Survey Professional Paper 272-E, p. 101-105.

Mass-Transfer Model

- How to find e_a?
  - From air temperature (T) , and dewpoint temperature (T_d) expressed in Celsius
  - Saturated vapor pressure (e_s) and the actual vapor pressure (e_a == e) are calculated using the formulas listed below:

$$e_s = 6.11 \times 10^{\left(\frac{7.5 \times T}{237.3 + T}\right)} \qquad e = 6.11 \times 10^{\left(\frac{7.5 \times T_d}{237.3 + T_d}\right)}$$

    The vapor pressure answers will be in units of millibars (*mb*) .
  - As a bonus, now can obtain relative humidity from:

$$rh = \frac{e}{e_s} \times 100$$

Mass-Transfer Model

- How to find e_o?
  - From water temperature (T_w) use the e_s formula, or

$$e_s = 6.11 \times 10^{\left(\frac{7.5 \times T}{237.3 + T}\right)}$$

  - Use a table of liquid properties:
    - http://atomickitty.ddns.net/documents/mytoolbox-server/FluidMechanics/WaterPropertiesSI/WaterPropertiesSI.html

Mass-Transfer Model

- How to find N?
  - Should make field observations over several episodes
    - 2 weeks in Winter, 2 weeks in Sprin … , use these to fit model to observations and recover N
  - Empirical relationships from literature
- Model is attractive for simulation because uses inputs that are generally available and/or easy to measure
  - Temperatures, Relative humidity, Wind speed
- Calibration is vital but suspect as climatic conditions depart from the calibration conditions

Energy-Budget Model

$$E = \frac{Q_s - Q_r + Q_a - Q_{ar} - Q_{bs} + Q_v - Q_x}{L\,(1 + R) + T_o}$$

where: E = evaporation, in centimeters per day;

Q_s = incoming solar radiation, in langleys per day;

Q_r = reflected solar radiation, in langleys per day;

Q_a = incoming long-wave radiation, in langleys per day;

Q_{ar} = reflected long-wave radiation, in langleys per day;

Q_{bs} = long-wave radiation from the water, in langleys per day;

Q_v = net energy advected into the lake, in langleys per day;

Q_x = change in stored energy, in langleys per day;

L = latent heat of vaporization, in calories per gram;

R = Bowen ratio; and

T_o = water-surface temperature, in degrees Celsius.

Energy-Budget Model

- How to determine R for latent heat calculation

$$R = 0.61 \frac{(T_o - T_a)}{(e_o - e_a)} P$$

where: T_a = air temperature, in degrees Celsius;

P = atmospheric pressure, in millibars; and

T_o, e_o, and e_a are as described previously.

Energy-Budget Model

- Other terms:
  - Incoming radiation – measure using a radiometer
  - Other radiation terms are estimated from methods in Anderson 1952 and Kolberg 1964
  - Stored and advected energy is temperature based:
    E ~ Temp*Volume

Anderson, E. R., 1952 (1954), Energy-budget studies, *in* Water-loss investigations, Lake Hefner studies, technical report: U.S. Geological Survey Professional Paper 269, p. 71-119.

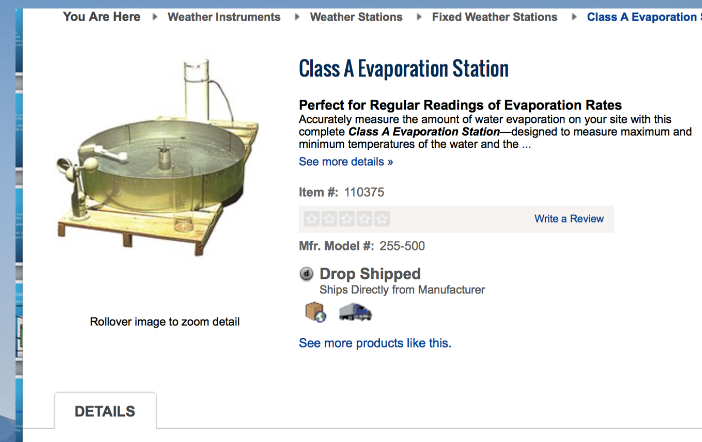
Koberg, G. E., 1964, Methods to compute long-wave radiation from the atmosphere and reflected solar radiation from a water surface: U.S. Geological Survey Professional Paper 272-F, p. 107-136.

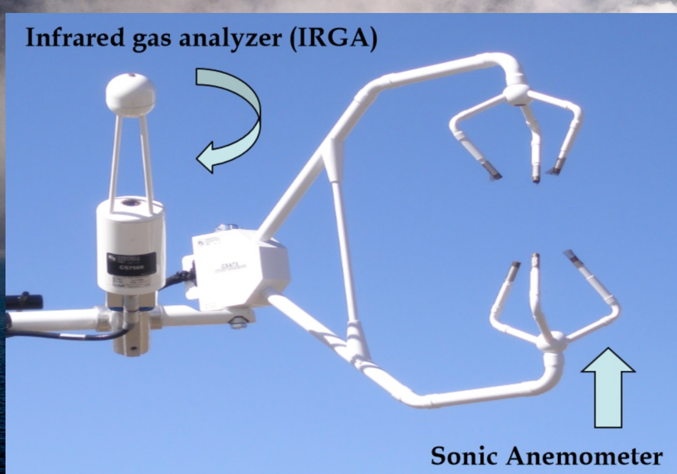
Energy-Budget Model

- Model is also attractive for simulation because uses inputs that re generally available and/or easy to measure
  - Temperatures, Relative humidity
  - Incoming radiation – measure using a radiometer
  - Advective fluxes: Temperature and volume (flows)
- Adaptations needed to use in non-open water setting, but applicable, also good for computational hydrology situations
- Blend the two approaches, deal with units, and have basis of models described in CMM and other references.

ET Measurement

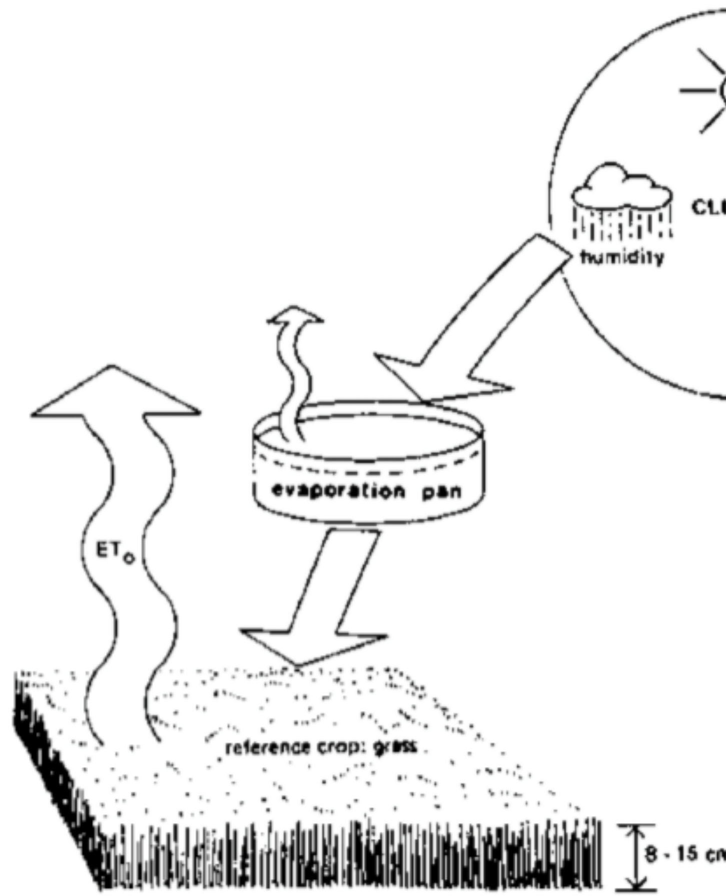
- Measurements
  - Evaporation Pans
    - Used worldwide
  - Flux Instruments
    - Eddy Covariance Instruments





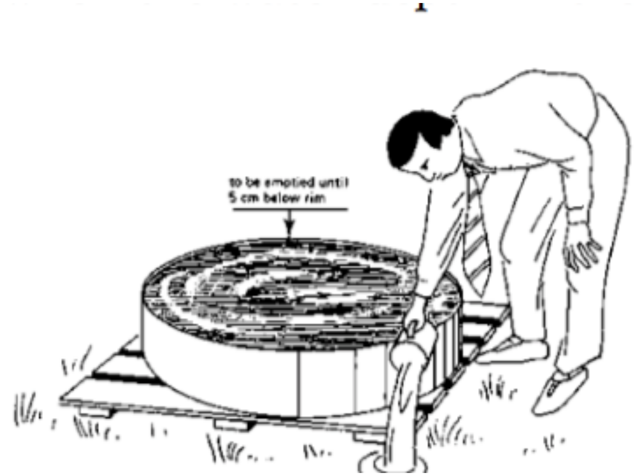
Evaporation Pans

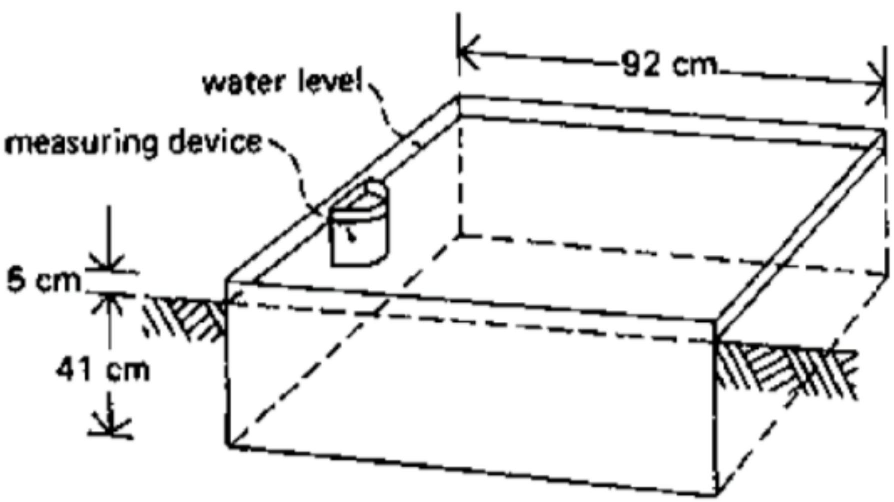
- Used in conjunction with lysimeter instruments to calibrate to crop type.
- Then make measurements with a pan or EC instrument



Evaporation Pans

- Class A - Circular.
- Colorado Sunken
  - Dug into ground, rectangular





- A small microprocessor, with sensors and pump controller can replace the person.
  - Program it to add water every 24 hours until full (easy to detect not full/full), record amount of make-up water (Hall Flow Detector); get air and water temp, and baromteric pressure, solar radiation

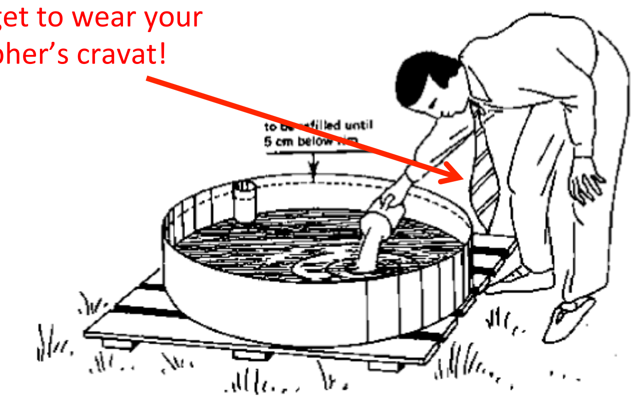
Evaporation Pan Operation (1 of 2)

- The pan is installed in the field
- The pan is filled with a known quantity of water
- The water is allowed to evaporate during a certain period of time (usually 24 hours).
  - The rainfall, if any, is measured simultaneously
- Every 24 hours, the remaining quantity of water (i.e. water depth) is measured

Evaporation Pan Operation (2 of 2)

- The amount of evaporation per time unit (the dffierence between the two measured water depths) is calculated; this is the pan evaporation: E_{pan} (in mm=24 hours)
- The E_{pan} is multiplied by a pan coecient, K_{pan}, to obtain the ET_o.
- Reset the pan for next time interval to desired level

Don’t forget to wear your hydrographer’s cravat!



Pan Constants

- Need to be determined by lysimeter or Eddy Covariance instruments

Table 2: Example of Pan Evaporation measurements and calculations

Item	Value
Pan type	Class A
Water depth in pan on day 1	150 mm
Water depth in pan on day 2	144 mm
Rainfall (during 24 hours)	0 mm
K_{pan}	0.75
Formula	$ET_o = K_{pan} \times E_{pan}$
Calculation	$E_{pan} = 150 - 144 = 6mm/day$
Result	$E_o = 0.75 \times 6 = 4.5mm/day$

Evapotranspiration – Models

- Models are used to estimate ET for practical cases where measurements are not available
  - Blaney-Criddle
  - Thornwaithe
  - Turk
- All similar in that they are correlations to averaged measurements at different locations
- All are just approximations, but are used in practice and when ET matters they may be only tool available

Blaney-Criddle Model

- Simple formula – Temperature and latitude driven only!
  - Estimates daily rate for a particular month

$$ET_o = p\ (0.46\ T_{mean} + 8)$$

  - Temperature is an average from daily values for a month

$$T_{max} = \frac{\sum T_{max\ \ daily}}{days}$$
$$T_{min} = \frac{\sum T_{min\ \ daily}}{days}$$
$$T_{mean} = \frac{T_{max} + T_{min}}{2}$$

Blaney-Criddle Model

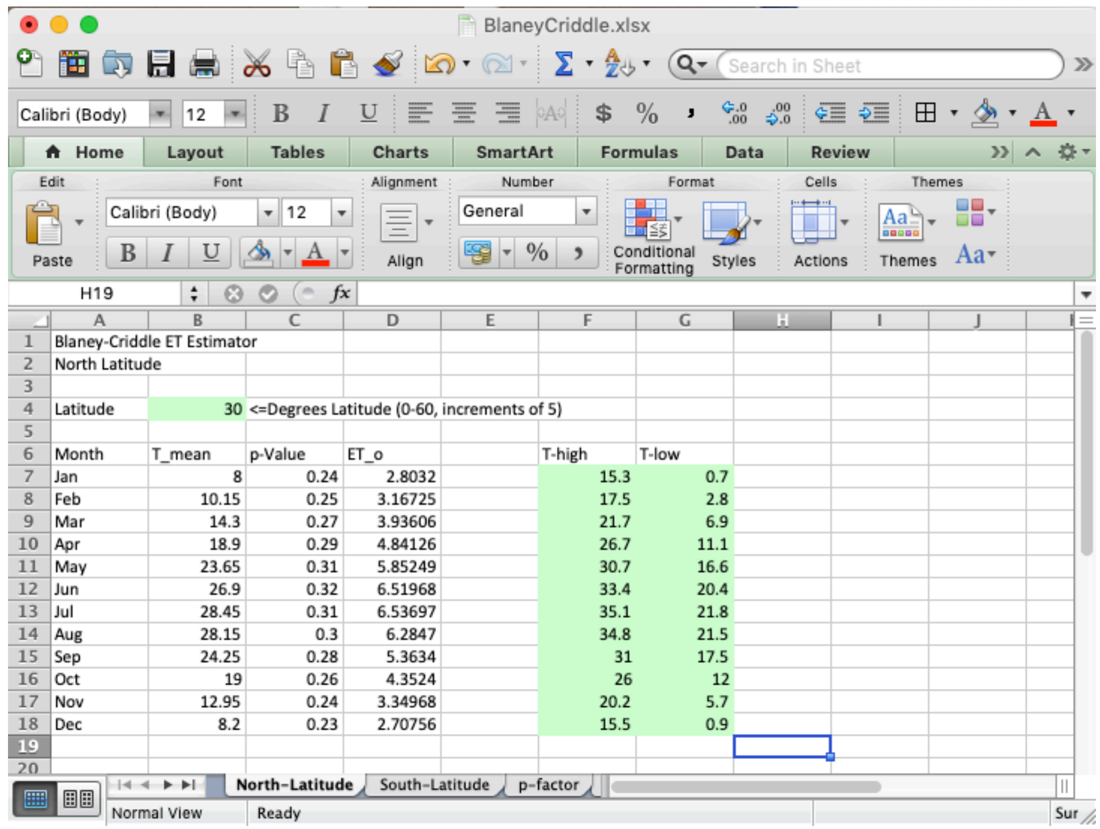
- P- value by latitude and month

Table 4: Blaney-Criddle p values by latitude

Lat (N)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Lat (S)	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun
60	.15	.20	.26	.32	.38	.41	.40	.34	.28	.22	.17	.13
55	.17	.21	.26	.32	.36	.39	.38	.33	.28	.23	.18	.16
50	.19	.23	.27	.31	.34	.36	.35	.32	.28	.24	.20	.18
45	.20	.23	.27	.30	.34	.35	.34	.32	.28	.24	.21	.20
40	.22	.24	.27	.30	.32	.34	.33	.31	.28	.25	.22	.21
35	.23	.25	.27	.29	.31	.32	.32	.30	.28	.25	.23	.22
30	.24	.25	.27	.29	.31	.32	.31	.30	.28	.26	.24	.23
25	.24	.26	.27	.29	.30	.31	.31	.29	.28	.26	.25	.24
20	.25	.26	.27	.28	.29	.30	.30	.29	.28	.26	.25	.25
15	.26	.26	.27	.28	.29	.29	.29	.28	.28	.27	.26	.25
10	.26	.27	.27	.28	.28	.29	.29	.28	.28	.27	.26	.26
5	.27	.27	.27	.28	.28	.28	.28	.28	.28	.27	.27	.27
0	.27	.27	.27	.27	.27	.27	.27	.27	.27	.27	.27	.27

Blaney-Criddle Model

- A spreadsheet-based tool to make Blaney-Criddle estimates is on the course server
- A google search will turn up similar cacluators
- Not too difficult to put into a program for long-term simulation use



Blaney-Criddle ET Estimator
North Latitude

Latitude 30 <=Degrees Latitude (0-60, increments of 5)

Month	T_mean	p-Value	ET_o		T-high	T-low
Jan	8	0.24	2.8032		15.3	0.7
Feb	10.15	0.25	3.16725		17.5	2.8
Mar	14.3	0.27	3.93606		21.7	6.9
Apr	18.9	0.29	4.84126		26.7	11.1
May	23.65	0.31	5.85249		30.7	16.6
Jun	26.9	0.32	6.51968		33.4	20.4
Jul	28.45	0.31	6.53697		35.1	21.8
Aug	28.15	0.3	6.2847		34.8	21.5
Sep	24.25	0.28	5.3634		31	17.5
Oct	19	0.26	4.3524		26	12
Nov	12.95	0.24	3.34968		20.2	5.7
Dec	8.2	0.23	2.70756		15.5	0.9

North-Latitude | South-Latitude | p-factor

Thornwaithe Model

- The Thornwaithe model is relatively simple like Blaney-Criddle, but has a few more terms:

1. Thornthwaite's Formula

The potential evapotranspiration (ET_p) per month or ten days is given by:

$$ET_p = 16(10\theta/I)^a \times F(\lambda)$$

Here, ET_p is given in millimeters per month.

θ	mean temperature of the period in question (°C) measured under shelter,
a	$6.75 \times 10^{-7} I^3 - 7.71 \times 10^{-5} I^2 + 1.79 \times 10^{-2} I + 0.49239$
I	annual thermal index, sum of twelve monthly thermal indexes i,
i	$(\theta/5)^{1.514}$
$F(\lambda)$	correction coefficient, function of the latitude and the month, given by Table A.1.1.

Correction Coefficient $F(\lambda)$ Depending on the Latitude and the Month[a]

Lat. N.	J	F	M	A	M	J	J	A	S	O	N	D
0	1.04	0.94	1.04	1.01	1.04	1.01	1.04	1.04	1.01	1.04	1.01	1.04
5	1.02	0.93	1.03	1.02	1.06	1.03	1.06	1.05	1.01	1.03	0.99	1.02
10	1.00	0.91	1.03	1.03	1.08	1.06	1.08	1.07	1.02	1.02	0.98	0.99
15	0.97	0.91	1.03	1.04	1.11	1.08	1.12	1.08	1.02	1.01	0.95	0.97
20	0.95	0.90	1.03	1.05	1.13	1.11	1.14	1.11	1.02	1.00	0.93	0.94
25	0.93	0.89	1.03	1.06	1.15	1.14	1.17	1.12	1.02	0.99	0.91	0.91
26	0.92	0.88	1.03	1.06	1.15	1.15	1.17	1.12	1.02	0.99	0.91	0.91
27	0.92	0.88	1.03	1.07	1.16	1.15	1.18	1.13	1.02	0.99	0.90	0.90
28	0.91	0.88	1.03	1.07	1.16	1.16	1.18	1.13	1.02	0.98	0.90	0.90
29	0.91	0.87	1.03	1.07	1.17	1.16	1.19	1.13	1.03	0.98	0.90	0.89
30	0.90	0.87	1.03	1.08	1.18	1.17	1.20	1.14	1.03	0.98	0.89	0.88
31	0.90	0.87	1.03	1.08	1.18	1.18	1.20	1.14	1.03	0.98	0.89	0.88
32	0.89	0.86	1.03	1.08	1.19	1.19	1.21	1.15	1.03	0.98	0.88	0.87
33	0.88	0.86	1.03	1.09	1.19	1.20	1.22	1.15	1.03	0.97	0.88	0.86
34	0.88	0.85	1.03	1.09	1.20	1.20	1.22	1.16	1.03	0.97	0.87	0.86
35	0.87	0.85	1.03	1.09	1.21	1.21	1.23	1.16	1.03	0.97	0.86	0.85
36	0.87	0.85	1.03	1.10	1.21	1.22	1.24	1.16	1.03	0.97	0.86	0.84
37	0.86	0.84	1.03	1.10	1.22	1.23	1.25	1.17	1.03	0.97	0.85	0.83
38	0.85	0.84	1.03	1.10	1.23	1.24	1.25	1.17	1.04	0.96	0.84	0.83
39	0.85	0.84	1.03	1.11	1.23	1.24	1.26	1.18	1.04	0.96	0.84	0.82
40	0.84	0.83	1.03	1.11	1.24	1.25	1.27	1.18	1.04	0.96	0.83	0.81
41	0.83	0.83	1.03	1.11	1.25	1.26	1.27	1.19	1.04	0.96	0.82	0.80
42	0.82	0.83	1.03	1.12	1.26	1.27	1.28	1.19	1.04	0.95	0.82	0.79
43	0.81	0.82	1.02	1.12	1.26	1.28	1.29	1.20	1.04	0.95	0.81	0.77
44	0.81	0.82	1.02	1.13	1.27	1.29	1.30	1.20	1.04	0.95	0.80	0.76
45	0.80	0.81	1.02	1.13	1.28	1.29	1.31	1.21	1.04	0.94	0.79	0.75
46	0.79	0.81	1.02	1.13	1.29	1.31	1.32	1.22	1.04	0.94	0.79	0.74
47	0.77	0.80	1.02	1.14	1.30	1.32	1.33	1.22	1.04	0.93	0.78	0.73
48	0.76	0.80	1.02	1.14	1.31	1.33	1.34	1.23	1.05	0.93	0.77	0.72
49	0.75	0.79	1.02	1.14	1.32	1.34	1.35	1.24	1.05	0.93	0.76	0.71
50	0.74	0.78	1.02	1.15	1.33	1.36	1.37	1.25	1.06	0.92	0.76	0.70
Lat. S.												
5	1.06	0.95	1.04	1.00	1.02	0.99	1.02	1.03	1.00	1.05	1.03	1.06
10	1.08	0.97	1.05	0.99	1.01	0.96	1.00	1.01	1.00	1.06	1.05	1.10
15	1.12	0.98	1.05	0.98	0.98	0.94	0.97	1.00	1.00	1.07	1.07	1.12
20	1.14	1.00	1.05	0.97	0.96	0.91	0.95	0.99	1.00	1.08	1.09	1.15
25	1.17	1.01	1.05	0.96	0.94	0.88	0.93	0.98	1.00	1.10	1.11	1.18
30	1.20	1.03	1.06	0.95	0.92	0.85	0.90	0.96	1.00	1.12	1.14	1.21
35	1.23	1.04	1.06	0.94	0.89	0.82	0.87	0.94	1.00	1.13	1.17	1.25
40	1.27	1.06	1.07	0.93	0.86	0.78	0.84	0.92	1.00	1.15	1.20	1.29
42	1.28	1.07	1.07	0.92	0.85	0.76	0.82	0.92	1.00	1.16	1.22	1.31
44	1.30	1.08	1.07	0.92	0.83	0.74	0.81	0.91	0.99	1.17	1.23	1.33
46	1.32	1.10	1.07	0.91	0.82	0.72	0.79	0.90	0.99	1.17	1.25	1.35
48	1.34	1.11	1.08	0.90	0.80	0.70	0.76	0.89	0.99	1.18	1.27	1.37
50	1.37	1.12	1.08	0.89	0.77	0.67	0.74	0.88	0.99	1.19	1.29	1.41

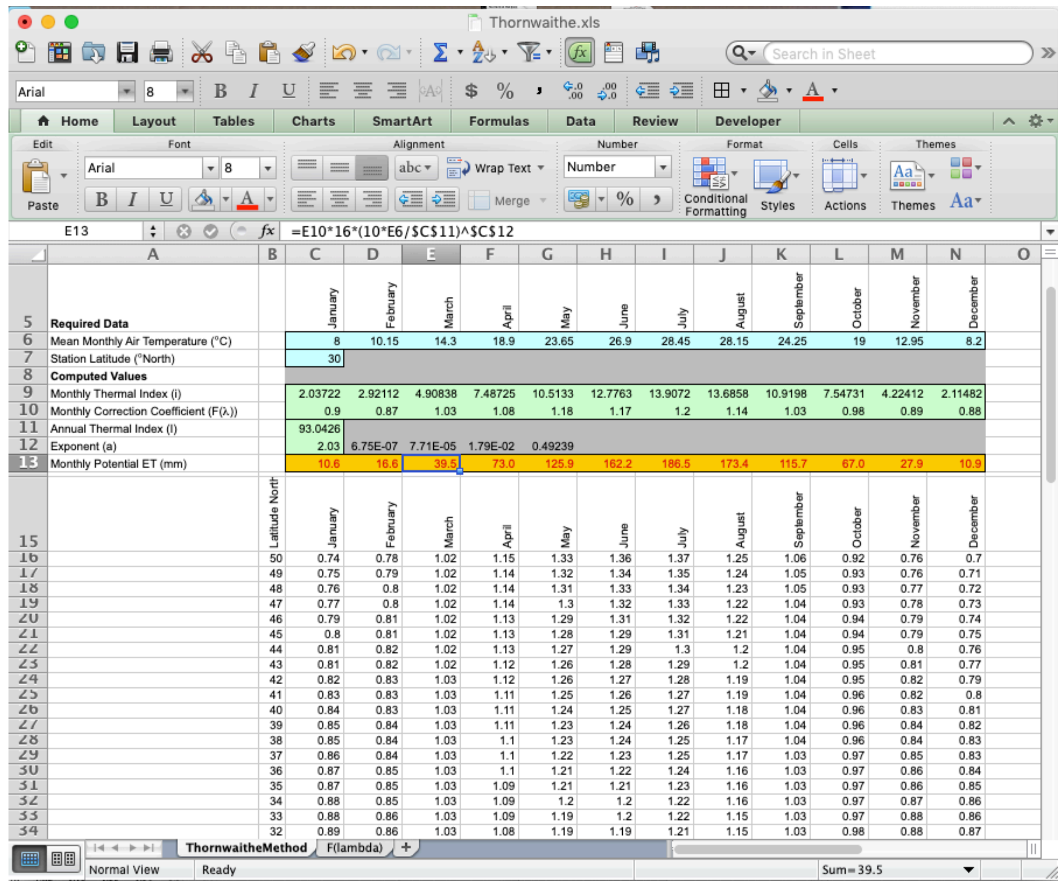
[a] Thornthwaite's formula, from Brochet and Gerbier (1974).

404

Table A.1.1

Thornwaithe Model

- A spreadsheet-based tool to make Thornwaithe estimates is on the course server
- A google search will turn up similar cacluators
- Not too difficult to put into a program for long-term simulation use



	B	January	February	March	April	May	June	July	August	September	October	November	December
Required Data													
Mean Monthly Air Temperature (°C)		8	10.15	14.3	18.9	23.65	26.9	28.45	28.15	24.25	19	12.95	8.2
Station Latitude (°North)		30											
Computed Values													
Monthly Thermal Index (i)		2.03722	2.92112	4.90838	7.48725	10.5133	12.7763	13.9072	13.6858	10.9198	7.54731	4.22412	2.11482
Monthly Correction Coefficient (F(λ))		0.9	0.87	1.03	1.08	1.18	1.17	1.2	1.14	1.03	0.98	0.89	0.88
Annual Thermal Index (I)		93.0426											
Exponent (a)		2.03	6.75E-07	7.71E-05	1.79E-02	0.49239							
Monthly Potential ET (mm)		10.6	16.6	39.5	73.0	125.9	162.2	186.5	173.4	115.7	67.0	27.9	10.9

Latitude North	January	February	March	April	May	June	July	August	September	October	November	December
50	0.74	0.78	1.02	1.15	1.33	1.36	1.37	1.25	1.06	0.92	0.76	0.7
49	0.75	0.79	1.02	1.14	1.32	1.34	1.35	1.24	1.05	0.93	0.76	0.71
48	0.76	0.8	1.02	1.14	1.31	1.33	1.34	1.23	1.05	0.93	0.77	0.72
47	0.77	0.8	1.02	1.14	1.3	1.32	1.33	1.22	1.04	0.93	0.78	0.73
46	0.79	0.81	1.02	1.13	1.29	1.31	1.32	1.22	1.04	0.94	0.79	0.74
45	0.8	0.81	1.02	1.13	1.28	1.29	1.31	1.21	1.04	0.94	0.79	0.75
44	0.81	0.82	1.02	1.13	1.27	1.29	1.3	1.2	1.04	0.95	0.8	0.76
43	0.81	0.82	1.02	1.12	1.26	1.28	1.29	1.2	1.04	0.95	0.81	0.77
42	0.82	0.83	1.03	1.12	1.26	1.27	1.28	1.19	1.04	0.95	0.82	0.79
41	0.83	0.83	1.03	1.11	1.25	1.26	1.27	1.19	1.04	0.96	0.82	0.8
40	0.84	0.83	1.03	1.11	1.24	1.25	1.27	1.18	1.04	0.96	0.83	0.81
39	0.85	0.84	1.03	1.11	1.23	1.24	1.26	1.18	1.04	0.96	0.84	0.82
38	0.85	0.84	1.03	1.1	1.23	1.24	1.25	1.17	1.04	0.96	0.84	0.83
37	0.86	0.84	1.03	1.1	1.22	1.23	1.25	1.17	1.03	0.97	0.85	0.83
36	0.87	0.85	1.03	1.1	1.21	1.22	1.24	1.16	1.03	0.97	0.86	0.84
35	0.87	0.85	1.03	1.09	1.21	1.21	1.23	1.16	1.03	0.97	0.86	0.85
34	0.88	0.85	1.03	1.09	1.2	1.2	1.22	1.16	1.03	0.97	0.87	0.86
33	0.88	0.86	1.03	1.09	1.19	1.2	1.22	1.15	1.03	0.97	0.88	0.86
32	0.89	0.86	1.03	1.08	1.19	1.19	1.21	1.15	1.03	0.98	0.88	0.87

Turc Model

- Turc's model is more elaborate, here U_m is mean relative humidity

2. Turc's Formula

Turc prefers different formulas according to whether the mean relative humidity is above or below 50%. If $U_m > 50\%$ (usual in temperate zones)

$$\mathrm{ET_p} \quad (\mathrm{mm}/10\,\mathrm{d}) = 0.13 \frac{\theta}{\theta + 15}(R_g + 50)$$

If $U_m < 50\%$

$$\mathrm{ET_p} \quad (\mathrm{mm}/10\,\mathrm{d}) = 0.13 \frac{\theta}{\theta + 15}(R_g + 50)\left[1 + \frac{50 - U_m}{70}\right]$$

θ mean temperature of the period in question (°C) measured under shelter,

R_g overall solar radiation $\simeq I_{g_0}(0.18 + 0.62\,h/H)$

h actual amount of sunshine in hours per day,

H maximum possible amount of sunshine (astronomical length of the day),

I_{g_0} direct solar radiation at the top of the atmosphere,

I_{g_0} and H are tabulated according to the latitude and the date on Tables A.1.2 and A.1.3.

Turc Model

- Turc's model tables:

Table A. 1.2.

Monthly I_{g_a} Values in Small Calories per cm² of Horizontal Surface Area and per Day[a]

Latitude North	30°	40°	50°	60°
January	508	364	222	87.5
February	624	495	360	215
March	764	673	562	432
April	880	833	764	676
May	950	944	920	880
June	972	985	983	970
July	955	958	938	908
August	891	858	800	728
September	788	710	607	487
October	658	536	404	262
November	528	390	246	111
December	469	323	180	55.5

[a] From Brochet and Gerbier (1974)

Table A.1.3.

Length of the Astronomical Day *H* (mean monthly values in hours per day)[a]

Latitude North	30°	40°	50°	60°
January	10.45	9.71	8.58	6.78
February	11.09	10.64	10.07	9.11
March	12.00	11.96	11.90	11.81
April	12.90	13.26	13.77	14.61
May	13.71	14.39	15.46	17.18
June	14.07	14.96	16.33	18.73
July	13.85	14.68	15.86	17.97
August	13.21	13.72	14.49	15.58
September	12.36	12.46	12.63	12.89
October	11.45	11.15	10.77	10.14
November	10.67	10.00	9.08	7.58
December	10.23	9.39	8.15	6.30

[a] From Brochet and Gerbier (1974)

Pennman-Monteith Model

- The original Penman model is a combination method in which the total evaporation rate is calculated by weighing the evaporation rate due to net radiation and the evaporation rate due to mass transfer, as follows (Ponce, 1989):

$$E = \frac{\Delta E_n + \gamma E_a}{\Delta + \gamma} \qquad (1)$$

- in which E = total evaporation rate; En = evaporation rate due to net radiation; Ea = evaporation rate due to mass transfer; Δ = saturation vapor pressure gradient, varying with air temperature; and γ = psychrometric constant, varying slightly with temperature. In Eq. 1, the mass-transfer evaporation rate is calculated with an empirical mass-transfer formula.

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  - Δ = saturation vapor pressure gradient, varying with air temperature;
  - γ = psychrometric constant, varying slightly with temperature.
  - The mass-transfer evaporation rate is calculated with an empirical mass-transfer formula.

Pennman-Monteith Model

- In the Penman-Monteith model the mass-transfer evaporation rate E_a is calculated based on physical principles. The original form of the Penman-Monteith equation, in dimensionally consistent units, is:

$$\rho\lambda E = \frac{\Delta \cdot H_n + \rho_a c_p (e_s - e_a) r_a^{-1}}{\Delta + \gamma^*}$$

in which

- $\rho\lambda E$ = total evaporative energy flux, in cal cm^{-2} s^{-1};
- Δ = saturation vapor pressure gradient, in mb °C^{-1};
- H = energy flux supplied externally, by net radiation, in cal cm^{-2} s^{-1};
- ρ_a = density of moist air, in gr cm^{-3};
- c_p = specific heat of moist air, in cal gr^{-1} °C^{-1};
- $(e_s - e_a)$ = vapor pressure deficit, in mb;
- r_a = external (aerodynamic) resistance, in s cm^{-1}; and
- γ^* = modified psychrometric constant, in mb °C^{-1}, equal to:

$$\gamma^* = \gamma \left(1 + \frac{r_s}{r_a}\right)$$

in which

- γ = psychrometric constant, in mb °C^{-1}, varying slightly with temperature, and
- r_s = internal (stomatal or surface) resistance, in s cm^{-1}.

Pennman-Monteith Model

- The reduced form of the Penman-Monteith equation, is:

$$E = \frac{\Delta E_n + \rho_a c_p (e_s - e_a) r_a^{-1} \rho^{-1} \lambda^{-1}}{\Delta + \gamma^*}$$

in which

- E = total evaporation rate, in cm s^{-1};
- E_n = evaporation rate due to net radiation, in cm s^{-1};
- ρ = density of water, in gr cm^{-3};
- λ = heat of vaporization of water, in cal gr^{-1};

  and

- Δ, γ^*, ρ_a, c_p, $(e_s - e_a)$, and r_a are in the same units as in Eq. 2.

Pennman-Monteith Model

- Physical constants:

The density of dry air at sea level is: ρ_{ad} = 1.2929 kg/m^3. The density of moist air can be approximated as follows:

$$\rho_a = \rho_{ad} \left(\frac{273}{273 + T} \right) \qquad (5)$$

in which T = air temperature, in °C.

For instance, at T = 20°C and sea level (standard atmospheric pressure):

$\rho_a = 0.0012046 \text{ gr cm}^{-3}$.

The specific heat of moist air, in the range 0°C ≤ T ≤ 40°C, is:

$c_p = 1.005 \text{ J gr}^{-1} \text{ °C}^{-1}$

Converting to calories:

$c_p = (1.005 \text{ J gr}^{-1} \text{ °C}^{-1})\ (0.239 \text{ cal/J}) = 0.2402 \text{ cal gr}^{-1} \text{ °C}^{-1}$.

Pennman-Monteith Model

- Evaporation Rates:

In evaporation units of cm d^{-1}, Eq. 4 is expressed as follows:

$$E = \frac{\Delta E_n + 86400\, \rho_a c_p (e_s - e_a) r_a^{-1} \rho^{-1} \lambda^{-1}}{\Delta + \gamma^*} \quad (6)$$

in which

- E = total evaporation rate (cm d^{-1});
- E_n = evaporation rate due to net radiation (cm d^{-1}); and
- Δ, γ^*, ρ_a, c_p, $(e_s - e_a)$, r_a, ρ, and λ are in the same units as Eqs. 2 and 4.

Equation 6 can be conveniently expressed in Penman form (Eq. 1) as follows:

$$E = \frac{\Delta E_n + \gamma^* E_a}{\Delta + \gamma^*} \quad (7)$$

in which E_a = evaporation rate due to mass transfer, in cm d^{-1}:

Pennman-Monteith Model

- Evaporation Rates:

Comparing Eqs. 6 and 7, the evaporation rate due to mass transfer is obtained:

$$E_a = \frac{86400\ \rho_a\ c_p\ (e_s - e_a)}{\rho\ \lambda\ \gamma\ (r_a + r_s)} \qquad (8)$$

Simplifying Eq. 8:

$$E_a = \frac{K\ (e_s - e_a)}{r_a + r_s} \qquad (9)$$

in which K = a constant varying with air temperature and atmospheric pressure, in units of s d^{-1} mb^{-1}, expressed as follows:

$$K = \frac{86400\ \rho_a\ c_p}{\rho\ \lambda\ \gamma} \qquad (10)$$

In Eq. 10, the units of ρ_a, c_p, ρ, λ, and γ are the same as in Eqs. 2 and 4.

Pennman-Monteith Model

- Evaporation Rates:

The psychrometric constant γ, in mb °C^{-1}, is:

$$\gamma = \frac{c_p\, p}{\lambda\, r_{MW}} \tag{11}$$

in which c_p = specific heat of moist air, in cal gr^{-1} °C^{-1}; p = atmospheric pressure, in mb; λ = heat of vaporization of water, in cal gr^{-1}; and r_{MW} = ratio of the molecular weight of water vapor to dry air: $r_{MW} = 0.622$.

Substituting Eq. 11 in Eq. 10:

$$K = \frac{86400\, \rho_a\, r_{MW}}{\rho\, p} \tag{12}$$

in which the constant K remains in units of s d^{-1} mb^{-1}.

Pennman-Monteith Model

- Evaporation Rates:

Replacing $r_{MW} = 0.622$ into Eq. 12:

$$K = \frac{53740.8\ \rho_a}{\rho\ p} \qquad (13)$$

in which the constant K remains in units of s d^{-1} mb^{-1}.

At $T = 20°C$ and standard atmospheric pressure (sea level): $\rho_a = 0.0012046$ gr cm^{-3}, $\rho = 0.99821$ gr cm^{-3}, and $p = 1013.25$ mb. Thus, the constant K in Eq. 13 reduces to: $K = 0.064$ s d^{-1} mb^{-1}, and Eq. 9 reduces to:

$$E_a = \frac{0.064\ (e_s - e_a)}{r_a + r_s} \qquad (14)$$

in which

- E_a = evaporation rate due to mass transfer, in cm d^{-1};
- $(e_s - e_a)$ = vapor pressure deficit, in mb;
- r_a = external (aerodynamic) resistance, in s cm^{-1}; and
- r_s = internal (stomatal) resistance, in s cm^{-1}.

Ev

del

The external (or aerodynamic) resistance r_a varies with the surface roughness (water, soil, or vegetation), being inversely proportional to wind speed. In other words, the external conductance (and thus, the evaporation rate) increases with wind speed, as postulated by Dalton (Ponce, 1989).

The external resistance for evaporation from open water can be estimated as follows:

$$r_a = \frac{4.72\,[\,\ln\,(z_m / z_o)\,]^2}{1 + 0.536\,v_2} \qquad (15)$$

in which

- r_a = external resistance, in s m^{-1};
- z_m = height at which meteorological variables are measured, in m;
- z_0 = aerodynamic roughness of the surface, in m; and
- v_2 = wind speed, in m s^{-1}, measured at 2-m height.

The external resistance r_a (s m^{-1}) for the reference crop (clipped grass 0.12-m high), for measurements of wind speed (m s^{-1}), temperature and humidity at a standardized height of 2 m is:

$$r_a^{rc} = \frac{208}{v_2} \qquad (16)$$

For instance, for v_2 = 200 km d^{-1} = 200000 m / 86400 s = 2.3148 m s^{-1}, the external or aerodynamic resistance of the reference crop is:

$$r_a^{rc} = \frac{208}{2.3148} = 89.85 \text{ s m}^{-1} = 0.8985 \text{ s cm}^{-1}. \qquad (17)$$

The internal (stomatal or surface) resistance r_s is inversely proportional to the leaf-area index L, i.e., the projected area of vegetation per unit ground area. An empirical relation is:

$$r_s = \frac{200}{L} \qquad (18)$$

in which r_s is in s m^{-1} and L is in m s^{-1}.

The leaf-area index L is empirically related to crop height h_c. Two examples are given here.

Leaf-area index for clipped grass

$L = 24\, h_c$, in which crop height h_c is in m, varying in the range $0.05 \le h_c \le 0.15$.

From Eq. 18, the stomatal resistance of the reference crop (clipped grass 0.12-m high) is:

$r_s^{rc} = 69.4$ s m^{-1} = 0.694 s cm^{-1}

Leaf-area index for alfalfa

$L = 5.5 + 1.5 \ln (h_c)$, in which crop height h_c is in m, varying in the range $0.1 \le h_c \le 0.5$.

From Eq. 18, the stomatal resistance of an alfalfa crop, with $h_c = 0.3$ m:

$r_s = 54.1$ s m^{-1} = 0.541 s cm^{-1}

Shuttleworth-Wallace Model

- The formula is based on an energy combination theory in which evaporation is calculated based on the resistances associated with the plants and with the soil or water in which they are growing.
- The equation is based on a one-dimensional model in which the transition between the asymptotic limits of bare substrate and closed canopy is evaluated.
- The equation is an improved version of the Penman-Monteith equation for evaporation and evapotranspiration.

Shuttleworth-Wallace Model

- The formula is:

$$\lambda E = C_c\, PM_c + C_s\, PM_s$$

- λE = latent heat flux from the complete crop (W m^{-2});
- C_c and C_s are coefficients,
- PM_c and PM_s are evaporation terms similar to the Penman-Monteith combination equation

Shuttleworth-Wallace Model

- The Shuttleworth-Wallace model has several intermediate calculations to paramaterise the

$$\lambda E = C_c \, PM_c + C_s \, PM_s$$

- λE = latent heat flux from the complete crop (W m^{-2});
- C_c and C_s are coefficients,
- PM_c and PM_s are evaporation terms similar to the Penman-Monteith combination equation

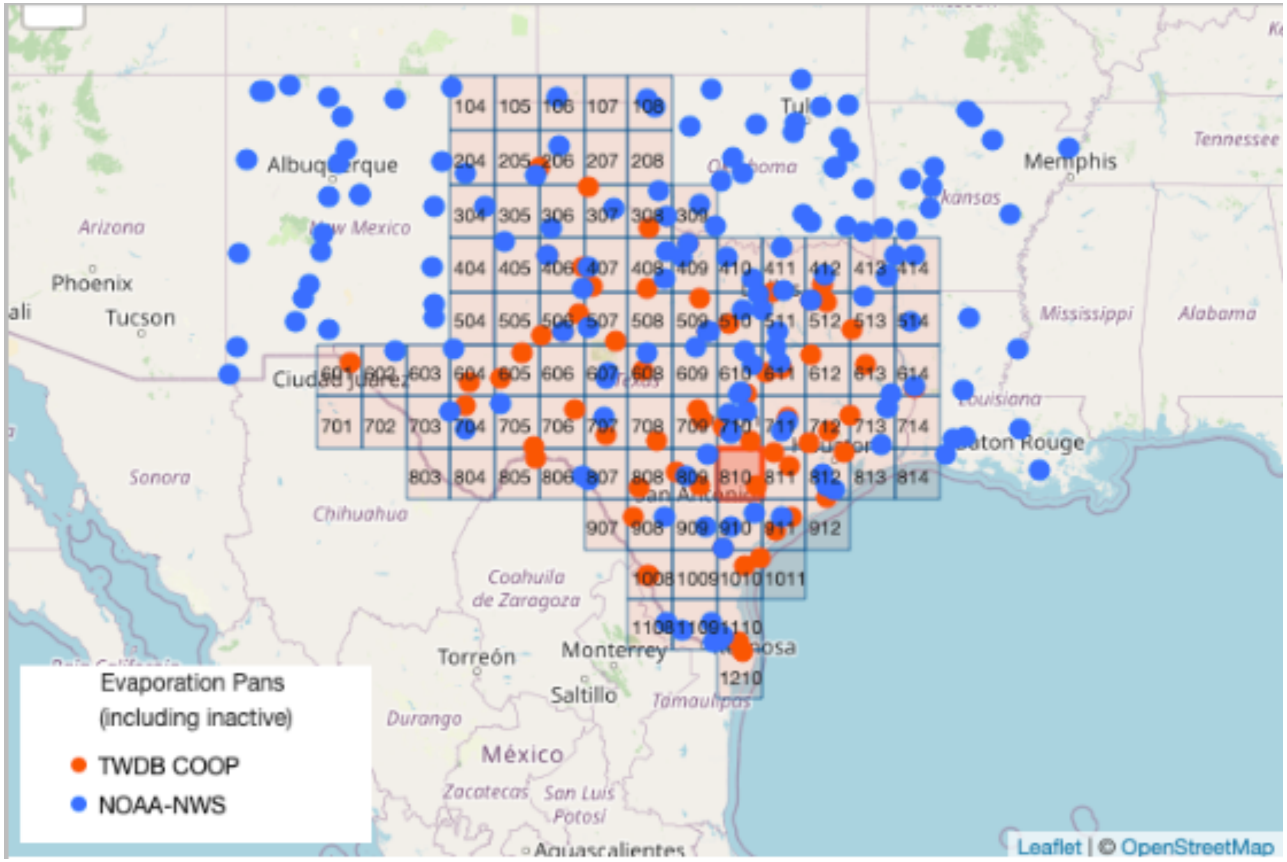
Data Science Approach

- In locations where data are available one can use a data science approach to estimate evaporation (gross or net) based on temperature and rainfall.

- Lets look at an example for Texas. We can obtain data from
  - https://waterdatafortexas.org/lake-evaporation-rainfall

Data Science Approach

- Coverage good – these are pan data



Data Science Approach

- Data available are:
  - Precipitation, Evaporation, Net Evaporation for all cells.
  - We would have to find temperature and solar radiation elsewhere if we intend to build a data model, perhaps a serial-correlation model, something like:

$$EVAP_{cell} = \phi\ (T_{3p}, P_{3p}, Month, R_{3p})$$