

CE 3354 Engineering Hydrology
Exam 1

1. For a watershed with a size of 120 km^2 , the following data on precipitation P , evaporation E and runoff Q are recorded in watershed mm.

Table 1: Monthly Precipitation (P), Evapotranspiration (E), and Runoff (Q)

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
P (mm)	250	205	165	50	5	0	0	5	10	55	65	190
E (mm)	5	25	30	50	80	100	150	70	60	20	10	5
Q (mm)	150	110	80	5	0	0	0	0	0	10	15	120

Determine:

- a) The month (end) when the amount of water stored in the basin is the largest.
- b) The month (end) when the amount of water stored in the basin is the smallest.
- c) The difference (in m^3) in the amount of water stored in the basin between these two extremes.
- d) The likely climate type (arid, humid temperate or humid tropical) one would expect to find this catchment.

Solution(s): Items a,b, and c are determined using a water budget approach. An excel spreadsheet is shown in Figure(s) 1, and 2

	A	B	C	D	E	F	G	H	I	J
1	ce3354-ex1-2025-2-problem1					$S_0(\text{mm})$	225	<= Changed until non-negative computed storage, one can answer questions on the change in storage terms as well		
2	DATA SUPPLIED									
3	Area (km ²)	120				COMPUTED VALUES				
4	MONTH	P(mm)	E(mm)	Q(mm)		$\Delta S(\text{mm})$	S(mm)		S(m ³)	
5	Jan	250	5	150		95	320		38400000	
6	Feb	205	25	110		70	390		46800000	
7	Mar	165	30	80		55	445	MAXIMUM VALUE	53400000	
8	Apr	50	50	5		-5	440		52800000	
9	May	5	80	0		-75	365		43800000	
10	Jun	0	100	0		-100	265		31800000	
11	Jul	0	150	0		-150	115		13800000	
12	Aug	5	70	0		-65	50		6000000	
13	Sep	10	60	0		-50	0	MINIMUM VALUE	0	
14	Oct	55	20	10		25	25		3000000	
15	Nov	65	10	15		40	65		7800000	
16	Dec	190	5	120		65	130		15600000	
17										
18								$\Delta S(\text{m}^3)$	53400000	

Figure 1: PR1 Water Budget

	A	B	C	D	E	F	G	H	I
1	ce3354-ex				$S_0(\text{mm})$			<= Changed until non-negative computed storage, one can answer questions on the change in storage terms as well	
2	DATA SUPPLY								
3	Area (km ²): 120				COMPUTED VALUES				
4	MONTH	P(mm)	E(mm)	Q(mm)	$\Delta S(\text{mm})$	S(mm)			S(m ³)
5	Jan	250	5	150	=B5-C5-D5	=G1+F5			=0.001*G5*\$B\$3*1000*1000
6	Feb	205	25	110	=B6-C6-D6	=F6+G5			=0.001*G6*\$B\$3*1000*1000
7	Mar	165	30	80	=B7-C7-D7	=F7+G6	MAXIMUM VALUE		=0.001*G7*\$B\$3*1000*1000
8	Apr	50	50	5	=B8-C8-D8	=F8+G7			=0.001*G8*\$B\$3*1000*1000
9	May	5	80	0	=B9-C9-D9	=F9+G8			=0.001*G9*\$B\$3*1000*1000
10	Jun	0	100	0	=B10-C10-D10	=F10+G9			=0.001*G10*\$B\$3*1000*1000
11	Jul	0	150	0	=B11-C11-D11	=F11+G10			=0.001*G11*\$B\$3*1000*1000
12	Aug	5	70	0	=B12-C12-D12	=F12+G11			=0.001*G12*\$B\$3*1000*1000
13	Sep	10	60	0	=B13-C13-D13	=F13+G12	MINIMUM VALUE		=0.001*G13*\$B\$3*1000*1000
14	Oct	55	20	10	=B14-C14-D14	=F14+G13			=0.001*G14*\$B\$3*1000*1000
15	Nov	65	10	15	=B15-C15-D15	=F15+G14			=0.001*G15*\$B\$3*1000*1000
16	Dec	190	5	120	=B16-C16-D16	=F16+G15			=0.001*G16*\$B\$3*1000*1000
17									
18								$\Delta S(\text{m}^3)$	=MAX(I5:I16)-MIN(I5:I16)

Figure 2: PR1 Water Budget Formulas

Item d requires internet research to learn about climate classification. The model I used is *Köppen climate classification*, Gupta's classification(s), and Mays' classifications follow closely. A good starting place is Wikipedia https://en.wikipedia.org/wiki/Kppen_climate_classification.

For the supplied monthly values we can make the following assertions:

- Rainfall (P) is highly concentrated in Jan–Mar and Dec.
- Dry season from May–Sep (very low P, high E).
- E is high in May–Sep, reaching a peak in July (150 mm) when P is 0.

These data indicate a distinct wet/dry seasonality.

The Köppen classification scheme:

- Distinct wet and dry seasons
- Wet summer is not present — rainfall is in winter.
- Dry summer with high E
- Total annual precipitation is moderate (1000 mm)

This pattern is typical of a Mediterranean Climate (Köppen: Csa or Csb) Csa: Hot, dry summer; mild, wet winter (likely match)

Characteristics:

- Summer drought
- 3+ months with $P < 30$ mm and $E > 60$ mm (May–Sep fits)
- Wet winters ($P > E$ in Jan–Mar, Dec)

The catchment is most likely in a Mediterranean climate (Köppen Csa), common in:

- Coastal California
- Southern Europe (e.g., Spain, Italy)
- Western Australia
- Cape region of South Africa
- Parts of central Chile

2. A watershed with a catchment area of $1mi^2$ converts about 60-percent of precipitation into streamflow, the remainder is lost. The watershed response equation is

$$k \frac{dQ}{dt} + Q(t) = P(t) \cdot A \cdot C \quad (1)$$

where $Q(t)$ is the streamflow leaving the catchment, $P(t)$ is the precipitation entering the catchment, A is the catchment area, C is the precipitation to streamflow conversion fraction, and k is the basin characteristic time constant.

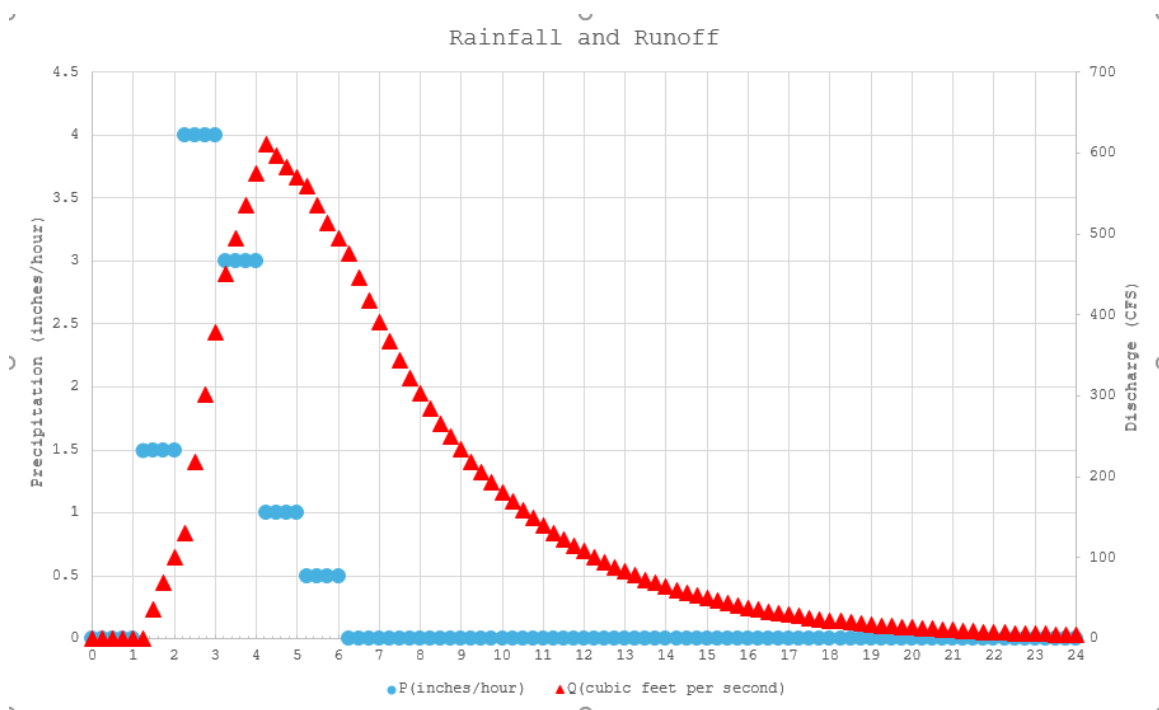


Figure 3: Rainfall-runoff plot for the catchment

Using the information in Figure 3 determine:

- The maximum discharge rate in cubic feet per second.
- The time when the maximum discharge occurs.
- The value in hours of the of the basin time constant k .
- The total volume in acre feet of precipitation entering the catchment (before any losses)
- The total volume in acre feet of discharge leaving the catchment

Solution(s): Items a, and b can be read directly from the supplied plot. They are about 600 cfs and 4.25 hours. We can get more accurate estimates when we simulate the watershed using the supplied equation, and fit the simulated chart to the provided chart.

To fit first build a finite-difference approximation as

$$k \frac{dQ}{dt} + Q(t) = P(t) \cdot A \cdot C$$

A should be supplied in acres assuming $P \cdot C$ is in inches per hour to produce output in CFS.

Move $Q(t)$ to RHS and divide by k

$$\frac{dQ}{dt} = \frac{P(t) \cdot A \cdot C - Q(t)}{k}$$

Then express the LHS as a difference quotient

$$\frac{Q(t+\Delta t) - Q(t)}{\Delta t} = \frac{P(t) \cdot A \cdot C - Q(t)}{k}$$

Isolate everything at the old time step to the RHS

$$Q(t + \Delta t) = Q(t) + \frac{\Delta t}{k} (P(t) \cdot A \cdot C - Q(t))$$

Code up into a spreadsheet, choose a small enough value of Δt and proceed.

Figure 4 is a screen capture of a portion of the spreadsheet.

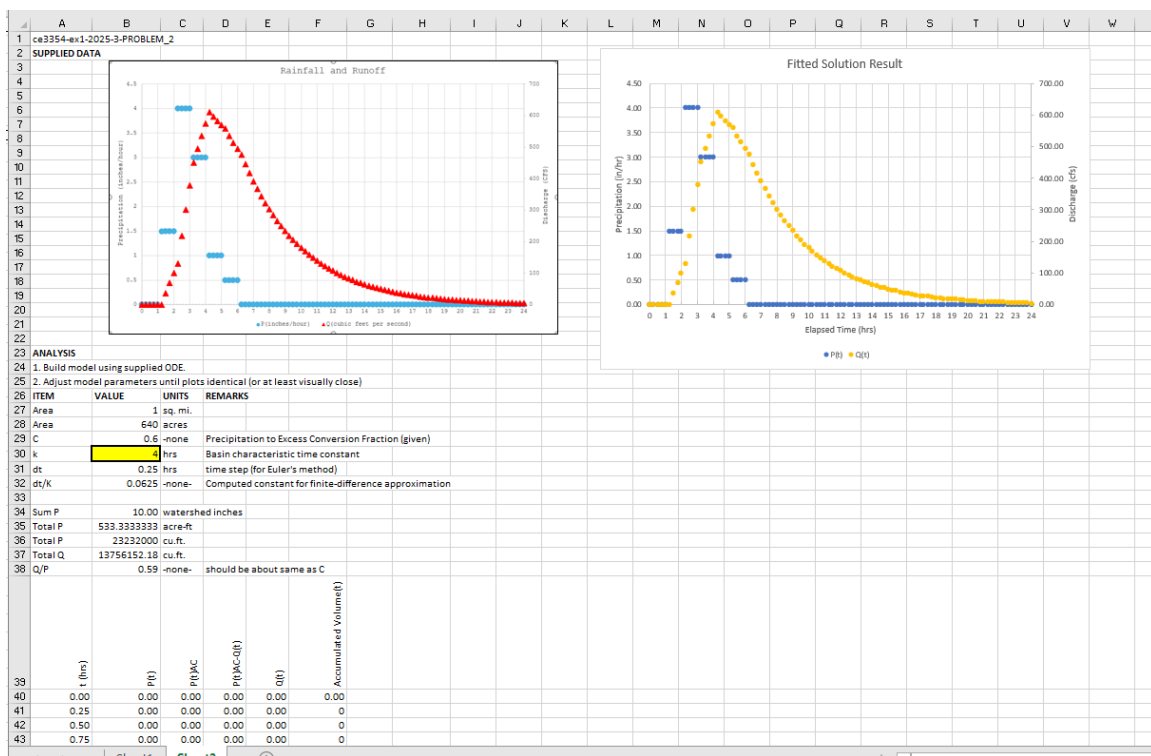


Figure 4: Rainfall-runoff plot for the catchment; supplied and computed charts side-by-side

Figure 5 is a screen capture of the computation portion of the spreadsheet.

	A	B	C	D	E	F	G	H	I	J	K	L	M
22													
23	ANALYSIS												
24	1. Build model using supplied ODE.												
25	2. Adjust model parameters until plots identical (or at least visually close)												
26	ITEM	VALUE	UNITS	REMARKS									
27	Area		1 sq. mi.										
28	Area	640	acres										
29	C	0.6	-none	Precipitation to Excess Conversion Fraction (given)									
30	k	4	hrs	Basin characteristic time constant									
31	dt	0.25	hrs	time step (for Euler's method)									
32	dt/K	0.0625	-none-	Computed constant for finite-difference approximation									
33													
34	Sum P	10.00	watershed inches										
35	Total P	533.3333333	acre-ft										
36	Total P	23232000	cu.ft.										
37	Total Q	13756152.18	cu.ft.										
38	Q/P	0.59	-none-	should be about same as C									
39	t (hrs)	P(t)	P(t)/AC	P(t)/AC-Q(t)	Q(t)	Accumulated Volume(t)							
40	0.00	0.00	0.00	0.00	0.00	0.00							
41	0.25	0.00	0.00	0.00	0.00	0							
42	0.50	0.00	0.00	0.00	0.00	0							
43	0.75	0.00	0.00	0.00	0.00	0							
44	1.00	0.00	0.00	0.00	0.00	0							
45	1.25	1.50	576.00	576.00	0.00	0							
46	1.50	1.50	576.00	540.00	36.00	16200							
47	1.75	1.50	576.00	506.25	69.75	63787.5							
48	2.00	1.50	576.00	474.61	101.39	140800.781							
49	2.25	4.00	1536.00	1404.95	131.05	245400.732							
50	2.50	4.00	1536.00	1317.14	218.86	402863.187							
51	2.75	4.00	1536.00	1234.82	301.18	636884.237							

Figure 5: Rainfall-runoff plot for the catchment; calculation portion. k is fit by trial-and-error updating

To demonstrate quality of the simulation, we change the computed chart background to transparent and overlay it onto the supplied graphic element. Figure 6 is the result. It is evident the fit is exact.

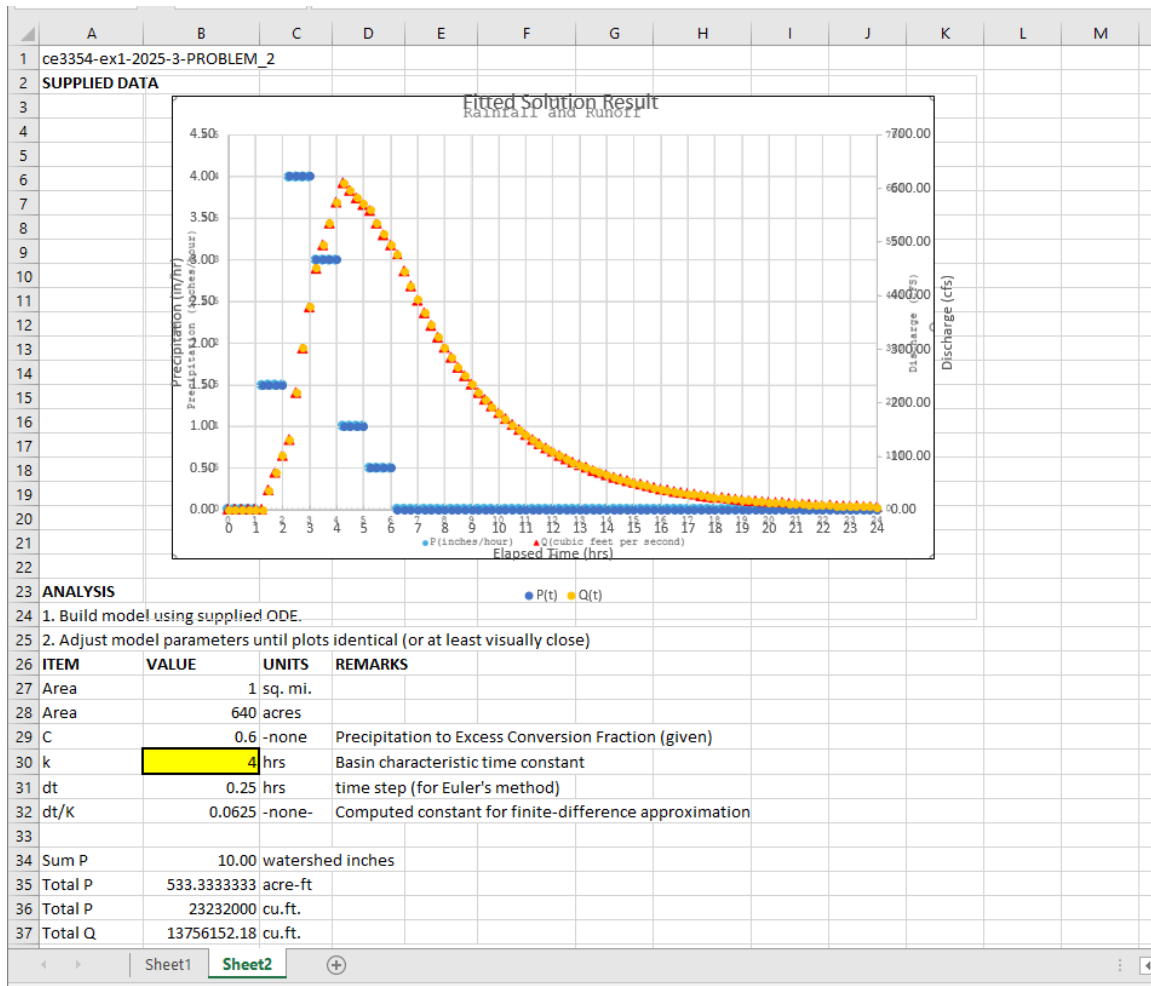


Figure 6: Rainfall-runoff plot for the catchment; Computed plot overlay and scaled so axes are same range.

Items c,d, and e appear directly or are computed from summary values in Figure 5.

c) value of k is 4 hours (to obtain the fitted chart)

d) Total volume in acre-feet of precipitation is **533.33** acre-feet

e) Total volume in acre feet of runoff is 13,756,152 cu.ft. divided by 43560 sq. ft./acre-ft to obtain **315.80 acre-feet**.

Lastly Figure 7 is a portion of the spreadsheet showing the formulas employed.

	A	B	C	D	E	F	
22							
23	ANALYSIS						
24	1. Build model using supplied data						
25	2. Adjust model parameters						
26	ITEM	VALUE	UNITS	REMARKS			
27	Area	1	sq. mi.				
28	Area	=640*B27	acres				
29	C	0.6	-none	Precipitation to Export			
30	k	4	hrs	Basin characteristic			
31	dt	0.25	hrs	time step (for Euler)			
32	dt/K	=B31/B30	-none-	Computed constant			
33							
34	Sum P	=SUM(B41:B137)*B31	watershed inches				
35	Total P	=(B34/12)*B28	acre-ft				
36	Total P	=B35*43560	cu.ft.				
37	Total Q	=F137	cu.ft.				
38	Total Q	=B37/43560	acre-feet				
39	Q/P	=B37/B36	-none-	should be about same			
40							
41	0	0	=B41*\$B\$28*\$B\$29	=C41-E41	0	=E41	
42	=A41+\$B\$31	0	=B42*\$B\$28*\$B\$29	=C42-E42	=E41+\$B\$32*D41	=0.5*(E42+E41)*(A42-A41)*3600)+F41	
43	=A42+\$B\$31	0	=B43*\$B\$28*\$B\$29	=C43-E43	=E42+\$B\$32*D42	=0.5*(E43+E42)*(A43-A42)*3600)+F42	
44	=A43+\$B\$31	0	=B44*\$B\$28*\$B\$29	=C44-E44	=E43+\$B\$32*D43	=0.5*(E44+E43)*(A44-A43)*3600)+F43	
45	=A44+\$B\$31	0	=B45*\$B\$28*\$B\$29	=C45-E45	=E44+\$B\$32*D44	=0.5*(E45+E44)*(A45-A44)*3600)+F44	
46	=A45+\$B\$31	1.5	=B46*\$B\$28*\$B\$29	=C46-E46	=E45+\$B\$32*D45	=0.5*(E46+E45)*(A46-A45)*3600)+F45	
47	=A46+\$B\$31	1.5	=B47*\$B\$28*\$B\$29	=C47-E47	=E46+\$B\$32*D46	=0.5*(E47+E46)*(A47-A46)*3600)+F46	
48	=A47+\$B\$31	1.5	=B48*\$B\$28*\$B\$29	=C48-E48	=E47+\$B\$32*D47	=0.5*(E48+E47)*(A48-A47)*3600)+F47	
49	=A48+\$B\$31	1.5	=B49*\$B\$28*\$B\$29	=C49-E49	=E48+\$B\$32*D48	=0.5*(E49+E48)*(A49-A48)*3600)+F48	
50	=A49+\$B\$31	4	=B50*\$B\$28*\$B\$29	=C50-E50	=E49+\$B\$32*D49	=0.5*(E50+E49)*(A50-A49)*3600)+F49	
51	=A50+\$B\$31	4	=B51*\$B\$28*\$B\$29	=C51-E51	=E50+\$B\$32*D50	=0.5*(E51+E50)*(A51-A50)*3600)+F50	

Figure 7: Rainfall-runoff formulas for the catchment

3. Using an appropriate NRCS 24-hour rainfall distribution

Determine:

- The cumulative rainfall depth (inches) for a 50-yr ARI storm in Lubbock, Texas.
- The rainfall intensity (inches/hour) for each half-hour increment of the storm.
- The maximum rainfall intensity (inches/hour) in any 30-minute interval.

Solution(s): An appropriate NRCS distribution for Lubbock, Texas is a Type-II storm. Figure 8 is a screen capture of a map showing that Lubbock is best represented by Type II.

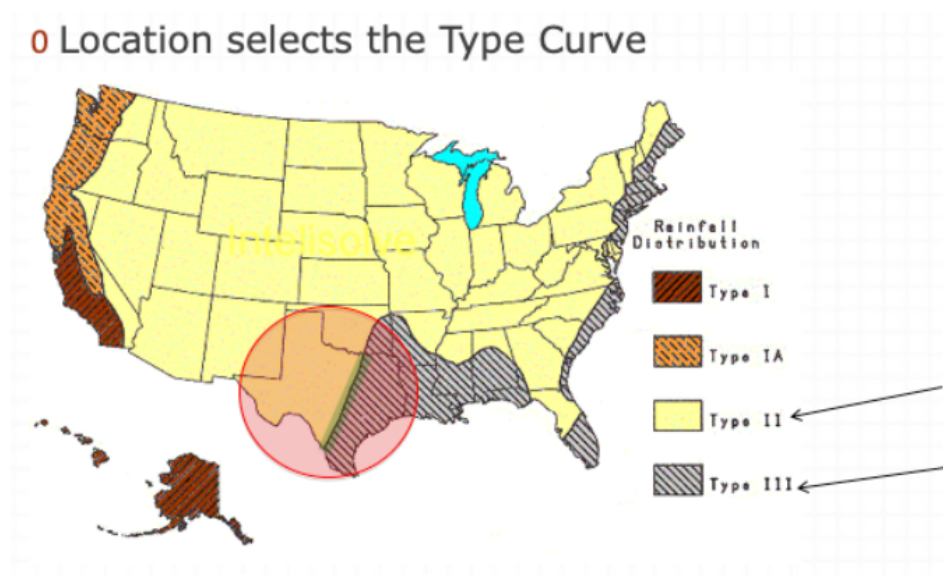


Figure 8: NRCS Type Curve Map

Item a) use NRCS PFDS to look up 24-hr storm depth. Using Partial Duration Series for Lubbock, TX value is **6.09 inches**. Figure 9 is a screen capture of the NRCS server access.

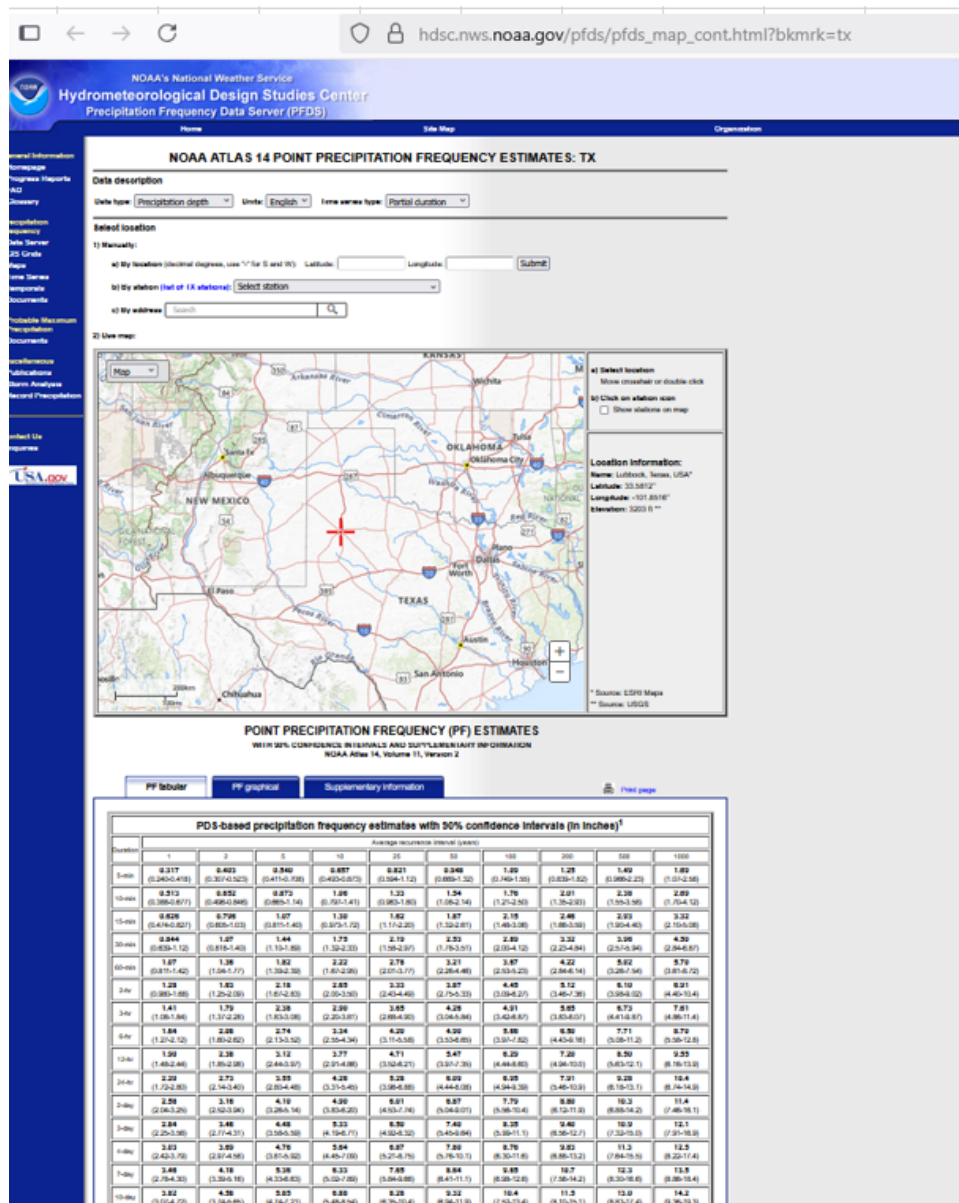


Figure 9: PFDS lookup for Lubbock, Texas

Item b) Use Script in notes, or spreadsheet implementation to obtain cumulative depth estimates each 1/2 hour. Backward difference to obtain intensity. Figure 10 is a screen capture of the python script for processing design storms (provided in course notes, and es2 solution file).

```
[6]: # Script to generate 1-minute hyetographs
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.interpolate import interp1d

hyetType = 'type2'
PT = 1.0 # total depth
#####
hour = [0,2,4,6,7,8,8.5,9,9.5,9.75,10,10.5,11,11.5,11.75,12,12.5,13,13.5,14,16,20,24,48]
minutes = [i*60 for i in hour]
hyets = {
    'type1': [0,0.035,0.076,0.125,0.156,0.194,0.219,0.254,0.303,0.362,0.515,0.583,0.624,0.654,0.669,0.682,0.706,0.727,0.748,0.767,0.83,0.926,1,1],
    'type1A': [0,0.05,0.116,0.206,0.268,0.425,0.48,0.52,0.55,0.564,0.577,0.601,0.624,0.645,0.655,0.664,0.683,0.701,0.719,0.736,0.8,0.906,1,1],
    'type2': [0,0.022,0.048,0.08,0.098,0.12,0.133,0.147,0.163,0.172,0.181,0.204,0.235,0.283,0.357,0.663,0.735,0.772,0.799,0.82,0.88,0.952,1,1],
    'type3': [0,0.02,0.043,0.072,0.089,0.115,0.13,0.148,0.167,0.178,0.189,0.216,0.25,0.298,0.339,0.5,0.702,0.751,0.785,0.811,0.886,0.957,1,1],
    'user': [0,0,0.4285,0.8571,1,0,1,0,1,0,1,0] # Adjust time scaling below if needed
}

if hyetType == 'user':
    user_time = [0,7,8,9,9.3333,10,24,48]
    minutes = [i*60 for i in user_time]
    hyet = hyets['user']
else:
    hyet = hyets.get(hyetType)
    f = interp1d(minutes, hyet, kind='linear')
    t24 = np.arange(0, 1455) # 48 hours in minutes
    depth = PT * f(t24) # Scaled cumulative rainfall depth
    intensity = np.diff(np.insert(depth, 0, 0)) * 60 # in/hr
    # Build DataFrame #####
    df = pd.DataFrame({
        "Time (min)": t24,
        "Cumulative Depth (in)": depth,
        "Intensity (in/hr)": intensity
    })
    # Optionally, write every nth row (e.g., every 15 minutes)
    n = 15
    df_thinned = df.iloc[::n]
    # Save to CSV #####
    df_thinned.to_csv(f"interpolated_output_{hyetType.upper()}.csv", index=False)
    #####
```

Figure 10: Script to generate Type II storm weights every 15 minutes

The output of the script is loaded into a spreadsheet to make the necessary computations to answer the remaining items. Figure 11 is a screen capture of such a spreadsheet.

Item b) based on differencing an interpolated NRCS storm, the intensity for each 1/2 hour is the column labeled I(in/hr) (rightmost in the figure), the maximum value in the spreadsheet, Item c) is 7.46 in/hr.¹

Figure 11 is a screen capture of the spreadsheet as formulas.

¹If you did not actually make the calculations and instead just reported the 30-min intensity from the PFDS, that value is 5.04 in/hour – partial credit for reporting this value

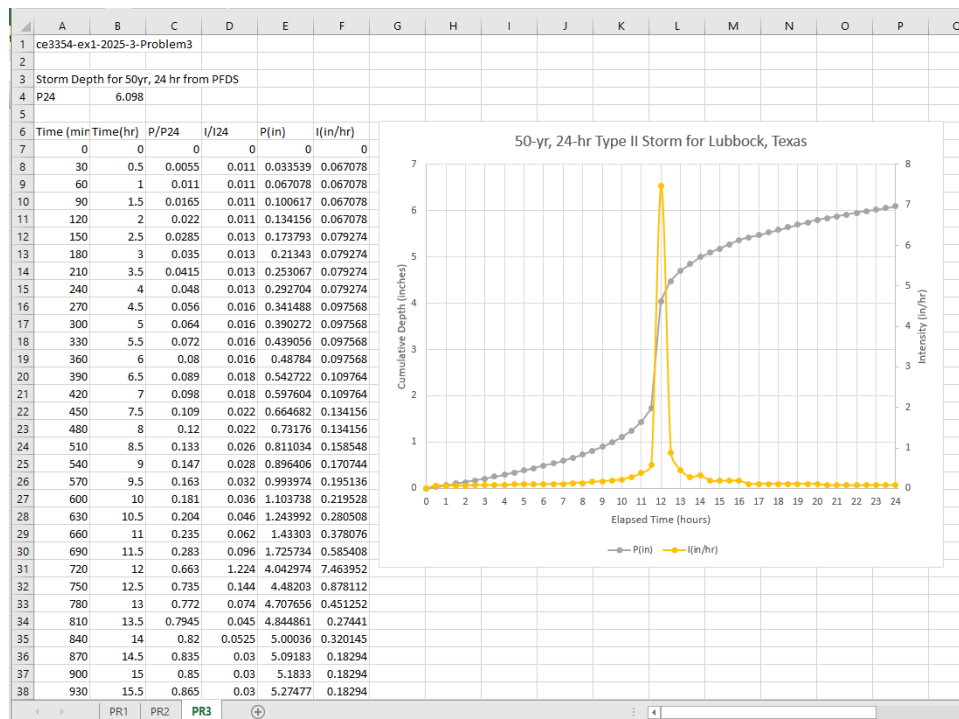


Figure 11: Spreadsheet showing scaled storm depths and intensities (backward differences) for Type II storm. The differencing was performed in the python pre-processing script - all the spreadsheet does is rescale the results.

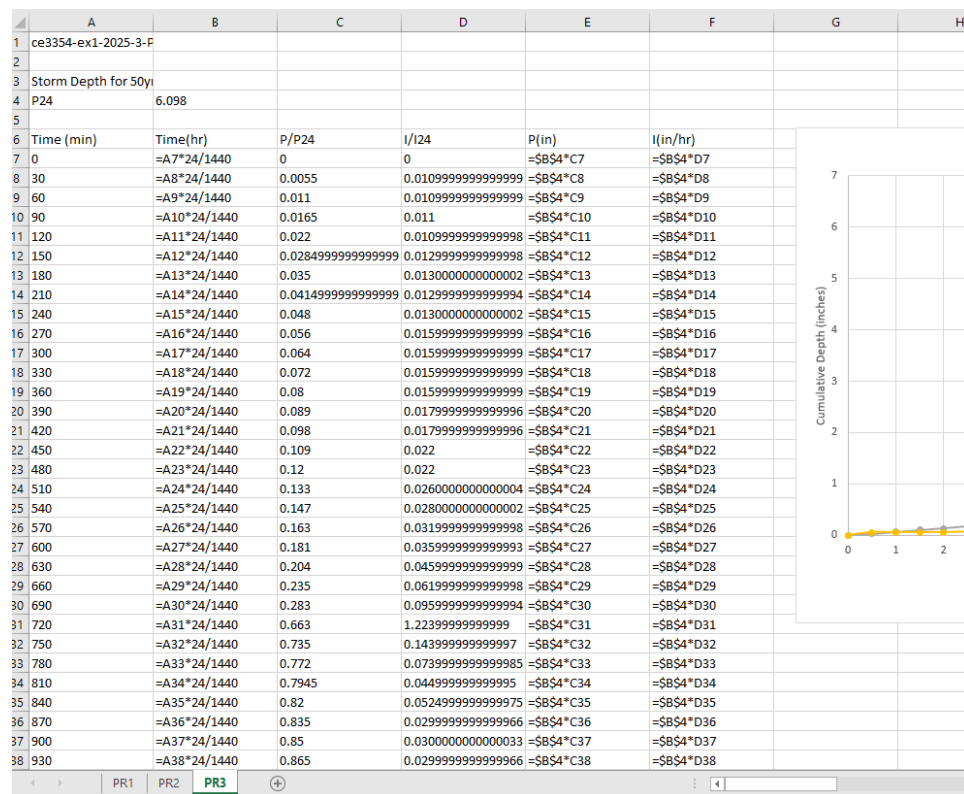


Figure 12: Spreadsheet showing scaled storm depths and intensities formulas for Type II storm

4. The relation between infiltration capacity in mm/hour and the time (in hours) since the start of the experiment as measured with an infiltrometer is depicted in Figure 13.

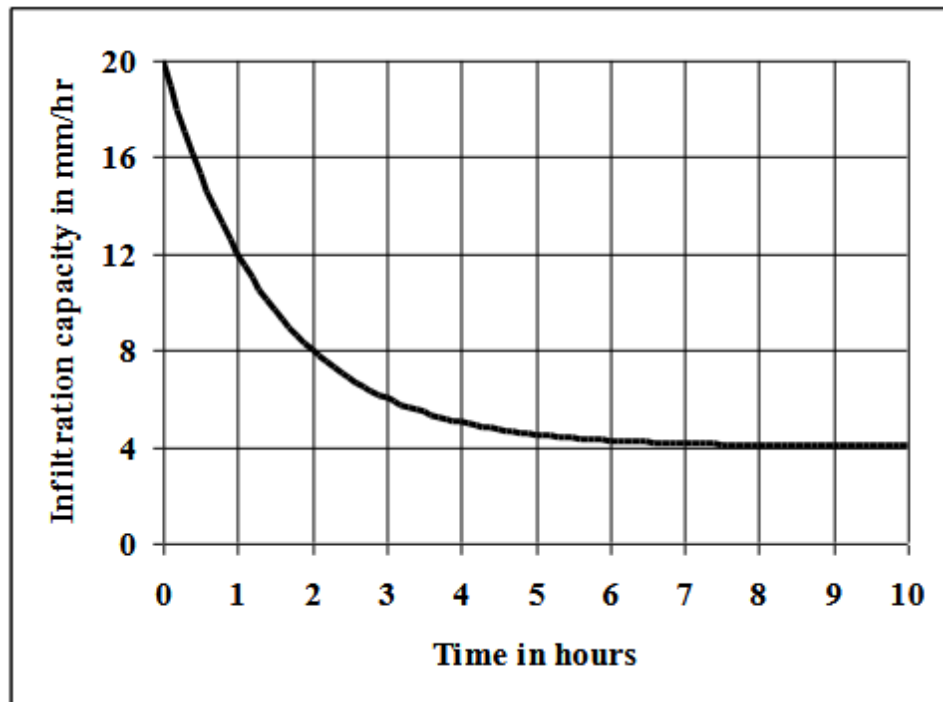


Figure 13: Infiltrometer data for some soil

The relationship is to be described with the Horton infiltration model

$$q(t) = f_c + (f_o - f_c)e^{-kt} \quad (2)$$

Determine:

- The equilibrium infiltration rate, f_c , in mm/hr.
- The initial (dry soil) infiltration rate, f_o , in mm/hr.
- The soil constant k .
- The total amount of water that will infiltrate into an initially dry soil during a rainstorm with a duration 60 minutes and a constant intensity of 20 mm/h.
- The total amount of water that will infiltrate into an initially dry soil during a rainstorm with a duration 480 minutes and a constant intensity of 12 mm/h.

Solution(s):

Use ES3 solution to construct a spreadsheet and fit infiltration curve as shown in Figure 14

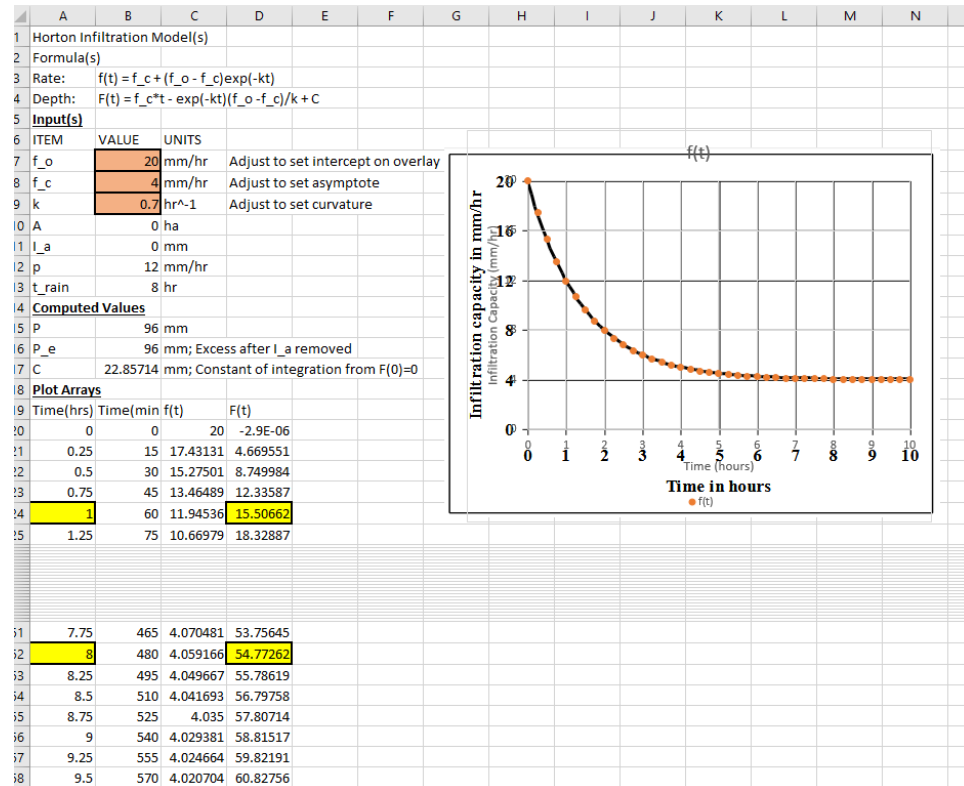


Figure 14: Fitted Infiltrimeter data for some soil on es3 solution spreadsheet tool. Change chart background to transparent, and overlay, resize to fit provided curve.

Results are:

- The equilibrium infiltration rate, f_c , is 4.0 mm/hr.
- The initial (dry soil) infiltration rate, f_o , is 20.0 mm/hr.
- The soil constant k , is $0.7 \frac{1}{hr}$.
- The total amount of water that will infiltrate into an initially dry soil during a rainstorm with a duration 60 minutes and a constant intensity of 20 mm/h is 15.5 mm.
- The total amount of water that will infiltrate into an initially dry soil during a rainstorm with a duration 480 minutes and a constant intensity of 12 mm/h is 54.7 mm