SOLUTION

CE 3354 Engineering Hydrology
Exercise Set 9

Exercises

1. Figure 1 is a plan view of a waste cell at a solid waste disposal site. Two borings are completed in the water table aquifer underlying the disposal site property.

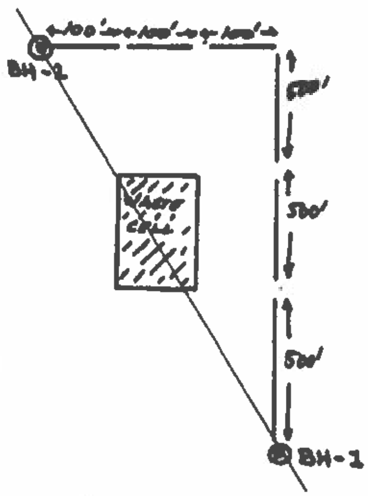


Figure 1: Plan View of an Waste Disposal Cell

The ground surface elevation at borehole BH-1 is 263.75 ft. The ground surface elevation at borehole BH-2 is 249.75 ft. The water table elevation in borehole BH-1 is 223.25 ft. The water table elevation in borehole BH-2 is 229.75 ft. The soil types in the area are silty-clay with a hydraulic conductivity of $K = 3 \times 10^{-5}\ \frac{ft}{sec}$ and an effective porosity of $n = 0.40$. A 100 x 500 foot waste cell is located between the two boreholes as shown. The bottom of the waste cell must be at least 5 feet above the water table.

Determine:

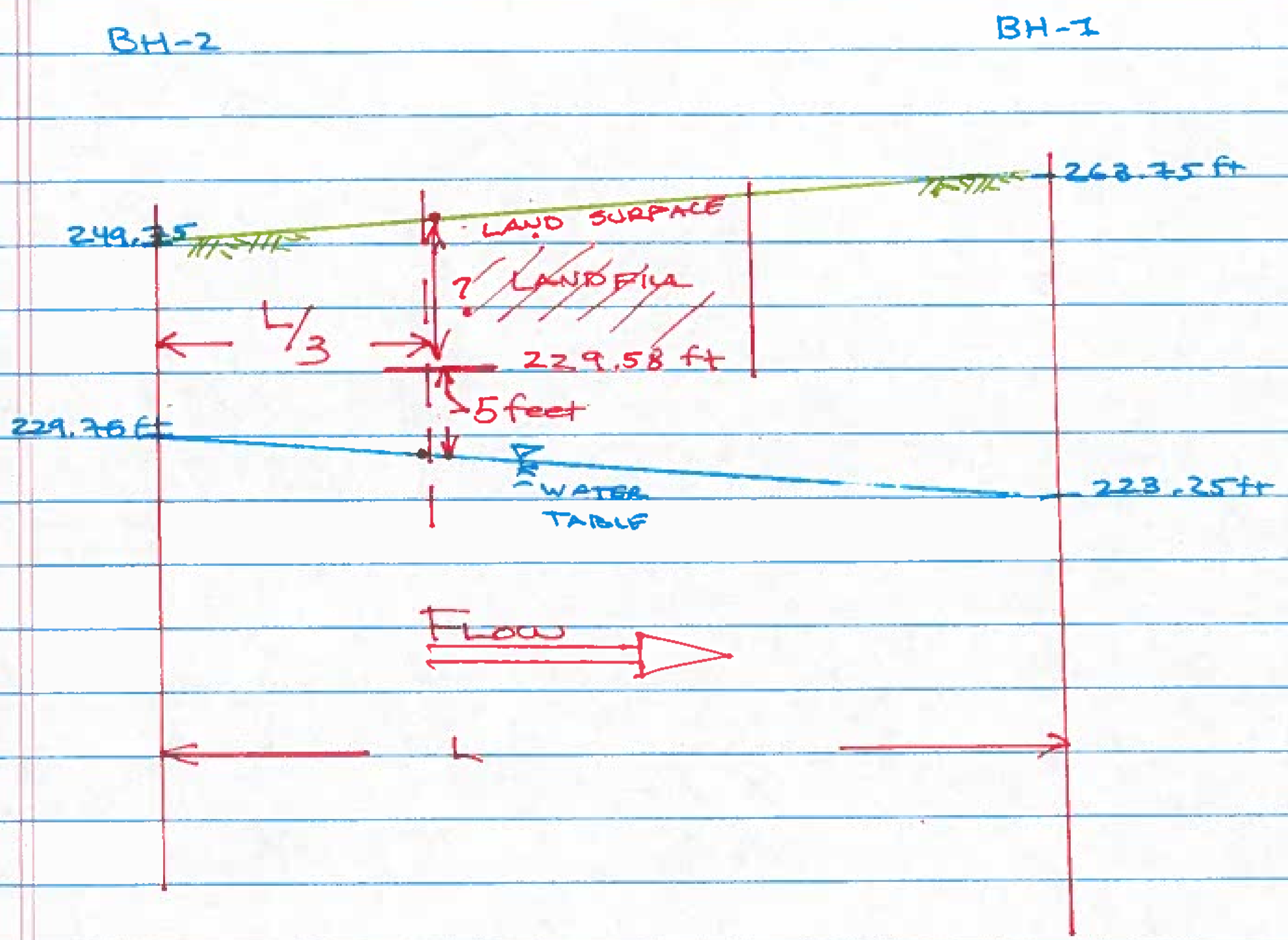
a) The hydraulic gradient (magnitude) from the provided information. 0.0039

b) The direction of groundwater flow from the provided information. FROM BH2 → BH1

c) The average linear (pore) velocity of groundwater. $2.925 \cdot 10^{-7}$ ft/sec

d) The minimum allowable elevation of the bottom of the waste cell. 229.58 ft

e) The anticipated travel time for contaminated leachate to reach the downstream borehole if the waste cell liner fails.



$L^2 = (300)^2 + (1500)^2 = 2{,}340{,}000 \text{ ft}^2$

$L = 1529.7 \text{ ft}$

$\frac{\Delta h}{\Delta l} = \frac{(229.75 - 223.25)}{1529.7} = \underline{\underline{0.0039}}$ hyd. gradient

$v = \frac{Q}{nA} = \frac{1}{n} K \frac{\Delta h}{\Delta l} = \frac{1}{0.40}\left(3 \cdot 10^{-5} \frac{\text{ft}}{\text{sec}}\right)(0.0039)$

$= 2.925 \cdot 10^{-7} \text{ ft/sec}$

LAND AT L/3

$$z_{LAND} = 249.25 + \frac{(263.75 - 249.25)}{\cancel{L}} \cdot \frac{\cancel{L}}{3}$$

$z_{LAND} = 254.08$ ft.

WATER TABLE A L/3

$$z_{WT} = 229.75 - \frac{(229.75 - 223.25)}{\cancel{L}} \cdot \frac{\cancel{L}}{3}$$

$= 227.58$

$z_{LAND} - z_{WT} = 254.08 - 227.58 = \cancel{29.5}$ ft

26.5ft

$\therefore z_{BOTTOM} = \cancel{(z_{LAND} - z_{WT}) + 5}$

$= z_{LAND} - (z_{LAND} - z_{WT}) + 5$

$= 254.08 -$ **26.5 + 5** $= \cancel{229.58 \text{ ft}}$

232.58 ft

TRAVEL TIME

$$t = \cancel{\frac{X}{V}} \frac{X}{V} = \left[\frac{2.925 \cdot 10^{-7} \text{ ft/sec } (3)}{1529.7 \text{ ft}}\right]^{-1}$$

$= 1.7432 \cdot 10^{9}$ sec $\left(\frac{1 \text{ day}}{86400 \text{ sec}}\right)$

$= 2.017 \cdot 10^{4}$ days

$= 55.2$ years

2. Figure 2 is a contour map of head in an aquifer system.

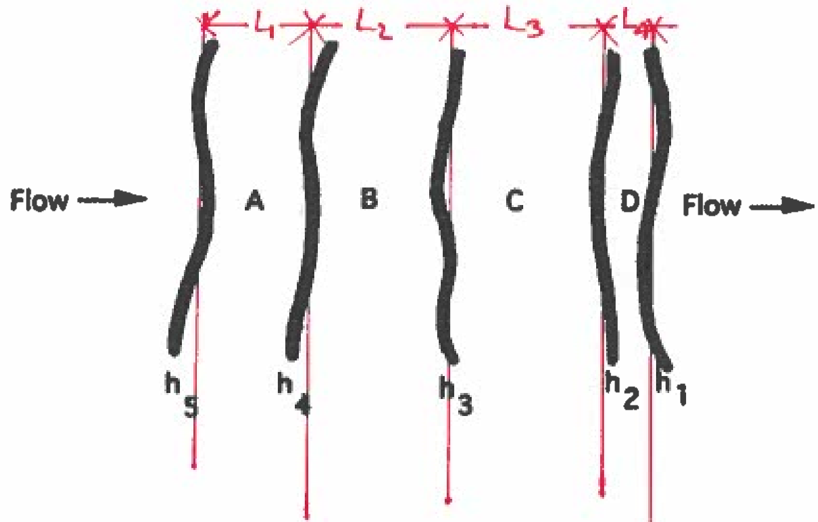


Figure 2: Elevation View of an Aquifer

The aquifer medium is isotropic and inflow equals outflow. The hydraulic conductivity of area A is $K = 1 \times 10^{-6}\ \frac{m}{sec}$

Determine:

USE ENGR SCALE:

(2) a) The hydraulic conductivity in area B $1.318 \cdot 10^{-6}\ \frac{m}{s}$ $L_1 = 22/40$ (1)

(2) b) The hydraulic conductivity in area C $1.455 \cdot 10^{-6}$ m/s $L_2 = 29/40$ (1)

(2) c) The hydraulic conductivity in area D $0.455 \cdot 10^{-6}$ m/s $L_3 = 32/40$ (1)

$L_4 = 10/40$ (1)

CONTINUNITY $\Rightarrow Q_1 = Q_2 = Q_3 = Q_4$

$$\frac{Q_1}{A} = K_1 \frac{h_5 - h_4}{L_1} = K_2 \frac{h_4 - h_3}{L_2} = K_3 \frac{h_3 - h_2}{L_3} = K_4 \frac{h_2 - h_1}{L_4} \quad (1)$$

$$= K_1 \cdot \frac{1}{22} = K_2 \cdot \frac{1}{29} = K_3 \cdot \frac{1}{32} = K_4 \cdot \frac{1}{10}$$

$$K_2 = K_1 \frac{29}{22} = 1.318 \cdot 10^{-6}\ m/s \quad (1)$$

$$K_3 = K_1 \frac{32}{22} = 1.455 \cdot 10^{-6}\ m/s \quad (1)$$

$$K_4 = K_1 \frac{10}{22} = 0.455 \cdot 10^{-6}\ m/s \quad (1)$$

3. Figure 3 is a profile (elevation) view of an aquifer system.

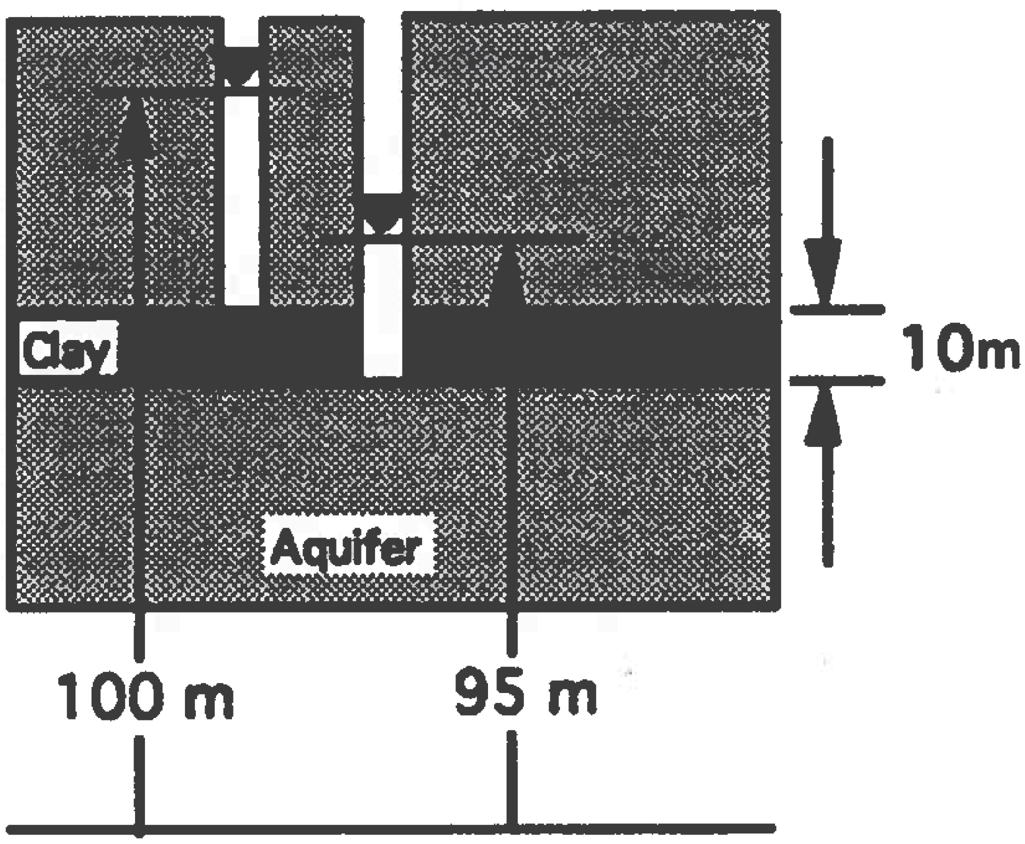


Figure 3: Elevation View of an Aquifer

The vertical hydraulic conductivity of the clay layer is $K_v = 1 \times 10^{-7}\ \frac{cm}{sec}$

Determine:

a) The distance (in meters) from the datum to the water level in the left piezometer. 100m (given) 1

b) The distance (in meters) from the datum to the right piezometer. 95m (given) 1

c) The vertical hydraulic gradient in the clay layer. ~~$1 \cdot 10^{-7} \frac{cm}{sec}$ (given)~~ 1

i=(100-95)/10=0.5 1

d) The specific discharge across the clay layer in cm/sec.

c) The direction of leakage.

$$q = K_v \frac{h_{left} - h_{RIGHT}}{thick} \quad 1$$

$$= \frac{(1 \cdot 10^{-7} \frac{cm}{s})(100\,m - 95\,m)}{10\,m} \quad 1$$

$$= \cancel{0.5} \cdot 10^{-8} \frac{cm}{s} \quad 1$$

5.0 E-08 cm/s

downward 1

4. Table 1 is a list of observations of piezometric heads in three observation wells that penetrate the same homogeneous, isotropic, confined aquifer of thickness $B = 20\ m$

Table 1: Noname USA Aquifer Data

Well ID	Easting (m)	Northing (m)	Head (m)
LJ-65-21-226	100.0	110.0	12.0
LJ-65-21-227	400.0	100.0	13.5
LJ-65-21-228	100.0	310.0	10.4

Drilling cuttings from the wells indicate that the effective porosity is $n = 0.20$, the hydraulic conductivity is $K = 15\ \frac{m}{day}$. The piezometric surface between the wells can be approximated as a plane.

Determine:

a) The hydraulic gradient indicated by the data (magnitude and direction).

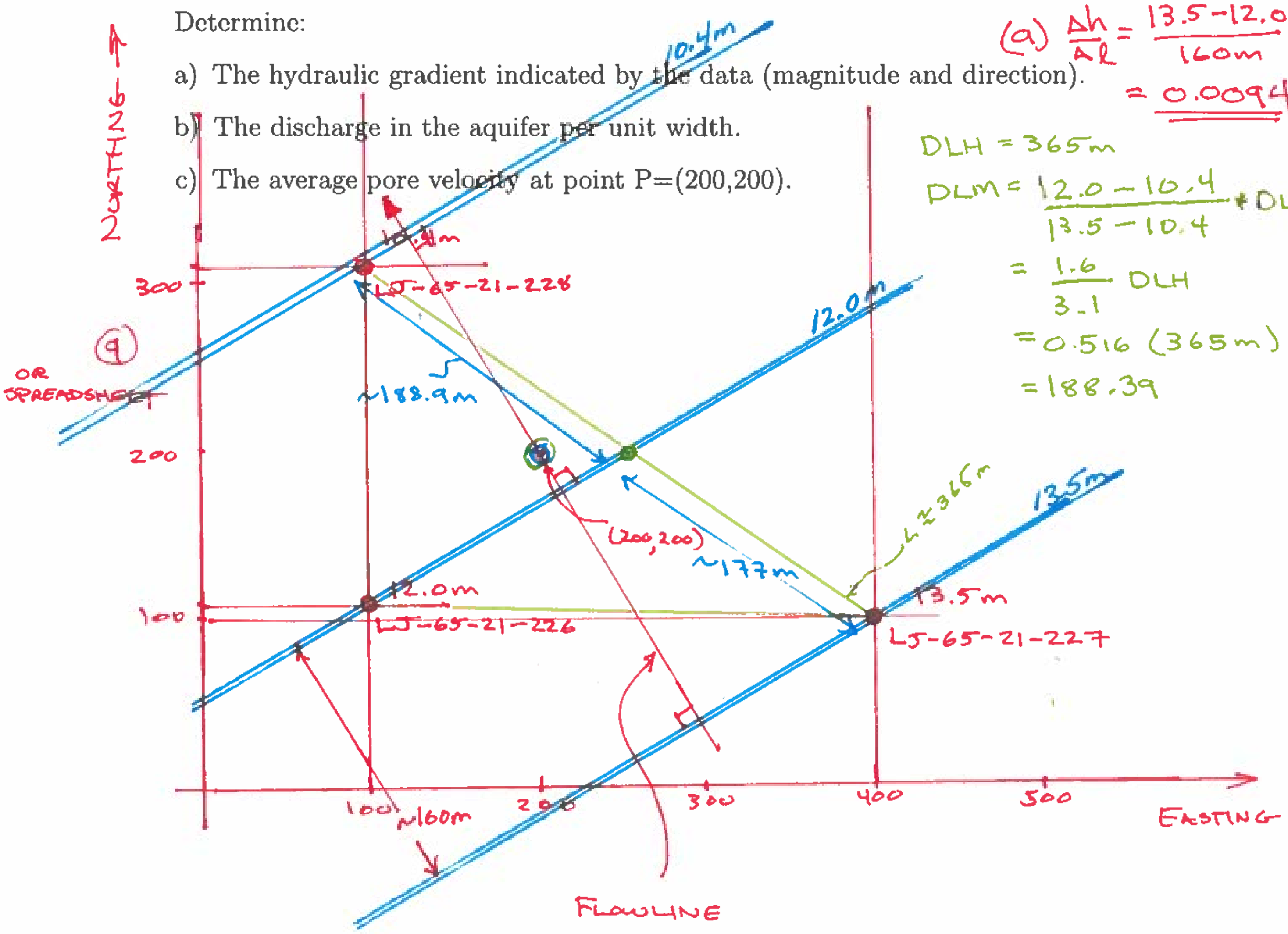
b) The discharge in the aquifer per unit width.

c) The average pore velocity at point P=(200,200).

(a) $\frac{\Delta h}{\Delta \ell} = \frac{13.5-12.0}{160m} = \underline{0.0094}$ (1)

DLH = 365m

$DLM = \frac{12.0-10.4}{13.5-10.4} * DLH = \frac{1.6}{3.1} DLH = 0.516\ (365m) = 188.39$



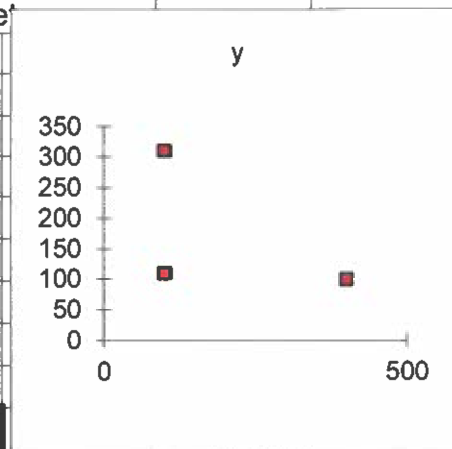
(b) $= \frac{Q}{W} = KB\frac{\Delta h}{\Delta \ell}$

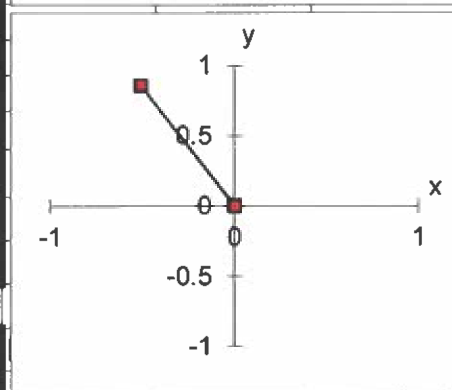
$= (\frac{15m}{d})(20m)(0.0094) = 2.8125 \frac{m^3}{m \cdot day}$ (2)

(c) $V = \frac{1}{n} K \frac{\Delta h}{\Delta \ell} = (\frac{15m}{d})(0.0094)(\frac{1}{0.2}) \approx 0.703\ m/day$ (2)

ALTERNATIVE TO GRAPHICAL ANALYSIS

	A	B	C	D	E	F	G	H	I	J
1		Instructions: Enter data for three wells in shaded area above left.								
2		Spreadsheet solves linear system and computes gradient. Plot shows								
3										
4										
5										
6		Groundwater Hydrology Gradient Spreadsheet								
7		Field Data from Three Wells								
8			x	y	head					
9		Well A	**100**	**110**	**12**					
10		Well B	**400**	**100**	**13.5**					
11		Well C	**100**	**310**	**10.4**					
12										
13		Hydraulic	**-0.005**	<-i.						
14		Gradient	**0.008**	<-j.						
15										
16		A-Matrix			b-vecto	x-vector				
17		100	110	1	12	0.0047				
18		400	100	1	13.5	-0.008				
19		100	310	1	10.4	12.407				
20		A-Inverse								
21		-0.0035	0.0033	0.0002						
22		-0.005	-2E-19	0.005						
23		1.9	-0.333	-0.567						
24										
25		Head	+	8.2833	**1750**	<-x				
26		Function	+	-17	**2125**	<-y				
27			+	12.407						
28				**3.69**	<= h(x,y)					
29										
30		This spreadsheet prepared by								
31		Theodore G. Cleveland, Ph.D., P.E.								
32		Fall, 1996. All rights reserved								
33										
34		Hydraulic Gradient Vector Magnitude								
35		Magnitude=	0.0093							
36		i=	j=							
37		0	0	Hydraulic Gradient						
38		-0.509212	0.8606	Direction Cosines						
39				x-dir.=>	-0.509					
40				y-dir.=>	0.861					





5. Table 2 is a list piezometric heads measured simultaneously in 13 wells penetrating an isotropic confined aquifer of thickness $B = 50\ m$, hydraulic conductivity $K = 20\ \frac{m}{day}$, and effective porosity $n = 0.23$.

Table 2: Somewhere USA Aquifer Data

Well ID	Easting (m)	Northing (m)	Head (m)
MW-01	4300	1000	34.6
MW-02	16500	3500	35.1
MW-03	7000	5100	32.8
MW-04	3000	6500	32.1
MW-05	11000	7000	31.5
MW-06	22000	6500	34.5
MW-07	8000	9000	33.3
MW-08	3200	11800	34.4
MW-09	18100	10000	34.3
MW-10	13500	12900	35.2
MW-11	4000	15500	35.2
MW-12	8700	16100	37.3
MW-12	19500	16300	36.3

Determine:

a) A contour map of the head distribution (1-meter contour intervals) (done using on-line tool)

b) Specific discharge (direction and magnitude) at location $A = (10000, 4000)$ $\nearrow \approx 0.01 \frac{m}{d}$

c) Specific discharge (direction and magnitude) at location $B = (16000, 11000)$ $\swarrow \approx 0.0033 \frac{m}{d}$

d) An estimate of total flow through the aquifer between wells MW-10 and MW-9. ≈ 0

e) An estimate of travel time for a conservative tracer introduced near well MW-12 to reach MW-5 ≈ 760 years

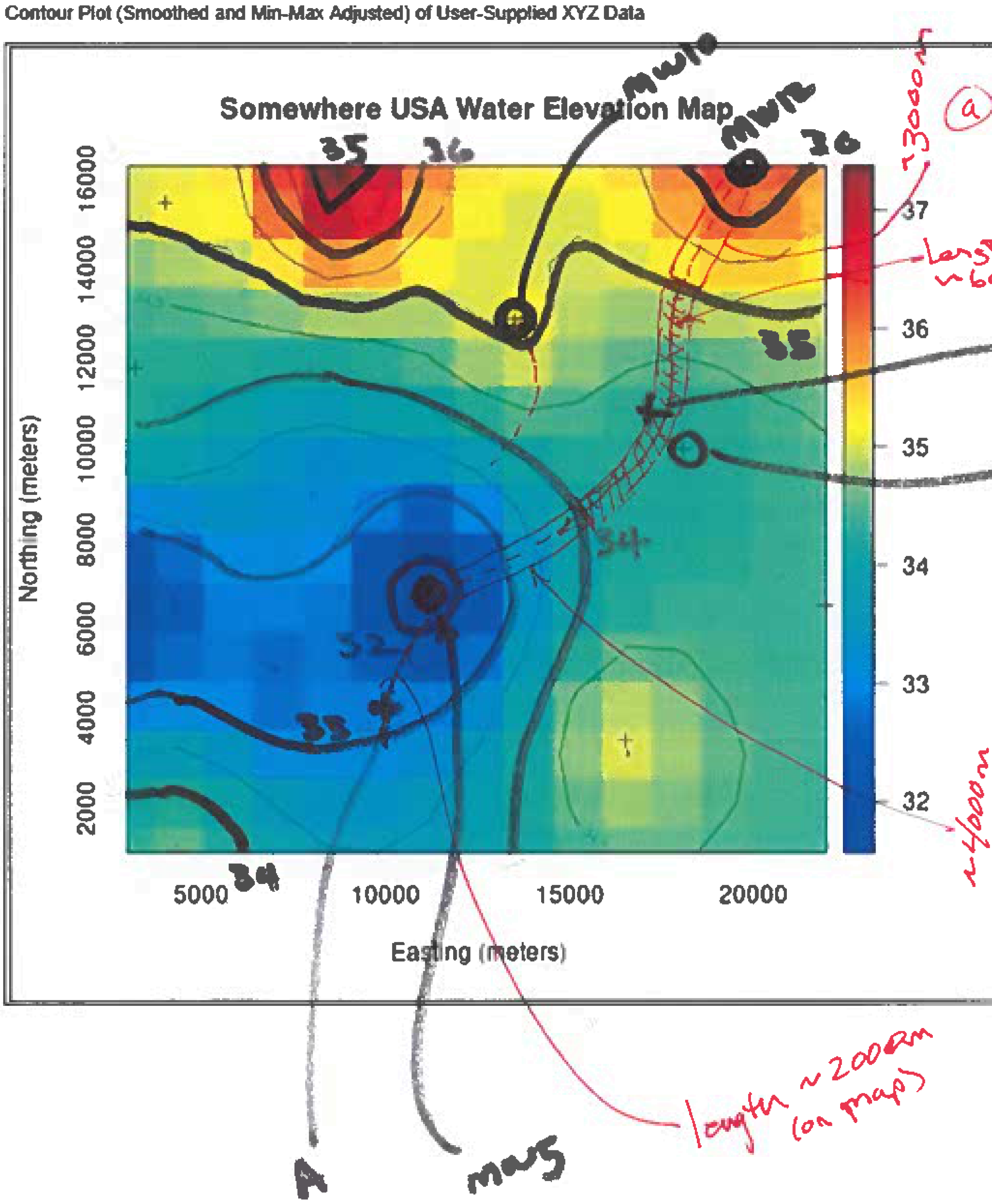
Not Secure http://54.243.252.9/cgi-bin/ordinarytools/FilledContourMap/XYZtoContour.p

Filled Contour Plotting (Using Python and R)

Machine Name : 54.243.252.9 (AWS East)
Run Date : Mon Aug 4 17:11:21 2025
Return Code : 0

----- INPUT VALUES -----	
Gridding Exponent =	2.0
Smoothing Parameter =	2.0
Grid Lines X =	15.0
Grid Lines Y =	15.0
Plot Title =	Somewhere USA Water Elevation Map
X-Axis Label =	Easting (meters)
Y-Axis Label =	Northing (meters)
XYZ Filename =	groundwatermap.txt
COMMAND TO RUN:	/usr/bin/Rscript XYZ2Contour.R
----- COMPUTED RESULT -----	
XYZ Absolute Path	/var/www/atkaws/toolbox/ordinarytools/FilledContourMap/working_files/groundwatermap.txt
Echo Input Parameter File	/var/www/atkaws/toolbox/ordinarytools/FilledContourMap/working_files/echoparms.txt

Contour Plot (Smoothed and Min-Max Adjusted) of User-Supplied XYZ Data



(a) (① MAKE MAP ②
② ANNONTATE

(b) $q = K \frac{\Delta h}{\Delta l}$ ②

$= 20 \frac{m}{d} \frac{(1.0 m)}{(2000 m)} = 0.01 m/d$

(c) $q = K \frac{\Delta h}{\Delta l}$

$= \frac{20 m}{d} \frac{(1.0 m)}{(6000 m)}$ ②

$= 0.0033 m/d$

(d) $q_{10-9} \approx 0$ ①

(NOT ON SAME FLOWLINE)

(e) distance ~ 11,000 m

pore velocity

$v \sim \frac{1}{n} K \frac{\Delta h}{\Delta l}$ ①

$= \frac{1}{0.23} \left(20 \frac{m}{d}\right) \left(\frac{36.5 - 31}{11000}\right)$

$= 0.039 m/d$

$t = \frac{x}{v} = \frac{11000}{0.039 m/d}$ ①

$= 278300.0$ days

$= \underline{762}$ years ②