

CE 3354 Engineering Hydrology
Exercise Set 2

Exercises

1. A raingage is located in a 2.5 acre impervious watershed with non initial abstraction. The gage records a catch of 1.0 inches of precipitation in one hour. The maximum intensity was 2.4 inches per hour for 10 minutes. Assume that 10 minutes is the characteristic time for which all parts of the watershed can contribute runoff to a discharge point.

Determine:

- a) Volume of rainfall in cubic feet for the watershed.
- b) Maximum (peak) discharge rate for the watershed.

Solution(s):

- a) Find volume of rainfall in cubic feet as product to total depth and watershed area.

$$V = A \times D = 2.5 \text{ ac} \cdot \frac{43560 \text{ ft}^2}{1 \text{ ac}} \cdot 1.0 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 9,074 \text{ ft}^3 \quad (1)$$

- b) Apply rational runoff equation to estimate peak discharge.

$$Q_{peak} = C \cdot i \cdot A = (1.0)(2.4)(2.5) = 6.0 \text{ cfs} \quad (2)$$

Where:

Q = peak discharge in cfs

C = runoff coefficient (1.0 for impervious surfaces, given)

i = rainfall intensity (2.4 in/hr, given maximum 10-min intensity) for the time of concentration (10 minutes, given)

A = area in acres (2.5 ac., given)

2. Consider the rainfall data in Table 1

Table 1: Somewhere USA Precipitation Data

| Time (minutes) | Cumulative Depth (inches) |
|----------------|---------------------------|
| 0.00 | 0.00 |
| 30.0 | 0.04 |
| 60.0 | 0.38 |
| 90.0 | 1.07 |
| 120. | 1.44 |
| 150. | 1.62 |
| 180. | 1.70 |

Determine:

- a) A depth (cumulative inches) hyetograph in 30 minute intervals (plot).
- b) An intensity (inches/hour) hyetograph in 30 minute intervals (plot).

Solution(s): Both items above are shown on Figure 1

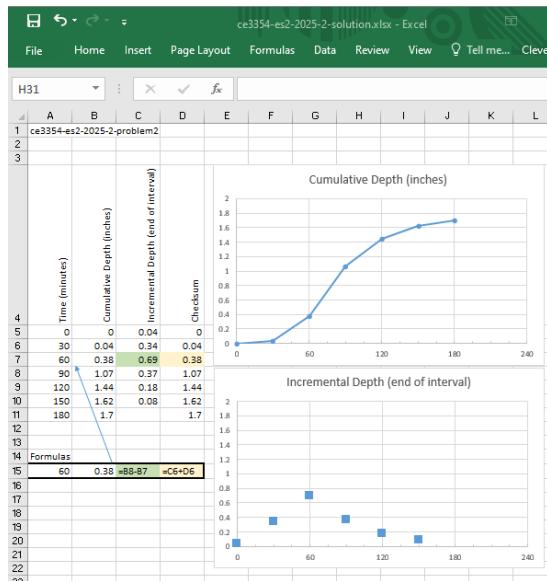


Figure 1: Excel screen capture for ES2-2 (both parts)

3. The intersection of US 408 and US 417 in Orange County, Florida at $28^{\circ}32'51.2''N$ $81^{\circ}15'28.5''W$ is the approximate centroid of that county. Using NOAA Atlas 14 (use the online PFDS tool)

Determine:

- The 1-hr rainfall depth for a 100-yr Annual Recurrence Interval (ARI).
- The 1-hr average rainfall intensity for a 100-yr Annual Recurrence Interval (ARI).
- The 6-hr rainfall depth for a 100-yr Annual Recurrence Interval (ARI).
- The 6-hr average rainfall intensity for a 100-yr Annual Recurrence Interval (ARI).

Solutions:

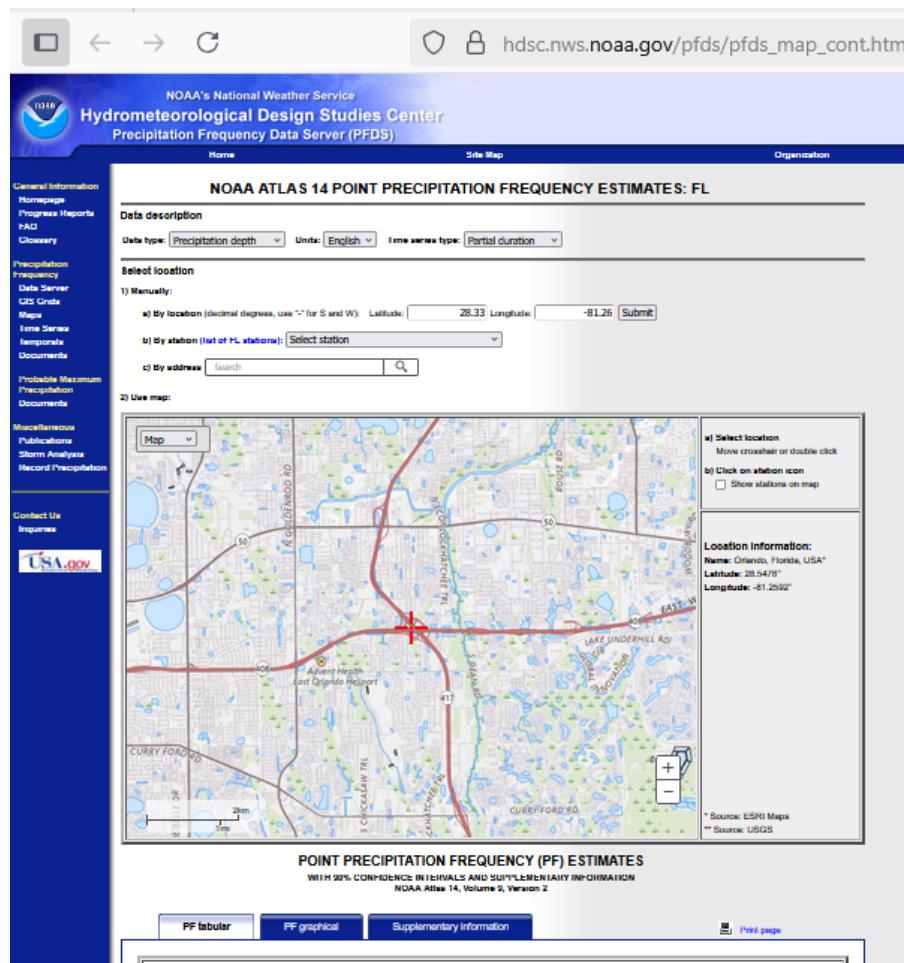


Figure 2: NOAA PFDS Locating the place of interest

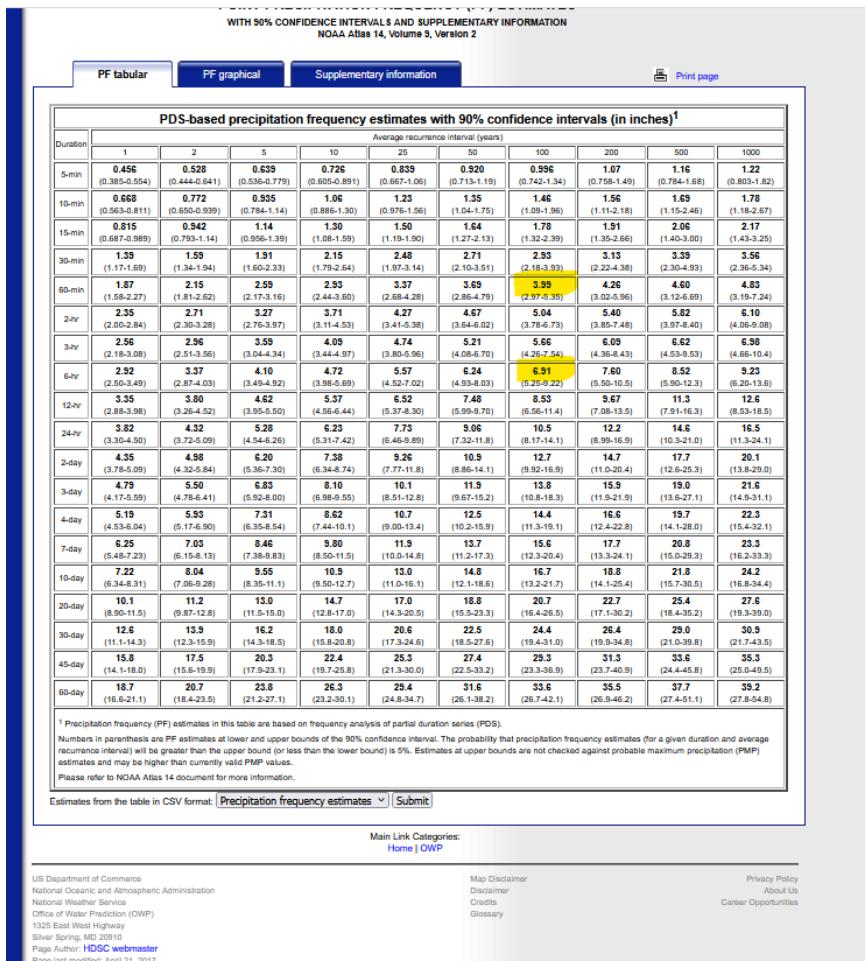


Figure 3: Depth for different recurrence intervals

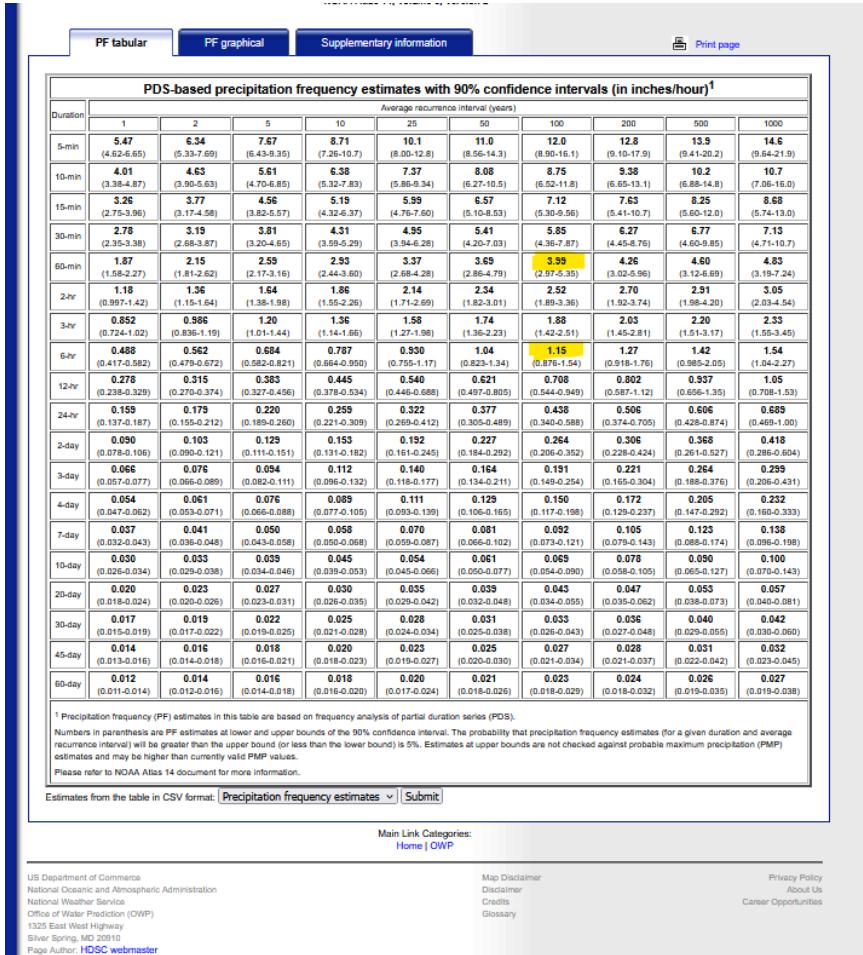


Figure 4: Intensity for different recurrence intervals

Table 2: Summary Florida Depths and Intensity for the intersection of US 408 and US 417 in Orange County, Florida

| Duration (hours) | Value | ARI (years) |
|------------------|------------------|-------------|
| 1.0 | 3.55 inches | 100 |
| 1.0 | 3.55 inches/hour | 100 |
| 6.0 | 6.91 inches | 100 |
| 6.0 | 1.15 inches/hour | 100 |

4. 55 mm of rain is recorded for a 6-hour storm by a raingage for a 10 km^2 watershed. The runoff from the storm indicates that only 45 mm of rain fell on the entire area.

- The areal reduction factor (ARF).
- Compare the result to Central Texas areal reduction factors

Solution(s):

Figure 5

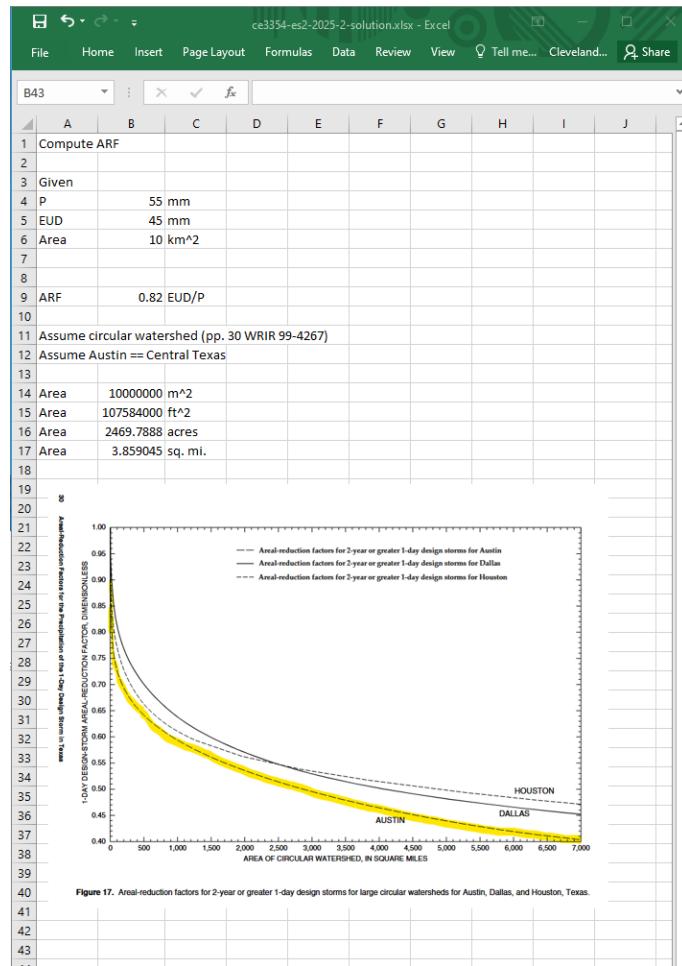


Figure 5: ARF computation and comparison to Central Texas

The reported ARF is consistent with plotted ARF for the Austin curve, but the reported is smaller than anticipated with Central Texas behavior.

5. Table 3 is precipitation data for a 6-hour storm in Somewhere Else USA.

Table 3: Somewhere Else USA Precipitation Data – End of Interval Catch

| Time (hours) | Cumulative Depth (inches) |
|--------------|---------------------------|
| 0.00 | 0.00 |
| 0.25 | 0.10 |
| 0.50 | 0.21 |
| 0.75 | 0.33 |
| 1.00 | 0.48 |
| 1.25 | 0.64 |
| 1.50 | 0.81 |
| 1.75 | 1.08 |
| 2.00 | 1.38 |
| 2.25 | 2.46 |
| 2.50 | 3.60 |
| 2.75 | 3.90 |
| 3.00 | 4.20 |
| 3.25 | 4.44 |
| 3.50 | 4.68 |
| 3.75 | 4.86 |
| 4.00 | 5.01 |
| 4.25 | 5.16 |
| 4.50 | 5.28 |
| 4.75 | 5.40 |
| 5.00 | 5.52 |
| 5.25 | 5.64 |
| 5.50 | 5.76 |
| 5.75 | 5.88 |
| 6.00 | 6.00 |

Determine:

- a) The average rainfall intensity (inches/hour) from hour 3:00 to hour 4:00.
- b) The average rainfall intensity (inches/hour) for the first half of the storm.
- c) The maximum rainfall intensity (inches/hour) in any one hour.
- d) The maximum rainfall intensity (inches/hour) in any 15-minute interval.
- e) The average rainfall intensity (inches/hour) for the last half hour of the storm.

Solution(s):

Figure 6 is a screen capture with the various storm values; formulas used are shown in the figure.

| | A | B | C | D | E | F | G | H | I | J |
|----|----------------------------------|-------------|----------------|-----------------------------|---------------------------|---|---|---|---|---|
| 1 | | | | | | | | | | |
| 2 | | | | | | | | | | |
| 3 | | | | | | | | | | |
| 4 | | Time(hours) | Depth (inches) | Increment (end of interval) | Interval Rate (inches/hr) | | | | | |
| 5 | | 0 | 0 | 0.1 | 0.4 | | | | | |
| 6 | 0.25 | 0.1 | 0.11 | 0.44 | | | | | | |
| 7 | 0.5 | 0.21 | 0.12 | 0.48 | | | | | | |
| 8 | 0.75 | 0.33 | 0.15 | 0.6 | | | | | | |
| 9 | 1 | 0.48 | 0.16 | 0.64 | | | | | | |
| 10 | 1.25 | 0.64 | 0.17 | 0.68 | | | | | | |
| 11 | 1.5 | 0.81 | 0.27 | 1.08 | | | | | | |
| 12 | 1.75 | 1.08 | 0.3 | 1.2 | | | | | | |
| 13 | 2 | 1.38 | 1.08 | 4.32 | | | | | | |
| 14 | 2.25 | 2.46 | 1.14 | 4.56 | | | | | | |
| 15 | 2.5 | 3.6 | 0.3 | 1.2 | | | | | | |
| 16 | 2.75 | 3.9 | 0.3 | 1.2 | | | | | | |
| 17 | 3 | 4.2 | 0.24 | 0.96 | | | | | | |
| 18 | 3.25 | 4.44 | 0.24 | 0.96 | | | | | | |
| 19 | 3.5 | 4.68 | 0.18 | 0.72 | | | | | | |
| 20 | 3.75 | 4.86 | 0.15 | 0.6 | | | | | | |
| 21 | 4 | 5.01 | 0.15 | 0.6 | | | | | | |
| 22 | 4.25 | 5.16 | 0.12 | 0.48 | | | | | | |
| 23 | 4.5 | 5.28 | 0.12 | 0.48 | | | | | | |
| 24 | 4.75 | 5.4 | 0.12 | 0.48 | | | | | | |
| 25 | 5 | 5.52 | 0.12 | 0.48 | | | | | | |
| 26 | 5.25 | 5.64 | 0.12 | 0.48 | | | | | | |
| 27 | 5.5 | 5.76 | 0.12 | 0.48 | | | | | | |
| 28 | 5.75 | 5.88 | 0.12 | 0.48 | | | | | | |
| 29 | 6 | 6 | 0 | 0 | | | | | | |
| 30 | | | | | | | | | | |
| 31 | a) average intensity hour 3 to 4 | | 0.768 in/hr | | | | | | | |
| 32 | b) average intensity 1st half | | 1.366154 in/hr | | | | | | | |
| 33 | c) Max intensity any one hour | | 2.496 in/hr | | | | | | | |
| 34 | d) Max intensity any 15 minute | | 4.56 in/hr | | | | | | | |
| 35 | e) Average last 1/2 hour | | 0.32 in/hr | | | | | | | |
| 36 | | | | | | | | | | |

Formulas:

| | | |
|----------------------------------|----------------|-------------------|
| a) average intensity hour 3 to 4 | 0.768 in/hr | =AVERAGE(D17:D21) |
| b) average intensity 1st half | 1.366154 in/hr | =AVERAGE(D5:D17) |
| c) Max intensity any one hour | 2.496 in/hr | =AVERAGE(D13:D17) |
| d) Max intensity any 15 minute | 4.56 in/hr | direct copy |
| e) Average last 1/2 hour | 0.32 in/hr | =AVERAGE(D27:D29) |

Figure 6: Various storm averages from Somewhere Else USA

6. Using the NRCS Type III rainfall distribution

Determine:

- The cumulative rainfall depth (inches) for half-hour increments for a 10 inch total depth, 24-hour storm.
- The rainfall intensity (inches/hour) for each half-hour increment of the storm.
- The maximum rainfall intensity (inches/hour) in any 30-minute interval.

Solution(s):

The python script below should interpolate the hyetographs onto the desired 15-minute space. Copy the output file into an Excel spreadsheet, and complete the exercise.

```
# Script to generate 1-minute hyetographs
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.interpolate import interp1d
hyetType = 'type3'
PT = 1.0 # total depth - change to suit problem
#####
hour = [0,2,4,6,7,8,8.5,9,9.5,9.75,10,10.5,11,11.5,11.75,12,12.5,13.0,13.6,14,16,20]
minutes = [i*60 for i in hour]
hyets = {
    'type1': [0,0,0.035,0.076,0.125,0.156,0.194,0.219,0.254,0.303,0.362,
              0.515,0.583,0.624,0.654,0.669,0.682,0.706,0.727,0.748,0.767,0.83,0.926,1,1],
    'type1A': [0,0.05,0.116,0.206,0.268,0.425,0.48,0.52,0.55,0.564,0.577,
               0.601,0.624,0.645,0.655,0.664,0.683,0.701,0.719,0.736,0.8,0.906,1,1],
    'type2': [0,0.022,0.048,0.08,0.098,0.12,0.133,0.147,0.163,0.172,0.181,
              0.204,0.235,0.283,0.357,0.663,0.735,0.772,0.799,0.82,0.88,0.952,1,1],
    'type3': [0,0.02,0.043,0.072,0.089,0.115,0.13,0.148,0.167,0.178,0.189,
              0.216,0.25,0.298,0.339,0.5,0.702,0.751,0.785,0.811,0.886,0.957,1,1],
    'user': [0,0,0.4285,0.8571,1.0,1.0,1.0,1.0] # Adjust time scaling below if
}
if hyetType == 'user':
    user_time = [0,7,8,9,9.3333,10,24,48]
    minutes = [i*60 for i in user_time]
    hyet = hyets['user']
else:
    hyet = hyets.get(hyetType)
f = interp1d(minutes, hyet, kind='linear')
```

```
t24 = np.arange(0, 1455) # 48 hours in minutes
depth = PT * f(t24) # Scaled cumulative rainfall depth
intensity = np.diff(np.insert(depth, 0, 0)) * 60 # in/hr
# Build DataFrame #####
df = pd.DataFrame({
    "Time (min)": t24,
    "Cumulative Depth (in)": depth,
    "Intensity (in/hr)": intensity
})
# Optionally, write every nth row (e.g., every 15 minutes)
n = 15
df_thinned = df.iloc[::n]
# Save to CSV #####
df_thinned.to_csv(f"interpolated_output_{hyetType.upper()}.csv", index=False)
#####
```

Figures 7 and 8 are spreadsheet fragments to illustrate the building of the tool. Once the tool(s) are built simply query arrays as needed to complete the exercise.

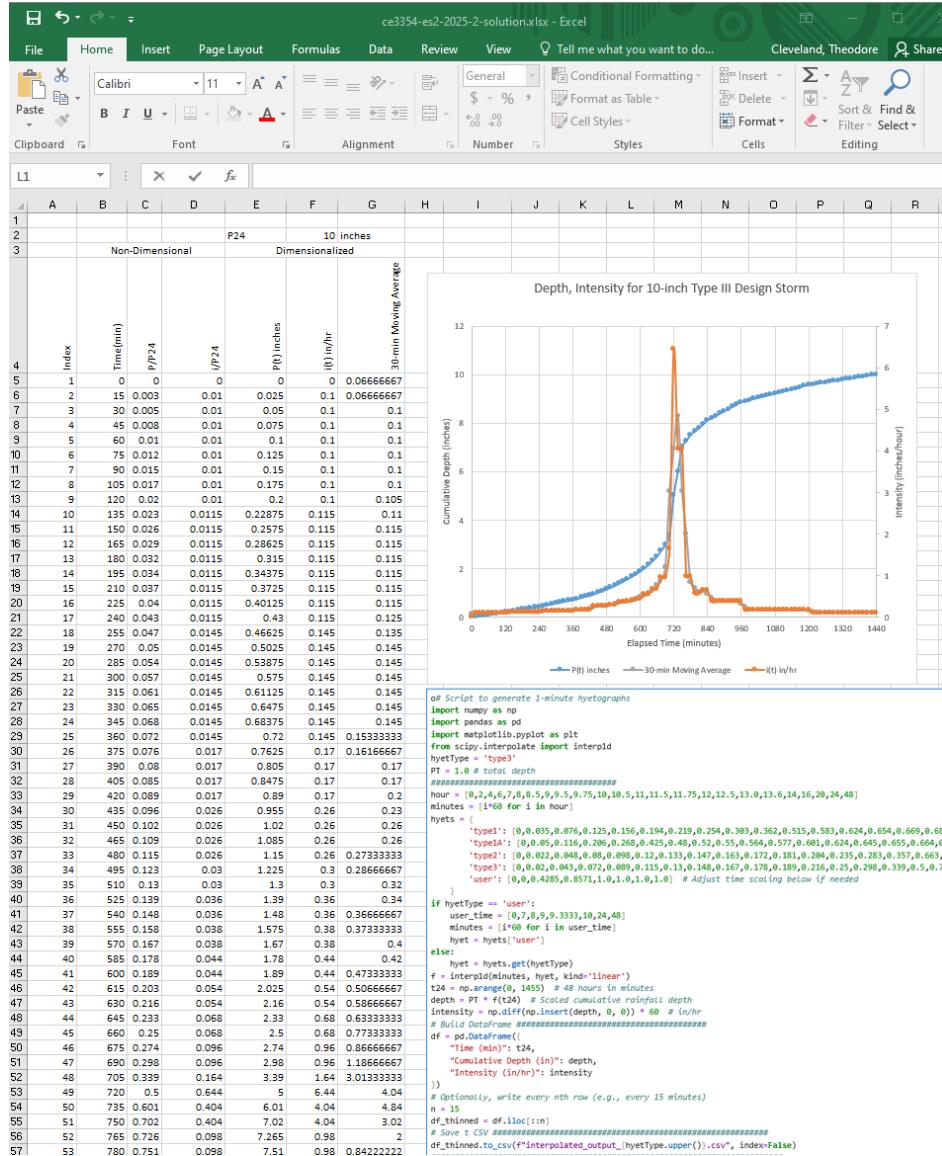


Figure 7: Type III Storm Spreadsheet

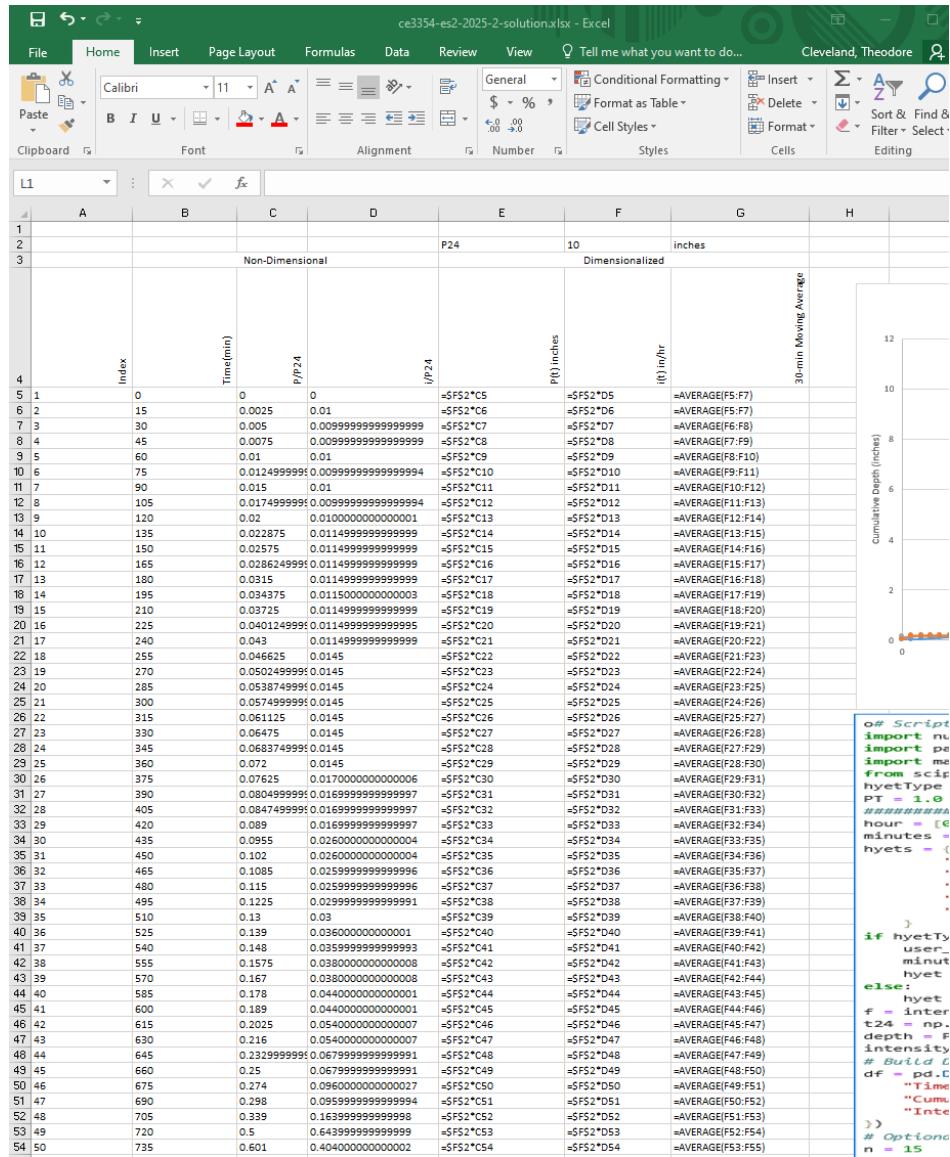


Figure 8: Type III Storm Spreadsheet (as formulas)

- a) The cumulative rainfall depth (inches) for half-hour increments for a 10 inch total depth, 24-hour storm. The column labeled **P(t) inches** is the cumulative depth for the design storm.
- b) The rainfall intensity (inches/hour) for each half-hour increment of the storm. The column labeled **i(t) in/hr** is the intensity for the design storm.
- c) The maximum rainfall intensity (inches/hour) in any 30-minute interval. The largest average of any 3 three adjacent rows would be the maximum 30-minute intensity. In the image its the column labeled 30-minute moving average. The largest value is 4.84 in/hr at time 735 minutes.(12:15 from the storm start)

7. Table 4 is intensity duration data for Somewhere USA.

Table 4: Somewhere USA Intensity-Duration

| Duration (minutes) | Intensity (inches/hour) |
|--------------------|-------------------------|
| 10.0 | 4.00 |
| 15.0 | 3.20 |
| 20.0 | 2.70 |
| 30.0 | 1.90 |
| 60.0 | 1.20 |
| 120. | 0.80 |
| 180. | 0.60 |

Determine:

- a) Plot the intensity duration on the plot type that produces a straight line.
- b) An equation (model) of the "best" straight line for these values.

Solution(s):

The attached spreadsheet in Figure 9 shows the analysis.

In this case the data, when plotted on log-log axes, align closely along a straight line. This suggests that the relationship between variables follows a power-law form of the type:

$$y = ax^b$$

In a log-log plot, such relationships appear linear because:

$$\log(y) = \log(a) + b \cdot \log(x)$$

This linearity in the transformed space indicates that a power-law model may be the most appropriate choice for describing the underlying behavior of the data.

The equation based on the plot (using Excel Trendline) is

$$I_{in/hr} = 18.925 \cdot T_c^{0.665} \text{ min} \quad (3)$$

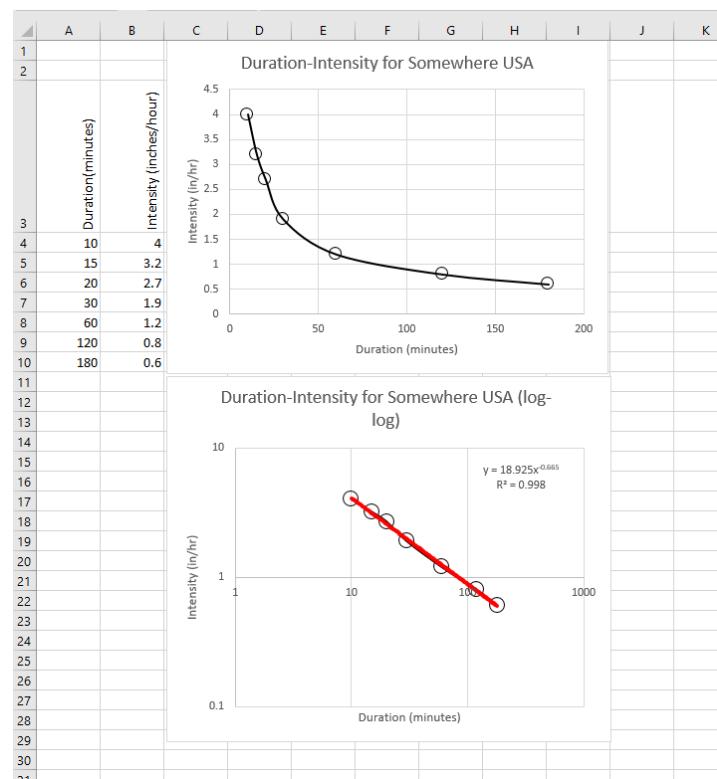


Figure 9: Spreadsheet screen capture with arithmetic and double log plots.

8. Table 5 is a measured 1-hour hyetograph for Baltimore, Maryland.

Table 5: Baltimore, Maryland Rainfall Data

| Time (minutes) | Intensity (cm/hour) |
|----------------|---------------------|
| 0.00 - 10.0 | 2.00 |
| 10.0 - 20.0 | 6.00 |
| 20.0 - 30.0 | 12.00 |
| 30.0 - 40.0 | 8.00 |
| 40.0 - 50.0 | 6.00 |
| 50.0 - 60.0 | 3.00 |

Determine:

- The average intensity in cm/hr.
- The net volume of rainfall in m^3 and liters if the watershed is $4,000 \text{ m}^2$
- An approximate Annual Recurrance Interval (ARI) for the measured event, using NOAA Atlas 14.

Solution(s):

a), b), and c) are shown in the spreadsheet in Figure 10 and the formulas used are in Figure 11

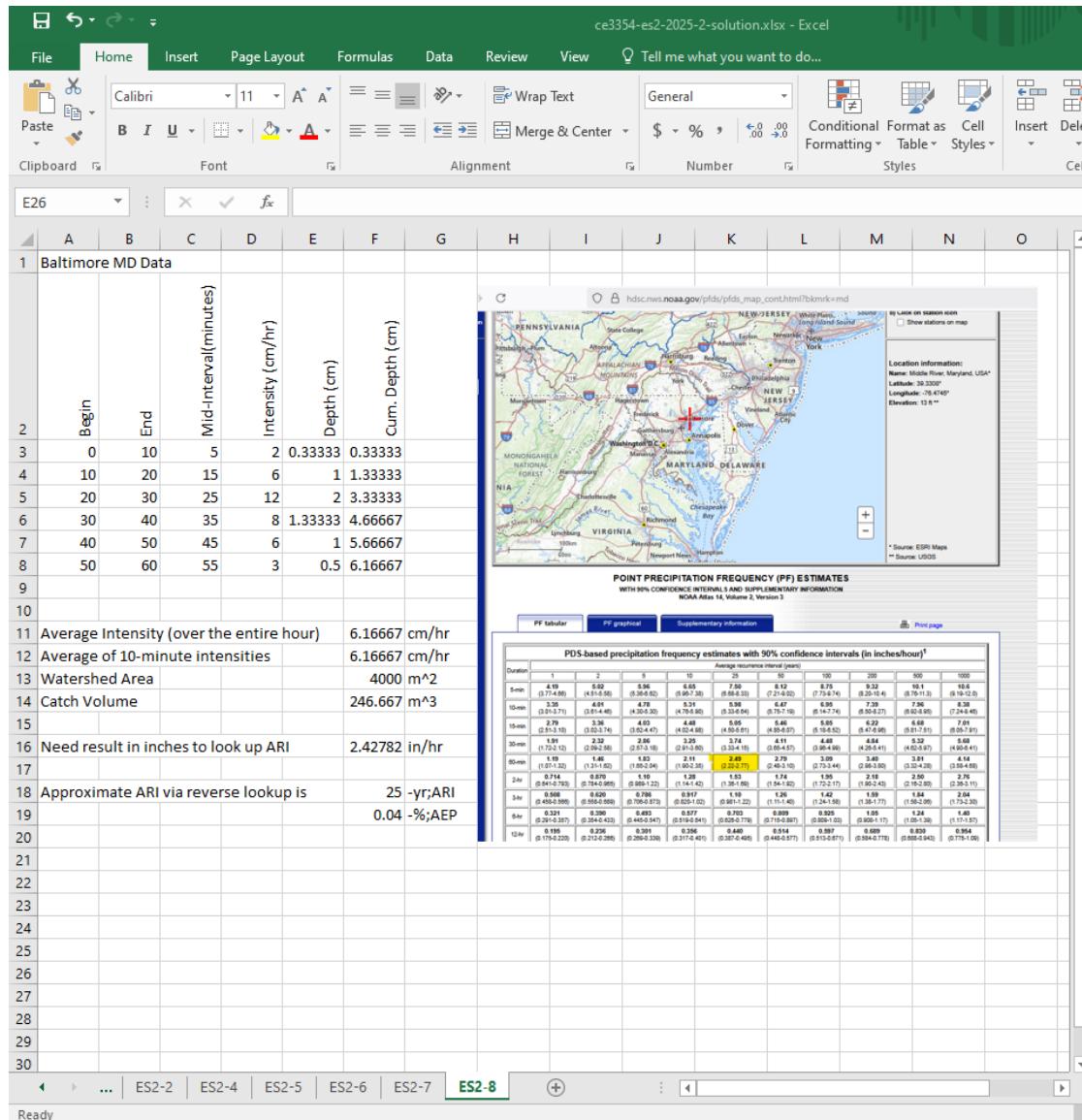


Figure 10: Baltimore, MD intensity data analysis

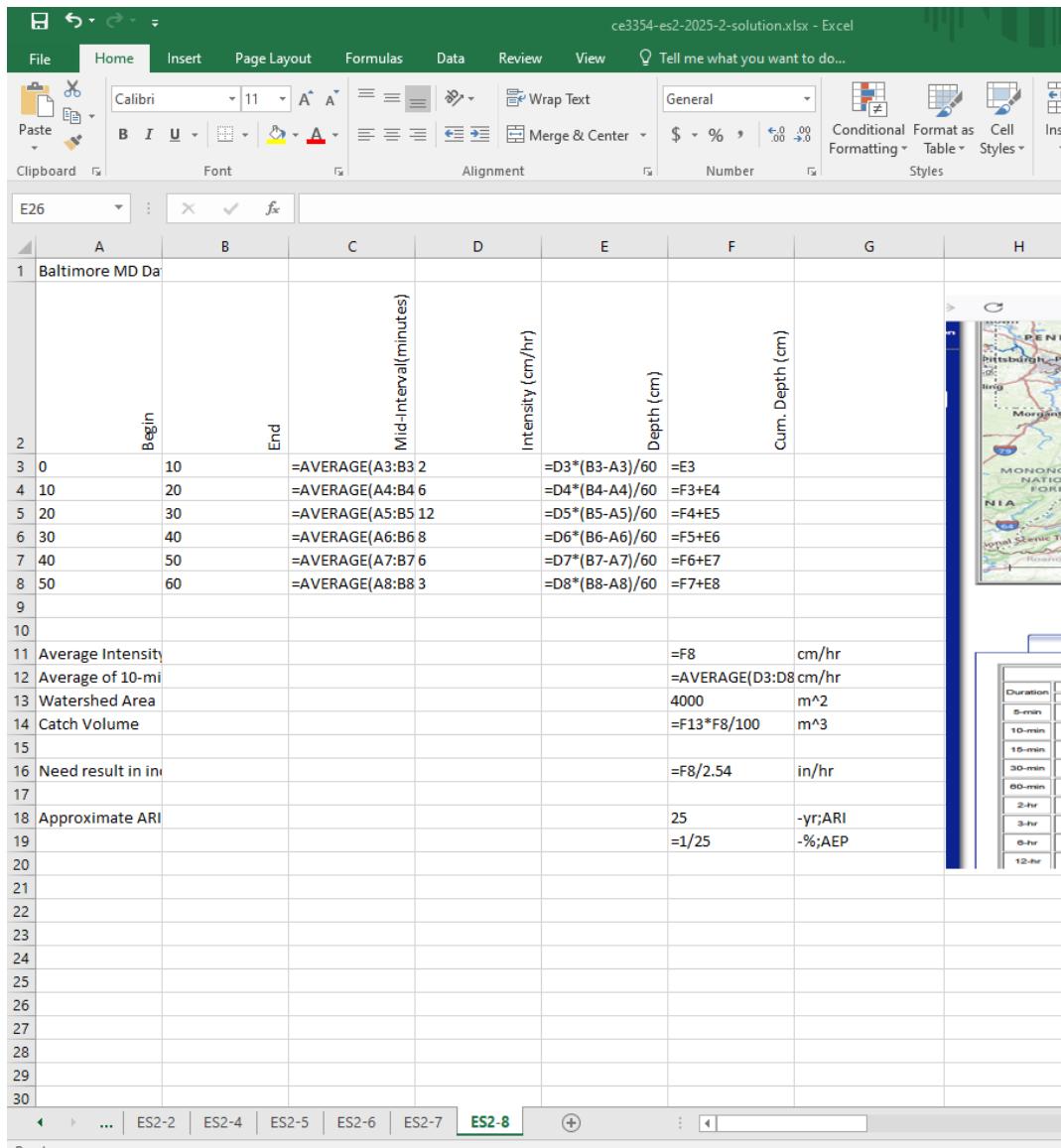


Figure 11: Baltimore, MD intensity data analysis (Excel formulas)

9. The map in Figure 12 shows the location of 6 rain gages and a watershed boundary. The rainfall depths for a certain storm are shown by each gage. An isohyetal map is displayed on the figure as is a linear distance scale.

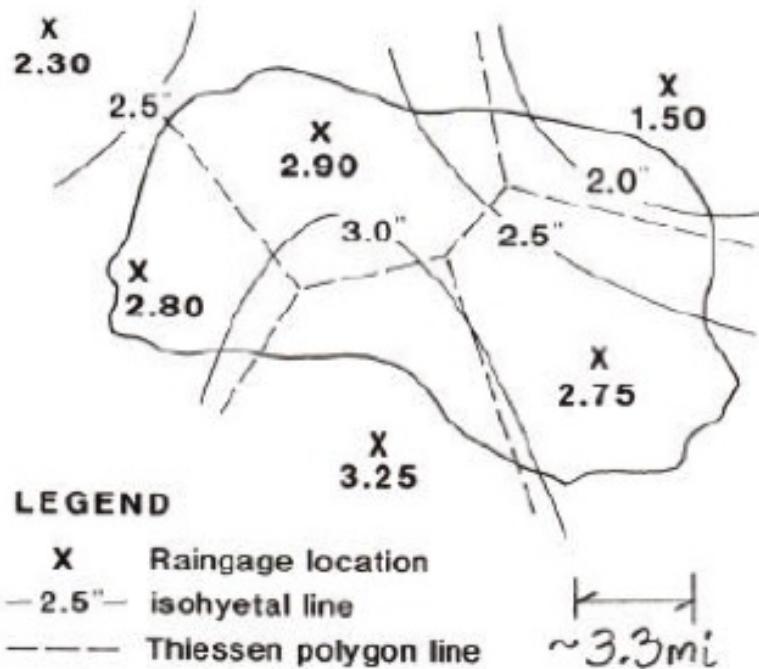


Figure 12: Raingages

Determine:

- The Thissen polygon boundaries – verify if the gage in the upper left corner is included in the polygon boundaries in the picture (i.e determine your own boundaries for the gages, do they agree with the drawing?).
- The polygon areas and compute the Thissen weights.
- The average weighted precipitation over the watershed (using the Thissen weights).
- The average weighted precipitation (using the Isohyets).

Solution(s):

For this problem the solution documents workflow using ‘ImageJ’ software, which is free to download. It has way more useful stuff than simple map measuring, but reflective of the kind of software that will have some durability.

- 1) Download and extract, install using the imageJ.exe file in the download folder, when extracted, move the folder to somewhere on your computer (or leave in Downloads).

Platform Independent
To install ImageJ on a computer with Java pre-installed, or to upgrade to the latest full distribution (including macros, plugins and LUTs), download the [ZIP archive](#) (6MB) and extract the ImageJ directory. Use the [Help>Update ImageJ](#) command to upgrade to newer versions.

Mac OS X
Download [ImageJ bundled with Java 8](#) (may need to work around Path Randomization). Instructions. With M1 (ARM) Macs, download [ImageJ bundled with Zulu OpenJDK 13.0.6](#).

Linux
Download [ImageJ bundled with Java 8](#). Instructions.

Windows
Download [ImageJ bundled with 64-bit Java 8](#). Instructions.

Documentation
Tiago Ferreira's comprehensive [ImageJ User Guide](#) is available as an 8MB PDF document and as a [ZIP archive](#). The online [JavaDoc API documentation](#) is also available as a [ZIP archive](#).

Source Code
The ImageJ [Java source](#) consists of 160,000 lines of code in 380 files ([LineCounter macro](#)). It is available [online](#) and as [zip archives](#).

Example Images
31 downloadable sample images and stacks are available in ImageJ's [File>Open Samples](#) submenu. These images, and more, are also available as a [3.2MB zip archive](#).

You can also browse the ImageJ download directory at imagej.net/ij/download/. Newer ImageJ distributions are available at <http://wsr.imagej.net/distros/>. Refer to the [Release Notes](#) for a list of new features and bug fixes.

[top](#) | [home](#) | [docs](#) | [download](#) | [plugins](#) | [resources](#) | [list](#) | [links](#)

Figure 13: ImageJ Download Landing Page (note the URL)

- 2) Start ImageJ and open the image file (in this case clip from the assignment)
Calibrate scale using a known distance on the map (in **Analyze** menu item set the scale after drawing a line on the 3.3 mi. scale bar. In the Set Scale dialog, enter 3.3 as the known distance)
- 3) Use **Polygon** tool to draw and measure various polygons.

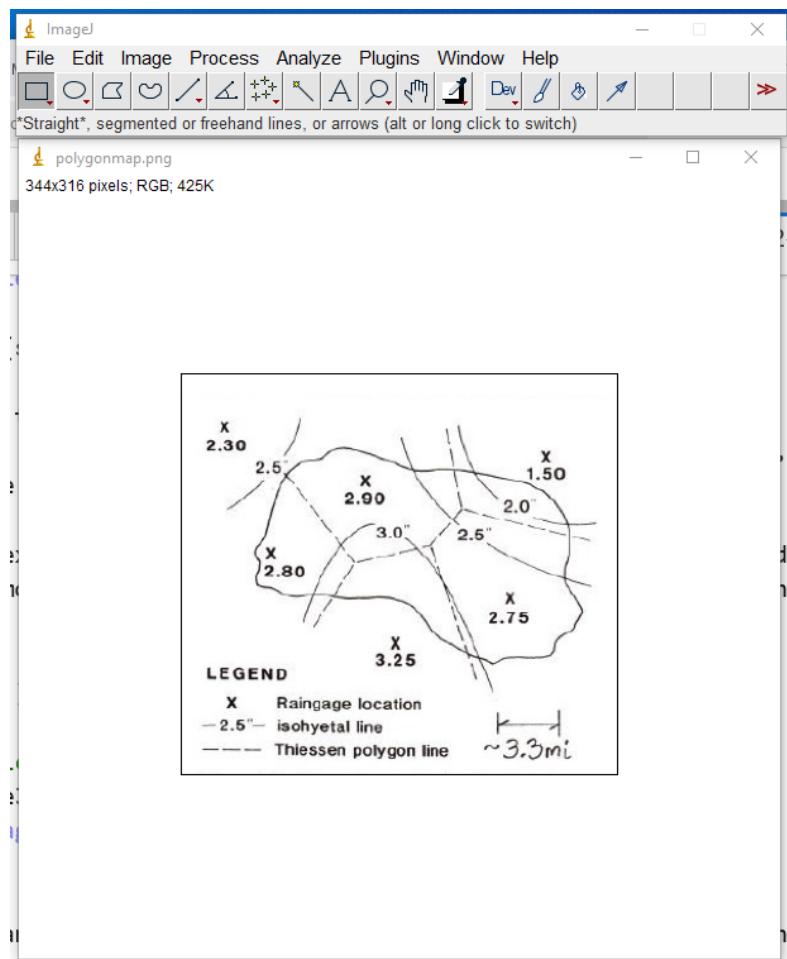


Figure 14: Image to process in ImageJ tool.

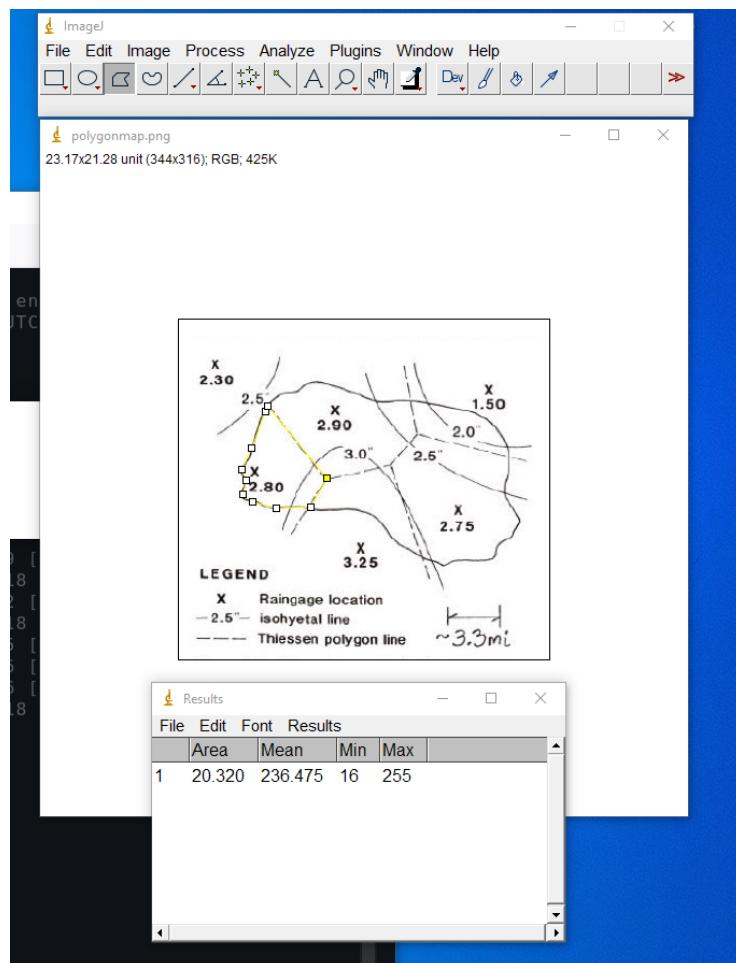


Figure 15: Polygon drawn and measurement completed.

- 4) Repeat 3) above until have the entire polygon field, record in a spreadsheet.
 a), b), c), d) are shown in Figures 16 and 17 using measurements made in ImageJ by the workflow depicted above.

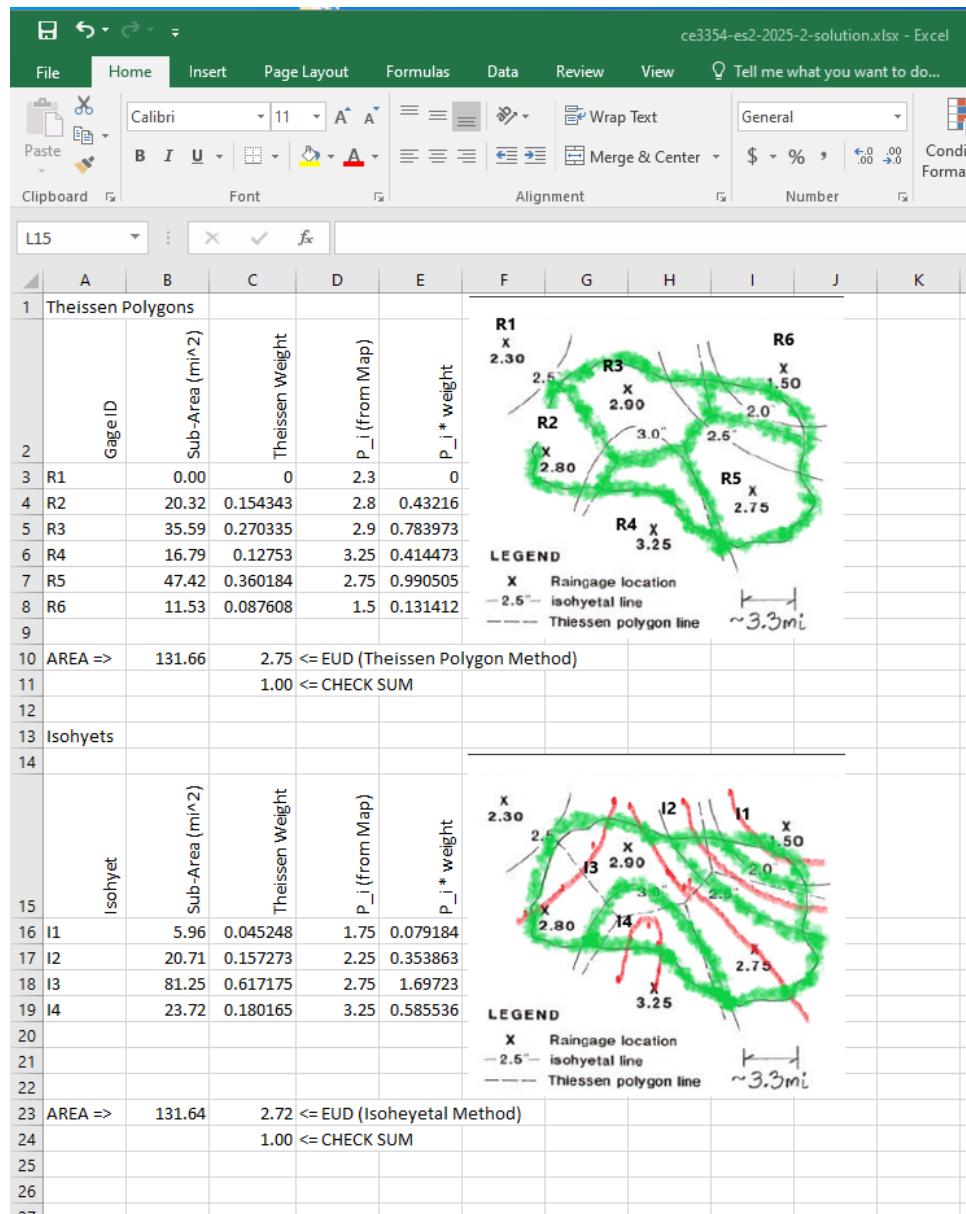


Figure 16: Excel spreadsheet showing EUD calculations.

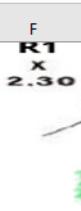
| | A | B | C | D | E | F |
|----|----------|-----------------|-----------------|---------------------|--------------|---|
| | Gage ID | Sub-Area (mi^2) | Theissen Weight | P_i (from Map) | P_i * weight | |
| 2 | | | | | | |
| 3 | R1 | 0 | =B3/\$B\$10 | 2.3 | =D3*C3 |  |
| 4 | R2 | 20.32 | =B4/\$B\$10 | 2.8 | =D4*C4 | |
| 5 | R3 | 35.591 | =B5/\$B\$10 | 2.9 | =D5*C5 | |
| 6 | R4 | 16.79 | =B6/\$B\$10 | 3.25 | =D6*C6 | |
| 7 | R5 | 47.42 | =B7/\$B\$10 | 2.75 | =D7*C7 | |
| 8 | R6 | 11.534 | =B8/\$B\$10 | 1.5 | =D8*C8 | |
| 9 | | | | | | |
| 10 | AREA => | =SUM(B3:B8) | =SUM(E3:E8) | <= EUD (Theissen Po | | |
| 11 | | | =SUM(C3:C8) | <= CHECK SUM | | |
| 12 | | | | | | |
| 13 | Isohyets | | | | | |
| 14 | | | | | | |
| 15 | | | | | | |
| 16 | I1 | 5.956216 | =B16/\$B\$23 | 1.75 | =D16*C16 |  |
| 17 | I2 | 20.70572 | =B17/\$B\$10 | 2.25 | =D17*C17 | |
| 18 | I3 | 81.25411 | =B18/\$B\$10 | 2.75 | =D18*C18 | |
| 19 | I4 | 23.719598 | =B19/\$B\$10 | 3.25 | =D19*C19 | |
| 20 | | | | | | |
| 21 | | | | | | |
| 22 | | | | | | |
| 23 | AREA => | =SUM(B16:B21) | =SUM(E16:E21) | <= EUD (Isohyetal N | | |
| 24 | | | =SUM(C16:C21) | <= CHECK SUM | | |
| 25 | | | | | | |
| 26 | | | =AVERAGE(D3:D8) | <=EUD (Arithmetic M | | |
| 27 | | | | | | |
| 28 | | | | | | |

Figure 17: Excel spreadsheet showing EUD calculations (formulas)

10. The map in Figure 18 shows the location of 8 rain gages and the watershed boundary. The rainfall depths for a certain storm are in Table 6. Use the Thiessen polygon method to determine the mean rainfall depth over the watershed for this storm event.

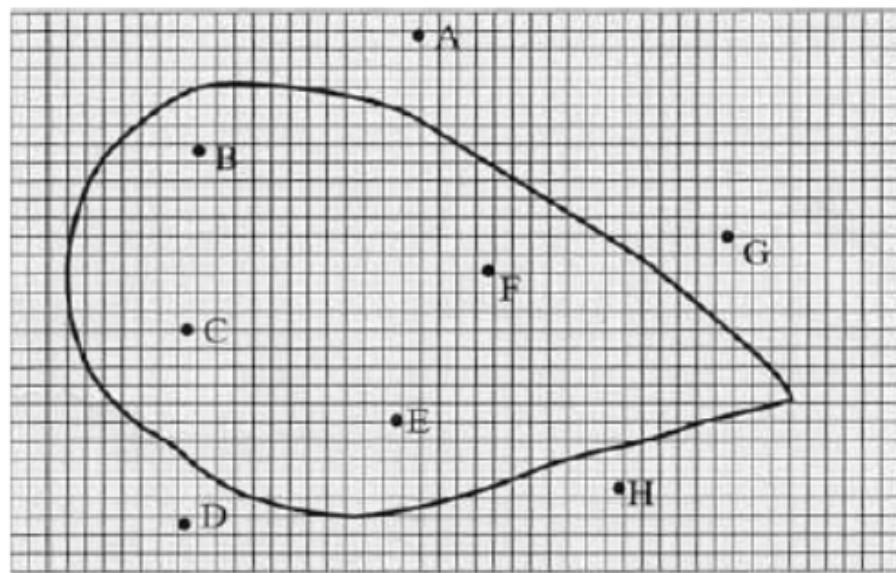


Figure 18: Nowhere Watershed Active Raingages

Table 6: Nowhere Watershed Precipitation

| Gage | Cumulative Depth (millimeters) |
|------|--------------------------------|
| A | 25.00 |
| B | 18.00 |
| C | 92.00 |
| D | 95.00 |
| E | 192.0 |
| F | 175.0 |
| G | 152.0 |
| H | 168.0 |

Determine:

- The mean rainfall depth over the watershed for this storm event using the arithmetic mean.
- The mean rainfall depth over the watershed for this storm event using the Thiessen polygon method.

Solution(s):

Again using ImageJ software to measure areas the results are displayed on Figures 19 and 20 below.

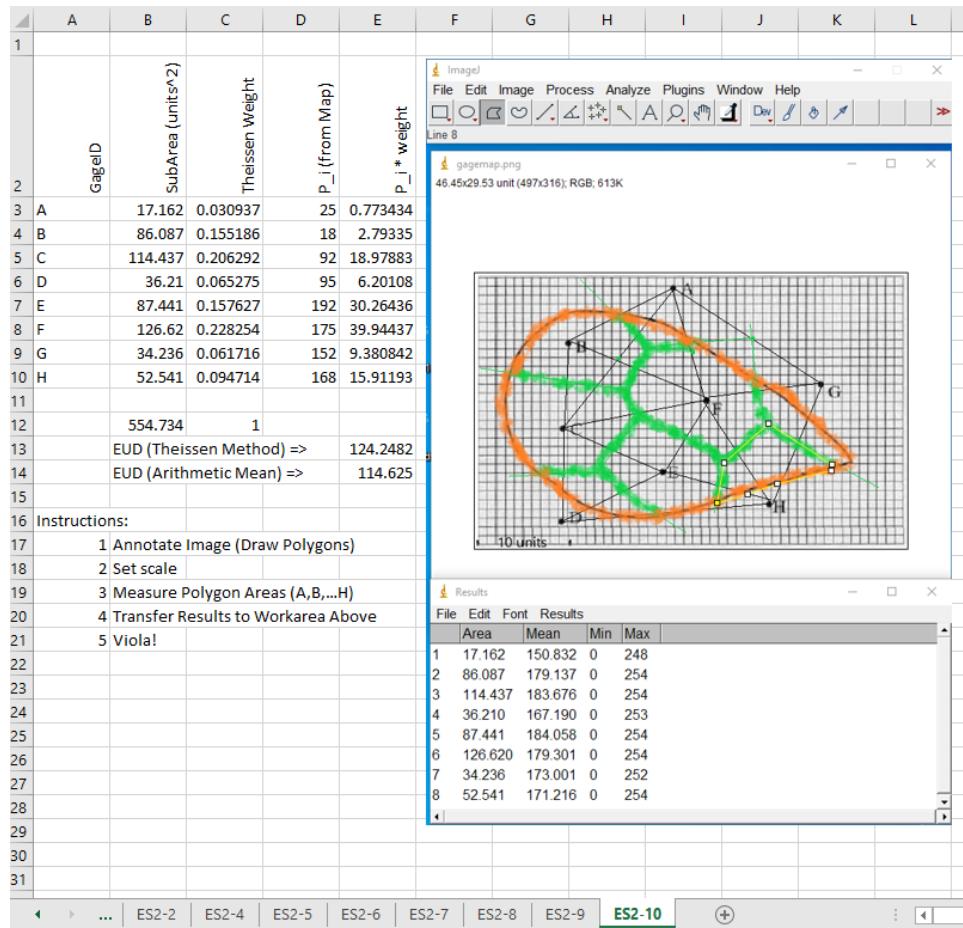


Figure 19: Excel spreadsheet showing EUD calculations.

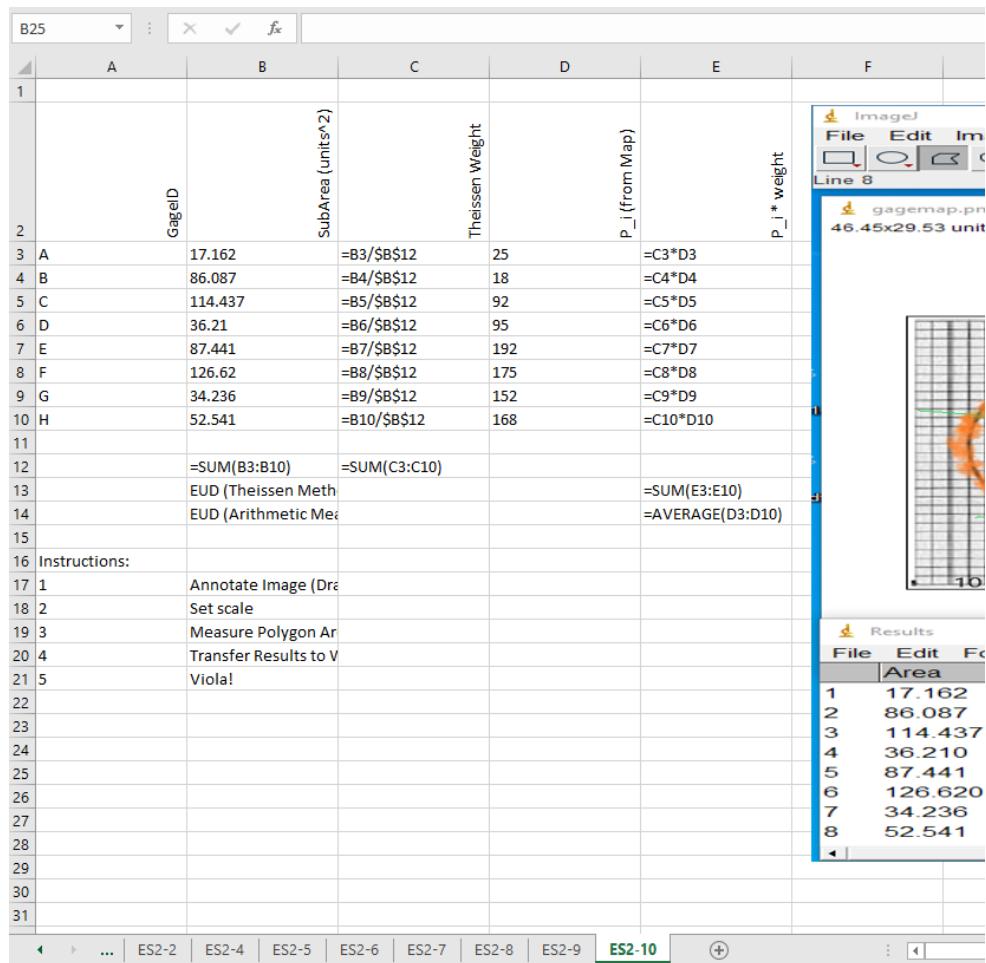


Figure 20: Excel spreadsheet showing EUD calculations (formulas)

11. The map excerpt in Figure 21, shows a stream gage labeled as U.S.G.S. no. 1. Various rain gages are shown as rectangles surrounding the catch for the gage for some time interval.

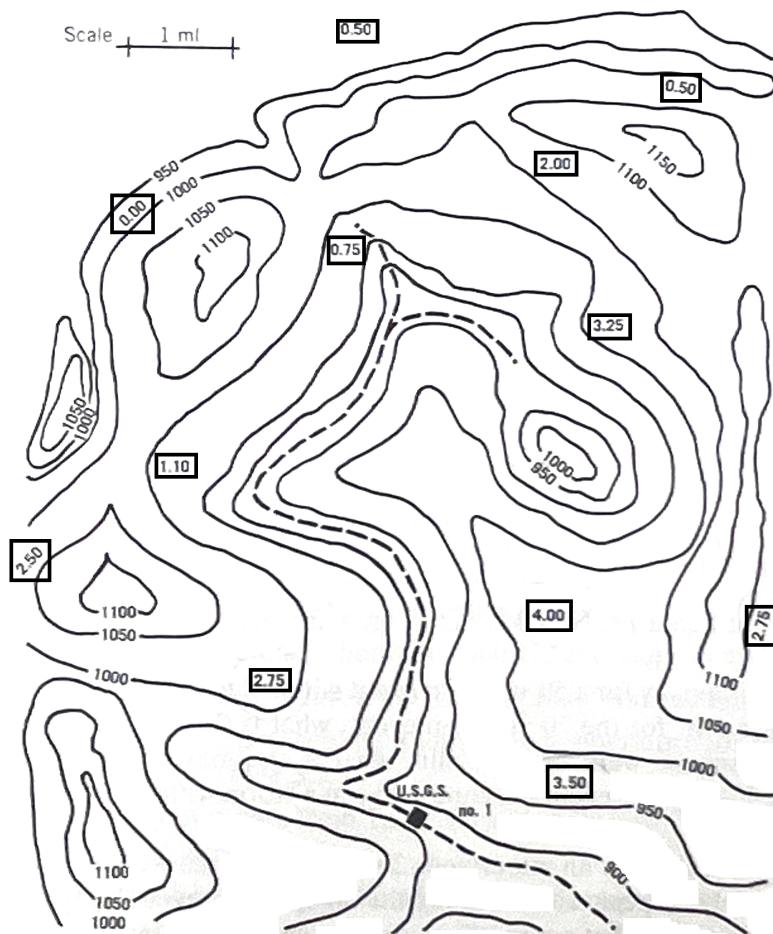


Figure 21: U.S.G.S. No. 1 Area Map

Determine:

- The drainage area boundary using watershed delineation principles.
- The drainage area in square miles.
- The average precipitation over the area by arithmetic mean.
- The average precipitation over the area by Thiessen polygon method.

Solution(s):

Again using ImageJ software to measure areas the results are displayed on Figures 22 and 23 below.

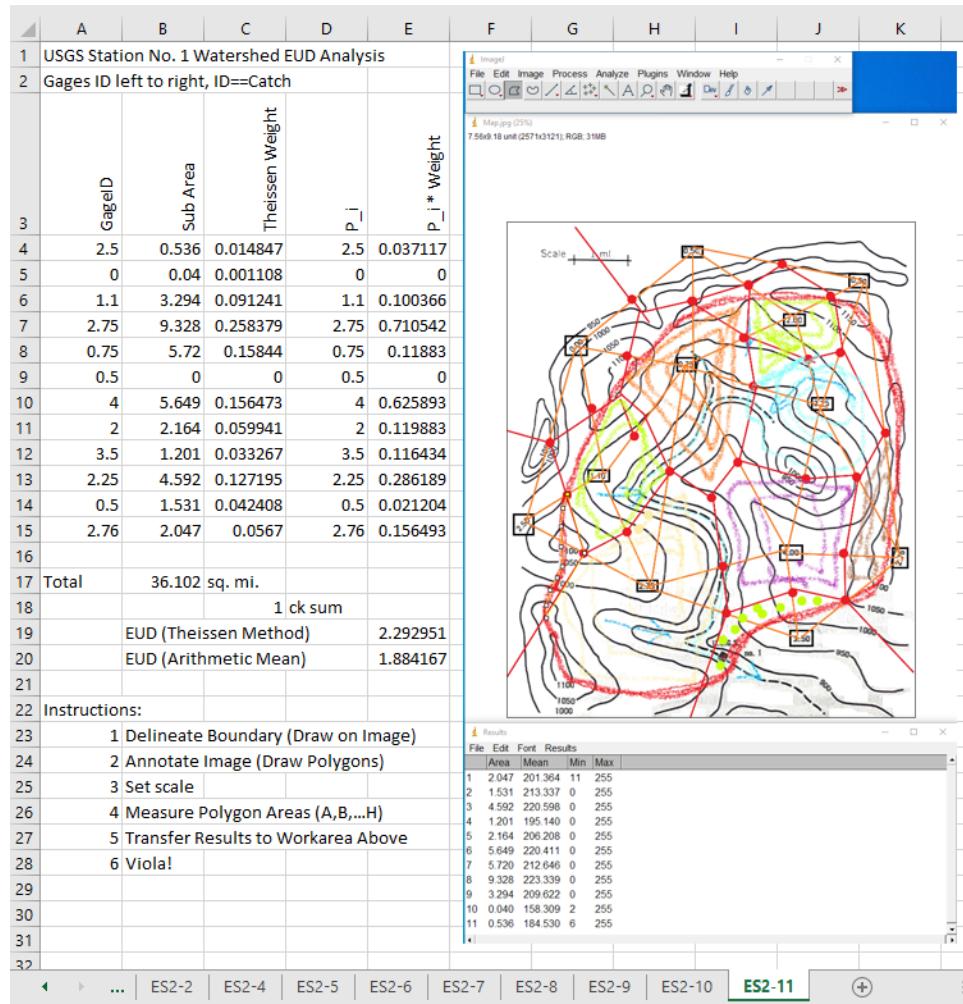


Figure 22: Excel spreadsheet showing EUD calculations.

| | A | B | C | D | E | F |
|----|------------------------|-----------------------|-----------------|--------|------------------|-------------------------|
| 1 | USGS Station No. 1 V | | | | | File Edit |
| 2 | Gages ID left to right | | | | | Map.jpg 7.56x9.18 ur |
| 3 | GageID | Sub Area | Theissen Weight | P_i | P_i * Weight | |
| 4 | 2.5 | 0.536 | =B4/\$B\$17 | 2.5 | =D4*C4 | |
| 5 | 0 | 0.04 | =B5/\$B\$17 | 0 | =D5*C5 | |
| 6 | 1.1 | 3.294 | =B6/\$B\$17 | 1.1 | =D6*C6 | |
| 7 | 2.75 | 9.328 | =B7/\$B\$17 | 2.75 | =D7*C7 | |
| 8 | 0.75 | 5.72 | =B8/\$B\$17 | 0.75 | =D8*C8 | |
| 9 | 0.5 | 0 | =B9/\$B\$17 | 0.5 | =D9*C9 | |
| 10 | 4 | 5.649 | =B10/\$B\$17 | 4 | =D10*C10 | |
| 11 | 2 | 2.164 | =B11/\$B\$17 | 2 | =D11*C11 | |
| 12 | 3.5 | 1.201 | =B12/\$B\$17 | 3.5 | =D12*C12 | |
| 13 | 2.25 | 4.592 | =B13/\$B\$17 | 2.25 | =D13*C13 | |
| 14 | 0.5 | 1.531 | =B14/\$B\$17 | 0.5 | =D14*C14 | |
| 15 | 2.76 | 2.047 | =B15/\$B\$17 | 2.76 | =D15*C15 | |
| 16 | | | | | | |
| 17 | Total | =SUM(B4:B15) | sq. mi. | | | |
| 18 | | | =SUM(C4:C15) | ck sum | | |
| 19 | | EUD (Theissen Meth) | | | =SUM(E4:E15) | |
| 20 | | EUD (Arithmetic Mea | | | =AVERAGE(D4:D15) | |
| 21 | | | | | | |
| 22 | Instructions: | | | | | |
| 23 | 1 | Delineate Boundary | | | | |
| 24 | 2 | Annotate Image (Draw) | | | | |
| 25 | 3 | Set scale | | | | |
| 26 | 4 | Measure Polygon Area | | | | |
| 27 | 5 | Transfer Results to V | | | | |
| 28 | 6 | Viola! | | | | |
| 29 | | | | | | |
| 30 | | | | | | |
| 31 | | | | | | |
| 32 | | | | | | |

Figure 23: Excel spreadsheet showing EUD calculations (formulas)