CE 3354 Engineering Hydrology
Exercise Set 1

Exercises

1. The mean annual precipitation for a certain 132 square-mile watershed is 26 inches. Assume that 21 percent of the annual precipitation reaches the watershed outlet as streamflow.

   Determine:

   a) The mean streamflow rate in acre-feet per year.

   b) The mean streamflow rate in cubic-feet per second.

   Solution(s):

	A	B	C
1	Streamflow from Precipitation		
2			
3	**Known:**		
4	**Quantity**	**Value**	**Units**
5	Watershed area	132	square miles
6	Mean annual precipitation	26	inches/year
7	Runoff coefficient	0.21	-
8			
9	**Unknown:**		
10	**Quantity**	**Value**	**Units**
11	Area	84480	acres
12	Precipitation Depth	2.17	feet
13	Precipitation Volume	183040	acre-feet
14			
15	Mean Streamflow Rate	38438	acre-ft/year
16	Mean Streamflow Rate	53.01	cfs
17	Mean Streamflow Rate	1.50	cms

Figure 1: Water Budget Spreadsheet

	A	B	C	D
1	Streamflow from Precipitation			
2				
3	**Known:**			
4	**Quantity**	**Value**	**Units**	
5	Watershed area	132	square miles	
6	Mean annual precipitation	26	inches/year	
7	Runoff coefficient	=21/100	-	
8				
9	**Unknown:**			
10	**Quantity**	**Value**	**Units**	
11	Area	=640*B5	acres	
12	Precipitation Depth	=B6/12	feet	
13	Precipitation Volume	=B11*B12	acre-feet	
14				
15	Mean Streamflow Rate	=B7*B13	acre-ft/year	
16	Mean Streamflow Rate	=B15*43560/(365	cfs	
17	Mean Streamflow Rate	=B16/(3.28^3)	cms	
18				
19				
20				
21				
22				
23				

Figure 2: Water Budget Spreadsheet Formulas

2. Figure 3 is a schematic of a 600-hectare farm; the land receives annual rainfall of 2500 mm. There is a river flowing through the farm land with inflow rate of 5 m^3/s and outflow rate of $4 m^3/s$. The annual water storage in the farm land increases by $2.5 \times 10^6 m^3$. Using the water budget concept, estimate the annual evaporation amount in millimeters.[1]

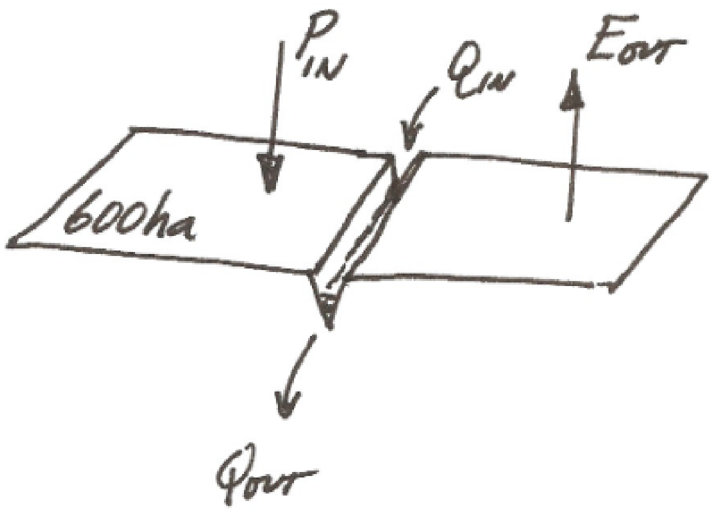


Figure 3: Schematic of Farmland

Solution(s):

[1] 1 hectare = 10,000 m^2

	A	B	C	D
1	A River Runs Through It			
2				
3	**Known:**			
4	**Quantity**	**Value**	**Units**	
5	Watershed area	600	hectares	
6	Mean annual precipitation	2500	mm	
7	River Inflow	5	m^3/s	
8	River Outflow	4	m^3/s	
9	Annual Storage Change	2.50E+06	m^3	
10				
11	**Unknown(s):**			
12	**Quantity**	**Value**	**Units**	
13	Watershed area	6000000	m^2	
14	Precip Volume	15000000	m^3	
15	River Inflow-Outflow	1	m^3/s	
16	River Inflow-Outflow	31536000	m^3	
17	Evaporation	4.40E+07	m^3	
18	Evaporation Depth	7.34	m	
19	Evaporation Depth	7339.33	mm	
20				

Figure 4: Water Budget Spreadsheet

	A	B	C	D
1	A River Runs Through It			
2				
3	**Known:**			
4	**Quantity**	**Value**	**Units**	
5	Watershed area	600	hectares	
6	Mean annual precipitation	2500	mm	
7	River Inflow	5	m^3/s	
8	River Outflow	4	m^3/s	
9	Annual Storage Change	2500000	m^3	
10				
11	**Unknown(s):**			
12	**Quantity**	**Value**	**Units**	
13	Watershed area	=B5*10000	m^2	
14	Precip Volume	=(B6/1000)*B13	m^3	
15	River Inflow-Outflow	=B7-B8	m^3/s	
16	River Inflow-Outflow	=86400*365*B15	m^3	
17	Evaporation	=B14+B16-B9	m^3	
18	Evaporation Depth	=B17/B13	m	
19	Evaporation Depth	=B18*1000	mm	
20				

Figure 5: Water Budget Spreadsheet Formulas

3. A reservoir has a surface area of 690 acres. Figure 6 shows the monthly inflow of surface water, outflows as releases from the reservoir via the spillway, direct precipitation into the reservoir, and evaporation from the reservoir. The reservoir water surface elevation was 701.0 feet on January 1. Determine the reservoir water surface elevation at the end of each month (i.e. complete the table)

Lake Woodlands									
Average Surface Area = 690 acres									
Month	Inflow (acre-feet)	Outflow (acre-feet)	Precipitation (inches)	Precipitation (acre-feet)	Evaporation (inches)	Evaporation (acre-feet)	Storage Net Change (acre-feet)	Elevation Change (feet)	Water Surface Elevation (feet)
December									701.00
January	1732	175	2.75	158.13	1.05	60.38	1654.75	2.40	703.40
February	1755	190	3.05		1.55				
March	872	232	3.76		2.05				
April	955	375	4.11		2.80				
May	708	525	2.70		3.75				
June	312	955	1.05		4.25				
July	102	1720	0.75		5.15				
August	37	2250	1.25		5.76				
September	175	1575	1.55		4.92				
October	575	550	3.79		3.02				
November	1250	175	4.53		1.75				
December	1875	125	5.01		0.60				

Figure 6: Tabular Water Budget Values

	A	B	C	D	E	F	G	H	I	J
1	Lake Woodlands									
2	Average Surface Area = 690 acres									
3	Month	Inflow (acre-feet)	Outflow (acre-feet)	Precipitation (inches)	Precipitation (acre-feet)	Evaporation (inches)	Evaporation (acre-feet)	Storage Net Change (acre-feet)	Elevation Change (feet)	Water Surface Elevation (feet)
4	December									701.00
5	January	1732	175	2.75	158.13	1.05	60.38	1654.75	2.40	703.40
6	February	1755	190	3.05	175.38	1.55	89.13	1651.25	2.39	705.79
7	March	872	232	3.76	216.20	2.05	117.88	738.33	1.07	706.86
8	April	955	375	4.11	236.33	2.80	161.00	655.33	0.95	707.81
9	May	708	525	2.70	155.25	3.75	215.63	122.63	0.18	707.99
10	June	312	955	1.05	60.38	4.25	244.38	-827.00	-1.20	706.79
11	July	102	1720	0.75	43.13	5.15	296.13	-1871.00	-2.71	704.08
12	August	37	2250	1.25	71.88	5.76	331.20	-2472.33	-3.58	700.50
13	September	175	1575	1.55	89.13	4.92	282.90	-1593.78	-2.31	698.19
14	October	575	550	3.79	217.93	3.02	173.65	69.28	0.10	698.29
15	November	1250	175	4.53	260.48	1.75	100.63	1234.85	1.79	700.08
21		Example Row showing formula entries for water balance								
22	January	1732	175	2.75	=D5*690/12	1.05	=F5*690/12	=B5+E5-C5-G5	=H5/690	=J4+I5
23										

Figure 7: Water Budget Spreadsheet for 690 acre Reservoir - results and representative record with formulas

4. The equation $k\frac{dQ}{dt} + Q(t) = I(t)$ is used to describe the response of streamflow to a constant rate of precipitation applied indefinitely on some watershed. Suppose that $Q(0) = 0$ and the watershed characteristic time constant is $k = 2$ hrs. $I(t) = 2$ for $t = [0, 12)$ hrs and then $I(t) = 0$ for $t = [12, 24]$ hrs.

   Determine:

   (a) Plot the values of $I(t)$ over the 24-hour perios, in 1-hour increments.

   (b) The necessary equation(s) to predict the response $Q(t)$ over the 24-hour period.

   (c) Plot the values of $Q(t)$ over the 24-hour period, in 1-hour increments.

   Solution(s):

   Use calculus, and by separation and integration determine the following expression

   Define inflow $I(t)$ and outflow $Q(t)$ as:

$$I(t) = \begin{cases} 0 & \text{for } t < 0 \\ 2 & \text{for } 0 \le t < \tau \\ 0 & \text{for } t \ge \tau \end{cases}$$

$$Q(t) = \begin{cases} 0 & \text{for } t \le 0 \\ 2\left[1 - e^{-t/k}\right] & \text{for } 0 < t \le \tau \\ 2\left[1 - e^{-t/k}\right] - 2\left[1 - e^{-(t-\tau)/k}\right] & \text{for } t > \tau \end{cases}$$

   where $\tau = 12$ hours and k is the reservoir coefficient (e.g., in hours).

   Alternatively create a finite-difference approximation as

   $k\frac{dQ}{dt} + Q(t) = I(t)$

   Move $Q(t)$ to RHS and divide by k

   $\frac{dQ}{dt} = \frac{I(t)-Q(t)}{k}$

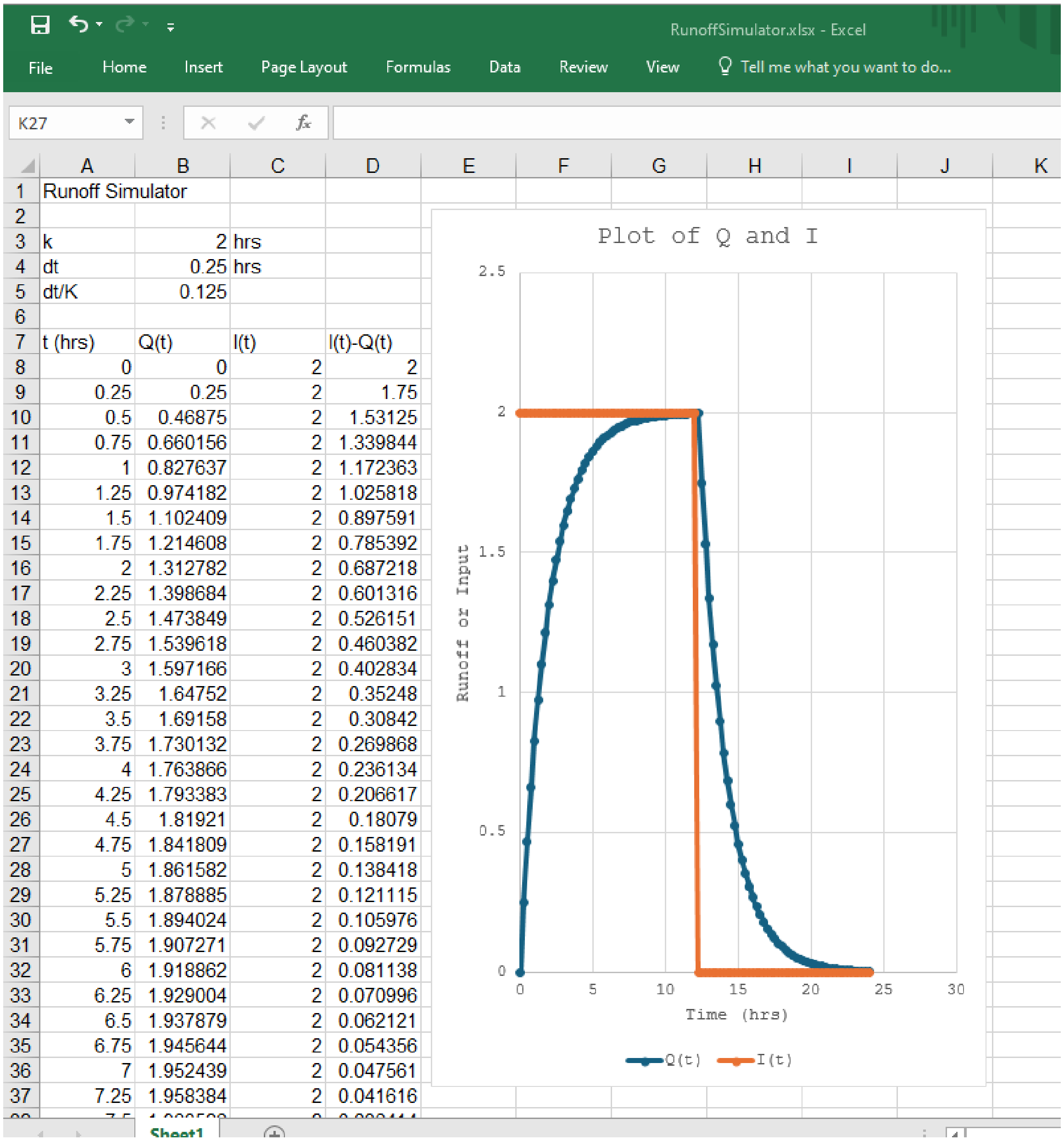
   Then express the LHS as a difference quotient

   $\frac{Q(t+\Delta t)-Q(t)}{\Delta t} = \frac{I(t)-Q(t)}{k}$

   Isolate everything at the old time step to the RHS

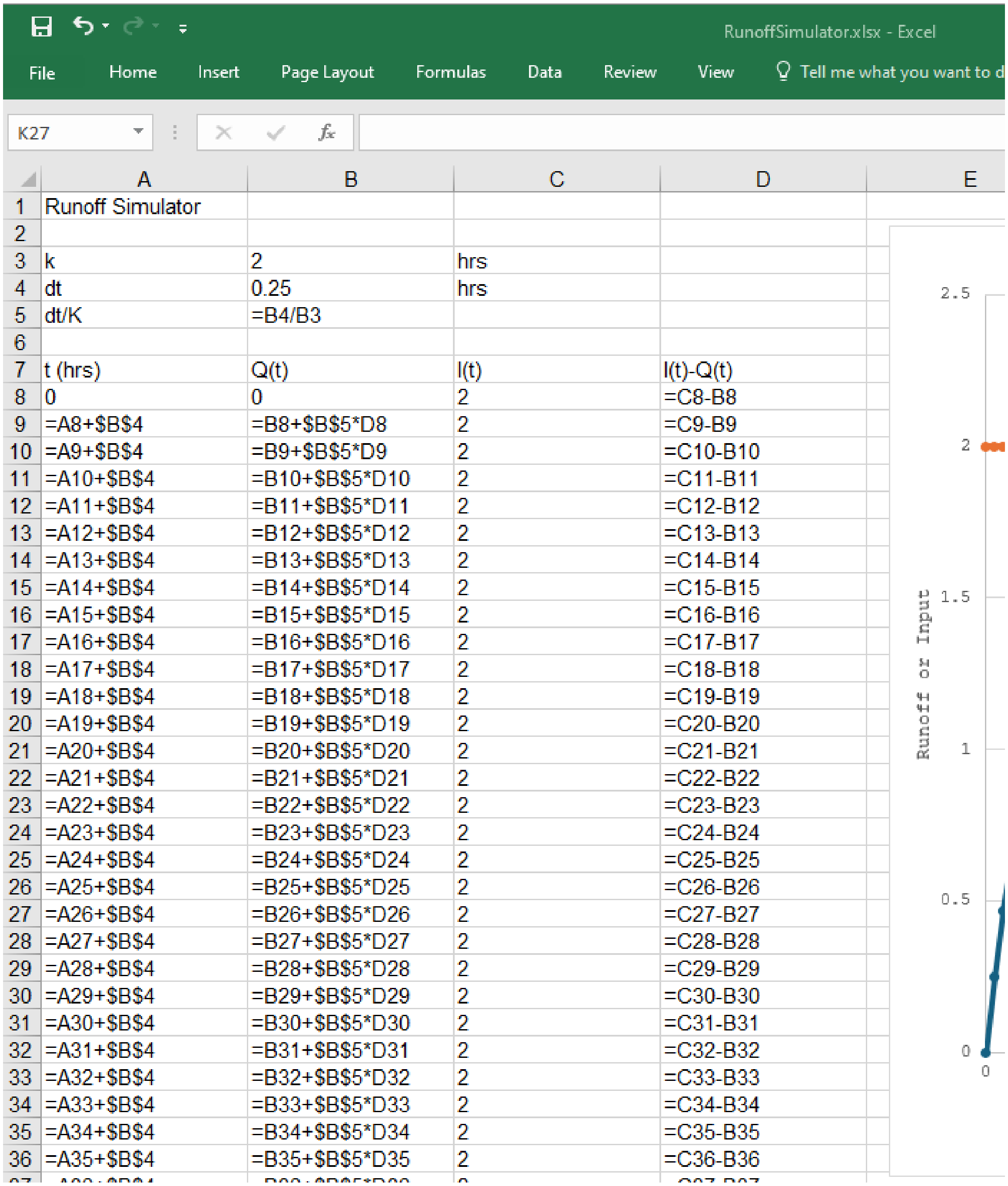
   $Q(t + \Delta t) = Q(t) + \frac{\Delta t}{k}(I(t) - Q(t))$

   Code up into a spreadsheet, choose a small enough value of Δt and proceede.



	A	B	C	D
1	Runoff Simulator			
2				
3	k	2	hrs	
4	dt	0.25	hrs	
5	dt/K	0.125		
6				
7	t (hrs)	Q(t)	I(t)	I(t)-Q(t)
8	0	0	2	2
9	0.25	0.25	2	1.75
10	0.5	0.46875	2	1.53125
11	0.75	0.660156	2	1.339844
12	1	0.827637	2	1.172363
13	1.25	0.974182	2	1.025818
14	1.5	1.102409	2	0.897591
15	1.75	1.214608	2	0.785392
16	2	1.312782	2	0.687218
17	2.25	1.398684	2	0.601316
18	2.5	1.473849	2	0.526151
19	2.75	1.539618	2	0.460382
20	3	1.597166	2	0.402834
21	3.25	1.64752	2	0.35248
22	3.5	1.69158	2	0.30842
23	3.75	1.730132	2	0.269868
24	4	1.763866	2	0.236134
25	4.25	1.793383	2	0.206617
26	4.5	1.81921	2	0.18079
27	4.75	1.841809	2	0.158191
28	5	1.861582	2	0.138418
29	5.25	1.878885	2	0.121115
30	5.5	1.894024	2	0.105976
31	5.75	1.907271	2	0.092729
32	6	1.918862	2	0.081138
33	6.25	1.929004	2	0.070996
34	6.5	1.937879	2	0.062121
35	6.75	1.945644	2	0.054356
36	7	1.952439	2	0.047561
37	7.25	1.958384	2	0.041616

Figure 8: Runoff Simulator Spreadsheet



	A	B	C	D
1	Runoff Simulator			
2				
3	k	2	hrs	
4	dt	0.25	hrs	
5	dt/K	=B4/B3		
6				
7	t (hrs)	Q(t)	I(t)	I(t)-Q(t)
8	0	0	2	=C8-B8
9	=A8+$B$4	=B8+$B$5*D8	2	=C9-B9
10	=A9+$B$4	=B9+$B$5*D9	2	=C10-B10
11	=A10+$B$4	=B10+$B$5*D10	2	=C11-B11
12	=A11+$B$4	=B11+$B$5*D11	2	=C12-B12
13	=A12+$B$4	=B12+$B$5*D12	2	=C13-B13
14	=A13+$B$4	=B13+$B$5*D13	2	=C14-B14
15	=A14+$B$4	=B14+$B$5*D14	2	=C15-B15
16	=A15+$B$4	=B15+$B$5*D15	2	=C16-B16
17	=A16+$B$4	=B16+$B$5*D16	2	=C17-B17
18	=A17+$B$4	=B17+$B$5*D17	2	=C18-B18
19	=A18+$B$4	=B18+$B$5*D18	2	=C19-B19
20	=A19+$B$4	=B19+$B$5*D19	2	=C20-B20
21	=A20+$B$4	=B20+$B$5*D20	2	=C21-B21
22	=A21+$B$4	=B21+$B$5*D21	2	=C22-B22
23	=A22+$B$4	=B22+$B$5*D22	2	=C23-B23
24	=A23+$B$4	=B23+$B$5*D23	2	=C24-B24
25	=A24+$B$4	=B24+$B$5*D24	2	=C25-B25
26	=A25+$B$4	=B25+$B$5*D25	2	=C26-B26
27	=A26+$B$4	=B26+$B$5*D26	2	=C27-B27
28	=A27+$B$4	=B27+$B$5*D27	2	=C28-B28
29	=A28+$B$4	=B28+$B$5*D28	2	=C29-B29
30	=A29+$B$4	=B29+$B$5*D29	2	=C30-B30
31	=A30+$B$4	=B30+$B$5*D30	2	=C31-B31
32	=A31+$B$4	=B31+$B$5*D31	2	=C32-B32
33	=A32+$B$4	=B32+$B$5*D32	2	=C33-B33
34	=A33+$B$4	=B33+$B$5*D33	2	=C34-B34
35	=A34+$B$4	=B34+$B$5*D34	2	=C35-B35
36	=A35+$B$4	=B35+$B$5*D35	2	=C36-B36

Figure 9: Runoff Simulator Spreadsheet with Formulas