CE 3354 Engineering Hydrology
Exercise Set 3

Exercises

1. Estimate the monthly evapotranspiration depths for Dallas (Tarrant County), Houston (Harris County), and San Angelo (Concho County) area using the Blaney-Criddle method.[1]

   Solution(s):

   We will need some city specific geographic data

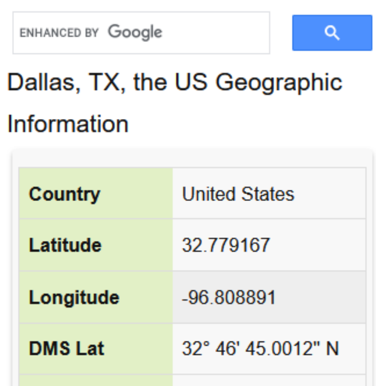


Figure 1: Latitude for Dallas, Texas

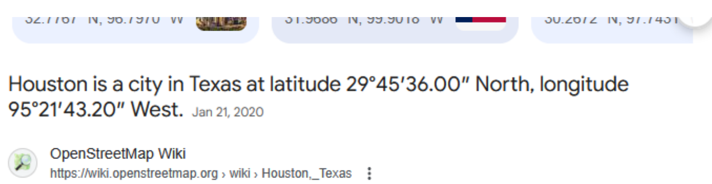


Figure 2: Latitude for Houston, Texas

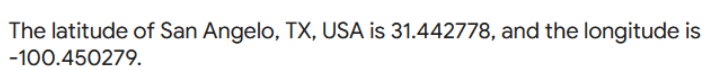


Figure 3: Latitude for San Angelo, Texas

   Then some temperature data

[1] A Google search should get you sufficient guidance to perform this exercise.

weatherspark.com/y/8813/Average-Weather-in-Dallas-Texas-United-States-Year-Round

Average	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
High	57°F	62°F	69°F	77°F	84°F	91°F	95°F	95°F	88°F	78°F	67°F	58°F
Temp.	47°F	50°F	58°F	66°F	74°F	82°F	85°F	86°F	78°F	68°F	57°F	48°F
Low	39°F	43°F	50°F	58°F	66°F	73°F	77°F	76°F	69°F	59°F	49°F	41°F

Figure 4: Mean Monthly Air Temperature for Dallas, Texas

weatherspark.com/y/9247/Average-Weather-in-Houston-Texas-United-States-Year-Round

The daily average high (red line) and low (blue line) temperature, with 25th to 75th and 10th to 90th percentile bands. The thin dotted lines are the corresponding average perceived temperatures.

Average	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
High	64°F	68°F	74°F	79°F	86°F	91°F	94°F	94°F	90°F	82°F	73°F	66°F
Temp.	54°F	57°F	63°F	70°F	77°F	82°F	84°F	85°F	80°F	73°F	63°F	56°F
Low	47°F	50°F	56°F	63°F	70°F	75°F	77°F	77°F	73°F	65°F	56°F	49°F

Figure 5: Mean Monthly Air Temperature for Houston, Texas

weatherspark.com/y/5240/Average-Weather-in-San-Angelo-Texas-United-States-Year-Round

percentile bands. The thin dotted lines are the corresponding average perceived temperatures.

Average	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
High	60°F	66°F	73°F	81°F	88°F	92°F	95°F	94°F	88°F	79°F	69°F	61°F
Temp.	46°F	50°F	58°F	67°F	74°F	81°F	84°F	83°F	76°F	66°F	55°F	47°F
Low	36°F	40°F	47°F	55°F	64°F	70°F	73°F	72°F	65°F	56°F	46°F	37°F

Figure 6: Mean Monthly Air Temperature for San Angelo, Texas

Now have data to employ the spreadsheet implementation; we should probably note units on the spreadsheet - based on class notes the values ET_o are in millimeters per day, so monthly depths would be these values multiplied 30 (for typical month length)

	A	B	C	D	E	F	G	H	I	J
1	Blaney-Criddle ET Estimator									
2	Location:	Dallas, Texas (Closest Latitude)								
3	North Latitude									
4										
5	Latitude	30	<=Degrees Latitude (0-60, increments of 5)							
6										
7	Month	T_mean	p-Value	ET_o		T-high	T-low		T-high (F)	T-low (F)
8	Jan	8.9	0.24	2.901		13.9	3.9		57	39
9	Feb	11.4	0.25	3.310		16.7	6.1		62	43
10	Mar	15.3	0.27	4.058		20.6	10.0		69	50
11	Apr	19.7	0.29	4.951		25.0	14.4		77	58
12	May	23.9	0.31	5.887		28.9	18.9		84	66
13	Jun	27.8	0.32	6.649		32.8	22.8		91	73
14	Jul	30.0	0.31	6.758		35.0	25.0		95	77
15	Aug	29.7	0.3	6.502		35.0	24.4		95	76
16	Sep	25.8	0.28	5.567		31.1	20.6		88	69
17	Oct	20.3	0.26	4.505		25.6	15.0		78	59
18	Nov	14.4	0.24	3.515		19.4	9.4		67	49
19	Dec	9.7	0.23	2.869		14.4	5.0		58	41
20										
21										
22										

Figure 7: Blaney-Criddle evapotranspiration for Dallas, Texas

A	B	C	D	E	F	G	H	I	J
Blaney-Criddle ET Estimator									
Location:	Houston, Texas (Closest Latitude)								
North Latitude									
Latitude	30	<=Degrees Latitude (0-60, increments of 5)							
Month	T_mean	p-Value	ET_o		T-high	T-low		T-high (F)	T-low (F)
Jan	13.1	0.24	3.361		17.8	8.3		64	47
Feb	15.0	0.25	3.725		20.0	10.0		68	50
Mar	18.3	0.27	4.437		23.3	13.3		74	56
Apr	21.7	0.29	5.210		26.1	17.2		79	63
May	25.6	0.31	6.124		30.0	21.1		86	70
Jun	28.3	0.32	6.731		32.8	23.9		91	75
Jul	29.7	0.31	6.718		34.4	25.0		94	77
Aug	29.7	0.3	6.502		34.4	25.0		94	77
Sep	27.5	0.28	5.782		32.2	22.8		90	73
Oct	23.1	0.26	4.837		27.8	18.3		82	65
Nov	18.1	0.24	3.913		22.8	13.3		73	56
Dec	14.2	0.23	3.339		18.9	9.4		66	49

Figure 8: Blaney-Criddle evapotranspiration for Houston, Texas

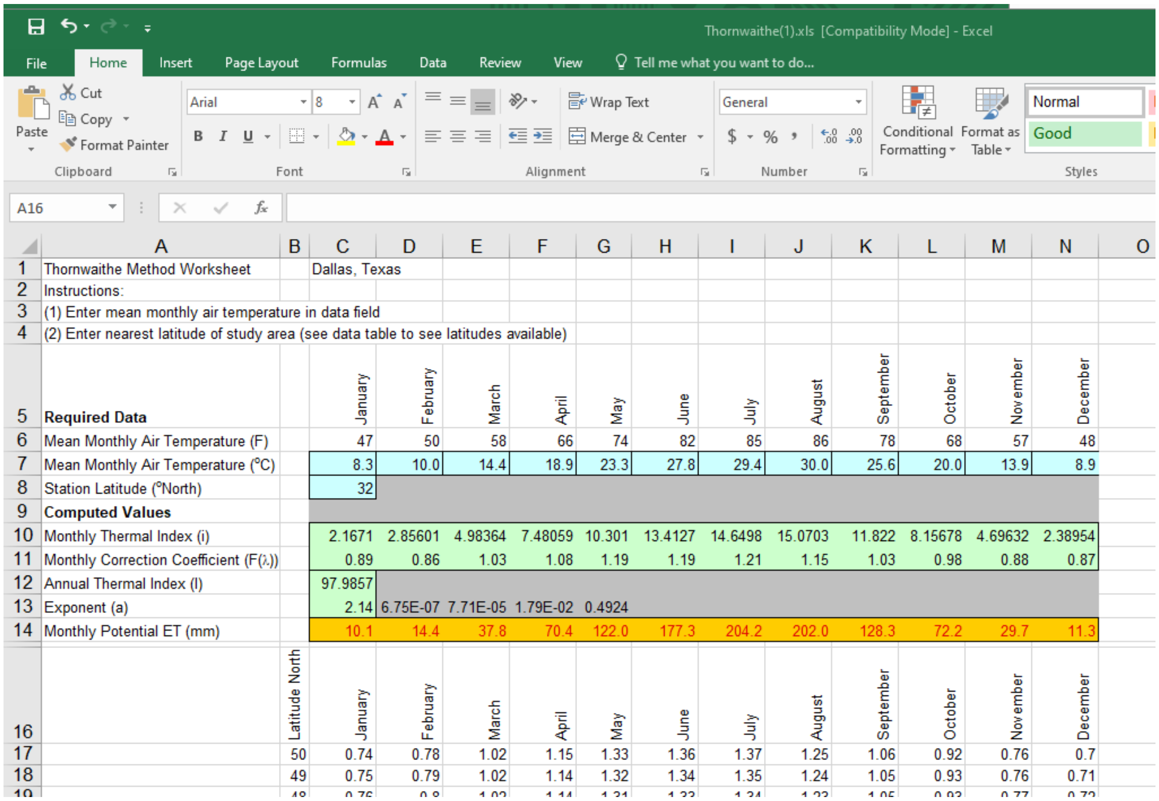
	A	B	C	D	E	F	G	H	I	J
1	Blaney-Criddle ET Estimator									
2	Location:	San Angelo, Texas (Closest Latitude)								
3	North Latitude									
4										
5	Latitude	30	<=Degrees Latitude (0-60, increments of 5)							
6										
7	Month	T_mean	p-Value	ET_o		T-high	T-low		T-high (F)	T-low (F)
8	Jan	8.9	0.24	2.901		15.6	2.2		60	36
9	Feb	11.7	0.25	3.342		18.9	4.4		66	40
10	Mar	15.6	0.27	4.092		22.8	8.3		73	47
11	Apr	20.0	0.29	4.988		27.2	12.8		81	55
12	May	24.4	0.31	5.966		31.1	17.8		88	64
13	Jun	27.2	0.32	6.567		33.3	21.1		92	70
14	Jul	28.9	0.31	6.600		35.0	22.8		95	73
15	Aug	28.3	0.3	6.310		34.4	22.2		94	72
16	Sep	24.7	0.28	5.424		31.1	18.3		88	65
17	Oct	19.7	0.26	4.439		26.1	13.3		79	56
18	Nov	14.2	0.24	3.484		20.6	7.8		69	46
19	Dec	9.4	0.23	2.839		16.1	2.8		61	37
20										
21										

Figure 9: Blaney-Criddle evapotranspiration for San Angelo, Texas

2. Estimate the monthly evapotranspiration depths for the San Angelo (Concho County) area using the Thornwaithe method.[2]

   Solution(s):

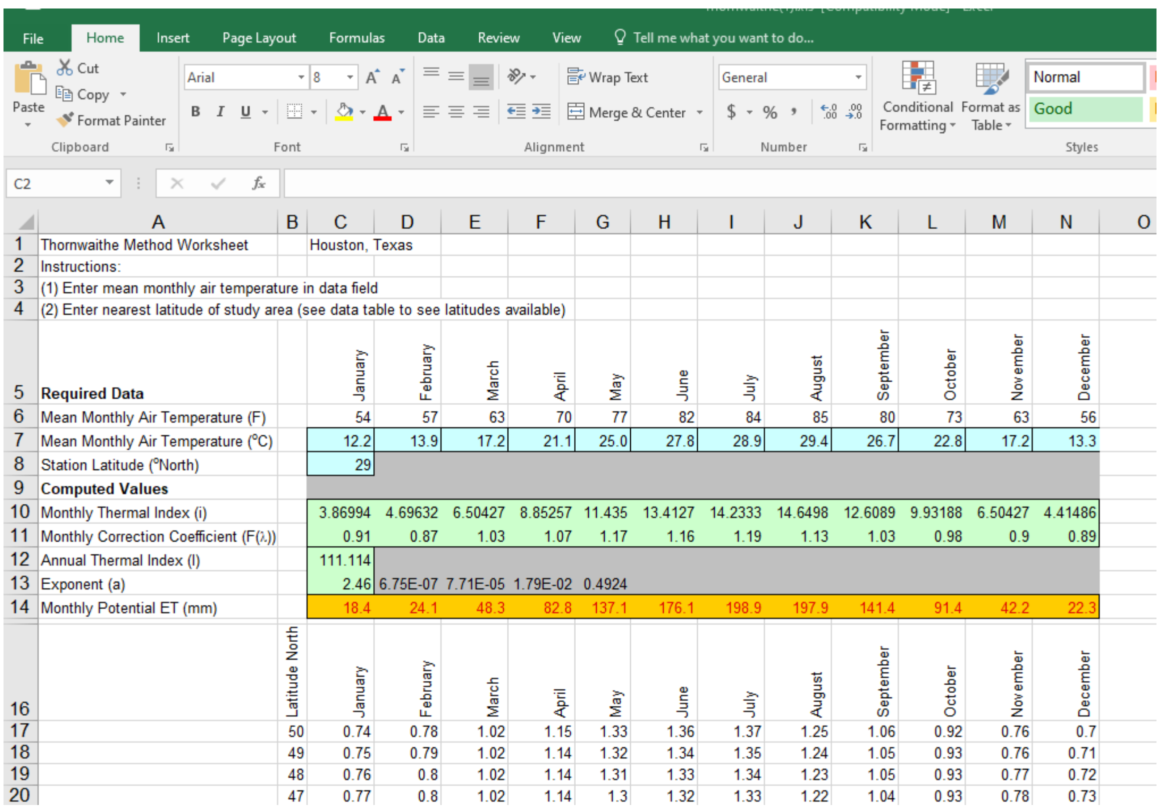
   Using same data sources, but a different tool we obtain



	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Thornwaithe Method Worksheet		Dallas, Texas											
2	Instructions:													
3	(1) Enter mean monthly air temperature in data field													
4	(2) Enter nearest latitude of study area (see data table to see latitudes available)													
5	**Required Data**		January	February	March	April	May	June	July	August	September	October	November	December
6	Mean Monthly Air Temperature (F)		47	50	58	66	74	82	85	86	78	68	57	48
7	Mean Monthly Air Temperature (°C)		8.3	10.0	14.4	18.9	23.3	27.8	29.4	30.0	25.6	20.0	13.9	8.9
8	Station Latitude (°North)		32											
9	**Computed Values**													
10	Monthly Thermal Index (i)		2.1671	2.85601	4.98364	7.48059	10.301	13.4127	14.6498	15.0703	11.822	8.15678	4.69632	2.38954
11	Monthly Correction Coefficient (F(λ))		0.89	0.86	1.03	1.08	1.19	1.19	1.21	1.15	1.03	0.98	0.88	0.87
12	Annual Thermal Index (I)		97.9857											
13	Exponent (a)		2.14	6.75E-07	7.71E-05	1.79E-02	0.4924							
14	Monthly Potential ET (mm)		10.1	14.4	37.8	70.4	122.0	177.3	204.2	202.0	128.3	72.2	29.7	11.3
16		Latitude North	January	February	March	April	May	June	July	August	September	October	November	December
17		50	0.74	0.78	1.02	1.15	1.33	1.36	1.37	1.25	1.06	0.92	0.76	0.7
18		49	0.75	0.79	1.02	1.14	1.32	1.34	1.35	1.24	1.05	0.93	0.76	0.71

Figure 10: Thornwaithe evapotranspiration for Dallas, Texas

[2]A Google search should get you sufficient guidance to perform this exercise.



	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	Thornwaithe Method Worksheet		Houston, Texas											
2	Instructions:													
3	(1) Enter mean monthly air temperature in data field													
4	(2) Enter nearest latitude of study area (see data table to see latitudes available)													
5	**Required Data**		January	February	March	April	May	June	July	August	September	October	November	December
6	Mean Monthly Air Temperature (F)		54	57	63	70	77	82	84	85	80	73	63	56
7	Mean Monthly Air Temperature (°C)		12.2	13.9	17.2	21.1	25.0	27.8	28.9	29.4	26.7	22.8	17.2	13.3
8	Station Latitude (°North)		29											
9	**Computed Values**													
10	Monthly Thermal Index (i)		3.86994	4.69632	6.50427	8.85257	11.435	13.4127	14.2333	14.6498	12.6089	9.93188	6.50427	4.41486
11	Monthly Correction Coefficient (F(λ))		0.91	0.87	1.03	1.07	1.17	1.16	1.19	1.13	1.03	0.98	0.9	0.89
12	Annual Thermal Index (I)		111.114											
13	Exponent (a)		2.46	6.75E-07	7.71E-05	1.79E-02	0.4924							
14	Monthly Potential ET (mm)		18.4	24.1	48.3	82.8	137.1	176.1	198.9	197.9	141.4	91.4	42.2	22.3
16		Latitude North	January	February	March	April	May	June	July	August	September	October	November	December
17		50	0.74	0.78	1.02	1.15	1.33	1.36	1.37	1.25	1.06	0.92	0.76	0.7
18		49	0.75	0.79	1.02	1.14	1.32	1.34	1.35	1.24	1.05	0.93	0.76	0.71
19		48	0.76	0.8	1.02	1.14	1.31	1.33	1.34	1.23	1.05	0.93	0.77	0.72
20		47	0.77	0.8	1.02	1.14	1.3	1.32	1.33	1.22	1.04	0.93	0.78	0.73

Figure 11: Thornwaithe evapotranspiration for Houston, Texas

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Thornwaithe Method Worksheet		San Angelo, Texas												
2	Instructions:														
3	(1) Enter mean monthly air temperature in data field														
4	(2) Enter nearest latitude of study area (see data table to see latitudes available)														
5	**Required Data**		January	February	March	April	May	June	July	August	September	October	November	December	
6	Mean Monthly Air Temperature (F)		46	50	58	67	74	81	84	83	76	66	47	56	
7	Mean Monthly Air Temperature (°C)		7.8	10.0	14.4	19.4	23.3	27.2	28.9	28.3	24.4	18.9	8.3	13.3	
8	Station Latitude (°North)		31												
9	**Computed Values**														
10	Monthly Thermal Index (i)		1.95216	2.85601	4.98364	7.8162	10.301	13.0087	14.2333	13.821	11.0526	7.48059	2.1671	4.41486	
11	Monthly Correction Coefficient (F(λ))		0.9	0.87	1.03	1.08	1.18	1.18	1.2	1.14	1.03	0.98	0.89	0.88	
12	Annual Thermal Index (I)		94.087												
13	Exponent (a)		2.06	6.75E-07	7.71E-05	1.79E-02	0.4924								
14	Monthly Potential ET (mm)		9.7	15.8	39.8	76.9	122.2	167.8	192.8	176.0	117.4	65.7	11.1	28.8	
16		Latitude North	January	February	March	April	May	June	July	August	September	October	November	December	
17		50	0.74	0.78	1.02	1.15	1.33	1.36	1.37	1.25	1.06	0.92	0.76	0.7	
18		49	0.75	0.79	1.02	1.14	1.32	1.34	1.35	1.24	1.05	0.93	0.76	0.71	
19		48	0.76	0.8	1.02	1.14	1.31	1.33	1.34	1.23	1.05	0.93	0.77	0.72	
20		47	0.77	0.8	1.02	1.14	1.3	1.32	1.33	1.22	1.04	0.93	0.78	0.73	

Figure 12: Thornwaithe evapotranspiration for San Angelo, Texas

3. Locate grid cells 506,410, and 812 at the TWDB lake evaporation database. Determine the long term monthly evaporation rates for the three cells. Compare these rates to the estimates you made above. These cells correspond approximately to

   - 410 == Dallas
   - 812 == Houston
   - 506 == San Angelo

   Solution(s):

   The next several pages show how to accomplish the database manipulation using ENGR-1330 methods.

Evaporation Analysis (ES3-3)

In [28]:
```
import sys
print(sys.executable)
print(sys.version)
print(sys.version_info)
# tested on aws lightsail instance 21 July 2020 using python38 kernel spec
```

```
/opt/jupyterhub/bin/python3
3.10.12 (main, May 27 2025, 17:12:29) [GCC 11.4.0]
sys.version_info(major=3, minor=10, micro=12, releaselevel='final', serial=0)
```

Using ENGR-1330 Methods

1. Get the data

In [29]:
```
import requests # Module to process http/https requests
import pandas as pd
remote_url="http://54.243.252.9/ce-3354-webroot/hydrohandbook/chapters/03-infiltration/all_quads_
rget = requests.get(remote_url, allow_redirects=True)  # get the remote resource, follow imbedded
open('all_quads_gross_evaporation.csv','wb').write(rget.content) # extract from the remote the co
import pandas as pd # Module to process dataframes (not absolutely needed but somewhat easier tha
evapdf = pd.read_csv("all_quads_gross_evaporation.csv",parse_dates=["YYYY-MM"]) # Read the file a
```

2. Compute monthly mean values for each cell in the database

In [30]:
```
# Extract month number from the datetime column
evapdf['Month'] = evapdf['YYYY-MM'].dt.month
# Group by month and compute average for each location
monthly_avg = evapdf.groupby('Month').mean(numeric_only=True)
# Optional: Add month names as labels
monthly_avg.index = monthly_avg.index.map(lambda m: pd.to_datetime(f"2020-{m:02}-01").strftime("%
```

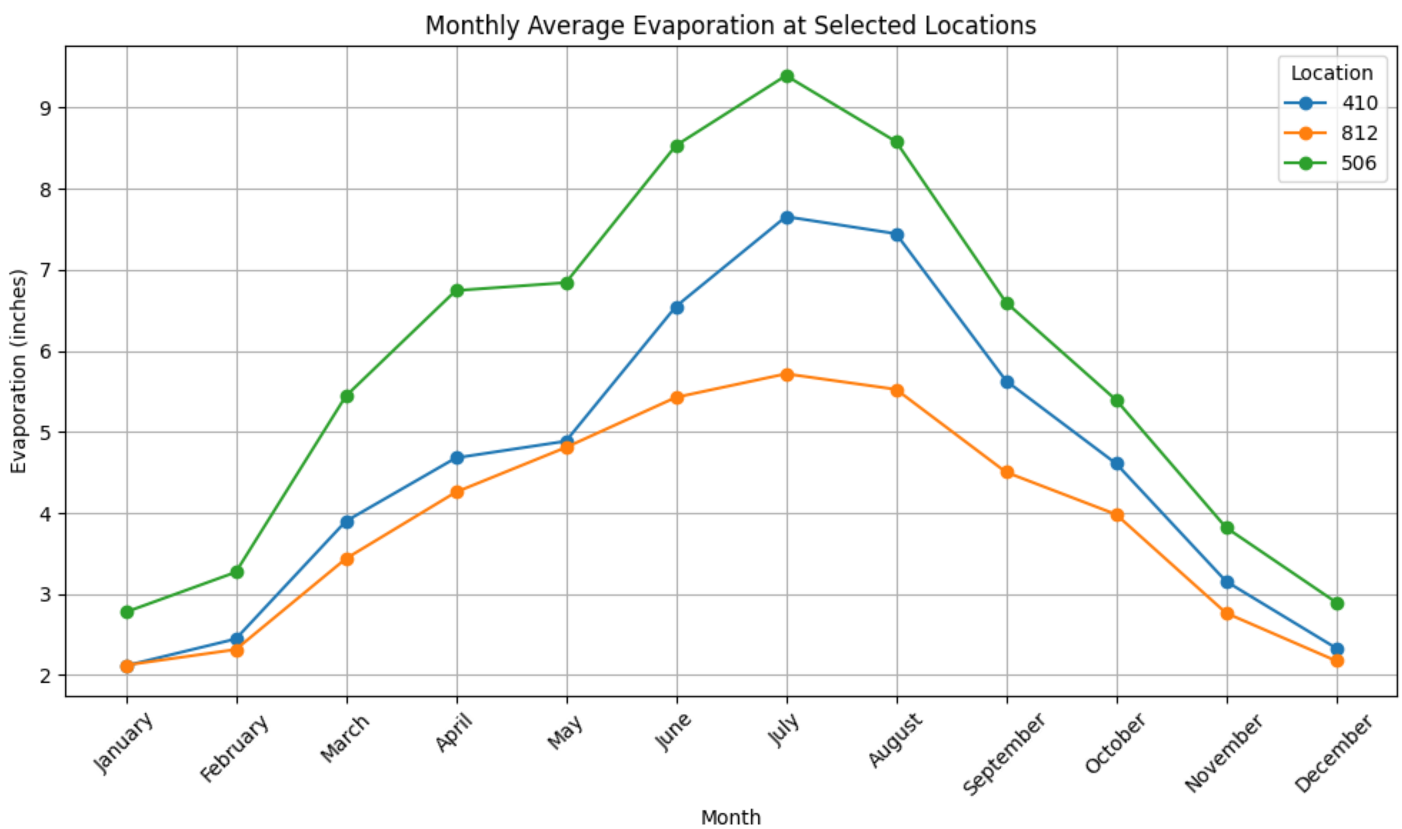
3. Extract the specific columns

In [31]:
```
print(monthly_avg[["410", "812", "506"]])
```

```
                410       812       506
Month
January    2.116061  2.123030  2.777424
February   2.446818  2.316212  3.270152
March      3.898333  3.436515  5.442727
April      4.680152  4.257576  6.744697
May        4.886515  4.809394  6.842121
June       6.548485  5.427121  8.534242
July       7.656818  5.717424  9.396818
August     7.445909  5.525606  8.582424
September  5.630909  4.503788  6.597424
October    4.610000  3.982727  5.394848
November   3.156212  2.765758  3.819545
December   2.333485  2.176364  2.897727
```

In [32]:
```
import matplotlib.pyplot as plt
# Define tick labels in correct order
```

```python
month_names = ["January", "February", "March", "April", "May", "June",
               "July", "August", "September", "October", "November", "December"]
# Select just the desired columns
selected = monthly_avg[["410", "812", "506"]]
# Plot all three on the same figure
selected.plot(kind='line', marker='o', figsize=(10, 6))
# Customize labels and title
plt.title("Monthly Average Evaporation at Selected Locations")
plt.xlabel("Month")
plt.ylabel("Evaporation (inches)")
plt.grid(True)
plt.legend(title="Location")
# Set tick locations and labels
plt.xticks(ticks=range(12), labels=month_names, rotation=45)
# Show the plot
plt.tight_layout()
plt.show()
```




```
In [ ]:
```

Comparing all the methods (for a summer month); The data science approach for July is 7-9 inches for the month, Thornwaithe is 192-204 millimeters, about 7.5-8.03 inches so comparable, Blaney-Criddle is 198-202 millimeters, about 7.8-7.9 inches, also comparable.

All methods return about same values for the region (at least for July) - the data science results have the advantage of being derived from actual measured values.

4. A storm of moderate intensity strikes a semi-urban watershed with predominantly loamy soils. The storm begins at time $t = 0$ and lasts for **3 hours** with a constant rainfall intensity of **15 mm/h**.

   The watershed has the following properties:

   - Area: 2 hectares
   - Slope: gentle (assume negligible effect)
   - Vegetative cover: 50% grass, 50% compacted dirt
   - Antecedent moisture conditions: dry (unless otherwise specified)
   - Initial abstraction: assume 5 mm where applicable

   A prior study suggests the following Horton parameters:

$$
\begin{aligned}
f_0 &= 5 \text{ mm/h} \quad \text{(initial infiltration capacity)} \\
f_c &= 1 \text{ mm/h} \quad \text{(final infiltration capacity)} \\
k &= 2.0 \text{ h}^{-1} \quad \text{(decay constant)}
\end{aligned}
$$

   Determine:

   a) The infiltration rate function $f(t)$ over the 3-hour duration using Horton's exponential decay equation: $f(t) = f_c + (f_0 - f_c)e^{-kt}$

   b) The cumulative infiltration depth $F(t)$, by integrating the rate function over time.

   c) Plot the rate and cumulative infiltration depth for every 15-minutes for the 3 hour storm.

   d) Report the total runoff depth as: Runoff = Rainfall Depth $- F(3 \text{ h})$

   Solution(s):

   a) Substitute supplied values:

$$
f(t) = 1.0 \text{ mm/hr} + (5.0 \text{ mm/hr} - 1.0 \text{ mm/hr}) \cdot e^{-2.0 \text{ hr}^{-1} \ t}
$$

   This function gets encoded into a spreadsheet.

   b) We are given the function:

$$
f(t) = f_c + (f_o - f_c)e^{-kt}
$$

   where:

- f_c is a constant (final value),
- f_o is a constant (initial value),
- k is a time constant,
- t is time.

We wish to compute the indefinite integral (antiderivative) of $f(t)$:

$$F(t) = \int f(t)\,dt = \int \left[f_c + (f_o - f_c)e^{-kt}\right] dt$$

We break this into parts and integrate term by term:

$$= f_c \int dt + (f_o - f_c) \int e^{-kt} dt$$

Now integrate:

$$= f_c t - \frac{f_o - f_c}{k} e^{-kt} + C$$

where C is the constant of integration, its value determined from $F(0) = 0$

$$\boxed{F(t) = \int f(t)\,dt = f_c t - \frac{f_o - f_c}{k} e^{-kt} + C}$$

This function also gets encoded into a spreadsheet.

c) and d) These two components are shown directly in the solution spreadsheet (Figures 13 and 14)

	A	B	C	D
1	Horton Infiltration Model(s)			
2	Formula(s)			
3	Rate:	f(t) = f_c + (f_o - f_c)exp(-kt)		
4	Depth:	F(t) = f_c*t - exp(-kt)(f_o -f_c)/k + C		
5	**Input(s)**			
6	ITEM	VALUE	UNITS	
7	f_o	5	mm/hr	
8	f_c	1	mm/hr	
9	k	2	hr^-1	
10	A	2	ha	
11	I_a	5	mm	
12	p	15	mm/hr	
13	t_rain	3	hr	
14	**Computed Values**			
15	P	45	mm	
16	P_e	40	mm; Excess after I_a removed	
17	C	2	mm; Constant of integration from F(0)=0	
18	**Plot Arrays**			
19	Time(hrs	Time(min)	f(t)	F(t)
20	0	0	5	0
21	0.25	15	3.426123	1.036939
22	0.5	30	2.471518	1.764241
23	0.75	45	1.892521	2.30374
24	1	60	1.541341	2.729329
25	1.25	75	1.32834	3.08583
26	1.5	90	1.199148	3.400426
27	1.75	105	1.12079	3.689605
28	2	120	1.073263	3.963369
29	2.25	135	1.044436	4.227782
30	2.5	150	1.026952	4.486524
31	2.75	165	1.016347	4.741826
32	3	180	1.009915	4.995042
33	**Total Runoff**			
34	Q	35.004958	mm	<= as watershed depth
35	V	700.09915	m^3	<=total volume

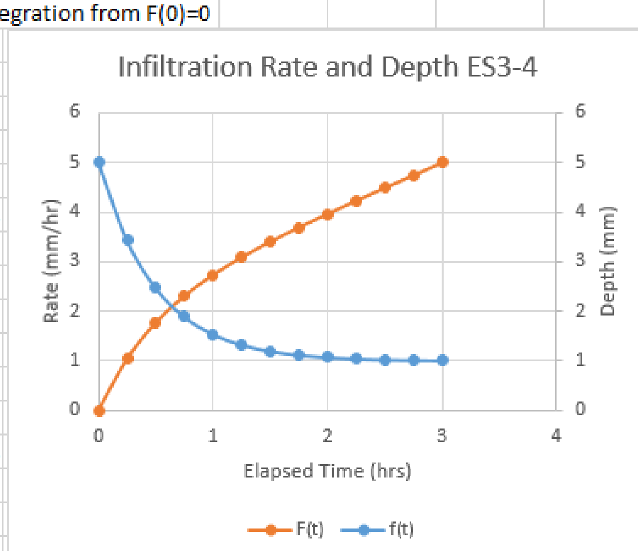


Figure 13: Excel screen capture for ES3-4

	A	B	C	D	E
1	Horton Infiltr				
2	Formula(s)				
3	Rate:	f(t) = f_c +			
4	Depth:	F(t) = f_c*			
5	**Input(s)**				
6	ITEM	VALUE	UNITS		
7	f_o	5	mm/hr		
8	f_c	1	mm/hr		
9	k	2	hr^-1		
10	A	2	ha		
11	I_a	5	mm		
12	p	15	mm/hr		
13	t_rain	3	hr		
14	**Computed Va**				
15	P	=B13*B12	mm		
16	P_e	=B15-B11	mm; Excess after I_a removed		
17	C	2	mm; Constant of integration from F(0		
18	**Plot Arrays**				
19	Time(hrs)	Time(min	f(t)	F(t)	
20	0	=A20*60	=$B$8+($B$7-$B$8)*EXP(-$B$9*A20)	=$B$8*A20-EXP(-$B$9*A20)*($B$7-$B$8)/$B$9+$B$17	
21	0.25	=A21*60	=$B$8+($B$7-$B$8)*EXP(-$B$9*A21)	=$B$8*A21-EXP(-$B$9*A21)*($B$7-$B$8)/$B$9+$B$17	
22	0.5	=A22*60	=$B$8+($B$7-$B$8)*EXP(-$B$9*A22)	=$B$8*A22-EXP(-$B$9*A22)*($B$7-$B$8)/$B$9+$B$17	
23	0.75	=A23*60	=$B$8+($B$7-$B$8)*EXP(-$B$9*A23)	=$B$8*A23-EXP(-$B$9*A23)*($B$7-$B$8)/$B$9+$B$17	
24	1	=A24*60	=$B$8+($B$7-$B$8)*EXP(-$B$9*A24)	=$B$8*A24-EXP(-$B$9*A24)*($B$7-$B$8)/$B$9+$B$17	
25	1.25	=A25*60	=$B$8+($B$7-$B$8)*EXP(-$B$9*A25)	=$B$8*A25-EXP(-$B$9*A25)*($B$7-$B$8)/$B$9+$B$17	
26	1.5	=A26*60	=$B$8+($B$7-$B$8)*EXP(-$B$9*A26)	=$B$8*A26-EXP(-$B$9*A26)*($B$7-$B$8)/$B$9+$B$17	
27	1.75	=A27*60	=$B$8+($B$7-$B$8)*EXP(-$B$9*A27)	=$B$8*A27-EXP(-$B$9*A27)*($B$7-$B$8)/$B$9+$B$17	
28	2	=A28*60	=$B$8+($B$7-$B$8)*EXP(-$B$9*A28)	=$B$8*A28-EXP(-$B$9*A28)*($B$7-$B$8)/$B$9+$B$17	
29	2.25	=A29*60	=$B$8+($B$7-$B$8)*EXP(-$B$9*A29)	=$B$8*A29-EXP(-$B$9*A29)*($B$7-$B$8)/$B$9+$B$17	
30	2.5	=A30*60	=$B$8+($B$7-$B$8)*EXP(-$B$9*A30)	=$B$8*A30-EXP(-$B$9*A30)*($B$7-$B$8)/$B$9+$B$17	
31	2.75	=A31*60	=$B$8+($B$7-$B$8)*EXP(-$B$9*A31)	=$B$8*A31-EXP(-$B$9*A31)*($B$7-$B$8)/$B$9+$B$17	
32	3	=A32*60	=$B$8+($B$7-$B$8)*EXP(-$B$9*A32)	=$B$8*A32-EXP(-$B$9*A32)*($B$7-$B$8)/$B$9+$B$17	
33	**Total Runoff**				
34	Q	=B16-D32	mm	<= as watershed depth	
35	V	=(B34/100	m^3	<=total volume	
36					
37					

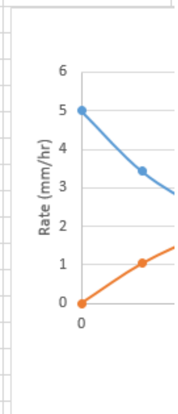


Figure 14: Excel screen capture for ES3-4 (formulas)

5. A storm of moderate intensity strikes a semi-urban watershed with predominantly loamy soils. The storm begins at time $t = 0$ and lasts for **3 hours** with a constant rainfall intensity of **15 mm/h**.

   The watershed has the following properties:

   - Area: 2 hectares
   - Slope: gentle (assume negligible effect)
   - Vegetative cover: 50% grass, 50% compacted dirt
   - Antecedent moisture conditions: dry (unless otherwise specified)
   - Initial abstraction: assume 5 mm where applicable

   An prior study suggests the following Green-Ampt parameters for the watershed:

$$\begin{aligned} \Delta\theta &= 0.25 \quad \text{(initial moisture deficit)} \\ \psi &= 110 \text{ mm} \quad \text{(wetting front suction head)} \\ K_s &= 3 \text{ mm/h} \quad \text{(saturated hydraulic conductivity)} \end{aligned}$$

   Determine:

   a) Use the Green-Ampt equation to estimate cumulative infiltration:

$$F = K_s t + \psi \Delta\theta \ln\left(1 + \frac{F}{\psi \Delta\theta}\right)$$

   Solve this equation iteratively (numerically or in Excel/Python) for $t = 3$ hours.

   b) Plot the Green-Ampt cumulative infiltration for every 15-minutes for the 3 hour storm.

   c) Report the total runoff depth as: Runoff = Rainfall Depth $- F(3 \text{ h})$

   Solution(s):

   a) We are given the following nonlinear equation that relates cumulative infiltration F to time t:

$$F = K_s t + \psi \Delta\theta \ln\left(1 + \frac{F}{\psi \Delta\theta}\right)$$

   where:

   - K_s is the saturated hydraulic conductivity (L/T),

- ψ is the capillary suction head (L),
- $\Delta\theta$ is the change in moisture content (dimensionless),
- t is time (T),
- F is the cumulative infiltration (L).

This equation cannot be solved explicitly for F because it appears on both sides of the equation and inside a logarithm. However, we can solve it numerically using an iterative approach or a root-finding method.

Numerical Solution Strategy

We recast the equation as a root-finding problem by defining a residual function:

$$f(F_{\text{guess}}) = F_{\text{guess}} - \left[K_s t + \psi\Delta\theta \ln\left(1 + \frac{F_{\text{guess}}}{\psi\Delta\theta}\right)\right]$$

We then seek the value of F_{guess} such that $f(F_{\text{guess}}) = 0$.

Solving in Excel Using Goal Seek

To solve the equation for a fixed time $t = 3$ hours using Excel:

(a) Create input cells for:
   - K_s
   - ψ
   - $\Delta\theta$
   - $t = 3$

(b) Create a cell labeled `F_guess` and assign it an initial value (e.g., 5 cm).

(c) In another cell, compute the right-hand side of the equation:

$$F(t) = K_s \cdot t + \psi \cdot \Delta\theta \cdot \ln\left(1 + \frac{\texttt{F\_guess}}{\psi \cdot \Delta\theta}\right)$$

(d) Compute the residual in a separate cell:

$$\texttt{Residual} = \texttt{F\_guess} - F(t)$$

(e) Use Excel's **Goal Seek**:

- Set `Residual` to 0
- By changing `F_guess`

This process will return the value of F that satisfies the infiltration equation for the given parameters.

b) and c) Figures 15 and 16 are the spreadsheet that performs the calculations. The Goal Seek portion is done row-by-row; the process could be automated by using the Excel Macro recorder.

	A	B	C	D	E	F
4	**Input(s)**					
5	Δθ	0.25				
6	ψ	110	mm/hr			
7	K	3	mm/hr			
8	p	15	mm/hr			
9	t_rain	3	hr			
10	I_a	5	mm			
11	A	2	ha			
12	**Computed Constant(s)**					
13	ψΔθ	27.5				
14	P	45	mm			
15	P_e	40	mm			
16	**Plot Arrays**					
17						Goal Seek: Target 0
18	t (hrs)	F*	K_s * t	1 + F/ψΔθ	F(t) (mm)	(F(t)-F*)^2
19	0	0.000	0	1	0.000	0.000
20	0.25	1.303	0.75	1.047391	1.303	0.000
21	0.5	2.565	1.5	1.093273	2.565	0.000
22	0.75	3.793	2.25	1.137932	3.793	0.000
23	1	4.992	3	1.181533	4.992	0.000
24	1.25	6.166	3.75	1.224236	6.166	0.000
25	1.5	7.318	4.5	1.266105	7.318	0.000
26	1.75	8.450	5.25	1.307275	8.450	0.000
27	2	9.565	6	1.34781	9.565	0.000
28	2.25	10.664	6.75	1.387772	10.664	0.000
29	2.5	11.748	7.5	1.427217	11.748	0.000
30	2.75	12.820	8.25	1.466195	12.820	0.000
31	3	13.880	9	1.50474	13.880	0.000
32	**Total Runoff**					
33	Q	27.180	mm	<= as watershed depth		
34	V	543.59	m^3	<=total volume		

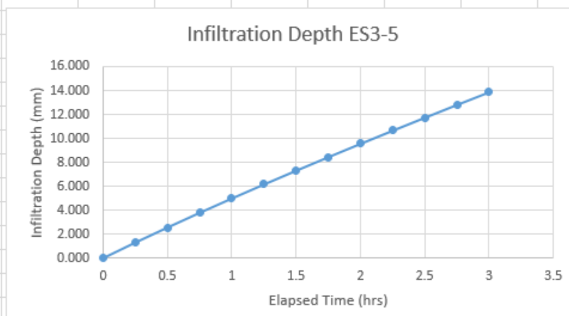
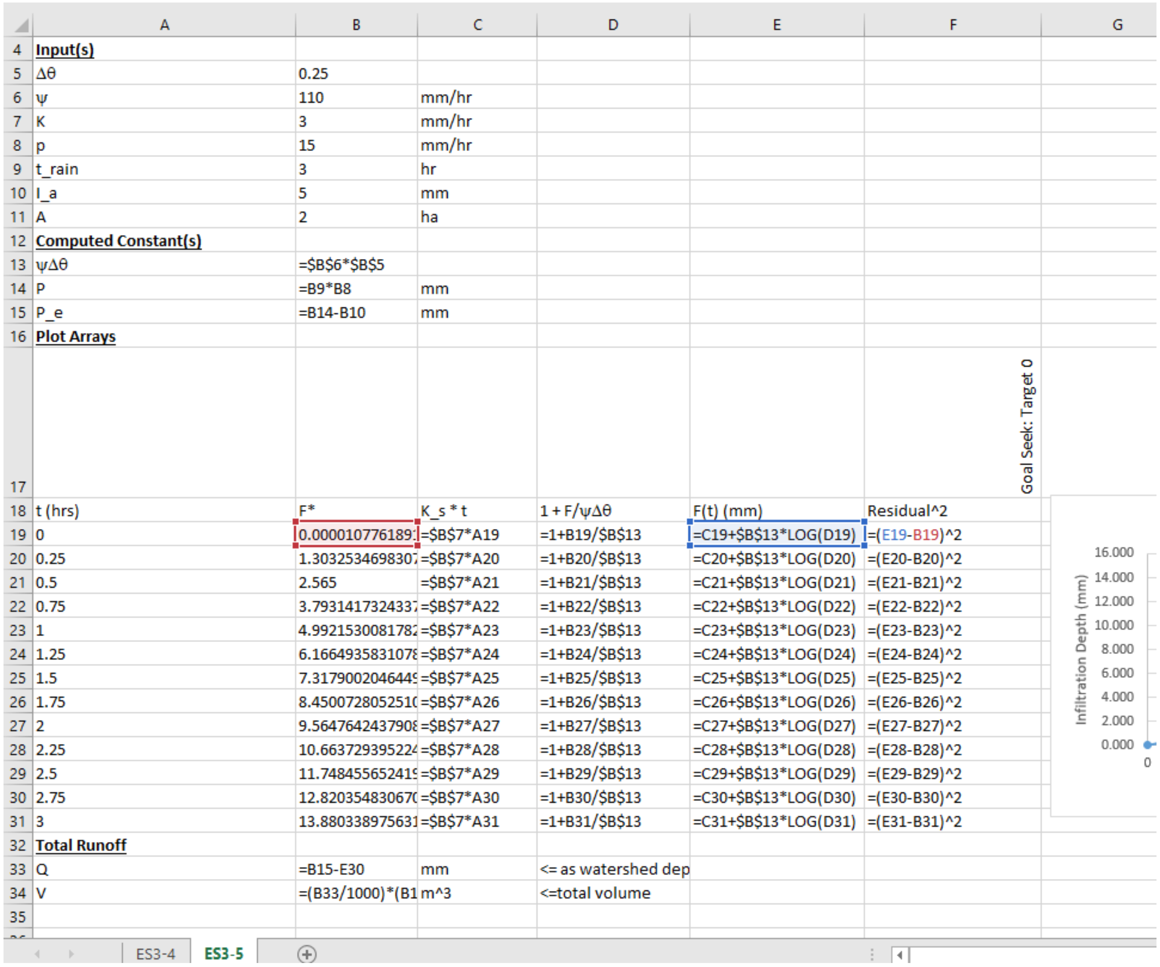


Figure 15: Excel screen capture for ES3-5



	A	B	C	D	E	F	G
4	**Input(s)**						
5	Δθ	0.25					
6	ψ	110	mm/hr				
7	K	3	mm/hr				
8	p	15	mm/hr				
9	t_rain	3	hr				
10	I_a	5	mm				
11	A	2	ha				
12	**Computed Constant(s)**						
13	ψΔθ	=$B$6*$B$5					
14	P	=B9*B8	mm				
15	P_e	=B14-B10	mm				
16	**Plot Arrays**						
17						Goal Seek: Target 0	
18	t (hrs)	F*	K_s * t	1 + F/ψΔθ	F(t) (mm)	Residual^2	
19	0	0.0000107761891	=$B$7*A19	=1+B19/$B$13	=C19+$B$13*LOG(D19)	=(E19-B19)^2	
20	0.25	1.3032534698307	=$B$7*A20	=1+B20/$B$13	=C20+$B$13*LOG(D20)	=(E20-B20)^2	
21	0.5	2.565	=$B$7*A21	=1+B21/$B$13	=C21+$B$13*LOG(D21)	=(E21-B21)^2	
22	0.75	3.7931417324337	=$B$7*A22	=1+B22/$B$13	=C22+$B$13*LOG(D22)	=(E22-B22)^2	
23	1	4.9921530081782	=$B$7*A23	=1+B23/$B$13	=C23+$B$13*LOG(D23)	=(E23-B23)^2	
24	1.25	6.1664935831078	=$B$7*A24	=1+B24/$B$13	=C24+$B$13*LOG(D24)	=(E24-B24)^2	
25	1.5	7.3179002046449	=$B$7*A25	=1+B25/$B$13	=C25+$B$13*LOG(D25)	=(E25-B25)^2	
26	1.75	8.4500728052510	=$B$7*A26	=1+B26/$B$13	=C26+$B$13*LOG(D26)	=(E26-B26)^2	
27	2	9.5647642437908	=$B$7*A27	=1+B27/$B$13	=C27+$B$13*LOG(D27)	=(E27-B27)^2	
28	2.25	10.663729395224	=$B$7*A28	=1+B28/$B$13	=C28+$B$13*LOG(D28)	=(E28-B28)^2	
29	2.5	11.748455652419	=$B$7*A29	=1+B29/$B$13	=C29+$B$13*LOG(D29)	=(E29-B29)^2	
30	2.75	12.820354830670	=$B$7*A30	=1+B30/$B$13	=C30+$B$13*LOG(D30)	=(E30-B30)^2	
31	3	13.880338975631	=$B$7*A31	=1+B31/$B$13	=C31+$B$13*LOG(D31)	=(E31-B31)^2	
32	**Total Runoff**						
33	Q	=B15-E30	mm	<= as watershed dep			
34	V	=(B33/1000)*(B1	m^3	<=total volume			
35							

Figure 16: Excel screen capture for ES3-5 (formulas)

6. A storm of moderate intensity strikes a semi-urban watershed with predominantly loamy soils. The storm begins at time $t = 0$ and lasts for **3 hours** with a constant rainfall intensity of **15 mm/h**.

   The watershed has the following properties:

   - Area: 2 hectares
   - Slope: gentle (assume negligible effect)
   - Vegetative cover: 50% grass, 50% compacted dirt
   - Antecedent moisture conditions: dry (unless otherwise specified)
   - Initial abstraction: assume 5 mm where applicable

   A prior study suggests the following NRCS CN parameters for the watershed:

   - Curve Number (CN): 75 (based on land use and hydrologic soil group B)
   - Total Rainfall: 45 mm over 3 hours

   Using the same watershed and storm conditions, Determine:

   a) Potential maximum retention:

   $$S = \frac{25400}{\text{CN}} - 254 \quad \text{(in mm)}$$

   b) Total runoff from the NRCS runoff equation:

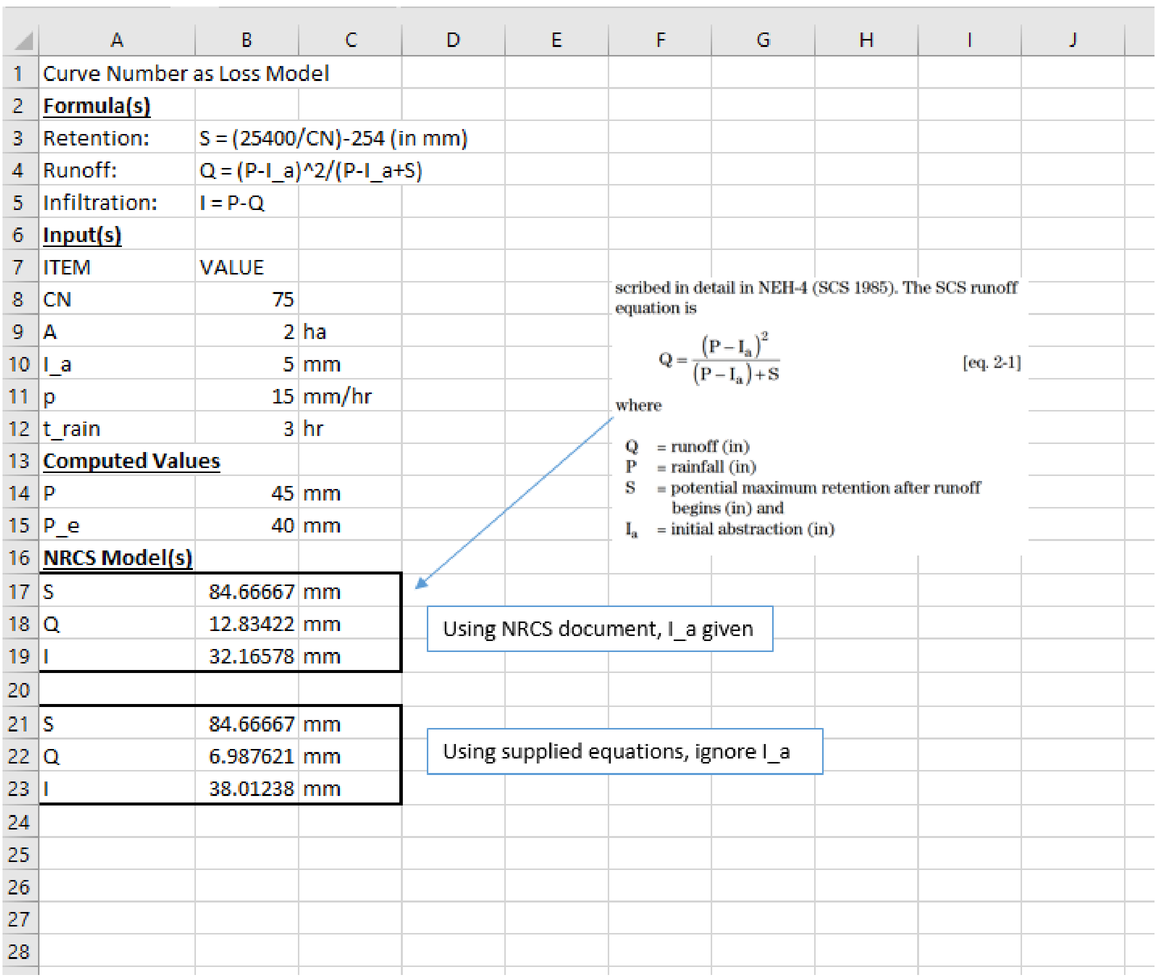
   $$Q = \frac{(P - 0.2S)^2}{P + 0.8S} \quad \text{for } P > 0.2S$$

   c) Total infiltration as:

   $$\text{Infiltration} = P - Q$$

   Solution(s):

   a), b), and c) are all incorporated into the spreadsheet depicted in Figures 17 and 18



	A	B	C
1	Curve Number as Loss Model		
2	**Formula(s)**		
3	Retention:	S = (25400/CN)-254 (in mm)	
4	Runoff:	Q = (P-I_a)^2/(P-I_a+S)	
5	Infiltration:	I = P-Q	
6	**Input(s)**		
7	ITEM	VALUE	
8	CN	75	
9	A	2	ha
10	I_a	5	mm
11	p	15	mm/hr
12	t_rain	3	hr
13	**Computed Values**		
14	P	45	mm
15	P_e	40	mm
16	**NRCS Model(s)**		
17	S	84.66667	mm
18	Q	12.83422	mm
19	I	32.16578	mm
20			
21	S	84.66667	mm
22	Q	6.987621	mm
23	I	38.01238	mm

scribed in detail in NEH-4 (SCS 1985). The SCS runoff equation is

$$Q = \frac{(P - I_a)^2}{(P - I_a) + S} \qquad \text{[eq. 2-1]}$$

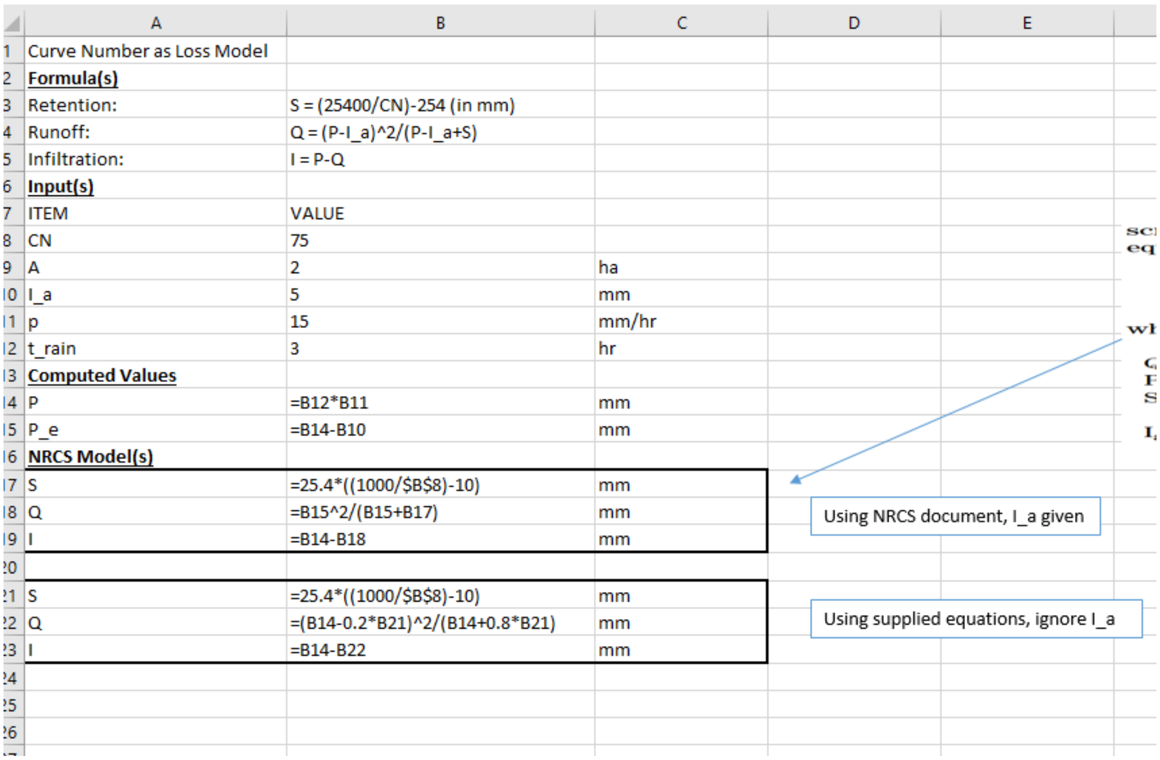
where

Q = runoff (in)
P = rainfall (in)
S = potential maximum retention after runoff begins (in) and
I_a = initial abstraction (in)

Using NRCS document, I_a given

Using supplied equations, ignore I_a

Figure 17: Excel screen capture for ES3-6



	A	B	C	D	E
1	Curve Number as Loss Model				
2	**Formula(s)**				
3	Retention:	S = (25400/CN)-254 (in mm)			
4	Runoff:	Q = (P-I_a)^2/(P-I_a+S)			
5	Infiltration:	I = P-Q			
6	**Input(s)**				
7	ITEM	VALUE			
8	CN	75			
9	A	2	ha		
10	I_a	5	mm		
11	p	15	mm/hr		
12	t_rain	3	hr		
13	**Computed Values**				
14	P	=B12*B11	mm		
15	P_e	=B14-B10	mm		
16	**NRCS Model(s)**				
17	S	=25.4*((1000/$B$8)-10)	mm		
18	Q	=B15^2/(B15+B17)	mm	Using NRCS document, I_a given	
19	I	=B14-B18	mm		
20					
21	S	=25.4*((1000/$B$8)-10)	mm		
22	Q	=(B14-0.2*B21)^2/(B14+0.8*B21)	mm	Using supplied equations, ignore I_a	
23	I	=B14-B22	mm		
24					
25					
26					

Figure 18: Excel screen capture for ES3-6 (formulas)

7. Compare infiltration results among the three methods.

   a) What causes the differences?

      The three models are built on different assumptions and emphasize different aspects of infiltration. CN Model is empirical and event-based, lumping all losses into a single abstraction value derived from land use and antecedent conditions. It assumes infiltration is front-loaded and limited by a fixed potential maximum retention.

      Green-Ampt is a physically based model that assumes a sharp wetting front, constant suction at the wetting front, and uniform soil properties. It models infiltration as a function of cumulative depth and soil moisture gradient.

      Horton's equation is semi-empirical, based on observed exponential decay of infiltration capacity over time, reflecting crusting or compaction during rainfall events.

   b) Which method is most sensitive to changes in soil properties?

      Green-Ampt is the most sensitive to soil properties because it explicitly uses Hydraulic conductivity, Suction head at the wetting front, and Moisture deficit (or porosity). Changes in texture, compaction, or porosity directly affect the infiltration curve.

      In contrast, Horton abstracts these effects into empirical decay constants, and CN wraps them into look-up tables, making them less directly sensitive.

   c) How would the results change under wet antecedent conditions?

      CN Model incorporates this directly through the Antecedent Moisture Condition (AMC), which reduces retention and increases runoff. Green-Ampt responds through a smaller initial moisture deficit, which increases infiltration rate early but levels off quickly. Horton's model assumes initial infiltration capacity is lower under wet conditions, so it starts lower and decays to the same minimum.

      In all models, wetter antecedent conditions generally result in: higher runoff, and lower total infiltration, but the mechanism varies by model.

   d) Suggest which model is most appropriate for:

      - Urban drainage design *CN Method; Simple, conservative, integrates land cover; widely accepted in practice*
      - Physically-based process modeling. *Green-Ampt; Captures infiltration physics, adaptable for layered soils and time-varying rainfall*

- Regional-scale hydrologic planning *Horton or CN; Both can be calibrated to regional behavior using observed runoff or soil maps; less parameter-intensive*