(Mechanics of Materials)

SOLIDS

the study of

$$\textbf{STRESS} = \frac{\textbf{Force}}{\textbf{Area}}$$

for the purpose of

(Engineering)

DESIGN

= choosing

1) Size (how big it is)

and

2) Material (what it's made of)

CE 3303 Solids Summary

The purpose of Solids (Mechanics of Materials) is to give you the tools to Design things for 1) adequacy to withstand loads (not fail), and 2) be economical (efficient).

External Forces (Loads & Supports) create Internal Forces – determine by making "cuts", drawing FBD's, and using Equilibrium Equations (Sum of Forces and Moments = ZERO).

- 1) Normal Forces (perpendicular to the "cut" surface)
- 2) Shear Forces (parallel to the "cut" surface) – 2 directions in 3-D
- 3) Moments (rotational forces) – 2 Bending Moments & 1 Torsion in 3-D

Internal Forces create STRESS = Force/Area. There are two basic types of Stress:

- 1) Normal Stress - σ - Axial = P/A and Bending = My/I
- 2) Shear Stress - τ Straight Line = VQ/IT and Torsion = Tp/J

STRESS is the same, no matter what the thing is made of.

STRESS creates STRAIN – Deformation. There are two basic types of Strain:

- 1) Normal Strain - ϵ - change in length (volume)
- 2) Shear Strain - γ - change in angles (shape)

STRAIN is NOT the same – it depends on what material the thing is made of – the Strength of the material

STRESS and STRAIN are related through Hooke's Law, which expresses the Strength of a material.

- Modulus of Elasticity E = σ/ϵ
- Shear Modulus G = τ/γ

There are basically two types of material – 1) Ductile and 2) Brittle, and they behave very differently

<u>Axial Load</u> (P or N, Tension or Compression) causes Normal Axial Stress which causes Elongation or Shortening = PL/AE
<u>Torsion</u> (Moment about a longitudinal axis) causes Shear Stress = Tp/J which causes Twisting = TL/JG
<u>Bending</u> (Moment about a centroidal axis) causes Normal Stress that varies from the Neutral Axis (Zero) to the extreme fiber – Compression on one side, Tension on the other. Bending can occur about two Neutral Axes (Combined)
<u>Transverse Shear</u> (V) is a component of Bending, and causes Shear Stress, τ = VQ/It that varies from Maximum near the N.A. to Zero at the extreme fibers. Shear can occur in two directions, and can combine when it does.

Superposition allows us to combine the stress caused by all four of these types of stress (actually six independent stresses – **<u>3 Normal</u>** – 1 Axial and 2 Bending & **<u>3 Shear</u>** – 1 Torsion and 2 Transverse – Combined Loadings (Stresses)
σ P/A My/I τ Tp/J VQ/It

Stresses (and Strains) vary as the coordinate system is rotated – our goal is to find the Maximum Stresses and design for them. We generally do this with Transformation Equations and Mohr's Circle.

Beams are Designed for Bending and checked for Shear, so that the stresses are less than allowable stress.

The Deflection of Beams is represented by the Elastic Curve – we used Integration and Superposition to determine it.

Columns tend to fail by Buckling (Bending) – Critical Buckling is affected by 3 factors – I(min), end conditions (k) and unsupported length (L). Eccentric Loads on Columns create Bending Moments, which seriously reduces column strength

Statically Indeterminant (too many unknowns) members must be analyzed with Equilibrium AND Deflection (Deformation) Compatibility

Average Stress: $\sigma_{ave} = \frac{P}{A}$ $\tau_{ave} = \frac{V}{A}$

Factor of Safety: $F.S. = \frac{F_{fail}}{F_{allow}} = \frac{\sigma_{fail}}{\sigma_{allow}} = \frac{\tau_{fail}}{\tau_{allow}}$

Polar moment of Inertia: $J_{solid} = \frac{\pi}{2}c^4$ $J_{tube} = \frac{\pi}{2}(c_o^4 - c_i^4)$

Torque in a Circular Shaft: $\tau = \frac{T\rho}{J}$ $\tau_{\max} = \frac{Tc}{J}$

Angle of Twist: $\phi = \sum \frac{TL}{JG}$

Hooke's Law: $\sigma = E \cdot \epsilon$ $\tau = G \cdot \gamma$ $G = \frac{E}{2(1+\nu)}$

Poisson's Ratio: $\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$

Strain: $\epsilon = \frac{\delta}{L}$ $\gamma = \frac{\pi}{2} - \theta'$

Deformation: $\delta_F = \frac{P \cdot L}{A \cdot E}$ $\delta_T = \alpha \cdot \Delta T \cdot L$ $\delta_{total} = \sum \delta$

Shear and Shear Flow: $\tau = \frac{VQ}{It}$ $q = \frac{VQ}{I} = \frac{F_{connx}}{s}$ $Q = \bar{y}'A'$

Axial Stress: $\sigma_a = \frac{P}{A}$

Bending Stress: $\sigma_{bend} = -\frac{My}{I}$ $\sigma_{\substack{bend\\ \max}} = \frac{Mc}{I}$

Thin Walled Pressure Vessels: $\sigma_{hoop} = \frac{pr_i}{t}$ $\sigma_{long} = \frac{pr_i}{2t}$

Mohr's Circle: $$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

Stress Transformations:
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Principal Stresses: $$tan2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \qquad \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Maximum In-Plane Shear Stress: $$tan2\theta_s = \frac{\sigma_y - \sigma_x}{2\tau_{xy}} \qquad \tau_{\substack{\max \\ \text{in-plane}}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

Design of Beams: $$S_{req'd} = \frac{M_{\max}}{\sigma_{allow}} = \frac{I}{c} \qquad \tau_{ave} = \frac{V_{\max}}{t_w d}$$

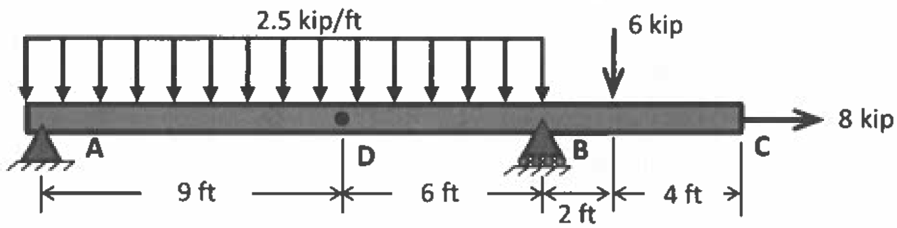
Elastic Curve: $$EI\theta(x) = \frac{dv}{dx} \qquad EIM(x) = \frac{d^2v}{dx^2} \qquad EIV(x) = \frac{d^3v}{dx^3}$$

Buckling – Critical Axial Load: $$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$ where: KL = the effective length, L_e

Slenderness Ratio: $\dfrac{KL}{r}$ where $r = \sqrt{\dfrac{I}{A}}$

Internal Forces - Deformable Body Equilibrium

Beam loaded as shown. D is a point within the beam (not a hinge). Reactions $\mathbf{A_y}$ = 17.95 kips and $\mathbf{B_y}$ = 25.55 kips have been determined. **(Hint:** There is another reaction at A.)



4) Draw a free body diagram for a cut at D. (5 pts)

5) Determine the internal shear force at D. (4 pts)

a. – 4.37 kip b. 0

c. – 4.55 kip d. -4.19 kip

6) Determine the internal normal force at D. (4 pts)

a. 0 b. 6.00 kip (T)

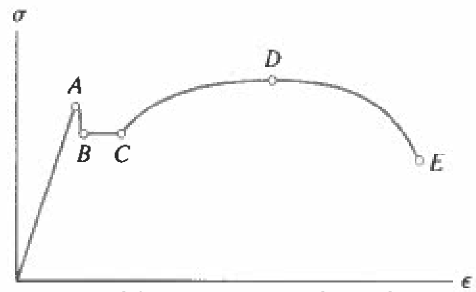
c. 8.00 kip (T) d. 8.00 kip (C)

7) Determine the internal moment at D. (4 pts)

a. + 65.2 kip·ft b. + 62.7 kip·ft

c. + 67.8 kip·ft d. + 60.3 kip·ft

Stress-Strain Relationships



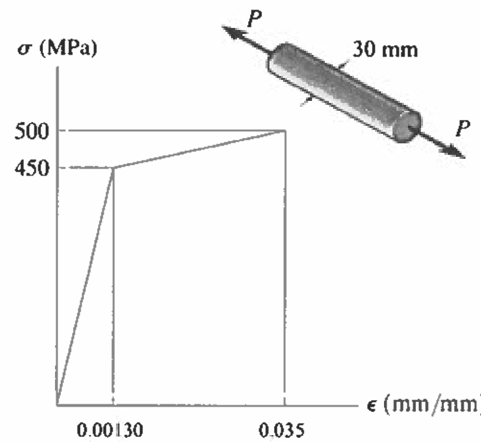
1) The point on the stress-strain diagram that represents ultimate stress is __ . (2 pts)

a. Point C b. Point D
c. Point B d. Point E

2) The point on the stress-strain diagram that represents yield stress is __ . (2 pts)

a. Point C b. Point A
c. Point D d. Point E

The specimen below has an unloaded length L = 250 mm. The Stress-Strain Diagram is shown.



3) Determine the modulus of elasticity, E, of the material. (2 pts)

a. 346 GPa b. 360 GPa

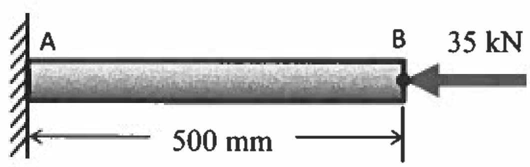
c. 320 GPa d. 333 GPa

4) Determine the normal strain of the specimen if it is loaded with a force P = 140 kN. (6 pts)

a. $549\ (10^{-6})\ ^{mm}/_{mm}$ b. $527\ (10^{-6})\ ^{mm}/_{mm}$

c. $572\ (10^{-6})\ ^{mm}/_{mm}$ d. $595\ (10^{-6})\ ^{mm}/_{mm}$

The system is loaded and supported as shown. The solid shaft has a diameter, d = 35 mm.
E = 200 GPa

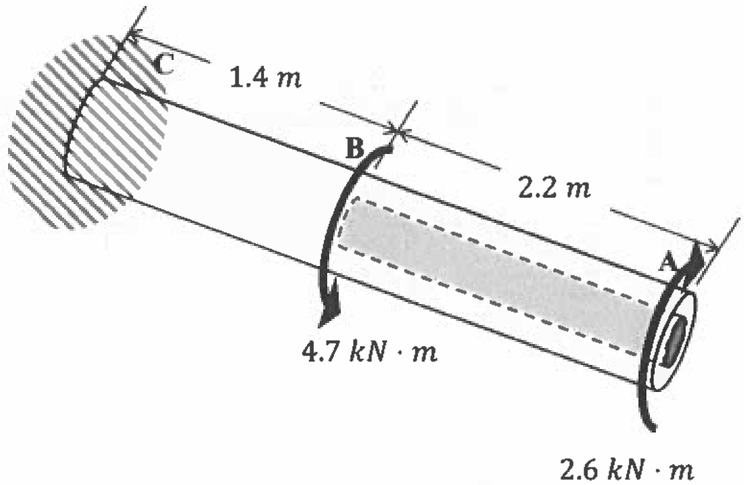


5) Determine Poisson's Ratio if the measured change in diameter under the load is Δd = +2.40 (10^{-3}) mm. (10 pts)

ν = ____________________

Torsional Shear Stress & Angle of Twist

The bar is loaded as shown. Section BC is solid with an outside diameter of 90 mm. Section AB is a tube with an outside diameter of 90 mm and a wall thickness of 10 mm. E = 123 GPa, G = 45 GPa



6) Determine the maximum shear stress in the section of the shaft between B and C. (6 pts)

a. 14.1 MPa　　b. 15.3 MPa

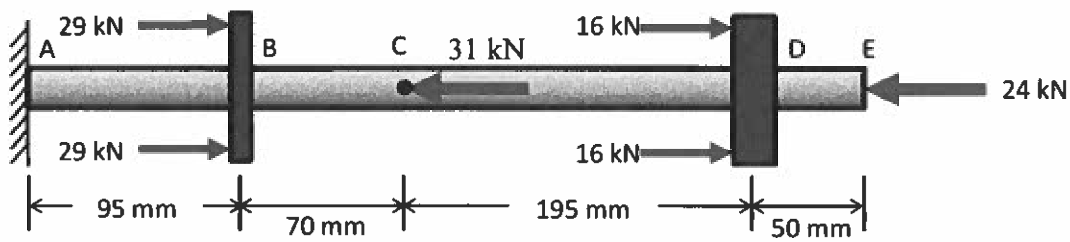
c. 15.9 MPa　　d. 14.7 MPa

7) Determine the angle of twist of point A relative to point B. (10 pts)

Angle of Twist = ____________________

Axial (Normal) Stress, Strain and Deformation

The system is loaded and supported as shown. The system is made of Titanium, E_T = 140 GPa. It is a solid shaft with diameter, d = 30 mm.



11) Calculate the axial normal stress in section BC, (5 pts)

a. 32.5 MPa (C)   b. 33.8 MPa (T)

c. 31.2 MPa (T)   d. 30.0 MPa (C)

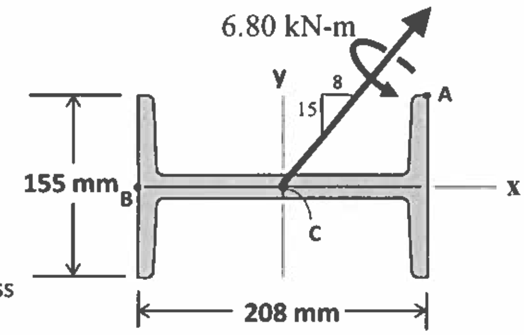
12) Calculate the displacement of point B relative to point D. (10 pts)

Displacement of B relative to D = ______________________

13) Calculate the axial normal strain in section DE. (5 pts)

a. -252 (10^{-6}) $^{mm.}/_{mm.}$ b. -243 (10^{-6}) $^{mm.}/_{mm.}$

c. -262 (10^{-6}) $^{mm.}/_{mm.}$ d. -233 (10^{-6}) $^{mm.}/_{mm.}$



Bending Stress and Flexure Formula

The cross section of a W200 x 36 is subjected the resultant internal bending moment as shown.
$I_x = 7.64\,(10^6)\ \text{mm}^4$, $I_y = 34.4\,(10^6)\ \text{mm}^4$

14) Determine the Direction of the bending stress at B. (2 pts)

a. + (Tension) b. – (Compression) c. No Stress

15) Calculate the magnitude of the bending stress at B. (4 pts)

a. 18.1 MPa b. 18.8 MPa

c. 0 d. 17.4 MPa

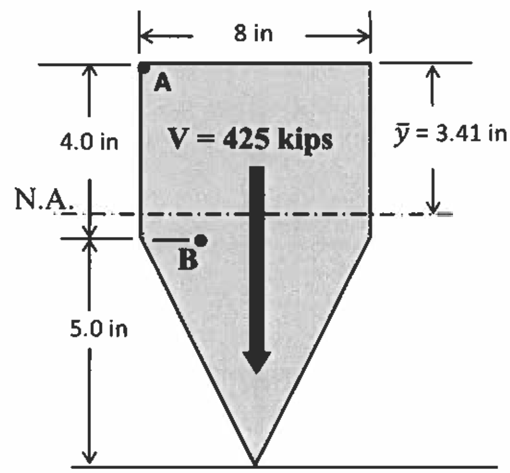
16) Calculate the bending stress at A. (8 pts)

Bending Stress @ A = ____________________

Transverse Shear

The cross-section has a moment of inertia about the neutral axis, $I = 236\ \text{in}^4$. Point B lies 4.0 inches below the top of the cross-section, 5.0 inches above the bottom of the cross-section.

17) Calculate the maximum transverse shear stress. (9 pts)



Maximum Shear Stress = ___________

18) Calculate the transverse shear stress at point B. (5 pts)

a. 10.6 ksi   b. 11.0 ksi

c. 10.2 ksi   d. 9.80 ksi

Thin Walled Pressure Vessels

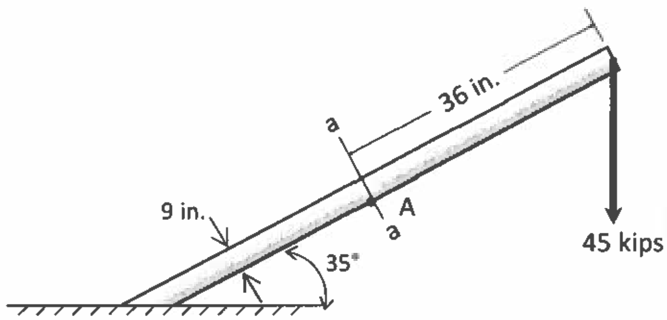
A pressurized ***cylindrical*** tank with a diameter 3.5 m, made of 25 mm thick steel, E = 200 GPa . The failure stress of the steel is 250 MPa.

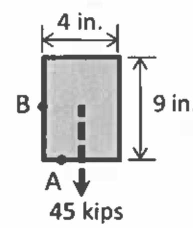
1) Calculate the maximum pressure the tank can contain with a Factor of Safety of 2.7. (5 pts)

Maximum Pressure = ___________________

Combined Loads and Stresses

The cantilever has a force applied at the center of the cross-section. This creates 3 types of stress at A.





Cross-section at a-a

2) Calculate the normal stress caused by the axial load at point A. (5 pts)

a. -0.661 ksi c. 0.688 ksi
b. 0.747 ksi d. -0.717 ksi

3) Calculate the normal stress caused by the bending moment at point A. (5 pts)

a. 27.6 ksi c. -25.6 ksi
b. -24.6 ksi d. 26.6 ksi

4) Calculate the transverse shear stress at point B (midheight of the cross-section). (5 pts)

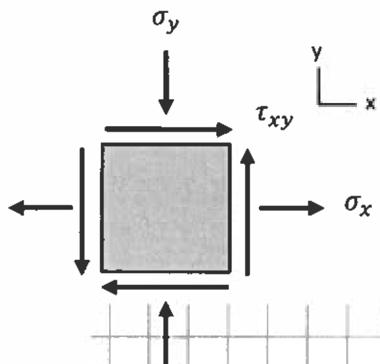
a. 1.54 ksi c. 1.70 ksi
b. 1.60 ksi d. 1.47 ksi

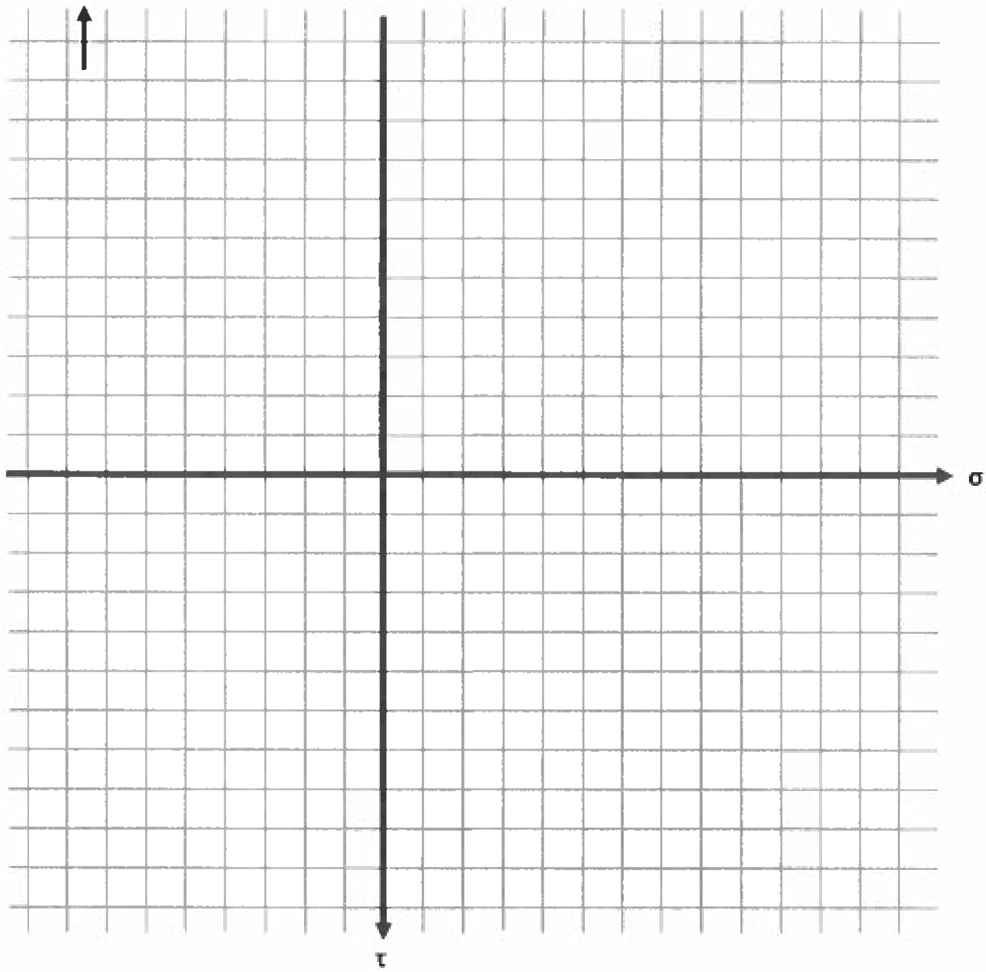
5) Determine the combined normal stress at point A (3 pts)

Normal Stress = ______________________

Mohr's Circle (12 pts)

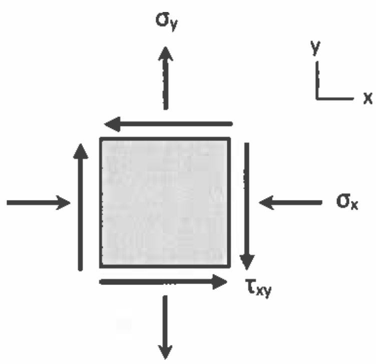
6) Draw Mohr's Circle for the Stress Element shown.
7) Show and give the stress values shown on the Stress Element (σ_x = 100 MPa, σ_y = 40 MPa, τ_{xy} = 50 MPa, signs shown on Stress Element),
8) Show and give the values of the 3 principal stresses,
9) Show and give their 3 angles of orientation,
10) Show and give the average normal stress (σ_{ave}),
11) Show and give the radius of Mohr's Circle (R)





Stress Transformation

The magnitudes of the state of stress at a point on a member was recorded as σ_x = 140 MPa, σ_y = 70 MPa, and τ_{xy} = 30 MPa.



12) Determine the normal stress acting in the x′ direction if the element is oriented 30° clockwise from its original position. (3 pts)

a. 59.1 MPa (C) c. 61.5 MPa (C)

b. 56.7 MPa (C) d. 63.9 MPa (C)

13) Determine the normal stress acting in the y′ direction if the element is oriented 30° clockwise from its original position. (3 pts)

a. 9.17 MPa (T) c. 8.48 MPa (C)

b. 8.14 MPa (C) d. 8.82 MPa (T)

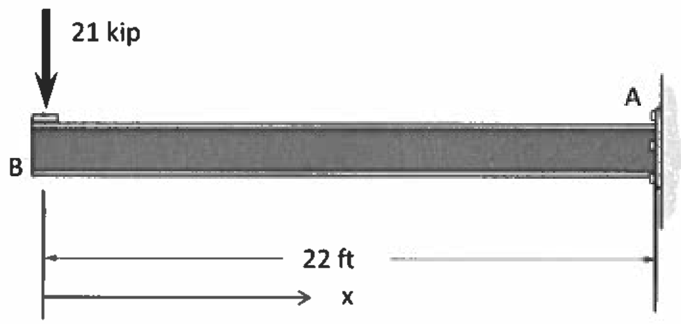
14) Determine the magnitude of the transformed in-plane shear stress if the element is oriented 30° clockwise from its original position. (3 pts)

a. 114 MPa c. 110 MPa

b. 102 MPa d. 106 MPa

Elastic Curve – Constants of Integration, Slope and Deflection

Given **the beam loaded as shown with a concentrated Load at point B**



The Moment Equation is: EI M(x) = $-21x$ kip-ft

The Slope Equation is: EI $\theta(x)$ = $-10.5x^2 + C_1$ kip-ft^2

The Deflection Equation is: EI v(x) = $-3.5x^3 + C_1x + C_2$ kip-ft^3

Evaluate the Constants of Integration. (4 pts each)

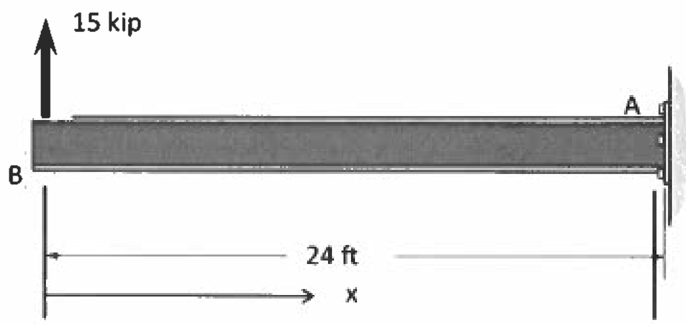
17) C_1 =

a. 5820 b. 5082
c. 0 d. -5280

18) C_2 =

a. -74536 b. -73654
c. -76543 d. -78635

Slope & Deflection - Given the beam loaded as shown. E = 29,000 ksi, I = 1670 in.^{4}



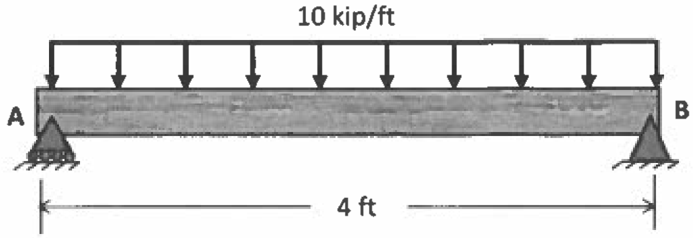
The Deflection Equation is: EIv(x) = $2.5x^3 - 4320x + 69120$ kip-ft^3

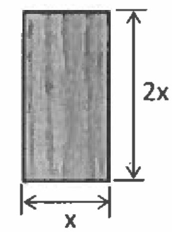
19) Calculate the deflection 15 ft left of point A. (8 pts)

Deflection = ______________________

Beam Design

The simply supported beam is made of timber that has an allowable bending stress of σ_{allow} = 1.60 ksi and an allowable shear stress of τ_{allow} = 210 psi. (The architect wants the beam with proportions shown.) (Hint: Draw a Shear and Moment Diagram to find V_{max} and M_{max})





cross-section

V |--|

M |--|

20) Calculate the required dimension, x, for bending stress. (6 pts)

a. 6.32 in c. 6.58 in

b. 6.08 in d. 5.84 in

21) Calculate the required dimension, x, for shear stress. (6 pts)

a. 8.45 in c. 9.14 in

b. 8.79 in d. 8.11 in

22) Rounding to the nearest inch, what dimension x should be used for the final beam design? (3 pts)

a. 6.00 in c. 8.00 in

b. 7.00 in d. 9.00 in

Column Buckling

An A-36 steel rod with a 4 inch diameter (circular) cross section (I = ???? in.^{4}) is to be used as a column. (E_{st} = 29,000 ksi, σ_Y = 36 ksi)

23) Determine the maximum allowable axial load the column can support without buckling if the 15 foot long column is fixed at one end and pinned at the other. (10 pts)

Maximum Axial Load = ____________________

24) Determine the maximum length of the column to support a 200 kip load without buckling. The column is pinned at both ends. (5 pts)

A) 11.2 ft
C) 12.0 ft
B) 11.6 ft
D) 10.8 ft