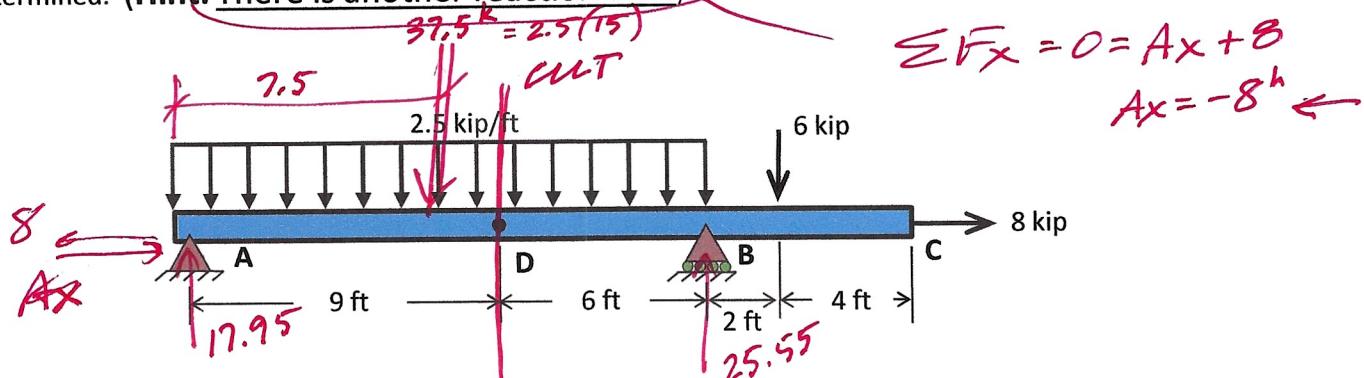


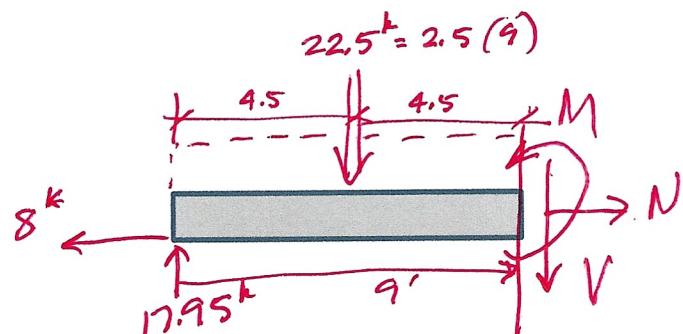
$$\sum M_A = 0 = -37.5(7.5) + B_y(15) - 6(17) \quad B_y = \frac{383.25}{15} = 25.55$$

Deformable Body Equilibrium

Beam loaded as shown. D is a point within the beam (not a hinge). Reactions $A_y = 17.95$ kips and $B_y = 25.55$ kips have been determined. (Hint: There is another reaction at A.)



- 4) Draw a free body diagram for a cut at D. (5 pts)



- 5) Determine the internal shear force at D. (4 pts)

- a. - 4.37 kip
- b. 0
- c. - 4.55 kip
- d. - 4.19 kip

$$+\uparrow \sum F_y = 0 = 17.95 - 22.5 - V \quad V = -4.55 \text{ kip}$$

- 6) Determine the internal normal force at D. (4 pts)

- a. 0
- b. 6.00 kip (T)
- c. 8.00 kip (T)
- d. 8.00 kip (C)

$$\sum F_x = 0 = -8 + N \quad N = +8 \text{ kip (T)}$$

- 7) Determine the internal moment at D. (4 pts)

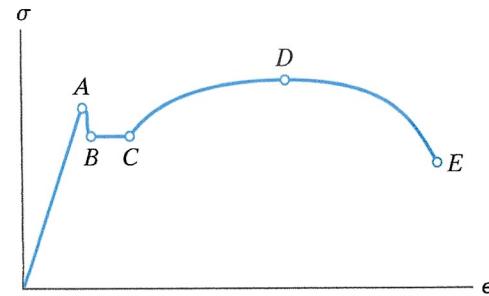
- a. + 65.2 kip·ft
- b. + 62.7 kip·ft
- c. + 67.8 kip·ft
- d. + 60.3 kip·ft

$$\textcircled{D} \sum M_{\text{cut}} = 0 = -17.95(9) + 22.5(4.5) + M \quad M = +60.3 \text{ kip·ft}$$

Stress-Strain Relationships

- 1) The point on the stress-strain diagram that represents ultimate stress is _____. (2 pts)

- a. Point C
b. Point D
c. Point B
d. Point E



- 2) The point on the stress-strain diagram that represents yield stress is _____. (2 pts)

- a. Point C
b. Point A
c. Point D
d. Point E

The specimen below has an unloaded length $L = 250$ mm. The Stress-Strain Diagram is shown.

- 3) Determine the modulus of elasticity, E , of the material.
(2 pts)

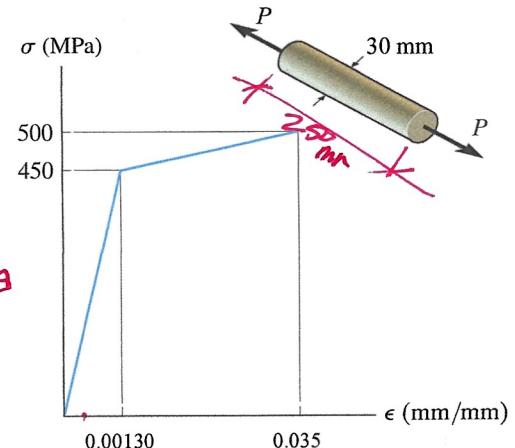
- a. 346 GPa
b. 360 GPa
c. 320 GPa
d. 333 GPa

$$\begin{aligned} &= \frac{\sigma}{E} \\ &= \frac{450}{0.00130} \\ &= 346.1 \text{ GPa} \\ &= 346154 \text{ MPa} \end{aligned}$$

- 4) Determine the normal strain of the specimen if it is loaded with a force $P = 140$ kN. (6 pts)

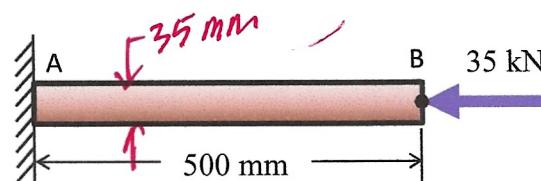
ME 511

$$\text{Strain} = \frac{572(10^{-6}) \text{ mm}}{572.2 \text{ E}^{-6} \text{ mm}} = \frac{E = \sigma/E}{E = P/A} = \frac{(140000)}{\frac{\pi}{4}(30)^2(346150)} = 0.000572$$



The system is loaded and supported as shown. The solid shaft has a diameter, $d = 35$ mm.

$E = 200$ GPa



$$E_{\text{long}} = \frac{P}{AE} = \frac{35000}{\frac{\pi}{4}(35)^2(200000)} = (0.0001819)$$

- 5) Determine Poisson's Ratio if the measured change in diameter under the load is $\Delta d = +2.40(10^{-3})$ mm. (10 pts)

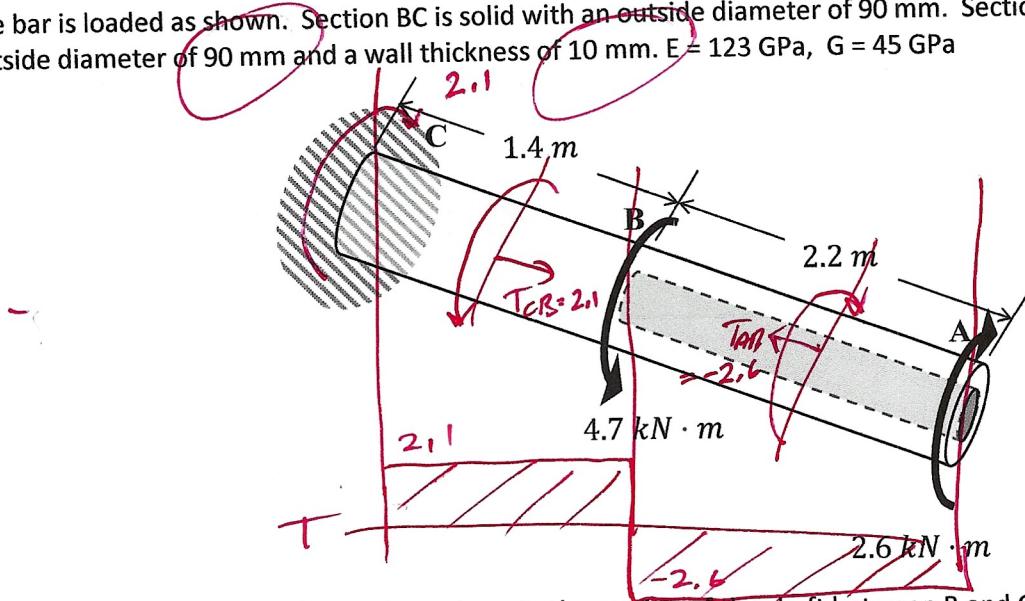
$$v = 0.377$$

$$\epsilon = \frac{d}{d_0} = \frac{2.4 \times 10^{-3} \text{ mm}}{35 \text{ mm}} = 0.0006857$$

$$v = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}} = -\frac{2.4 \times 10^{-3}}{0.0001819} = 0.3770$$

Torsional Shear Stress & Angle of Twist

The bar is loaded as shown. Section BC is solid with an outside diameter of 90 mm. Section AB is a tube with an outside diameter of 90 mm and a wall thickness of 10 mm. $E = 123 \text{ GPa}$, $G = 45 \text{ GPa}$

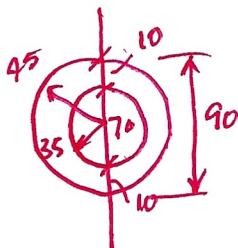


- 6) Determine the maximum shear stress in the section of the shaft between B and C. (8 pts)

MC

$$\text{Maximum shear stress} = 14.7 \text{ MPa}$$

$$T = \frac{T_P}{J} = \frac{2.1(1000)(1000)}{6.441E6} \frac{\text{N} \cdot \text{m}}{\text{mm}^3} = 14.67 \frac{\text{N}}{\text{mm}^2} \text{ MPa}$$



$$J_{BC} = \frac{\pi}{2} \left(\frac{90}{2}\right)^4 = 6.441 E 6 \text{ mm}^4$$

$$J_{AB} = \frac{\pi}{2} \left(\left(\frac{90}{2}\right)^4 - \left(\frac{70}{2}\right)^4\right) = 4.084 E 6 \text{ mm}^4$$

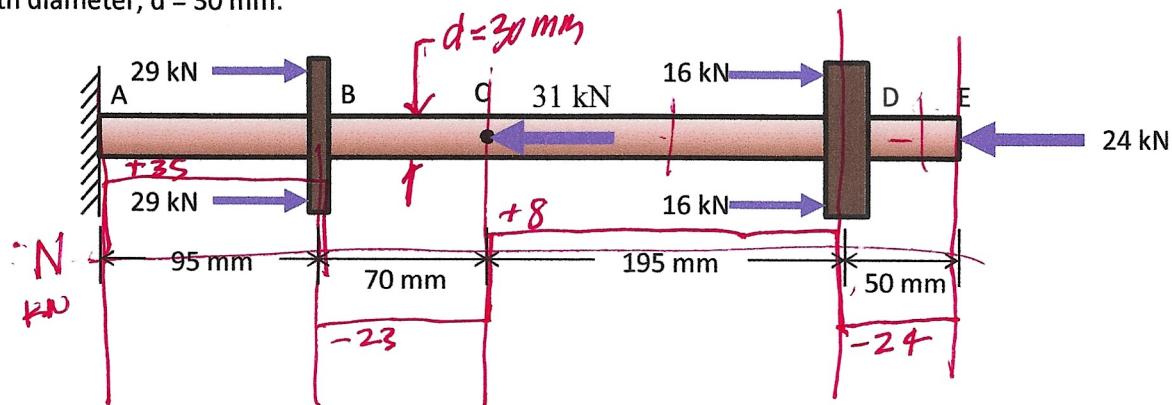
- 7) Determine the angle of twist of point A relative to point B. (8 pts)

$$\text{Angle of Twist} = -0.0311 \text{ rad}$$

$$\phi_{AB} = \frac{TL}{JG} = \frac{-2.6(1000)(1000)(2200)}{4.084E6(45000) \frac{\text{N}}{\text{mm}}} = -0.03112 \text{ rad}$$

Axial (Normal) Stress, Strain and Deformation

The system is loaded and supported as shown. The system is made of Titanium, $E_T = 140 \text{ GPa}$. It is a solid shaft with diameter, $d = 30 \text{ mm}$.



11) Calculate the axial normal stress in section BC, (6 pts)

$$\sigma = \frac{P}{A} = \frac{-23000}{\pi/4(30)^2} = -32.54 \text{ MPa (c)}$$

MC \Rightarrow

Normal stress in section BC = -32.5 MPa (c)

12) Calculate the displacement of point B relative to point D. (10 pts)

$$\delta_{BC} = \frac{PL}{AE} = -\frac{23000(70)}{\pi/4(30)^2(140000)} = -0.01627 \text{ mm}$$

$$\delta_{CD} = \frac{+8000(195)}{\pi/4(30)^2(140000)} = +0.01576 \text{ mm}$$

Displacement of B relative to D = -505E-6 mm

13) Calculate the axial normal strain in section DE. (4 pts)

a. $-252 (10^{-6}) \text{ mm/mm}$

c. $-262 (10^{-6}) \text{ mm/mm}$

b. $243 (10^{-6}) \text{ mm/mm}$

d. $-233 (10^{-6}) \text{ mm/mm}$

$$\begin{aligned} E = \frac{\sigma}{\epsilon} &= \frac{\sigma}{E} = \frac{P/AE}{E} \\ &= -\frac{24000}{\pi/4(30)^2(140000)} \\ &= 242.5 E-6 \text{ mm/mm} \end{aligned}$$

$$M_y = \frac{15}{17}(6.8) = 6.0 \text{ kN-m}$$

Bending Stress and Flexure Formula

The cross section of a W200 x 36 is subjected to the resultant internal bending moment as shown.

$$I_x = 7.64 (10^6) \text{ mm}^4, I_y = 34.4 (10^6) \text{ mm}^4$$

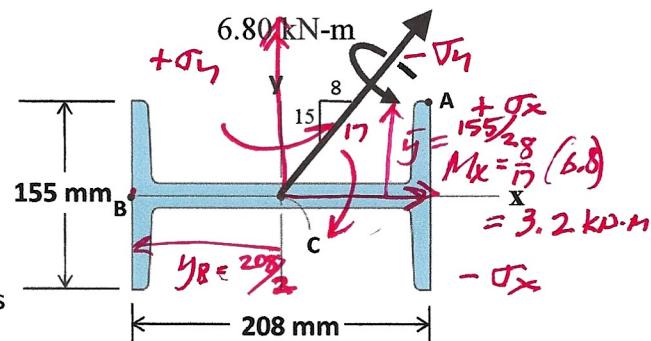
$$+\sigma_y + \sigma_x$$

- 14) Determine the direction of the bending stress at B. (2 pts)

a. + (Tension)

b. - (Compression)

c. No Stress



- 15) Calculate the magnitude of the bending stress at B. (4 pts)

a. 18.1 MPa

b. 18.8 MPa

c. 0

d. 17.4 MPa

$$\sigma_B = \frac{M_y y}{I_y} = \frac{6.0(1000)(1000)(\frac{208}{2})}{34.4E6} = 18.14 \text{ MPa}$$

- 16) Calculate the bending stress at A. (8 pts)

$$\begin{aligned} \sigma_A &= +\sigma_x - \sigma_y \\ &= +\frac{M_x y}{I_x} - \frac{M_y y}{I_y} \\ &= \frac{3.2(1000)(1000)(\frac{155}{2})}{7.64E6} - 18.14 \text{ MPa} = +14.32 \text{ MPa} \end{aligned}$$

Bending Stress @ A = $\frac{+14.3 \text{ MPa}}{+14.3 \text{ MPa}}$

Transverse Shear

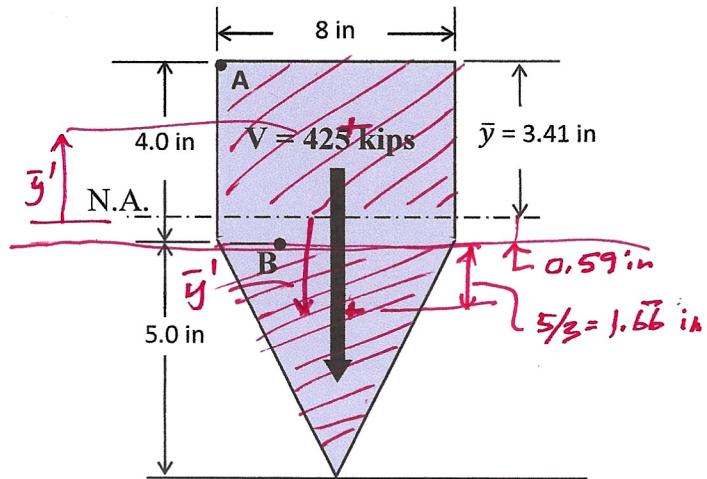
The cross-section has a moment of inertia about the neutral axis, $I = 236 \text{ in}^4$. Point B lies 4.0 inches below the top of the cross-section, 5.0 inches above the bottom of the cross-section.

- 17) Calculate the maximum transverse shear stress. (10 pts)

$$\tau = \frac{VQ}{I\epsilon} = \frac{425 \left(\frac{3.41}{2}\right)(3.41)(8)}{(236)(8)}$$

$$= 10.47 \text{ ksi}$$

Maximum Shear Stress = 10.5 ksi



- 18) Calculate the transverse shear stress at point B. (4 pts)

- a. 10.6 ksi b. 11.0 ksi

- c. 10.2 ksi d. 9.80 ksi

$$\tau_B = \frac{VQ}{I\epsilon} = \frac{425 \left(\frac{5}{3} + 0.59 \right) \left(\frac{1}{2} \left(\frac{5}{3} \right) (8) \right)}{236 (8)}$$

$$= 10.16 \text{ ksi}$$

Thin Walled Pressure Vessels

**CHECK
SOLUTION**

A pressurized **cylindrical** tank with a diameter 3.5 m, made of 25 mm thick steel, $E = 200 \text{ GPa}$. The failure stress of the steel is 250 MPa.

- 1) Calculate the maximum pressure the tank can contain with a Factor of Safety of 2.7. (6 pts)

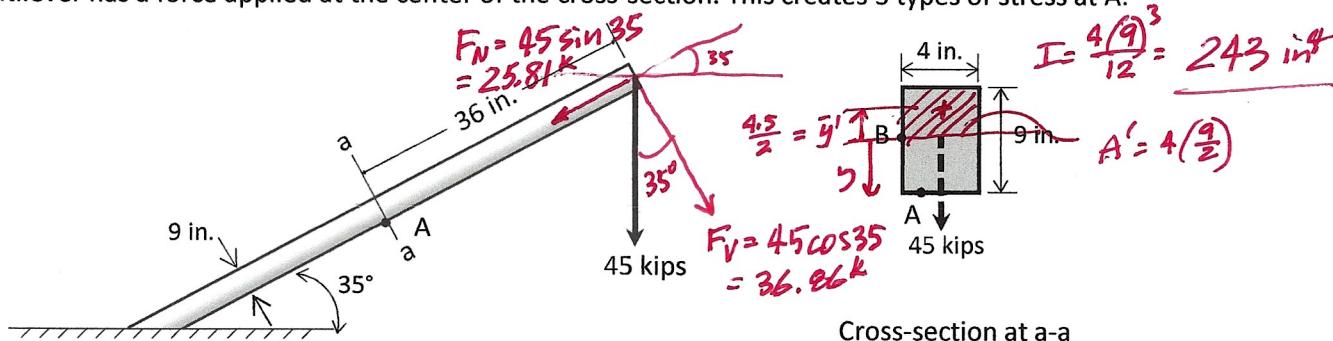
$$\sigma = \frac{Pr}{t} \quad P = \frac{\sigma t}{r} = \frac{250(25)}{343.8} = 3.623 \text{ MPa}$$

$$\div 2.7 = 1.342 \text{ MPa}$$

Maximum Pressure = 1.34 MPa

Combined Loads and Stresses

The cantilever has a force applied at the center of the cross-section. This creates 3 types of stress at A.



- 2) Calculate the normal stress caused by the axial load at point A. (3 pts)

- a. -0.661 ksi c. 0.688 ksi
 b. 0.747 ksi d. -0.717 ksi

$$\sigma_{Ax} = \frac{P}{A} = \frac{25.81}{9(4)} = 0.7170 \text{ ksi}$$

- 3) Calculate the normal stress caused by the bending moment at point A. (3 pts)

- a. 27.6 ksi c. -25.6 ksi
 b. -24.6 ksi d. 26.6 ksi

$$\sigma_{Bend} = \frac{My}{I} = \frac{-36.86(36)\left(\frac{9}{2}\right)}{243} = -24.57 \text{ ksi}$$

- 4) Calculate the transverse shear stress at point B (midheight of the cross-section). (3 pts)

- a. 1.54 ksi c. 1.70 ksi
 b. 1.60 ksi d. 1.47 ksi

$$\tau = \frac{VQ}{It} = \frac{36.86(4)\left(\frac{9}{2}\right)\left(\frac{4.5}{2}\right)}{243(4)} = 1.536 \text{ ksi}$$

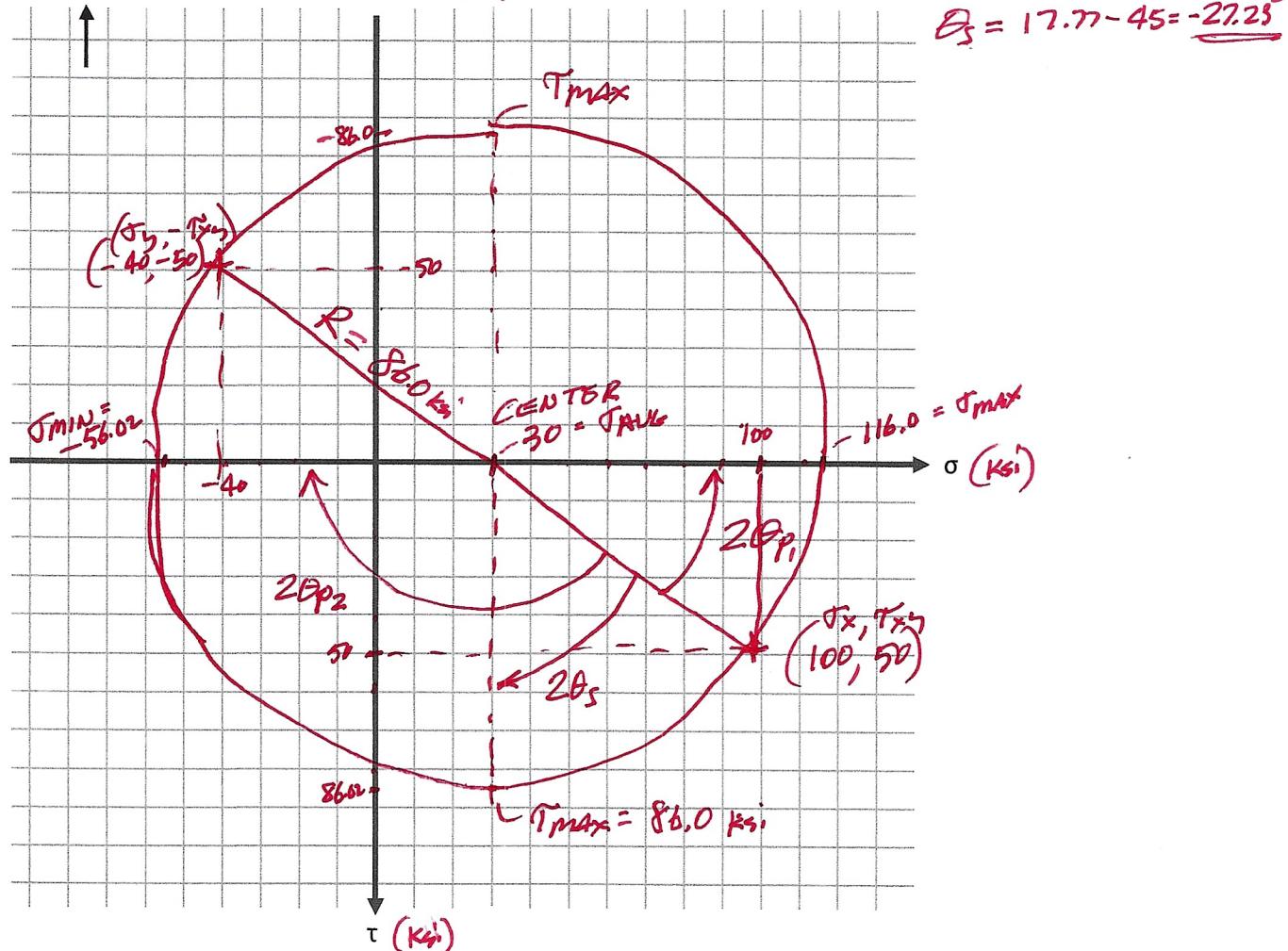
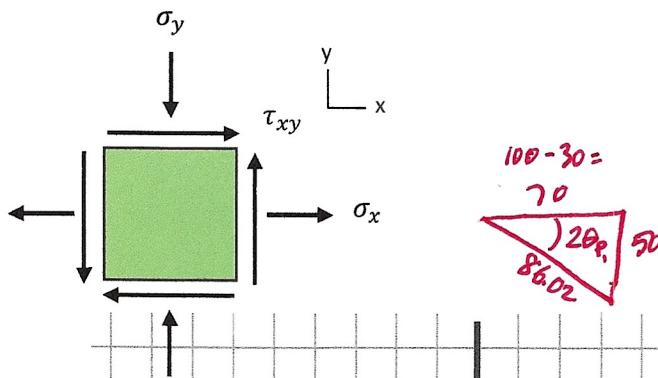
- 5) Determine the combined normal stress at point A (3 pts)

Normal Stress = -25.3 ksi (C)
 means

$$\sigma_A = -24.57 - 0.717 = -25.29 \text{ ksi}$$

Mohr's Circle (15 pts)

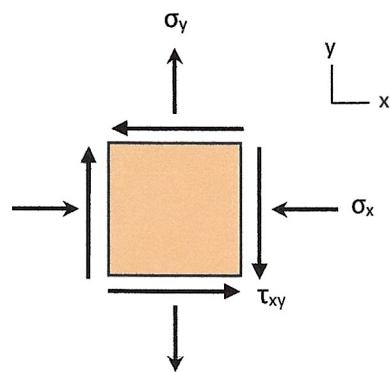
- 6) Draw Mohr's Circle for the Stress Element shown.
- 7) Show and give the stress values shown on the Stress Element ($\sigma_x = 100 \text{ MPa}$, $\sigma_y = 40 \text{ MPa}$, $\tau_{xy} = 50 \text{ MPa}$, signs shown on Stress Element),
- 8) Show and give the values of the 3 principal stresses,
- 9) Show and give their 3 angles of orientation,
- 10) Show and give the average normal stress (σ_{ave}),
- 11) Show and give the radius of Mohr's Circle (R)



Stress Transformation

The magnitudes of the state of stress at a point on a member was recorded as $\sigma_x = 140 \text{ MPa}$, $\sigma_y = 70 \text{ MPa}$, and $\tau_{xy} = 30 \text{ MPa}$.

$$\begin{aligned}\sigma_x &= -140 \text{ MPa} \\ \sigma_y &= +70 \text{ MPa} \\ \tau_{xy} &= -30 \text{ MPa}\end{aligned}$$



$$\begin{aligned}\sigma'_x &= -\frac{140+70}{2} + -\frac{140-70}{2}\cos 2(-30) + (-30)\sin 2(-30) \\ &= (-35) + (-52.5) + (25.98) \\ &= \boxed{-61.52 \text{ MPa (C)}}\end{aligned}$$

 -30°

- 12) Determine the normal stress acting in the x' direction if the element is oriented 30° clockwise from its original position. $\boxed{3}$ (5 pts)

- a. 59.1 MPa (C)
- b. 56.7 MPa (C)
- c. $\boxed{61.5 \text{ MPa (C)}}$
- d. 63.9 MPa (C)

$$\sigma'_{y'} = (-35) - (-52.5) - (25.98) = \boxed{-8.481 \text{ MPa (C)}}$$

- 13) Determine the normal stress acting in the y' direction if the element is oriented 30° clockwise from its original position. $\boxed{3}$ (5 pts)

- a. 9.17 MPa (T)
- b. 8.14 MPa (C)
- c. $\boxed{8.48 \text{ MPa (C)}}$
- d. 8.82 MPa (T)

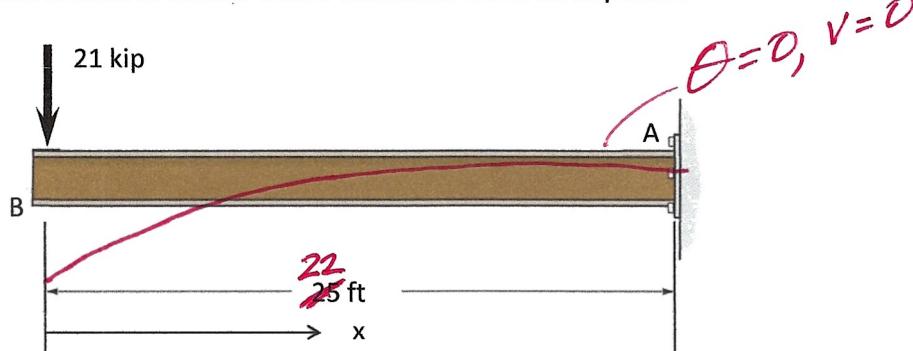
$$\begin{aligned}\tau'_{x'y'} &= -\frac{90.93}{2} + -15 = \boxed{-105.9 \text{ MPa}} \\ &= -\frac{140-70}{2}\sin 2(-30) + (-30)\cos 2(-30)\end{aligned}$$

- 14) Determine the magnitude of the transformed in-plane shear stress if the element is oriented 30° clockwise from its original position. $\boxed{3}$ (5 pts)

- a. 114 MPa
- b. 102 MPa
- c. 110 MPa
- d. $\boxed{106 \text{ MPa}}$

Elastic Curve – Constants of Integration, Slope and Deflection

Given the beam loaded as shown with a concentrated Load at point B

The Moment Equation is: $EI M(x) = -21x$ kip-ft

$$\theta(x) = 22, \theta(0) = -10.5(22)^2 + C_1$$

$$C_1 = +5082$$

The Slope Equation is: $EI \theta(x) = -10.5x^2 + C_1$ kip-ft²The Deflection Equation is: $EI v(x) = -3.5x^3 + C_1x + C_2$ kip-ft³Evaluate the Constants of Integration. ⁴ (5 pts each)

17) $C_1 =$

a. 5820

c. 0

b. 5082

d. -5280

18) $C_2 =$

a. -74536

c. -76543

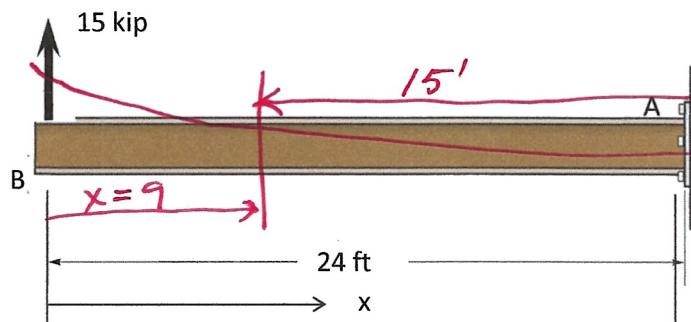
b. -73654

d. -78635

$$\theta(x) = 22, v(0) = -3.5(22)^3 + 5082(22) + C_2$$

$$-37268 + 111804$$

$$C_2 = -74536$$

Given the beam loaded as shown. $E = 29,000$ ksi, $I = 1670$ in.⁴The Deflection Equation is: $EI v(x) = 2.5x^3 - 4320x + 69120$ kip-ft³19) Calculate the deflection 15 ft left of point A. ⁸ (10 pts)

$$= 2.5(9)^3 - 4320(9) + 69120 =$$

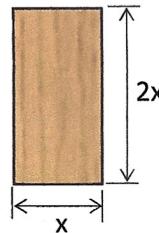
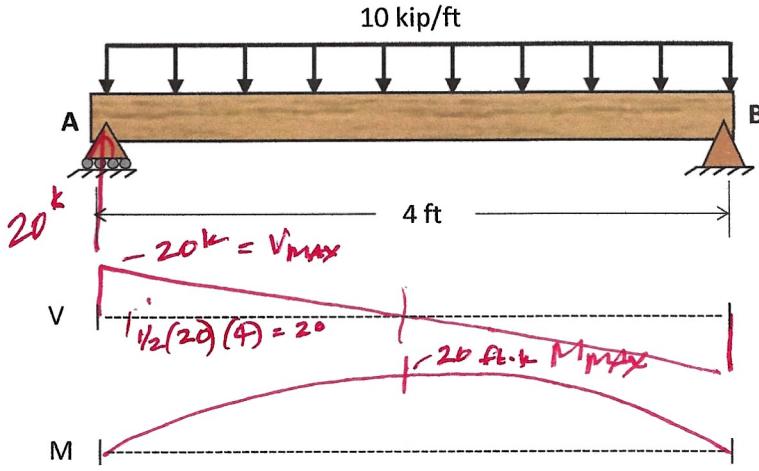
$$= 32062.5 \frac{\text{kip} \cdot \text{in}^3}{(29000)(1670)}$$

$$= +1.144 \text{ in}$$

Deflection = +1.14 in

Design of Beams

The simply supported beam is made of timber that has an allowable bending stress of $\sigma_{allow} = 1.60 \text{ ksi}$ and an allowable shear stress of $\tau_{allow} = 210 \text{ psi}$. (The architect wants the beam with proportions shown.)
 (Hint: Draw a Shear and Moment Diagram to find V_{max} and M_{max})



cross-section

$$I = \frac{bh^3}{12} \quad S = \frac{I}{c}$$

$$S = \frac{bh^3}{12} / h/2 = \frac{bh^2}{6}$$

$$= \frac{(x)(2x)^3}{6} = \frac{4}{6}x^3$$

$$= \frac{2}{3}x^3$$

20) Calculate the required dimension, x, for bending stress. (9 pts)

a. 6.32 in

c. 6.58 in

b. 6.08 in

d. ~~6.07 in~~
5.84

$\sigma = \frac{M}{S}$

$S = \frac{M}{\sigma} = \frac{20(12)}{1.60}$

$= \frac{2}{3}x^3$

$x = \sqrt[3]{\frac{3}{2}(150)}$

$= 6.08 \text{ in}$

21) Calculate the required dimension, x, for shear stress. (9 pts)

a. 8.45 in

c. 9.14 in

b. 8.79 in

d. 8.11 in

rectangle
 $C_T = 1.5 \frac{V}{A}$

$A = \frac{1.5V}{C_T} = \frac{1.5(20)}{0.210 \text{ ksi}} = 142.86 \text{ in}^2$

$= 2x^2$

$x = \sqrt{\frac{142.86}{2}} = 8.452 \text{ in}$

22) Rounding to the nearest inch, what dimension x should be used for the final beam design?

(2 pts)

a. 6.00 in

c. 8.00 in

b. 7.00 in

d. 9.00 in

9.0"

Column Buckling Load

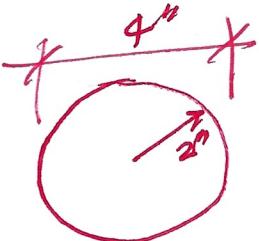
$$I = \frac{\pi}{4} r^4 = \frac{\pi}{4} \left(\frac{4}{2}\right)^4 = 12.566 \text{ in}^4$$

An A-36 steel rod with a 4 inch diameter (circular) cross section ($I = ????$ in. 4) is to be used as a column. ($E_{st} = 29,000$ ksi, $\sigma_y = 36$ ksi)

- 23) Determine the maximum allowable axial load the column can support without buckling if the 15 foot long column is fixed at one end and pinned at the other. (10 pts)

$$P_{cr} \cdot P_{un} \quad K = 0.7$$

Maximum Axial Load = 227 kip



$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 (29000)(12.566)}{(0.7)(15)(12))^2}$$

$$= 226.6 \frac{\text{k}}{\text{in}^2}$$

- 24) Determine the maximum length of the column to support a 200 kip load without buckling. The column is pinned at both ends. (5 pts)

$$K = 1.0$$

(A) $\frac{9.61}{11.2} \text{ ft}$

(C) $\frac{9.25}{12.0} \text{ ft}$

(B) $\frac{8.53}{11.6} \text{ ft}$

(D) $\frac{8.89}{10.8} \text{ ft}$

$$(KL)^2 = \frac{\pi^2 EI}{P}$$

$$(1.0)^2 (L)^2 = \frac{\pi^2 (29000)(12.566)}{200 \frac{\text{k}}{\text{in}^2}} = (17984 \text{ in}^2)$$

$$L = \sqrt{\frac{17984}{134.1}} = 134.1 \text{ in} / 12 = \boxed{11.18'}$$