

■ Work It Out: Rectilinear Motion (Solution)

Problem:

An object experiences rectilinear acceleration $a(t) = 10 - 2t$.
How far does it travel in 6 seconds if its initial velocity is $v(0) = 10$ m/s?

Step 1. Write down what's given

Acceleration: $a(t) = 10 - 2t$

Initial velocity: $v(0) = 10$ m/s

Initial position: $x(0) = 0$ (assume origin)

Time interval: $0 \leq t \leq 6$ s

Step 2. Relate acceleration to velocity

$$a(t) = dv/dt = 10 - 2t$$

Integrate:

$$v(t) = \int (10 - 2t) dt = 10t - t^2 + C$$

Step 3. Apply initial condition for velocity

At $t = 0$, $v(0) = 10$:

$$10 = (10 \cdot 0 - 0^2) + C \rightarrow C = 10$$

$$\text{So: } v(t) = 10t - t^2 + 10$$

Step 4. Relate velocity to position

$$v(t) = dx/dt$$

Integrate:

$$x(t) = \int (10t - t^2 + 10) dt$$

$$x(t) = 5t^2 - (1/3)t^3 + 10t + C$$

Step 5. Apply initial condition for position

At $t = 0$, $x(0) = 0$:

$$0 = (5 \cdot 0^2 - (1/3) \cdot 0^3 + 10 \cdot 0) + C \rightarrow C = 0$$

$$\text{So: } x(t) = 5t^2 - (1/3)t^3 + 10t$$

Step 6. Evaluate displacement

At $t = 6$:

$$x(6) = 5(36) - (1/3)(216) + 10(6)$$

$$x(6) = 180 - 72 + 60 = 168 \text{ m}$$

Step 7. Check units and reasonableness

Units: $x(t)$ is in meters ■

Displacement after 6 seconds is 168 m ■

The object starts with positive velocity and slows (since acceleration decreases), but still covers a significant positive distance.

■ **Final Answer:**

The object travels **168 meters** in 6 seconds.