CE 4353/5360 October 10, 2016

Homework 5, Due October 19

1. Generate the Q/Q_{full}, V/V_{full} vs. y/d curves known as the hydraulic elements graph for partially full flow in circular pipes. You can assume a pipe diameter, n, and slope. Use at least 25 pairs of points to define the curves. You can show the graph on the same page as the Excel table. Remember to show your name on your spreadsheet printout, and to provide sample calculations along one row.
2. Problem 4.1 with Q = 3200 cfs.
3. Repeat 4.1 for a rectangular channel with b=40 ft
4. Problem 4.2 with 17 cfs.
5. Problem 4.11 with $Q_{max} = 1.2$ m^3/sec and $Q_{min} = 0.3$ m^3/sec.

Hydraulic Elements Graph

Can assume the following

d = 1 ft
n = 0.013
S = 0.005

y/d	y (ft)	θ (rad)	A (ft^2)	P (ft)	R (ft)	Q (cfs)	V (fps)	Q/Qfull	V/Vfull
0.00	0.00	0.000	0.000	0.000	0.000	0.000	0	0	0
0.02	0.02	0.568	0.004	0.284	0.013	0.002	0.453	0.001	0.141
0.04	0.04	0.805	0.011	0.403	0.026	0.008	0.714	0.003	0.222
0.06	0.06	0.990	0.019	0.495	0.039	0.018	0.930	0.007	0.289
0.08	0.08	1.147	0.029	0.574	0.051	0.033	1.119	0.013	0.348
0.10	0.10	1.287	0.041	0.644	0.064	0.053	1.290	0.021	0.401
0.12	0.12	1.415	0.053	0.707	0.075	0.077	1.447	0.031	0.450
0.14	0.14	1.534	0.067	0.767	0.087	0.106	1.593	0.042	0.495
0.16	0.16	1.646	0.081	0.823	0.099	0.140	1.729	0.056	0.538
0.18	0.18	1.753	0.096	0.876	0.110	0.179	1.857	0.071	0.577
0.20	0.20	1.855	0.112	0.927	0.121	0.221	1.978	0.088	0.615
0.22	0.22	1.953	0.128	0.976	0.131	0.268	2.093	0.106	0.651
0.24	0.24	2.048	0.145	1.024	0.142	0.319	2.201	0.126	0.684
0.26	0.26	2.140	0.162	1.070	0.152	0.374	2.305	0.148	0.717
0.28	0.28	2.230	0.180	1.115	0.161	0.433	2.403	0.171	0.747
0.30	0.30	2.319	0.198	1.159	0.171	0.495	2.496	0.196	0.776
0.32	0.32	2.405	0.217	1.203	0.180	0.560	2.585	0.222	0.804
0.34	0.34	2.490	0.235	1.245	0.189	0.629	2.670	0.249	0.830
0.36	0.36	2.574	0.255	1.287	0.198	0.700	2.751	0.277	0.855
0.38	0.38	2.657	0.274	1.328	0.206	0.775	2.828	0.307	0.879
0.40	0.40	2.739	0.293	1.369	0.214	0.851	2.902	0.337	0.902
0.42	0.42	2.820	0.313	1.410	0.222	0.930	2.971	0.368	0.924
0.44	0.44	2.901	0.333	1.451	0.229	1.011	3.038	0.400	0.944
0.46	0.46	2.981	0.353	1.491	0.237	1.094	3.101	0.433	0.964
0.48	0.48	3.062	0.373	1.531	0.243	1.178	3.160	0.466	0.983
0.50	0.50	3.142	0.393	1.571	0.250	1.263	3.216	0.500	1.000
0.52	0.52	3.222	0.413	1.611	0.256	1.349	3.269	0.534	1.016
0.54	0.54	3.302	0.433	1.651	0.262	1.436	3.319	0.568	1.032
0.56	0.56	3.382	0.453	1.691	0.268	1.523	3.366	0.603	1.046
0.58	0.58	3.463	0.472	1.731	0.273	1.610	3.409	0.637	1.060
0.60	0.60	3.544	0.492	1.772	0.278	1.697	3.449	0.672	1.072
0.62	0.62	3.626	0.512	1.813	0.282	1.783	3.486	0.706	1.084
0.64	0.64	3.709	0.531	1.855	0.286	1.869	3.520	0.740	1.094
0.66	0.66	3.793	0.550	1.897	0.290	1.953	3.551	0.773	1.104
0.68	0.68	3.878	0.569	1.939	0.293	2.035	3.578	0.806	1.112
0.70	0.70	3.965	0.587	1.982	0.296	2.115	3.602	0.837	1.120
0.72	0.72	4.053	0.605	2.026	0.299	2.193	3.622	0.868	1.126
0.74	0.74	4.143	0.623	2.071	0.301	2.267	3.639	0.898	1.131
0.76	0.76	4.235	0.640	2.118	0.302	2.339	3.652	0.926	1.135
0.78	0.78	4.330	0.657	2.165	0.304	2.406	3.661	0.953	1.138
0.80	0.80	4.429	0.674	2.214	0.304	2.469	3.666	0.977	1.140
0.82	0.82	4.531	0.689	2.265	0.304	2.527	3.666	1.000	1.140
0.84	0.84	4.637	0.704	2.319	0.304	2.579	3.662	1.021	1.139
0.86	0.86	4.749	0.719	2.375	0.303	2.625	3.653	1.039	1.136
0.88	0.88	4.868	0.732	2.434	0.301	2.663	3.638	1.054	1.131
0.90	0.90	4.996	0.745	2.498	0.298	2.692	3.616	1.066	1.124
0.92	0.92	5.136	0.756	2.568	0.294	2.711	3.586	1.073	1.115
0.94	0.94	5.293	0.766	2.647	0.289	2.717	3.547	1.076	1.103
0.96	0.96	5.478	0.775	2.739	0.283	2.706	3.493	1.071	1.086
0.98	0.98	5.716	0.782	2.858	0.274	2.669	3.415	1.057	1.062
1.00	1.00	6.283	0.785	3.142	0.250	2.526	3.216	1.000	1.000
						Q_{full}	V_{full}		

$$\theta = 2\cos^{-1}\left(1-2\frac{y}{d}\right)$$

$$A = (\theta - \sin\theta)\frac{d^2}{8}$$

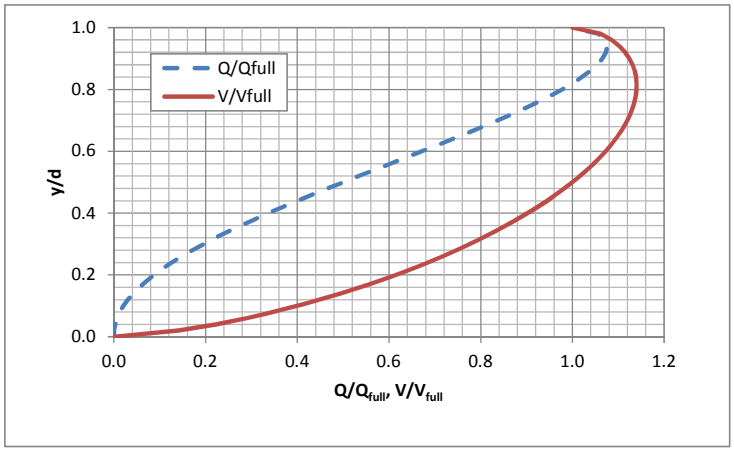
$$P = \frac{\theta d}{2}$$

$$R = \frac{A}{P}$$

$$Q = \frac{1.49}{n} AR^{2/3} S^{1/2}$$

$$V = \frac{Q}{A}$$

20 points
16 for calculations
4 for graph



2] 4.1 Given: Trapezoidal channel $b = 40\,ft$, $m = 3$, $S_o = 0.002$
$n = 0.025$ $Q = 3200\,cfs$
Find (i) y_o & y_c, Slope steep or mild?
(ii) Repeat for $n = 0.012$. Does y_c change?

(i) y_o: $Q = \frac{1.49}{n} A R^{2/3} S_o^{1/2}$

$$A R^{2/3} = \frac{nQ}{1.49 S_o^{1/2}}$$

[6] $$A R^{2/3} = A\left(\frac{A}{P}\right)^{2/3} = \frac{A^{5/3}}{P^{2/3}} = \frac{[y_o(b+m y_o)]^{5/3}}{[b+2y_o(1+m^2)^{1/2}]^2} = \frac{nQ}{1.49 S_o^{1/2}}$$

[4] $$\frac{[y_o(40ft + 3y_o)]^{5/3}}{[40ft + 2y_o(1+3^2)^{1/2}]^{2/3}} = \frac{0.025(3200cfs)}{1.49(0.002)^{1/2}} = 1201$$

w/Goal Seek → $\boxed{y_o = 6.82ft}$

y_c: $\frac{Q^2}{g} \frac{B}{A_c^3} = 1$

[6] $$\frac{Q^2}{g} = \frac{A_c^3}{B_c} = \frac{[y_c(b+m y_c)]^3}{b + 2m y_c}$$

$$\frac{(3200cfs)^2}{32.2 ft/s^2} = \frac{[y_c(40ft + 3y_c)]^3}{40ft + 2(3)y_c} = \frac{[40y_c + 3y_c^2]^3}{40 + 6y_c}$$

w/Goal Seek → $\boxed{y_c = 5.10ft}$

[1] $y_o > y_c$ slope is mild

(ii) Repeat for $n = 0.012$

y_c does not change, n is not in y_c equation

[6] $$\frac{nQ}{1.49 S_o^{1/2}} = \frac{0.012(3200cfs)}{1.49(0.002)^{1/2}} = 576.3$$

Relate to Eq. [4] → Goal Seek

$\boxed{y_o = 4.59\,ft}$

[1] $y_o < y_c$, slope is steep

3] Given: Repeat 4.1 for rectangular X-section
$b = 40$ ft, $n = 0.025$, $Q = 3200$ cfs, $S_o = 0.002$

Find: (i) y_o & y_c, is slope steep or mild?
(ii) Repeat for $n = 0.012$

(i) y_o: $Q = \frac{1.49}{n} A R^{2/3} S_o^{1/2}$

$$AR^{2/3} = \frac{Qn}{1.49 S_o^{1/2}}$$

$$AR^{2/3} = \frac{A^{5/3}}{P^{2/3}} = \frac{(b y_o)^{5/3}}{(b+2y_o)^{2/3}} = \frac{nQ}{K_n S_o^{1/2}}$$

[A] $\frac{[(40\text{ft})(y_o)]^{5/3}}{[40\text{ft}+2y_o]^{2/3}} = \frac{0.025(3200\text{ cfs})}{1.49(0.025)^{1/2}} = 1201$

[6] w/ Goal Seek $y_o = 8.92$ ft

y_c: $y_c = \left[\frac{q^2}{g}\right]^{1/3}$

[6] $q = \frac{Q}{b} = \frac{3200\text{ cfs}}{40\text{ ft}} = 80\text{ ft}^2/\text{sec}$

$$y_c = \left[\frac{(80\text{ ft}^2/\text{s})^2}{32.2\text{ ft/s}^2}\right]^{1/3} = 5.84\text{ ft}$$

[1] $y_o > y_c$, slope is mild

(ii) Repeat for $n = 0.012$

[6] $\frac{nQ}{1.49 S_o^{1/2}} = \frac{0.012(3200\text{ cfs})}{1.49(0.002)^{1/2}} = 576.3$

relate to [A] LHS, use Goal Seek

$y_o = 5.46$ ft

y_c does not change

[1] $y_o < y_c$, slope is steep

4 | 4.2 modified Given: Concrete culvert, $n=0.015$, $D=36in$, $S=0.002$

$Q = 17$ cfs

Find: (i) y_0 & y_c Is slope steep or mild?

(ii) Repeat for $S=0.02$

(i) y_0: $AR^{2/3} = \frac{nQ}{1.49 S_0^{1/2}}$

$A = (\theta - \sin\theta)\frac{d^2}{8}$

$P = \frac{\theta d}{2}$

$AR^{2/3} = A\left(\frac{A}{P}\right)^{2/3} = \frac{A^{5/3}}{P^{2/3}}$

[A] $\theta = 2\cos^{-1}\left(1 - 2\frac{y}{d}\right) = 2\cos^{-1}\left[1 - 2\left(\frac{y_0}{3ft}\right)\right]$

6 $\frac{\left[(\theta - \sin\theta)\frac{d^2}{8}\right]^{5/3}}{\left[\frac{\theta d}{2}\right]^{2/3}} = \frac{nQ}{1.49 S_0^{1/2}}$

[B] $\frac{\left[(\theta - \sin\theta)\frac{(3ft)^2}{8}\right]^{5/3}}{\left[\frac{\theta (3ft)}{2}\right]^{2/3}} = \frac{(0.015)(17cfs)}{1.49(0.002)^{1/2}} = 3.83$

Used Goal seek $\boxed{y_0 = 1.77ft}$ $\theta = 3.51$ rad

y_c: $\frac{Q^2}{g} = \frac{A_c^3}{B_c}$

$\theta = 2\cos^{-1}\left[1 - 2\left(\frac{y_c}{3ft}\right)\right]$

6 $B_c = d\sin\frac{\theta}{2}$

$\frac{Q^2}{g} = \frac{(17cfs)^2}{(32.2ft/s^2)} = \frac{\left[(\theta - \sin\theta)\frac{(3ft)^2}{8}\right]^3}{\left[(3ft)\sin\frac{\theta}{2}\right]}$

Used Goal Seek $\boxed{y_c = 1.32ft}$ $\theta = 2.89$ rad

1 $y_0 > y_c$, slope is mild

(ii) repeat $S_0 = 0.02$

$\frac{nQ}{1.49 S_0^{1/2}} = \frac{0.015(17cfs)}{1.49(0.02)^{1/2}} = 1.210$

6 relate to [A] & [B] above use Goal Seek

$\boxed{y_0 = 0.92ft}$, $\theta = 2.36$ rad

1 $\boxed{y_0 < y_c, \text{ slope is steep}}$

5] 4.11 modified Given: Concrete sewer for $Q_{max} = 1.2\ m^3/sec$, $Q_{min} = 0.3\ m^3/sec$
$S_o = 0.0018$

Find: Diameter d
check V for self-cleaning

(1) Estimate diameter (as in text)

$$d = 1.56\left[\frac{n Q}{K_n S_o^{1/2}}\right]^{3/8} = 1.56\left[\frac{(0.015)(1.2\ m^3/sec)}{(1.0)(0.0018)^{1/2}}\right]^{3/8} = 1.56(0.424)^{3/8}$$

[A] $d = 1.13\ m\left(\frac{3.28\ ft}{m}\right) = 3.71\ ft\left(\frac{12\ in}{ft}\right) = 44.5\ in$

Say $d = 48\ in$ as nominal pipe size (or 1.22 m)

$AR^{2/3} = \frac{n Q}{K_n S_o^{1/2}} = 0.424$ from above

[B] $\theta = 2\cos^{-1}\left[1 - 2\left(\frac{y}{d}\right)\right] = 2\cos^{-1}\left[1 - 2\left(\frac{y}{1.22\ m}\right)\right]$

$A = (\theta - \sin\theta)\frac{d^2}{8} = (\theta - \sin\theta)\frac{(1.22\ m)^2}{8}$

$P = \frac{\theta d}{2} = \frac{\theta (1.22\ m)}{2}$

$R = \frac{A}{P} \Rightarrow AR^{2/3} = 0.424$ target

Use Goal Seek $\Rightarrow$ **$y = 0.83\ m$** $A = 0.843\ m^2$ $\theta = 3.87\ rad$

$$V_{max} = \frac{Q_{max}}{A} = \frac{1.2\ m^3/sec}{0.843\ m^2} = 1.42\ m/sec > 0.61\ m/sec$$

↳ OK at Q_{max}

At $Q_{min} = 0.3\ m^3/sec$

[B] $\frac{n Q}{K_n S_o^{1/2}} = \frac{0.015(0.3\ m^3/sec)}{1(0.0018)^{1/2}} = 0.106$

Same Eqs. Used Goal Seek, $AR^{2/3}$ target $= 0.106$

$y_{min} = 0.37\ m$ $A_{min} = 0.300\ m^2$ $\theta = 2.53\ rad$

$$V = \frac{Q}{A} = \frac{0.3\ m^3/sec}{0.30\ m^2} = 1.0\ m/sec > 0.61\ m/sec$$

↳ OK at Q_{min}