Homework 3, Due September 25

1. A hydraulic jump occurs in a rectangular channel. The channel is 20 ft wide, and the upstream depth is 3.5 ft at a flow rate of 2500 cfs. Find the downstream depth in ft and the head loss across the jump.

2. Problem 3.1 modified. A hydraulic jump is to be formed in a trapezoidal channel with a base width of 20 ft and side slopes of 2:1. The upstream depth is 1.35 ft and Q=1100 cfs.
[a] Find the downstream depth and the head loss in the jump. Solve by Figure 3.2.
[b] Now that both y_1 and y_2 are known for the trapezoidal channel, calculate both M_1 and M_2 to show M_1 = M_2.

3. Problem 3.2 modified.
[a] Determine the sequent depth for a hydraulic jump in a 3-ft diameter storm sewer with a flow depth of 0.65 ft at a discharge of 6.5 cfs. Solve by Figure 3.3.
[b] Now that both y_1 and y_2 are known, calculate both M_1 and M_2 to show M_1 = M_2.

4. Problem 3.4 modified. A flume with triangular cross section contains water flowing at a depth of 0.15 m and at a discharge of 0.35 m^3/s. The side slopes of the flume are 2:1. Determine the sequent depth for a hydraulic jump.

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1. Given: Hydraulic jump in rectangular channel.

$$b = 20 \text{ ft} \quad y_1 = 3.5 \text{ ft} \quad Q = 2500 \text{ cfs}$$

Find: y_2 Δ head loss across jump

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8 Fr_1^2}\right]$$

$$Fr_1 = \frac{V}{\sqrt{g y_1}}$$

[5]

$$V_1 = \frac{Q}{A_1} = \frac{Q}{b y_1} = \frac{2500 \text{ cfs}}{(20 \text{ ft})(3.5 \text{ ft})} = 35.7 \text{ ft/sec}$$

$$Fr_1 = \frac{35.7 \text{ ft/sec}}{\left[(32.2 \text{ ft/s}^2)(3.5 \text{ ft})\right]^{1/2}} = 3.36$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \left(1 + 8(3.36)^2\right)^{1/2}\right] = 4.28$$

$$y_2 = 4.28\, y_1 = 4.28(3.5 \text{ ft})$$

$$\boxed{y_2 = 15.0 \text{ ft}}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 3.5 \text{ ft} + \frac{(35.7 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)}$$

$$E_1 = 23.3 \text{ ft}$$

$$E_2 = y_2 + \frac{V_2^2}{2g}$$

[5]

$$V_2 = \frac{Q}{b y_2} = \frac{2500 \text{ cfs}}{(20 \text{ ft})(15 \text{ ft})} = 8.33 \text{ ft/sec}$$

$$E_2 = 15 \text{ ft} + \frac{(8.33 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)}$$

$$E_2 = 16.1 \text{ ft}$$

$$E_1 - E_2 = 23.3 \text{ ft} - 16.1 \text{ ft}$$

$$\boxed{E_1 - E_2 = 7.2 \text{ ft}}$$

$$\frac{E_L}{E_1} = \frac{7.2 \text{ ft}}{23.3 \text{ ft}} = 0.31$$

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3.1 modified | Given: Hydraulic jump in trapezoidal channel

$$b = 20 \text{ ft}, \quad m = 2 \text{ on side slopes}$$
$$y_1 = 1.35 \text{ ft} \quad Q = 1100 \text{ cfs}$$

Find: [a] y_2 & head loss in jump by Fig 3.2

[b] find M_1 & M_2; show $M_1 = M_2$

[a] Fig. 3.2 need $Z_{trap} = \dfrac{Q \, m^{3/2}}{g^{1/2} \, b^{5/2}}$

$$= \dfrac{(1100 \text{ cfs})(2)^{3/2}}{(32.2 \text{ ft/s})^{1/2}(20 \text{ ft})^{5/2}} = 0.31$$

$$\Rightarrow \dfrac{m y_1}{b} = \dfrac{2(1.35 \text{ ft})}{20 \text{ ft}} = 0.135 \Bigg\} \to \dfrac{y_2}{y_1} = 6.3$$

$$y_2 = 6.3 \, y_1 = 6.3 (1.35 \text{ ft})$$

$$\boxed{y_2 = 8.5 \text{ ft}}$$

$$E_1 = y_1 + \dfrac{V_1^2}{2g} = y_1 + \dfrac{Q^2}{2g A_1^2}$$

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$$A_1 = y_1(b + m y_1) = (1.35 \text{ ft})(20 \text{ ft} + 2(1.35 \text{ ft})) = 30.6 \text{ ft}^2$$

$$E_1 = 1.35 \text{ ft} + \dfrac{(1100 \text{ cfs})^2}{2(32.2 \text{ ft/s})(30.6 \text{ ft}^2)^2} = 21.4 \text{ ft}$$

$$E_2 = y_2 + \dfrac{Q^2}{2g A_2^2}$$

$$A_2 = (8.5 \text{ ft})[20 \text{ ft} + 2(8.5 \text{ ft})] = 314 \text{ ft}^2$$

$$E_2 = 8.5 + \dfrac{(1100 \text{ cfs})^2}{2(32.2 \text{ ft/s})(314 \text{ ft}^2)^2} = 8.7 \text{ ft}$$

8.5
722.5 + 409.4 119.5

$$E_1 - E_2 = 21.4 \text{ ft} - 8.7 \text{ ft}$$

$$\boxed{E_1 - E_2 = 12.7 \text{ ft}}$$

[b] $M_1 = \dfrac{b y_1^2}{2} + \dfrac{m y_1^3}{3} + \dfrac{Q^2}{g y_1(b + m y_1)}$

$$= \dfrac{(20 \text{ ft})(1.35 \text{ ft})^2}{2} + \dfrac{2(1.35 \text{ ft})^3}{3} + \dfrac{(1100 \text{ cfs})^2}{(32.2 \text{ ft/s})(1.35 \text{ ft})[20 \text{ ft} + 2(1.35 \text{ ft})]}$$

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$$= 18.23 \text{ ft}^3 + 1.64 \text{ ft}^3 + 1226.2 \text{ ft}^3$$

$$\boxed{M_1 = 1246 \text{ ft}^3}$$

$$M_2 = \dfrac{b y_2^2}{2} + \dfrac{m y_2^3}{3} + \dfrac{Q^2}{g y_2(b + m y_2)}$$

$$= \dfrac{20(8.5 \text{ ft})^2}{2} + \dfrac{2(8.5 \text{ ft})^3}{3} + \dfrac{(1100 \text{ cfs})^2}{(32.2 \text{ ft/s})(8.5 \text{ ft})[20 \text{ ft} + 2(8.5 \text{ ft})]}$$

$$= 656.1 \text{ ft}^3 + 354.3 \text{ ft}^3 + 128.2 \text{ ft}^3$$

$$\boxed{M_2 = 1251 \text{ ft}^3}$$

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3.2 modified | Given: Hydraulic jump in 3-ft diameter storm sewer

$$y_1 = 0.65 \text{ ft} \qquad Q = 6.5 \text{ cfs}$$

Find: [a] y_2 by Fig 3.3

[b] M_1 & M_2, so $M_1 = M_2$

[a] $$Z_{circ} = \frac{Q}{g^{1/2} d^{5/2}} = \frac{6.5 \text{ cfs}}{(32.2 \text{ ft/s}^2)^{1/2} (3 \text{ ft})^{5/2}} = 0.073$$

Fig 3.3 $\quad \dfrac{y_1}{d} = \dfrac{0.65 \text{ ft}}{3 \text{ ft}} = 0.22 \Rightarrow \dfrac{y_2}{y_1} = 1.5$

$$y_2 = 1.5\, y_1 = 1.5\,(0.65 \text{ ft})$$

$$\boxed{y_2 = 0.98 \text{ ft}}$$

$$M_1 = \left[3 \sin\frac{\theta_1}{2} - \sin^3\left(\frac{\theta_1}{2}\right) - 3\left(\frac{\theta_1}{2}\right)\cos\left(\frac{\theta_1}{2}\right)\right]\frac{d^3}{24} + \frac{Q^2}{g d^2 \left(\frac{\theta - \sin\theta}{8}\right)}$$

$$\theta = 2\cos^{-1}\left[1 - 2\left(\frac{y_1}{d}\right)\right]$$

$$\theta_1 = 2\cos^{-1}\left[1 - 2\frac{(0.65 \text{ ft})}{(3 \text{ ft})}\right] = 1.94 \text{ rad}$$

$$M_1 = \left[3\sin\frac{1.94}{2} - \sin^3\left(\frac{1.94}{2}\right) - 3\left(\frac{1.94}{2}\right)\cos\left(\frac{1.94}{2}\right)\right]\frac{(3 \text{ ft})^3}{24} + \frac{(6.5 \text{ cfs})^3}{(32.2 \text{ ft/s}^2)(3 \text{ ft})^2 \left[\frac{1.94 - \sin(1.94)}{8}\right]}$$

$$= [2.47 - 0.56 - 1.64]\frac{27}{24} + 1.16$$

$$= 0.30 + 1.16 = \underline{1.46 \text{ ft}^3}$$

$$\theta_2 = 2\cos^{-1}\left[1 - 2\left(\frac{1.1 \text{ ft}}{3 \text{ ft}}\right)\right] = 2.43 \text{ rad}$$

$$M_2 = \left[3\sin\left(\frac{2.43}{2}\right) - \sin^3\left(\frac{2.43}{2}\right) - 3\left(\frac{2.43}{2}\right)\cos\left(\frac{2.43}{2}\right)\right]\frac{(3 \text{ ft})^2}{24} + \frac{(6.5 \text{ cfs})^2}{(32.2 \text{ ft/s}^2)(3 \text{ ft})^2 \left[\frac{2.43 - \sin(2.43)}{8}\right]}$$

$$= [2.81 - 0.82 - 1.27]\frac{27}{24} + 0.66$$

$$\boxed{M_2 = 1.47 \text{ ft}^3}$$

3.4 modified | Given: Triangular flume   m = 2 side slopes

$$Q = 0.35 \, m^3/s \qquad y_1 = 0.15 \, m$$

Find: y_2 sequent depth

Can use Fig 3.5

$$F_{r_1} = \frac{V_1}{\sqrt{g D_1}}$$

$$D_1 = \frac{A_1}{B_1} = \frac{m y_1^2}{2 m y_1} = \frac{y_1}{2} = \frac{0.15 \, m}{2} = 0.075 \, m$$

$$V_1 = \frac{Q}{A_1} = \frac{0.35 \, m^3/s}{2(0.15 \, m)^2} = 7.78 \, m/sec$$

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$$F_{r_1} = \frac{7.78 \, m/sec}{[(9.81 \, m/s^2)(0.075 \, m)]^{1/2}} = 9.1$$

Fig 3.5 $\dfrac{y_2}{y_1} = 4.9$

$$y_2 = 4.9 \, y_1 = 4.9 \, (0.15 \, m)$$

$$y_2 = 0.74 \, m$$

3.4 modified | Given: Triangular flume   m = 2 side slopes