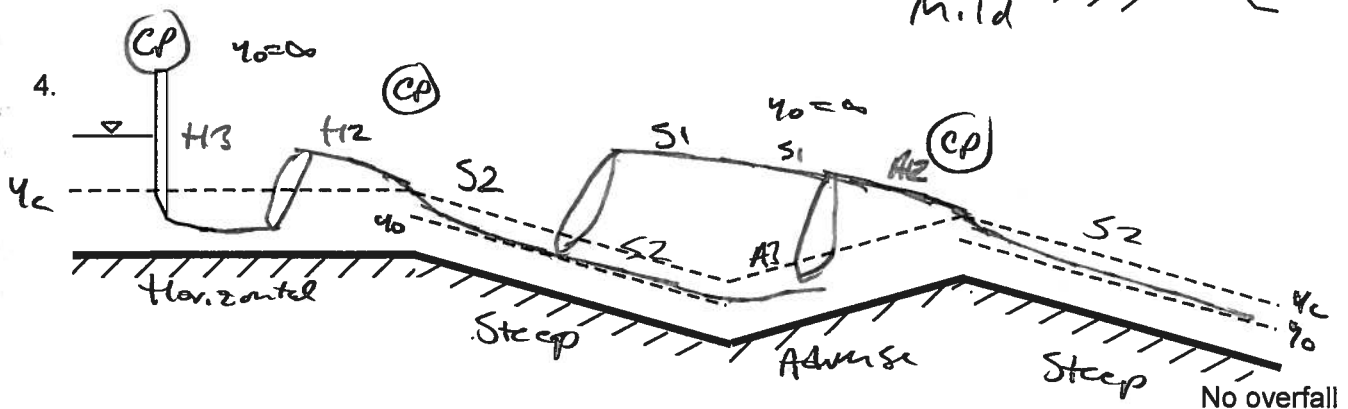HW 7, Due October 27, 2014
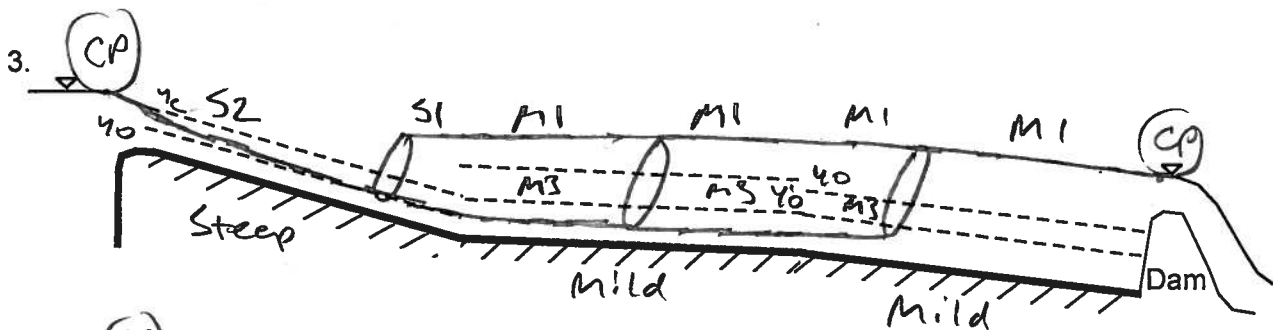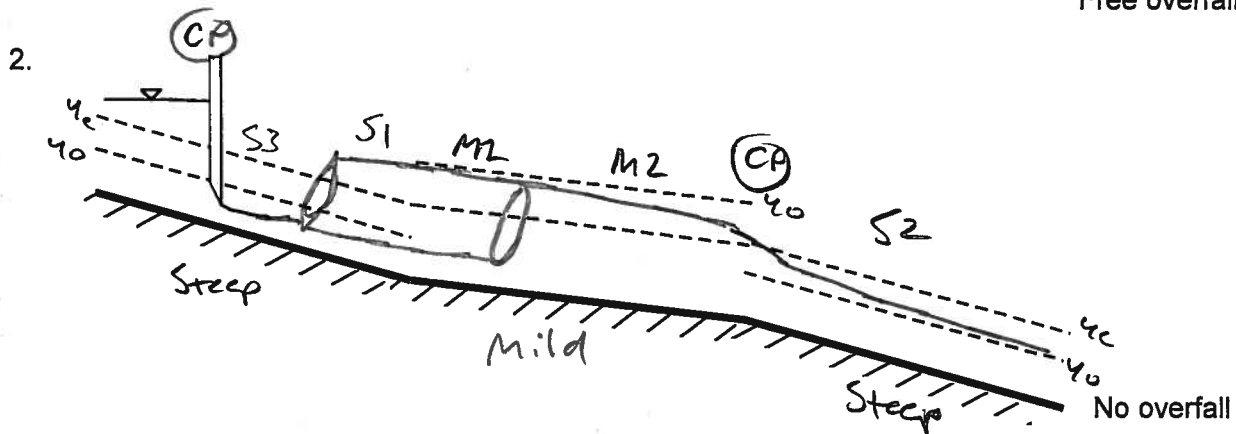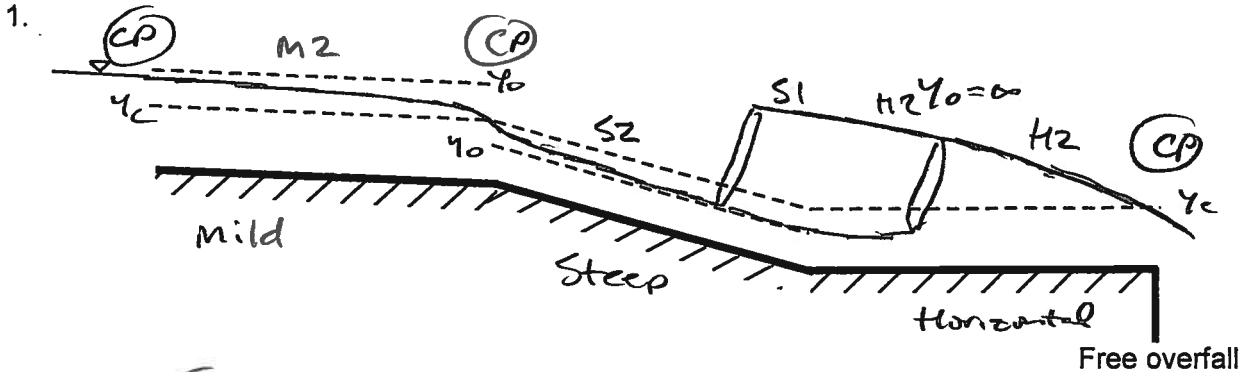
Problems 1-4.  Assume the following channel slopes occur in a prismatic open channel.  Note where the water surface begins on the left.
[a] Label $y_c$ and $y_o$ for each reach (state if $y_o$ is infinite).
[b] Label the bed slopes.
[c] Label the control points.
[d] Draw a possible gradually varied flow profile and label all the curves.

1.



2.



3.



4.

**5.2** Given: Reservoir discharges into long trapezoidal channel.
$b = 20$ ft, $m = 3$, $n = 0.025$, $S_0 = 0.001$.
Reservoir water surface 10 ft above invert of channel entrance

Find: $Q$

① Assume steep slope.

Find $y_c$

$$\frac{Q^2 B_c}{g A_c^3} = 1 \implies \frac{Q^2}{g} = \frac{A_c^3}{B_c} \qquad (\alpha = 1)$$

$$H = y_c + \frac{Q^2}{2g A_c^2} = y_c + \frac{A_c^3}{B_c} \frac{1}{2 A_c^2}$$

$$H = y_c + \frac{A_c}{2 B_c} = y_c + \frac{y_c(b + m y_c)}{2(b + 2 m y_c)}$$

$$10\text{ft} = y_c + \frac{y_c(20\text{ft} + 3 y_c)}{2(20\text{ft} + 2(3) y_c)}$$

$\boxed{y_c = 7.54\text{ft}}$ — using Goal Seek in Excel

$A_c = 321\text{ ft}^2 \qquad B_c = 65.2\text{ ft} \qquad P = b + 2 y_c (1 + m^2)^{1/2} = 20\text{ft} + 2(7.54\text{ft})\sqrt{10}$

$$P = 67.7\text{ ft}$$

$$R_c = \frac{A_c}{P_c} = 4.75\text{ ft}$$

$$Q = \left[ \frac{g A_c^3}{B_c} \right]^{1/2} = \left[ \frac{(32.2\text{ft/s}^2)(321\text{ft}^2)^3}{65.2\text{ft}} \right]^{1/2} = 4040\text{ cfs}$$

$$S_c = \left[ \frac{Q n}{1.49 A_c R_c^{2/3}} \right]^2 = \left[ \frac{4040\text{ cfs}(0.025)}{1.49(321\text{ft}^2)(4.75\text{ft})^{2/3}} \right]^2$$

$\boxed{S_c = 0.0056} > S_0 = 0.001$, so slope is mild

$$H_0 = y_0 + \frac{Q^2}{2g A_0^2} = y_0 + \frac{Q^2}{2g \left[ y_0(b + m y_0) \right]^2}$$

$$10\text{ft} = y_0 + \frac{Q^2}{2(32.2\text{ft/s}^2)\left[ y_0(20\text{ft} + 3 y_0) \right]^2} \qquad \textcolor{blue}{[1]}$$

$$Q = \frac{1.49}{n} A_0 R_0^{2/3} S_0^{1/2} = \frac{1.49}{n} \frac{A_0^{5/3}}{P_0^{2/3}} S_0^{1/2}$$

$$Q = \frac{1.49}{0.025} \left[ \frac{y_0(20\text{ft} + 3 y_0)^{5/3}}{\left[ 20\text{ft} + 2 y_0(1 + 3^2)^{1/2} \right]^{2/3}} \right] (0.001)^{1/2} \qquad \textcolor{blue}{[2]}$$

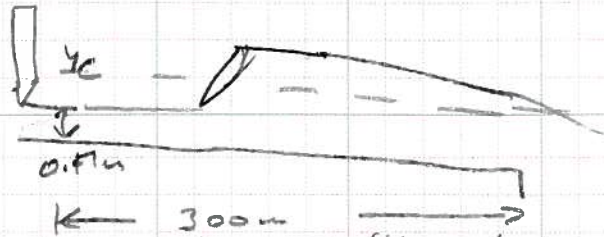Using Goal Seek in Excel $\boxed{y_0 = 9.43\text{ft}} \quad \boxed{Q = 2750\text{ cfs}}$

20 Total

10

10

**5.5** Given: Rectangular channel $b = 6.1 \text{ m}$ $n = 0.014$ $S_0 = 0.001$

$$Q = 17 \, m^3/sec$$



Find: Water surface profiles & hydraulic jump location

$$q = \frac{Q}{b} = \frac{17 \, m^3/s}{6.1 \, m} = 2.79 \, m^2/sec$$

$$y_c = \left(\frac{q^2}{g}\right)^{1/3} = \left[\frac{(2.79 \, m^2/s)^2}{9.81 \, m/s^2}\right]^{1/3} = \underline{0.925 \, m}$$

$$Q = \frac{1}{n} A R^{2/3} S_0^{1/2} = \frac{1}{n} by \left(\frac{by}{b+2y}\right)^{2/3} S_0^{1/2}$$

$$17 \, m^3/sec = \frac{1}{0.014} \frac{[(6.1m)y_0]^{5/3}}{[6.1m+2y_0]^{2/3}} (0.001)^{1/2}$$

$$\underline{y_0 = 1.31 \, m} - \text{Goalseek in Excel}$$

**Direct Step Method**

$$A_i = by_i = (6.1m)y_i$$
$$P_i = b+2y_i = (6.1m)+2y_i$$
$$R_i = A_i/P_i$$
$$V_i = \frac{Q}{A_i} = \frac{17 \, m^3/sec}{A_i}$$
$$E_i = y_i + \frac{V^2}{2g} = y_i + \frac{V^2}{2(9.81 \, m/s^2)}$$
$$S_{E_i} = \left[\frac{n V_i}{R_i^{2/3}}\right]^2 = \left[\frac{0.014 V_i}{R_i^{2/3}}\right]^2$$

See spreadsheet

$$S_{E_{bar}} = \frac{S_{E_1} + S_{E_2}}{2}$$

$$\Delta E = E_2 - E_1$$

$$\Delta x = \frac{\Delta E}{S_0 - S_{E_{bar}}}$$

For Hydraulic Jump - need momentum function

$$M = Ah_c + \frac{Q^2}{gA} = \frac{by^2}{2} + \frac{Q^2}{gby} = \frac{(6.1m)y^2}{2} + \frac{(17 \, m^3/s)^2}{(9.81 \, m/s^2)(6.1m)y}$$

Intersection $M_u = 8.58 \, m^3$ @ $x = 56.5m \rightarrow y_u = 0.67 m$

$M_d = 8.58 \, m^3$ @ $x = 56.5m \rightarrow y_d = 1.24 m$

$$F_{r_1} = \frac{V_1}{(gy_1)^{1/2}} = \frac{(4.16/s)}{[(9.81 \, m/s^2)(0.67m)]^{1/2}} = 1.62$$

Check $\dfrac{y_2}{y_1} = \dfrac{1}{2}\left[-1 + \sqrt{1 + 8\,F_{r_1}^2}\right]$

$\qquad = \dfrac{1}{2}\left[-1 + \sqrt{1 + 8(1.62)^2}\right] = 1.85$

$y_2 = 1.85\, y_1 = 1.85\,(0.67\text{m})$

$\boxed{y_2 = 1.24\text{m}}$ ✓

## 5.5

Given: Rectangular channel
b = 6.1 m
n = 0.014
$S_o$ = 0.001
300 m upstream from overfall
y = 0.47 m
Q = 17 m³/sec
Find: Water surface profiles and location of hydraulic jump
Direct Step Method
Δy (m) = 0.02 m
Moving upstream from overfall

| | $y_o$ (m) | $A_o$ (m²) | $P_o$ (m) | $R_o$ (m) | Q (m³/s) |
|---|---|---|---|---|---|
| | 1.31 | 7.98 | 8.72 | 0.92 | 17.00 |
| $y_c$ (m) | | | | | |
| | 0.93 | | | | |



| y (m) | A (m²) | P (m) | R (m) | V (m/s) | E (m) | $S_e$ | $S_{bar}$ | ΔE (m) | Δx (m) | Σx (m) | $M_d$ (m³) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.93 | 5.67 | 7.98 | 0.71 | 3.00 | 1.38769 | 0.002765 | | | | 300.00 | 7.83 |
| 0.95 | 5.80 | 8.00 | 0.72 | 2.93 | 1.38862 | 0.002593 | 0.002679 | 0.00093 | -0.55 | 299.45 | 7.84 |
| 0.97 | 5.92 | 8.04 | 0.74 | 2.87 | 1.39072 | 0.002435 | 0.002514 | 0.00210 | -1.39 | 298.06 | 7.85 |
| 0.99 | 6.04 | 8.08 | 0.75 | 2.82 | 1.38390 | 0.002290 | 0.002382 | 0.00317 | -2.33 | 295.73 | 7.87 |
| 1.01 | 6.16 | 8.12 | 0.76 | 2.76 | 1.39806 | 0.002156 | 0.002223 | 0.00418 | -3.40 | 292.33 | 7.89 |
| 1.03 | 6.28 | 8.16 | 0.77 | 2.71 | 1.40313 | 0.002033 | 0.002095 | 0.00508 | -4.64 | 287.69 | 7.92 |
| 1.05 | 6.41 | 8.20 | 0.78 | 2.65 | 1.40905 | 0.001919 | 0.001976 | 0.00592 | -6.06 | 281.63 | 7.96 |
| 1.07 | 6.53 | 8.24 | 0.79 | 2.60 | 1.41576 | 0.001814 | 0.001867 | 0.00670 | -7.73 | 273.89 | 8.01 |
| 1.09 | 6.65 | 8.28 | 0.80 | 2.56 | 1.42319 | 0.001717 | 0.001765 | 0.00743 | -9.70 | 264.19 | 8.05 |
| 1.11 | 6.77 | 8.32 | 0.81 | 2.51 | 1.43129 | 0.001626 | 0.001671 | 0.00810 | -12.07 | 252.12 | 8.11 |
| 1.13 | 6.89 | 8.36 | 0.82 | 2.47 | 1.44001 | 0.001542 | 0.001584 | 0.00873 | -14.94 | 237.18 | 8.17 |
| 1.15 | 7.02 | 8.40 | 0.84 | 2.42 | 1.44933 | 0.001464 | 0.001503 | 0.00931 | -18.52 | 218.66 | 8.23 |
| 1.17 | 7.14 | 8.44 | 0.85 | 2.38 | 1.45918 | 0.001391 | 0.001427 | 0.00985 | -23.07 | 195.59 | 8.30 |
| 1.19 | 7.26 | 8.48 | 0.86 | 2.34 | 1.46954 | 0.001323 | 0.001357 | 0.01038 | -29.05 | 166.54 | 8.38 |
| 1.21 | 7.38 | 8.52 | 0.87 | 2.30 | 1.48038 | 0.001259 | 0.001291 | 0.01084 | -37.26 | 129.27 | 8.46 |
| 1.23 | 7.50 | 8.56 | 0.88 | 2.27 | 1.49165 | 0.001200 | 0.001229 | 0.01128 | -49.20 | 80.08 | 8.54 |
| 1.25 | 7.63 | 8.60 | 0.89 | 2.23 | 1.50335 | 0.001144 | 0.001172 | 0.01169 | -68.12 | 11.95 | 8.63 |
| 1.27 | 7.75 | 8.64 | 0.90 | 2.19 | 1.51543 | 0.001092 | 0.001118 | 0.01208 | -102.66 | -90.71 | 8.72 |
| 1.29 | 7.87 | 8.68 | 0.91 | 2.16 | 1.52788 | 0.001043 | 0.001067 | 0.01245 | -185.54 | -276.26 | 8.82 |
| 1.31 | 7.99 | 8.72 | 0.92 | 2.13 | 1.54067 | 0.000997 | 0.001020 | 0.01279 | -653.34 | -929.60 | 8.92 |

Moving downstream from sluice gate

| y (m) | A (m²) | P (m) | R (m) | V (m/s) | E (m) | $S_e$ | $S_{bar}$ | ΔE (m) | Δx (m) | Σx (m) | $M_u$ (m³) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.47 | 2.87 | 7.04 | 0.41 | 5.93 | 2.26202 | 0.022829 | | | | 0.00 | 10.95 |
| 0.49 | 2.99 | 7.08 | 0.42 | 5.69 | 2.13872 | 0.020019 | 0.021424 | -0.12330 | 6.04 | 6.04 | 10.59 |
| 0.51 | 3.11 | 7.12 | 0.44 | 5.46 | 2.03194 | 0.017652 | 0.018836 | -0.10878 | 5.99 | 12.02 | 10.26 |
| 0.53 | 3.23 | 7.16 | 0.45 | 5.26 | 1.93925 | 0.015644 | 0.016648 | -0.09270 | 5.92 | 17.95 | 9.97 |
| 0.55 | 3.36 | 7.20 | 0.47 | 5.07 | 1.85862 | 0.013930 | 0.014787 | -0.08063 | 5.85 | 23.80 | 9.70 |
| 0.57 | 3.48 | 7.24 | 0.48 | 4.89 | 1.78840 | 0.012458 | 0.013194 | -0.07022 | 5.76 | 29.55 | 9.46 |
| 0.59 | 3.60 | 7.28 | 0.49 | 4.72 | 1.72720 | 0.011187 | 0.011823 | -0.06120 | 5.66 | 35.21 | 9.25 |
| 0.61 | 3.72 | 7.32 | 0.51 | 4.57 | 1.67385 | 0.010084 | 0.010636 | -0.05335 | 5.54 | 40.75 | 9.05 |
| 0.63 | 3.84 | 7.36 | 0.52 | 4.42 | 1.62737 | 0.009122 | 0.009603 | -0.04647 | 5.40 | 46.15 | 8.88 |
| 0.65 | 3.97 | 7.40 | 0.54 | 4.29 | 1.58694 | 0.008279 | 0.008701 | -0.04043 | 5.25 | 51.40 | 8.72 |
| 0.67 | 4.09 | 7.44 | 0.55 | 4.16 | 1.55184 | 0.007538 | 0.007908 | -0.03510 | 5.08 | 56.48 | 8.58 |
| 0.69 | 4.21 | 7.48 | 0.56 | 4.04 | 1.52146 | 0.006883 | 0.007210 | -0.03038 | 4.89 | 61.37 | 8.45 |
| 0.71 | 4.33 | 7.52 | 0.58 | 3.93 | 1.49528 | 0.006302 | 0.006592 | -0.02618 | 4.68 | 66.05 | 8.34 |
| 0.73 | 4.45 | 7.56 | 0.59 | 3.82 | 1.47284 | 0.005785 | 0.006044 | -0.02244 | 4.45 | 70.50 | 8.24 |
| 0.75 | 4.58 | 7.60 | 0.60 | 3.72 | 1.45375 | 0.005324 | 0.005555 | -0.01909 | 4.19 | 74.69 | 8.15 |
| 0.77 | 4.70 | 7.64 | 0.61 | 3.62 | 1.43766 | 0.004911 | 0.005118 | -0.01608 | 3.91 | 78.60 | 8.08 |
| 0.79 | 4.82 | 7.68 | 0.63 | 3.53 | 1.42429 | 0.004541 | 0.004726 | -0.01338 | 3.59 | 82.19 | 8.02 |
| 0.81 | 4.94 | 7.72 | 0.64 | 3.44 | 1.41335 | 0.004207 | 0.004374 | -0.01094 | 3.24 | 85.43 | 7.96 |
| 0.83 | 5.06 | 7.76 | 0.65 | 3.36 | 1.40462 | 0.003905 | 0.004056 | -0.00873 | 2.88 | 88.29 | 7.92 |
| 0.85 | 5.19 | 7.80 | 0.66 | 3.28 | 1.39790 | 0.003632 | 0.003768 | -0.00672 | 2.43 | 90.72 | 7.89 |
| 0.87 | 5.31 | 7.84 | 0.68 | 3.20 | 1.39300 | 0.003384 | 0.003508 | -0.00490 | 1.95 | 92.67 | 7.86 |
| 0.89 | 5.43 | 7.88 | 0.69 | 3.13 | 1.38976 | 0.003158 | 0.003271 | -0.00324 | 1.43 | 94.10 | 7.84 |
| 0.91 | 5.55 | 7.92 | 0.70 | 3.06 | 1.38803 | 0.002953 | 0.003056 | -0.00173 | 0.84 | 94.94 | 7.83 |
| 0.93 | 5.67 | 7.98 | 0.71 | 3.00 | 1.38769 | 0.002765 | 0.002859 | -0.00034 | 0.18 | 95.12 | 7.83 |
| 0.95 | 5.80 | 8.00 | 0.72 | 2.93 | 1.38862 | 0.002593 | 0.002679 | 0.00093 | -0.55 | 94.56 | 7.84 |

Δx (m) changes sign, because passed critical depth
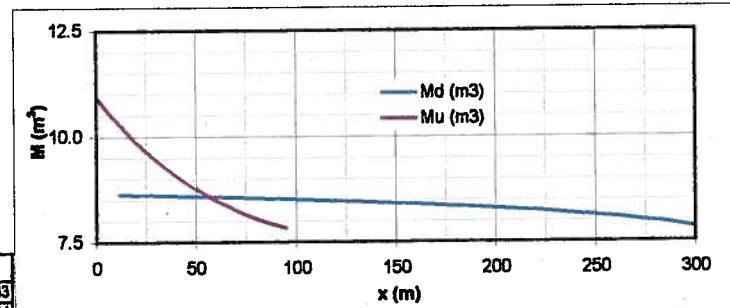
5.5

Moving downstream from sluice gate

| $\sum x$ (m) | y (m) |
|---|---|
| 0.00 | 0.47 |
| 6.04 | 0.49 |
| 12.02 | 0.51 |
| 17.95 | 0.53 |
| 23.80 | 0.55 |
| 29.55 | 0.57 |
| 35.21 | 0.59 |
| 40.75 | 0.61 |
| 46.15 | 0.63 |
| 51.40 | 0.65 |
| 56.48 | 0.67 |
| 35.25 | 0.59 |
| 38.04 | 0.60 |
| 40.79 | 0.61 |
| 43.51 | 0.62 |
| 46.20 | 0.63 |
| 48.84 | 0.64 |
| 51.45 | 0.65 |
| 54.02 | 0.66 |
| 56.54 | 0.67 |
| 56.55 | 1.24 |
| 80.08 | 1.23 |
| 129.27 | 1.21 |
| 166.54 | 1.19 |
| 195.59 | 1.17 |
| 218.66 | 1.15 |
| 237.18 | 1.13 |
| 252.12 | 1.11 |
| 264.19 | 1.09 |
| 273.89 | 1.07 |
| 281.63 | 1.05 |
| 287.69 | 1.03 |
| 292.33 | 1.01 |
| 295.73 | 0.99 |
| 298.06 | 0.97 |
| 299.45 | 0.95 |
| 300.00 | 0.93 |