CE 4353/5360

Homework 3

1. A hydraulic jump occurs in a rectangular channel. The channel is 20 ft wide, and the upstream depth is 3.5 ft at a flow rate of 2500 cfs. Find the downstream depth in ft and the head loss across the jump.

2. Problem 3.1 modified. A hydraulic jump is to be formed in a trapezoidal channel with a base width of 20 ft and side slopes of 2:1. The upstream depth is 1.35 ft and Q=1100 cfs.
[a] Find the downstream depth and the head loss in the jump. Solve by Figure 3.2.
[b] Now that both y_1 and y_2 are known for the trapezoidal channel, calculate both M_1 and M_2 to show $M_1 = M_2$.

3. Problem 3.2 modified.
[a] Determine the sequent depth for a hydraulic jump in a 3-ft diameter storm sewer with a flow depth of 0.65 ft at a discharge of 6.5 cfs. Solve by Figure 3.3.
[b] Now that both y_1 and y_2 are known, calculate both M_1 and M_2 to show $M_1 = M_2$.

4. Problem 3.4 modified. A flume with triangular cross section contains water flowing at a depth of 0.15 m and at a discharge of 0.35 m^3/s. The side slopes of the flume are 2:1. Determine the sequent depth for a hydraulic jump.

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ASCE STUDENT CHAPTER

ENGINEERING TEXAS TECH

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1) Given: Hydraulic jump in rectangular channel.

$b = 20\ ft \quad y_1 = 3.5\ ft \quad Q = 2500\ cfs$

Find: y_2 & head loss across jump

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8 Fr_1^2}\right]$$

$$Fr_1 = \frac{V}{\sqrt{g y_1}}$$

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$$V_1 = \frac{Q}{A_1} = \frac{Q}{b y_1} = \frac{2500 cfs}{(20 ft)(3.5 ft)} = 35.7\ ft/sec$$

$$Fr_1 = \frac{35.7\ ft/sec}{[(32.2 ft/s^2)(3.5 ft)]^{1/2}} = 3.36$$

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + (1 + 8(3.36)^2)^{1/2}\right] = 4.28$$

$$y_2 = 4.28\, y_1 = 4.28(3.5 ft)$$

$$\boxed{y_2 = 15.0\ ft}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = 3.5 ft + \frac{(35.7 ft/s)^2}{2(32.2 ft/s^2)}$$

$$E_1 = 23.3\ ft$$

$$E_2 = y_2 + \frac{V_2^2}{2g}$$

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$$V_2 = \frac{Q}{b y_2} = \frac{2500 cfs}{(20 ft)(15 ft)} = 8.33\ ft/sec$$

$$E_2 = 15 ft + \frac{(8.33 ft/s)^2}{2(32.2 ft/s^2)}$$

$$E_2 = 16.1\ ft$$

$$E_1 - E_2 = 23.3 ft - 16.1 ft$$

$$\boxed{E_1 - E_2 = 7.2\ ft}$$

$$\frac{E_L}{E_1} = \frac{7.2 ft}{23.3 ft} = 0.31$$

3.1 modified | Given: Hydraulic jump in trapezoidal channel

$b = 20\text{ft}$, $m = 2$ on side slopes

$y_1 = 1.35\text{ft}$ $Q = 1100\text{ cfs}$

Find: [a] y_2 & head loss in jump by Fig 3.2

[b] Find M_1 & M_2; show $M_1 = M_2$

[a] Fig. 3.2 need $Z_{trap} = \dfrac{Q m^{3/2}}{g^{1/2} b^{5/2}}$

$$= \frac{(1100\text{cfs})(2)^{3/2}}{(32.2\text{ft/s}^2)^{1/2}(20\text{ft})^{5/2}} = 0.31$$

$$\Rightarrow \frac{m y_1}{b} = \frac{2(1.35\text{ft})}{20\text{ft}} = 0.135 \Big\} \rightarrow \frac{y_2}{y_1} = 6.3$$

$$y_2 = 6.3\, y_1 = 6.3\,(1.35\text{ft})$$

$$\boxed{y_2 = 8.5\ \text{ft}}$$

$$E_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{Q^2}{2gA_1^2}$$

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$$A_1 = y_1(b + m y_1) = (1.35\text{ft})(20\text{ft} + 2(1.35\text{ft})) = 30.6\text{ft}^2$$

$$E_1 = 1.35\text{ft} + \frac{(1100\text{cfs})^2}{2(32.2\text{ft/s}^2)(30.6\text{ft}^2)^2} = 21.4\text{ft}$$

$$E_2 = y_2 + \frac{Q^2}{2gA_2^2}$$

$$A_2 = (8.5\text{ft})\left[20\text{ft} + 2(8.5\text{ft})\right] = 314\ \text{ft}^2$$

8.5
722.5 + 409.4 + 119.5

$$E_2 = 8.5 + \frac{(1100\text{cfs})^2}{2(32.2\text{ft/s}^2)(314\ \text{ft}^2)^2} = 8.7\ \text{ft}$$

$$E_1 - E_2 = 21.4\text{ft} - 8.7\ \text{ft}$$

$$\boxed{E_1 - E_2 = 12.7\ \text{ft}}$$

[b] $$M_1 = \frac{b y_1^2}{2} + \frac{m y_1^3}{3} + \frac{Q^2}{g y_1(b + m y_1)}$$

$$= \frac{(20\text{ft})(1.35\text{ft})^2}{2} + \frac{2(1.35\text{ft})^3}{3} + \frac{(1100\text{cfs})^2}{(32.2\text{ft/s}^2)(1.35\text{ft})\left[20\text{ft} + 2(1.35\text{ft})\right]}$$

$$= 18.23\ \text{ft}^3 + 1.64\ \text{ft}^3 + 1226.2\ \text{ft}^3$$

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$$\boxed{M_1 = 1246\ \text{ft}^3}$$

$$M_2 = \frac{b y_2^2}{2} + \frac{m y_2^3}{3} + \frac{Q^2}{g y_2 (b + m y_2)}$$

$$= \frac{20(8.5\text{ft})^2}{2} + \frac{2(8.5\ \text{ft})^3}{3} + \frac{(1100\text{cfs})^2}{(32.2\text{ft/s}^2)(8.5\text{ft})\left[20\text{ft} + 2(8.5\text{ft})\right]}$$

$$= 656.4\text{ft}^3 + 354.3\ \text{ft}^3 + 128.2\ \text{ft}^3$$

$$\boxed{M_2 = 1251\ \text{ft}^3}$$

3.2 modified | Given: Hydraulic jump in 3-ft diameter storm sewer

$y_1 = 0.65\text{ ft} \qquad Q = 6.5\text{ cfs}$

Find: [a] y_2 by Fig 3.3
[b] M_1 & M_2, so $M_1 = M_2$

[a] $$z_{circ} = \frac{Q}{g^{1/2} d^{5/2}} = \frac{6.5\text{ cfs}}{(32.2\text{ ft/s}^2)^{1/2}(3\text{ ft})^{5/2}} = 0.073$$

Fig 3.3 $$\frac{y_1}{d} = \frac{0.65\text{ ft}}{3\text{ ft}} = 0.22 \Rightarrow \frac{y_2}{y_1} = 1.5$$

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$$y_2 = 1.5\,y_1 = 1.5\,(0.65\text{ ft})$$

$$\boxed{y_2 = 0.98\text{ ft}}$$

$$M_1 = \left[3\sin\frac{\theta_1}{2} - \sin^3\left(\frac{\theta_1}{2}\right) - 3\left(\frac{\theta_1}{2}\right)\cos\left(\frac{\theta_1}{2}\right)\right]\frac{d^3}{24} + \frac{Q^2}{gd^2\left(\frac{\theta_1 - \sin\theta}{8}\right)}$$

$$\theta = 2\cos^{-1}\left[1 - 2\left(\frac{y_1}{d}\right)\right]$$

$$\theta_1 = 2\cos^{-1}\left[1 - 2\,\frac{(0.65\text{ ft})}{(3\text{ ft})}\right] = 1.94\text{ rad}$$

$$M_1 = \left[3\sin\frac{1.94}{2} - \sin^3\left(\frac{1.94}{2}\right) - 3\left(\frac{1.94}{2}\right)\cos\left(\frac{1.94}{2}\right)\right]\frac{(3\text{ ft})^3}{24} + \frac{(6.5\text{ cfs})^3}{(32.2\text{ ft/s}^2)(3\text{ ft})^2\left[\frac{1.94 - \sin(1.94)}{8}\right]}$$

$$= \left[2.47 - 0.56 - 1.64\right]\frac{27}{24} + 1.16$$

$$= 0.30 + 1.16 = \underline{1.46\text{ ft}^3}$$

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$$\theta_2 = 2\cos^{-1}\left[1 - 2\left(\frac{1.1\text{ ft}}{3\text{ ft}}\right)\right] = 2.43\text{ rad}$$

$$M_2 = \left[3\sin\left(\frac{2.43}{2}\right) - \sin^3\left(\frac{2.43}{2}\right) - 3\left(\frac{2.43}{2}\right)\cos\left(\frac{2.43}{2}\right)\right]\frac{(3\text{ ft})^2}{24} + \frac{(6.5\text{ cfs})^2}{(32.2\text{ ft/s}^2)(3\text{ ft})^2\left[\frac{2.43 - \sin(2.43)}{8}\right]}$$

$$= \left[2.81 - 0.82 - 1.27\right]\frac{27}{24} + 0.66$$

$$\boxed{M_2 = 1.47\text{ ft}^3}$$

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3.4 modified Given: Triangular flume $m = 2$ side slopes

$Q = 0.35\ m^3/s \qquad y_1 = 0.15\ m$

Find: y_2 sequent depth

Can use Fig 3.5

$$Fr_1 = \frac{V_1}{\sqrt{g D_1}}$$

$$D_1 = \frac{A_1}{B_1} = \frac{m y_1^2}{2 m y_1} = \frac{y_1}{2} = \frac{0.15\ m}{2} = 0.075\ m$$

$$V_1 = \frac{Q}{A_1} = \frac{0.35\ m^3/s}{2(0.15\ m)^2} = 7.78\ m/sec$$

$\boxed{10}$

$$Fr_1 = \frac{7.78\ m/sec}{[(9.81\ m/s^2)(0.075\ m)]^{1/2}} = 9.1$$

Fig. 3.5 $\quad \frac{y_2}{y_1} = 4.9$

$$y_2 = 4.9\ y_1 = 4.9\ (0.15\ m)$$

$$y_2 = 0.74\ m$$