

## CHAPTER SIX

### DISCOUNT RATE

*6-1 CAPITAL FORMATION* A Pacific islander fishing with a spear may obtain enough fish for his daily needs but find himself unable to catch enough extra for a cash income. His entire effort is being spent in producing consumer goods. If he were able at the price of a temporary sacrifice in his diet to forego fishing and spend his time making a fishing lure, he would with the completion of this task be able to catch enough fish for his own use plus extra for sale. By shifting some of his production from consumer to capital goods, he has improved his lot in the long run through a temporary sacrifice.

Basic resources (land, labor, minerals, water, etc.) can be used to produce consumer goods used to satisfy immediate human needs. They can also be used to produce capital goods, the intermediate goods used in the production of more goods. As capital goods are continually produced, the formation of capital provides the production base permitting a long-term improvement in human welfare.

Water resources development is essentially a capital formation process, and two questions must be answered in planning capital formation:

- 1 How much sacrifice in current consumption should be made to increase production of capital goods?
- 2 How much of each kind of capital goods should be produced?

The analyses described in the other chapters of this book seek the optimal project design answering the second question. The answer to the first question is the subject at hand.

Clearly, the greater the sacrifice in consumption, the more basic resources can be devoted to producing capital goods and the faster capital will be formed. The rate of economic development is directly tied to a willingness to sacrifice. The discount rate for project planning is essentially a measure of this willingness.

**6-2 DISCOUNT RATE VS. INTEREST RATE** While the interest rate may be loosely defined as any expression of the time value of capital, a more precise definition distinguishes between interest rate and discount rate. Interest is the fee one producer pays to use the capital of another. The interest rate is determined by the capital market. It must be considered in financial analysis (Sec. 22-1). An enterprise seeking to improve its own welfare will pay an interest rate determined by the least expensive source of capital and use that rate (unless the intangible determinates of company policy dictate otherwise) to evaluate its investment opportunities, using appropriate discounting techniques. It will make money by investing in projects having a rate of return exceeding its borrowing rate.

The reasoning for investing past earnings follows a slightly different line. These earnings can either be distributed to the owners for their own consumption or outside investment or kept within the enterprise to finance capital expansion. The decision process for allocating earnings between dividends to owners and capital expansion is basically that described in presenting the rate-of-return discounting technique (Sec. 2-10). Management will devote earnings to capital formation starting with the project yielding the highest return but will be careful that funds remaining for distribution to the owners are not reduced to a level that will discourage further investment in the company. Selection of the cutoff point is basically a value judgment by management based on what is good for the company. The cutoff point is in effect the company discount rate.

A discount rate is the expression of the time value of capital used in equivalence calculations comparing alternatives. The rate is essentially a value judgment based on a compromise between present consumption and capital formation from the viewpoint of the decision maker. For public works planning, this means the viewpoint of the people as a whole (Sec. 1-3). The ideal discount rate would achieve a rate of capital formation maximizing total social welfare.

Many viewpoints have been expressed on what is the best discount rate from the public point of view. Some advocate public planning based on a zero discount rate because interest is not directly charged on tax dollars or because of the distrust of usury prevalent in Christian and socialist traditions. However, without discounting, the planner would con-

sider a perpetual income of a penny per year (summing to infinity) preferable to a million dollars tomorrow if both could be obtained through the same investment. Some advocate discounting with the interest rate paid to borrow funds for project financing. However, this approach ignores the opportunity cost of tax funds and makes project merit dependent on the borrowing power of the financing agency. Some advocate discounting with the internal rate of return for the least profitable project which can be financed without exhausting available funds. However, this approach is no help in answering the basic capital formation question of optimum budget size when funds are obtained from tax revenues.

**6-3 OPPORTUNITY COST** The fundamental purpose of economic analysis is to determine whether a particular addition to existing investment capital is worthwhile. Capital formation requires two kinds of sacrifice. Some resources will be used which would otherwise have gone into the formation of some other type of productive capital; the opportunity cost of their use equals the productivity they would have had in the investment foregone. Some resources would have otherwise been consumed; their opportunity cost is determined by the value placed by consumers on the required sacrifice.

The value of sacrificed consumption becomes the key to discount rate selection because this sacrifice comes closest to representing the marginal-opportunity cost of additional capital formation. Market analysis may be used to estimate the value of the sacrifice to the individual. Determination of the value to society as a whole is embroiled in assessing collective and external effects of increased production.

**6-4 THE CAPITAL MARKET** Just as other markets provide a starting point for evaluating other inputs to public works projects, the interest rate determined by the capital market provides a starting point in discount rate selection. As a factor in production, the price of capital, as is the price of all the other factors, is a market equilibrium position determined by the interaction of supply and demand.

The supply of capital depends on the investment of savings. An individual divides his income between consumption and savings according to his preference as represented by the indifference map of Fig. 6-1. The indifference lines represent combinations of present consumption and uniform annual future income among which the individual has no preference and thus is indifferent. Because uniform annual future income equals the product of the amount invested and the interest rate,  $CD$  equals  $i(DE)$ . The available alternatives range between consuming the entire income at

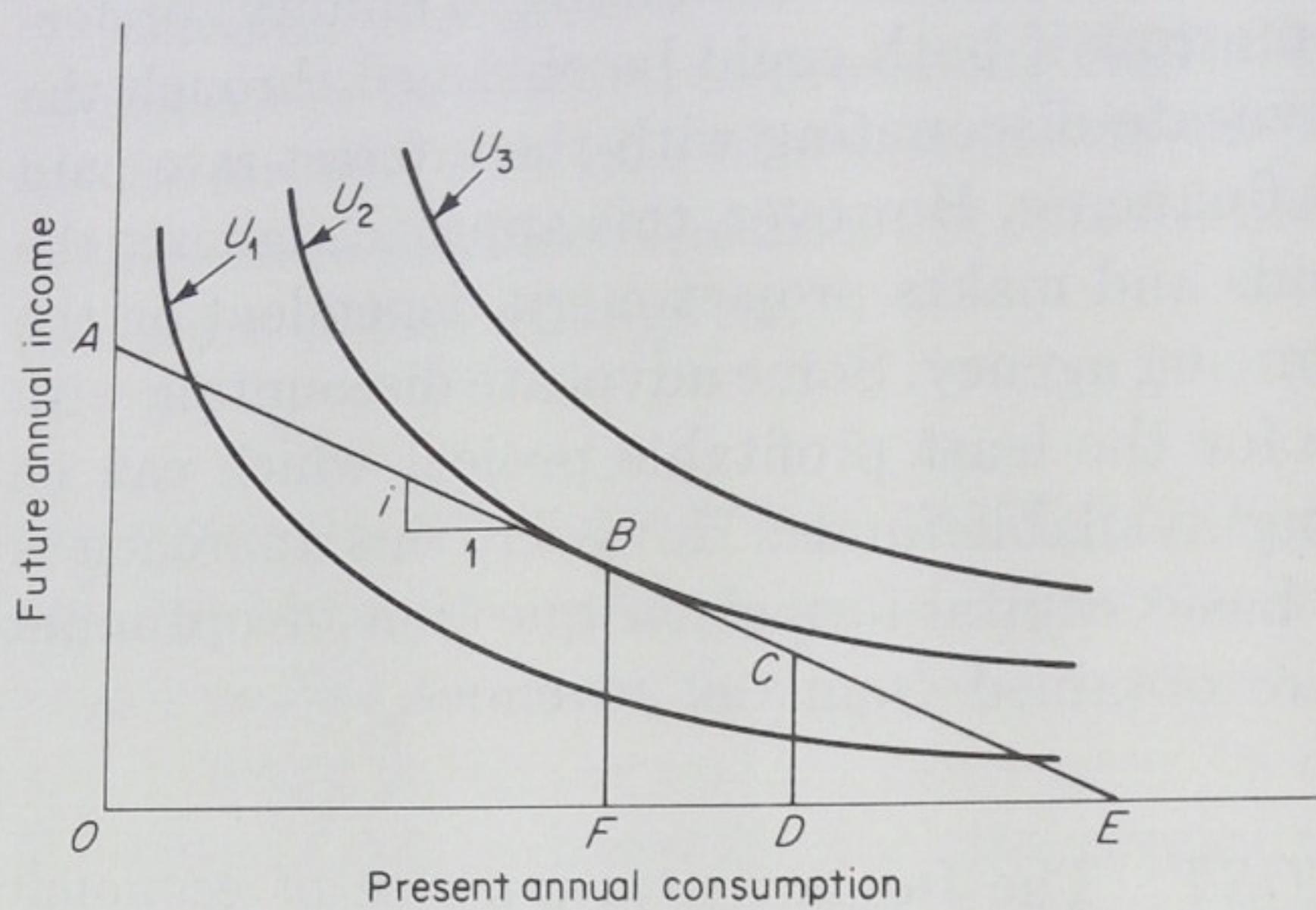


FIGURE 6-1 Representative indifference map between consumption and future income.

point  $E$  and saving the entire income at point  $A$ . For the income  $OE$ , the alternatives fall on the line  $ABCE$ . The individual whose preferences are represented by Fig. 6-1 and whose income equals  $OE$  will enjoy the greatest utility from his income at point  $B$ , for at this point he has reached his highest possible indifference line  $U_2$ . He will consume  $OF$  and save  $FE$  at interest rate  $FB/FE$ . At point  $B$ , his willingness to substitute future interest income for present consumption is exactly equal to the interest rate. In a uniform market, every other individual will have this same marginal rate of substitution between consumption and savings because all will receive the same rate of interest even though people vary widely in the fraction of their income saved because of indifference line differences. Aggregate savings equal the sum of the individual savings. The aggregate marginal rate of substitution is identical with that for each individual.

If the interest rate exceeded  $FB/FE$ , the line representing available alternatives would be steeper and the higher utility of another indifference curve could be attained. The change in savings by a particular individual would depend on the nature of the individual's indifference map. Those saving toward a fixed objective may prefer extra consumption if a higher interest rate were to allow them to reach their goal by saving a smaller portion of their total income. Nevertheless, for most individuals and certainly for a modern industrial society as a whole, savings increase with interest received. Thus, the investment-capital-supply curve will slope upward to the right as shown on Fig. 6-2.

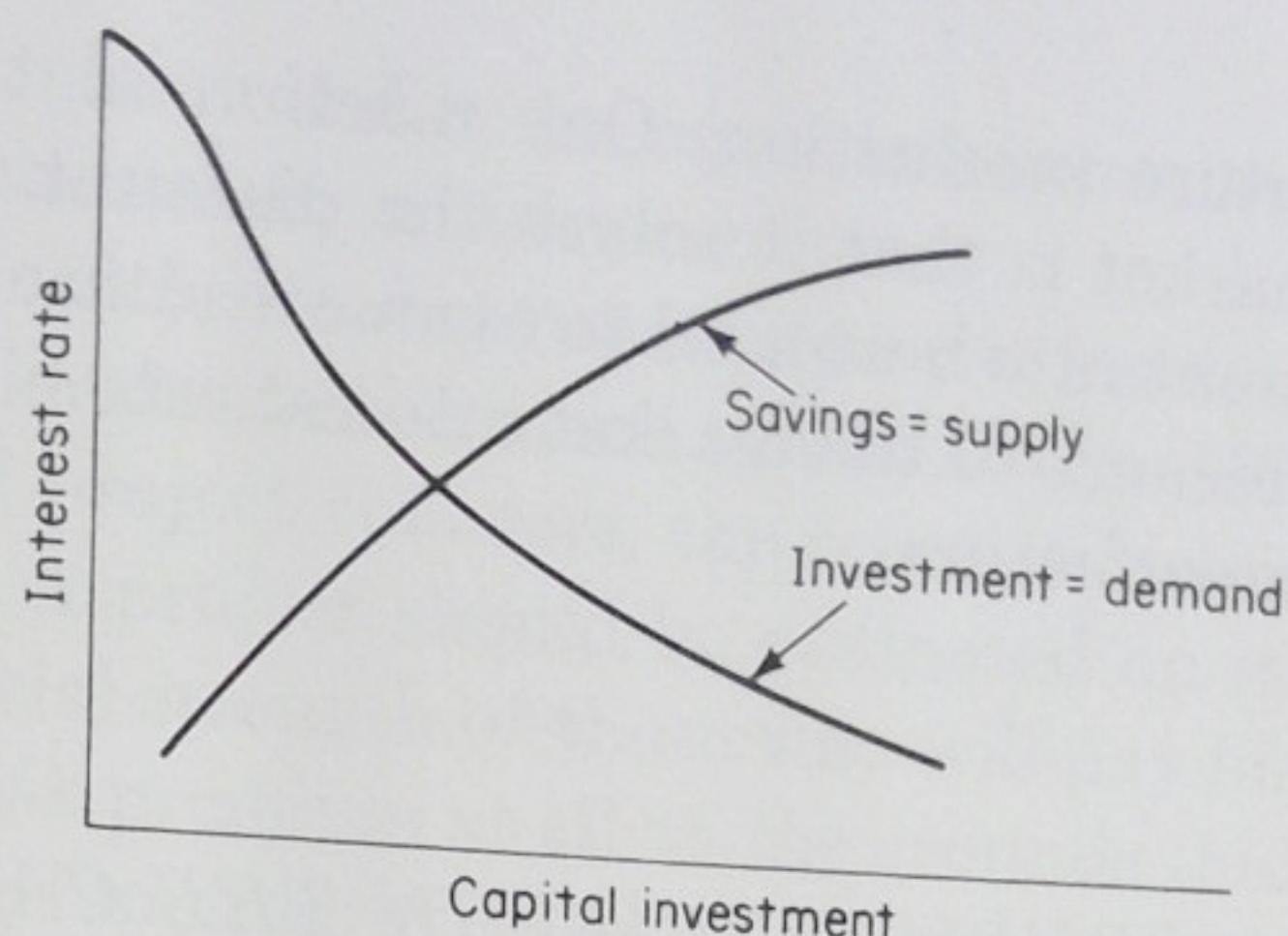


FIGURE 6-2 Supply and demand curves in the capital market.

The demand for capital depends on the available investment opportunities. Each investment opportunity has an internal rate of return. Theoretically, all available investment alternatives could be ranked in order of their internal rate of return. The demand curve would intersect the ordinate of Fig. 6-2 at the internal rate of return of the top project on this list. Other points on the demand curve have an ordinate determined by the rate of return on the marginal project and an abscissa determined by the total investment required to implement all projects having an internal rate of return higher than that indicated. The curve would be unlikely to reach the horizontal axis because any investment providing a perpetual income has a rate of return exceeding zero.

Some have postulated that savings will continue as long as men earn income but that investment opportunities become fewer as the most desirable projects are built. When all investment opportunities have been utilized, economic stagnation will result. However, investment history does not give much substance to this worry as the human capacity to consume and the technical ability to produce seem to expand even faster than general economic growth. Per capita investment in the economically most developed countries far exceeds that in the most primitive. Marginal internal rates of return exceeding 10 percent prevail in American industry. There may be reason to doubt the social desirability of current consumption patterns, but there is little reason to doubt the ability of man's wants and his technical ability to produce to expand into the indefinite future at a rate sufficient to absorb all available savings.

The capital market balances the supply and demand of capital to establish an equilibrium interest rate. An excess of savings reduces interest rates as occurred immediately after World War II. An excess of investment opportunity causes interest rates to rise as has occurred more recently. Either way, the market mechanism allocates the time pattern of consumption by the sacrifice (*FE* in Fig. 6-1) induced to obtain capital for

future production. One reaction to this accomplishment of the capital market is that it solves the discount rate problem by providing the discount rate best used to evaluate alternatives. However, the normative significance of the market interest rate hinges on the consequences of market imperfections.

**6-5 DEFICIENCIES IN THE CAPITAL MARKET** The financial structure of modern government has become so closely entwined with the capital market that government is both the major participant and the regulator charged with keeping the interest rate in line with politically determined economic policy. The first effect has to do with government fiscal policy. A large federal deficit requires heavy borrowing and increases interest rates by increasing the demand for the capital supplied by lenders. The effect is amplified as government spending induces private investment which must also be financed by borrowed money. Spending induces investment by those who sell their goods and services to the government as well as by those who are provided more government produced goods and services. Overall, government participation in the capital market is far too large to maintain the approximation of perfect competition.

In addition, the federal government regulates the supply of money by its monetary policy.<sup>1</sup> By buying and selling federal reserve notes and modifying the rules on borrowing and lending which financial institutions must follow, the government may decrease the supply of money and thus increase rates (a tight money policy) or increase the supply of money and thus reduce interest rates (a loose money policy). Discounting with the market interest rate would cause the desirability of alternative investments to fluctuate with federal monetary policy. Since this policy may be varied by the government at will, the net benefit of alternative investments becomes a manipulated rather than a normative measure of project worth. The arbitrariness is reduced by monetary policy and planning being functions of different government agencies, but the basic argument still holds.

The capital market is also affected by market interest rates having two components: the time value of money and a risk premium. A lender traditionally charges a higher rate of interest to those less likely to repay. The rate of interest charged to a borrower (such as by the United States government) whose ability to repay is in absolutely no doubt may be thought of as risk free or pure interest. It is the market evaluation of the

<sup>1</sup> Paul A. Samuelson, "Economics: An Introductory Analysis," 5th ed. (New York: McGraw-Hill Book Company, 1961), pp. 644-662.

time value of money. As the doubt about ability to repay increases, the interest rate will correspondingly increase by a risk premium.

A risk premium may be added to the discount rate to account for risk. Because economic analysis seeks to evaluate the merit of a given project rather than the financial strength of project sponsors, the relevant risk is the probable success of the project. A project should be evaluated on its own merits rather than by the financial strength of those who will pay for it. Some may argue against using a risk premium at all on the grounds that risk is not properly a compounding function of time, but there is little justification for selecting a risk premium on any basis other than the attributes of the investment under consideration.

The capital market is also influenced by the debt aversion of prospective borrowers. People and institutions tend to be more hesitant about spending borrowed than earned capital. A project yielding a low return might be favored over one yielding a higher return if the latter must be financed by borrowing. Debt aversion stems from the threat of extensive borrowing to the solvency of the borrower and the higher interest rate which must be paid on essential subsequent loans. Legal limitations limit borrowing by local government and public utilities. Whether legally or self-imposed, debt aversion distorts the equilibrium point of the capital market.

A difference exists between borrowing and lending rates. An individual pays a higher rate of interest when he borrows than he gets when he lends his money to a bank. Much of the difference can be explained by the difference in risk premium required for the two parties. Institutions which use their money in more productive investments will grow financially stronger and be required to pay a smaller risk premium on borrowed money.

One of the most telling criticisms of the capital market is that it determines how much the present generation wants to save for the future, but it cannot consult the desires of future generations on either the amount or kind of savings. The selection of the proper kind of savings has been a major concern of the conservation movement. The world is changing. Man is replacing an old environment with a new. He must be careful to prevent the economic forces producing the change from sacrificing those noneconomic values of the old environment essential to his well-being.

Many believe planning using a cutoff rate of return equal to the relatively high market interest rate would unduly restrict capital investment.<sup>1</sup> The capital market is the product of many individual investment decisions. They suggest that society may be willing to save more as a group than people will save as individuals. Individuals may be willing to commit

<sup>1</sup> Stephen A. Marglin, *Economic Factors Affecting System Design*, in Arthur Maass et al., "Design of Water-resource Systems" (Cambridge, Mass.: Harvard University Press, 1962), pp. 194-205.

more of their own income to savings if they could be sure everyone else were also doing so. If a lower social discount rate at which society would be willing to save as a group exists, it is the discount rate which should be used to evaluate public projects financed from enforced group savings (taxes).<sup>1</sup>

The chief argument against discounting, for selecting public works alternatives with a low group-determined social interest rate, lies in the need to distinguish between two types of capital. Liquid capital is money available for investment. Plant capital is goods used to produce other goods. Theoretically, the decision to use a discount rate for planning in the public sector that is lower than the market rate used in the private sector tends to slow capital formation by diverting capital from the private sector to less productive investment in the public sector. Furthermore, the resultant accumulation of public plant capital may force overall savings from easily convertible liquid capital to a form of plant which may in later years be found to be either incapable of supplying then current needs or technologically obsolete. Unusable savings are as bad as no savings at all.

**6-6 BASIC APPROACHES FOR DEALING WITH MARKET DEFICIENCIES** Two basic approaches exist on how to deal with capital market deficiencies in developing a discount rate. Some say the defects are too great for the market interest to have any normative significance whatsoever and the discount rate must be selected by some other method. Others recognize the defects in the market but can find no better method. They believe the government should concentrate on formulating policies to correct market defects in order to better coordinate public with private investment rather than pick what must be a somewhat arbitrary rate for its own planning.<sup>2</sup> Financial pressures force the private sector to use the interest rate rather than a normative discount rate. Unless the two rates are identical, project design and justification will depend on who does the planning, a condition reducing economic efficiency.

**6-7 SPECIFIC APPROACHES FOR PICKING A DISCOUNT RATE** A number of methods for picking a discount rate for use in planning by public agencies have been suggested, and the resulting values vary over a wide range. The first four methods follow a market-based approach. The fifth does not.

<sup>1</sup> Maynard M. Hufschmidt, John V. Krutilla, Julius Margolis, and Stephen A. Marglin, "Standards and Criteria for Formulating and Evaluating Federal Water Resources Developments," Report of Panel of Consultants to the Bureau of the Budget, Washington, 1961, pp. 11-23.

<sup>2</sup> Jack Hirshleifer, James C. DeHaven, and Jerome W. Millman, "Water Supply: Economics, Technology, and Policy" (Chicago: The University of Chicago Press, 1960), pp. 121-123.

- 1 One approach is to discount with the market interest rate for risk-free investment. The rate is estimated and adjusted as necessary to correct for known market defects and market allowance for currency inflation. Because government bonds are one of the most risk-free investments available and market interest rate varies with the duration that capital is committed, the interest rate on recently issued government bonds having a maturity period approximately equal to anticipated project life, normally 50 to 100 years, provides a good estimate. The average interest rate on old outstanding government bonds is not particularly relevant to current investment, but all outstanding bonds were considered in selecting the discount rate used by United States government agencies until 1968. In that year, the average discount rate on new government bonds was about 4.625 percent while the average rate on all outstanding bonds was about 3.25 percent.
- 2 Most rapid capital formation comes by making the most productive investments. If funds were committed to the project yielding the highest return first and then to subsequent projects in order of rate of return, the internal rate of return of the last project selected before funds ran out would be the marginal internal rate of return. The existence of a marginal internal rate of return implies a budgetary constraint or capital rationing because otherwise the funds would not run out. History has shown funds available to public agencies to be chronically less than what could potentially be invested. In order to maximize the rate of capital formation, it is important to recognize that the productivity of capital recovered and reinvested may substantially depart from that of the original investment. Public works projects must consider the reinvestment of those benefits received by the general public, while private industry recovers and reinvests the benefits from its own investment. The overall rate of return is the discount rate equating the present worth of the investment with the present worth of the consumption (by year in which it occurs) plus the present worth of invested and reinvested capital at the end of the period of analysis. The marginal internal rate of return will vary widely among agencies and over time, but a rate which has been suggested for highway planning is 7.0 percent.<sup>1</sup>
- 3 In order to preserve the merits of a discount rate based on productivity but avoid the weaknesses of one based on a politically controlled budgetary constraint, a third approach is to use the marginal internal rate of return for private companies engaged in comparable activity. The comparable activity requirement limits comparison to investments of

<sup>1</sup> Eugene L. Grant, Interest and the Rate of Return on Investments, *Highway Res. Board, Spec. Rept. 56*, 1960.

comparable capital intensity and risk, both factors which have a substantial effect on market interest rate. Efficiency is promoted if all economic planning is based on the same discount rate. If federal monetary control is not used to bring public and private discount rates in line, one approach would be to use the private rate deliberately for public planning. Eugene L. Grant states a common discount rate used by public utilities to be 7.0 percent after income taxes.<sup>1</sup> Hirshleifer has estimated the marginal internal rate of productivity in private companies engaged in water resources type of activity to be about 5.0 percent after income taxes. This is equivalent to about 8.3 percent for tax-free public investment. He rounds this upward to 10.0 percent to include other taxes and because the government usually undertakes the less desirable and hence more risky projects.<sup>2</sup>

- 4 A fourth approach is to tie the discount rate to the source of funds rather than to productivity. This yields a discount rate equal to the interest paid on borrowed funds for governments using bond financing. For pay-as-you-go financing, tax money has an opportunity cost equaling its productivity had the taxpayer been able to keep it. Rather elaborate statistical studies have been made to estimate tax incidence (who it is who really pays the taxes) and the spending and investment habits of the incident groups in order to determine the social cost of federal financing.<sup>3</sup> Economic efficiency is promoted if the government invests money in projects yielding returns larger than those from taxpayer investment foregone, but tying discount rates to borrowing causes some confusion between financial and economic analysis. Krutilla and Eckstein concluded from their study of federal tax revenue that the social cost of federal financing is about 6.0 percent.
- 5 A final approach is to pick a social discount rate on the grounds that the capital market is too badly distorted to have any normative significance. It:

assumes the capital market to be imperfect, to be rife with rationing, ignorance, differential tax treatments, reluctance to finance investment from external funds, slow adjustment processes, etc., which destroy the normative significance of actual rates found in the market. . . . Once the interest rates in the markets are denied their normative role, the rate for public decision-making must be derived from other considerations. It can be derived from individual revealed preference, from a planner's preference

<sup>1</sup> *Ibid.*, p. 83.

<sup>2</sup> Hirshleifer, DeHaven, and Milliman, *op. cit.*, pp. 144-148.

<sup>3</sup> John V. Krutilla and Otto Eckstein, "Multiple Purpose River Development" (Baltimore: The John Hopkins Press, 1958), pp. 78-130.

model, or from a vision revealed in a dream, it is a value judgment, pure and simple.<sup>1</sup>

Those who make such a value judgment usually produce a low discount rate with the goal of increasing capital investment for the benefit of future generations.

Whenever market evaluation is discarded, some alternative must be substituted. Some economists have postulated that the political decision-making process both is able to and in fact actually has made meaningful decisions on discount rate selection. Other economists distrust both the market and the political mechanisms and prefer to rely on a value judgment by some decision maker.<sup>2</sup> The discount rate may be selected around a conference table or through the use of a mathematical model designed to achieve a desired rate of economic growth. While advocates of a social discount rate do not agree as to how such a rate should be established, they favor use of a low discount rate to increase savings in the form of more public works projects. A typical value is 2.0 percent.

#### 6-8 CURRENT DISCOUNTING PRACTICE

Until 1968, the official policy of the United States government has been:

The interest rate to be used in plan formulation and evaluation for discounting future benefits and computing costs, or otherwise converting benefits and costs to a common time basis shall be based upon the average rate of interest payable by the Treasury on interest-bearing marketable securities of the United States outstanding at the end of the fiscal year preceding such computation which, upon original issue, had terms to maturity of 15 years or more. Where the average rate so calculated is not a multiple of one-eighth of 1 percent, the rate of interest shall be the multiple of one-eighth of 1 percent, next lower than such average rate.<sup>3</sup>

From 1960 through 1965, this rate rose approximately 0.125 percent per year to 3.25 percent and remained at that value through 1968. "Securities outstanding at the end of the fiscal year" have a lower average interest rate than securities sold within the year in periods of rising interest rate. In 1968 the Federal Water Resources Council recommended a policy change

<sup>1</sup> Otto Eckstein, *A Survey of the Theory of Public Expenditure Criteria*, in National Bureau of Economic Research, "Public Finances: Needs, Sources, and Utilization" (Princeton, N.J.: Princeton University Press, 1961), p. 503.

<sup>2</sup> More discussion is found in Marglin, *Economic Factors*, *op. cit.*, p. 197.

<sup>3</sup> The President's Water Resources Council, *Policies, Standards, and Procedures in the Formulation, Evaluation, and Review of Plans for Use and Development of Water and Related Land Resources*, 87th Cong., 2d Sess., Sen. Doc. 97, 1962, p. 12.

to using a rate based on securities sold within the year and rounded to the nearest one-eighth of 1 percent provided that the maximum change in any 1 year shall be limited to one-quarter of 1 percent. When the recommendation was followed in 1969, the rate changed to 4.625 percent.

Federal agencies have used higher rates for discounting benefits and private costs than they have for public costs because private parties must pay higher interest rates for borrowed money, but the practice has been discontinued because of the planning distortions produced by multiple discount rates. The use of a risk-free rate implies adjustment for risk and uncertainty by corrections to the time streams of benefits and costs rather than by adding a risk component to the discount rate. Discounting practice varies among nonfederal planning groups, but most of them use higher rates because of their higher cost of borrowing. Private groups tend to use still higher rates. Thus, diversity in discounting practice still distorts project evaluation among planning groups.

#### 6-9 CONSEQUENCES OF DISCOUNTING AT A LOW RATE

In making the value judgments necessary in discount rate selection, it is necessary to evaluate the consequences of using a low rate.

- 1 When the private discount rate exceeds the public discount rate, the private sector evaluates the present worth of assets at a higher discount rate and thus gives them a lower present worth. The project analysis evaluates present worth at a low discount rate and thus gives a high value. The value of a project will thus be overstated if its present worth at a low discount rate is compared with the value given other assets in the private sector by a market which implicitly uses a higher discount rate. While economy studies should evaluate all assets on a common basis, the difficulty is especially acute with respect to right-of-way. A major bias in favor of project construction results if the future income on benefitted land is discounted at a low rate, while the future income foregone on land purchased for right-of-way is implicitly discounted at a higher rate by using market prices for land in figuring project cost.<sup>1</sup> The way to achieve consistency is to evaluate right-of-way by a shadow price based on discounting expected income from the land at the lower rate. Such evaluation can readily be made for rural areas where annual farm income is predictable but becomes more complex for urban fringe areas where the predicted time stream of earned income is more uncertain.
- 2 Planning with a low discount rate increases the optimum project size,

<sup>1</sup> For an example, see Otto Eckstein, "Water Response Development: The Economics of Project Evaluation" (Cambridge, Mass.: Harvard University Press, 1958), pp. 146-148.

favors an earlier time of construction, and is unfavorable to stage construction. The projects selected become bigger and more inflexible to adjustment with changing technoeconomic conditions. A low discount rate increases savings for the future, but it also increases the risk that large amounts of capital will be tied to projects unusable by future generations and, to the extent this happens, defeats the whole purpose of capital formation.

- 3 Planning with a low discount rate is also highly favorable to project justification. The lower the discount rate and the longer the anticipated project life, the more likely project benefits are to exceed the cost. A low discount rate increases resource allocation to, and the capital intensity of investment within, those sectors in which it is used.
- 4 Planning with a low discount rate will also normally show many more projects to be economically justified than funds are available to build. A sophisticated method of project ranking (Sec. 9-12) could overcome this difficulty by picking the best of the justified projects, but actual project selection is more likely to be placed on some basis other than economic merit.

Use of a low discount rate is favorable to the construction of public works projects and the interests which profit by project construction, but excessive diversion of resources to the public sector is detrimental to economic and even social efficiency and thus the long-run welfare of the nation. Solutions to pressing current needs may have to be sacrificed for the benefit of those living in the distant future. It is not possible to defend any exact discount rate for use in government planning dogmatically, but too low a rate definitely has serious adverse consequences to national economic growth.

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**PROBLEMS**

- 6-1 Assume the lines on the indifference map between consumption  $C$  and future income  $F$  are represented by  $V = C^5(F + 50)$ .
- Derive an expression for  $C$  as a function of annual income  $I$  and rate of return  $i$  on saving  $S$ .
  - Derive an expression for the minimum income  $I_m$  required for a person to save anything.
  - How much will a person earning \$5,000 annually save if he can obtain a return of 5 percent?
  - How much will his savings increase if his income doubles?
  - If instead, the rate of return doubles, how much will his savings increase?
- 6-2 Capital investment in a certain water resources development is expected to produce an annual return of 15 percent. Capital formulation may be summarized by the table:

Year	(1) Capital stock	(2) Returns	(3) Investment	(4) Consumption
1	\$1,000		\$150	
2	1,000 + (3) for year 1			

Complete the table for 10 years if no returns are invested to increase capital stock.

- 6-3 Complete the table of Prob. 2 assuming \$75 is invested annually. What is the average annual growth rate of capital stock? Of consumption?
- 6-4 Complete the table of Prob. 2 assuming half the returns are invested annually. What is the average annual growth of capital stock? Of consumption?
- 6-5 Complete the table of Prob. 2 assuming consumption is 100 in the first year and increases at a rate of 4 percent annually thereafter. Also assume capital stock depreciates at a rate of 3 percent annually,  $C_t = 0.97C_{t-1} + I_{t-1}$ . In which year is capital stock a maximum? In which year does consumption become so large as to prevent further investment?
- 6-6 Complete the table of Prob. 2 assuming consumption is 80 in the first year and the growth rate of 4 percent and depreciation rate of 3 percent apply. What is the average annual growth rate of investment? of capital stock?
- 6-7 Compare the solutions of Probs. 5 and 6. What do the results say about the role of "belt-tightening" in economic development? What

would be the implications of accepting a lower rate of return on long-term investment vs. continuing short-term investment yielding 15 percent, as far as economic development is concerned?

- 6-8 A water resources project having a 50-year life has an initial construction cost of \$10 million and an annual cost of \$250,000. The planning discount rate is 4.5 percent. Right-of-way requirements can be provided by an expenditure of \$2 million. The land obtained produces a net annual income of \$200,000. Annual project benefits are \$1,200,000. What is the project benefit-cost ratio? What rate of return are landowners obtaining? What would the project benefit-cost ratio be if the analysis were based on income lost from the land rather than purchase expenditure?