An Introduction of

Nays

A Public Domain Solver for Unsteady Flow and River Geomorphology

iRIC Project Co-Chairman

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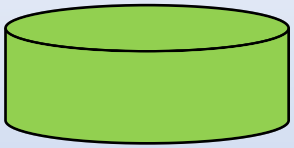
What is iRIC ?

iRIC

international River Interface Corporative

A free software for supporting the numerical modeling of river morphodynamics



Preprocessing

- Read field data
- Grid generation
- Cell condtion
- Computational condition

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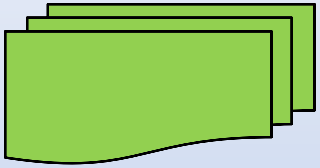
Solver

- Nays
- Morpho2D
- FaSTMECH
- SToRM



Postprocessing

- Visualization
- Output the result
  - text data
  - figures (jpg, png etc)
- Animation on Google Earth

What can we do by using Nays?

Computation of flow, bed deformation and bank erosion in rivers

Unsteady flow calculation
in general coordinate grid

Bed Deformation
Bannk Erosion

Cell Condition
- Obstacle
- fixed – movable bed
- vegetation

Model Descriptions

- 2D Shallow Water Equation
- General Coordinate Grid System
- Moving Boundary Fitted Coordinate
- Turbulence Model (zero-eq. model, k-ε model)
- Bed load + Suspended load
- Secondary Flow for Bed load Transport
- Bank Erosion Model
- Unsteady Computation
- Finite Difference Method
- Uniform Sediment
- Cell Condition(obstacle, fixed bed, vegetation etc)

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Calculation Procedure

1. Specify the solver by XML file.
2. Generate the computational grid.
   - .csv .tpo file format
   - cell condition
3. Set the computational condition.
   - initial condition, boundary condition
   - computational method
4. Run the nays solver.
5. Visualize the computational result.
   - iRIC animation
   - Animation on Google Earth

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Sand Bar Evolution in Sine-Generated Curve

Exercise 2

Flow Computation with River Training Structure

Exercise 3

Flood Calculation in Real Scale River

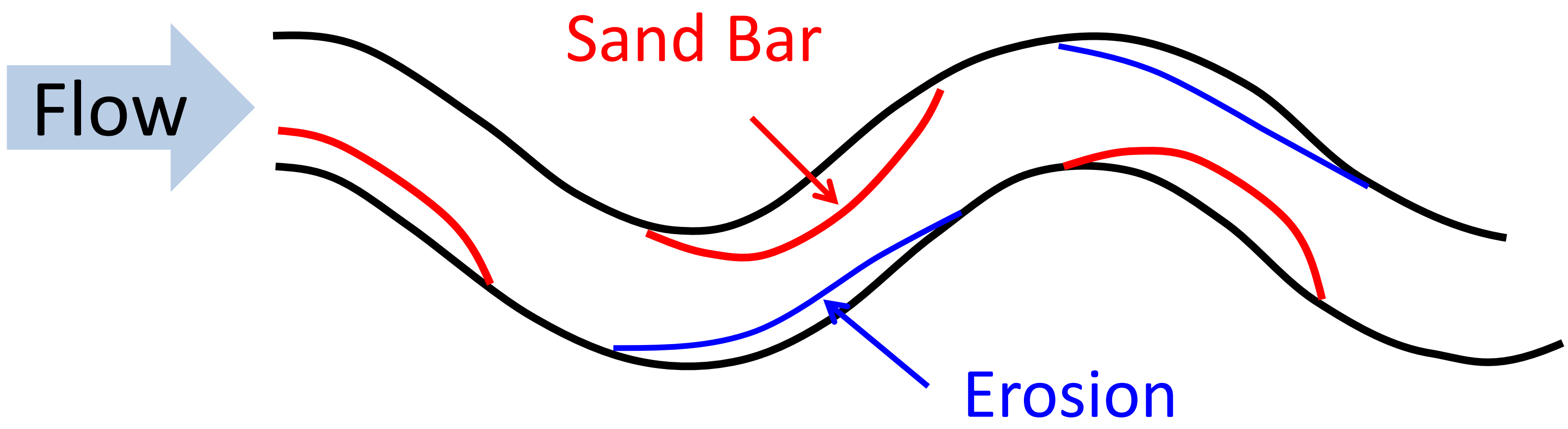
Sand Bar Evolution

in Sine-Generated Curve Channel

EXERCISE 1

Exercise 1

- Computation of flow and sand bar evolution in sine-generated curve channel

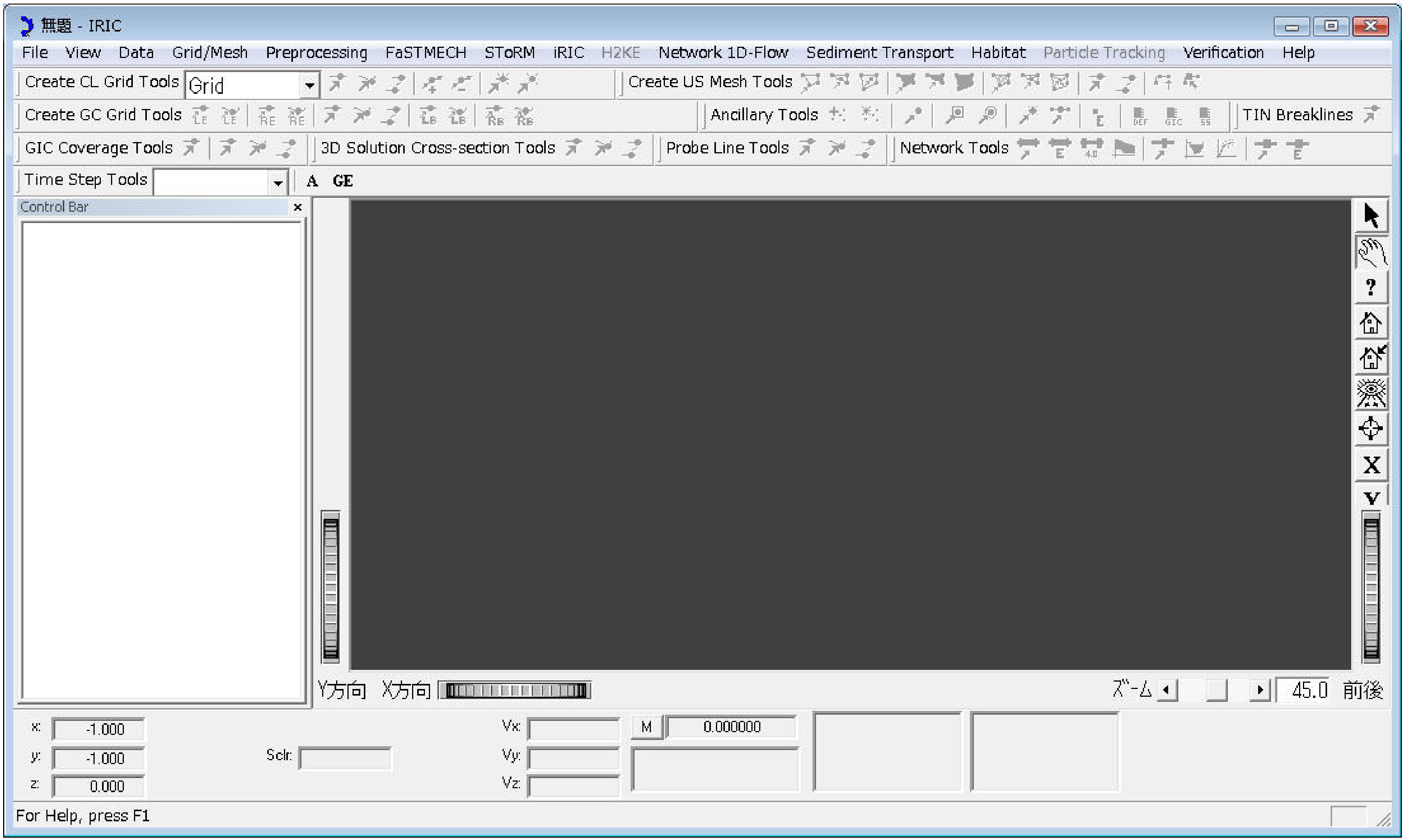


Start iRIC

- Click following icon.

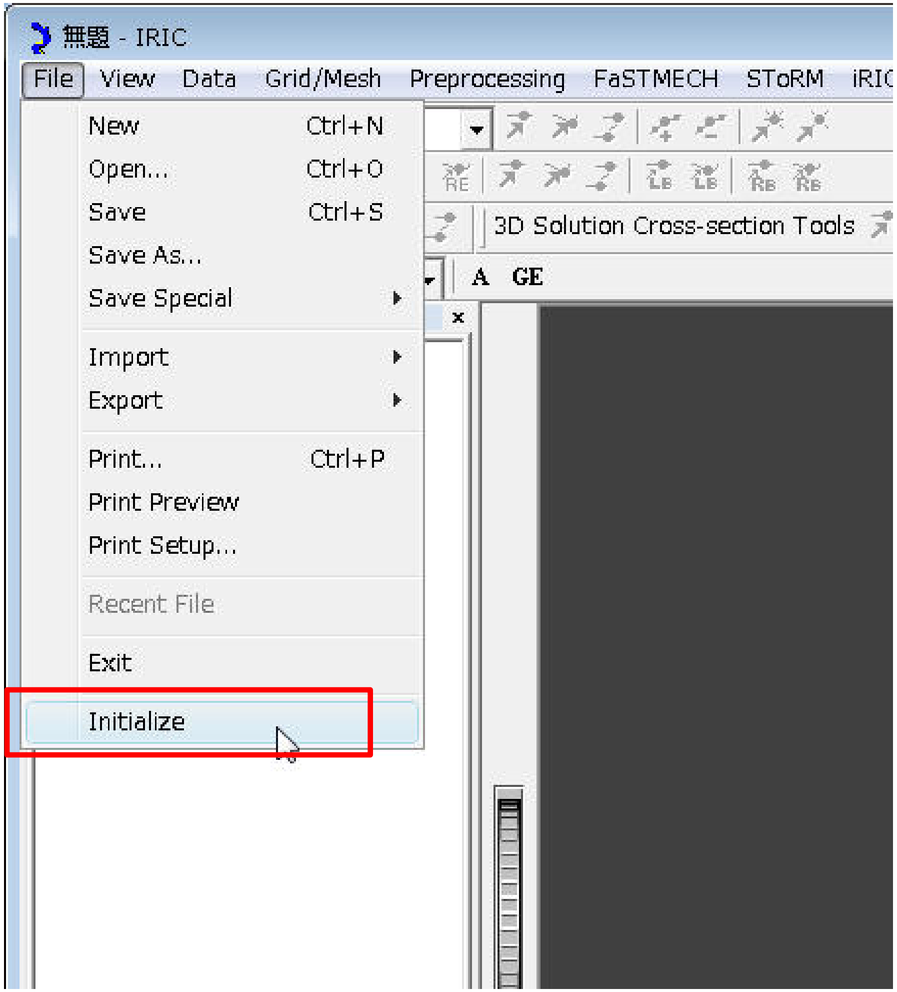
Main Window of iRIC

- Specify the solver in which you want to use by XML file.



Read XML file(1)

- Specify the solver in which you want to use by XML file.
- File → Initialize



- Nays → Nays.xml
- Morpho2D
  → Morpho2D.xml
- FaSTMECH, SToRM
  → not need to read

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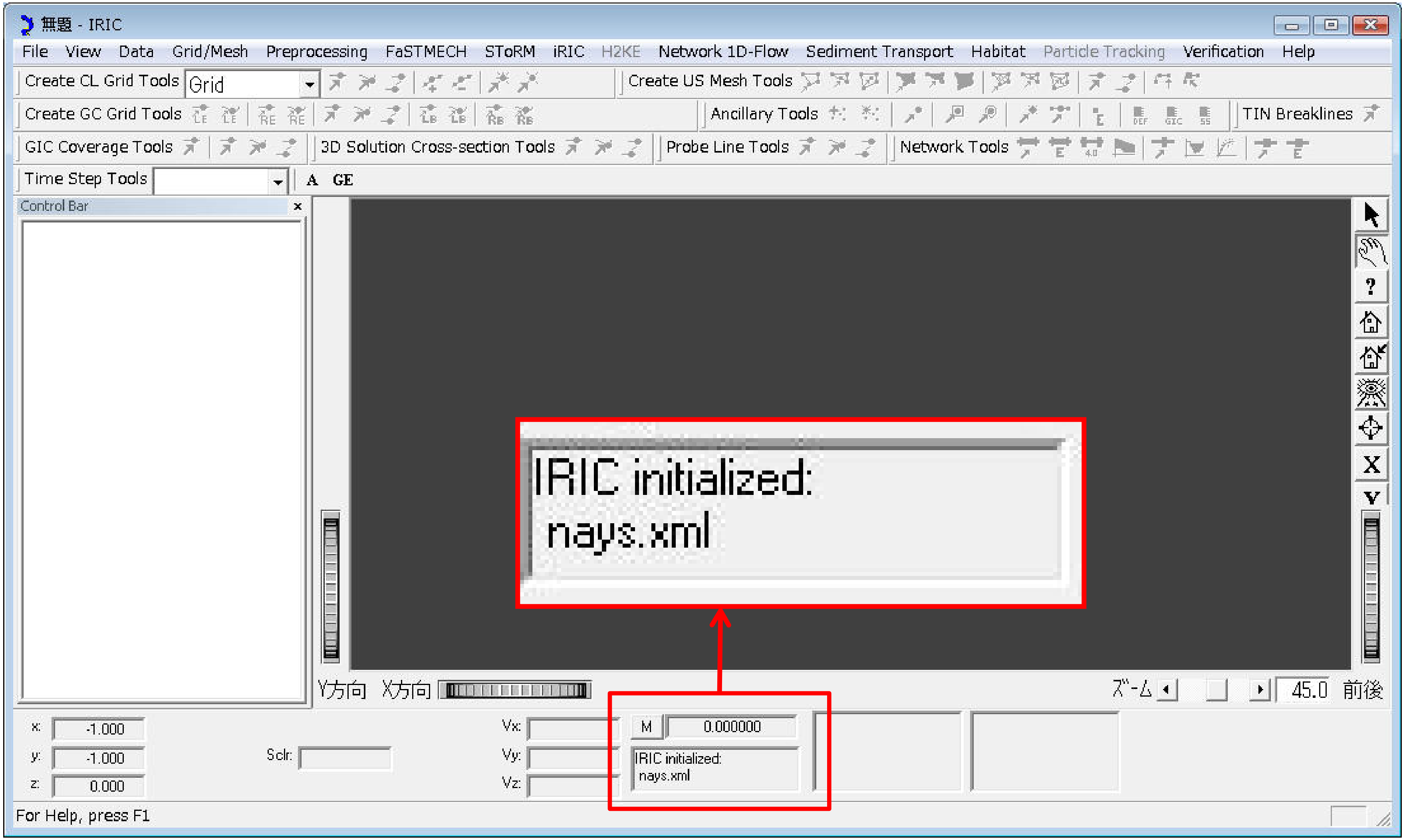
Read XML file(2)

- The solver is registered as follows by XML file.



Read the grid, bed elevation data

- Nays can read the two format of grid data.

1. Grid data formatted as CSV

・The number of grid point, x, y coordinate and bed elevation data are written.

・This file is useful, if you already generate the grid data.

2. Bed elevation data formatted as TPO

・Only x, y coordinate and bed elevation are written.

・This file will be used for generating the computational grid from bed elevation data.

Grid data format of CSV file

	A	B	C	D		
1	IMAX	JMAX	KMAX			
2	37	9	2			
3	I	J	K	X	Y	Z
4	0	0	0	0.08589	-0.16045	-0.001
5	1	0	0	0	-0.15	0
6	2	0	0	0.170855	-0.1913	-0.002
7	3	0	0	0.255914	-0.24213	-0.004
8	4	0	0	0.34292	-0.3081	-0.005
9	5	0	0	0.43692	-0.3871	-0.006
10	6	0	0	0.543914	-0.46813	-0.007
11	7	0	0	0.668855	-0.5423	-0.009
12	8	0	0	0.81289	-0.59445	-0.01
13	9	0	0	0.97	-0.613	-0.011
14	10	0	0	1.12688	-0.59351	-0.012

Grid number of I, j direction
（I:flow direction, J:transverse direction）

Grid number of I direction

Grid number of j direction

grid number of k direction

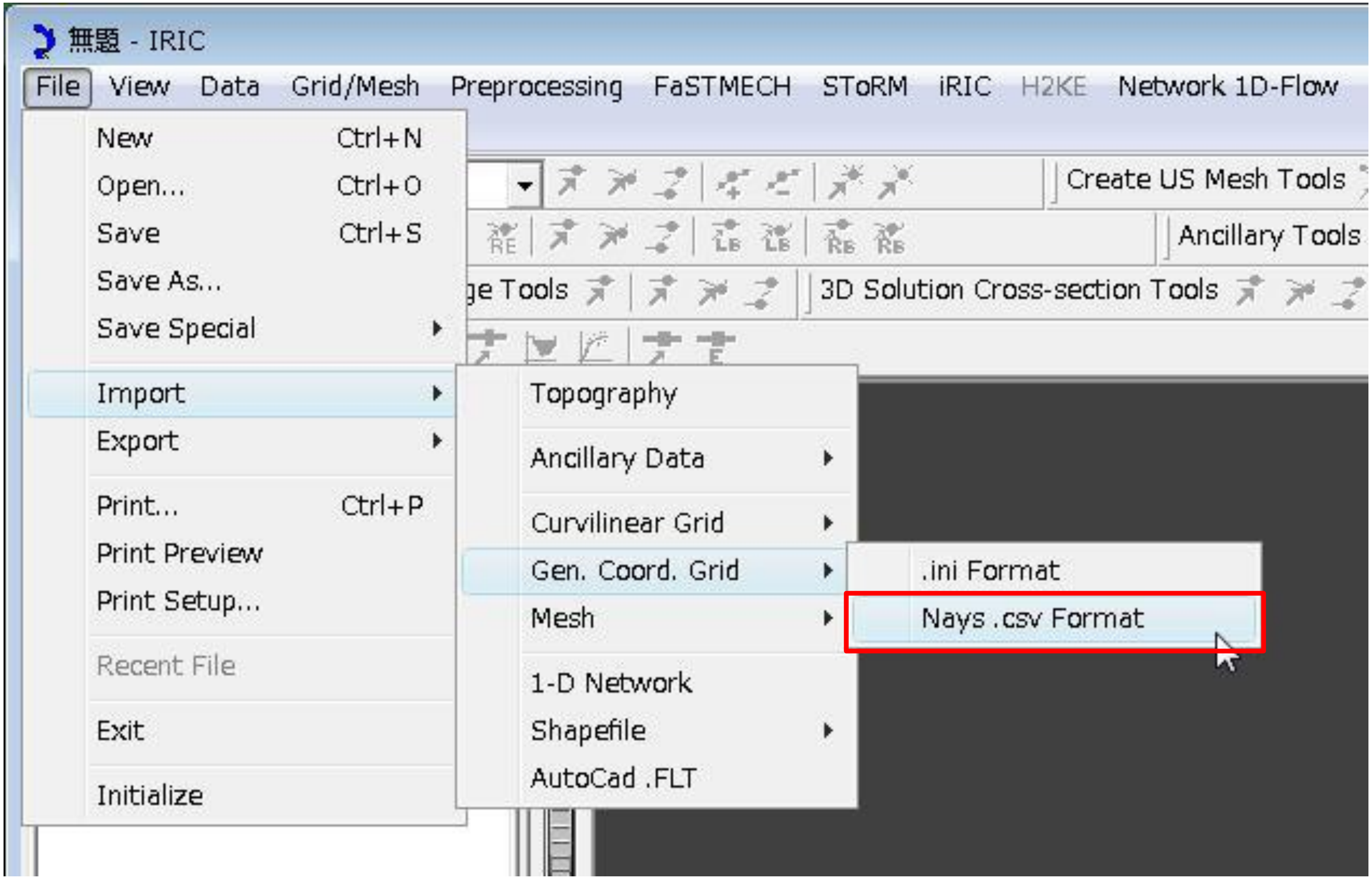
X

Y

Bed elevation

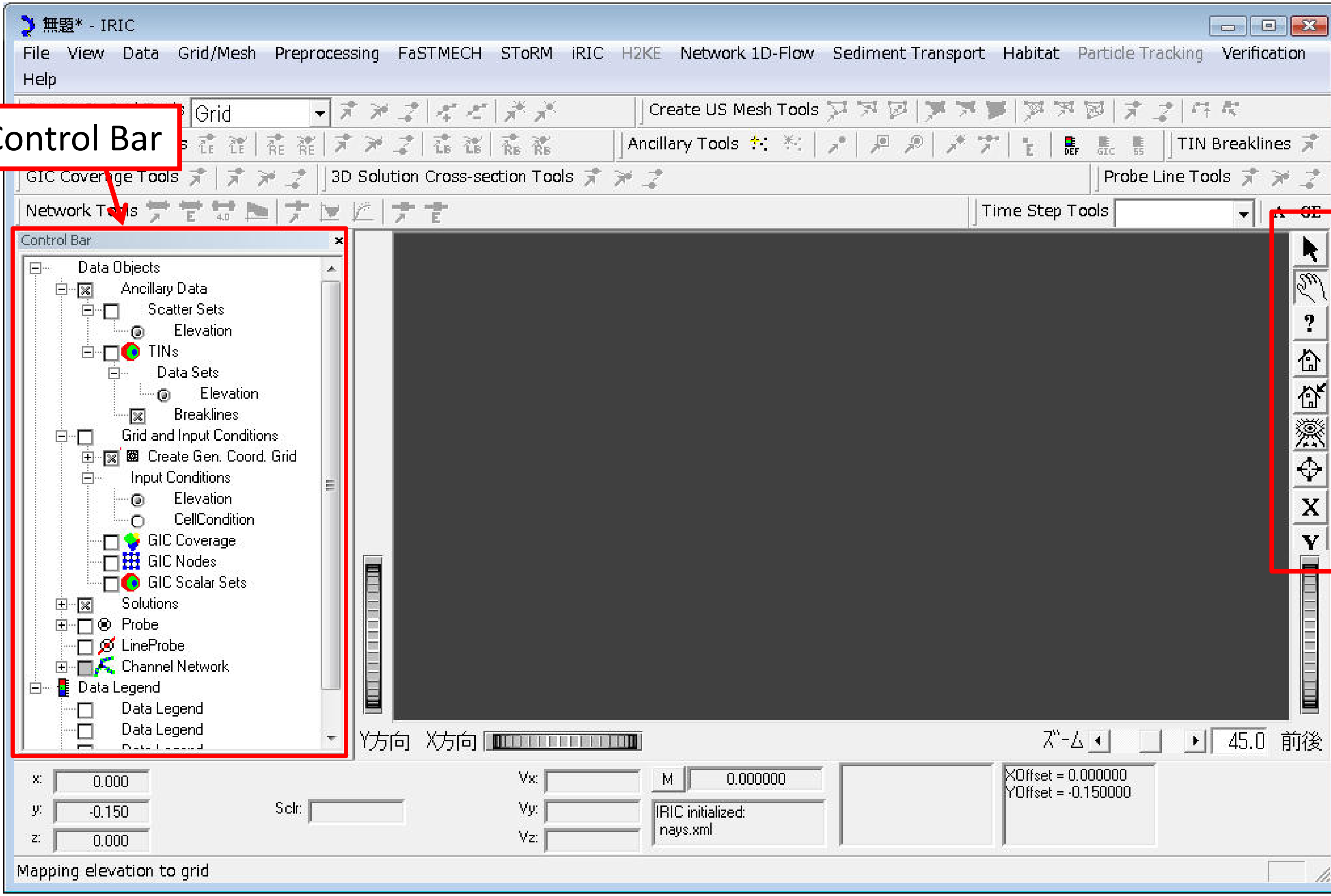
Read Grid Data

- "File"→"Import"→"Gen. Coord. Grid"→"Nays .csv Format"



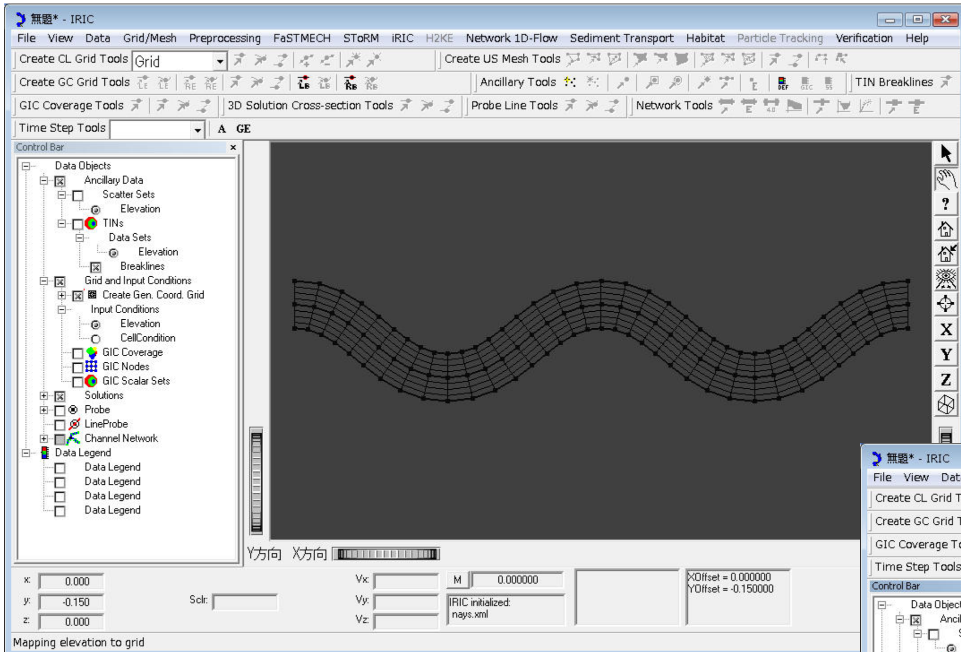
Computational Grid

- You can see the computational grid generated from csv grid data.



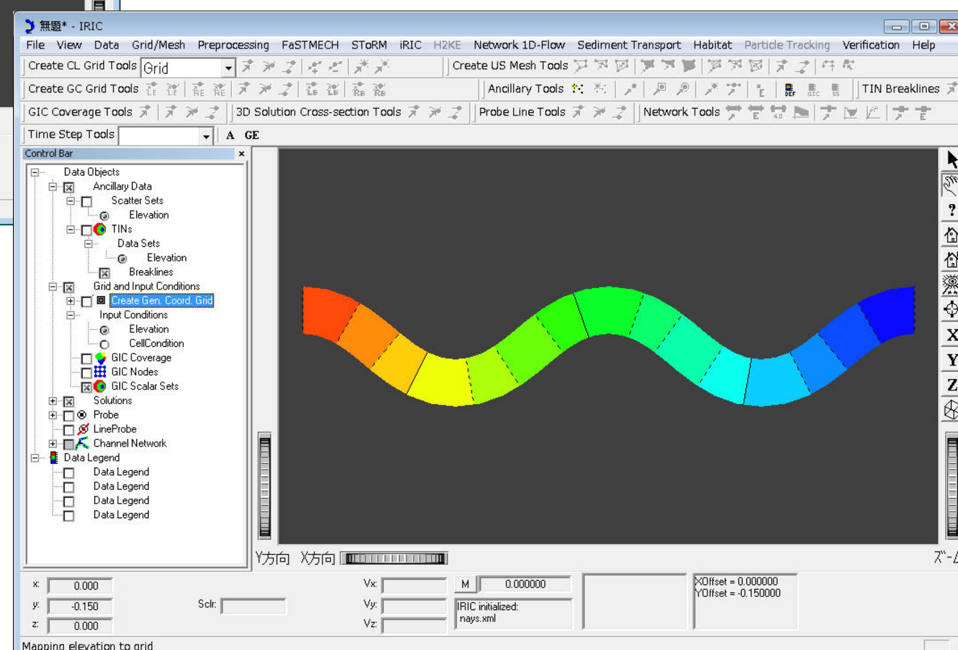
Computational Grid

- Check the computational grid and initial bed configuration.



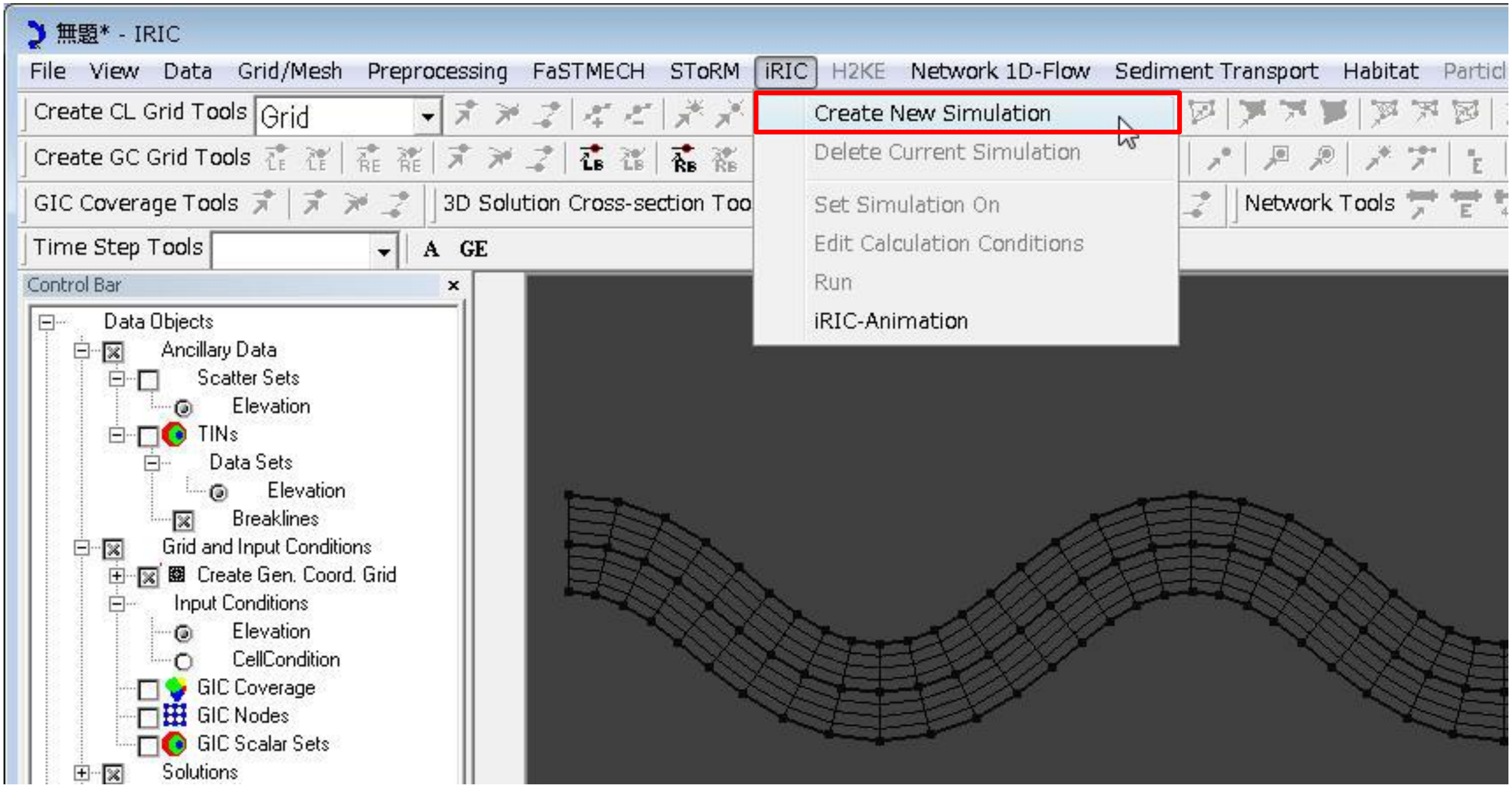
← "grid and Input Conditions"

"GIC Scalar Sets "
in "Grid and Input Conditions" →



Set Computational Conditions(1)

- Create the file for new simulation in "iRIC"→"Create New Simulation".



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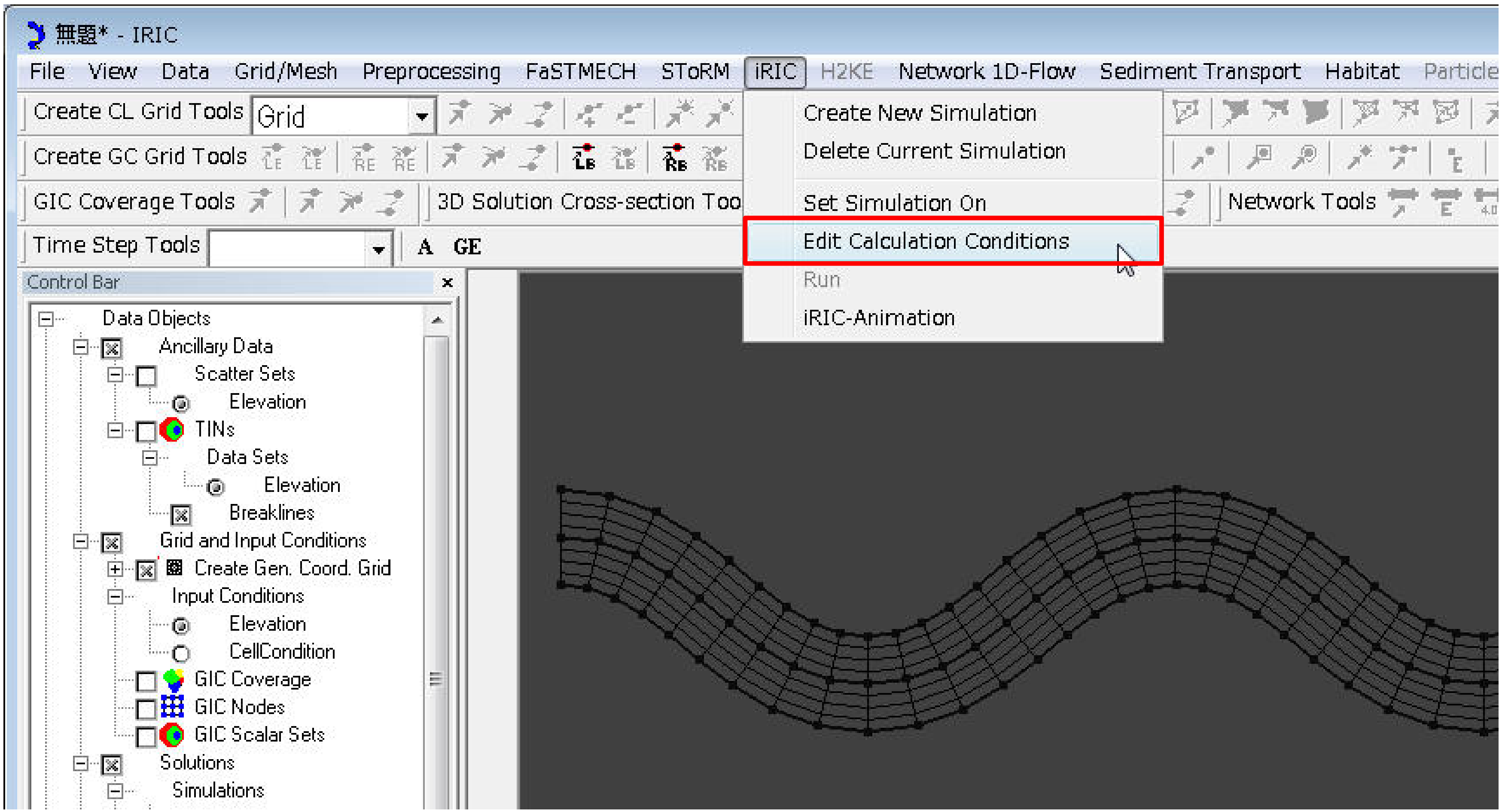
Set Computational Conditions(2)

- The computational condition can be determined on "Edit Calculation Conditions" in "iRIC"



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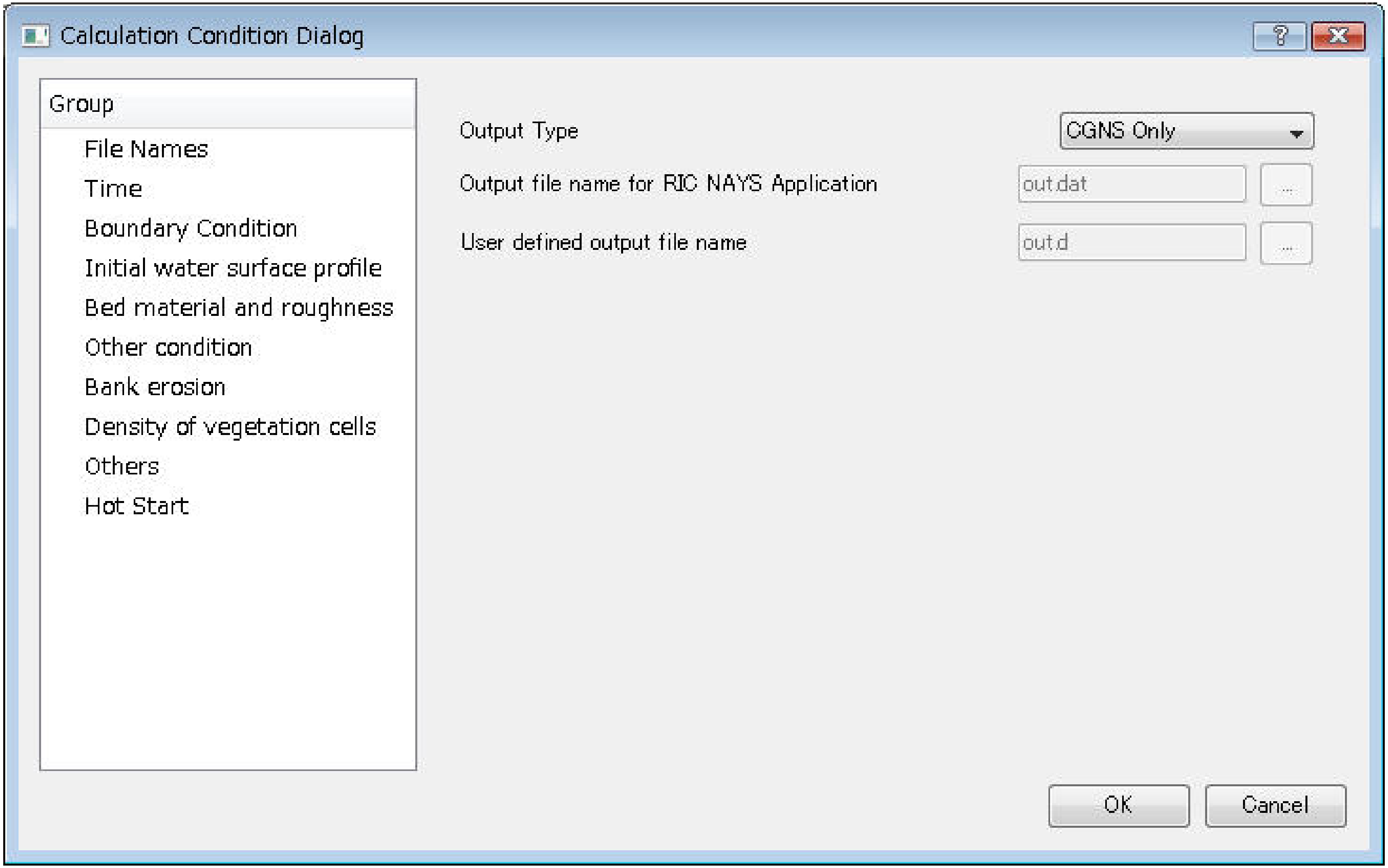
Set Computational Conditions(2)

- Window for setting computational conditions.



Set Computational Condition (3)

- **File Names**
  Output type : CGNS and RIC-Nays

- **Time**
  Output time interval : 10 sec
  Calculation time step : 0.02 sec
  Discharge time series : Import from “Discharge.txt”

- **Boundary conditions**
  Water surface at downstream : Uniform flow
  Slope : calculated from geometric data

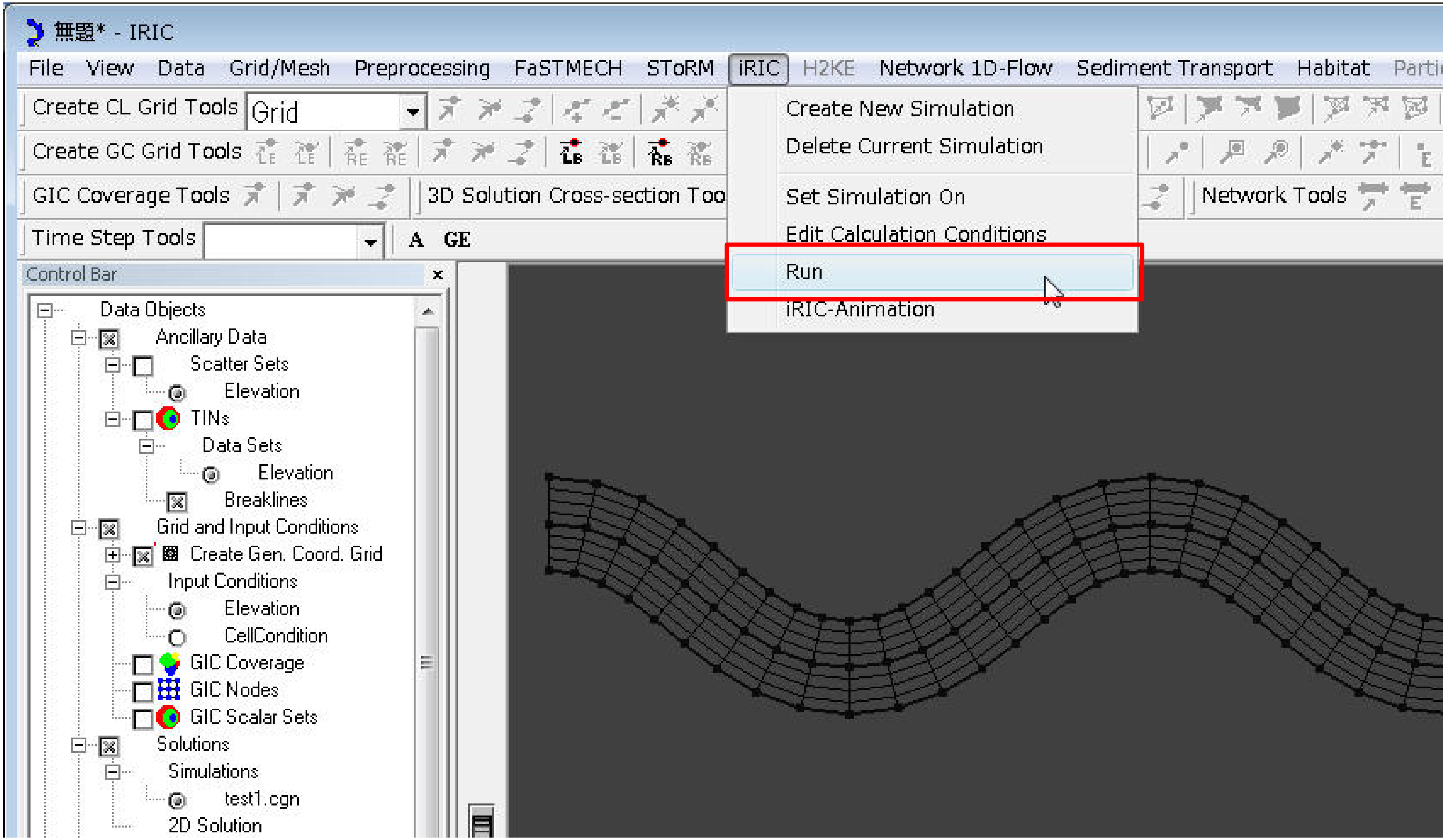
- **Initial water surface profiles**
  Initial water surface : uniform flow

- **Other condition**
  Turbulent model : zero-equation model
  Finite differential method of the advection terms : CIP Method

Other computational conditions are set to default.

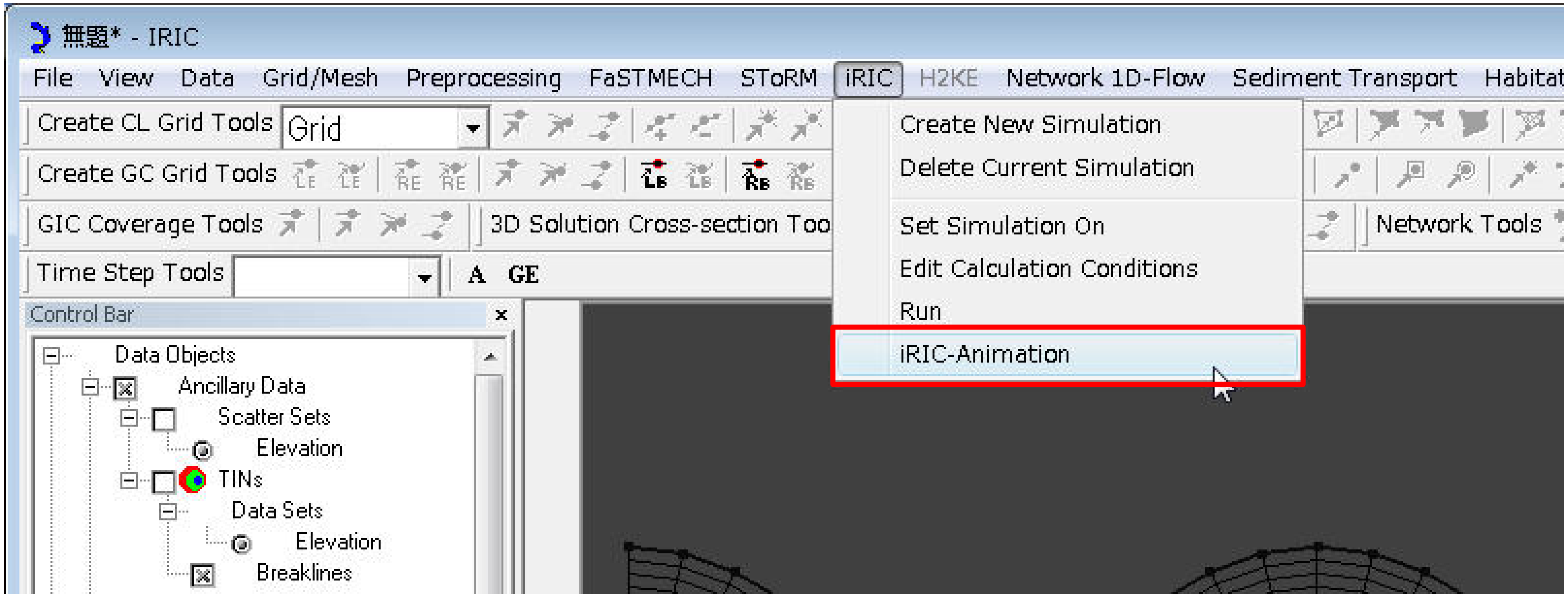
Start computation by Nays Model

- "iRIC"→"Run"

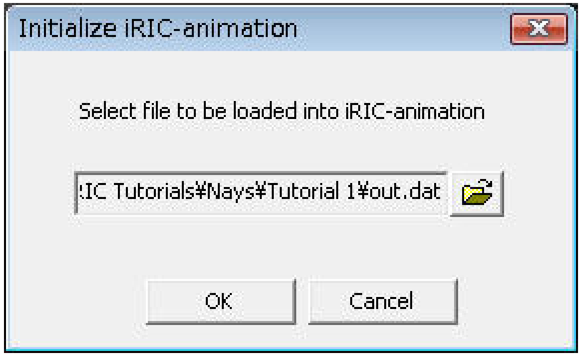


How to Visualize?

- It is useful to use 'iRIC-Animation' for visualization of unsteady results.

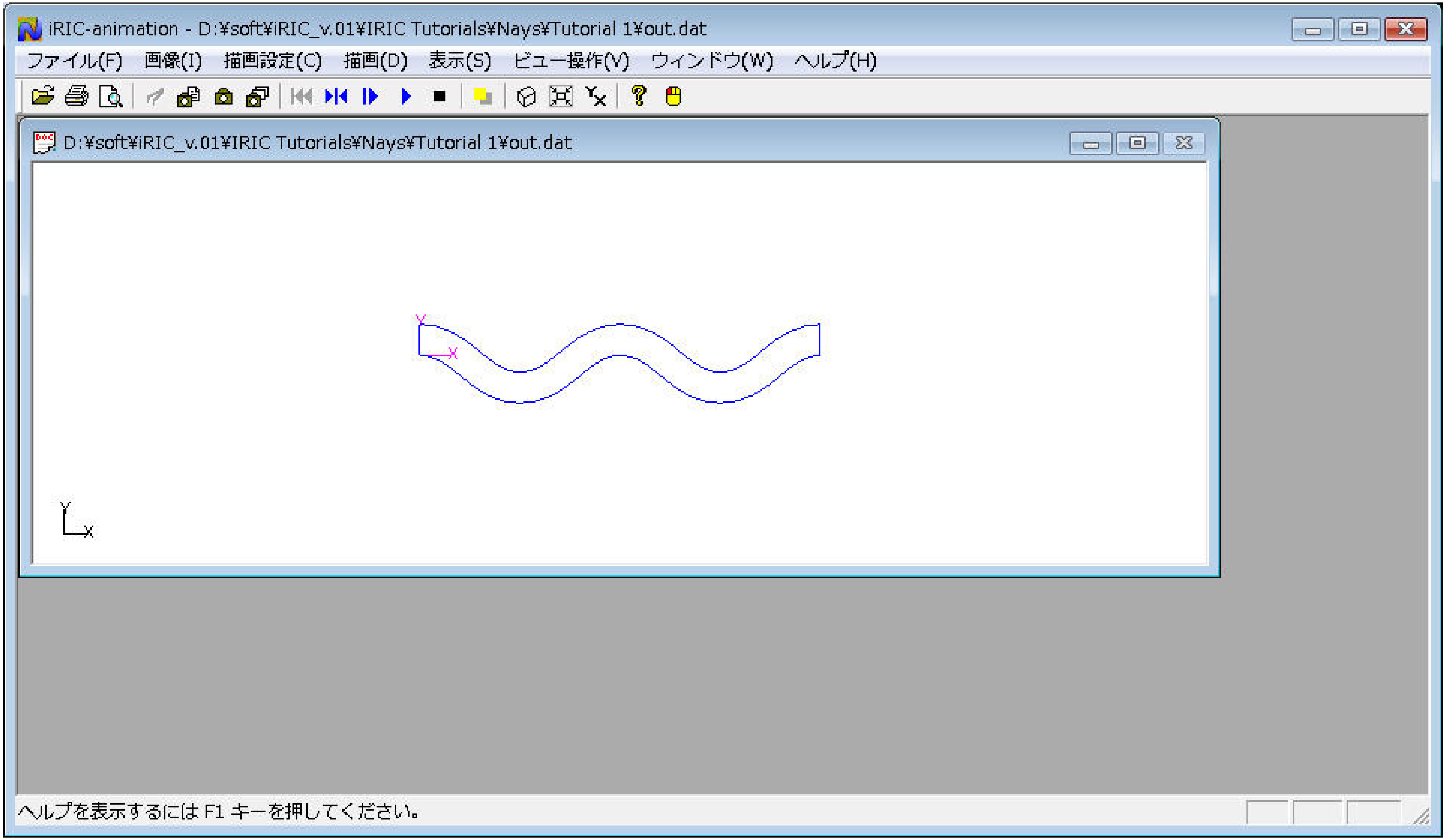


You can specify the output
file for visualization in this dialog. →



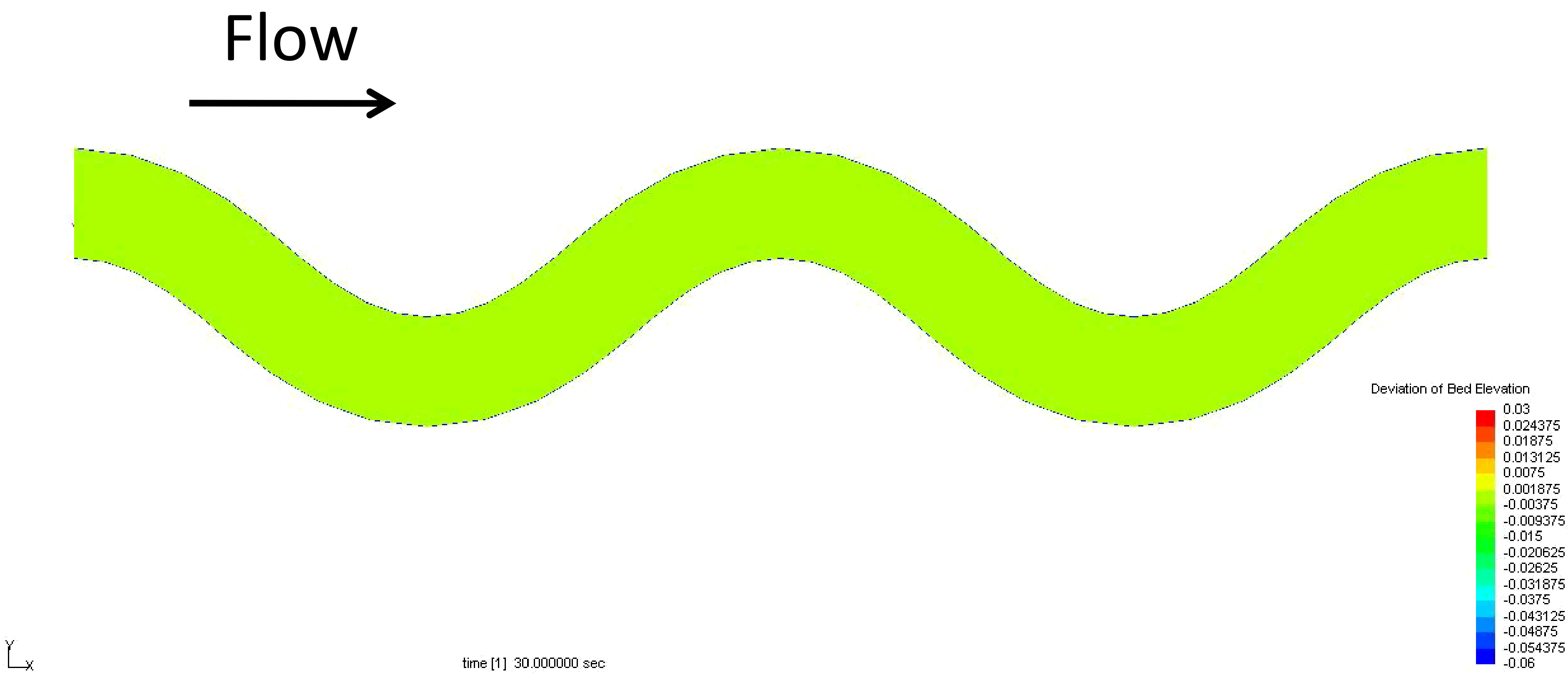
Visualization ～iRIC-Animation～

- Visualization of the computational result by iRIC Animation.



Computational Result

- Animation of bed evolution process

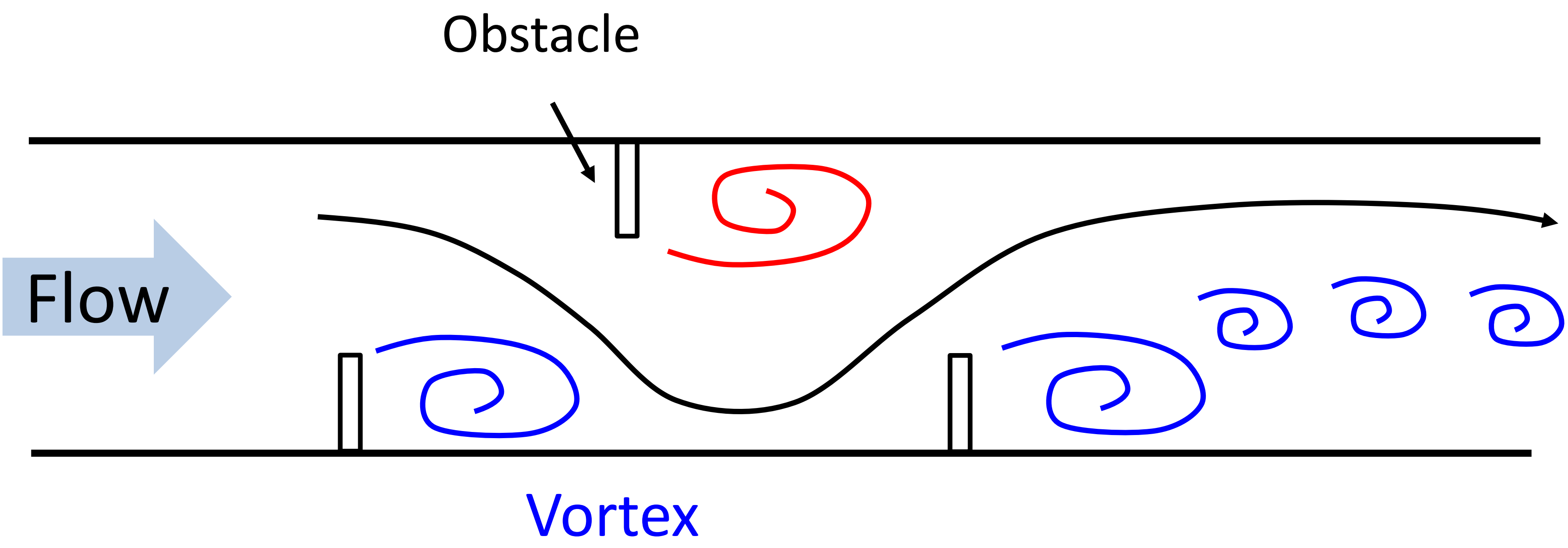


Flow with River Training Structures

EXERCISE 2

Exercise 2

- Flow computation with structures



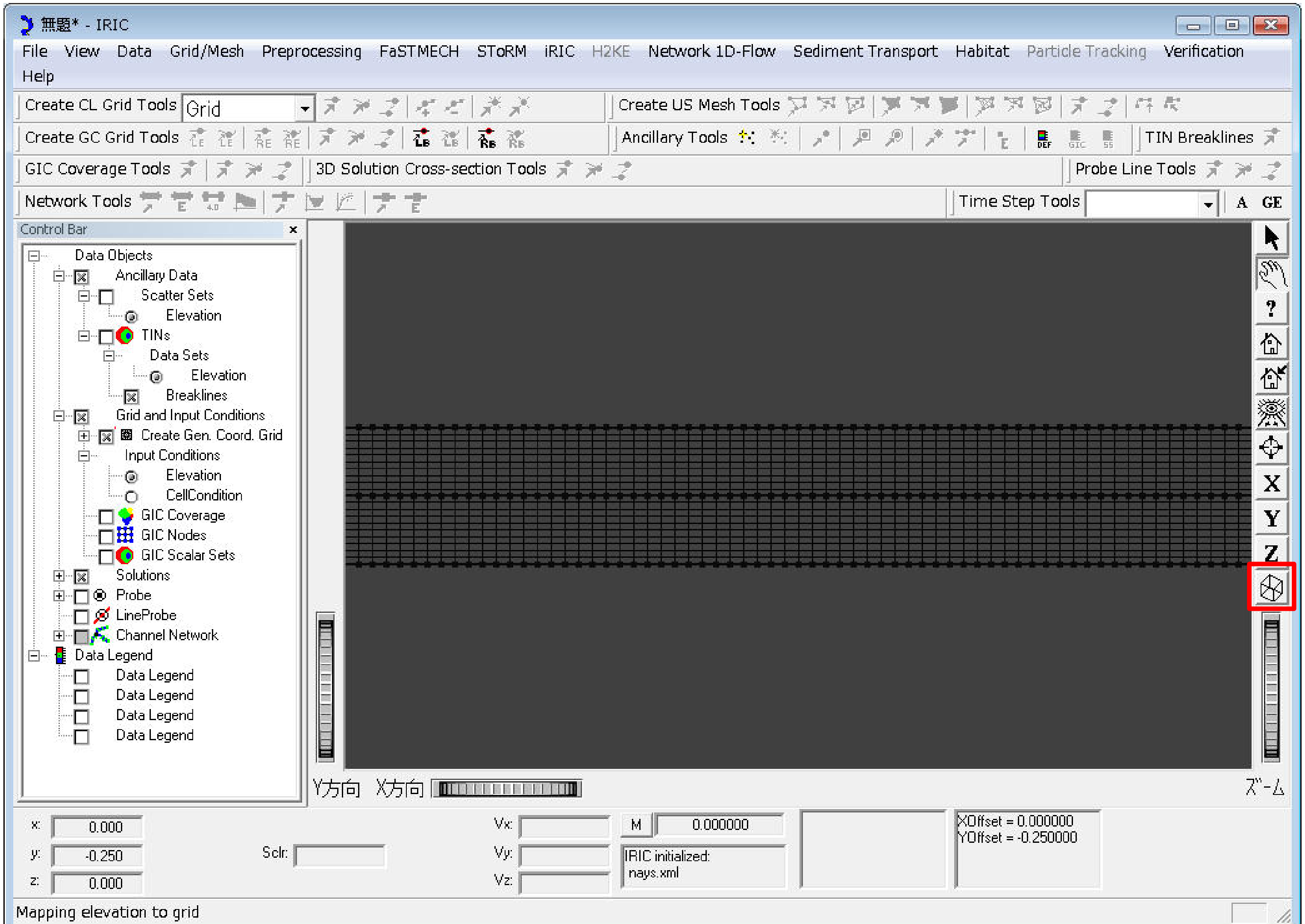
Cell Condition

- Nays model can set the cell condition as follows.

- Obstacle
- Fixed or Movable bed
- Low water channel or
  High water channel
- Density of Vegetation
  (dense, middle, coarse)

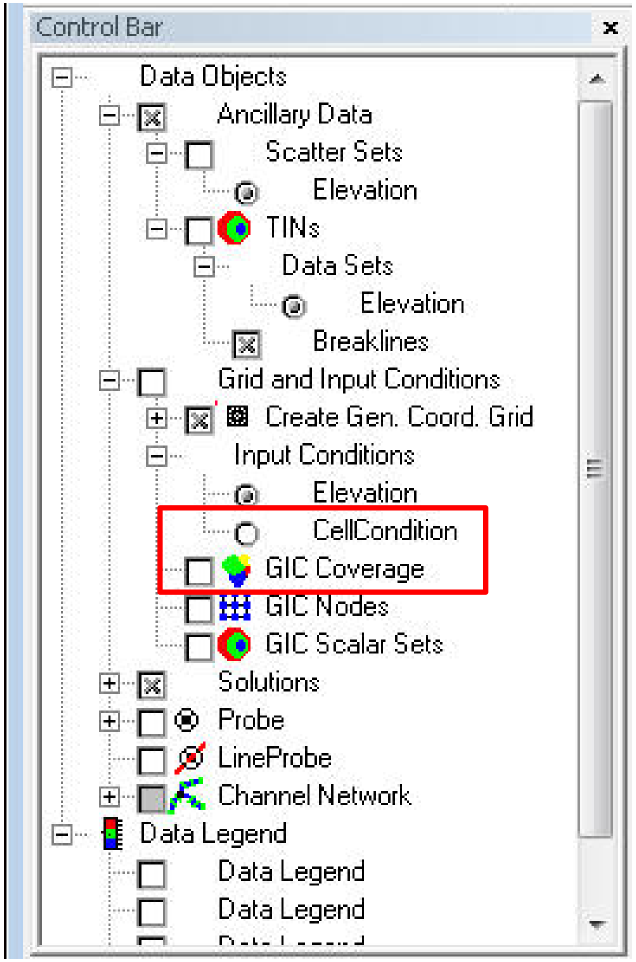
Set Cell Condition(1)

- Cell condition can be set on computational grid.



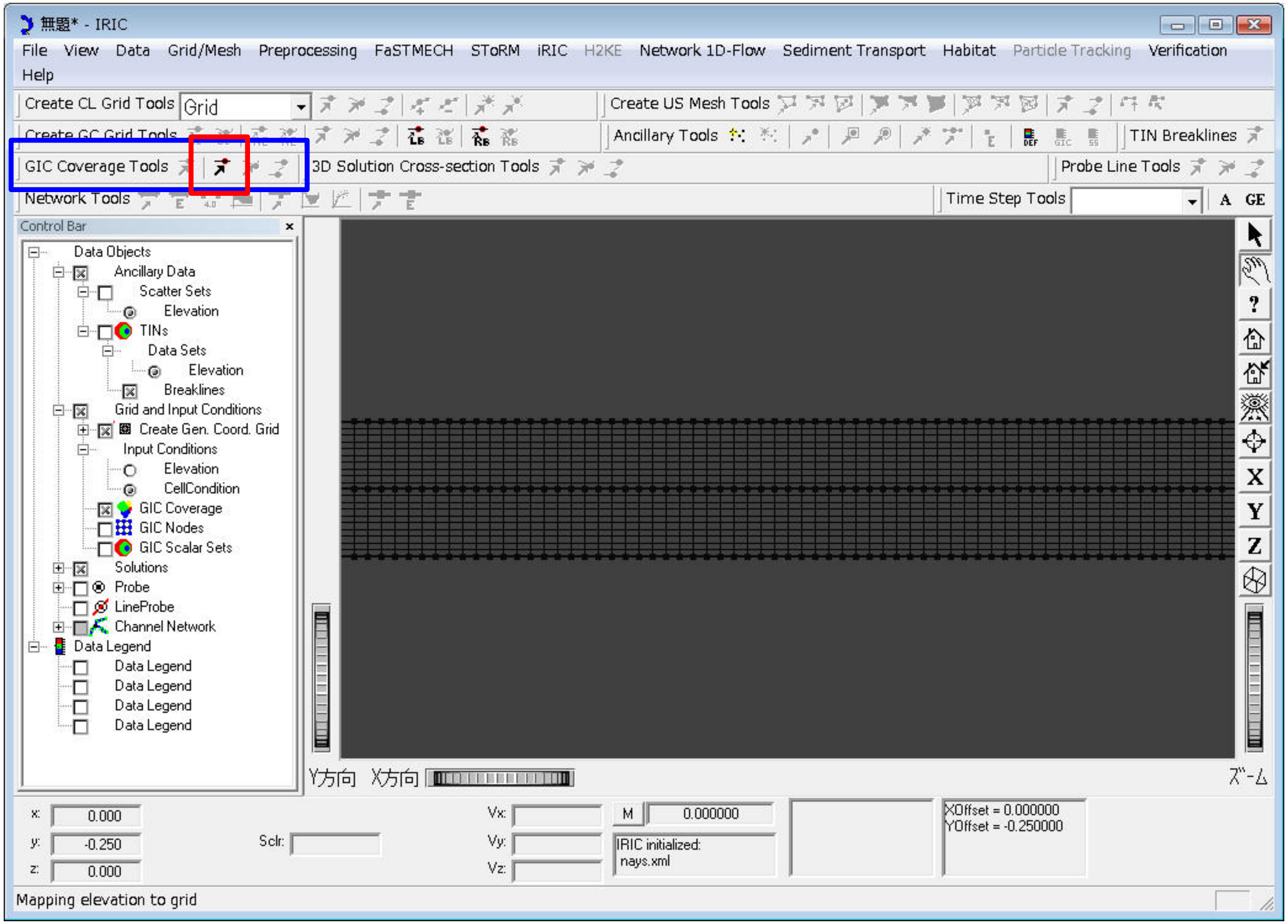
Set Cell Condition(2)

- Check "GIC Coverage" by Control Bar→"Grid and Input Condition" →"Cell Condition"



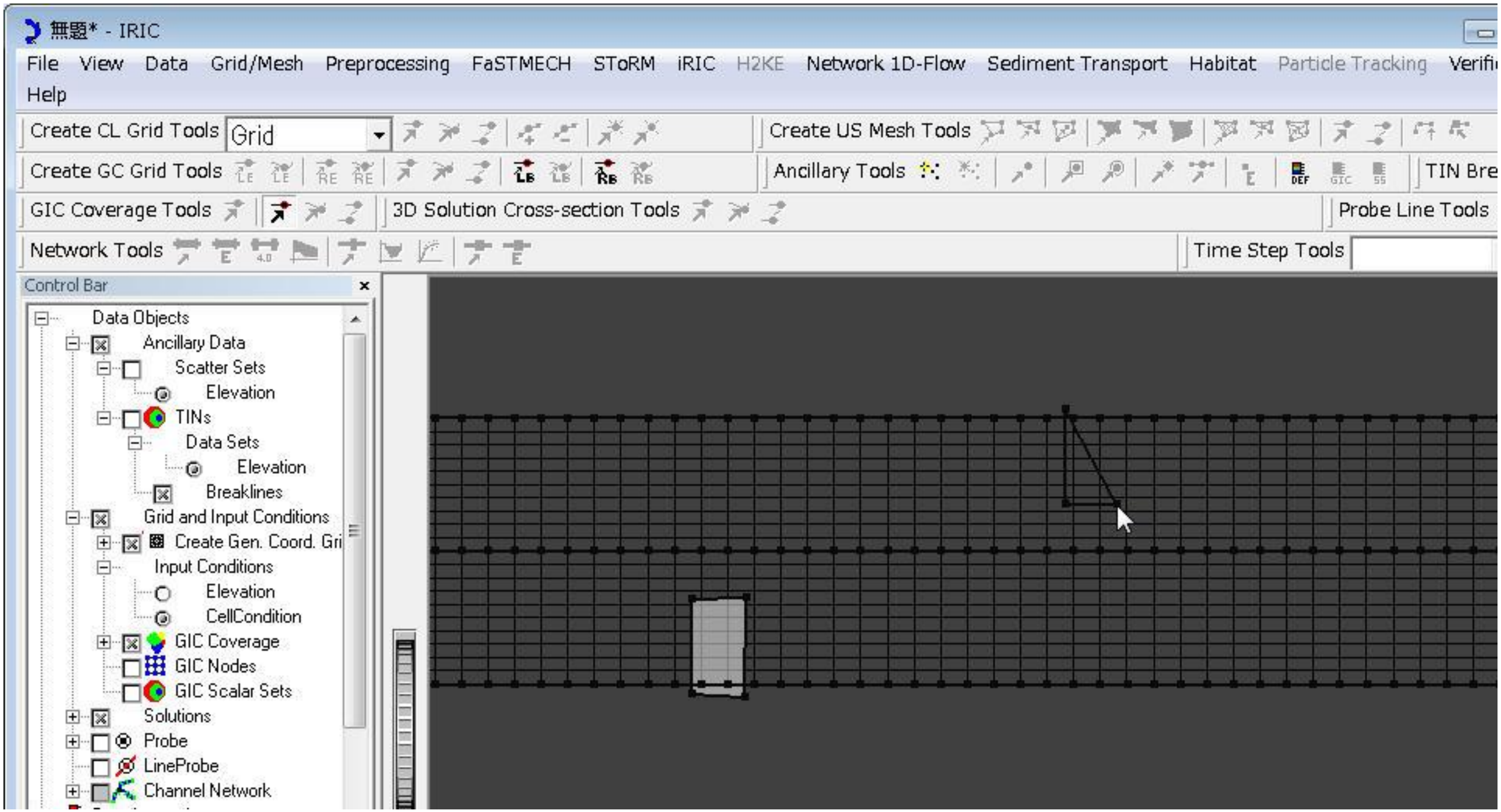
Set Cell Condition(3)

➢ The area in which you want to specify the cell condition can be determined by using "GIC Coverage Tools"



Set Cell Condition(4)

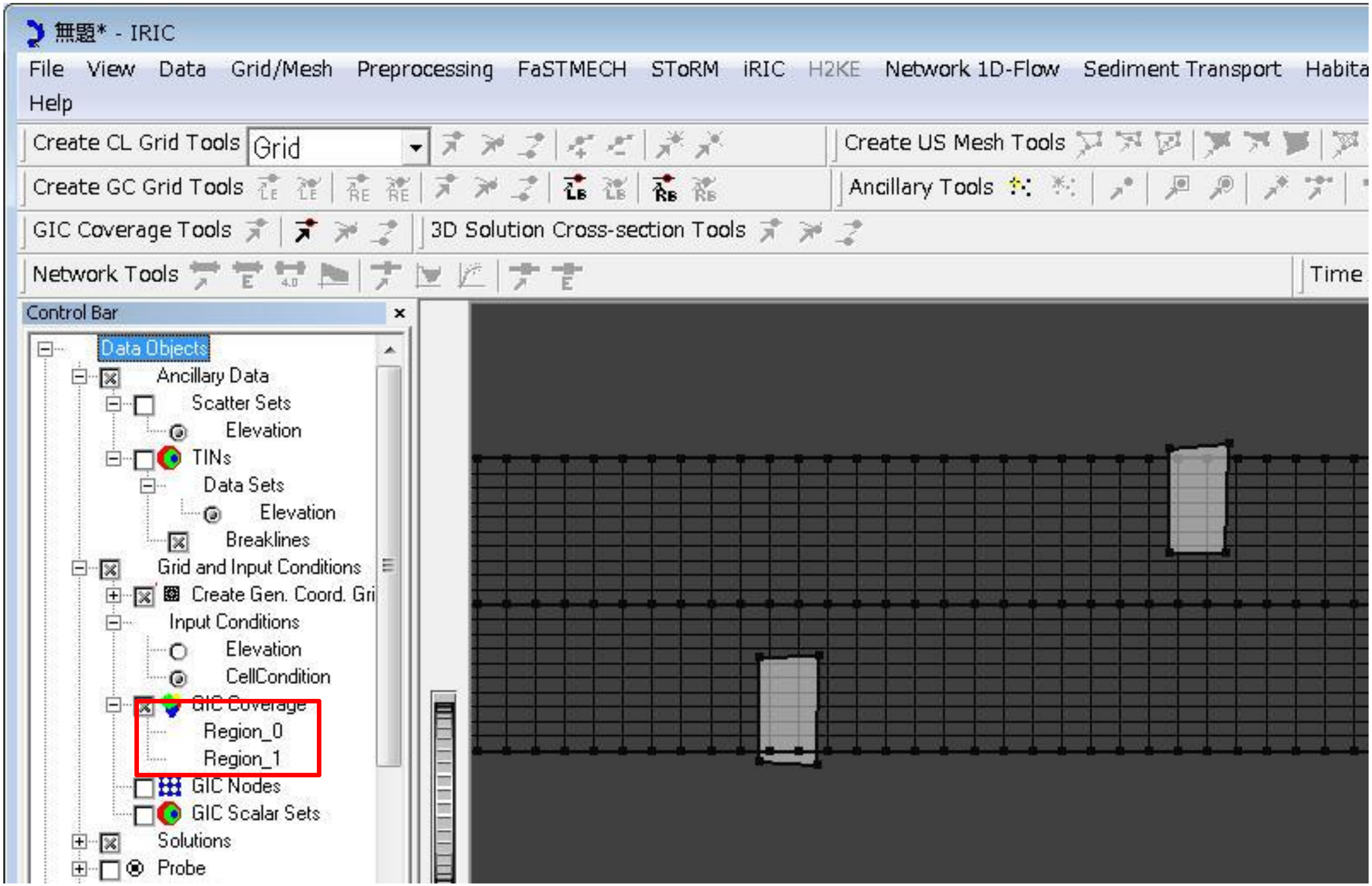
- You can enclose the area as follows.



Left click on the mouse : determine the points
Return key on keyboard : Finish the surrounding the area

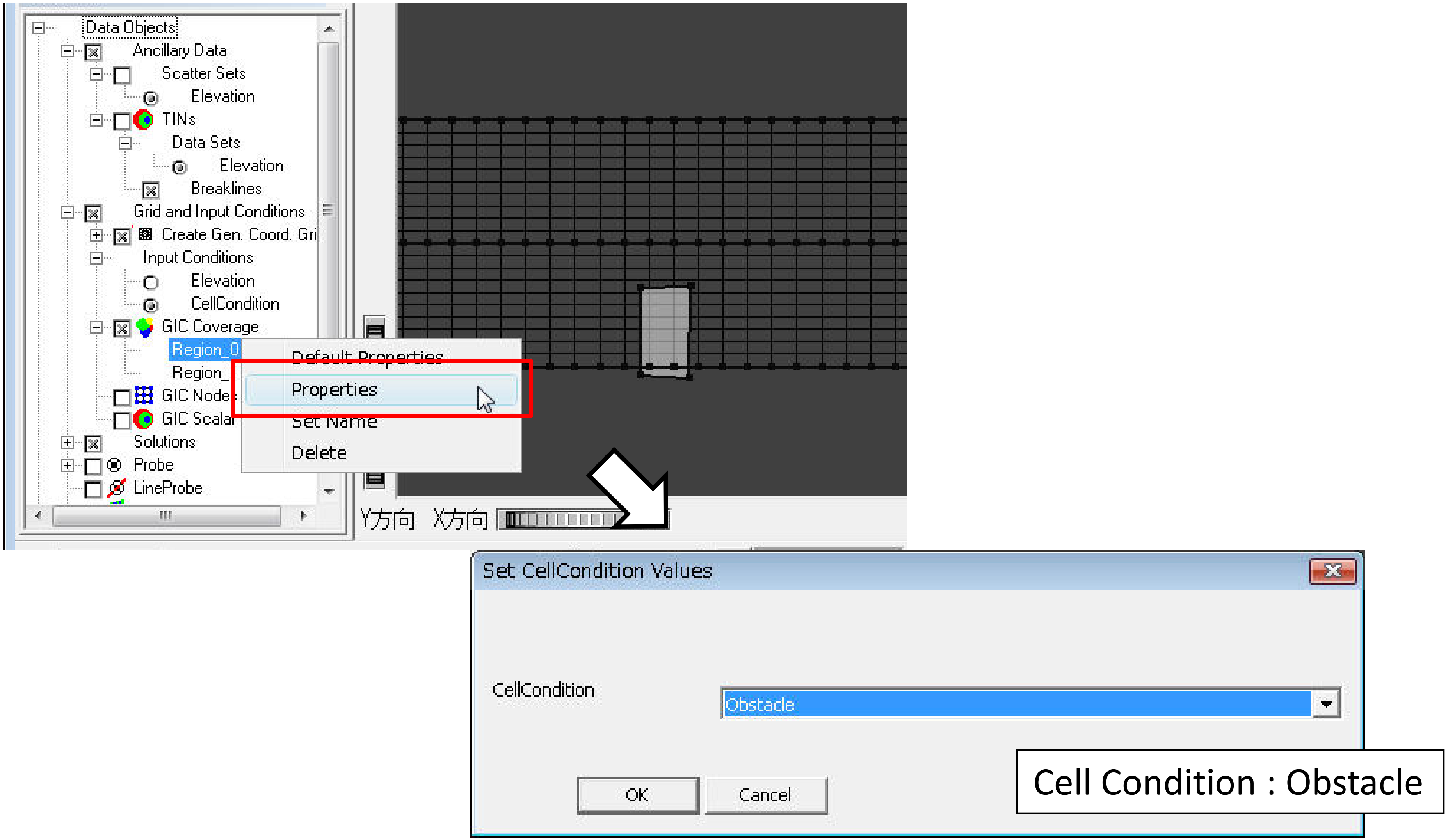
Set Cell Condition(5)

➢ The surrounding areas are registered as ‘region’ in ‘GIC Coverage’.



Set Cell Condition(6)

➢ You can specify the cell condition by selecting ‘Properties’.

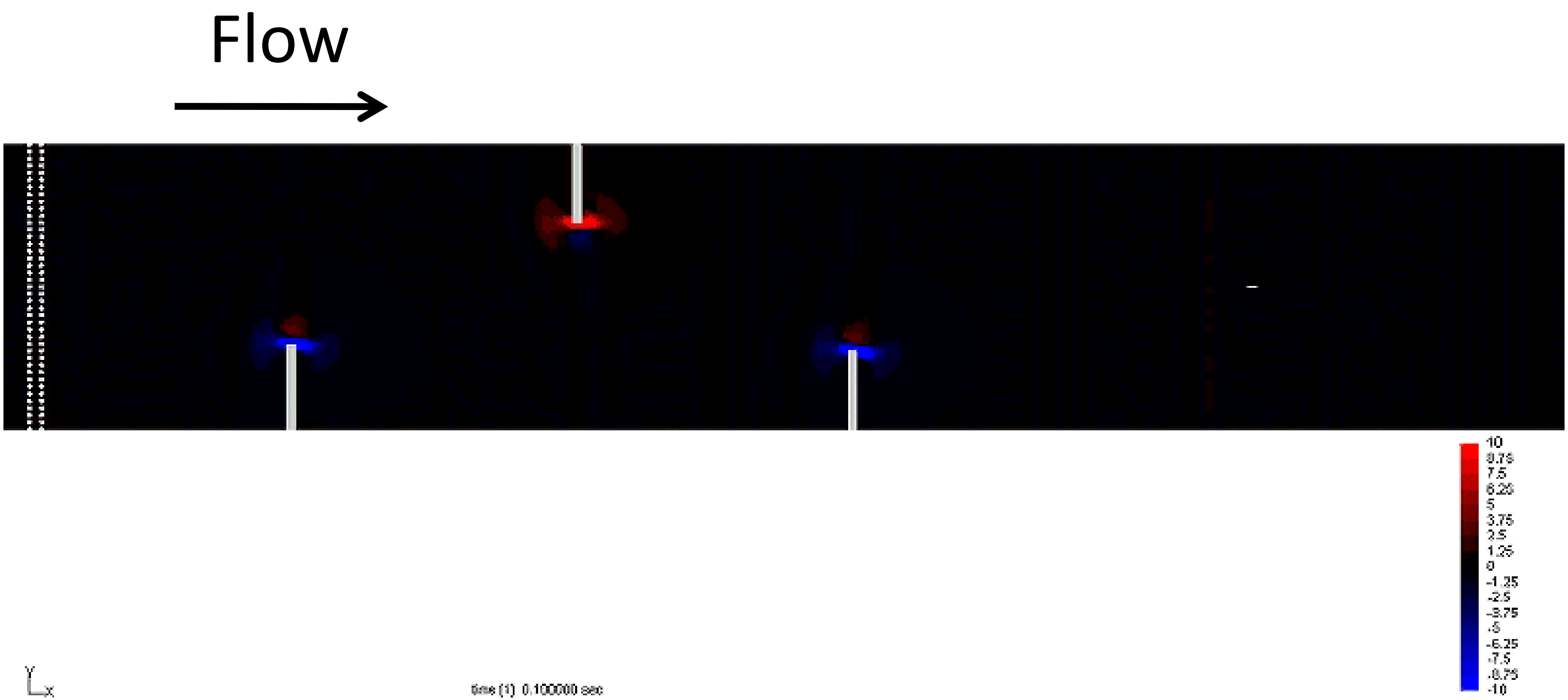


Set Computational Conditions

- File Names
  Output type : CGNS and RIC-Nays
- Time
  Output time interval : 0.1 sec
  Calculation time step : 0.002 sec
  Start time of output : 0 sec
  Start time of bed deformation : negative value
  Discharge time series : Import from "Discharge.txt"
- Boundary conditions
  Water surface at downstream : Uniform flow
  Slope : calculated from geometric data
- Initial water surface profiles
  Initial water surface : uniform flow
- Other condition
  Turbulent model : zero-equation model
  Finite differential method of the advection terms : CIP Method

Computational Result

- Animation of flow with obstacles by particle tracers and vortices.



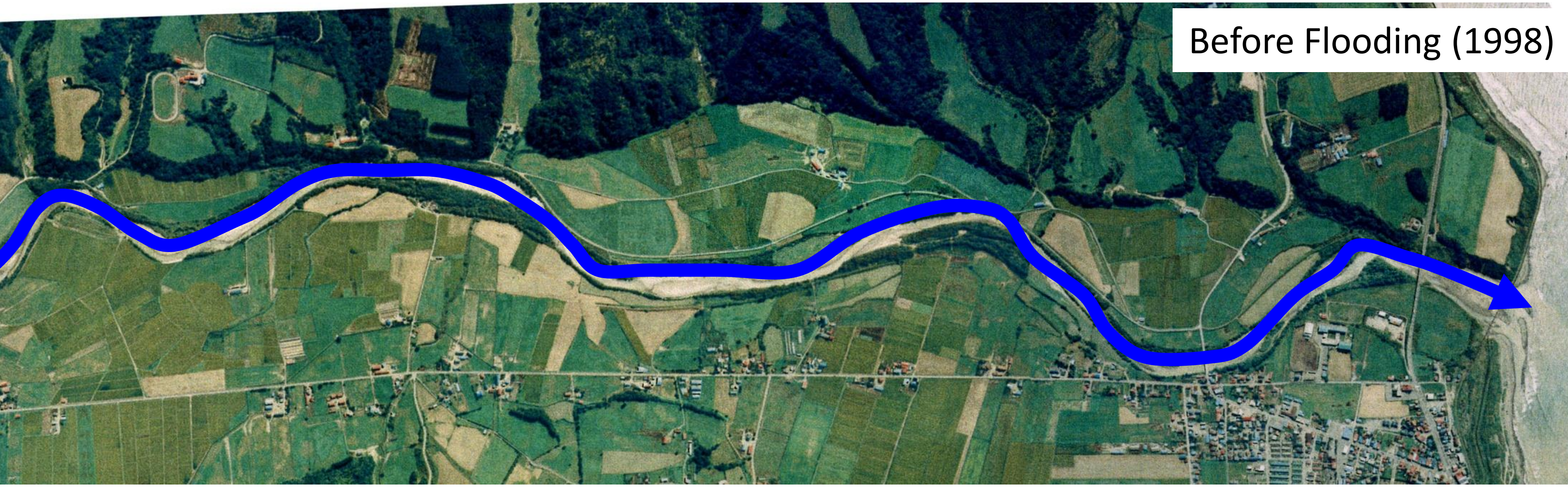
Flood Calculation in Real Scale River

Flooding of Appetsu River in Hokkaido, Japan, 2003

EXERCISE 3

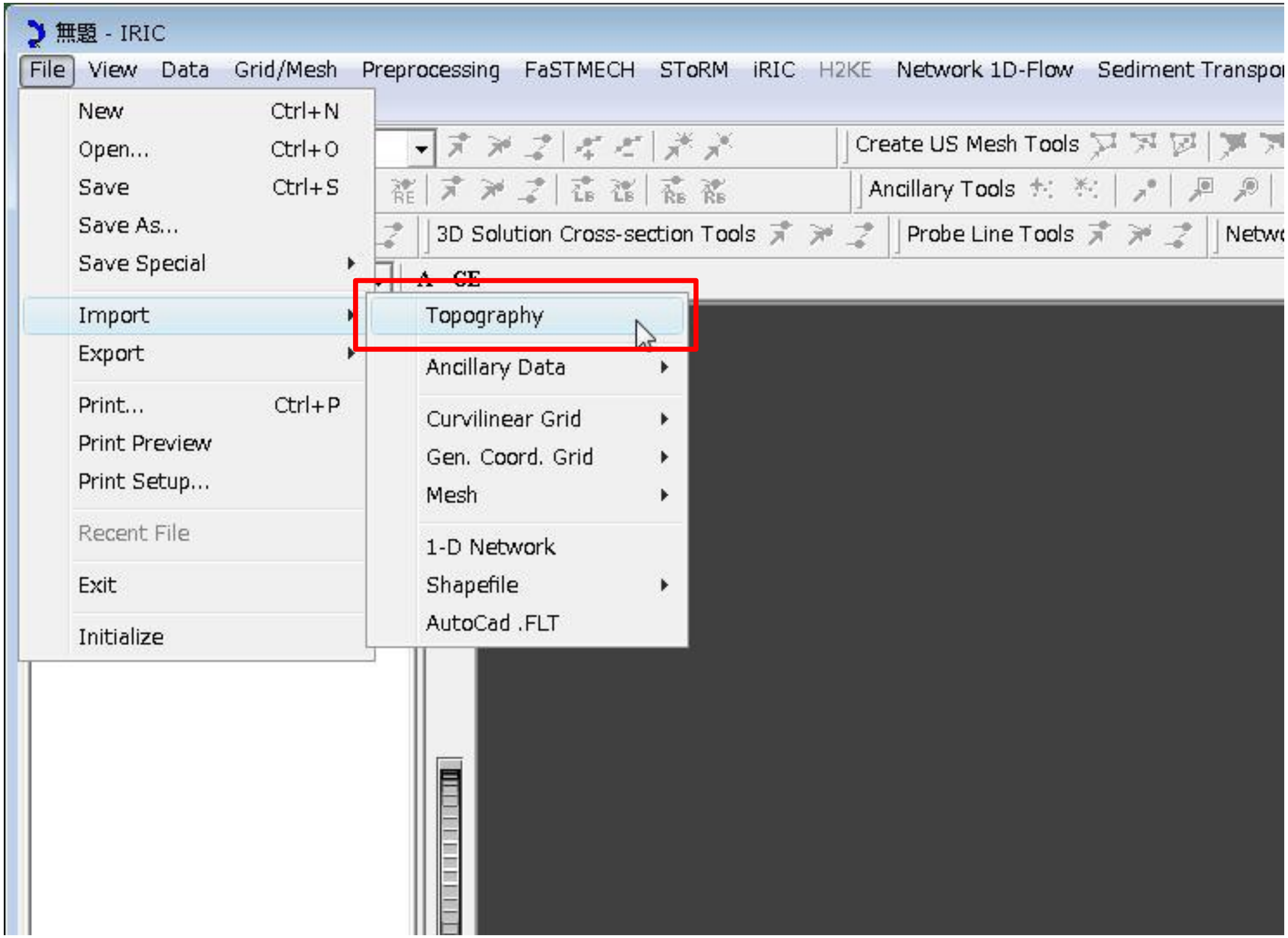
Exercise 3

- Flooding of Appetsu river in Hokkaido, Japan, 2003.



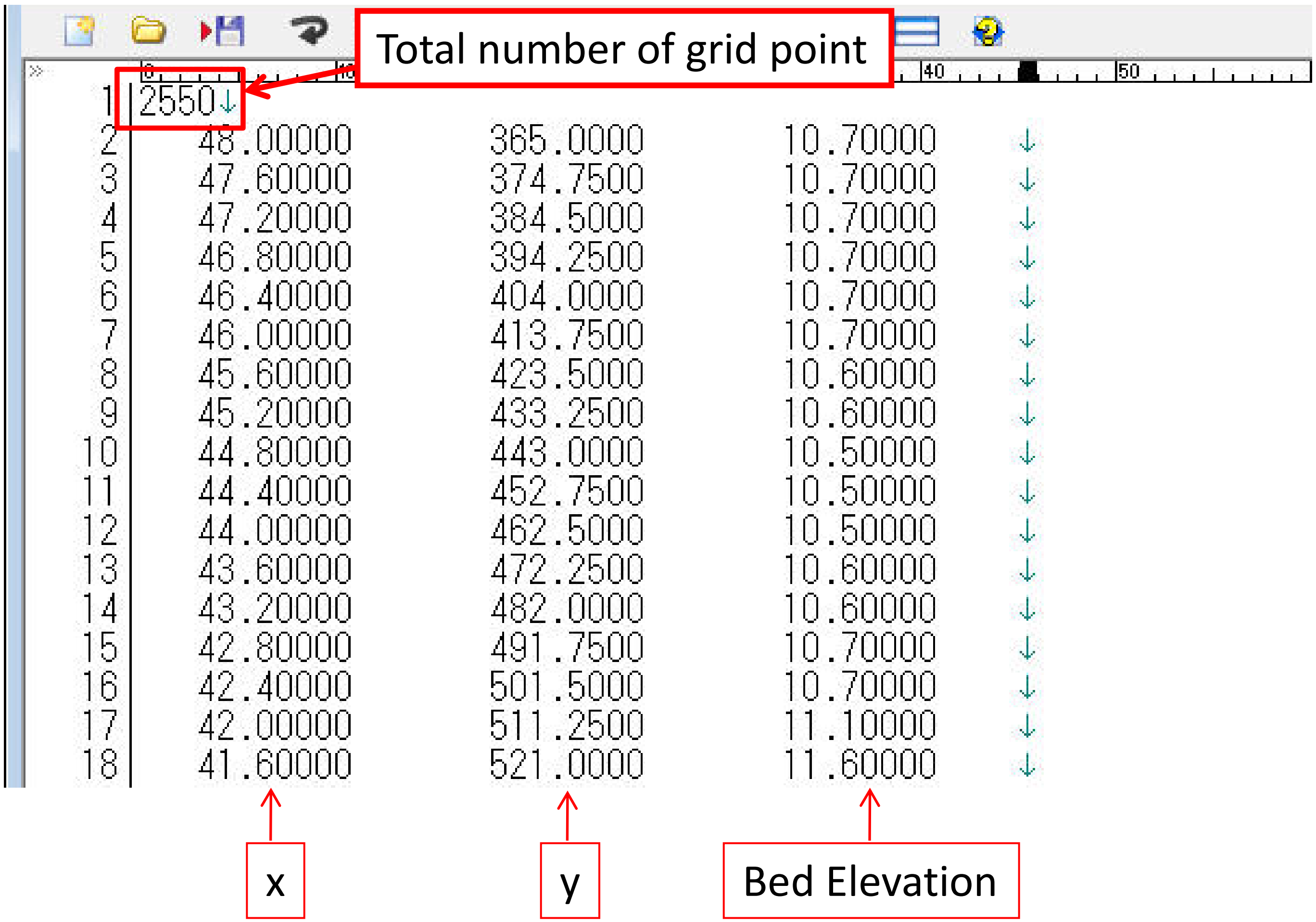
Grid Generation(1) Read the Bed Elevation Data

➢ iRIC can make the computational grid from data of x, y, z like DEM data.



Grid Generation（2） .tpo File Format

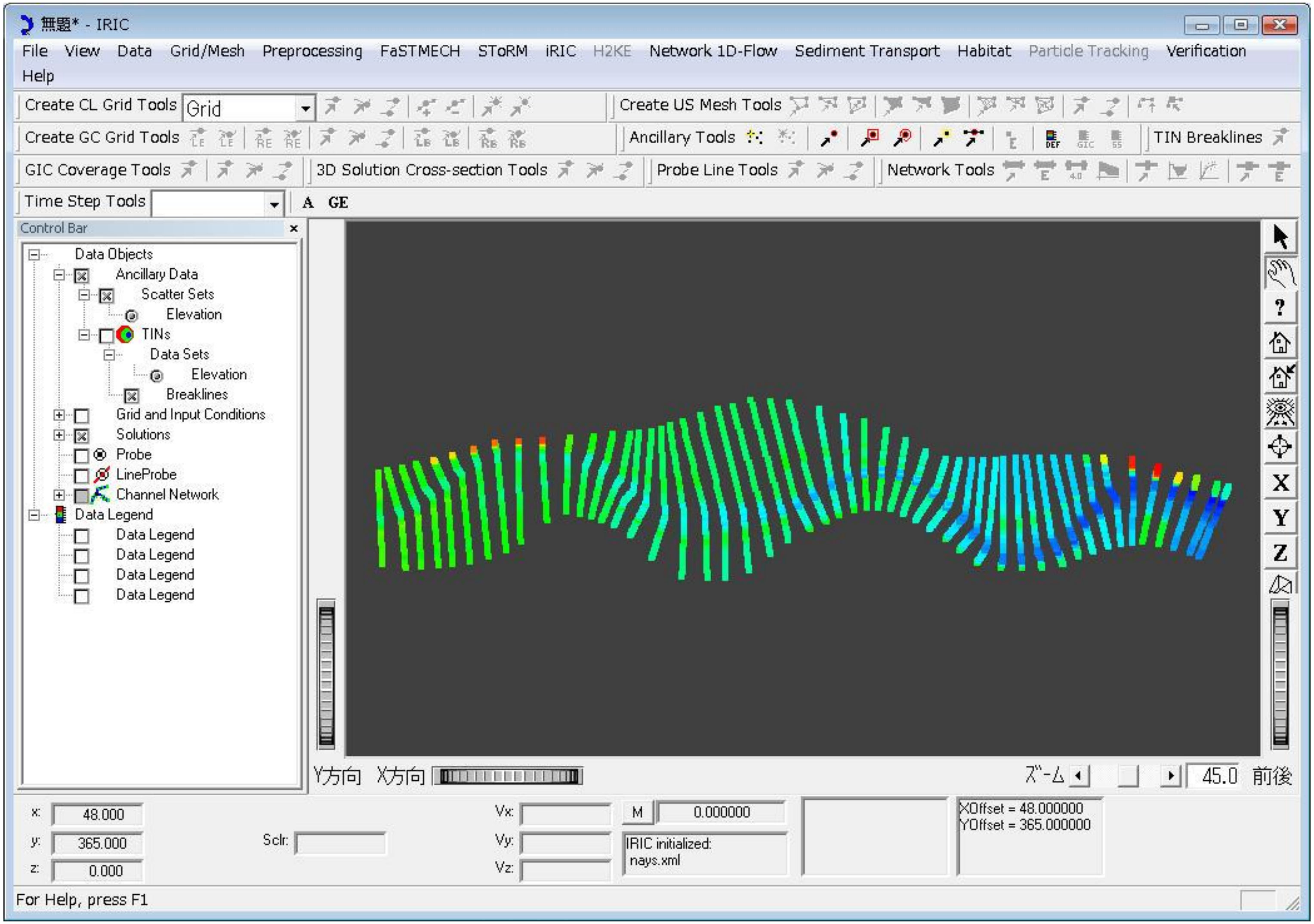
- The file of .tpo is text file.
- Total number of grid point, coordinate data (x, y) and bed elevation data are written in this file.



1	2550		
2	48.00000	365.0000	10.70000
3	47.60000	374.7500	10.70000
4	47.20000	384.5000	10.70000
5	46.80000	394.2500	10.70000
6	46.40000	404.0000	10.70000
7	46.00000	413.7500	10.70000
8	45.60000	423.5000	10.60000
9	45.20000	433.2500	10.60000
10	44.80000	443.0000	10.50000
11	44.40000	452.7500	10.50000
12	44.00000	462.5000	10.50000
13	43.60000	472.2500	10.60000
14	43.20000	482.0000	10.60000
15	42.80000	491.7500	10.70000
16	42.40000	501.5000	10.70000
17	42.00000	511.2500	11.10000
18	41.60000	521.0000	11.60000

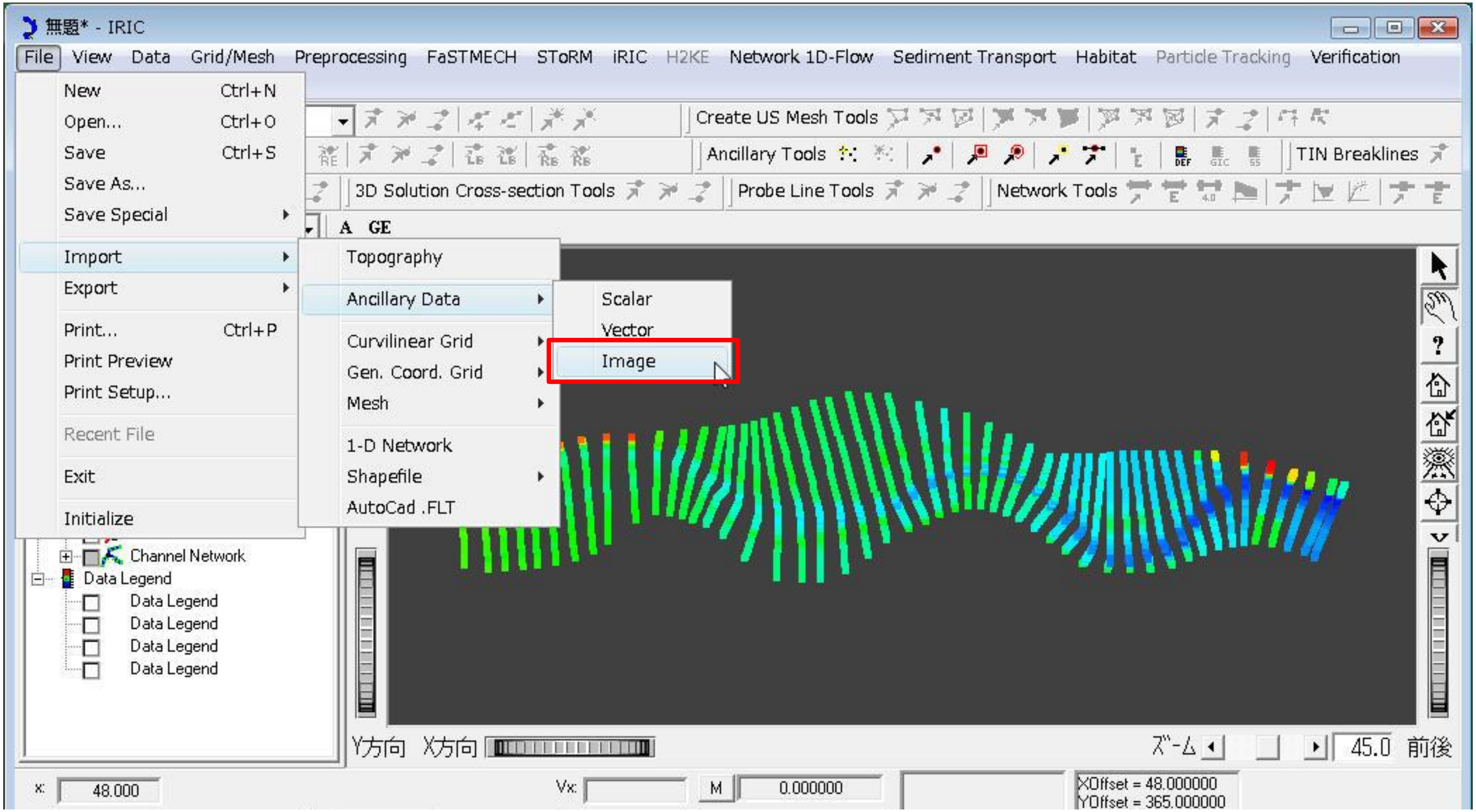
Grid Generation(3)

➢ The point data of tpo file are displayed in main window.



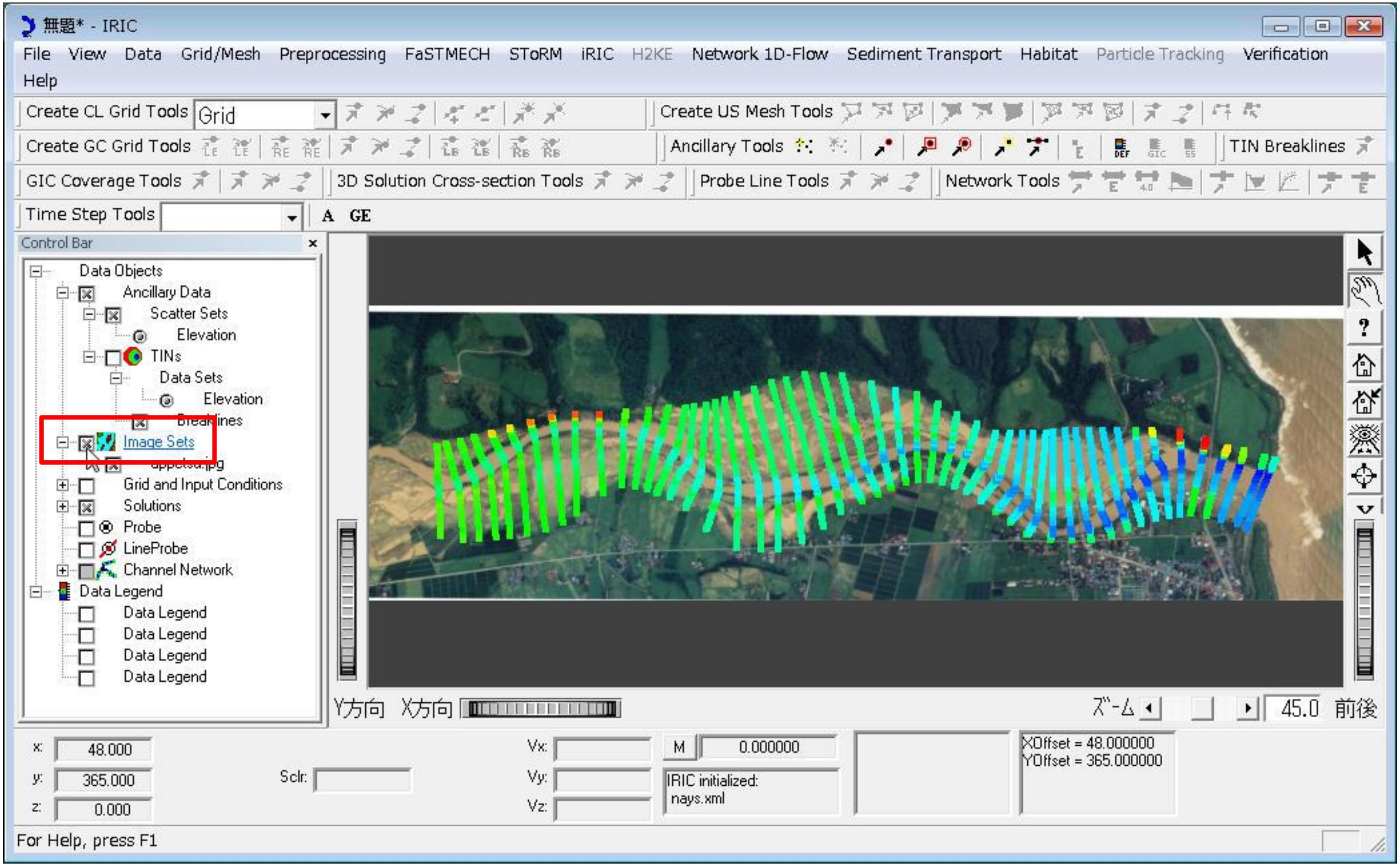
Grid Generation (4) Read Background Image

- Background image of river helps you to make the computational grid.
- Read the photo file from "File"→"Import"→"Ancillary Data"→"Image"



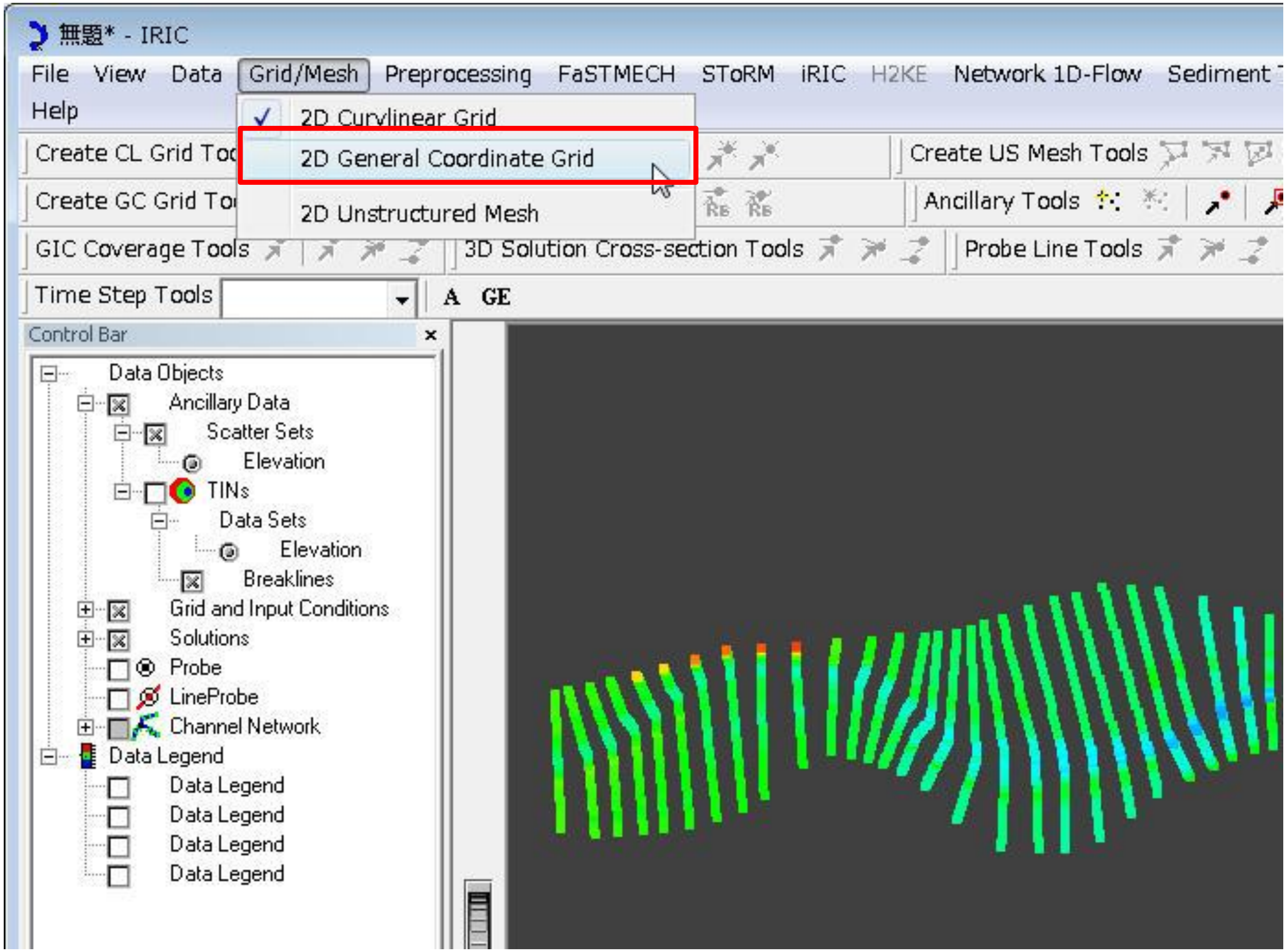
Grid Generation(4) Read Background Image

➢ Display the image by "Image Set" in "Control Bar"



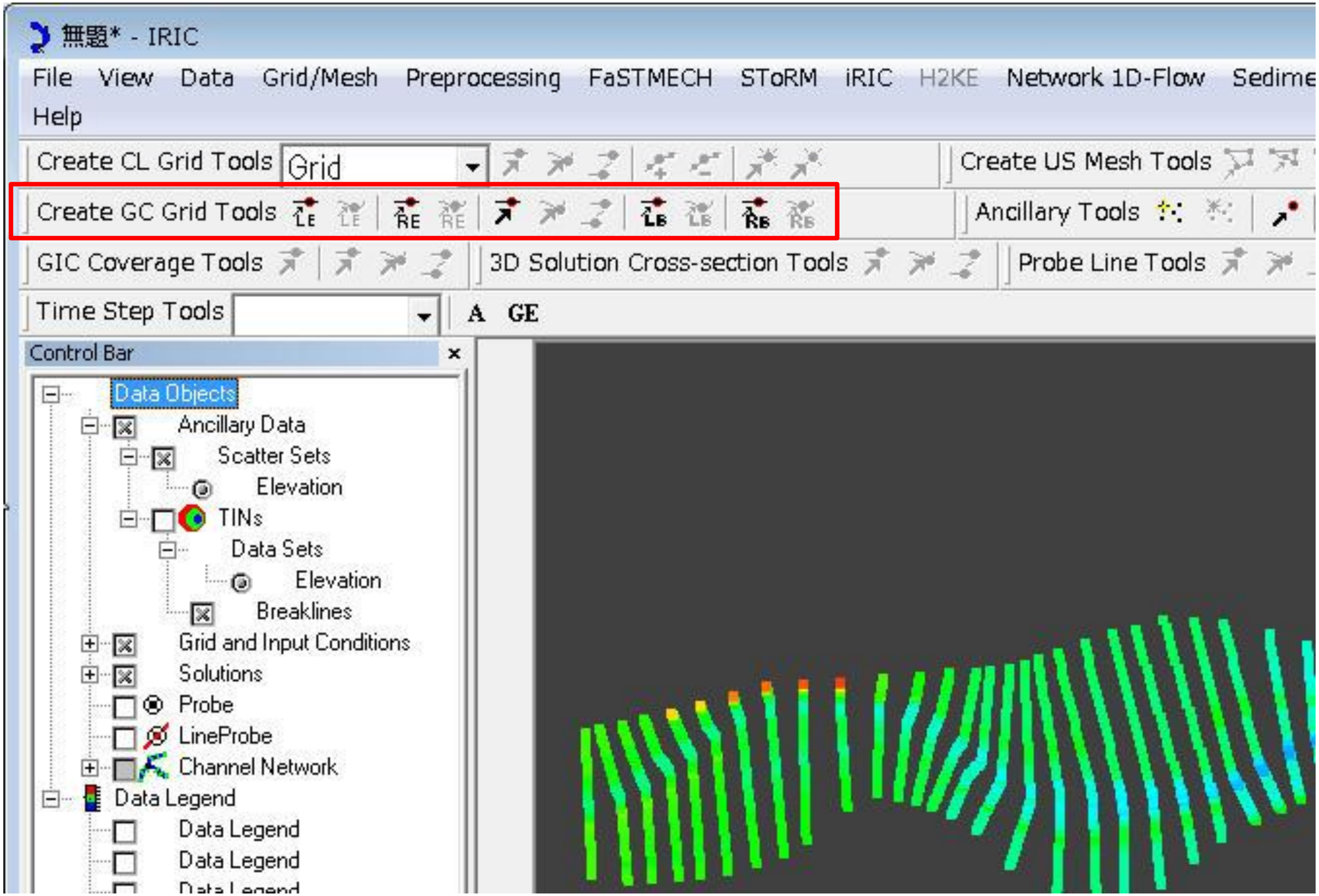
Grid Generation(5)

- General coordinate grid system is used in nays model.
- Select “2D General Coordinate Grid” in “File”→”Grid/Mesh”.



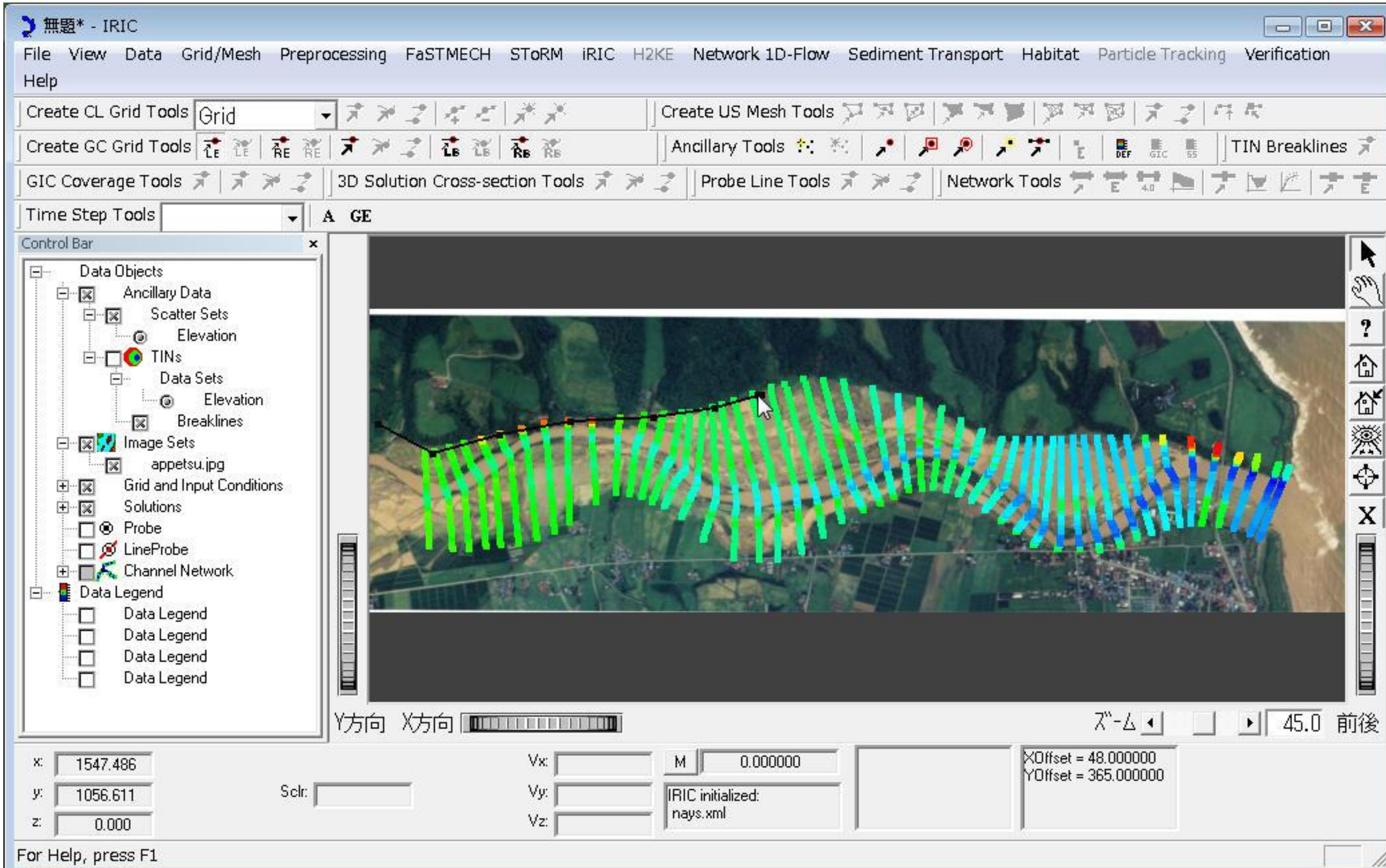
Grid Generation(6)

- Specify the left, right bank and center line of river by “Create GC Grid Tool”



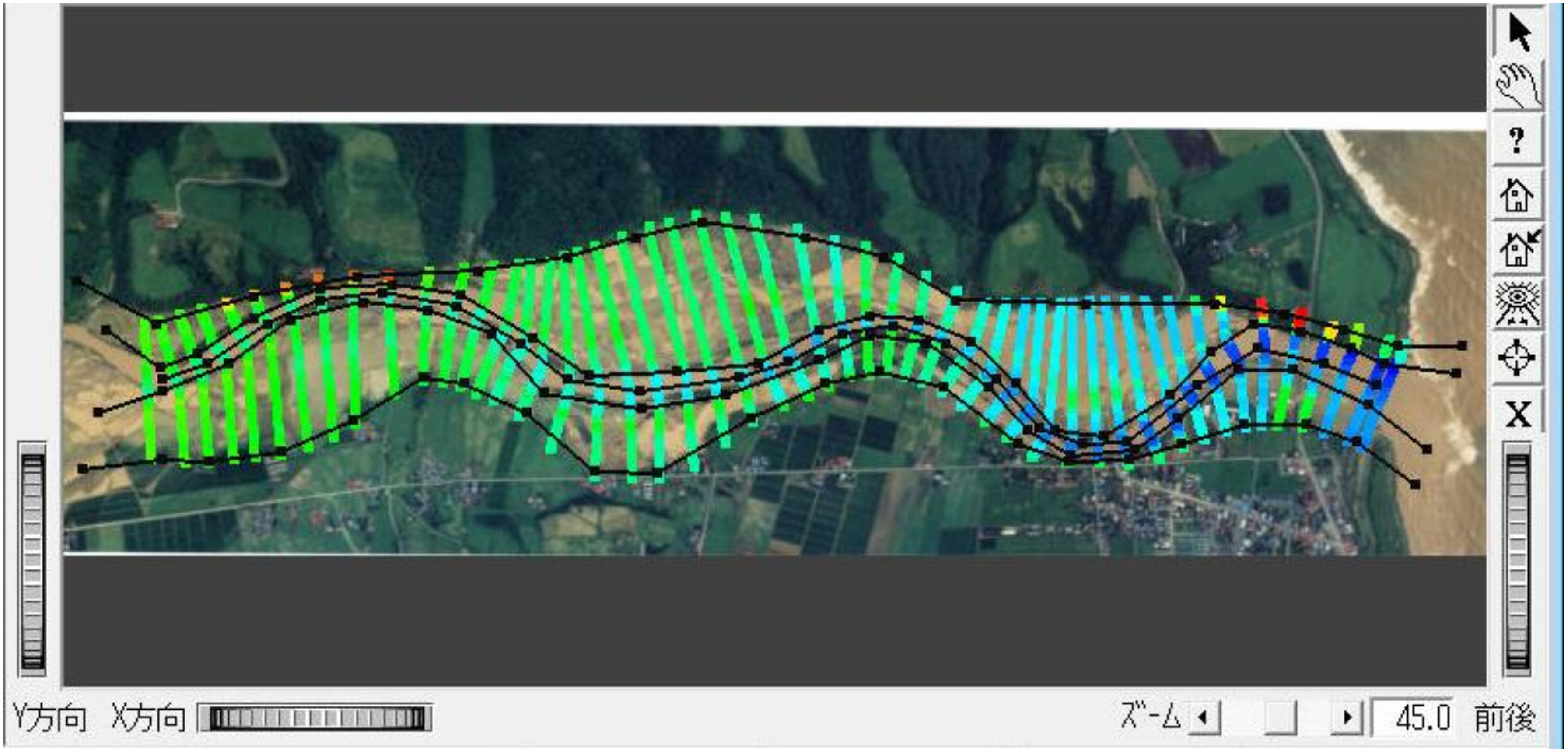
Grid Generation(7)

- LE: left edge, RE: Right edge, LB: Left bank, RB: Right bank
  "Arrow": Center line of river



Grid Generation(8) Attention

- First and end point of the lines except center line should be located out of the bed elevation data.
- All points except above points have to be located in the existing elevation data.
- The line can not cross each other.



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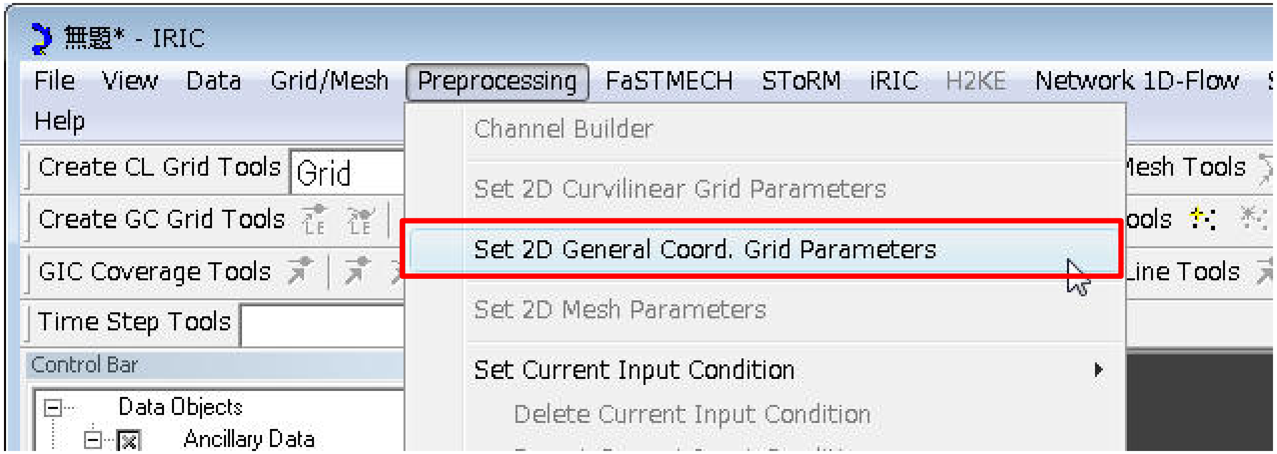
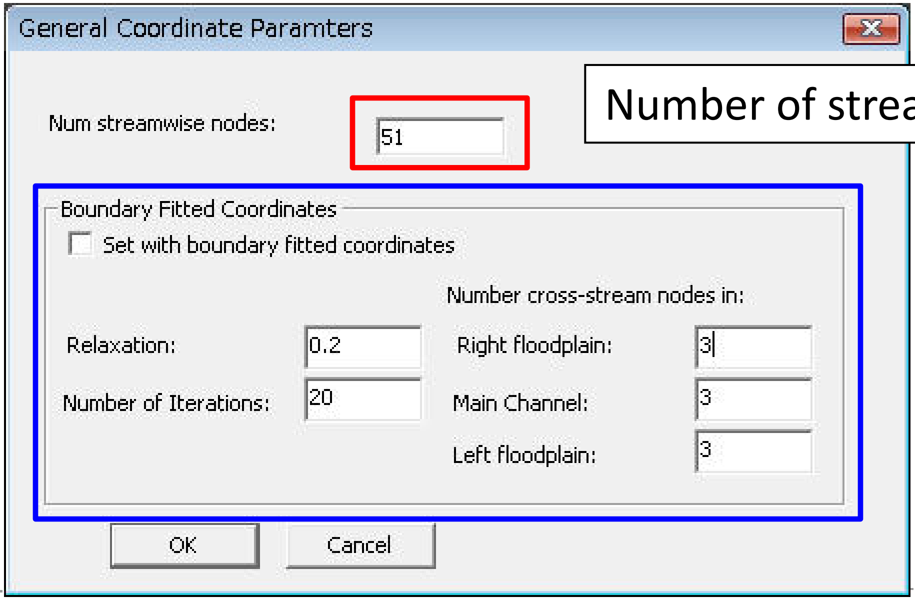
Grid Generation(9)　Generate grid in flow direction

- “Preprocessing”→”Set 2D General Coord. Grid Parametes”





Number of streamwise noeds

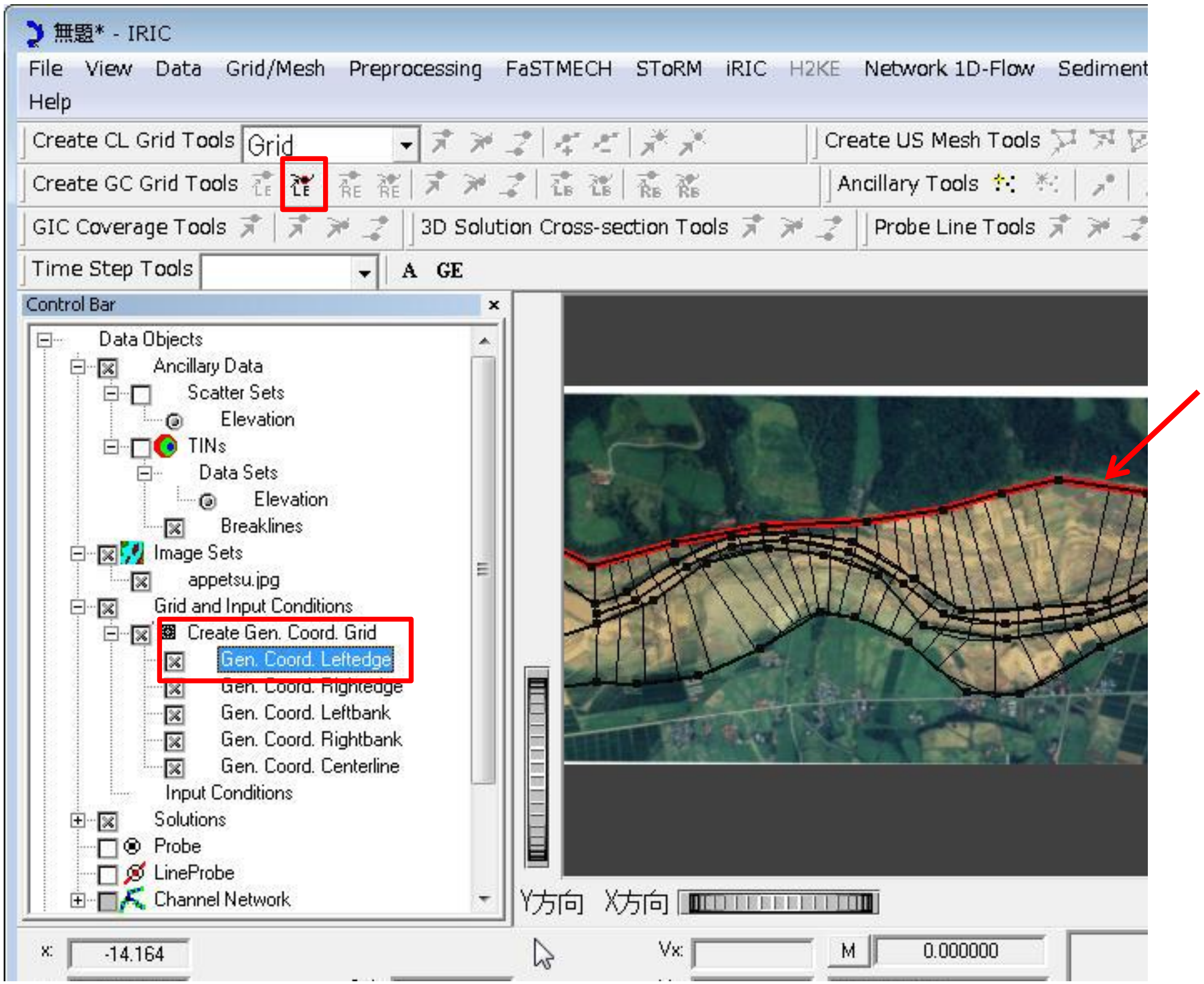
Condition of grid in transverse direction;
It is better to generate the grid in
streamwise direction in advance.

Grid Generation(9)　Generate grid in flow direction

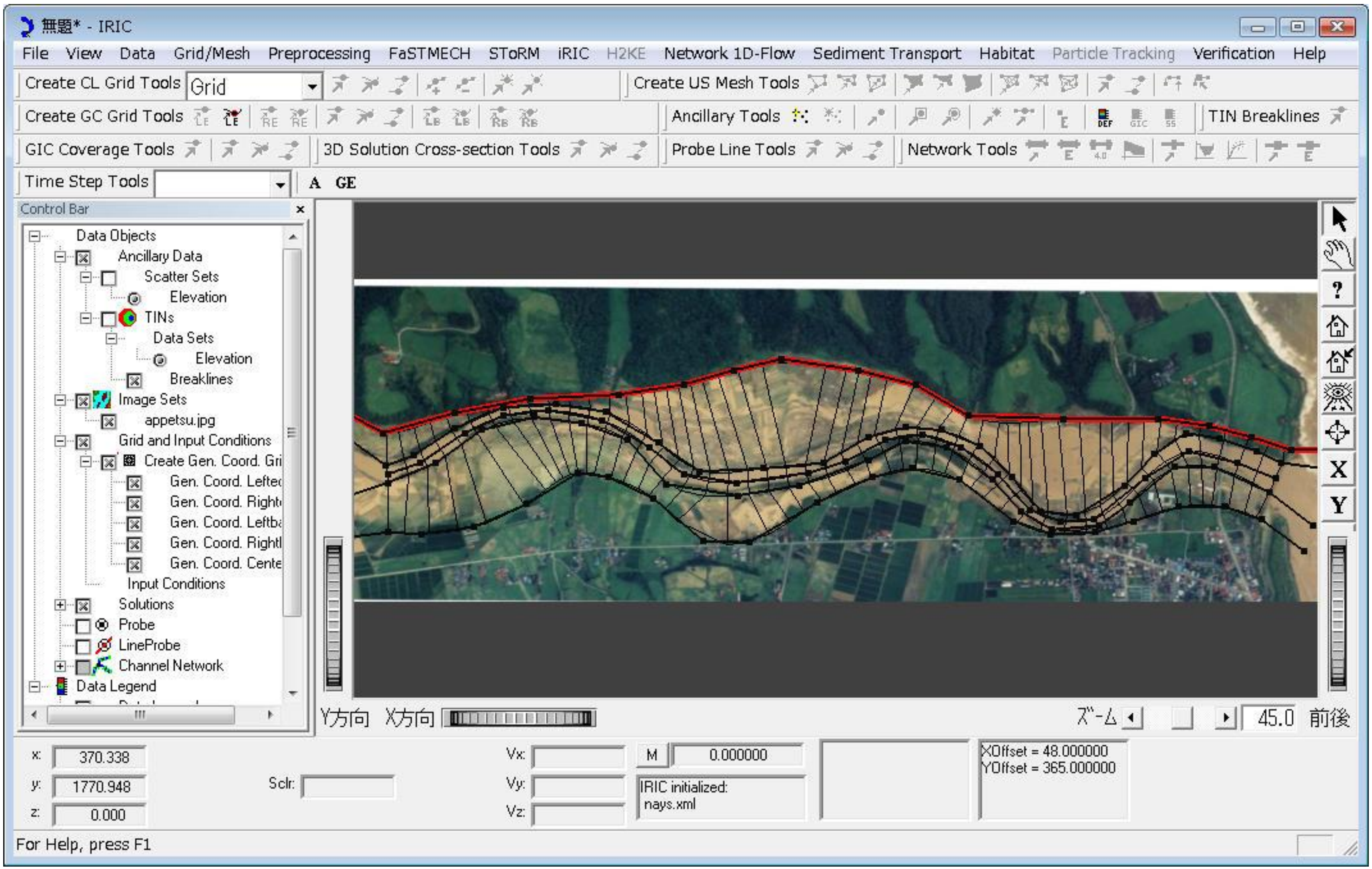
- Check the computational grid in streamwise direction.
- You can correct the computational grid.

Grid Generation(10) Correction of Grid

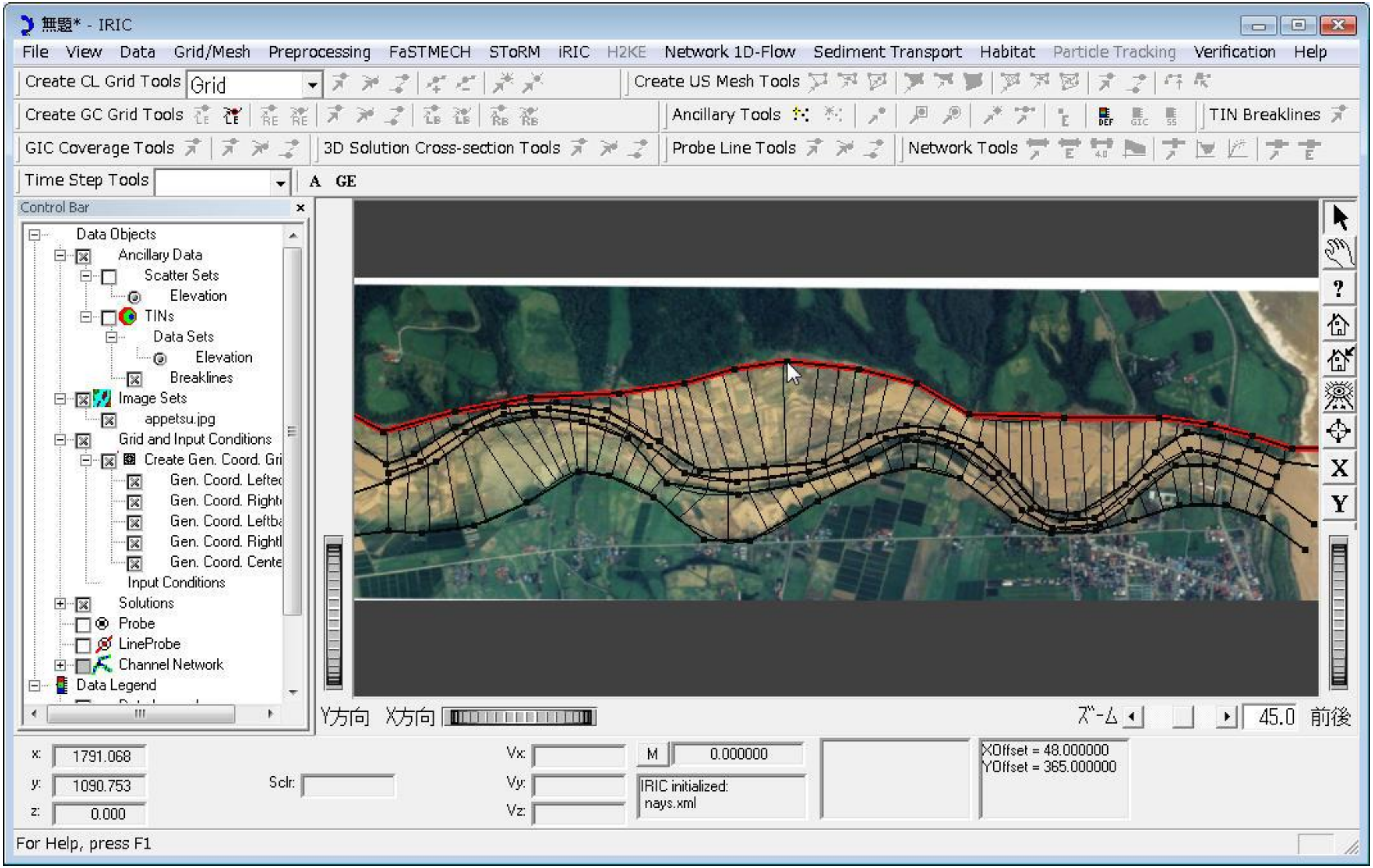
- Select the line in which you want to change in "Create Gen. Coord Grid"



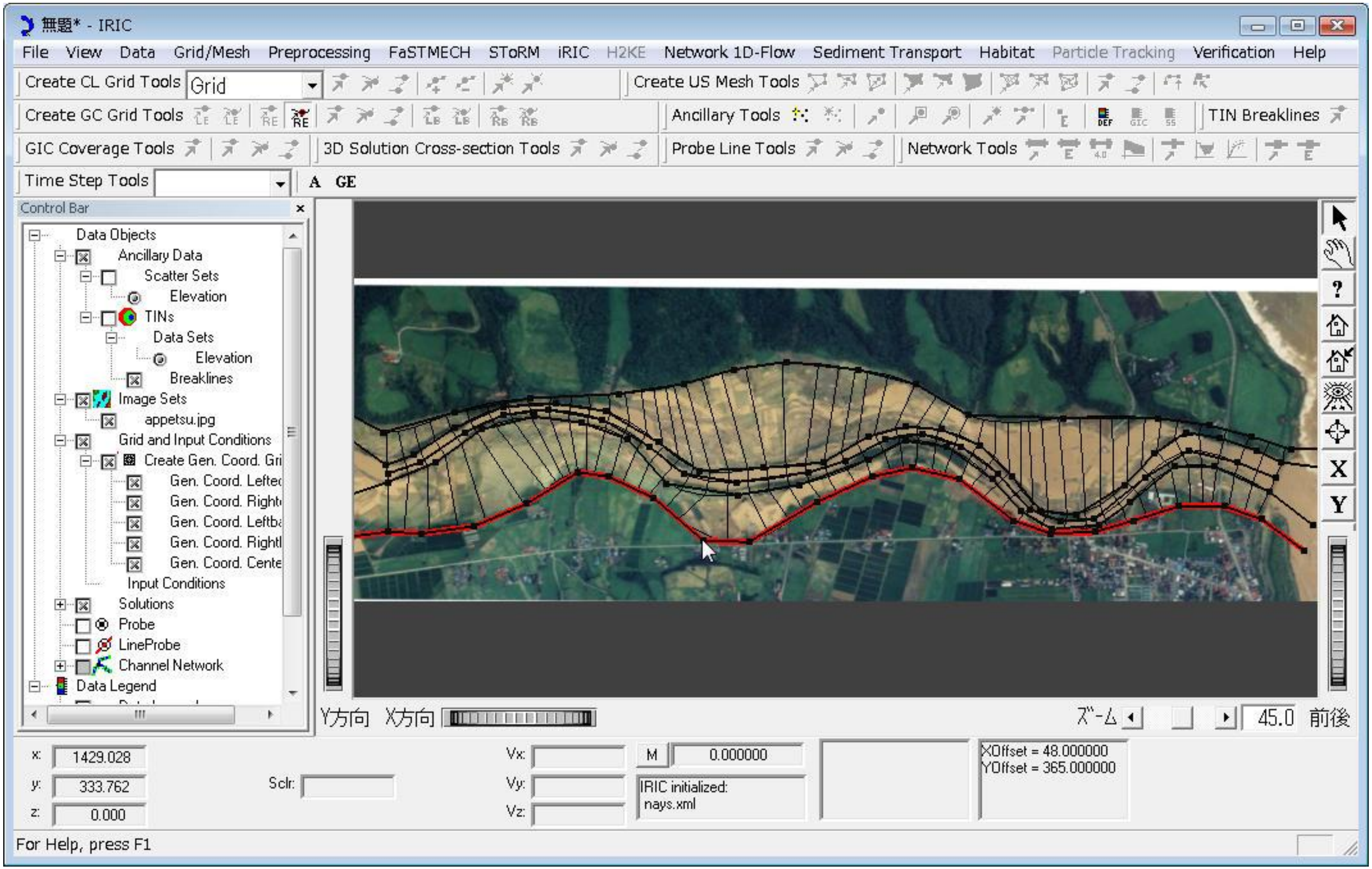
Grid Generation(11) Correction of Grid



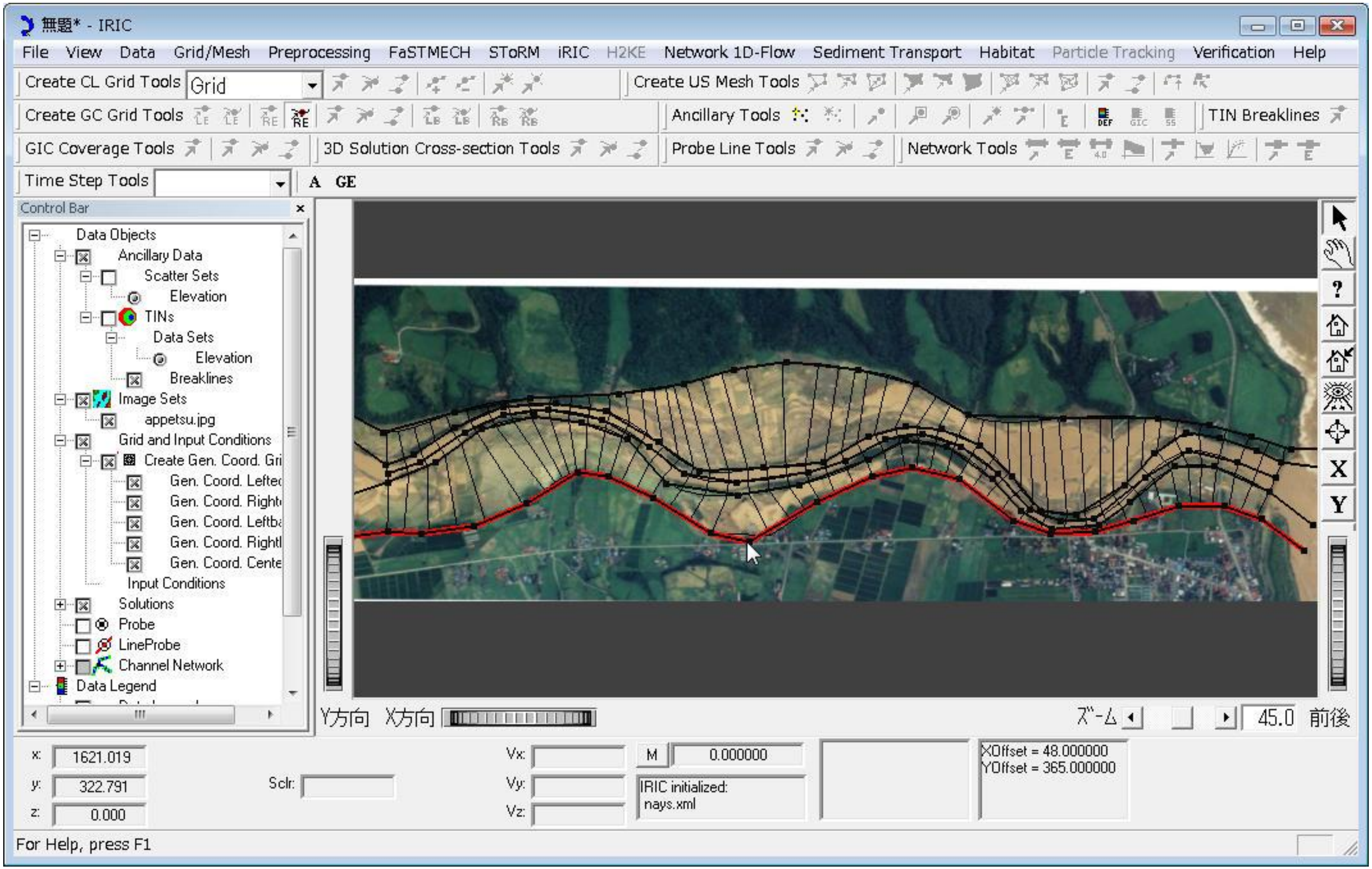
Grid Generation（11） Correction of Grid



Grid Generation（11） Correction of Grid



Grid Generation（11） Correction of Grid



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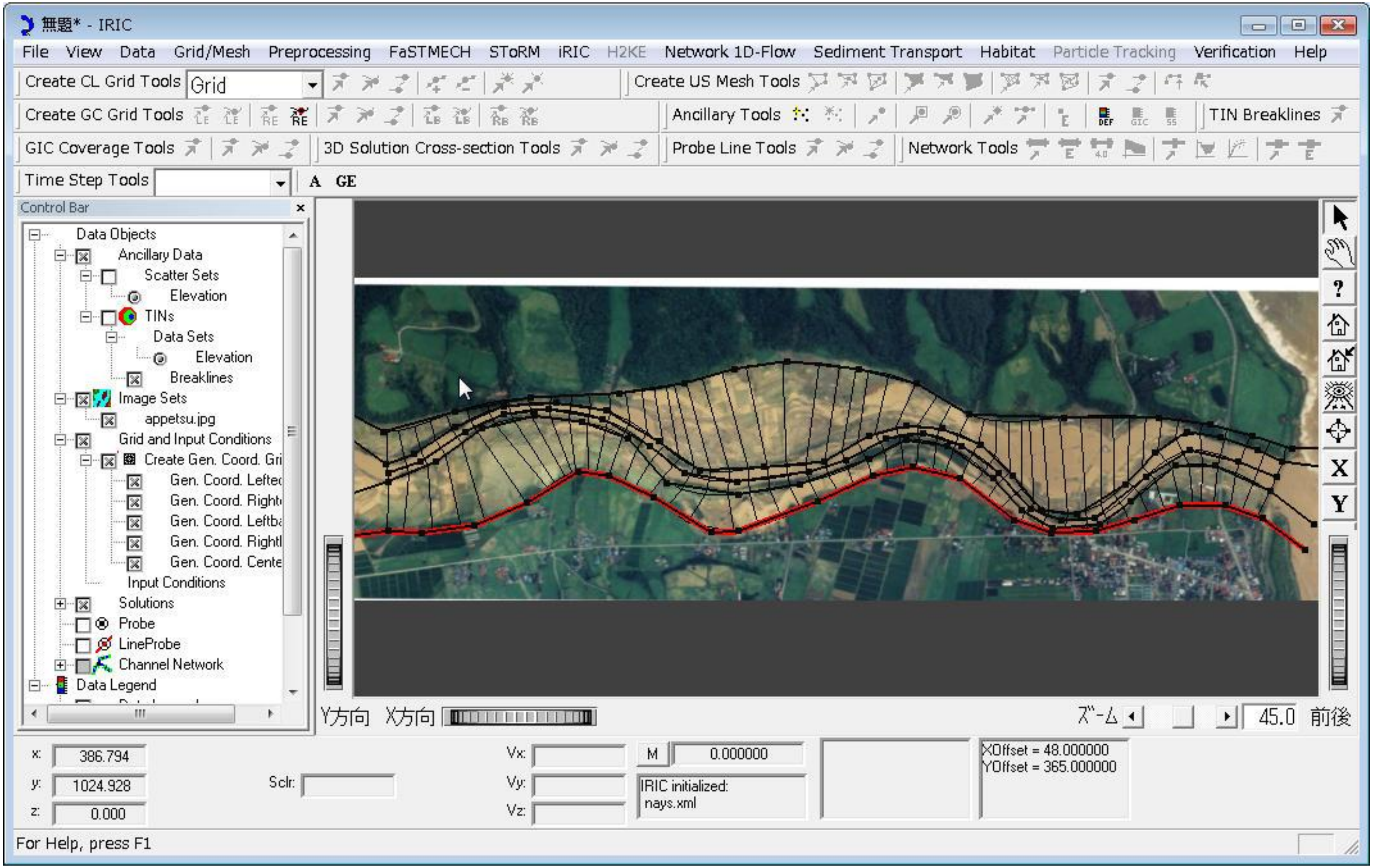
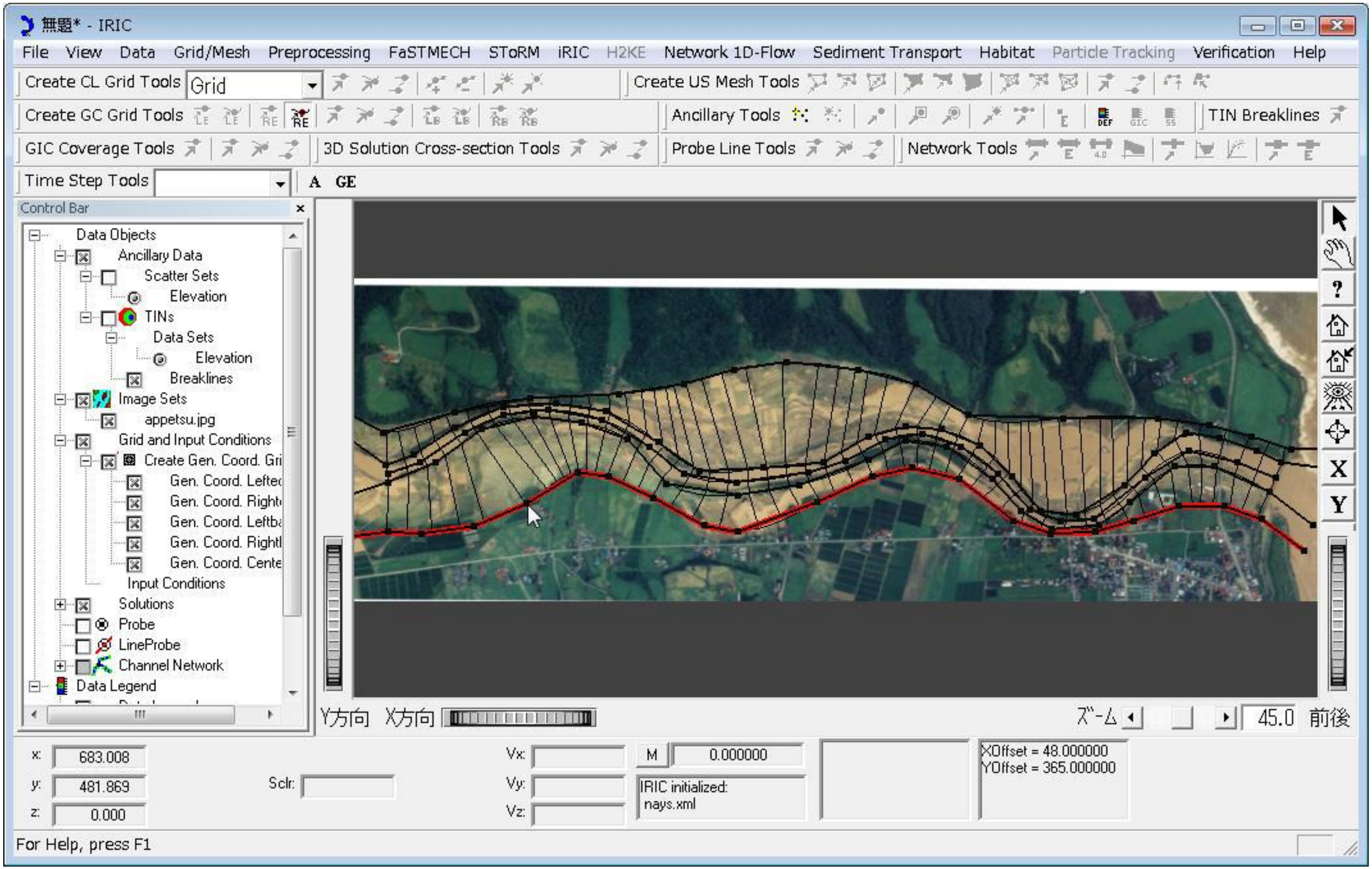
Grid Generation（11） Correction of Grid



Grid Generation(11) Correction of Grid



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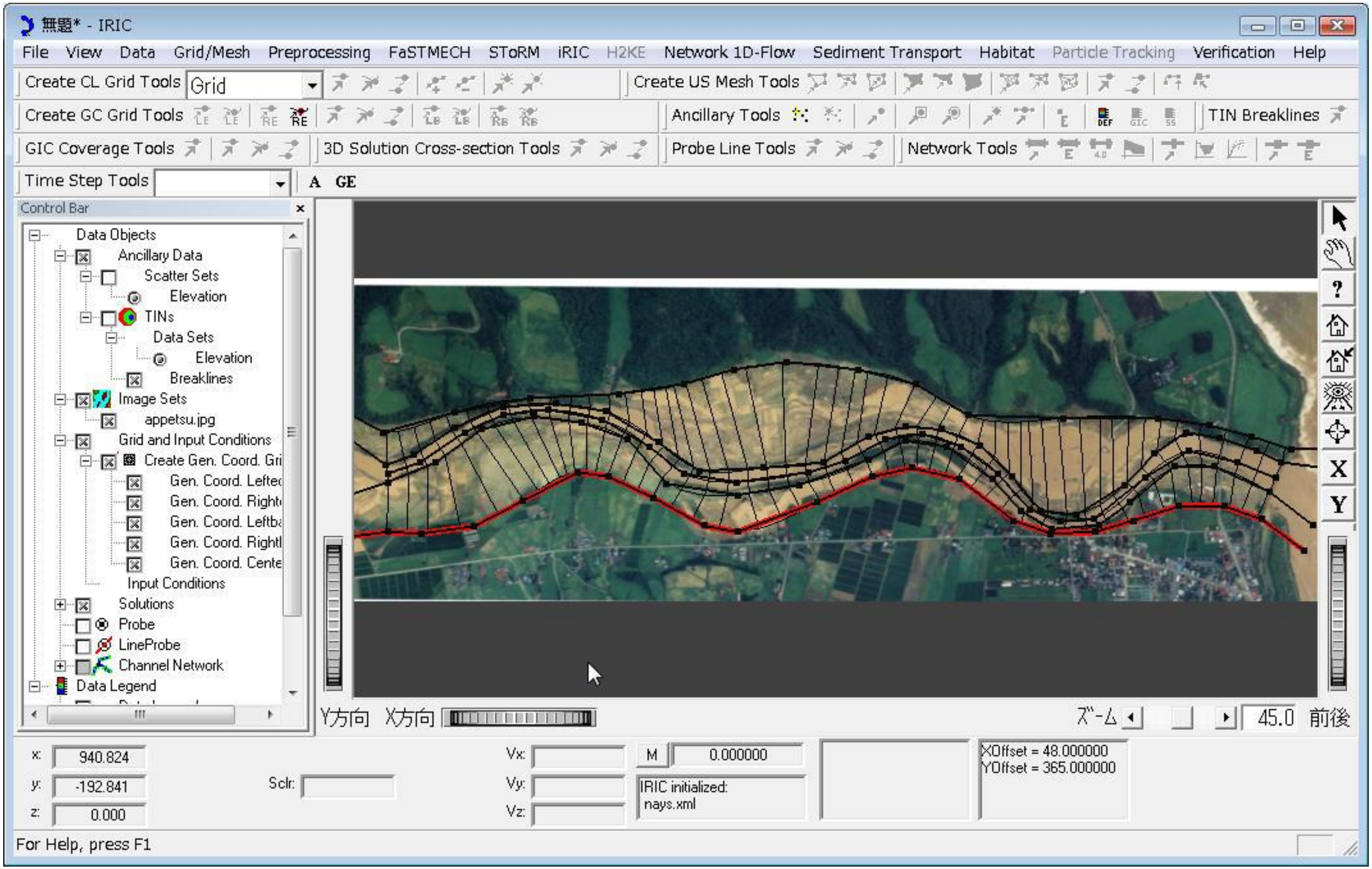
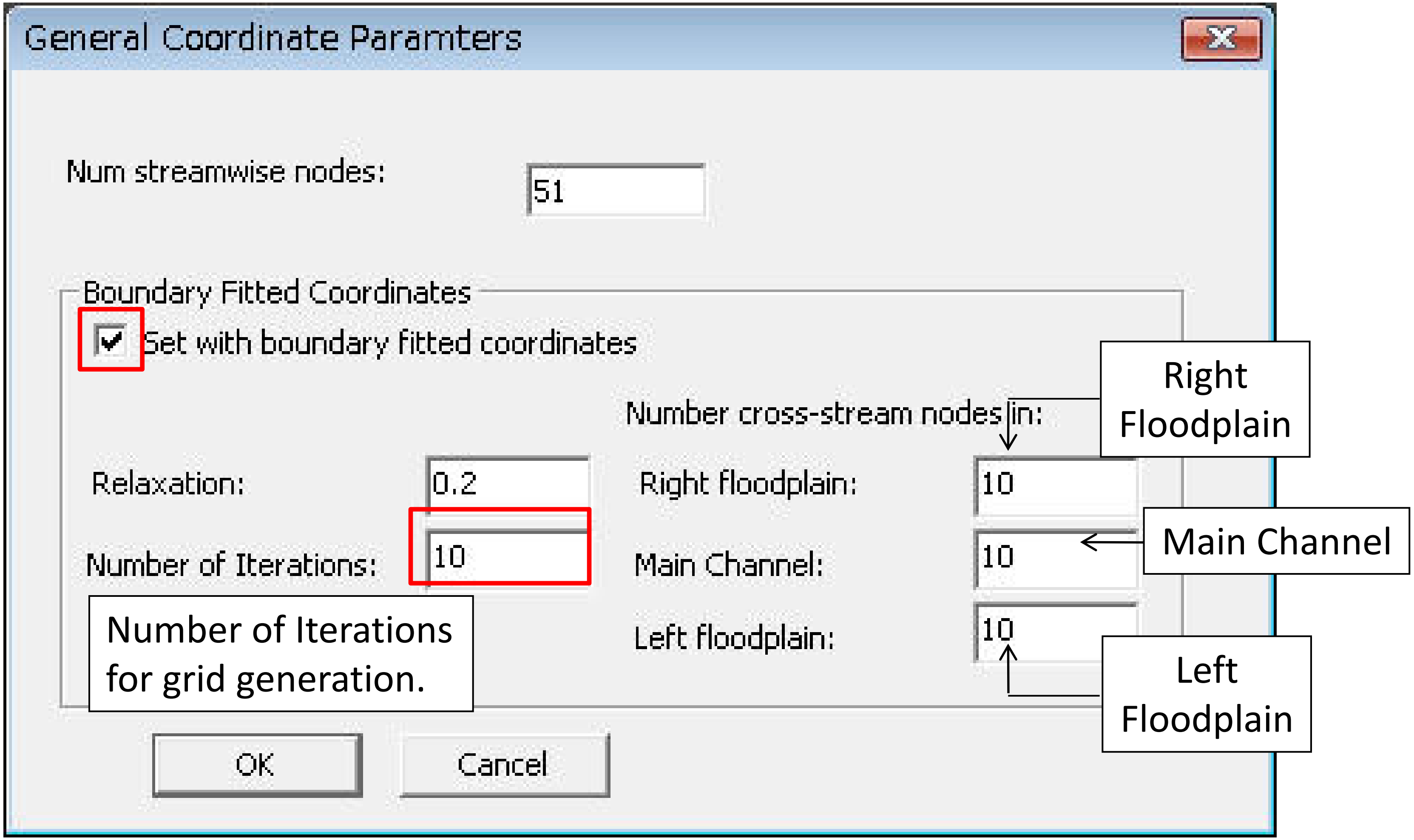
Grid Generation（11） Correction of Grid



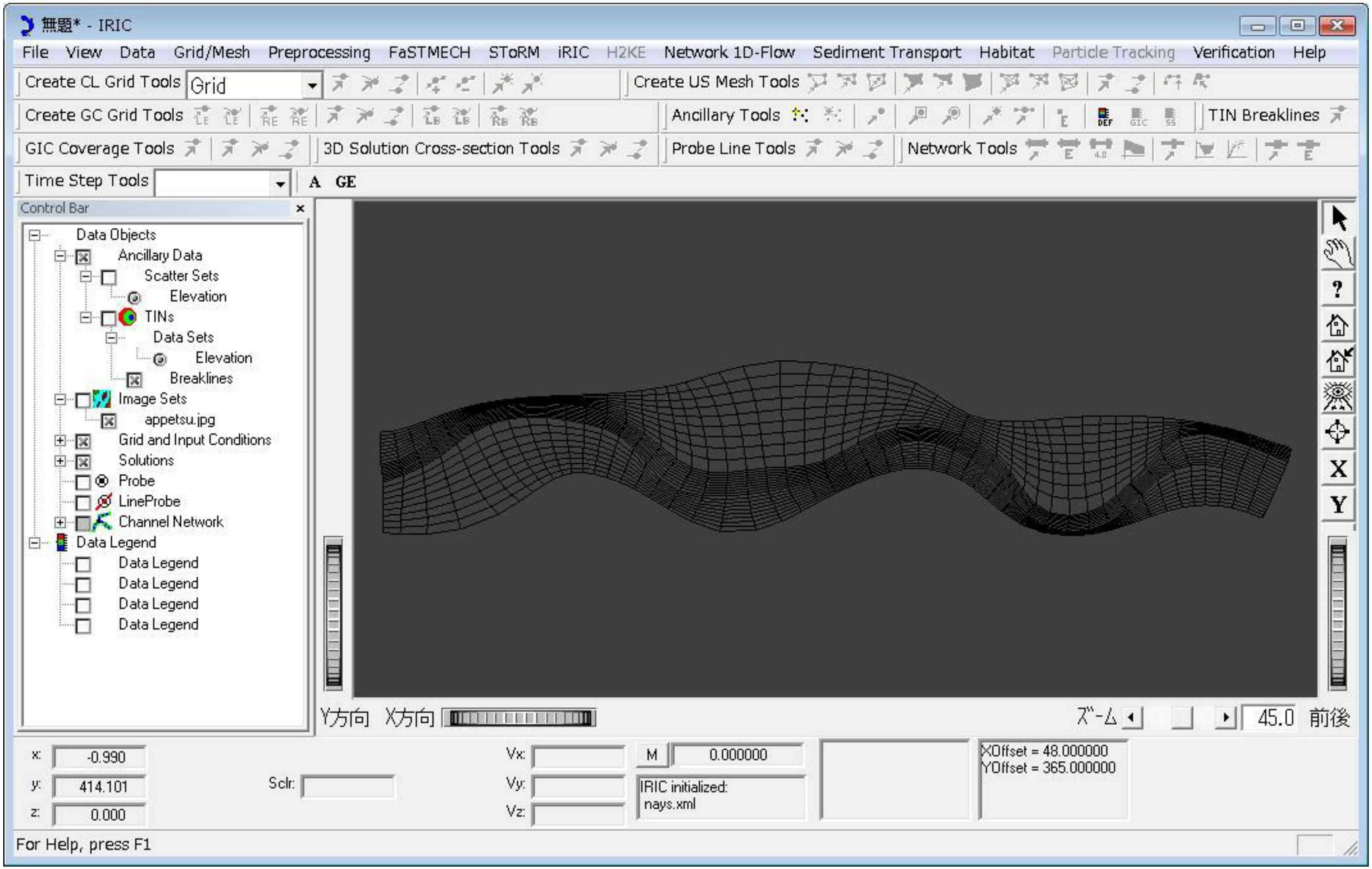
Grid Generation (12) Generate the grid in j direction

- "Preprocessing"→"Set 2D General Coord. Grid Parametes"



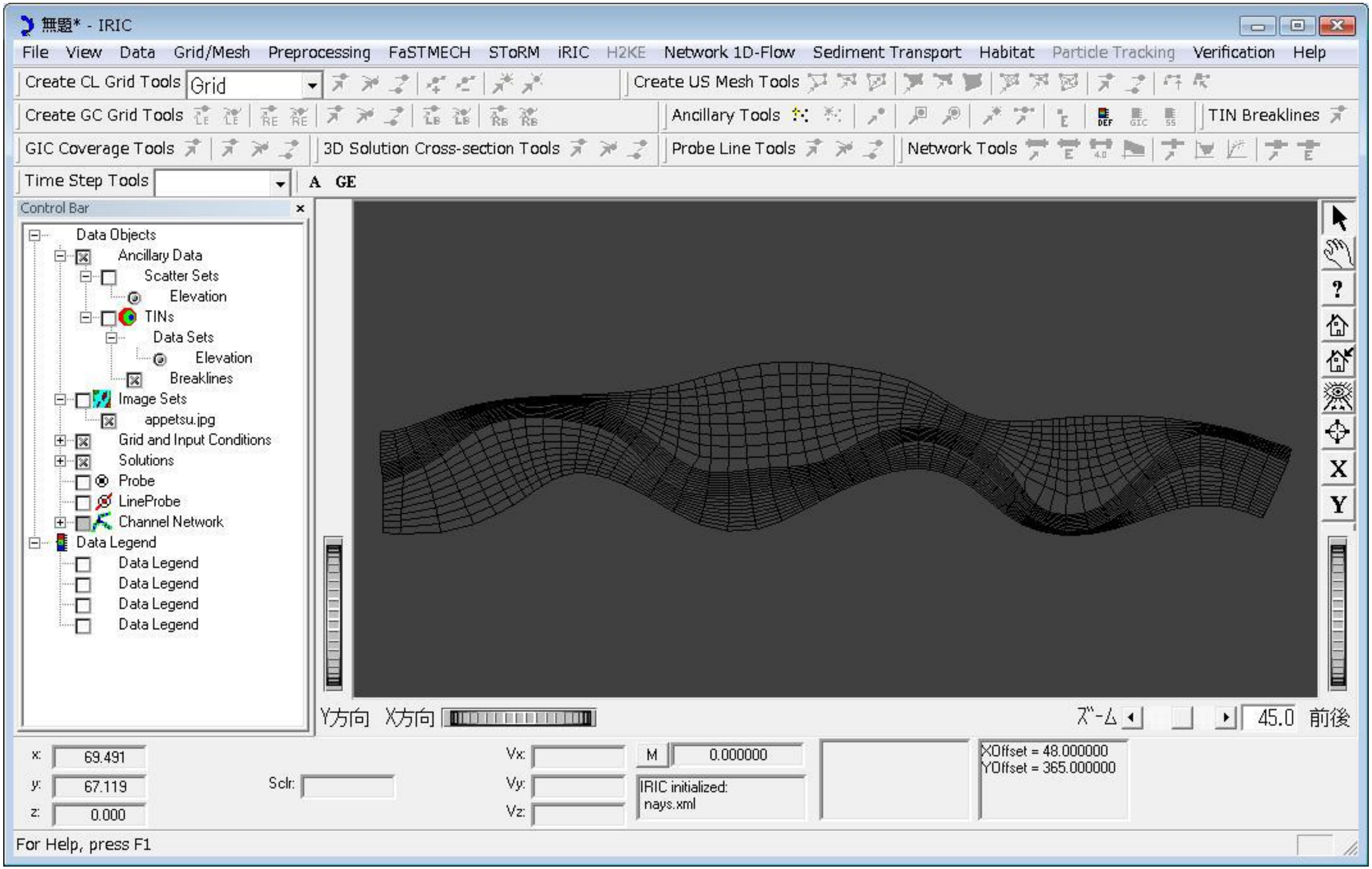
Grid Generation（12） Generate the grid in j direction

- Iteration number=1



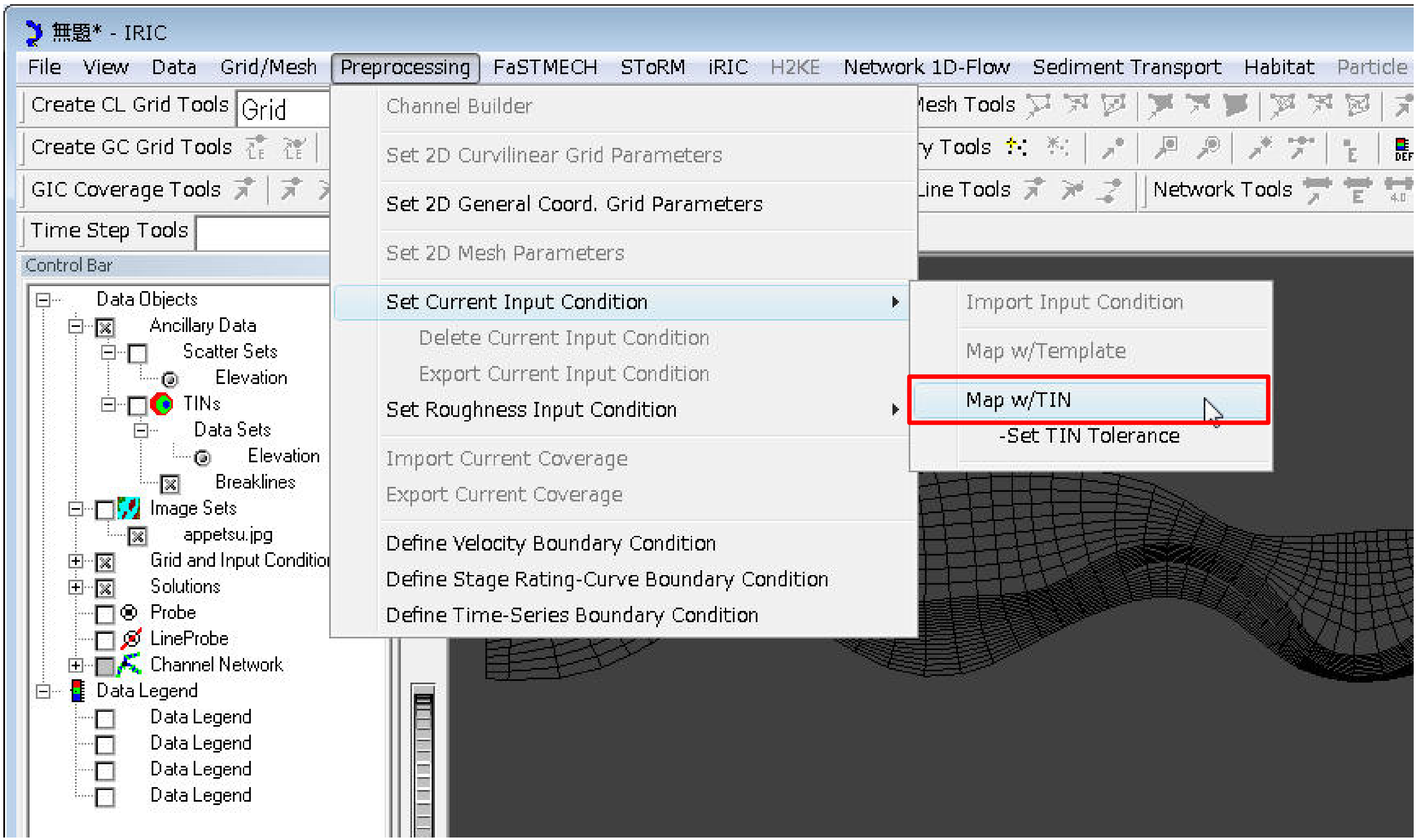
Grid Generation（12） Generate the grid in j direction

- Iteration number=20



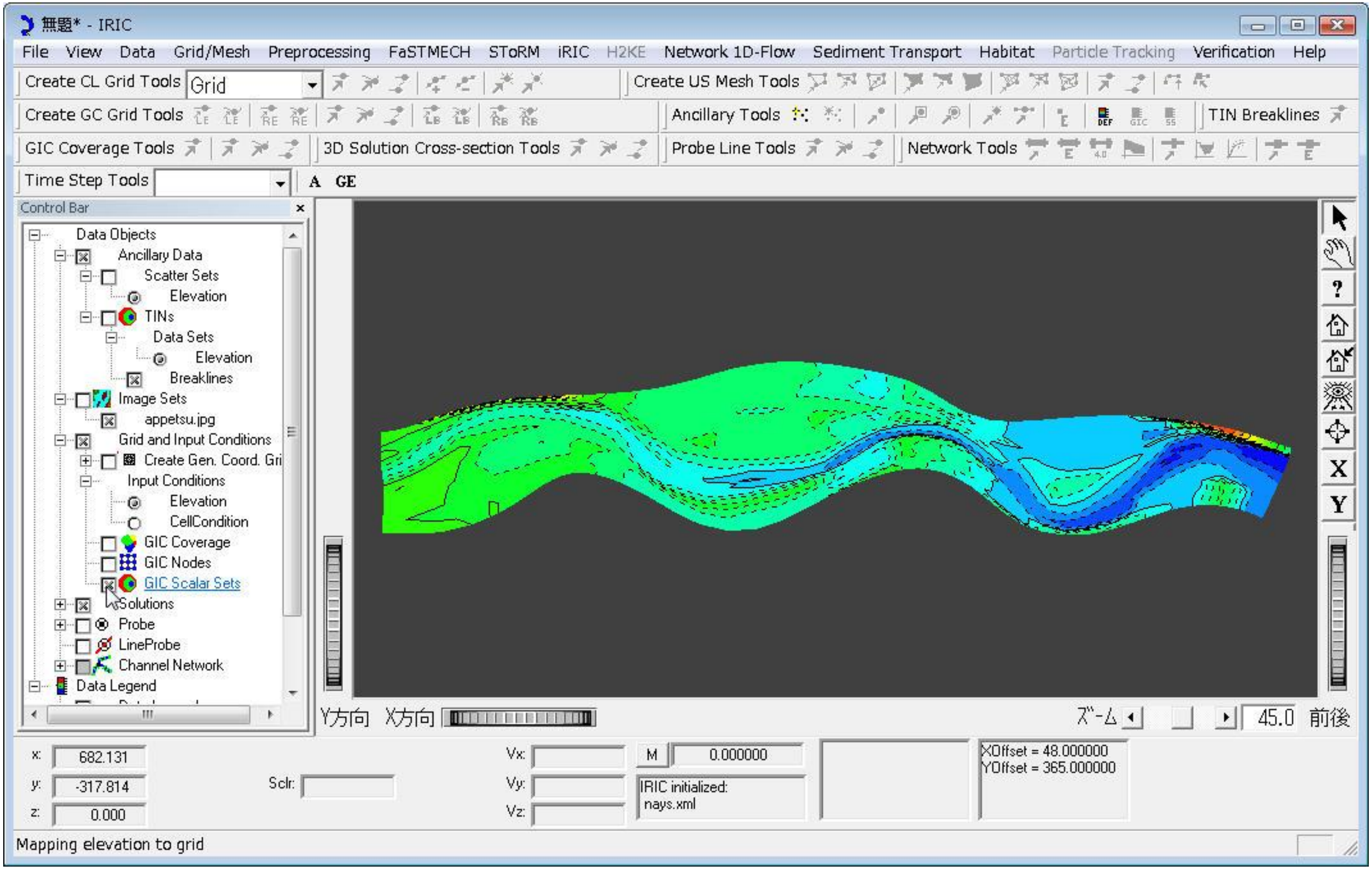
Grid Generation(13) Interpolation of Elevation Data

➢ Interpolate the bed elevations to computational grid by "Preprocessing"→"Set Current Input Condition"→"Map w/TIN"



Grid Generation(14)

➤ Bed elevation of computational grid interpolated from tpo file data.



Set Computational Conditions

- File Names
  Output type : CGNS and RIC-Nays

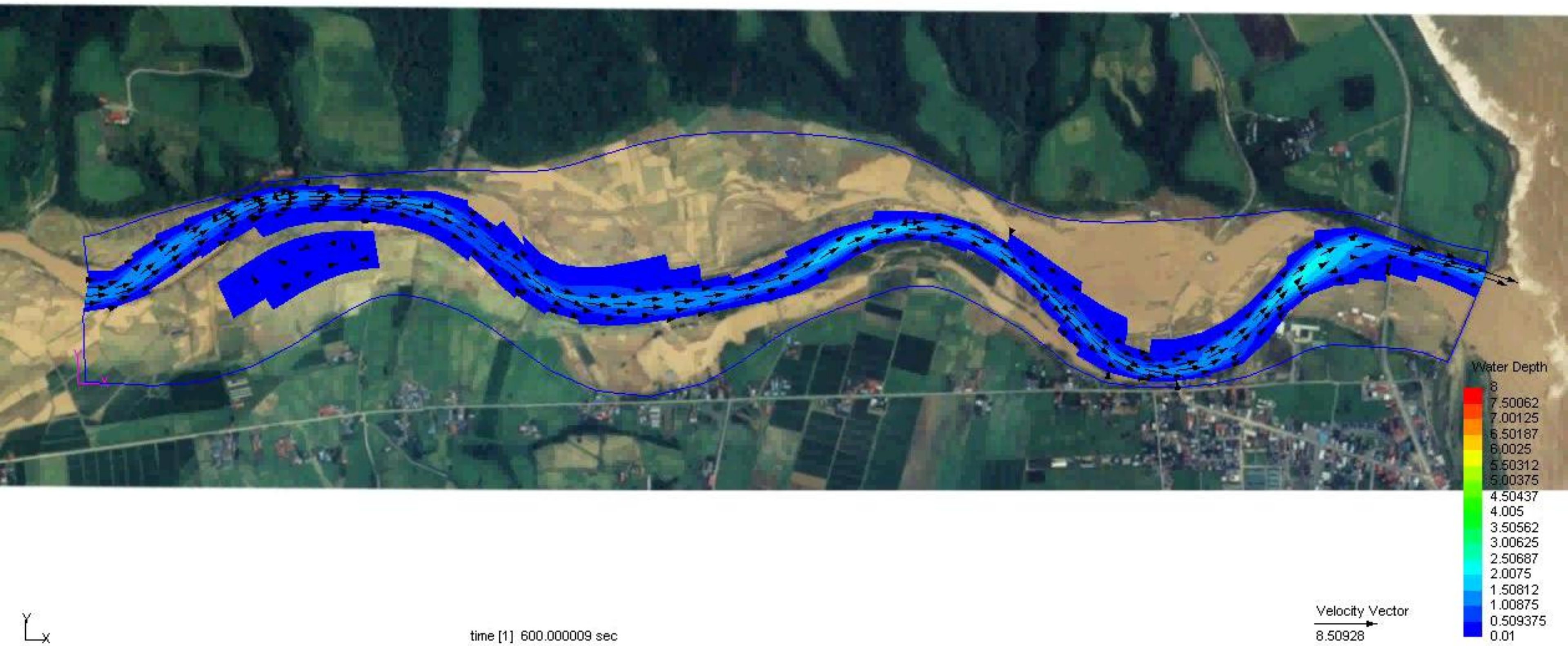
- Time
  Output time interval : 600 sec
  Calculation time step : 0.01 sec
  Start time of output : 0 sec
  Start time of bed deformation : negative value
  Discharge time series : Import from "Discharge.txt"

- Boundary conditions
  Water surface at downstream : Uniform flow
  Slope : calculated from geometric data

- Initial water surface profiles
  Initial water surface : uniform flow

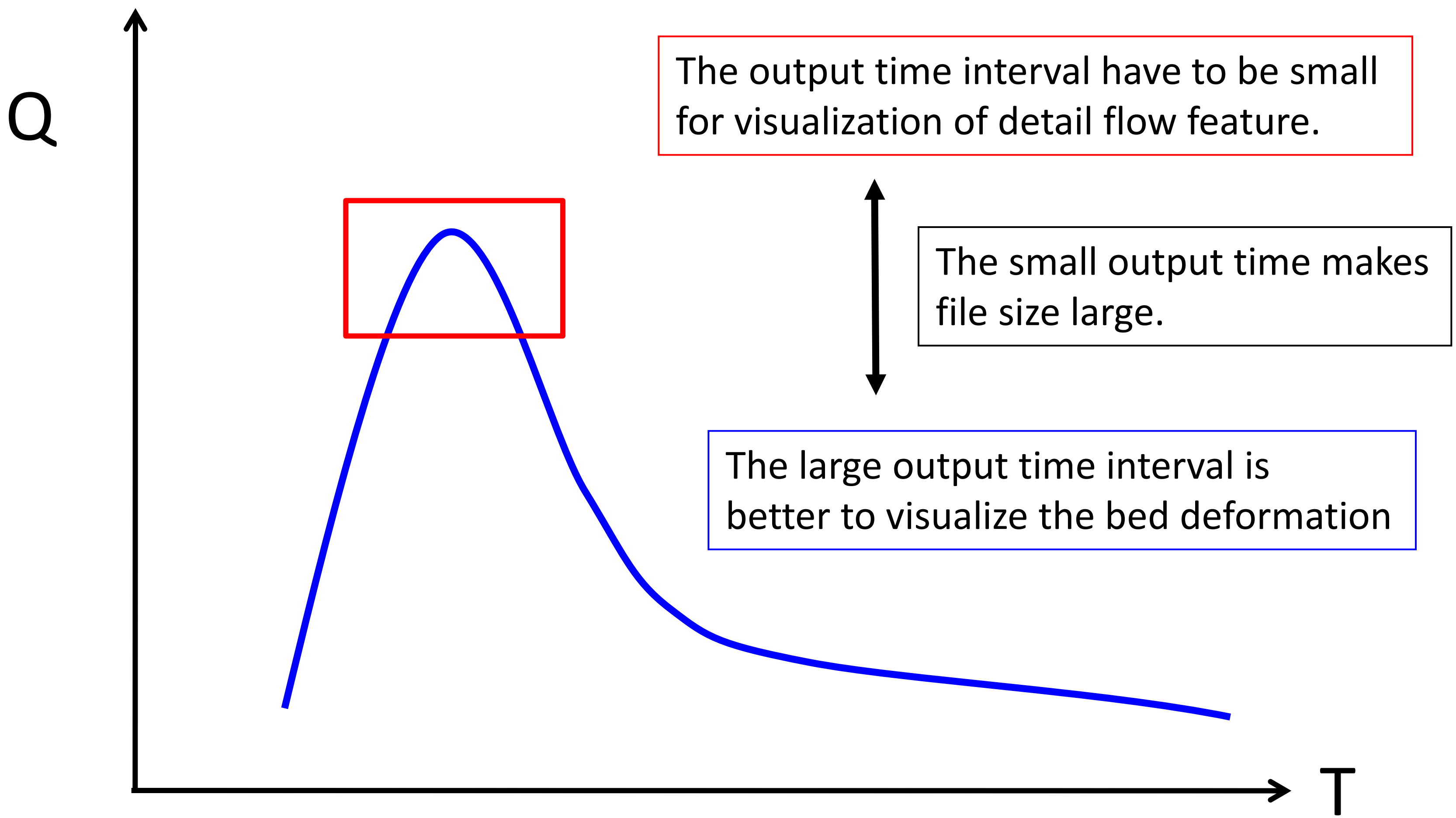
- Other condition
  Turbulent model : zero-equation model
  Finite differential method of the advection terms : CIP Method

Computational Result
- Water depth and Flow Vector -



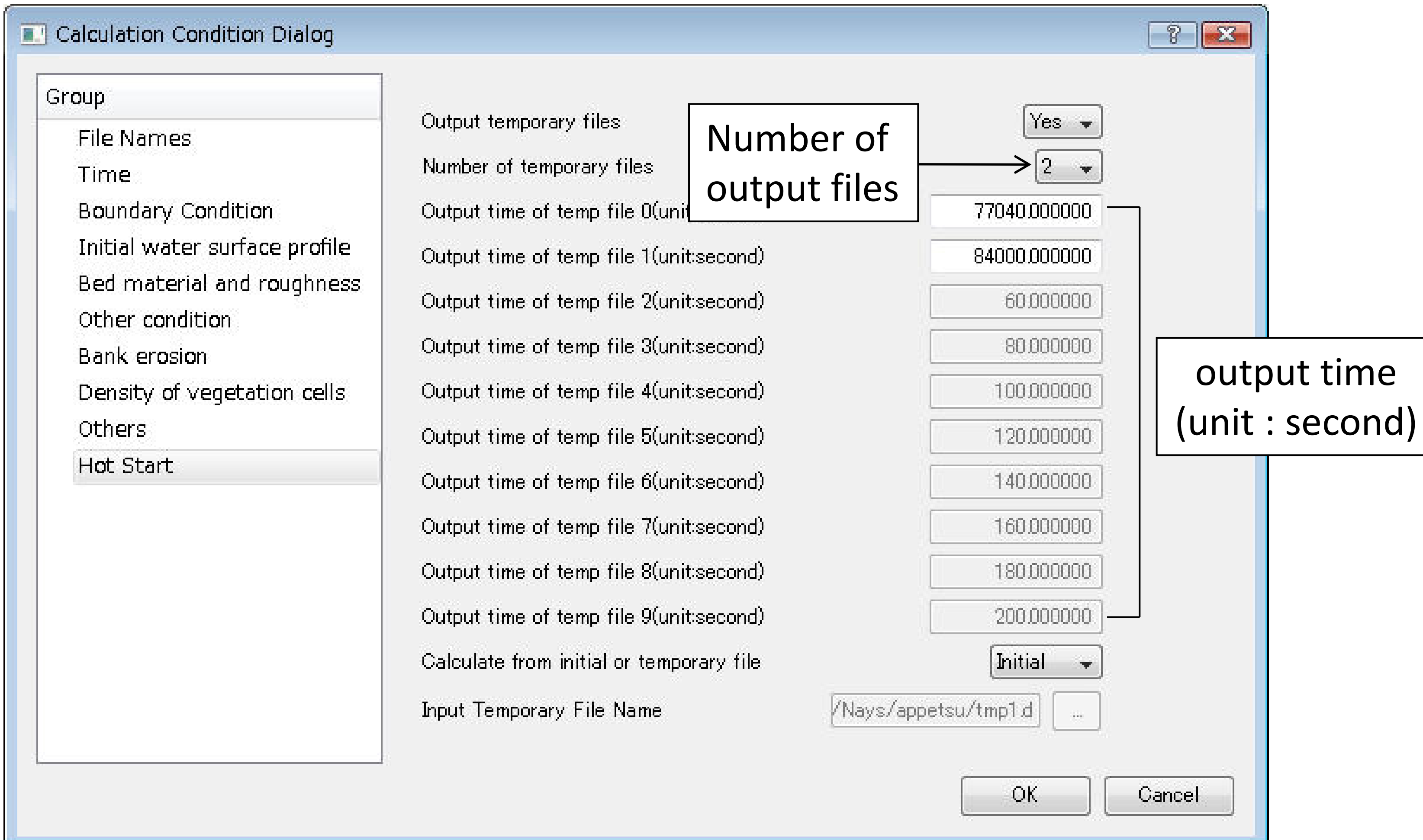
Hot Start(1)

- Nays can recalculate the computation from any point of hydrograph.



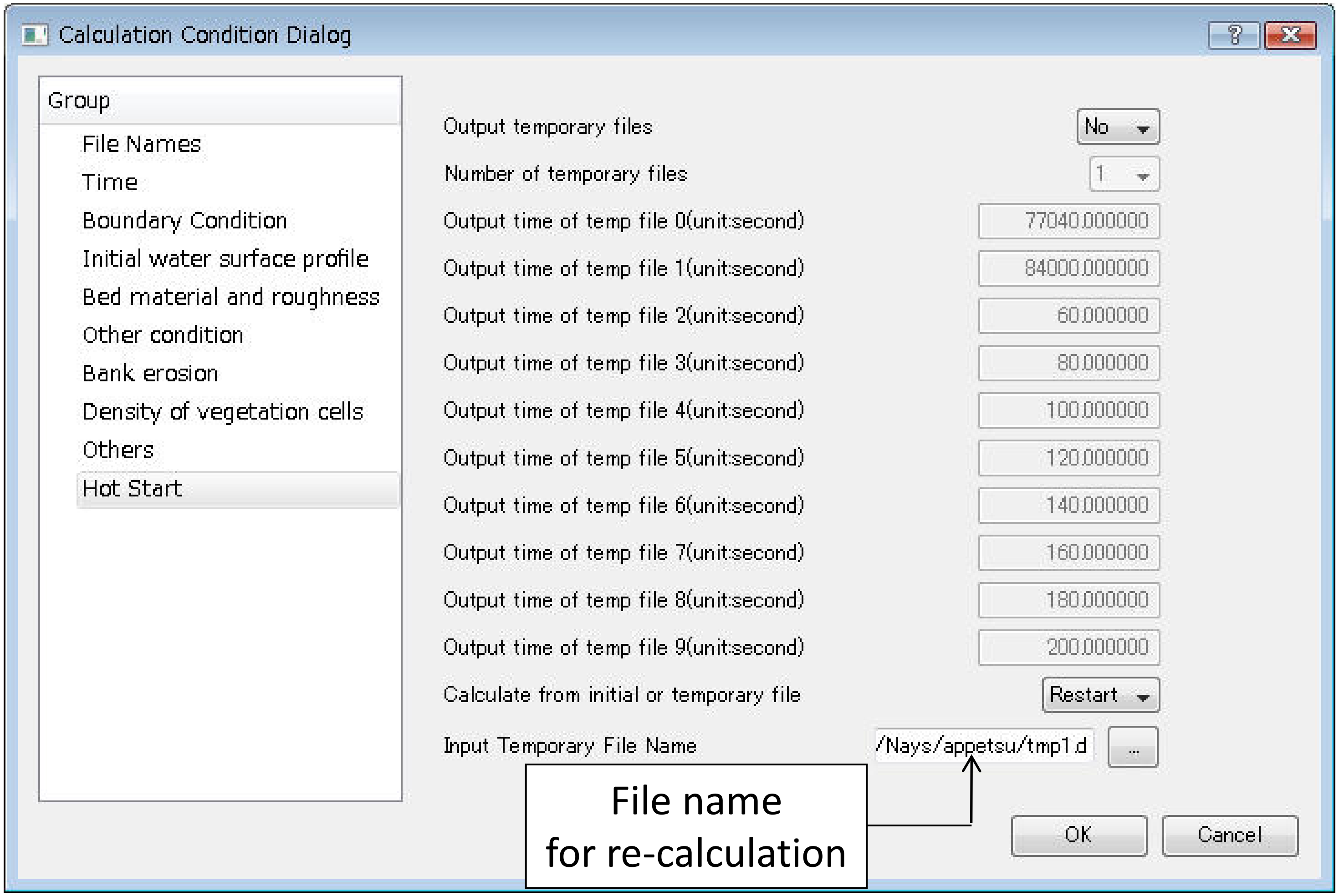
Hot Start(2)　Temporary Output File

- You can determine the number of temp file and output time.
- File names of temp file are automatically set as tmp0.d to tmp9.d.

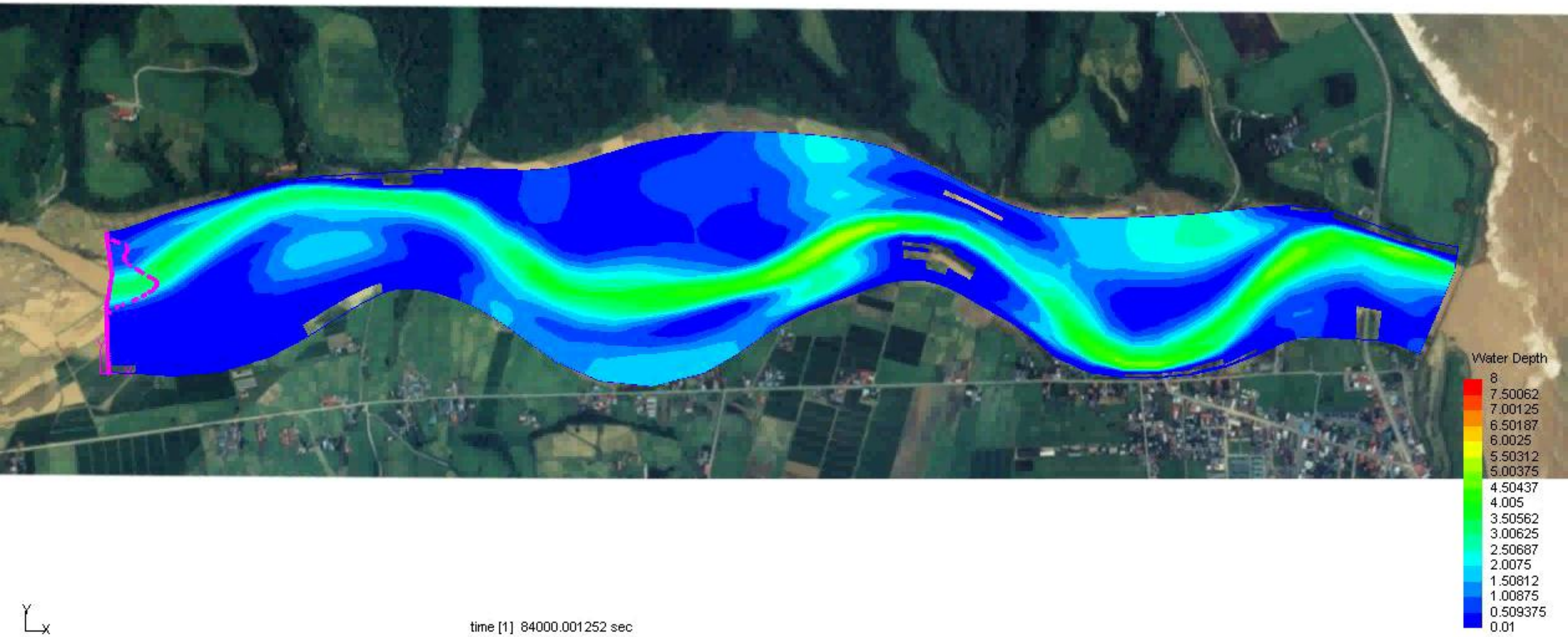


Hot Start(3) Re-calculation

- The temporary output file have to be determined, if you want to recalculate.



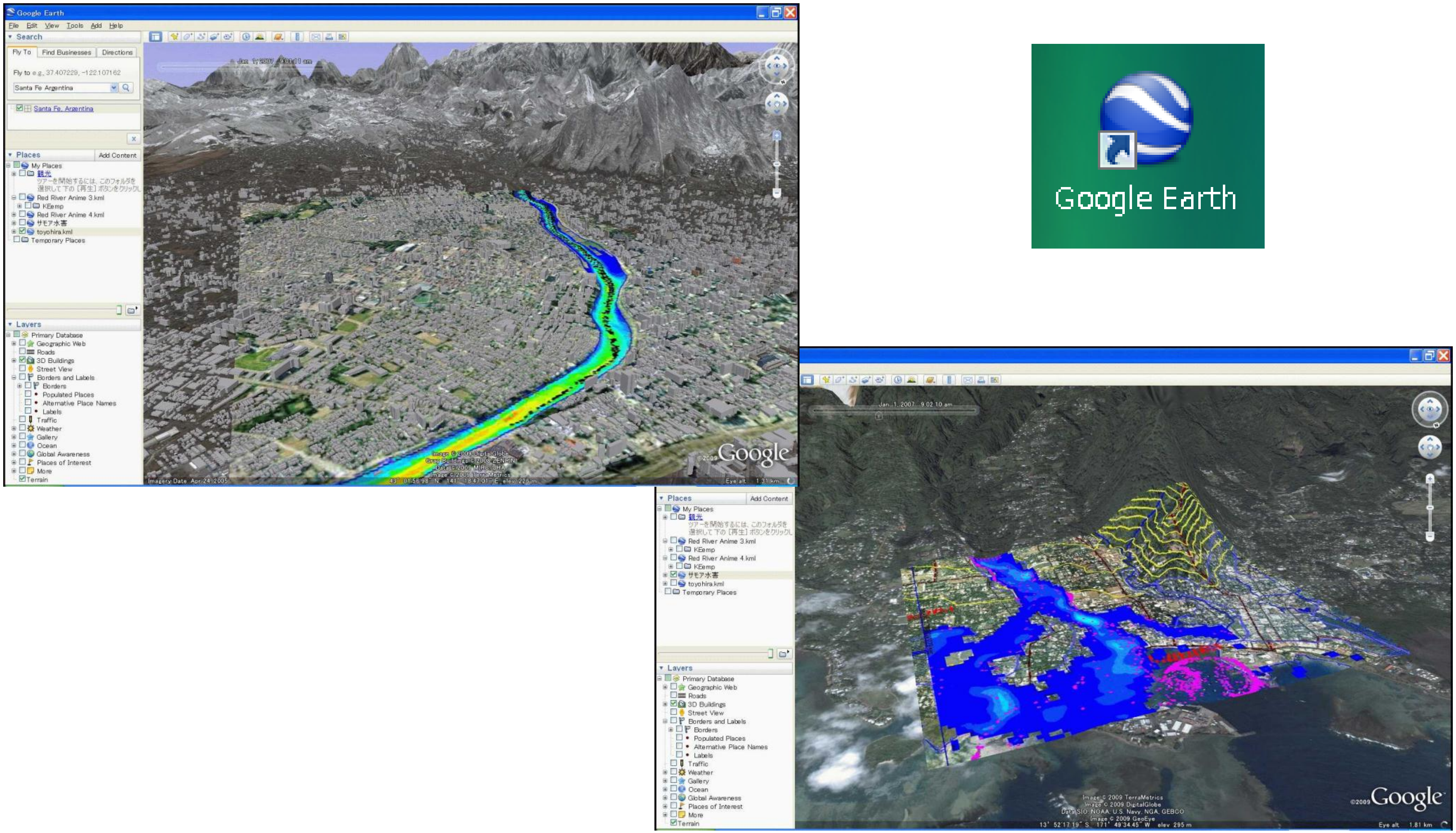
Computational Result
- Flow in Maximum Discharge -



International
River Interface Corporative
iRIC

Visualize the result on Google Earth

- You can see the animation of result on Google Earth.



Thank you
for your attention !!

BASIC EQUATIONS USED IN NAYS

2D Shallow Water Equatoins

Momentum in x-direction

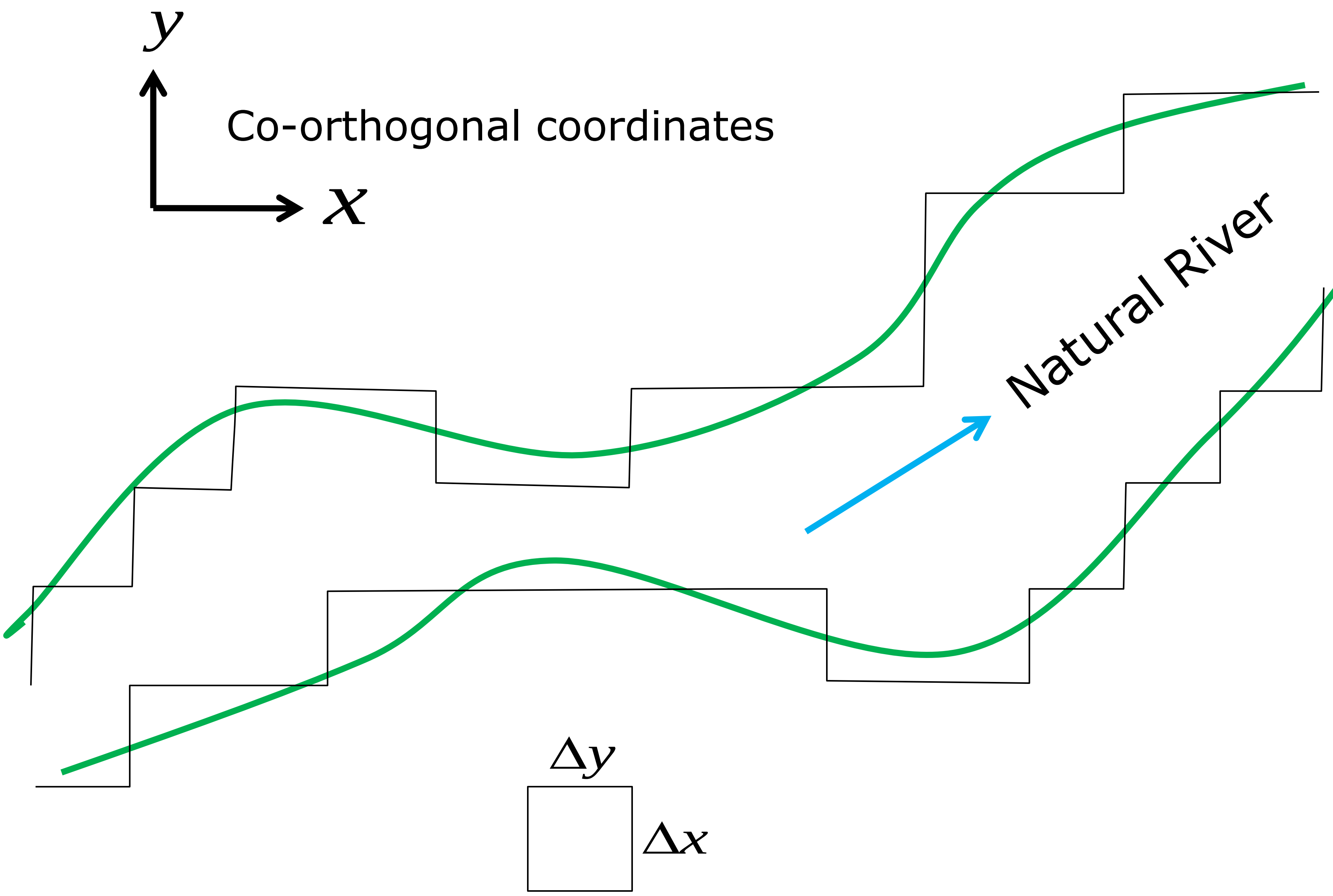
$$\frac{\partial\left(uh\right)}{\partial t}+\frac{\partial\left(u^2h\right)}{\partial x}+\frac{\partial\left(uvh\right)}{\partial y}=-gh\frac{\partial H}{\partial x}-\frac{\tau_x}{\rho}+D_x$$

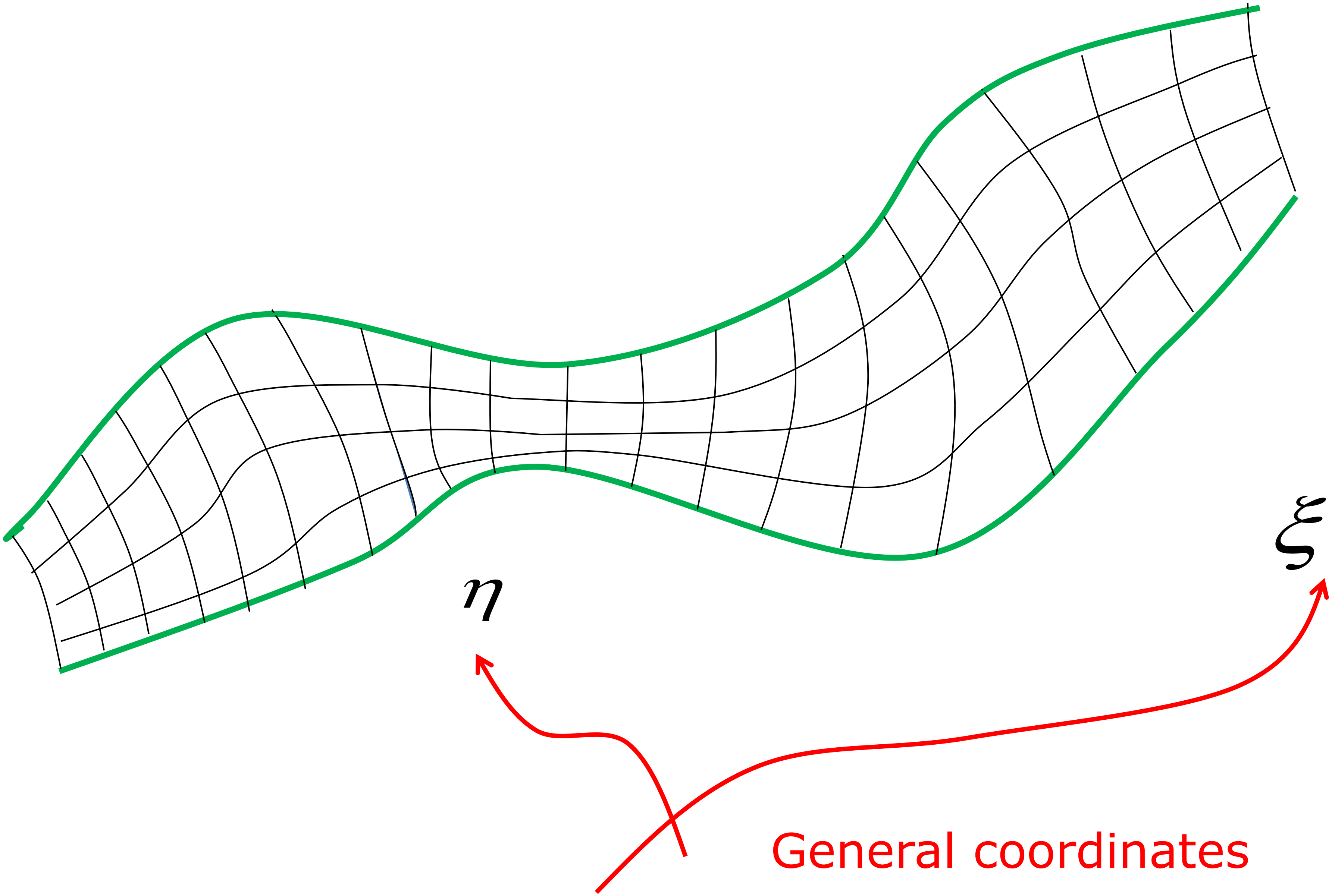
Momentum in y-direction

$$\frac{\partial\left(vh\right)}{\partial t}+\frac{\partial\left(uvh\right)}{\partial x}+\frac{\partial\left(v^2h\right)}{\partial y}=-gh\frac{\partial H}{\partial y}-\frac{\tau_y}{\rho}+D_y$$

Continuity

$$\frac{\partial h}{\partial t}+\frac{\partial(uh)}{\partial x}+\frac{\partial(vh)}{\partial y}=0$$


y
Co-orthogonal coordinates
x
Natural River
Δy
Δx



General coordinates

$$\frac{\partial}{\partial \xi} = \frac{\partial x}{\partial \xi}\frac{\partial}{\partial x} + \frac{\partial y}{\partial \xi}\frac{\partial}{\partial y}$$
$$\frac{\partial}{\partial \eta} = \frac{\partial x}{\partial \eta}\frac{\partial}{\partial x} + \frac{\partial y}{\partial \eta}\frac{\partial}{\partial y}$$

$$\begin{pmatrix} \dfrac{\partial}{\partial \xi} \\ \dfrac{\partial}{\partial \eta} \end{pmatrix} = \begin{pmatrix} x_\xi & y_\xi \\ x_\eta & y_\eta \end{pmatrix} \begin{pmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \end{pmatrix}$$

In which, $x_\xi = \dfrac{\partial x}{\partial \xi}$, $x_\eta = \dfrac{\partial x}{\partial \eta}$, $y_\xi = \dfrac{\partial y}{\partial \xi}$, $y_\eta = \dfrac{\partial y}{\partial \eta}$

Therefore,

$$\begin{pmatrix} \dfrac{\partial}{\partial \xi} \\ \dfrac{\partial}{\partial \eta} \end{pmatrix} = \frac{1}{\xi_x \eta_y - \xi_y \eta_x} \begin{pmatrix} \eta_y & -\eta_x \\ -\xi_y & \xi_x \end{pmatrix} \begin{pmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \end{pmatrix} = \begin{pmatrix} x_\xi & y_\xi \\ x_\eta & y_\eta \end{pmatrix} \begin{pmatrix} \dfrac{\partial}{\partial x} \\ \dfrac{\partial}{\partial y} \end{pmatrix}$$

If we define

$$J = \xi_x \eta_y - \xi_y \eta_x \longrightarrow \frac{1}{J} \begin{pmatrix} \eta_y & -\eta_x \\ -\xi_y & \xi_x \end{pmatrix} = \begin{pmatrix} x_\xi & y_\xi \\ x_\eta & y_\eta \end{pmatrix}$$

2D Flow Equations in General Coordinate Grid

$$\frac{\partial}{\partial t}\left(\frac{h}{J}\right)+\frac{\partial}{\partial \xi}\left(\frac{hu^\xi}{J}\right)+\frac{\partial}{\partial \eta}\left(\frac{hu^\eta}{J}\right)=0$$

$$\frac{\partial u^\xi}{\partial t}+u^\xi\frac{\partial u^\xi}{\partial \xi}+u^\eta\frac{\partial u^\xi}{\partial \eta}+\alpha_1 u^\xi u^\xi+\alpha_2 u^\xi u^\eta+\alpha_3 u^\eta u^\eta=$$

$$-g\left[(\xi_x^2+\xi_y^2)\frac{\partial H}{\partial \xi}+(\xi_x\eta_x+\xi_y\eta_y)\frac{\partial H}{\partial \eta}\right]$$

$$-\frac{C_d u^\xi}{hJ}\sqrt{(\eta_y u^\xi-\xi_y u^\eta)^2+(-\eta_x u^\xi+\xi_x u^\eta)^2}+D^\xi$$

$$\frac{\partial u^\eta}{\partial t}+u^\xi\frac{\partial u^\eta}{\partial \xi}+u^\eta\frac{\partial u^\eta}{\partial \eta}+\alpha_4 u^\xi u^\xi+\alpha_5 u^\xi u^\eta+\alpha_6 u^\eta u^\eta=$$

$$-g\left[(\eta_x\xi_x+\eta_y\xi_y)\frac{\partial H}{\partial \xi}+(\eta_x^2+\eta_y^2)\frac{\partial H}{\partial \eta}\right]$$

$$-\frac{C_d u^\eta}{hJ}\sqrt{(\eta_y u^\xi-\xi_y u^\eta)^2+(-\eta_x u^\xi+\xi_x u^\eta)^2}+D^\eta$$

in which,

$$\alpha_1 = \xi_x \frac{\partial^2 x}{\partial \xi^2} + \xi_y \frac{\partial^2 y}{\partial \xi^2}, \quad \alpha_2 = 2\left(\xi_x \frac{\partial^2 x}{\partial \xi \partial \eta} + \xi_y \frac{\partial^2 y}{\partial \xi \partial \eta}\right), \quad \alpha_3 = \xi_x \frac{\partial^2 x}{\partial \eta^2} + \xi_y \frac{\partial^2 y}{\partial \eta^2} \tag{28}$$

$$\alpha_4 = \eta_x \frac{\partial^2 x}{\partial \xi^2} + \eta_y \frac{\partial^2 y}{\partial \xi^2}, \quad \alpha_5 = 2\left(\eta_x \frac{\partial^2 x}{\partial \xi \partial \eta} + \eta_y \frac{\partial^2 y}{\partial \xi \partial \eta}\right), \quad \alpha_6 = \eta_x \frac{\partial^2 x}{\partial \eta^2} + \eta_y \frac{\partial^2 y}{\partial \eta^2} \tag{29}$$

$$D^\xi = \left(\xi_x \frac{\partial}{\partial \xi} + \eta_x \frac{\partial}{\partial \eta}\right)\left[\nu_t \left(\xi_x \frac{\partial u^\xi}{\partial \xi} + \eta_x \frac{\partial u^\xi}{\partial \eta}\right)\right] + \left(\xi_y \frac{\partial}{\partial \xi} + \eta_y \frac{\partial}{\partial \eta}\right)\left[\nu_t \left(\xi_y \frac{\partial u^\xi}{\partial \xi} + \eta_y \frac{\partial u^\xi}{\partial \eta}\right)\right] \tag{30}$$

$$D^\eta = \left(\xi_x \frac{\partial}{\partial \xi} + \eta_x \frac{\partial}{\partial \eta}\right)\left[\nu_t \left(\xi_x \frac{\partial u^\eta}{\partial \xi} + \eta_x \frac{\partial u^\eta}{\partial \eta}\right)\right] + \left(\xi_y \frac{\partial}{\partial \xi} + \eta_y \frac{\partial}{\partial \eta}\right)\left[\nu_t \left(\xi_y \frac{\partial u^\eta}{\partial \xi} + \eta_y \frac{\partial u^\eta}{\partial \eta}\right)\right] \tag{31}$$

Turbulence Model for the Flow

1. ν_t = constant

2. Zero equation model $\nu_t = \frac{1}{6} u_* h$

3. Standard $k - \varepsilon$ model

$k{-}\epsilon$ Model

$$\nu_t = C_\mu \frac{k^2}{\epsilon} \tag{1}$$

$$\frac{\partial k}{\partial t} + u\frac{\partial k}{\partial x} + v\frac{\partial k}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial y}\right) + P_h + P_{kv} - \epsilon \tag{2}$$

$$\frac{\partial \epsilon}{\partial t} + u\frac{\partial \epsilon}{\partial x} + v\frac{\partial \epsilon}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\nu_t}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\nu_t}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial y}\right) + C_{1\epsilon}\frac{\epsilon}{k}P_h + P_{\epsilon v} - C_{2\epsilon}\frac{\epsilon^2}{k} \tag{3}$$

where

$$P_h = \nu_t\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right] \tag{4}$$

$$P_{kv} = C_k\frac{u_*^3}{h}, \qquad P_{\epsilon v} = C_\epsilon\frac{u_*^4}{h^2} \tag{5}$$

$$u_* = \sqrt{C_f(u^2+v^2)} \tag{6}$$

$$C_k = \frac{1}{\sqrt{C_f}}, \qquad C_\epsilon = 3.6\frac{C_{2\epsilon}}{C_f^{3/4}}\sqrt{C_\mu} \tag{7}$$

C_μ	$C_{1\epsilon}$	$C_{2\epsilon}$	σ_k	σ_ϵ
0.09	1.44	1.92	1.0	1.3

In the range of $30 < y^+ < 100$,

$$\frac{k}{u_*^2} = \frac{1}{\sqrt{C_\mu}}, \qquad \epsilon = \frac{u_*^3}{\kappa y} \tag{8}$$

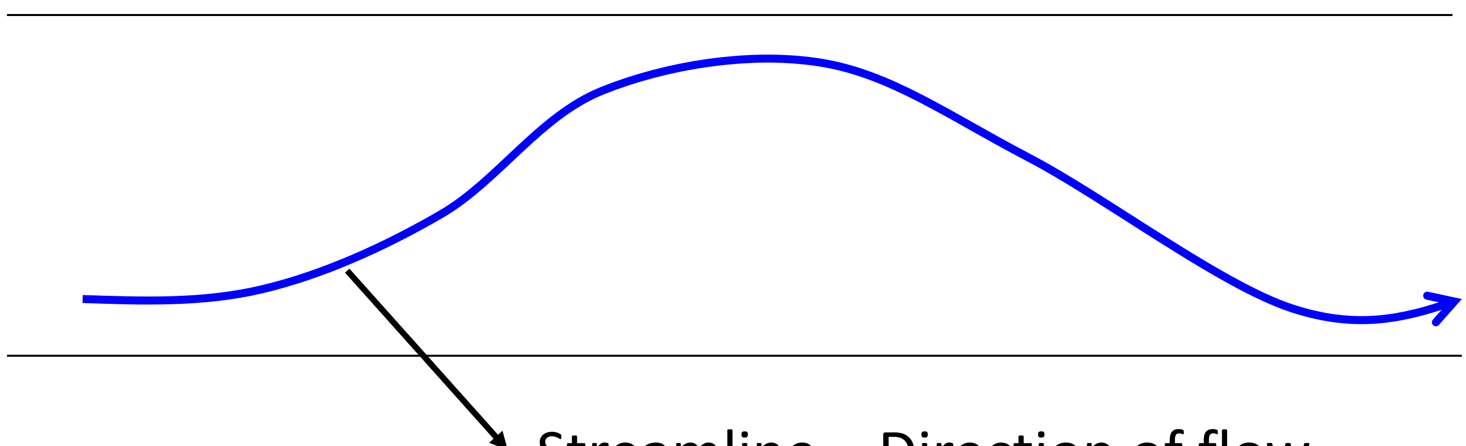
where y is distance from wall and, $y^+ = \dfrac{y u_*}{\nu}$. Therefore,

$$\nu_t = C_\mu \frac{k^2}{\epsilon} = C_\mu \frac{\kappa y}{u_*^3} \frac{u_*^4}{C_\mu} = \kappa y u_* = \kappa \nu y^+ \tag{9}$$

BASIC IDEA OF BED DEFORMATION MODEL AND BANK EROSION MODEL IN NAYS

2-dimensional Bed Deformation

$$\frac{\partial z_b}{\partial t}+\frac{1}{1-\lambda}\left(\frac{\partial q_{bx}}{\partial x}+\frac{\partial q_{by}}{\partial y}\right)=0$$

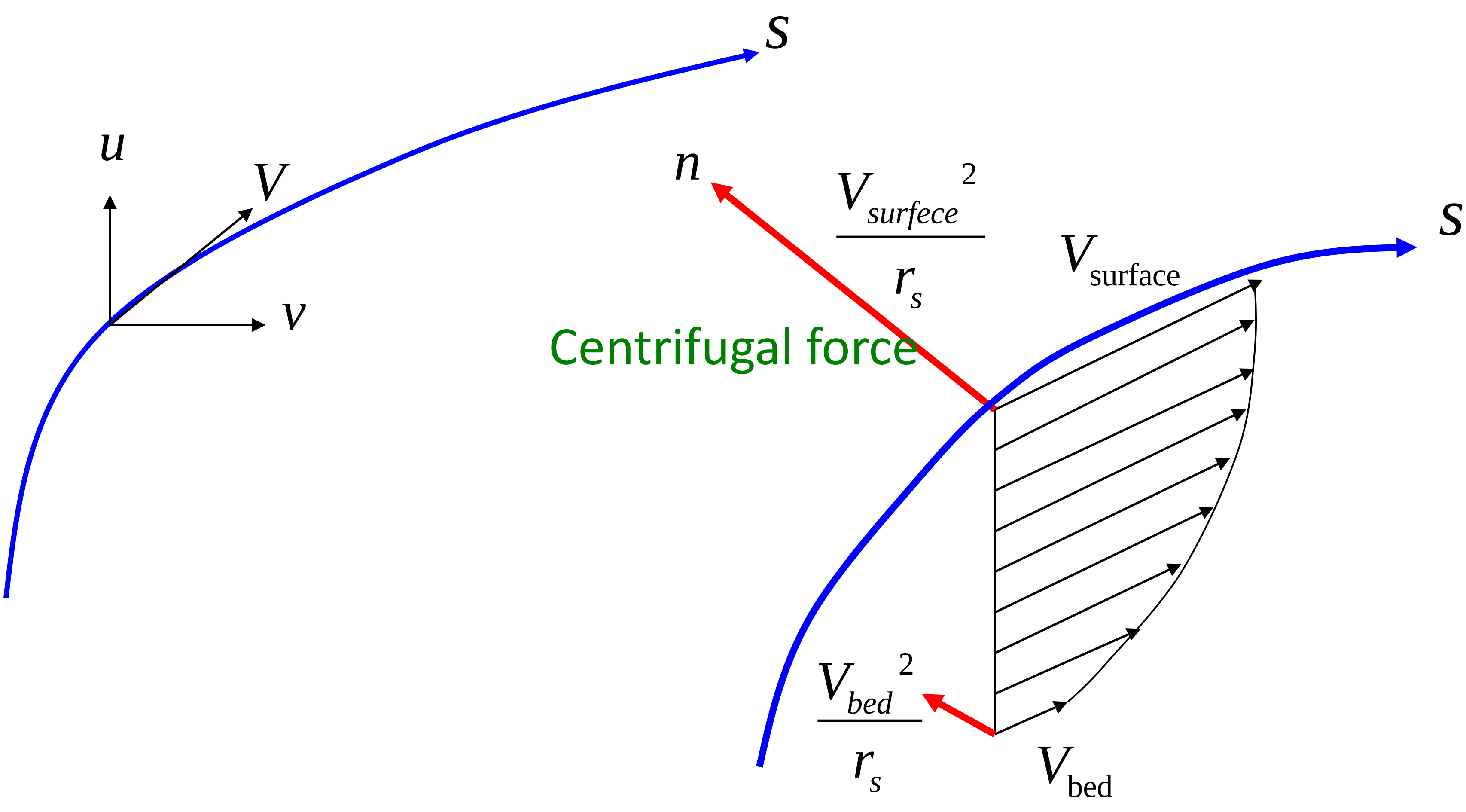


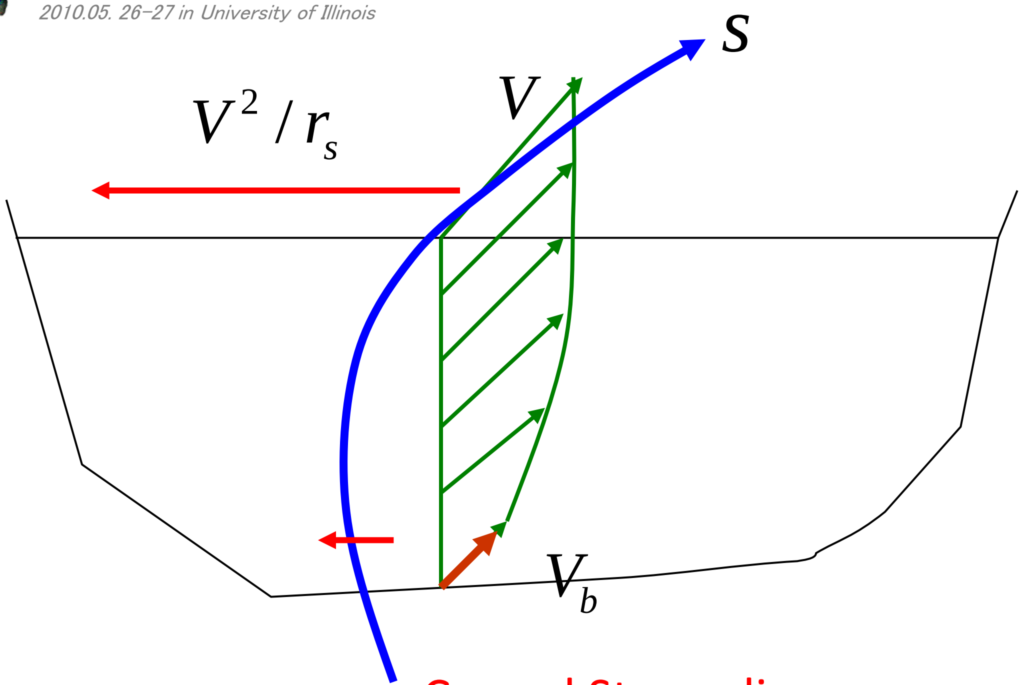
Streamline = Direction of flow

=Direction of sediment transport

??? $\longrightarrow$ Not so simple!

Streamline Curvature and Secondary Flow





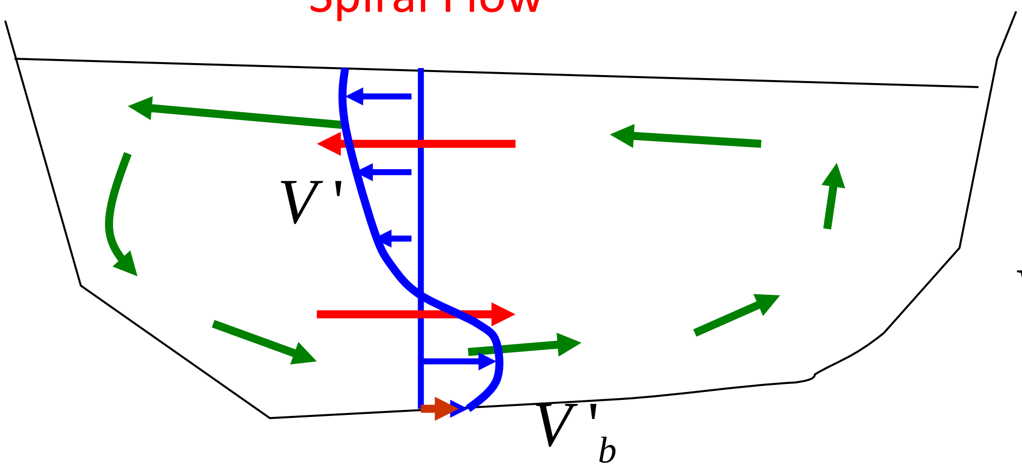
Engelund, F., Flow and bed topography in channe bends, Proc. Am. Soc. Civ. Eng. Hydraul. Div., 100, 1631-1648, 1974.

$$V'_b = \frac{hV_b}{r_s} N_* \approx 7.0$$

V Flow Velocity along Streamline

V_b Flow velocity at the bottom along the streamline

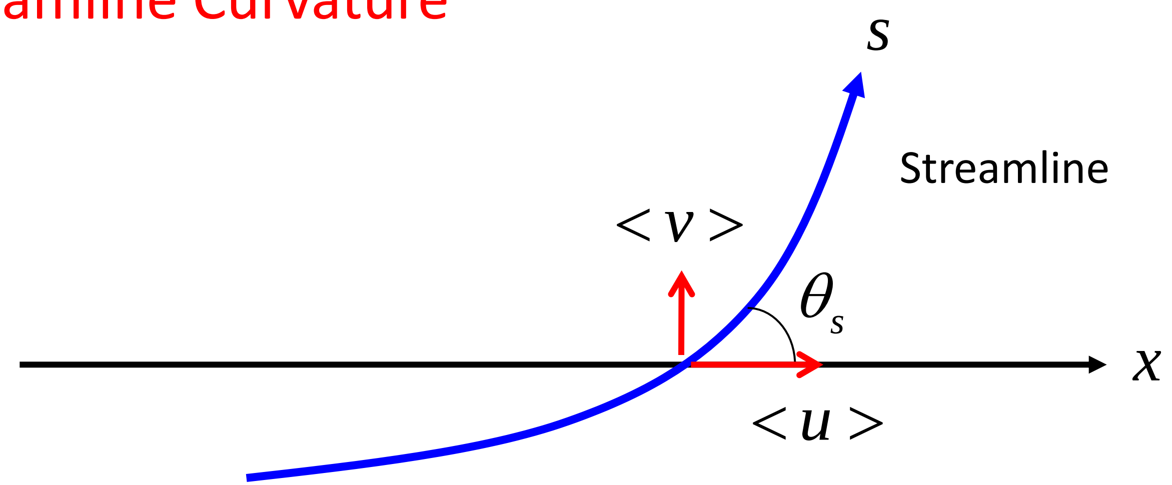
Secondary Flow or Spiral Flow



V' Secondary Flow Perpendicular to the Streamline

V'_b Secondary Flow Perpendicular to the Streamline at the bottom

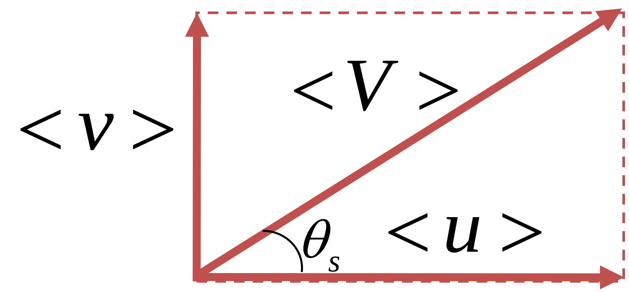
Streamline Curvature



$$\frac{1}{r_s} = \frac{\partial \theta_s}{\partial s} \qquad \tan\theta_s = \frac{<v>}{<u>} \equiv T$$

$$\frac{1}{r_s} = \frac{\partial}{\partial s}\left[\tan^{-1}(T)\right] = \frac{1}{1+T^2}\frac{\partial T}{\partial s}$$

$$\frac{1}{1+T^2} = \frac{<u>^2}{<u>^2 + <v>^2} = \frac{<u>^2}{<V>^2}$$

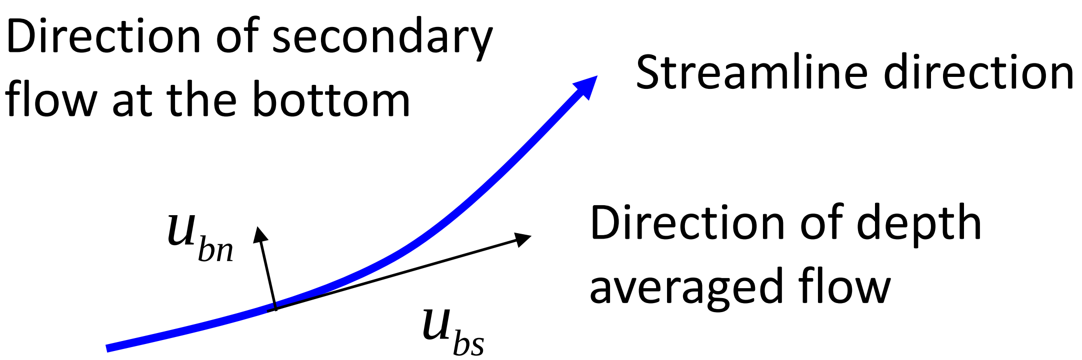


$$\frac{\partial T}{\partial s} = \frac{\partial}{\partial s}\left(\frac{<v>}{<u>}\right) = \frac{<u>\dfrac{\partial <v>}{\partial s} - <v>\dfrac{\partial <u>}{\partial s}}{<u>^2}$$

$$\frac{\partial}{\partial s} = \frac{\partial x}{\partial s}\frac{\partial}{\partial x} + \frac{\partial y}{\partial s}\frac{\partial}{\partial y} = \frac{<u>}{<V>}\frac{\partial}{\partial x} + \frac{<v>}{<V>}\frac{\partial}{\partial y}$$

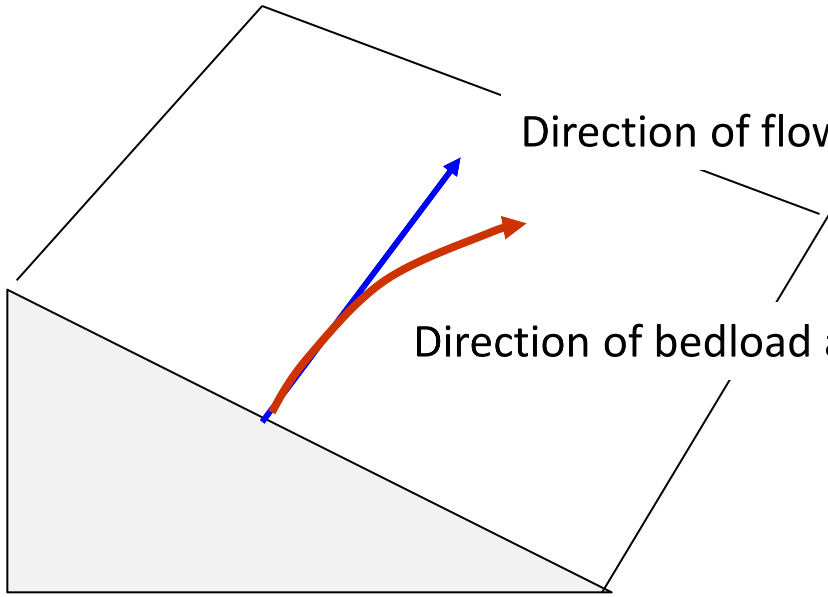
$$\frac{\partial T}{\partial s} = \frac{1}{<u>^2<V>}\left[<u>\left\{<u>\frac{\partial <v>}{\partial x} + <v>\frac{\partial <v>}{\partial y}\right\} - <v>\left\{<u>\frac{\partial <u>}{\partial x} + <v>\frac{\partial <u>}{\partial y}\right\}\right]$$

$$\frac{1}{r_s} = \frac{1}{<V>^3}\left[<u>^2 \frac{\partial <v>}{\partial x} + <u><v> \frac{\partial <v>}{\partial y} - <v><u> \frac{\partial <u>}{\partial x} - <v>^2 \frac{\partial <u>}{\partial y}\right]$$



$$u_b = u_{bs}\cos\theta_s - u_{bn}\sin\theta_s$$
$$v_b = u_{bs}\sin\theta_s + u_{bn}\cos\theta_s$$

Gravity Effect from Transverse Bed Slope



Watanabe's equation.

$$q_{bx} = q_b \left[\frac{u_b}{V_b} - \gamma \left(\frac{\partial z_b}{\partial x} + \cos\theta_s \frac{\partial z_b}{\partial y} \right) \right]$$

$$q_{by} = q_b \left[\frac{v_b}{V_b} - \gamma \left(\frac{\partial z_b}{\partial y} + \cos\theta_s \frac{\partial z_b}{\partial x} \right) \right]$$

Hasegwa's formula

$$\gamma = \sqrt{\frac{\tau_{*c}}{\mu_s \mu_k \tau_{*c}}}$$

Bank Erosion Model (Non-cohesive Bank)