CE 5362 Surface Water Modeling

1D/2D Hydrodynamic Models

Open Channel Flow

- Review of important terms
- Derive (plausibility argument) St. Venant (1D-spatial)
- Algorithms
  - Steady – variable space step
  - Steady – fixed space step

Review of Terms

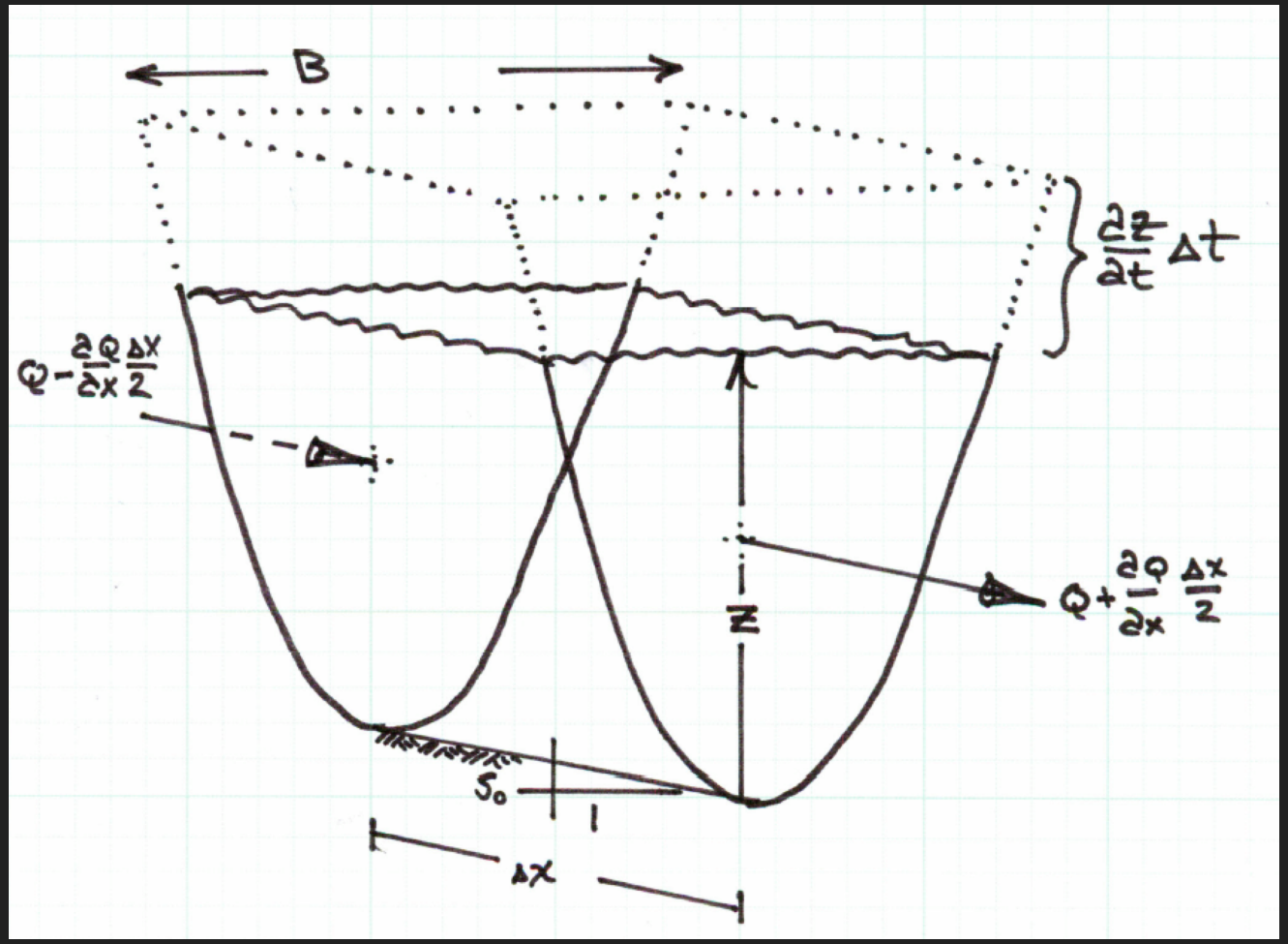
- Steady flow
  - Uniform
  - Gradually varied
  - Rapidly varied
- Unsteady flow

St. Venant Equations

- Continuity
  - Lateral inflow
- Momentum
  - Pressure
  - Gravity drive
  - Friction Loss

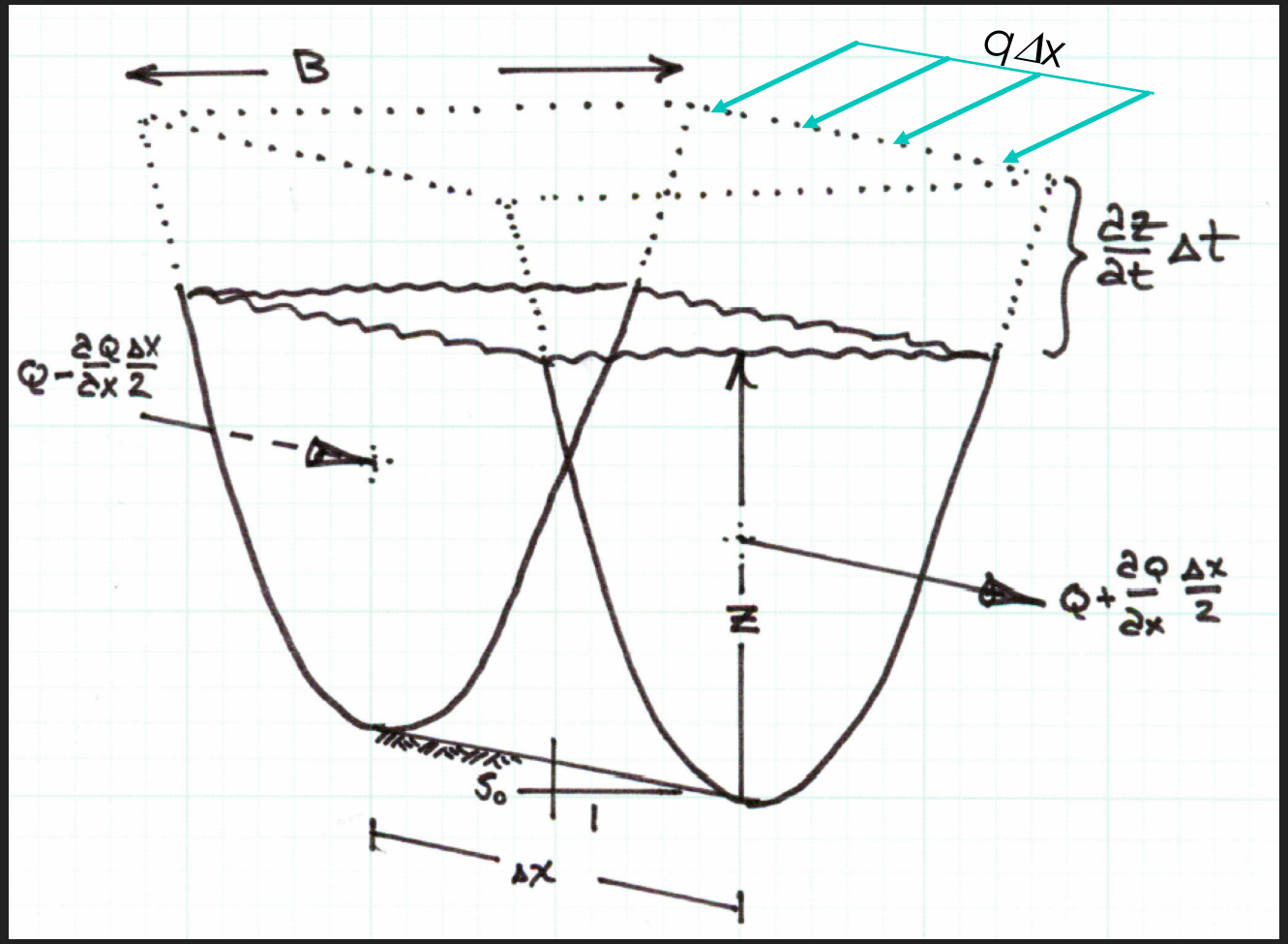
Computational Cell

- Topwidth (*B*) is a function of depth (and geometry)
- Here using z as flow depth variable



Conservation of Mass

- M_{in} = mass flow into cell
  - $q * \Delta x$ = lateral mass inflow
- M_{out} = mass flow out of cell
- $M_{storage}$ = rate of mass stored in cell



Conservation of Mass

- M_{in} = mass flow into cell

$$\dot{M}_{in} = \rho * (Q - \frac{\partial Q}{\partial x} * \frac{\Delta x}{2}) * \Delta t$$

$+\rho q\ \Delta x \Delta t$

Conservation of Mass

- M_{out} = mass flow out of cell

$$\dot{M}_{out} = \rho * (Q + \frac{\partial Q}{\partial x} * \frac{\Delta x}{2}) * \Delta t$$

Conservation of Mass

- $M_{storage}$ = rate of mass stored in cell

$$\dot{M}_{storage} = \rho * (\frac{\partial z}{\partial t} * \Delta t) * B(z) * \Delta x$$

Conservation of Mass

- Assemble the conservation equation:

$M_{in} - M_{out} - M_{storage} = 0$

$$\rho * (\frac{\partial z}{\partial t} * \Delta t) * B(z) * \Delta x = \rho * (Q - \frac{\partial Q}{\partial x} * \frac{\Delta x}{2}) * \Delta t - \rho * (Q + \frac{\partial Q}{\partial x} * \frac{\Delta x}{2}) * \Delta t$$

$+\rho q\ \Delta x \Delta t$

Conservation of Mass

$$\rho * (\frac{\partial z}{\partial t} * \Delta t) * B(z) * \Delta x = \rho * (Q - \frac{\partial Q}{\partial x} * \frac{\Delta x}{2}) * \Delta t - \rho * (Q + \frac{\partial Q}{\partial x} * \frac{\Delta x}{2}) * \Delta t$$

$+\rho q\ \Delta x \Delta t$

- Collapse and simplify (divide by $\rho \Delta x \Delta t$)

$$(\frac{\partial z}{\partial t}) * B(z) + \frac{\partial Q}{\partial x} = q$$

Conservation of Momentum

- Change of momentum in the cell is equal to the net momentum entering the cell plus the sum of the forces on liquid in the cell.