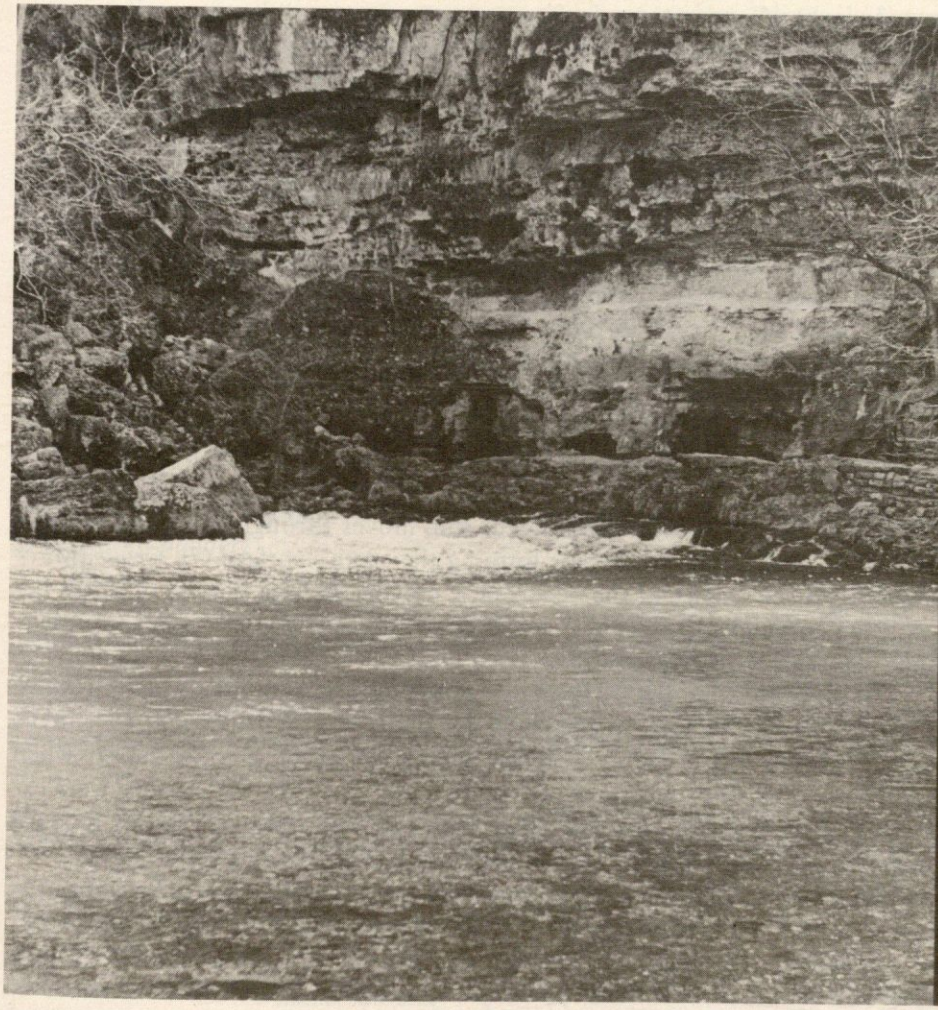


Big Spring in Carter County Missouri. The spring rises from the Eminence Dolomite and has an annual average flow of 438 cubic feet per second. (Photo by Mr. James E. Vandike of the State of Missouri, Department of Natural Resources, Division of Geology and Land Survey, Rolla, Missouri)

3

Groundwater

3-1 General Considerations

As was shown in Fig. 2-3, page 24 groundwater, in general, is that portion of the earth's water occuring below the surface of the earth. Groundwater (that is, underground water) is one of the earth's most important resources. Its use accounts for approximately 40% of the water used on the earth with the exception of that used for hydropower generation and cooling thermal power plants. Despite its wide use, groundwater is also the least understood resource. There are many common misconceptions about groundwater, including the belief that it runs in rivers beneath the surface of the earth. The only groundwater occurrence resembling the concept of a surface stream are the large solution caverns often found in massive limestone formations. The Carlsbad Caverns in New Mexico and the Meramac Caverns in Missouri are well-known examples of solution caverns in regions underlain by limestone.

Groundwater, which makes up about 14% of the earth's fresh water and amounts to approximately 4 million cu km, moves through the openings that exist within the natural materials forming the earth's surface (7). *Groundwater hydrology* is the science of the occurrence, movement, and quality of water beneath the surface of the earth. It has become increasingly important to understand the science of groundwater.

Groundwater has always been one of the most important sources of water supply. Virtually all parts of the earth are underlain by water, and wells have been constructed throughout recorded history to provide a water supply when surface water was not readily available. In early times, the water wells were excavated by hand and were rarely more than a few meters deep. Today, modern water wells are drilled and in some localities are more than 300 m deep.

Water wells have played an increasingly important role in irrigation as long as water has been available, energy has been relatively cheap, and prices for agricultural products have been high enough to make irrigation systems feasible. Favorable economic conditions between 1950 and 1970 led to extensive irrigation development using well water in the high plains of Texas, the central valley of California, central Arizona, eastern Washington, eastern Colorado, western Kansas, and central Oklahoma. In all these areas, pumping rates have substantially exceeded natural recharge rates, resulting in continuously dropping groundwater elevations. As the groundwater levels have dropped, the energy required to maintain the pumping rates has increased correspondingly. Moreover, the larger initial cost of constructing and maintaining deeper wells, and the sharp increases in the cost of energy since 1973, have resulted in significant increases in the cost of the pumped water. Because of this, lands formerly irrigated have been abandoned in many areas. In other areas, such as the central valley of California, the falling groundwater levels have produced substantial subsidence of the land surface.

Other activities that have influenced attention given to groundwater development include the injection of liquid wastes into very deep aquifers for permanent disposal and the use of groundwater for heat storage. Disposal of waste products in locations where products leached from them enter the groundwater and are transported toward wells or springs often contaminates water supplies and causes serious concern.

All these activities make it important for engineers to be aware of the hydraulics of groundwater.

3-2 Occurrence of Groundwater

Figure 3-1 illustrates the occurrence of water beneath the earth's surface. In the *unsaturated zone*, which adjoins the earth's surface, the spaces between soil and rock particles are filled with air, water, and water vapor. In the *saturated zone*, the interconnected openings of the supporting material are almost invariably filled with water. In a strict technical sense groundwater is that within the saturated zone. The soil or rock containing the saturated zone is called an *aquifer*.

The *soil zone*, the upper part of the unsaturated zone, is generally less than 1 or 2 m in thickness and encompasses that part of the earth's surface used by plant roots, rodents, eartn burrowing insects, and worms. It often is altered by the activities of both man and nature. Within this zone, water may be present in the form of *hygroscopic water* (water absorbed from the air), *capillary water* (water held in the soil by capillary action), or *gravitational water* (water draining downward through the soil). The lowest part of the unsaturated zone contains the *capillary zone*. In this zone, water is pulled from the saturated zone by capillary action within the voids of the porous material, which may be rock,

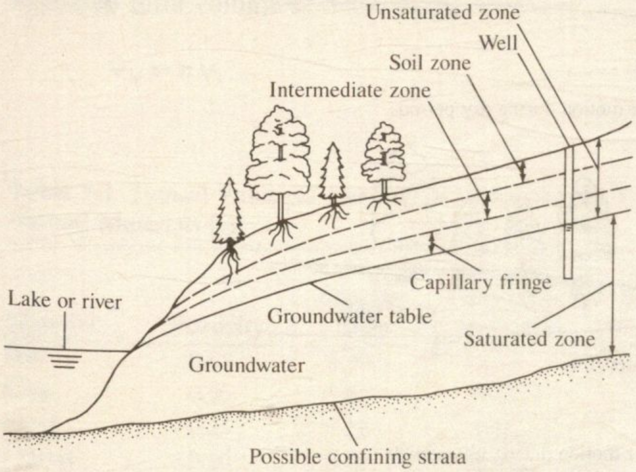


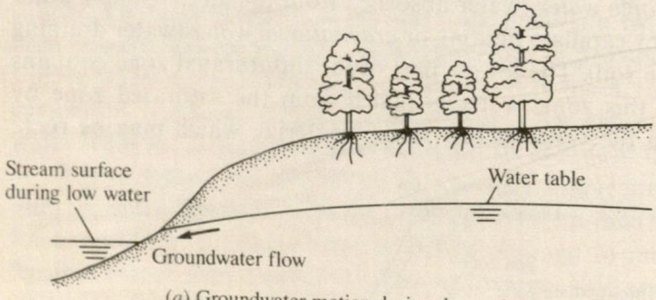
Figure 3-1 Schematic of the occurrence of groundwater

gravel, or soil depending on the local geology. The smaller the voids in the capillary zone, the higher the water is pulled and the thicker the capillary zone becomes. The water in this capillary zone is under negative gauge pressure, since the upper surface of the saturated zone is normally at atmospheric pressure.

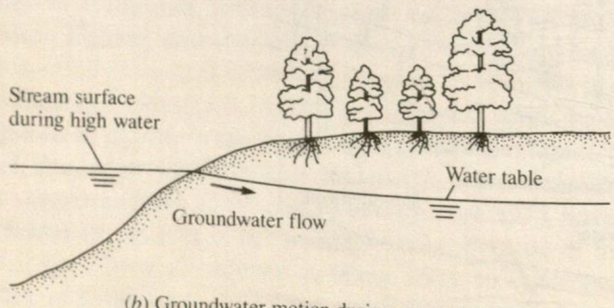
The *intermediate zone* is essentially a connecting zone between the soil zone and the capillary zone. Water passes through this zone under the action of gravity and moves downward to the saturated zone.

As we discussed in Sec. 2-3, and as shown in Fig. 2-3, groundwater is replenished from precipitation that infiltrates through the earth's surface. If sufficient infiltration occurs to fill the "field capacity," deep infiltration occurs, and groundwater replenishment takes place. *Deep infiltration* is sometimes referred to as *percolation*, or the motion of water downward through the soil zone.

Within the unsaturated zone, the movement of water is complicated by the presence of capillary and gravitational forces. If layers of low vertical permeability exist, the influence of these gravity and capillary forces can produce lateral movement through the soil called *interflow*. This interflow ends where the layer terminates and can result in flow directly from the unsaturated soil to channels or drains.



(*a*) Groundwater motion during dry periods



(*b*) Groundwater motion during wet periods

Figure 3-2 Behavior of groundwater during periods of high and low streamflow (a) Groundwater flow during dry periods (b) Groundwater flow during wet periods

The thickness of the unsaturated zone varies depending on the amount of water in storage. After significant rainfall and infiltration, the top of the saturated level will be high. During dry periods, as stored groundwater drains to streams or other water bodies, the saturated level will generally fall.

Below the groundwater table, water generally moves very slowly. As shown in Fig. 3-2, water can flow either into or out of streams, springs, or lakes depending on the gradient of the groundwater surface. Springs occur wherever the groundwater table intersects the earth's surface. Springs can be intermittent if the water table rises and falls above and below the spring's elevation, since the amount of rainfall and infiltration vary throughout the year. In some locations, streams gain water from groundwater inflow during the rainy season and lose water to the groundwater zone during the dry season.

Water within the saturated zone fills the natural voids that occur within the solid material, which may be soil, gravel, or rock depending on the local geology. If voids make up a large percent of the bulk volume of the solid material, a greater volume of water can be contained. Thus, an aquifer of saturated gravel contains a much greater volume of water in a unit volume of the solid material than does an equal volume of hard rock. All natural materials are porous to some degree. Table 3-1 lists typical values of *porosity* (volume of voids divided by total volume) for several natural materials. The equation used to calculate porosity (n) is

$$n = \frac{\forall_v}{\forall_t} \tag{3-1}$$

where $\forall_v$ is the volume of the voids, and $\forall_t$ is the total volume.

Using the definition of porosity, the volume of water $\forall_w$ contained in a saturated total volume of material $\forall_t$ is

$$\forall_w = n\forall_t \tag{3-2}$$

Table 3-1 Typical Values of Mechanical Properties for Various Natural Materials (6)

Material	Porosity	Specific Yield	Specific Retention	Hydraulic Conductivity (m/day)
Soil	0.55	0.40	0.15	10^{-3}–5
Clay	0.50	0.02	0.48	10^{-7}–10^{-4}
Sand	0.25	0.22	0.03	0.06–120
Gravel	0.20	0.19	0.01	100–7000
Limestone	0.20	0.18	0.02	10^{-4}–5000
Sandstone	0.11	0.06	0.05	10^{-5}–0.5
Basalt	0.11	0.08	0.03	10^{-8}–1000
Granite	0.001	0.0009	0.0001	10^{-8}–5

However, during pumping or draining of a given aquifer, not all the water originally held during saturated conditions will be drained. Part of the water will be held within the solids by molecular attraction. The physical forces (surface tension) creating this attraction are the same as those that pull water from the water table forming the capillary fringe. In groundwater terminology, the volume of water contained in the ground is divided into two parts. The fraction that drains from the ground under the action of gravity is called *specific yield*, whereas the fraction retained as a film around particle surfaces or in very small openings is called *specific retention.* Porosity is related to specific yield (S_y) and specific retention (S_r):

$$n = S_y + S_r \tag{3-3}$$

$$S_y = \frac{\forall_d}{\forall_t} \tag{3-4}$$

$$S_r = \frac{\forall_r}{\forall_t} \tag{3-5}$$

where $\forall_r$ is the volume retained, and $\forall_d$ is the volume drained from the total volume $\forall_t$. Table 3-1 lists typical values of specific yield and specific retention.

3-3 Principles of Groundwater Flow

Water can flow through all natural materials. However, the velocity at which it can move is inversely proportional to the size of the openings through which it moves. Figure 3-3 illustrates a situation in which water from one reservoir is flowing to another through a conduit filled with a permeable material. If the energy equation* is written between point 1 and a point inside the pipe, the following equation arises:

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V^2}{2g} + \frac{p}{\gamma} + z + h_L \tag{3-6}$$

where V = average velocity within the conduit at the point of interest (V is further defined as Q/A, where Q is the flow rate through the conduit, and A is the cross-sectional area of the conduit.)

V_1 = the average velocity across a section at point 1

p = pressure at the point of interest in the conduit

p_1 = the pressure at point 1 (atmospheric)

z = the elevation of the centerline of the conduit at x

* This form of the one-dimensional energy equation should be familiar from the student's first course in fluid mechanics. It is presented again in a different content in Sec. 5-2, page 241.

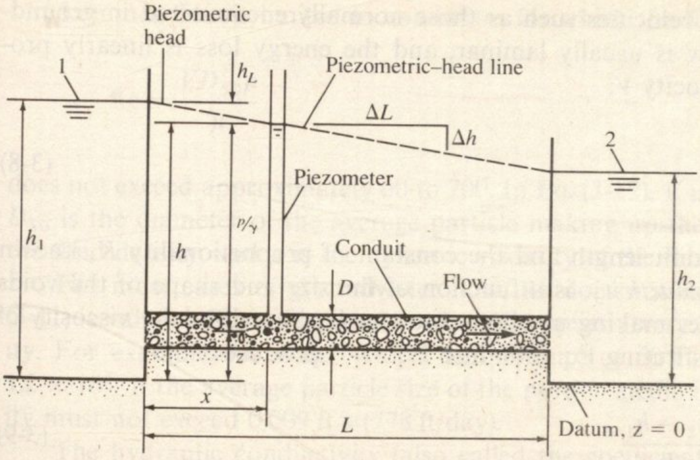


Figure 3-3 Flow through a conduit filled with permeable material

z_1 = the elevation of point 1
g = acceleration due to gravity
γ = specific weight of the flowing fluid
h_L = energy loss which occurs as a result of the flow between the two points

For most flow through porous media, the average water velocity is very small (in the range of inches or feet per day). Thus, the terms involving the square of the velocity are also very small and, because the other terms in the equation are much larger by comparison, can usually be neglected.

The term $(p/\gamma + z)$ is referred to as the piezometric head, and we shall use h to represent it. The piezometric head is the distance above the datum to which the water surface would rise if free to do so (See Fig. 3-3). Thus, if a vertical tube (a piezometer) is connected to the horizontal pipe as shown in Fig. 3-3, the water in the tube will rise until its free surface is p/γ above the centerline of the pipe. The broken line in Fig. 3-3 represents the piezometric head for the water in the pipe between reservoirs 1 and 2. We will consider piezometric head further as we examine cases of groundwater flow.

If the velocity terms are neglected in Eq. 3-6, we have (between points 1 and 2)

$$z_1 - z_2 = h_1 - h_2 = h_L \tag{3-7}$$

since $p_1 = p_2$ = atmospheric pressure.

For very small velocities such as those normally encountered in groundwater flow, the flow is usually laminar, and the energy loss is linearly proportional to the velocity V:

$$h_L = \frac{LV}{K} \tag{3-8}$$

where L is the conduit length and the constant of proportionality K, known as the *hydraulic conductivity,* is a function of the size and shape of the voids between the particles making up the porous media as well as the viscosity of the water (13). Substituting Eq. (3-8) into Eq. (3-7) produces

$$V = -K\frac{h_2 - h_1}{L} \tag{3-9}$$

Equation (3-9) can be written as

$$V = -K\frac{dh}{dL} \tag{3-10}$$

because the derivative dh/dL (the gradient of the piezometric head) is a constant for the situation shown in Fig. 3-3, where the average velocity V is constant throughout the length of the tube.

Using the principle of continuity, Eq. (3-10) can be written as

$$Q = -KA\frac{dh}{dL} \tag{3-11}$$

where Q = the total rate of flow through the cross-sectional area A

Equations (3-10) and (3-11) are two forms of Darcy's law. In 1856, Henri Darcy first concluded that velocity of flow through a porous media was directly proportional to the gradient of piezometric head (hydraulic gradient) (8). The negative sign arises because we have assumed the positive direction of velocity is in the direction of decreasing piezometric head.

As we noted, Eq. (3-8), and thus Darcy's law, Eq. (3-11), are valid only for laminar flow. In developing Eq. (3-11), we have defined the velocity V as the average velocity calculated as if the fluid were flowing throughout the entire cross-sectional area occupied by both solid particles and the openings between particles. Defined this way, average velocity is sometimes referred to as the Darcy velocity. The actual velocity of flow in the tortuous path formed by the interconnected openings would be larger than the Darcy velocity because the cross-sectional area formed by the openings between particles is much smaller than the total area of the conduit. The cross-sectional area of voids can be calculated by multiplying the porosity by the conduit cross-sectional area.

Laminar flow is limited to conditions for which the Reynolds number

$$\mathrm{Re} = \frac{VD_{50}\rho}{\mu} \tag{3-12}$$

does not exceed approximately 60 to 700. In Eq. (3-12), V is the Darcy velocity, D_{50} is the diameter of the average particle making up the porous media, ρ is the fluid density, and μ is the dynamic viscosity of the fluid.

For most cases of groundwater flow, Re does not exceed unity and Eq. (3-11) is valid. A Reynolds number of unity implies a very small average velocity. For example, for water with a temperature of 60°F, the ratio of $\rho/\mu = 1.2 \times 10^5$. If the average particle size of the porous media is 1/64 in., the velocity must not exceed 0.009 ft/s (778 ft/day).

The hydraulic conductivity (also called the coefficient of permeability or simply permeability) has the dimensions of a velocity and is generally thought of as a measure of the permeability or impermeability of a porous material. Table 3-1, page 111, lists typical values of hydraulic conductivity for various natural materials and shows that a wide variation can exist for all materials depending on their state. Basalt, for example, can vary in hydraulic conductivity from 10^{-8} to 1000 m per day. The lower value is applicable to unfractured basalt in its natural state. Because rock such as basalt is frequently fractured as a result of weathering, cooling, or other geologic processes, openings and passageways develop that provide paths with much less resistance to water movement than is provided by the voids in the unfractured material. A large value of hydraulic conductivity is typical of a highly fractured rock.

Expansive soils, such as some clays, also exhibit wide ranges in hydraulic conductivity depending on their state. When the clay is very dry, cracks may form that provide paths along which water readily flows, and the resulting hydraulic conductivity can be large. Once the clay becomes wet again, it may swell significantly, reducing the size of the interstitial openings and greatly reducing the hydraulic conductivity.

The viscosity of groundwater has a definite effect on the hydraulic conductivity, as might be expected for laminar flow. Generally, values of hydraulic conductivity are given for a temperature of 20°C (68°F). The hydraulic conductivity is approximately proportional to viscosity, and values of K at temperatures other than 20°C can be computed by

$$K_t = K_{20}\left(\frac{\mu_{20}}{\mu_t}\right) \tag{3-13}$$

where t and 20 refer to the temperature of interest and 20°C, respectively.

Darcy's law is valid for most conditions of groundwater flow. However, for some cases where the openings between particles are very small, such as in dense clays, the effects of relative electrical charges between clay and water

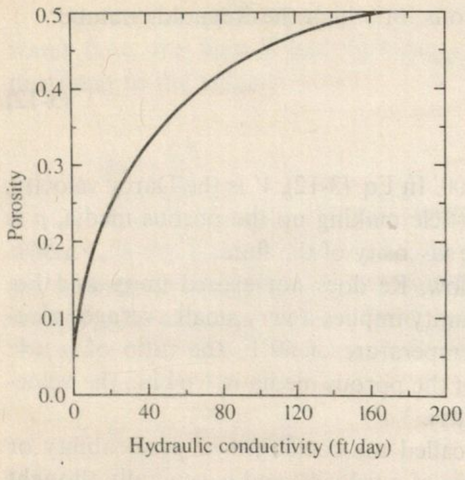


Figure 3-4 Variation of hydraulic conductivity with porosity (17)

molecules can be important, and a nonlinearity between flow rate and hydraulic gradient may exist.

For a given material, the hydraulic conductivity tends to decrease as porosity decreases. This is illustrated in Fig. 3-4, which shows the variation of hydraulic conductivity of mixed-grain sand for different porosities. The data for Fig. 3-4 were developed by measuring the permeability for given sands with varying degrees of vibratory compaction (17). In the compaction process, the volume of solids remains constant, but the volume of the voids decreases with additional compaction.

Figure 3-3 is schematically similar to a device commonly used to measure hydraulic conductivity in the laboratory. However, it is difficult, if not impossible, to obtain "undisturbed" samples of natural material that can be tested in the laboratory. Hydraulic conductivities for use in analyzing the motion of groundwater in its natural condition are usually determined by field pumping tests, which we will describe later. The so-called undisturbed samples are obtained by driving a cylindrical tube into the material to retrieve a core or by core drilling in the case of rock.

3-4 One-Dimensional Steady Groundwater Flow

Equations (3-9), (3-10), and (3-11) can be readily applied to steady one-dimensional groundwater flows. Figure 3-5 illustrates a situation in which an aquifer has been fully penetrated by a trench. The river is assumed to maintain a constant head (or water-surface elevation) h_1. Water is to be pumped at a constant rate Q to maintain a constant head h_2. The thickness of the aquifer

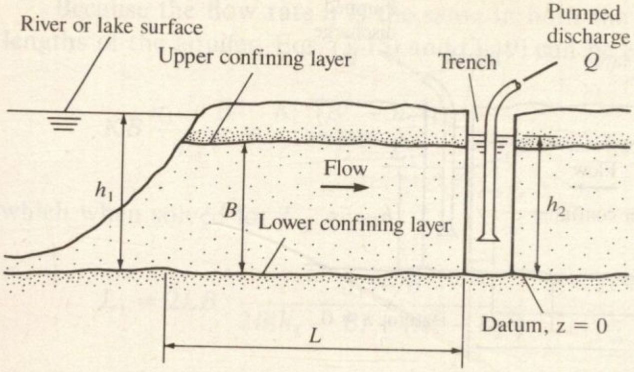


Figure 3-5 Flow to a trench through a confined aquifer

is B, and we assume that the layers above and below the aquifer are level and have a much smaller hydraulic conductivity than the aquifer. The datum $z = 0$ is the elevation of the lower confining layer. Thus, the aquifer is known as a *confined aquifer*.

Application of Eq. (3-11) gives

$$q = -KB\frac{h_2 - h_1}{L} \tag{3-14}$$

where q is the pumping rate per unit (foot or meter) of trench length. The total pumped discharge Q is equal to q multiplied by the length of the trench. *Note*: The area through which flow occurs is simply B for a unit width of the aquifer.

This solution assumes the aquifer thickness B is constant, water flows only from the aquifer on the river side of the trench, the hydraulic conductivity of the porous material within the aquifer is constant throughout, and nonuniform flow conditions near the trench and the river occur over a distance much smaller than L. Thus, the flow is assumed to be essentially one-dimensional through the aquifer.

If the aquifer is not fully confined (the groundwater surface does not touch the upper confining layer throughout), as in Fig. 3-6 on the next page, Eq. (3-11) can still be used to analyze the flow. Partially confined aquifers are typically encountered at construction sites where groundwater is being pumped to maintain dry conditions within an excavation, a process called dewatering.

The flow through the aquifer between the river and the point where the aquifer no longer flows full (the portion that is confined) is analyzed in a manner similar to that used for Fig. 3-5. The following equation arises:

$$q = KB\frac{h_1 - B}{L_1} \tag{3-15}$$

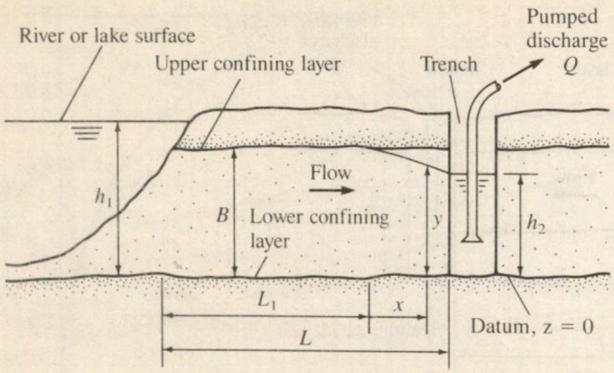


Figure 3-6 Flow to a trench through a partially confined aquifer

The form of the gradient term in Eq. (3-15) arises since the piezometric head at L_1 is equal to the aquifer depth B. To analyze flow along the remaining length of the aquifer where the velocity is not constant, the differential form of Darcy's law, Eq. (3-10) must be used:

$$q = VA = -KA\frac{dh}{dx} \tag{3-16}$$

In this case, the area A (with unit width) is a variable equal to y. The piezometric head h at a distance x from the section at L_1 is likewise equal to y if the bottom of the aquifer is assumed to be horizontal.

Thus, Equation (3-16) becomes

$$q = -Ky\frac{dy}{dx} \tag{3-17}$$

which can be integrated over the distance from L_1 to L as

$$q\int_{L_1}^{L} dx = -K\int_{B}^{h_2} y\,dy \tag{3-18}$$

which upon integration becomes

$$q = K\frac{B^2 - h_2{}^2}{2(L - L_1)} \tag{3-19}$$

showing that the surface of the saturated portion of the aquifer between L_1 and L is parabolic.

Because the flow rate q is the same in both the confined and unconfined lengths of the aquifer, Eqs. (3-15) and (3-19) can be equated, giving

$$KB\frac{h_1 - B}{L_1} = \frac{K}{2} \cdot \frac{(B^2 - h_2^{\,2})}{L - L_1} \tag{3-20}$$

which when solved for L_1, gives

$$L_1 = 2LB \cdot \frac{h_1 - B}{2B(h_1 - B) + (B^2 - h_2^{\,2})} \tag{3-21}$$

Substituting Eq. (3-21) into Eq. (3-15) gives

$$q = \frac{K}{2L}\left[2B(h_1 - B) + (B^2 - h_2^{\,2})\right] \tag{3-22}$$

In the analysis of flow in Fig. 3-6, we have made the same assumptions as were used in developing Eq. (3-14). Moreover, we have assumed that the flow is still essentially one-dimensional in the unconfined zone, a valid assumption as long as velocities are small and the slope dy/dx is not large.

The foregoing analysis of one-dimensional groundwater flows is applied to the following two examples, in which the bottom of the aquifer is at constant elevation. A sloping aquifer can be analyzed similarly if it is recognized that h in Eqs. (3-10) and (3-11) is in reality the piezometric head $(p/\gamma + z)$. Thus, Eq. (3-14) can be applied to the situation shown in Fig. 3-7 to yield

$$q = -KB\frac{h_2 + z_2 - h_1 - z_1}{L} \tag{3-23}$$

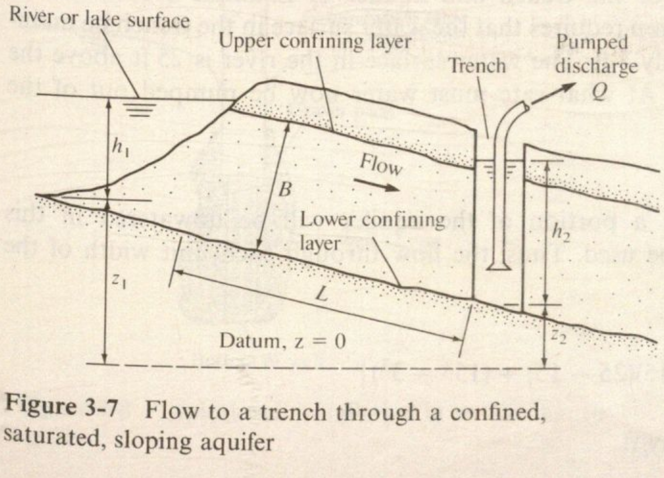


Figure 3-7 Flow to a trench through a confined, saturated, sloping aquifer

EXAMPLE 3-1 A trench 1000 ft long is to be excavated parallel to and 800 ft from a river. An aquifer, similar to that shown in Fig. 3-5, exists and is known to have a hydraulic conductivity of 15 ft per day and a depth of 15 ft. If the water level in the trench must be kept 10 ft below the water level in the river (but still above the top of the aquifer), determine the rate at which water must be pumped from the trench.

SOLUTION Since the aquifer is confined throughout its length, Eq. (3-14) is applicable. Thus, the flow through each unit width of the aquifer is

$$q = -15(15)\frac{-10}{800}$$

$$= 15(15)\frac{10}{800} = 2.81 \text{ ft}^3/\text{day}$$

The rate at which water must be pumped from the trench is

$$Q = 2.81(1000) = 2810 \text{ ft}^3/\text{day}$$

$$= 0.0325 \text{ ft}^3/\text{s}$$

$$= 14.6 \text{ gal/min}$$

Note: This solution assumes no flow from that portion of the aquifer on the side of the trench away from the river. Such a situation would occur only after a long period of pumping when the water table on the land side had subsided to a near equilibrium position at height h_2, or if a barrier, such as a sheet-piling wall, were constructed to prevent flow from that side of the trench. ■

EXAMPLE 3-2 For the trench and aquifer of Example 3-1, assume a change in the construction requires that the water surface in the trench be maintained at a depth of only 3 ft. The water surface in the river is 25 ft above the bottom of the aquifer. At what rate must water now be pumped out of the trench?

SOLUTION Since a portion of the aquifer will be unwatered in this case, Eq. (3-22) must be used. Thus, the flow through each unit width of the aquifer is

$$q = \frac{15}{2(800)}[2(15)(25 - 15) + (15^2 - 3^2)]$$

$$= 4.84 \text{ ft}^3/\text{day/ft}$$

The rate at which water must be pumped from the trench is

$$Q = 4.84(1000) = 4840 \text{ ft}^3/\text{day}$$
$$= 0.056 \text{ ft}^3/\text{s}$$
$$= 25 \text{ gal/min} \quad \blacksquare$$

3-5 Well Hydraulics

Wells are the principle means by which groundwater is extracted. Wells such as Fig. 3-8 are designed to penetrate an aquifer and to deliver a desired rate of water flow. Wells have been used throughout recorded history to obtain a water supply. In arid lands, villages were developed around wells that yielded a dependable water supply. For the most part, older wells were dug by hand and penetrated only shallow aquifers. Because shallow aquifers are easily polluted by infiltration of contaminated runoff, discharge from inefficient or faulty sewage-treating septic tanks, or leakage from underground storage tanks, many shallow wells have had to be abandoned and replaced by much deeper modern wells that penetrate aquifers carrying water of good quality. A modern well includes a hole that is normally drilled, a casing that prevents caving in of the

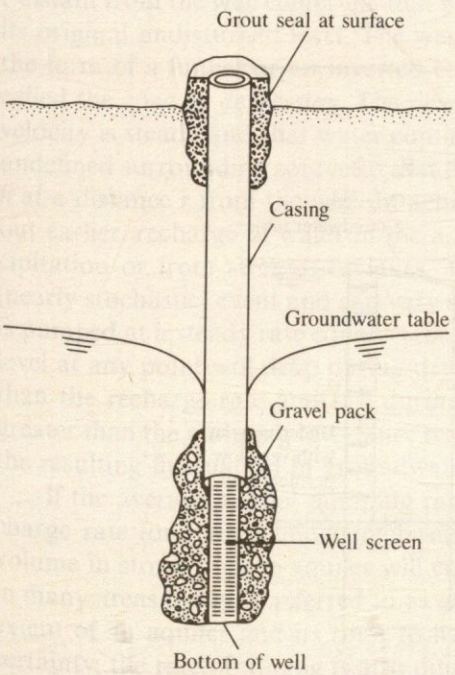


Figure 3-8 Typical well installation in an unconfined aquifer

sides or inflow of water from an undesirable aquifer, a pump and its appurtenances, and a section of perforated casing (well screen) that is located so that water is drawn from the desired level. Figure 3-8 shows a typical modern well installation. The grout seal at the surface prevents possibly contaminated surface runoff from following a low-resistance path along the casing and eventually reaching the source aquifer. The gravel pack around the screen provides a local zone of large hydraulic conductivity, which enhances flow to the well screen.

Unconfined Steady Flow

Equation (3-11) can be used to analyze axially symmetric flow into a well in a manner similar to that used to analyze the previous cases of one-dimensional flow. Figure 3-9 illustrates the flow conditions for a well that fully penetrates the aquifer. The velocity of the flow (Darcy velocity) is steady and equal to V at a distance r from the center of the well, and the depth of saturated flow at that point is h. The original depth of groundwater is h_o. The radius of the well is r_w, and the steady-state height of the water in the well is h_w. We will assume

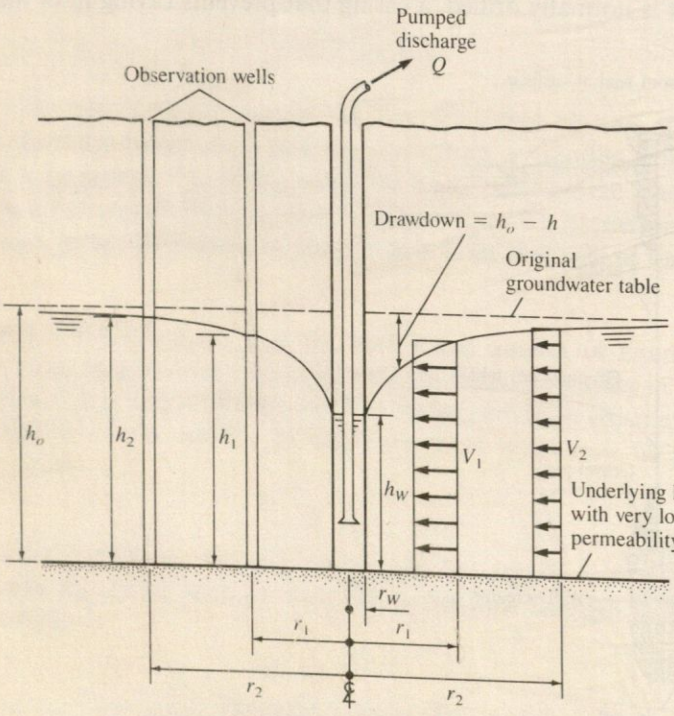


Figure 3-9 Axially-symmetric steady flow to a well in an unconfined aquifer

that the aquifer is homogeneous and underlain by an impervious layer, but unconfined on top, and that the flow is steady.

These wells are normally relatively shallow. Equation (3-11) becomes

$$Q = 2\pi K r h \frac{dh}{dr} \tag{3-24}$$

where the area A has been replaced by $2\pi rh$, and since the positive direction of r is opposite to the positive direction of velocity, the gradient dh/dr is equal to $-dh/dL$. Equation (3-24) can be integrated as

$$Q \int_{r_1}^{r_2} \left(\frac{1}{r}\right) dr = 2\pi K \int_{h_1}^{h_2} h\, dh \tag{3-25}$$

to yield

$$Q = \frac{\pi K(h_2{}^2 - h_1{}^2)}{\ln\left(\frac{r_2}{r_1}\right)} \tag{3-26}$$

The *drawdown* of the water table is $h_o - h$ and is the vertical distance at a point r distant from the well centerline that pumping has lowered the water table from its original undisturbed level. The water surface formed by the drawdown has the form of a funnel or an inverted cone as shown in Fig. 3-9. This surface is called the *cone of depression*. The development of Eq. (3-26) has assumed that velocity is steady and that water continually moves toward the well from some undefined surrounding source so that the water level remains at a constant level h at a distance r from the well. In actuality, this seldom occurs. As we pointed out earlier, recharge of water in the aquifer comes from the infiltration of precipitation or from streams or lakes. Thus, the infiltration is a very unsteady (nearly stochastic) event and can vary significantly from year to year. If the well is pumped at a steady rate equal to the average recharge rate, the average water level at any point will drop during dry periods when the pumping rate is more than the recharge rate and rise during wet periods when the recharge rate is greater than the pumping rate. Since recharge is a slow process for most aquifers, the resulting fluctuation in groundwater levels is also a slow process.

If the average annual pumping rate is greater than the average annual recharge rate for the groundwater basin, the level of the groundwater and the volume in storage in the aquifer will continuously fall. This situation, common in many areas today, is referred to as *groundwater mining*. Because the absolute extent of an aquifer and its total recharge rate are difficult to determine with certainty, the rate of mining is also difficult to determine. In general, Eq. (3-26) gives reasonably accurate results if the maximum drawdown is not more than about one half the aquifer depth.

If Eq. (3-26) were used to estimate the water depth in the well, substituting r_w for r_1, then headloss due to flow through the well screen would need to be considered. A headloss occurs as water flows from the aquifer through the well screen and into the well. If r_w is substituted for r in Eq. (3-26) an estimate is obtained for the groundwater depth immediately outside the well casing. To obtain an estimate of the water depth in the well for a given Q, it is necessary to subtract the headloss from h_w the water depth outside the well. Figure 3-9 does not show this headloss. To size the pump to be used for the required rate of flow Q, it is necessary to subtract the headloss due to flow through the screen from the quantity h_w to determine the water surface elevation within the well.

To use Eq. (3-26) to estimate the hydraulic conductivity of an aquifer, information obtained during a pumping test is required. A well is constructed as shown in Fig. 3-8, and one or two smaller observation wells are drilled at different distances from the pumped well, such as r_1 and r_2 as shown in Fig. 3-9. The water levels are observed and recorded before pumping at a known rate begins, and the approximately steady-state values achieved after continuous pumping are substituted into Eq. (3-26) to obtain estimates of K. However, because the shape of the drawdown surface usually varies with time, values of K determined through using Eq. (3-26) must be considered approximations. If the rate of decline of the groundwater levels becomes slow, the indicated value of hydraulic conductivity can be reasonable. However, if the groundwater levels continue to fall rapidly, the flow is obviously very unsteady, and errors in the value of hydraulic conductivity, as determined from the steady-state equations, can be significant. Experience has shown that a concept called the radius of influence can often be invoked where the surface of the groundwater table at an appreciably large distance from the well (usually 500 r_w or approximately 1000 ft) is assumed to be at a constant elevation. Values of Q, h_1, and h_2 (measured during pumping) are substituted into Eq. (3-26) to calculate the hydraulic conductivity. This assumption of a radius of influence provides reasonably accurate values of the hydraulic conductivity if the observation well located at r_2 is sufficiently far from the pumped well and if conditions are such that the observed water table at r_2 is declining at a very slow rate.

The development of Eq. (3-26) tacitly assumed that the velocity is always horizontal, which is definitely not the case near the well. Moreover, velocities become large near the well and may actually reach turbulent rather than laminar-flow conditions, with the result that resistance to flow is greater, and the gradient dh/dr of the water table can be much steeper than is given by Darcy's equation. Thus, drawdown outside the well casing is always greater than that predicted by Eq. (3-26) although using the equation gives a reasonable approximation for an aquifer having large hydraulic conductivity such as would occur for coarse gravels. Using Eq. (3-26) to calculate ordinates of the groundwater surface will not yield accurate values for points near the well. However, for values of r greater than approximately $1.5h_o$, using Eq. (3-26) will usually provide satisfactory values for groundwater elevations.

EXAMPLE 3-3 A 48-in. diameter well ($r_w = 2$ ft) is drilled as shown in Fig. 3-9. The water table is initially at a depth of 300 ft above the bottom of the aquifer, which has a hydraulic conductivity of 0.80 ft/day. If 100 gal/min (19,251 ft^3/day) is pumped from the well, what will be the depth of water in the well if the drawdown is essentially zero in the observation well 1000 ft from the well? What is the drawdown at a distance of 300 ft from the well?

SOLUTION The aquifer is unconfined, so Eq. (3-26) can be used. Equation (3-26) is first solved for h_1. Because we have only one observation well, h_w must then be substituted for h_1. Thus, the depth in the well will be approximately

$$h_w = \left[(300)^2 - \left(\frac{19{,}251}{0.80\,\pi}\right)\ln\left(\frac{1000}{2}\right)\right]^{1/2} = 206 \text{ ft}$$

Note: The flow rate has been converted to ft^3/day for use in Eq. (3-26) with the hydraulic conductivity in ft/day. The actual value of h_w will be somewhat less than 206 ft because of headloss through the screen.

Equation (3-26) can be rewritten to calculate the drawdown at 300 ft from the well.

$$h_o^2 - h^2 = \frac{Q}{\pi K}\ln\left(\frac{r_o}{r}\right)$$

Thus, $$h_o^2 - h^2 = \frac{19{,}251}{0.80\,\pi}\ln\left(\frac{1000}{300}\right) = 9222$$

$$h = [(300)^2 - 9222]^{1/2} = 284 \text{ ft}$$

The drawdown at 300 ft from the well is

$$h_o - h = 300 - 284 = 16 \text{ ft}$$ ■

Confined Steady Flow

Equation (3-11) can also be used to analyze the behavior of the hydrostatic pressures (piezometric head) in a confined aquifer under pressure (artesian conditions), such as that illustrated in Fig. 3-10 on the next page. In this case, the aquifer is confined both above and below by essentially impermeable layers. Equation (3-11) becomes

$$Q = 2\pi KBr\,\frac{dh}{dr} \tag{3-27}$$

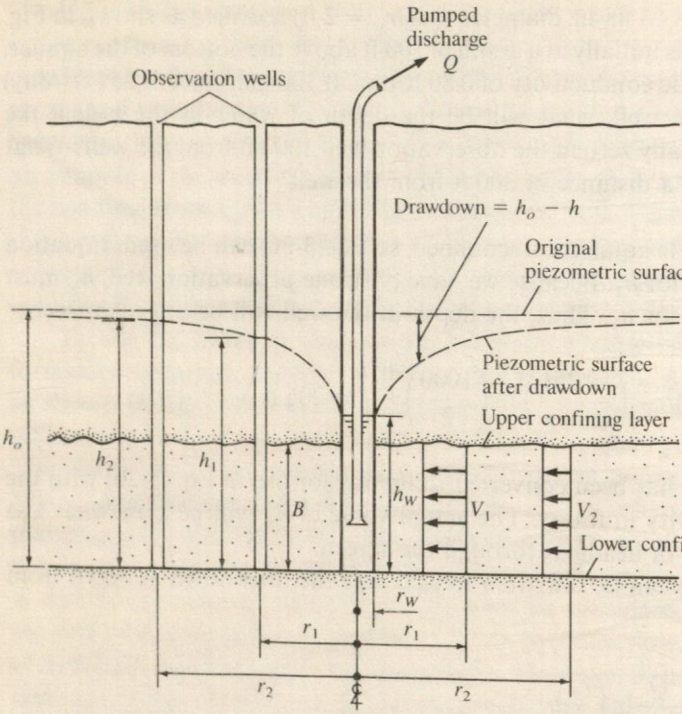


Figure 3-10 Axially symmetric steady flow to a well in a confined aquifer

which can be integrated as

$$Q \int_{r_1}^{r_2} \left(\frac{1}{r}\right) dr = 2\pi KB \int_{h_1}^{h_2} dh \tag{3-28}$$

or

$$Q = 2\pi KB \frac{h_2 - h_1}{\ln \dfrac{r_2}{r_1}} \tag{3-29}$$

where h_2 and h_1 refer to the piezometric heads at r_2 and r_1, respectively.

In this case, we have assumed that the depth of saturation does not change after pumping. Only the piezometric head declines in the area around the well, and we assume that it does not fall below the top of the aquifer. For the confined aquifer the drawdown $h_o - h$ forms a geometric surface which is the top of the piezometric head. The inverted cone formed by the drawdown is *analogous to the cone of depression* formed by the groundwater surface in Fig. 3-9, page 122.

In actuality, if pumping takes place at a rate greater than that at which recharge of the aquifer occurs, portions of the aquifer could be dewatered, and the conditions assumed for development of Eq. (3-29) would no longer exist.

However, a case like this can be at least approximately analyzed using an approach similar to that used to develop Eq. (3-22) for the flow conditions shown in Fig. 3-6, page 118.

Frequently, large reductions in piezometric head within the aquifer are produced by intensive groundwater pumping. The reduction in pressure results in compression of the aquifer by the weight of overburden, which originally was partially supported by the groundwater under pressure. For very thick aquifers, the amount of compression may be large and can result in significant subsidence of the land surface. Subsidence of more than 30 ft has been measured in areas such as the San Joaquin valley in California where extensive pumping has taken place (10). The withdrawal of oil from oil-bearing strata has produced the same effect in some locations, such as near Long Beach, California, where subsidence of the coastal area has been enough to result in flooding by Pacific Ocean tides (10).

EXAMPLE 3-4 Determine the hydraulic conductivity of a confined aquifer from which water is being pumped by a fully penetrating well. The aquifer is 100 ft thick, and the well is being pumped at a rate of 1500 gal/min. Water surfaces in two observation wells 700 ft and 70 ft from the pumped well are respectively 1 ft and 10 ft below their levels prior to the beginning of pumping.

SOLUTION Since the aquifer is fully confined, Eq. (3-29) may be rearranged to obtain

$$K = \frac{Q \ln (r_2/r_1)}{2\pi B(h_2 - h_1)}$$

In order to maintain consistent units, the 1500 gal/min must be divided by 7.48 gal/ft^3.

$$\text{Thus,} \quad K = \frac{(1500/7.48) \ln (700/70)}{2\pi(100)(9)} = 0.0816 \text{ ft/min} = 118 \text{ ft/day} \quad \blacksquare$$

Unsteady Well Hydraulics

As we noted in the previous section, drawdown of an aquifer is not a steady-state occurrence. Even in very infrequent situations where all the conditions assumed for the development of Eq. (3-26) exist, a long period of pumping would be required before the cone of depression around the well would reach a steady-state shape. To obtain accurate estimates of hydraulic conductivity for conditions where the flow is appreciably unsteady, analyzing the flow to the well under unsteady conditions is necessary. Theis analyzed this situation in 1935 (18).

Equation (3-11) is not valid for this situation, since h is a function of time, as is A for the case of an unconfined aquifer. Instead, Darcy's law must be expressed as a partial differential equation including time as a variable. The partial differential equation cannot be solved exactly, but for a confined aquifer, fully penetrated by a well, a solution can be obtained in the form of an exponential integral and can be expressed as follows (18):

$$h_o - h = \left(\frac{Q}{4\pi KB}\right) W(u) \tag{3-30}$$

where $h_o - h$ = drawdown at a point r distance from the well
Q = pumping rate
B = aquifer thickness
K = hydraulic conductivity
$W(u)$ = a dimensionless mathematical function called the well function

The well function $W(u)$ which is dimensionless can be expressed in a series as

$$W(u) = -0.577216 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \cdots \tag{3-31}$$

where the argument u is

$$u = \left(\frac{S}{4KB}\right)\frac{r^2}{t} \tag{3-32}$$

and r = distance between the pumped and observation wells
t = time after pumping starts
S = a dimensionless constant called the *storage coefficient*

In Eqs. (3-30) and (3-32), the units for K, Q, S, r, and t must be chosen consistently. Thus, if Q is measured in ft^3/sec, K should be measured in ft/sec, r measured in ft, t measured in sec, and B in ft. Various forms of Eq. (3-30) and (3-32) have been published where the units are not consistent, and constants of proportionality are required.

The dimensionless constant S, called the storage coefficient, is defined as the volume of water yielded by a prism of the aquifer having a projected unit area in the horizontal plane if withdrawal is sufficient to cause a unit drop in the hydrostatic head. For an unconfined aquifer, the drained water comes primarily from gravity drainage of the aquifer, and for this case, the storage coefficient is essentially equal to the specific yield Sy.

For the case of a confined aquifer, the volume of water draining out to produce a unit drop in head comes entirely from expansion of the water and compression of the aquifer. The volume expansion of water for a unit change in head is very small. For a confined aquifer having a porosity of 0.2 and containing water at 15°C, approximately 3×10^{-7} m^3 of water are forced out of 1 m^3 of aquifer for a 1 m decline in head. If the aquifer were 100 m thick, the storage coefficient would be

$$S = 100(3 \times 10^{-7}) = 3 \times 10^{-5}$$

The storage coefficient of most aquifers ranges from 0.00001 to 0.001. The more compressible the aquifer, the larger the value of the storage coefficient.

In general, it is necessary to solve Eqs. (3-30), (3-31), and (3-32) for the hydraulic conductivity K and the storage coefficient S for a set of measurements of h at a given distance r from the well for periods of time t after pumping begins at a constant rate Q. Theis devised a convenient graphical method by which the solution can be achieved. The graphical solution utilizes the fact that the terms in parentheses in Eqs. (3-30) and (3-32) are constants for a given aquifer and an observation well. As a result, graphs of $W(u)$ versus u and $(h_o - h)$ versus (r^2/t) will have geometrically similar shapes. In the graphical solution, graphs of $W(u)$ versus u are prepared using Eq. (3-31) or Table 3-2, which presents calculated values of $W(u)$ as functions of u. The measured values of $h_o - h$, r, and t are used to separately plot the graph of $(h_o - h)$ versus (r^2/t). The two graphs are then superimposed and shifted as necessary, holding the coordinate axes parallel, until the curves are coincident. A point on the coincident curves is selected, and numerical values of $W(u)$, u, $h_o - h$, and r^2/t are

Table 3-2 Values of $W(u)$ (6)

u	**1.0**	**2.0**	**3.0**	**4.0**	**5.0**	**6.0**	**7.0**	**8.0**	**9.0**
$\times 1$	0.219	0.049	0.013	0.0038	0.0011	0.00036	0.00012	0.000038	0.00
$\times 10^{-1}$	1.82	1.22	0.91	0.70	0.56	0.45	0.37	0.31	0.26
$\times 10^{-2}$	4.04	3.35	2.96	2.68	2.47	2.30	2.15	2.03	1.92
$\times 10^{-3}$	6.33	5.64	5.23	4.95	4.73	4.54	4.39	4.26	4.14
$\times 10^{-4}$	8.63	7.94	7.53	7.25	7.02	6.84	6.69	6.55	6.44
$\times 10^{-5}$	10.94	10.24	9.84	9.55	9.33	9.14	8.99	8.86	8.74
$\times 10^{-6}$	13.24	12.55	12.14	11.85	11.63	11.45	11.29	11.16	11.04
$\times 10^{-7}$	15.54	14.85	14.44	14.15	13.93	13.75	13.60	13.46	13.34
$\times 10^{-8}$	17.84	17.15	16.74	16.46	16.23	16.05	15.90	15.76	15.65
$\times 10^{-9}$	20.15	19.45	19.05	18.76	18.54	18.35	18.20	18.07	17.95
$\times 10^{-10}$	22.45	21.76	21.35	21.06	20.84	20.66	20.50	20.37	20.25
$\times 10^{-11}$	24.75	24.06	23.65	23.36	23.14	22.96	22.81	22.67	22.55
$\times 10^{-12}$	27.05	26.36	25.96	25.67	25.44	25.26	25.11	24.97	24.86
$\times 10^{-13}$	29.36	28.66	28.26	27.97	27.75	27.56	27.41	27.28	27.16
$\times 10^{-14}$	31.66	30.97	30.56	30.27	30.05	29.87	29.71	29.58	29.46
$\times 10^{-15}$	33.96	33.27	32.86	32.58	32.35	32.17	32.02	31.88	31.76

recorded for that point. The aquifer parameters are then determined from the following equations derived from Eqs. (3-30) and (3-32):

$$\frac{Q}{4\pi KB} = \frac{h_o - h}{W(u)} \tag{3-33}$$

$$\frac{S}{4KB} = \frac{u}{r^2/t} \tag{3-34}$$

Rearranging Eqs. (3-33) and (3-34) yields

$$KB = \frac{Q}{4\pi}\left(\frac{W(u)}{h_o - h}\right) \tag{3-35}$$

and
$$S = 4KB\left(\frac{u}{r^2/t}\right) \tag{3-36}$$

In these equations, the product KB is frequently used as a single variable called *transmissivity*. Special forms of the Theis method have been developed for wells that only partially penetrate the aquifer, for wells in unconfined aquifers, and for wells in leaky confined aquifers (3, 5, 8, 11, and 20).

A solution for unsteady flow to a well in an unconfined aquifer is difficult because the depth of groundwater flow is not constant but decreases with both time and nearness to the well. Moreover, the streamlines become steeper near the well, making the velocity deviate from the horizontal. If, however, the drawdown $h_o - h$ is small compared to h_o, the Theis method gives approximate but adequate results for the unconfined aquifer.

A more simple version of Eq. (3-30) can be written for the particular case where u has a value less than 0.01. For this condition only, the first two terms of Eq. (3-31) are important. Substituting those two terms into Eq. (3-30) and substituting Eq. (3-32) for u gives

$$h_o - h = \frac{Q}{4\pi KB} \ln \frac{2.25KBt}{r^2 S} \tag{3-37}$$

Equation (3-37) can be satisfactorily used to directly evaluate the transmissivity KB from pumping-test data as long as values of u do not exceed 0.01, a condition that will be more common for very thick aquifers or aquifers having large values of hydraulic conductivity.

EXAMPLE 3-5 Determine the transmissivity (KB) and the storage coefficient S for a confined aquifer from which 500 gal/min is being pumped. The water level measurements shown in the accompanying table were made at an observation well 100 ft from the pumped well.

(1) Time		(2) Distance to Water Level	(3) Drawdown	(4)
(hr)	(min)	(ft)	(ft)	(r^2/t)
0	0	10.2	0	
1	60	11.7	1.5	166.67
2	120	13.5	3.3	83.33
4	240	24.0	13.8	41.67
6	360	31.5	21.3	27.78
8	480	37.8	27.6	20.83
12	720	46.7	36.5	13.89

SOLUTION Drawdown is computed as the distance from the initial water level, as column 3 of the accompanying table shows. Using the given data on time and drawdown and the given 100 ft radius, values of r^2/t are computed, as shown in column 4, for each measured water level. As shown in the second table values of the argument u are chosen arbitrarily. Values of $W(u)$ are calculated using Eq. (3-31) or, for the chosen values of the argument u, Table 3-2.

u	$W(u)$
0.1	1.823
0.2	1.226
0.3	0.906
0.4	0.702
0.5	0.560
0.6	0.457
0.8	0.310

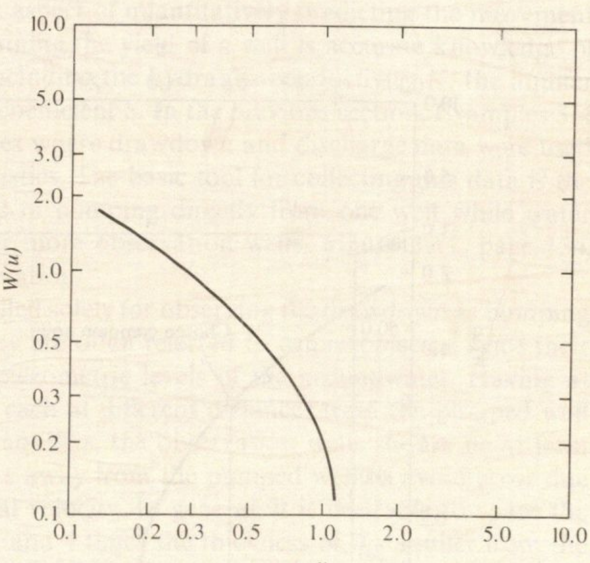


Figure A

The values of $W(u)$ and u are then plotted on log-log paper, as shown in Fig. A, whereas values of drawdown $(h_o - h)$ are plotted against r^2/t, as in Fig. B. The two figures are then superimposed and positioned until the curves closely coincide, as in Fig. C.

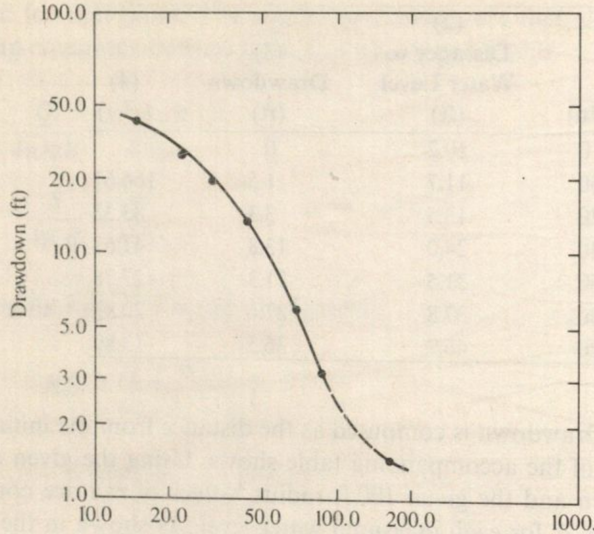


Figure B

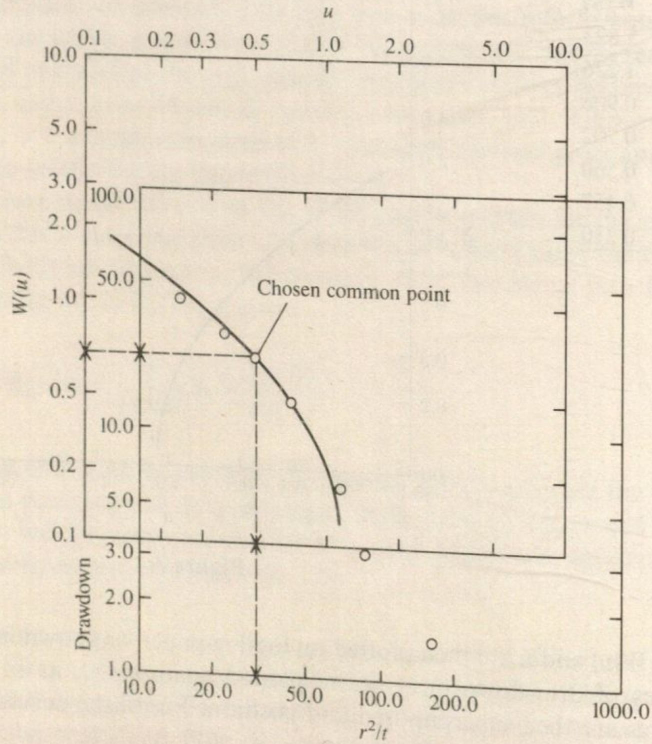


Figure C

For the common point shown in Fig. C, the following values of the variables are read

$$u = 0.500, \qquad W(u) = 0.59, \qquad \frac{r^2}{t} = 28.0, \qquad h_o - h = 22$$

Equation (3-35) then yields

$$KB = \frac{(500/7.48)(0.59/22)}{4\pi} = 0.143 \text{ ft}^2/\text{min}$$

and Eq. (3-36) gives

$$S = \frac{4(0.143)(0.500)}{28} = 0.010$$

■

3-6 Pumping Tests

The most important aspect of quantitatively predicting the movement of groundwater or in determining the yield of a well is accurate knowledge of the aquifer characteristics, including the hydraulic conductivity K, the aquifer thickness B, and the storage coefficient S. In the previous section, Examples 3-4 and 3-5 presented simple cases where drawdown and discharge data were used to estimate aquifer characteristics. The basic tool for collecting this data is the pumping test, which consists of pumping directly from one well while water levels are recorded in one or more observation wells. Figure 3-11, page 134, shows a pumping test in operation.

Observation wells are drilled solely for observing the drawdown as pumping of the test well proceeds. They are often referred to as piezometers, since their sole function is to monitor piezometric levels of the groundwater. Having at least three observation wells each at different distances from the pumped well is desirable. For unconfined aquifers, the observation wells should be at least 1.5 times the aquifer thickness away from the pumped well to avoid error due to the effects of nonhorizontal velocity. In general, it is desirable to space the wells approximately at 1.5, 2, and 4 times the thickness of the aquifer from the test well. Financial considerations will often limit the number of observation wells that can be constructed. In many cases, there may be existing wells nearby that can be used as observation wells.

Determining K, B, and S from pumping-test data is based on the following assumptions, most of which are also inherent in the development of the steady and unsteady well-hydraulic equations:

1. The aquifer is homogeneous, isotropic, and of infinite horizontal extent.
2. The flow in the aquifer is horizontal only.

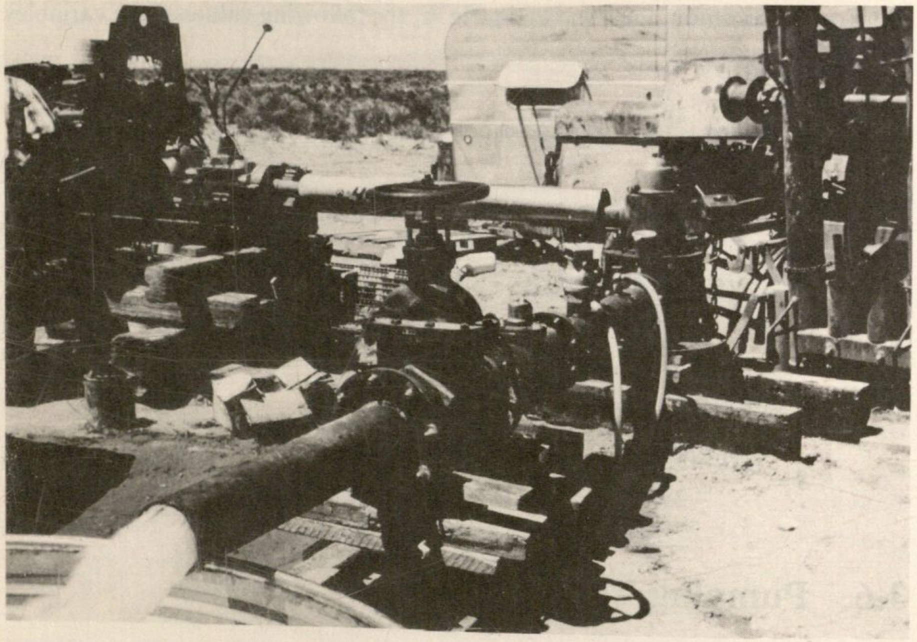


Figure 3-11 Pumping test in progress. Pump drive is in left background. Tank in foreground is used as volume measurement tank for use in measuring pumping rate. (Photo courtesy of U.S. Geological Survey, Boise, Idaho)

3. For unsteady conditions, water is released from storage in the aquifer in immediate response to a drop in piezometric head.
4. The only flow in the aquifer is that produced by the well.
5. Pumping is at a constant rate.
6. The volume of water initially in the well that is removed by pumping during unsteady conditions is much smaller than the volume coming from the aquifer.
7. The well completely penetrates the aquifer and is open to inflow throughout the entire depth of the aquifer.

For steady-state conditions Eqs. (3-26) and (3-29) are used to compute the hydraulic conductivity of the aquifer for unconfined and confined conditions, respectively. In theory, of course, the water levels in the observation wells will never reach equilibrium. However, they often approach steady-state conditions sufficiently close to yield accurate estimates of K. If at least two observation wells are used, an inherent advantage arises: Even though the water table is falling, the difference in drawdown between the two observation wells will be nearly constant. Under these conditions, Eqs. (3-26) or (3-29) will give reasonably constant values of KB for each of the simultaneous drawdown measurements.

Once KB has been determined using the steady-state equations, Eqs. (3-30), (3-31), and (3-32) can be used to calculate the storage coefficient S. With a measured value of drawdown for an observation well at a particular time, Eq. (3-30) can be solved for $W(u)$. Table 3-2 can then be entered with the value of $W(u)$ to obtain an associated value of u. Using the resulting value of u, the computed KB, and the associated values of r and t, Eq. (3-32) can then be solved for S.

The procedures developed by Theis, and demonstrated in Example 3-5, should be used whenever unsteady effects are important and the various drawdown observations do not produce consistent values of KB using the pertinent steady-state equations.

Several techniques have been developed for approximately evaluating local aquifer characteristics using so-called rate of use or slug tests. In these techniques, a volume of water is quickly removed from the well, and the rate of rise of the water surface in the well is carefully observed after the water removal. The rate of rise can then be related to the local value of the hydraulic conductivity. These techniques are much cheaper and quicker to perform than pumping tests and are valuable for use in preliminary investigations. Bouwer (2) describes these tests and their use in detail.

3-7 Recharging or Injection Wells

In areas where recharge of aquifers is desirable, it may sometimes be accomplished by excavation of surface material to expose the aquifer and directing surface runoff to it. Alternatively, a well may be used to inject water into the aquifer. Wells are frequently used for injecting water (and sometimes steam) into oil-bearing strata to aid in recovering oil and to prevent subsidence of the aquifer and the overlying ground. Figure 3-12, on the next page, shows a well in an unconfined aquifer into which water is being injected at a rate Q. Assume that the well fully penetrates the aquifer and that the screen is long enough to allow flow into the full depth of the unbounded aquifer. Darcy's law is used to analyze the steady-state flow as in Sec. 3-5, for the well pumping from the unconfined aquifer. The following equation arises from Eq. 3-26:

$$h = \left[h_w{}^2 - \frac{Q}{\pi K} \ln\left(\frac{r}{r_w}\right) \right]^{1/2} \tag{3-38}$$

Equation (3-38) can be used to calculate the coordinates of the mounding by solving the equation for h for chosen values of r. As was the case in using Eq. (3-26), the calculated ordinates will be approximate near the well, where the actual velocities are not horizontal. The rise of the groundwater surface at any point is $h - h_o$ and the resulting surface can be called the *cone of mounding*.

If pumping is stopped, the flow becomes unsteady and the mound gradually flattens out, approaching the level h_o after a theoretically infinite time. Obviously, for the real case of a bounded aquifer, the groundwater table will rise above h_o reflecting the volume of recharge.

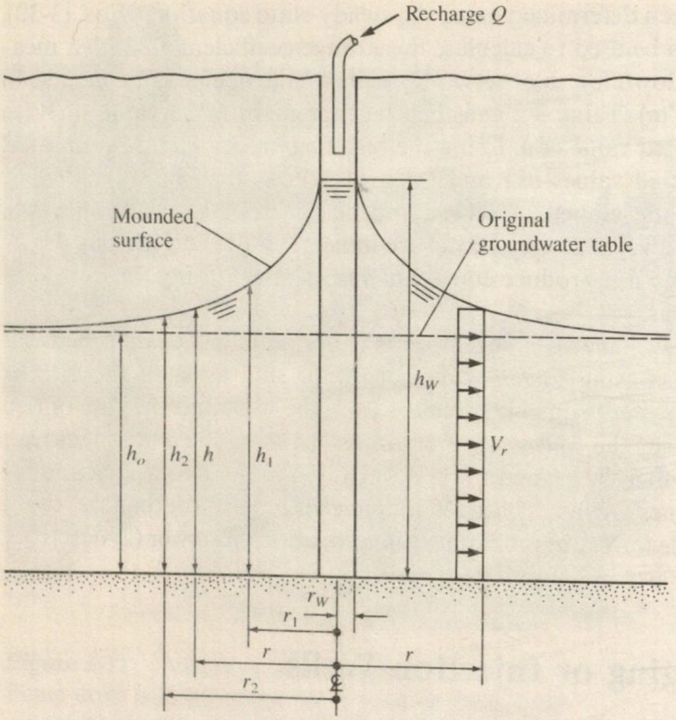


Figure 3-12 Flow from an injection or recharge well to an unconfined aquifer

Recharging a confined aquifer can be readily analyzed by the method used to analyze a pumping well. The following equation is formed:

$$h = h_w - \frac{Q}{2\pi KB} \ln\left(\frac{r}{r_w}\right) \tag{3-39}$$

where h again refers to piezometric head, as in Eq. (3-29).

3-8 Boundaries of Aquifers

In the analysis of steady and unsteady well hydraulics, the assumption was tacitly made that the aquifer extended an infinite distance radially in all directions. In actuality, although some aquifers are very large, none totally satisfy the assumption. All aquifers are bounded both in the horizontal and vertical directions. Layers of low-hydraulic conductivity can exist both above and below the aquifer and may effectively cut off vertical flow. Horizontal flow is affected

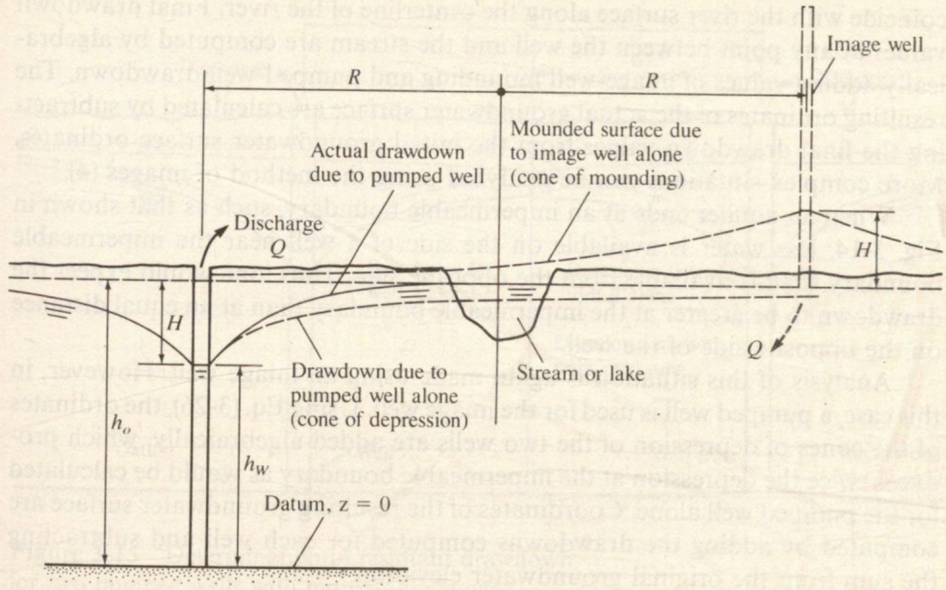


Figure 3-13 Analysis of drawdown for a pumped well near a recharging body of water

by at least two types of boundaries. A recharge boundary is one where water enters the aquifer providing a source of supply. Figure 3-13 illustrates a case where the aquifer is recharged by a perennial stream or lake. If a well is located as shown in Fig. 3-13, pumping will cause drawdown that will ultimately reach the river. The continuous pumping induces recharge from the stream, and the cone of depression cannot fall below the water surface at the stream as long as the streamflow is sufficient to satisfy flow to the aquifer. Such a stream is commonly referred to as a "losing" stream. In some cases, the stream can be dried up by flow to the aquifer.

Drawdown around a well near a recharging body of water can be analyzed using a technique known as the method of images (13). In the analysis, the river is treated as a line source, and a recharge well (negative image of a pumped well) is placed across the river and at the same distance from the river as the pumped well, as in Fig. 3-13. The final steady-state drawdown at any point is obtained by adding the drawdown calculated for the image well (negative) and the drawdown calculated for the pumped well. Since both wells are equidistant from the plane of symmetry (a vertical plane along the river), the cone of depression due to the pumped well is as far below the river surface as the cone of mounding due to the image well is above the river surface. Thus, when image-well and pumped-well drawdowns (calculated with Eqs. (3-38) and (3-26), respectively) are added, the calculated cone of depression due to the pumped well will coincide with the river surface along the centerline of the river. Final drawdown values at any point between the well and the stream are computed by algebraically adding values of image-well mounding and pumped-well drawdown. The resulting ordinates of the actual groundwater surface are calculated by subtracting the final drawdown values from the initial groundwater surface ordinates. More complex situations can be analyzed using the method of images (4).

When an aquifer ends at an impermeable boundary, such as that shown in Fig. 3-14, less water is available on the side of a well near the impermeable boundary than is available from the opposite side. Thus, one would expect the drawdown to be greater at the impermeable boundary than at an equal distance on the opposite side of the well.

Analysis of this situation is again made using an image well. However, in this case, a pumped well is used for the image well. Using Eq. (3-26), the ordinates of the cones of depression of the two wells are added algebraically, which produces twice the depression at the impermeable boundary as would be calculated for the pumped well alone. Coordinates of the resulting groundwater surface are computed by adding the drawdowns computed for each well and subtracting the sum from the original groundwater elevation.

3-9 Well Fields

If wells are constructed near one another, the drawdown of each well will be affected by the other wells in a manner that produces increased drawdown. The effect is illustrated in Fig. 3-15, where the drawdown curves of the wells overlap.

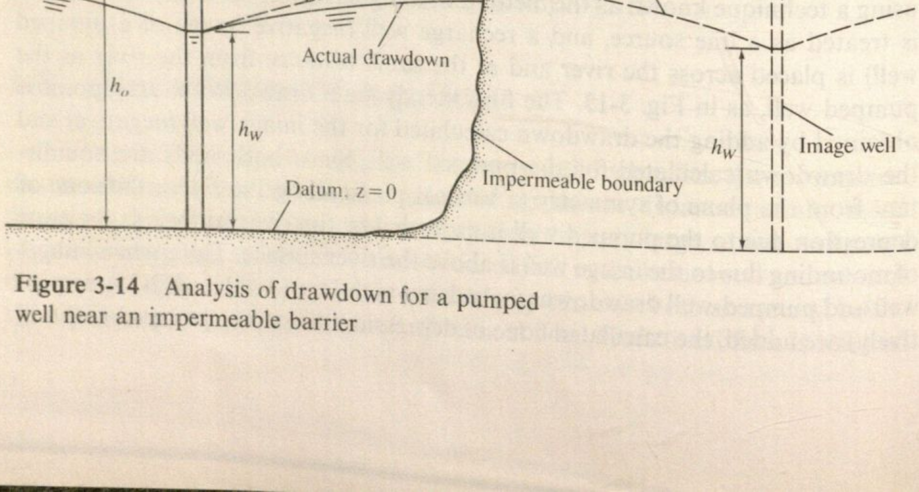


Figure 3-14 Analysis of drawdown for a pumped well near an impermeable barrier

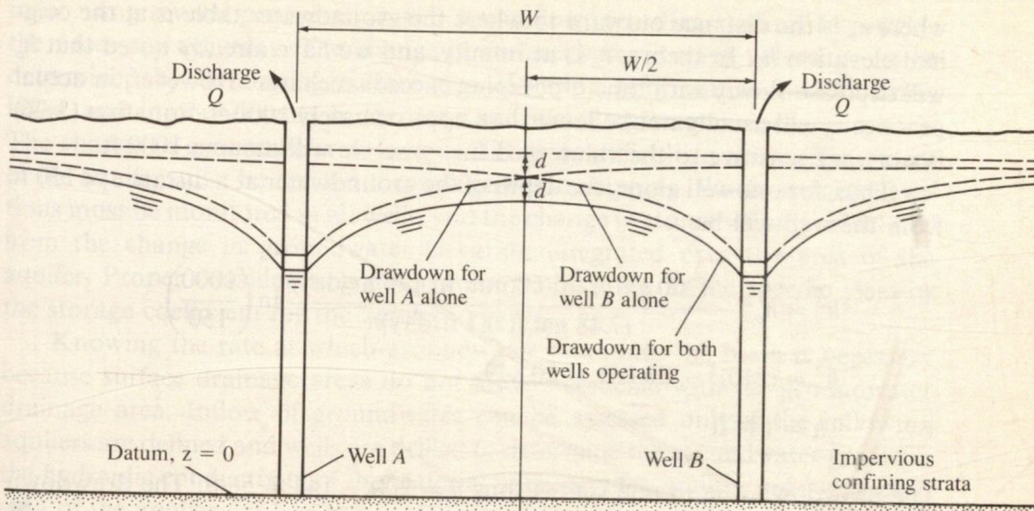


Figure 3-15 Determination of resultant drawdown for two pumped wells with mutual interference

To calculate the drawdown due to both wells at any point, an independent drawdown is calculated (using Eq. (3-26) for an unconfined aquifer or Eq. (3-29) for a confined aquifer) for each well neglecting the effect of any other well. The actual drawdown at any point is then calculated as the algebraic sum of the calculated independent drawdown values for that point. Fig. 3-15 graphically illustrates this process for two wells, but it can readily be applied to any number of wells. In general, the total drawdown predicted by this method gives accurate results provided the total drawdown does not exceed approximately one half the height of the original piezometric head. This restriction is usually stated because the combination of limitations assumed in using Darcy's law, Eq. (3-11), may be violated for greater drawdown.

EXAMPLE 3-6 Calculate the drawdown half way between the two wells shown in Fig. 3-15 if the wells are 300 ft apart, the original groundwater table is 210 ft above the bottom of the aquifer, and the hydraulic conductivity of the aquifer is 1.0 ft/day. Each well is pumped at a rate of 200 gal/min.

SOLUTION Equation (3-26) can be used to calculate the drawdown for each well. Thus, for one well, we can write

$$h_o^2 - h^2 = \frac{Q}{\pi K} \ln\left(\frac{r_o}{r}\right)$$

where r_o is the distance outward to where the groundwater table is at the original elevation h_o. In theory, r_o is at infinity, and we have already noted that h_o will decrease slowly with time if pumping exceeds recharge. However, in actual practice r_o will usually not be more than approximately 1000 ft. Equation (3-26) is not very sensitive to the value used for r_o, so we will assume 1000 ft.

Thus, for one well alone, the depth of the groundwater at a distance of 150 ft from the well will be

$$h^2 = h_o^{\ 2} - \frac{(200\text{ gal/min})(60\text{ min/hr})(24\text{ hr/day})}{(7.48\text{ gal/ft}^3)(1\text{ ft/day})\pi} \ln\left(\frac{1000}{150}\right)$$

$$h^2 = (210)^2 - 23{,}251 = 20{,}849$$

$$h = 144\text{ ft}$$

The drawdown due to one well is thus $d = 200 - 144 = 56$ ft. The drawdown due to the two wells will be $d + d = 56 + 56 = 112$ ft. ■

3-10 Groundwater Recharge and Safe Yield

As we mentioned, the extent of a groundwater aquifer is difficult to determine with certainty. A detailed study of the geologic structure must be made to determine if and where the aquifer, if it is confined, strikes the surface. Since some aquifers may extend for several hundred miles, such as the Ogallala aquifer in Kansas and Oklahoma, the outcrops may be at a great distance from the point of interest. Even confined aquifers receive some recharge from infiltration at the earth's surface, part of which will eventually leak through the upper confining strata and into the aquifer.

A water balance equation for all water in a drainage area can be written as

$$Q_P \cdot \Delta t = (Q_G - Q_s - E_T \cdot A_s) \cdot \Delta t + P \cdot A_s - \Delta S_G - \Delta S_S \qquad (3\text{-}40)$$

where Q_P = average rate of groundwater withdrawal from the aquifer during the time interval Δt

P = average precipitation depth over the drainage area during the time interval Δt

A_s = surface drainage area contributing to the aquifer

Q_s = average rate at which surface flow leaves the area during Δt

E_T = average evapotranspiration rate over the drainage area during Δt

Q_G = rate at which groundwater flows into the area

ΔS_G = change in volume of stored groundwater during Δt

ΔS_S = change in surface water storage during Δt

All terms in Eq. (3-40) are difficult to evaluate with accuracy. The average precipitation P and the streamflow Q_s must be determined by direct measurement using several precipitation and streamflow recording stations to provide the necessary accuracy. The change in surface-water storage volume must be determined by developing surface-area elevation information for all ponds, lakes, and impoundments in the area and installing stage recorders in each. The change in groundwater volume is still more difficult to assess. The extent of the aquifer must be defined through geologic mapping. Water-surface elevations must be monitored in all wells, and the change in stored volume calculated from the change in groundwater elevation integrated over the area of the aquifer. Proper consideration must of course be given to the specific yield or the storage coefficient for the aquifer.

Knowing the rate at which groundwater flows into the basin is necessary because surface drainage areas do not always coincide with the groundwater drainage area. Inflow of groundwater can be assessed only if the inflowing aquifers are defined and wells are drilled to determine the groundwater gradient, the hydraulic conductivity of the material in the aquifer, and the cross-sectional area of the aquifer.

The area-average evapotranspiration rate E_T can only be estimated. An average rate must usually be estimated based on assumed parameters for the different types of vegetation. Frequently, evapotranspiration rates are assumed to equal evaporation rates, and evaporation pans are located and monitored within the drainage area.

The term *safe yield* is defined as the rate at which groundwater can be pumped from the basin without endangering the groundwater supply. Since, as we have pointed out, precipitation is erratic, recharge and groundwater levels are also erratic. Figure 3-16 shows the fluctuation in groundwater level at an observation well and precipitation at a nearby station. Recharge of the aquifer

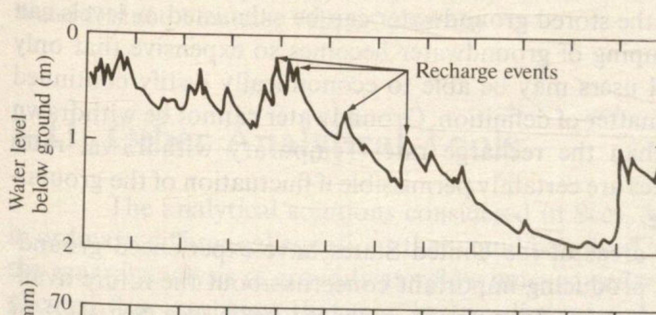


Figure 3-16 Fluctuations of observed water table at an observation well on the coastal plain of North Carolina (6)

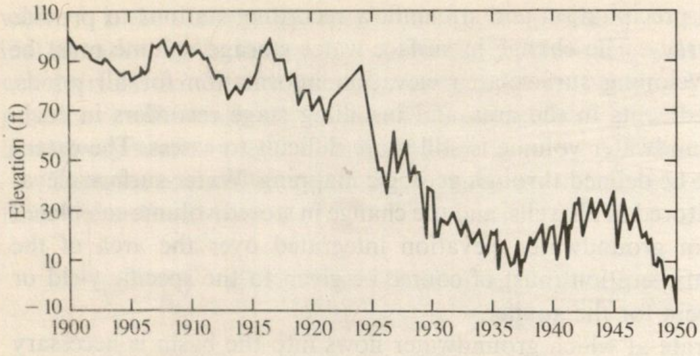


Figure 3-17 Long-term declines in groundwater levels in the Santa Ana River Basin in California* (14).

* The short-term fluctuations are the result of changes in pumping rate.

was erratic, but discharge occurred continuously, as is shown in Fig. 3-16 by the declining water surface from August through November when relatively little rain fell. Observations made over several years will show that average levels fluctuate significantly from year to year depending on the amount of average annual rainfall. Moreover, the long-period observations, like those shown in Fig. 3-17, will exhibit continuous declines in average water-table levels when average pumping rates exceed long-term average recharge.

The safe yield of an aquifer is generally considered a matter of economics. If declines in groundwater levels, which result in deeper and more expensive wells and greater energy costs for pumping, do not make the use of the water uneconomical, pumping will likely continue despite the declines. However, groundwater levels will continue to decline where pumping rates exceed recharge rates; eventually, the stored groundwater can be exhausted or levels can drop so far that the pumping of groundwater becomes so expensive that only municipal and industrial users may be able to economically justify continued use. Thus, safe yield is a matter of definition. Groundwater cannot be withdrawn indefinitely at greater than the recharge rate. Temporary withdrawal rates greater than recharge rates are certainly permissible if fluctuation of the groundwater table is acceptable.

As we noted, many areas of the United States have experienced groundwater-table drawdowns, producing important concerns about the future availability of groundwater supply. As a result, recharge wells (see Sec. 3-7) or spreading basins have been constructed to help recharge the depleted aquifers. Figure 3-17 shows the historical declines in the groundwater levels in the Santa Ana River basin in Southern California. To help recharge the groundwater, large spreading basins were constructed along the Santa Ana River. Surface runoff is impounded in these spreading basins and allowed to infiltrate the underlying aquifer. Some of these basins are shown in Fig. 3-18 during a period when recharge was occurring.

Figure 3-18 Spreading basins on the Santa Ana River for recharging the underlying groundwater basin. (Photo courtesy of the Orange County Water District, Fountain Valley, California)

3-11 Other Analytical Tools

The analytical solutions considered in Secs. 3-4, 3-5, and 3-6 resulted in ordinary differential equations that could be solved in closed form. However, the general analysis of groundwater flow must consider significantly more complicated flow problems that may be two-dimensional, three-dimensional, and unsteady. In these cases, numerical solutions are often the only ones possible. A wide variety of programs for digital computers (so-called groundwater models) have been developed for special purposes (1, 9, 12).

The development of these programs is beyond the level of this text. However, to illustrate the type of mathematical analysis required and the general concept of groundwater modeling, we will consider briefly the numerical modeling.

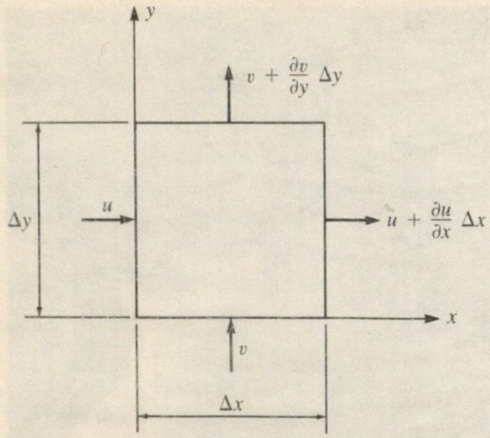


Figure 3-19 Control volume for two-dimensional flow in a horizontal plane.

To develop the general partial differential equation for flow of groundwater, we will consider two-dimensional flow in a horizontal plane. Figure 3-19 illustrates a control volume in the x–y plane. For steady flow, the rate of flow into and out of the control volume must be equal. Thus,

$$u\,\Delta y + v\,\Delta x - \left(u + \frac{\partial u}{\partial x}\,\Delta x\right)\Delta y - \left(v + \frac{\partial v}{\partial y}\,\Delta y\right)\Delta x = 0 \tag{3-41}$$

where u and v are the velocity components and $\partial u/\partial x$, and $\partial v/\partial y$ are their rates of change in the x and y directions, respectively. Equation (3-41) reduces to

$$-\frac{\partial u}{\partial x}\,\Delta x\,\Delta y - \frac{\partial v}{\partial y}\,\Delta x\,\Delta y = 0$$

or

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3-42}$$

which is the familiar differential form of the equation of continuity. Equation (3-10), Darcy's equation, tells us that the velocity components u and v can be expressed as

$$u = -K\,\frac{\partial h}{\partial x} \tag{3-43}$$

and

$$v = -K\,\frac{\partial h}{\partial y} \tag{3-44}$$

Substituting Eqs. (3-43) and (3-44) into Eq. (3-42) then yields

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \tag{3-45}$$

as long as the hydraulic conductivity K is a constant throughout the aquifer. Equation (3-45) is a classic partial differential equation known as the Laplace equation. It is linear in the piezometric head h, and its solution depends solely on the values of h on the boundaries of the flow field in the x–y plane. Thus, the value of h at any point within a flow field can be determined uniquely in terms of the values of h on the boundaries.

Many classic solutions have been developed for Eq. (3-45) for the flow of groundwater (2, 3, 20). Moreover, the same equation arises in many other areas of interest such as hydrodynamics, elasticity, electricity, and the flow of heat (19). You may also find classical solutions to this equation in several texts on applied mathematics (15, 16).

Equations (3-43) and (3-44) show that the velocity of flow is normal to the lines of constant piezometric head. In the derivations of Eqs. (3-26) and (3-29) for flow to wells from unconfined and confined aquifers, respectively, we have used this fact but without calling attention to it. For example, in Figs. 3-5, 3-6, 3-7, 3-9, and 3-10, the velocity is in the plane of the page, and we assumed it to be horizontal in our analyses. The velocity is normal to lines of constant h, which in Figs. 3-5, 3-6, and 3-7, would be normal to the page. Figure 3-20 is an example of a hypothetical flow in which the velocity vector V has the x and y components u and v, respectively, and is normal to the lines of constant h. V is directed in the direction of decreasing h, which agrees with our formulation of Darcy's equation.

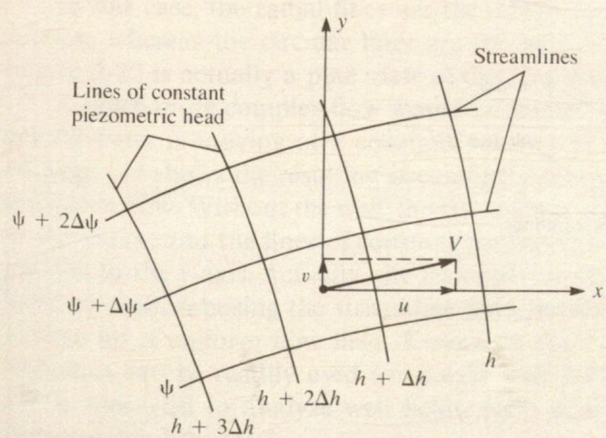


Figure 3-20 A hypothetical flow net for groundwater flow in the x–y plane.

If lines are drawn tangent to the velocity vector at every point in a steady flow field, a set of lines called *streamlines* is formed that are normal to the lines of constant piezometric head. This family of lines (streamlines) is called the streamfunction ψ, and in steady flow, the streamlines mark the paths of the flowing particles of fluid.

Because the streamlines are everywhere normal to the lines of constant piezometric head, the velocity components can be expressed in terms of the streamfunction ψ as

$$u = \frac{\partial \psi}{\partial y} \tag{3-46}$$

$$v = -\frac{\partial \psi}{\partial x} \tag{3-47}$$

Substituting Eqs. (3-46) and (3-47) into Eq. (3-42) yields

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \tag{3-48}$$

which again is the Laplace equation, this time expressed in terms of the streamfunction rather than the piezometric head.

Since streamlines are lines that are everywhere tangent to the velocity vectors, there can be no flow across the streamlines in steady flow, and the rate of flow is constant between any two streamlines. Figure 3-21 illustrates the relationship between the streamfunction and the velocity. We can determine

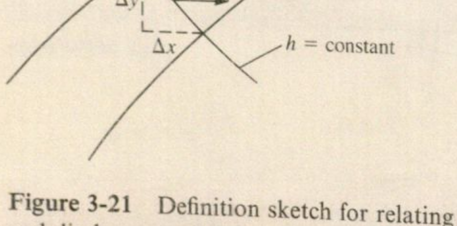


Figure 3-21 Definition sketch for relating velocity and discharge to the stream function

the rate of flow between the streamlines as follows where the flow field is assumed to have a depth of 1:

$$v\,\Delta n = u\,\Delta x - v\,\Delta y \tag{3-49}$$

or

$$u\,\Delta x - v\,\Delta y = \Delta q \tag{3-50}$$

where q is the unit discharge, Δn is the spacing of the streamlines along a line normal to the velocity vector, and Δx and Δy are the projections of the normal increment Δn on the x–y plane. In differential form, Eq. (3-49) becomes

$$u\,dx - v\,dy = dq \tag{3-51}$$

Substituting Eqs. (3-46) and (3-47) for u and v, respectively, we have

$$\frac{\partial\psi}{\partial x}\,dx + \frac{\partial\psi}{\partial y}\,dy = dq \tag{3-52}$$

Equation (3-52) implies that

$$dq = d\psi \tag{3-53}$$

or that in two-dimensional flow, the value of the streamfunction is numerically equal to the unit discharge, and that the increment in unit discharge between two streamlines is equal to the change in the value of the streamfunction between two streamlines.

The use of streamlines, lines of constant piezometric head, and the definition of the streamfunction provide a graphic means of illustrating complex two-dimensional groundwater flows. For example, Fig. 3-22 on the next page shows the plan view of flow toward a well in an infinite aquifer.

In this case, the radial lines are the streamlines and show the direction of motion, whereas the circular lines are the lines of constant piezometric head. Figure 3-22 is actually a plan view of the flow pattern shown in Fig. 3-10.

A much more complex flow would be created by a well in an aquifer where groundwater is moving at a constant velocity in the x direction. Figure 3-23 on page 149 shows the resulting streamline and constant piezometric head lines for such a flow. Without the well, the streamlines would be straight lines parallel to the x-axis, and the lines of constant piezometric head would be straight and parallel to the y-axis. Actually, the flow pattern shown in Fig. 3-23 was formulated by superimposing the streamline pattern of Fig. 3-22 onto the streamline pattern for a uniform flow field. Known as the method of superposition, this technique can be readily used to analyze well fields (13), and in Secs. 3-8 and 3-9, it was used to analyze well fields, wells near a river, and wells near an impermeable barrier.

Equation (3-48) provides the basis for numerical solution of complex two-dimensional groundwater flows. It can be expressed as a finite-difference or

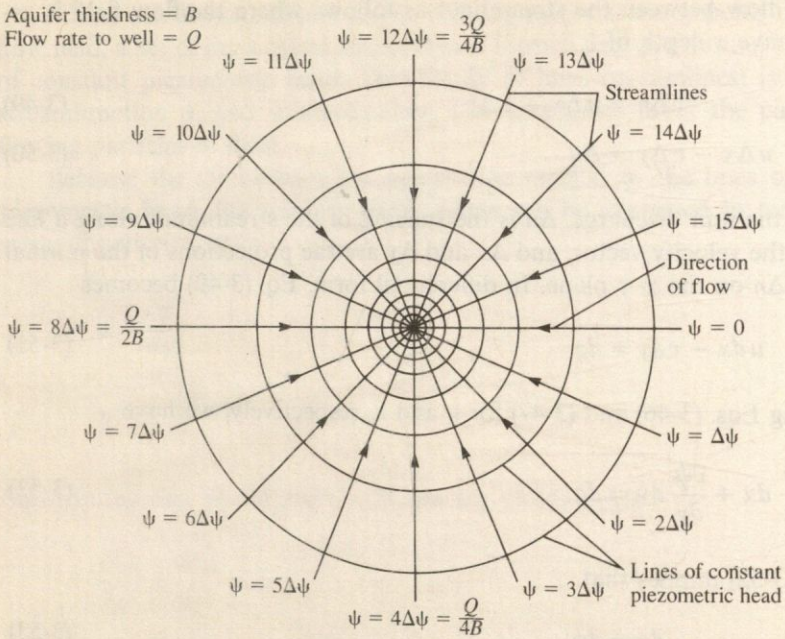


Figure 3-22 Streamlines and lines of constant piezometric head for flow to a well in an infinite aquifer

finite-element approximation (3). Figure 3-24 shows a finite-difference network in the x–y plane. In the finite-difference approximation, values of ψ or h will be known at points all around the outside boundary of the flow field. To develop a solution that will give velocities and directions of flow, the value of ψ or h at each interior node in the field must be determined. For convenience, we will consider only the streamfunction ψ in our development, but the same analysis could be performed using only h. To formulate the finite-difference equation, the partial derivatives $\partial^2\psi/\partial x^2$ and $\partial^2\psi/\partial y^2$ must be approximated numerically. This is done by assuming that the variation in ψ is linear between nodes (intersections of the x- and y-coordinate lines) in the field, an assumption whose accuracy depends on the spacing of the finite-difference grid. Thus, the first-order differential $\partial\psi/\partial x$ is approximated at a point halfway between point i, j and point $i + 1, j$ (Fig. 3-24) as

$$\frac{\partial\psi}{\partial x} = \frac{\psi_{i+1,j} - \psi_{i,j}}{\Delta x} \qquad (3\text{-}54)$$

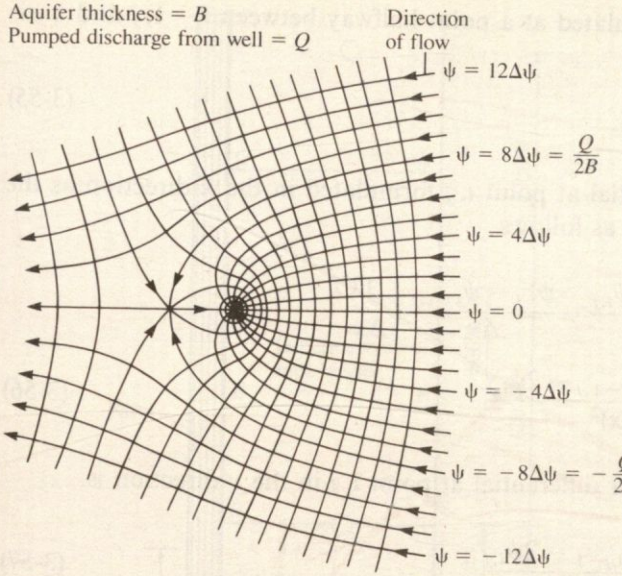


Figure 3-23 Streamlines and lines of constant piezometric head for a pumped well in a uniform groundwater flow parallel to the x axis

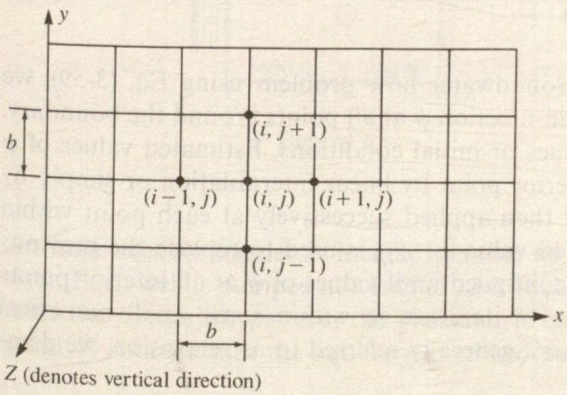


Figure 3-24 Square finite difference mesh for development of Eq. (3-59)

A similar expression formulated at a point halfway between $i-1, j$ and i, j is

$$\frac{\partial \psi}{\partial x} = \frac{\psi_{i,j} - \psi_{i-1,j}}{\Delta x} \tag{3-55}$$

The second-order differential at point i, j formulated in the x direction as the rate of change in $\partial\psi/\partial x$ is as follows

$$\frac{\partial^2 \psi}{\partial x^2} = \left[\frac{\psi_{i+1,j} - \psi_{i,j}}{\Delta x} - \frac{\psi_{i,j} - \psi_{i-1,j}}{\Delta x}\right]\frac{1}{\Delta x}$$

or

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\psi_{i+1,j} + \psi_{i-1,j} - 2\psi_{i,j}}{(\Delta x)^2} \tag{3-56}$$

Similarly, the second-order differential at point i, j in the y direction is

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{\psi_{i,j+1} + \psi_{i,j-1} - 2\psi_{i,j}}{(\Delta y)^2} \tag{3-57}$$

Substituting Eqs. (3-56) and (3-57) into Eq. (3-48) yields

$$\frac{\psi_{i+1,j} + \psi_{i-1,j} - 2\psi_{i,j}}{b^2} + \frac{\psi_{i,j+1} + \psi_{i,j-1} - 2\psi_{i,j}}{b^2} = 0 \tag{3-58}$$

where b has been substituted for Δx and Δy, since we are using a square grid with a mesh spacing of b.

After rearranging and combining terms, Eq. (3-58) becomes

$$\psi_{i,j} = \frac{\psi_{i+1,j} + \psi_{i-1,j} + \psi_{i,j+1} + \psi_{i,j-1}}{4} \tag{3-59}$$

To solve a two-dimensional groundwater flow problem using Eq. (3-59), we first establish values of the streamfunction ψ at all points around the boundary. These values are boundary values or initial conditions. Estimated values of ψ are then calculated at each interior point by linear interpolation or simply by assumption. Equation (3-59) is then applied successively at each point within the flow field, and the calculated value for $\psi_{i,j}$ is used to replace the previous value of ψ at point i, j. This is continued until values of ψ at all interior points remain constant from iteration to iteration to within some small numerical tolerance. This numerical process, generally referred to as relaxation, we demonstrate in Example 3-7.

EXAMPLE 3-7 A line of wells is to be drilled in an alluvial aquifer as shown in Fig. A. The wells are equally spaced along a line 600 ft from the river.

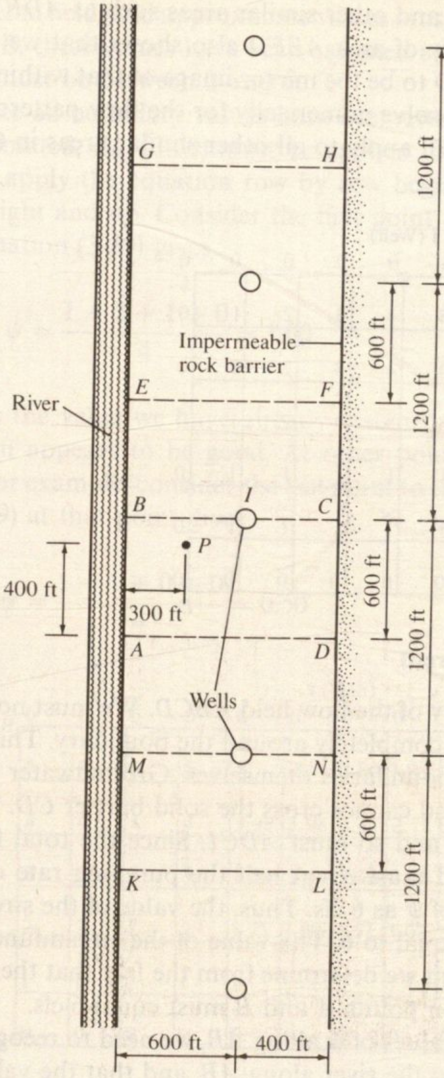


Figure A

An impermeable barrier is located parallel to the river and 1000 ft from the river. Each well is pumped at a rate of 12 ft^3/s. Using numerical methods, determine the streamline pattern, the piezometric head pattern, and the velocity at point P. Assume that the aquifer is confined with a depth of 100 ft.

SOLUTION Considering the geometry of the flow field will show us that the flow pattern in *AEFD* will have a flow pattern exactly like that within *EGHF*. Thus, we need to solve for the flow pattern within *AEFD* only, since that solution will apply to *EGHE* and other similar areas such as *ADKL*. Further consideration of the geometry of area *AEFD* also shows that we can expect the flow pattern within *ABCD* to be the mirror image of that within *BEFC* or *ADMN*. Therefore, we need to solve numerically for the flow pattern within *ABCD* only, since that solution will apply to all other similar areas in the flow field.

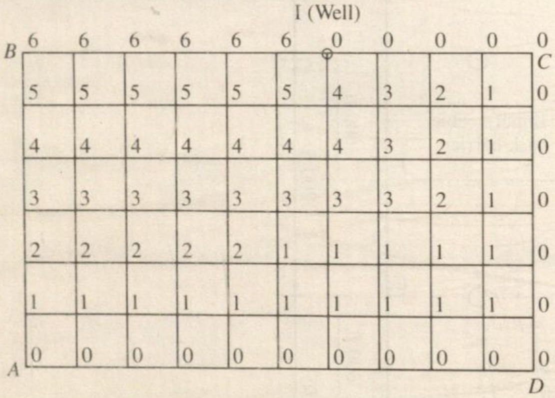


Figure B

Figure B shows the boundary of the flow field *ABCD*. We must now establish values of the streamfunction completely around the boundary. This is done by first considering flow at the boundaries themselves. Groundwater will flow from the river toward the well and cannot cross the solid barrier *CD*. Thus, *BI* must be a bounding streamline and so must *ADCI*. Since the total flow rate passing through the *ABCD* field must equal half the pumping rate of well *I*, we can set the maximum value of ψ as 6 cfs. Thus, the value of the streamfunction at all points on line *BI* is equal to 6. The value of the streamfunction is 0 along the streamline *ADCI* which we determine from the fact that the total difference in the value of ψ between points *A* and *B* must equal 6 cfs.

To establish the boundary values of ψ along *AB*, we need to recognize that the streamlines will be normal to the river along *AB*, and that the value of the streamfunction will vary linearly along *AB* from 0 at *A* to 6 at *B*. We arrive at this conclusion by realizing that the flow out of the river into the aquifer is uniform along the river. We can arbitrarily choose the size of the finite-difference mesh we wish to use in our solution. A very fine grid will produce the greatest accuracy in determining the flow pattern but also will require greater effort to solve Eq. (3-59). Conversely, a coarse finite-difference grid gives less accuracy but provides for a more speedy solution because fewer numbers are involved. In this case, we will arbitrarily use a spacing created by dividing the line *AD* into ten increments each 100 ft long. Using the same spacing in the vertical direction, line *AB* is divided into six equal increments. As shown, the value of the difference in streamfunction between successive meshes along *AB* must equal 1. The boundary values of the streamfunction are now established entirely around the field, and approximate values must be assumed for all interior points. In Fig. B, these values have been assumed arbitrarily, considering only that all values must be between 0 and 6.

Once all boundary values have been established and initial interior values approximated, Eq. (3-59) must be applied to each interior point. In this case, we will apply the equation row by row beginning at the bottom and moving to the right and up. Consider the first point at the left of the first row.

Equation (3-59) gives

$$\psi = \frac{1 + 2 + 1 + 0}{4} = 1.00$$

which is the value we have already assumed for that point; therefore, our assumption appears to be good. At other points, however, our guess is not as good. For example, consider the last point to the right on the first row. Applying Eq. (3-59) at that point gives

$$\psi = \frac{1 + 1 + 0 + 0}{4} = 0.50$$

							I (Well)					
B	6	6	6	6	6	6	0	0	0	0	0	C
	5	5	5	5	5	4.74	2.80	1.88	1.19	0.53	0	
	4	4	4	4	3.99	3.98	3.47	2.72	1.86	0.93	0	
	3	3	3	3	2.94	2.91	2.90	2.39	1.70	0.86	0	
	2	2	2	2	1.75	1.69	1.67	1.67	1.42	0.73	0	
	1	1	1	1	1	1	1	1	1	0.50	0	
A	0	0	0	0	0	0	0	0	0	0	0	D

Figure C

Thus, the assumed value of 1 at that point must be replaced by 0.50. We make this replacement before applying Eq. (3-59) to the next point. Figure C shows the results of applying Eq. (3-59) to all interior points in succession for one complete iteration. Thus, Fig. C provides a first iteration to the solution. To complete the solution, Eq. (3-59) is applied successively to each point until the interior values of ψ remain constant from iteration to iteration to within a chosen numerical tolerance. Figure D shows the interior values of ψ after 29 iterations. The interior values at the end of the 29th iteration changed less than 0.01 from the 28th iteration. The values of ψ shown in Fig. D are, thus, a solution of our problem to within a numerical precision of ± 0.01 cfs.

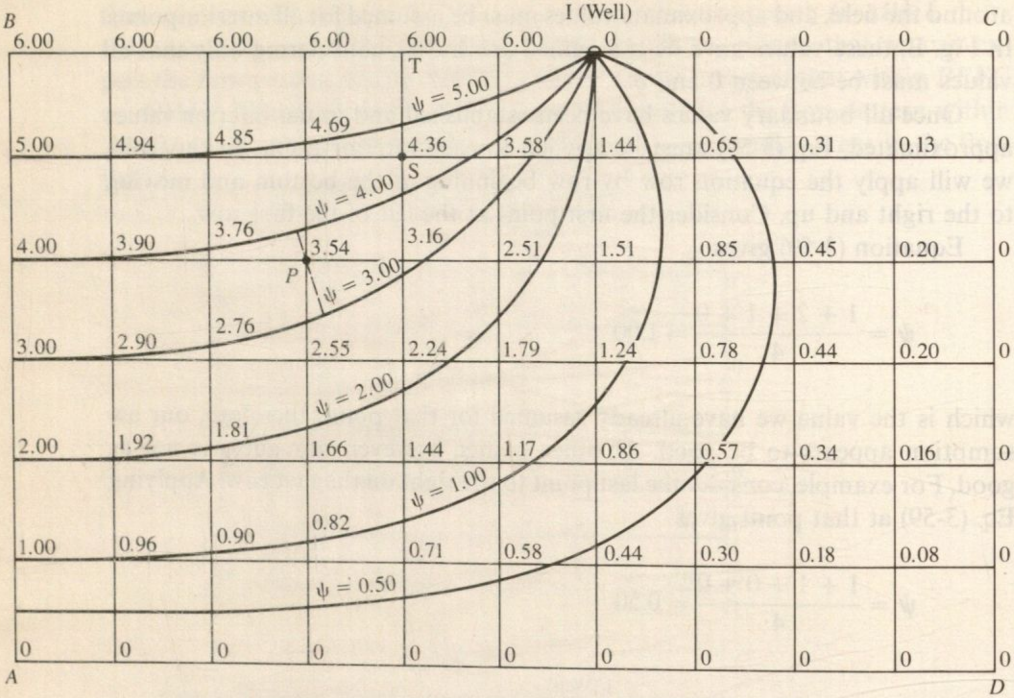


Figure D

The velocity at point P can be calculated by using Eq. (3-46) and (3-47) to calculate, respectively, the x and y components of velocity. The partial derivatives are calculated from the numerical values in Fig. D as

$$\left(\frac{\partial \psi}{\partial x}\right)_P = \frac{(3.16 - 3.54) + (3.54 - 3.76)}{200} = -\frac{0.6}{200} = -0.003$$

$$v = -\frac{\partial \psi}{\partial x}\frac{1}{B} = \frac{0.003}{100} = 0.00003 \text{ ft/s}$$

$$\left(\frac{\partial \psi}{\partial y}\right)_P = \frac{(4.69 - 3.54) + (3.54 - 2.55)}{200} = \frac{2.14}{200} = 0.0107$$

$$u = \frac{\partial \psi}{\partial y}\frac{1}{B} = \frac{0.0107}{100} = 0.000107 \text{ ft/s}$$

The thickness of the aquifer $B = 100$ had to be included because flow takes place throughout the entire depth of the aquifer. Thus, the y component of velocity is 0.00003 ft/s in the positive y direction, whereas the x component of velocity is 0.000107 ft/s in the positive x direction. The total velocity is

$$V = \sqrt{u^2 + v^2} = \sqrt{(0.00003)^2 + (0.000107)^2} = 0.00011 \text{ ft/s}$$

The velocity at point P can be determined graphically as well. Streamlines can be drawn as shown in Fig. D. To locate the streamlines, interpolation was performed along each vertical and horizontal grid line in Fig. D. For example, to locate the streamline for $\psi = 5$, between points S and T in Fig. D the streamline must be above point S, a distance of

$$\Delta y = 100 \frac{5 - 4.36}{6 - 4.36} = 39.0 \text{ ft}$$

Similar interpolation was done at each remaining vertical grid line that would be crossed by the $\psi = 5$ streamline. The remaining streamlines were located using the same method but for each of the other streamlines, interpolation had to be performed along the horizontal grid lines as well. The resulting streamline pattern is sketched in Fig. D. An additional streamline $\psi = 0.5$ has been located as well to provide more detail of the flow pattern.

The velocity at point P can also be determined using the principle of continuity. Since the streamfunction is numerically and dimensionally equal to unit discharge, we can determine the average velocity between the streamlines by dividing the total discharge between two streamlines by the normal distance between them. The normal spacing between the lines $\psi = 4$ and $\psi = 5$ at point P on Fig. D scales as 88 ft. Thus, the velocity is

$$V = \frac{1}{88(100)} = 0.00011 \text{ ft/s}$$

which agrees with the value calculated using Eq. (3-46) and (3-47). ■

The streamline pattern sketched in Fig. D of Example 3-7 is a good representation of the two-dimensional flow pattern, particularly near the left side of the field. However, near the well (point I), the streamlines develop a strong curvature, and the spacing of the streamlines appears somewhat irregular. This is because the grid spacing is too coarse in that region to give accurate values of the streamfunction at each mesh point (the numerical approximation to the partial derivatives in Eq. (3-48) is not accurate because of the rapid curvature). The mesh could be subdivided near the well, and Eq. (3-59) could be applied to that finer grid just as it was applied to the total grid in producing the numbers shown on Fig. D for Example 3-7. The result would be a better definition of the streamfunction where rapid curvature occurs.

The streamline pattern of Example 3-7 shows that the velocity is very small in the region to the right near the impermeable barrier because the streamlines are widely spaced there. Theoretically the velocity must be zero at points C and D and infinite at the well (point I).

The numerical procedures discussed in Example 3-7 are used to model steady-state flow patterns for complex groundwater flow patterns where more simple closed-form solutions cannot be obtained. Contaminant transport can also be modeled to the degree that chemical and physical actions between the aquifer material, the groundwater, and the contaminant are understood and amenable to analytical modeling.

Problems

3-1 How many cubic meters of water are contained in a 100-m^3 volume of saturated clay having a porosity of 0.48?

3-2 In a laboratory test, a one cubic foot sample of an aquifer was found to weigh 85 lb. After being allowed to drain thoroughly, the sample weighed 73 lb. After being crushed and thoroughly dried, the sample weighed 51 lb. If the sample was saturated initially, calculate the indicated specific yield, porosity, specific retention, and specific gravity of the solids.

3-3 Estimate the number of acre-feet of water that could theoretically be withdrawn from a saturated sandstone aquifer with an average surface area of 1500 sq mi and an average depth of 4500 ft.

3-4 If a spill of radioactive material occurred over an aquifer, estimate the length of time required for the material to reach a river 2 mi away if the aquifer is

a. Gravel
b. Clay
c. Sandstone
d. Granite

Groundwater elevation at the spill site is 100 ft above the water surface in the river.

3-5 Make the same estimate as requested in Prob. 3-4, but assume the aquifer is badly fractured basalt.

3-6 A permeameter (similar to the one shown in Fig. 3-3, page 113) was used to test three different materials. The horizontal tube of the permeameter is 5 ft long with an inside diameter of 4 in. The head measurements were 185 in., 77 in., and 39 in. for h_1, and 34 in., 35 in., and 36 in. for h_2 for the three materials, respectively. What was the indicated hydraulic conductivity for each sample if the flow rate was 0.227 gal/hr for each test?

3-7 The indicated hydraulic conductivity for a material tested in the permeameter of Prob. 3-6 is 10 ft/day for a gradient of 0.005. If the porosity of the material is 0.30, what is the average velocity of flow within the voids of the sample?

3-8 In using a permeameter like that shown in Fig. 3-3, page 113, it is sometimes convenient to make a "falling-head" test. The horizontal tube is 1 m long and 10 cm in diameter and is filled with a sample material. During a test, the upstream head h_1 falls from 200 to 180 cm in 4 hr, while h_2 is held constant at 15 cm. The diameter of the vertical tube in which h_1 is measured is 40 cm. What is the indicated average hydraulic conductivity? Estimate the range between maximum and minimum values of hydraulic conductivity.

3-9 Laboratory testing indicates that a sample of an aquifer material has a porosity of 0.5. The sample is recompacted and tested in the permeameter whose physical dimensions are as given in Prob. 3-8. The differential head during the permeameter test is 100 cm, and 2 liters of water were discharged in 10 min. Was the porosity of the recompacted sample different from its original value?

3-10 In Fig. 3-5, page 117, the aquifer thickness is 10 ft, the hydraulic conductivity of the aquifer is 0.06 m/day, and the river water surface is 7 m above the bottom of the aquifer. If it is necessary to maintain a water level in the trench not more than 10 m deep, at what rate must water be pumped out of a 100-m long trench? The distance between the edge of the trench and the river is 300 m. Assume all water comes from the river.

3-11 A trench 100 ft long is excavated parallel to a river, as shown in Fig. 3-5, page 117. The aquifer thickness B is 19 ft, and the trench is 400 ft from the river. During a pumping test, a 10-ft differential head was maintained while pumping was steady at a rate of 10 gal/min. What is the indicated hydraulic conductivity of the aquifer?

3-12 A trench 100 ft long is excavated 300 ft away and is parallel to a river bank. A sand aquifer is 20 ft thick and similar to that shown in Fig. 3-6, page 118. The river water surface is 30 ft above the horizontal bottom of the aquifer. If water depth in the trench must be maintained at 6 ft, at what rate must water be pumped from the trench? How far from the trench will the aquifer cease to be saturated?

3-13 An aquifer slopes away from a river, as shown in Fig. 3-7, page 119. The hydraulic conductivity of the aquifer which is 20 ft thick is known to be 0.1 ft/day. The water is 20 ft deep in the river, and the bottom of the trench is 30 ft below the bottom of the river and 600 ft away from the edge of the river. The trench is 40 ft deep. Will water flow out of the trench without pumping? If so, what will be the rate of flow?

3-14 A 10-in. diameter well is drilled to the bottom of an unconfined aquifer. The water table is originally 500 ft above the bottom of the aquifer whose hydraulic conductivity is known to be 0.25 ft/day. If 100 gal/min is pumped from the well, what will be the equilibrium depth of water in the well if the radius of influence is 1000 ft? How much drawdown should be expected at a distance of 250 ft from the well?

3-15 In a pumping test, 100 gal/min is pumped from a 12-in. diameter well. The groundwater table is originally 300 ft above the bottom of the unconfined aquifer. Drawdown in the well is 110 ft and 5 ft at a distance of 400 ft from the well when approximately steady-state conditions are achieved. What is the indicated hydraulic conductivity for the aquifer?

3-16 In a confined aquifer (see Fig. 3-10, page 126), a 50-cm diameter well fully penetrates the aquifer. The hydraulic conductivity of the aquifer was approximately 0.8 m/day. Originally, the piezometric head was 300 m above the top of the aquifer, which is 30 m thick. If 0.8 m^3/min is pumped from the well, what should be the depth of water in the well? What will the height of the piezometric head be (above the top of the aquifer) at a distance of 50 m from the well? Assume the radius of influence to be 1000 ft.

3-17 A confined aquifer has a thickness of 85 ft. An 86-in. diameter well is drilled to the bottom of the aquifer, as shown in Fig. 3-10, page 126. If the hydraulic conductivity of the aquifer is 10 ft/day, and if water is originally 170 ft deep in the well, calculate approximately the maximum rate of flow that can be pumped from the well without dewatering any part of the aquifer. Assume a reasonable value for the radius of influence. What will be the average velocity (Darcy velocity) of flow in the aquifer at a distance of 20 ft from the center of the well?

3-18 The following measurements were made at an observation well 70 ft from a well being pumped at 150 gal/min. Determine the storage coefficient and transmissivity of the aquifer being pumped.

Time (hr)	Distance From Top of Well to Water (ft)
0	15.5
0.02	15.6
0.03	15.7
0.04	15.8
0.06	16.0
0.11	16.3
0.25	16.8
0.43	17.1
1.00	17.7
3.90	18.5

3-19 The following data were taken during the pumping test of a well that was pumped at a rate of 0.41 ft^3/s. The two observation wells were located

320 ft and 640 ft from the well. Calculate the storage coefficient and the transmissivity of the confined aquifer.

Time (hr)	Drawdown (ft)	
	Well at 320 ft	Well at 640 ft
0.024	0.27	0.06
0.120	0.54	0.32
0.240	0.81	0.48
1.200	1.23	0.88
2.400	1.41	1.06
12.000	1.83	1.47
24.000	2.01	1.65
120.000	2.43	2.07
240.000	2.61	2.25

3-20 For Prob. 3-19, assume that the argument of the well function u is less than 0.01, and solve for the transmissivity and the storage coefficient for each of the drawdown values. Compare this with the values obtained in the Theis method used in Prob. 3-19.

3-21 A groundwater recharge well, as shown in Fig. 3-11, page 134, is injected with a flow of 100 gal/min. If the water level in the well is originally 100 ft deep and rises to an elevation 15 ft above the original water table, estimate the hydraulic conductivity of the unconfined aquifer. Calculate the distance that water will be mounded above the original water table at a distance of 80 ft from the well. The well diameter is 24 in.

3-22 What is the volume of water contained above the original water surface if injection of 30 gal/min into an injection well results in a quasi-steady water surface in the 24-in. diameter well 30 ft above the original ground water table? The original water depth is 200 ft. Assume that the mounded surface essentially coincides with the original groundwater table at 600 ft from the well.

3-23 A 60-in. diameter well is located 300 ft from a river and fully penetrates a gravel aquifer having a hydraulic conductivity of 40 ft/day. It is pumped at a rate of 400 gal/min. Calculate coordinates and plot the equilibrium groundwater surface along a line perpendicular to the river bank and passing through the well. Assume a 1000-ft radius of influence on the side away from the river. The river water surface is 15 ft above the bottom of the aquifer.

3-24 For the well and pumping conditions described in Prob. 3-23, calculate the drawdown to be expected at a point halfway between the well and the river and for a point 150 ft from the well on the side away from the river.

3-25 For the well and pumping conditions given in Prob. 3-23, calculate the drawdown to be expected at a point 100 ft from the well and 300 ft from the river.

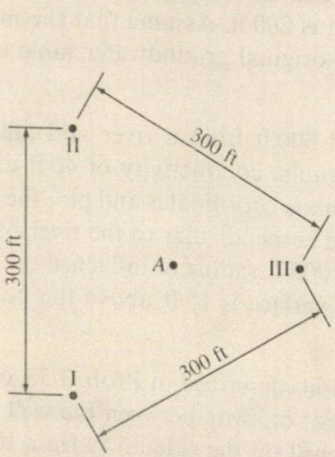
3-26 A well is pumped to unwater an excavation site in an unconfined aquifer. The water level is originally 10 ft below the top of the well. The well extends to the bottom of the aquifer and is 100 ft deep. A sheet-pile wall is driven completely through the aquifer at a distance of 80 ft from the well. If the hydraulic conductivity of the aquifer is 0.8 ft/day, calculate the drawdown at the sheet-pile wall and at a point 80 ft away from the well but on the side away from the wall. The pumping rate is 7.5 gal/min. Assume a radius of influence of 1000 ft on the side away from the wall, and that the wall is long enough to fully cut off flow in the aquifer.

3-27 For the well conditions given in Prob. 3-26, assume that the pumping rate is increased to 150 gal/min. What will the drawdown become at the wall under the new pumping rate?

3-28 Two wells are drilled 1300 ft apart. Both fully penetrate a confined aquifer 800 ft thick. They are both pumped at a rate of 20 gal/min. If the aquifer has a hydraulic conductivity of 3×10^{-2} ft/day, and the piezometric head is originally 400 ft above the top of the aquifer, calculate the drawdown to be expected halfway between the wells. Assume a radius of influence of 1000 ft.

3-29 Three wells are drilled into an unconfined aquifer. The level, impervious bottom of the aquifer is 500 ft below the earth's surface. The groundwater table is initially 100 ft below the top of the wells, all of which are at the same elevation. Hydraulic conductivity of the aquifer is 20 ft/day. The three wells are arranged as shown in the accompanying figure and are all pumped at a rate of 5000 gal/min. Assume a radius of influence of 1000 ft.

(continued p. 161)

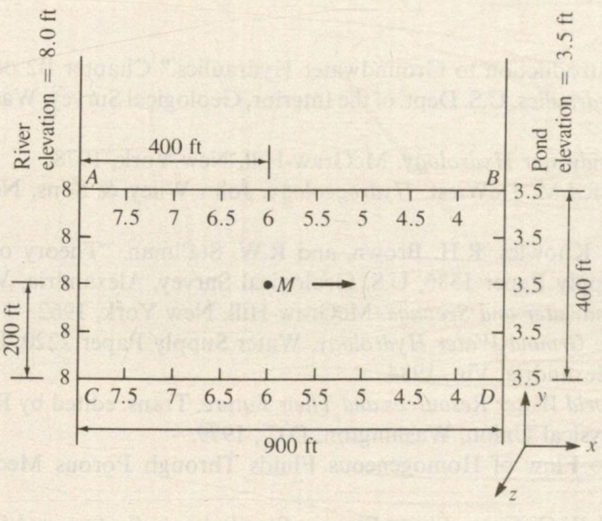


Layout of well field

Calculate the drawdown at point *A* midway between the three wells. Calculate the ordinates and plot the equilibrium groundwater surface along a line between wells I and II.

3-30 A drainage basin 400 sq mi in area completely covers a limestone aquifer, which averages 300 ft in thickness. Estimate the total volume of water stored in the aquifer if it were saturated. If average annual precipitation over the area is 22 in./yr, evapotranspiration averages 8 in./yr, and runoff is 20% of precipitation, estimate the annual yield that could be pumped from groundwater without a long-term overdraft.

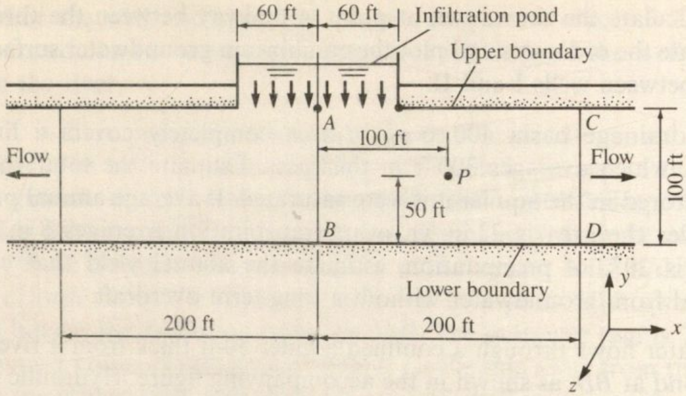
3-31 Water flows through a confined aquifer 50-ft thick from a river at *AC* to a pond at *BD*, as shown in the accompanying figure. Hydraulic conductivity is 4 ft/day. Piezometric heads are as shown on the boundaries of the flow field. Using Eq. (3-59) (and using piezometric head instead of streamfunction), determine the values of the piezometric head for the interior of the field. What is the velocity of flow at point *M*? Determine the flow rate between the river and the pond.



PROBLEM 3-31

3-32 Flow infiltrates from a pond to a confined aquifer as shown in the figure on the next page. The infiltration rate is a constant 12 in./day. Assuming that the flow is two-dimensional and that the upper and lower boundaries are level and parallel, establish values of the streamfunction around the boundary. Dividing the vertical dimension of the aquifer into five finite-difference increments and using a square finite-difference grid, calculate the value of the streamfunction at each interior grid point. What is the velocity

(continued p. 162)



PROBLEM 3-32

at point P? What is the velocity at any point along CD? Hydraulic conductivity is 0.1 ft/day.

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