CE 5395 February 4, 2020

Analytical solute transport homework
Due February 11, 2020

1. An instantaneous release of biodegradable organics occurs in a 1-D aquifer. Assume that the mass spilled is 1.0 kg over a 10 m^2 area normal to the flow direction, $\alpha = 1.0$ m, the seepage velocity is 1.0 m/day and the half-life of the decaying contaminant is 33 years. Compute the maximum concentration at 100 m from the source.

2. An accidental spill from a point source introduced 10 kg of contaminant mass to an aquifer. The seepage velocity in the aquifer is 0.1 ft/day in the x direction. The longitudinal dispersion coefficient $D_L = 0.01$ ft^2/day, the lateral and vertical dispersion coefficients are $D_y = D_z = 0.001$ ft^2/day.

(a) Calculate the maximum concentration at $x = 100$ ft and $t = 5$ years.

(b) Calculate the concentration at point $x = 200$ ft, $y = 5$ ft, $z = 2$ ft, 5 years after the spill.

3. Domenico & Schwartz (1998) developed a model for a planar source that accounts for the source geometry with longitudinal. lateral and vertical spreading. The steady state model was applied at the plane of symmetry where y = z = 0 (see Figure 6.8). The model is to be applied to the case of a continuous source that has been leaking contaminant into an aquifer for 15 years. The source had a width Y and a depth Z of 6 m, the initial concentration of the source was 10 mg/L, the seepage velocity is 0.057 m/day, and the longitudinal, transverse, and vertical dispersivities were estimated at 1 m, 0.1 m, and 0.01 m respectively. Calculate the present contaminant concentration at x = 200 m from source. using the Domenico model.

4. Repeat Example 6.2 from Chapter 6 for benzene (with a retardation factor, $R = 2$, and decay coefficient of 0.0005/d) for a 10,000 μg/L spill.

6.7 Given: Instantaneous release of biodegradable organic, 1-D flow

$M = 1.0\ kg \quad A = 10\ m^2$ normal to x-direction

$d_x = 1.0\ m \quad V_x = 1.0\ m/d \quad t_{1/2} = 33\ yr$

Find: C_{max} @ 100 m from source

$$C(x,t) = \frac{M}{(4\pi D_x t)^{1/2}} \exp\left[-\frac{(x-V_x t)^2}{4 D_x t} - \lambda t\right]$$

$$M = \frac{m}{A} = \frac{1.0\ kg}{10\ m^2} = 0.1\ kg/m^2$$

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{33\ yr} = 0.021\ yr^{-1}\left(\frac{1\ yr}{365\ d}\right) = 5.8\times10^{-5}\ d^{-1}$$

$$D_x = d_x V_x + \underbrace{D^*}_{0} = (1.0\ m)(1\ m/d) = 1\ m^2/d$$

t to get C_{max} @ $x = 100\ m$

$$t = \frac{x}{V_x} = \frac{100\ m}{1\ m/d} = 100\ d$$

when C_{max} @ $x = 100$, $x - V_x t = 0$

10

$$C(100\ m, 100\ d) = \frac{0.1\ kg/m^2}{[4\pi(1\ m^2/d)(100\ d)]^{1/2}} \exp\left[-0 - (5.8\times10^{-5}\ d^{-1})100\ d\right]$$

$$= \left(2.82\times10^{-3}\ \frac{kg}{m^3}\right)(0.994)$$

$$= 2.8\times10^{-3}\ \frac{kg}{m^3}\left(\frac{1\ m^3}{1000\ L}\right)\left(\frac{10^6\ mg}{kg}\right)$$

$$\boxed{C(100\ m, 100\ d) = 2.80\ mg/L}$$

6.9 Point source spill $m = 10$ kg $V_x = 0.1$ ft/d
$D_x = 0.01$ ft²/d, $D_y = D_z = 0.001$ ft²/d   no decay given

[a] find C_{max} @ $x = 100$ ft, and $t_1 = 5$ years

$$C(x,y,z,t) = \frac{C_0 V_0}{8(\pi t)^{3/2}(D_x D_y D_z)^{1/2}} \exp\left[-\frac{(x-vt)^2}{4D_x t} - \frac{y^2}{4D_y t} - \frac{z^2}{4D_z t} - \lambda t\right]$$

on centerline for C_{max} exp [ ] term = 0 $\lambda = 0$
$C_0 V_0 = m = 10$ kg

12d

- $x = 100$ ft

C_{max} when $t = \frac{100\text{ ft}}{V_x} = \frac{100\text{ ft}}{0.1\text{ ft/d}} = 1000$ d

$$C_{max}(100\text{ft},0,0,1000\text{d}) = \frac{10\text{ kg}}{8[(\pi)(1000\text{d})]^{3/2}\sqrt{(0.01\text{ft}^2/\text{d})(0.001\text{ft}^2/\text{d})^2}}$$

$$= 0.0710 \frac{\text{kg}}{\text{ft}^3}\left(\frac{3.28\text{ ft}}{\text{m}}\right)^3$$

$$= 2.51 \frac{\text{kg}}{\text{m}^3}\left(\frac{10^6\text{ mg}}{\text{kg}}\right)\left(\frac{1\text{ m}^3}{1000\text{ L}}\right)$$

$$\boxed{C_{max}(100,0,0,1000\text{d}) = 2510 \frac{\text{mg}}{\text{L}}}$$

- $t = 5$ yr $\left(365 \frac{\text{d}}{\text{yr}}\right) = 1825$ d

Peak at $x = V_x t = (0.1\text{ ft/d})(1825\text{ d}) = 183$ ft

$$C_{max}(183\text{ft},0,0,1825\text{d}) = \frac{10\text{ kg}}{8[(\pi)(1825\text{d})]^{3/2}\sqrt{(0.01\text{ft}^2/\text{d})(0.001\text{ft}^2/\text{d})^2}}$$

$$= 0.0288 \frac{\text{kg}}{\text{ft}^3}\left(\frac{3.28\text{ ft}}{\text{m}}\right)^3$$

$$= 1.02 \frac{\text{kg}}{\text{m}^3}\left(\frac{10^6\text{ mg}}{\text{kg}}\right)\left(\frac{1\text{ m}^3}{1000\text{ L}}\right)$$

$$\boxed{C_{max}(183,0,0,1825\text{d}) = 1020 \frac{\text{mg}}{\text{L}}}$$

[b] Find $C(200\text{ft}, 5\text{ft}, 2\text{ft}, 1825\text{d})$

$$C(200,5,2,1825\text{d}) = C_{max}(1825\text{d}) \exp\left[-\frac{(200\text{ft} - 0.1\text{ft/d}(1825\text{d}))^2}{4(0.01\text{ft}^2/\text{d})(1825\text{d})} - \frac{(5\text{ft})^2}{4(0.001\text{ft}^2/\text{d})(1825\text{d})} - \frac{(2\text{ft})^2}{4(0.001\frac{\text{ft}^2}{\text{d}})(1825\text{d})}\right]$$

$$= 1020 \frac{\text{mg}}{\text{L}} \exp[-4.20 - 3.42 - 0.55]$$

$$\boxed{C(200,5,2,1825\text{d}) = 0.289 \frac{\text{mg}}{\text{L}}}$$

10

6-10 **Given:** Continuous source for 15 yr. $Y = Z = 6\,m$, $C_0 = 10\,mg/L$
$V_x = 0.057\,m/d$ $\alpha_x = 1\,m$ $\alpha_y = 0.1\,m$ $\alpha_z = 0.01\,m$

Find: C @ x = 200 m @ t = 15 yr using Domenico Model

Assume y = z = 0, so should be highest concentration

$$\frac{C(x,0,0,t)}{C_0} = \frac{1}{2}\,\mathrm{erfc}\left[\frac{(x-v_x t)}{2(\alpha_x v_x t)^{1/2}}\right]\left\{\mathrm{erf}\left[\frac{Y}{4(\alpha_y x)^{1/2}}\right]\mathrm{erf}\left[\frac{Z}{4(\alpha_z x)^{1/2}}\right]\right\}$$

$$\frac{C(200\,m,0,0,15\,yr)}{C_0} = \frac{1}{2}\,\mathrm{erfc}\left[\frac{200\,m-(0.057\,m/d)(5475\,d)}{2[(1\,m)(0.057\,m/d)(5475\,d)]^{1/2}}\right]\mathrm{erf}\left[\frac{6\,m}{4[(0.1\,m)(200\,m)]^{1/2}}\right]\mathrm{erf}\left[\frac{6\,m}{4[(0.01\,m)(200\,m)]^{1/2}}\right]$$

$$= \frac{1}{2}\,\mathrm{erfc}[-3.17]\;\mathrm{erf}[0.335]\;\mathrm{erf}[1.06]$$

$\mathrm{erf}(-\beta) = -\mathrm{erf}(\beta)$
$\mathrm{erfc}(\beta) = 1-\mathrm{erf}(\beta)$
$\mathrm{erfc}(-\beta) = 1-\mathrm{erf}(-\beta) = 1+\mathrm{erf}(\beta)$

$$\frac{C(200\,m,0,0,15\,yr)}{C_0} = \frac{1}{2}[1+1][0.354][0.865]$$

$$= 0.306$$

$$C(200\,m,0,0,15\,yr) = 0.306\,(10\,mg/L)$$

$$\underline{= 3.1\,mg/L}$$

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ASCE STUDENT CHAPTER

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6.24 Given: Benzene, $R = 2$, $C_0 = 10000$ mg/L
$A = 10\,m^2$, $D_x = 1\,m^2/d$, $D_y = 0.1\,m^2/d$, $v_x = 1\,m/d$, $\lambda = 0.0005\,d^{-1}$

Find: [a] time for centroid of mass to reach $x = 75$ m
[b] C_{max} @ 75m
[c] x & y dimensions of plume there

[a] $t = \dfrac{Rx}{v_x} = \dfrac{2(75\,m)}{1\,m/d}$

2

$\boxed{t = 150\,d}$

[b] $C_{max} = \dfrac{C_0 A}{4\pi t (D_x' D_y')^{1/2}} e^{-\lambda t}$

$D_x' = \alpha_x v_x' = \alpha_x \dfrac{v_x}{R} = \dfrac{D_x}{R} = \dfrac{1\,m^2/d}{2} = 0.5\,m^2/d$

4

$D_y' = \alpha_y v_x' = \dfrac{D_y}{R} = \dfrac{0.1\,m^2/d}{2} = 0.05\,m^2/d$

10

$C_{max} = \dfrac{(10000\,mg/L)(10\,m^2)}{4\pi(150\,d)\left[(0.5\,m^2/d)(0.05\,m^2/d)\right]^{1/2}} e^{-(0.0005\,d^{-1})(150\,d)}$

$\boxed{C_{max} = 312\,mg/L}$

[c] Plume dimensions

$\sigma_x = (2 D_x' t)^{1/2}$
$= \left[2(0.5\,m^2/d)(150\,d)\right]^{1/2}$

4

$\sigma_x = 12.2\,m$

$\boxed{x\text{-dimension} = 3\sigma_x = 36.7\,m}$

$\sigma_y = (2 D_y' t)^{1/2}$
$= \left[2(0.05\,m^2/d)(150\,d)\right]^{1/2}$

$\sigma_y = 3.87\,m$

$\boxed{y\text{-dimension} = 3\sigma_y = 11.6\,m}$