CIVE 3331 - ENVIRONMENTAL ENGINEERING
Spring 2003

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Purpose: Lecture #9 CIVE3331

Groundwater

Groundwater supplies 1/3 – 2/3 of the world's drinking water. Typically it is high quality water because of natural filtration. Chemical contaminants are not filtered. Mineral and metal contaminants can be a problem. For example, arsenic in drinking water is a cost problem in the USA, but in other nations (Bangladesh for instance) it causes early mortality, morbidity, and loss of economic productivity. Once a groundwate3r system is contaminated it is extremely difficult to restore.

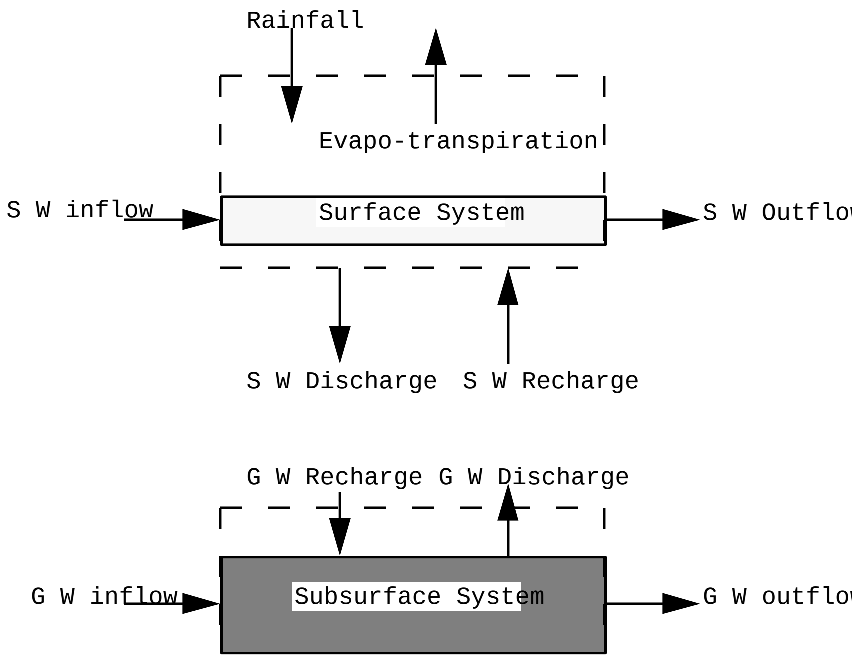


Figure 1. Hydrologic System Diagram

The groundwater system of interest is the shaded (bottom) system in Figure 1. To protect the system engineers must take greta care to understand the “recharge” arrow and reduce contaminants that can go into the system that the system does not “treat” (more later).

Aquifers

Groundwater “reservoirs” are usually called “aquifers”. The word aquifer is used to describe a geologic unit that can store and transmit water. The key words are “geologic”, “store”, and “transmit.” Figure 2 is a sketch of the groundwater system from the surface of the earth down into the gorund.

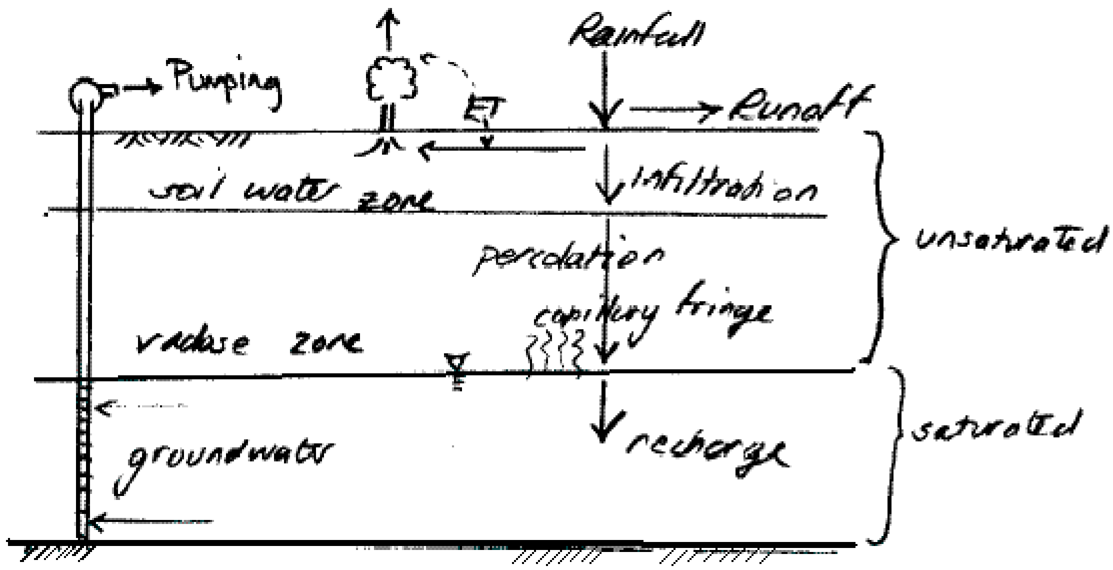


Figure 2. Soil Profile

There are several different "kinds" of aquifers depending on what defines the "upper" flow boundary. Hydrologists classify aquifers into three general types: confined, unconfined (water table), and leaky. All three are idealizations, unconfined are by far the most common, but confined and leaky conceptualizations are used for hydraulic design equally as much as unconfined.

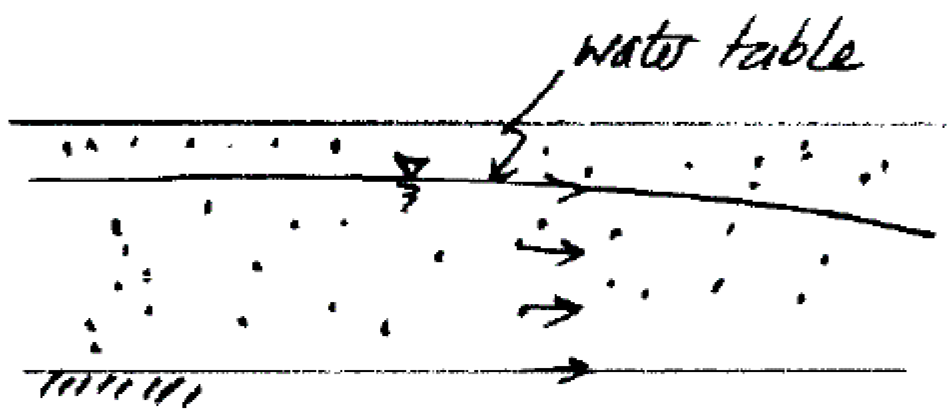


Figure 3. Unconfined Aquifer

An unconfined aquifer has a water table as its upper flow boundary. The water table is sometimes called the phreatic surface. The water table for all practical purposes is the hydraulic grade line, so it is also the piezometric surface (except near wells and springs).

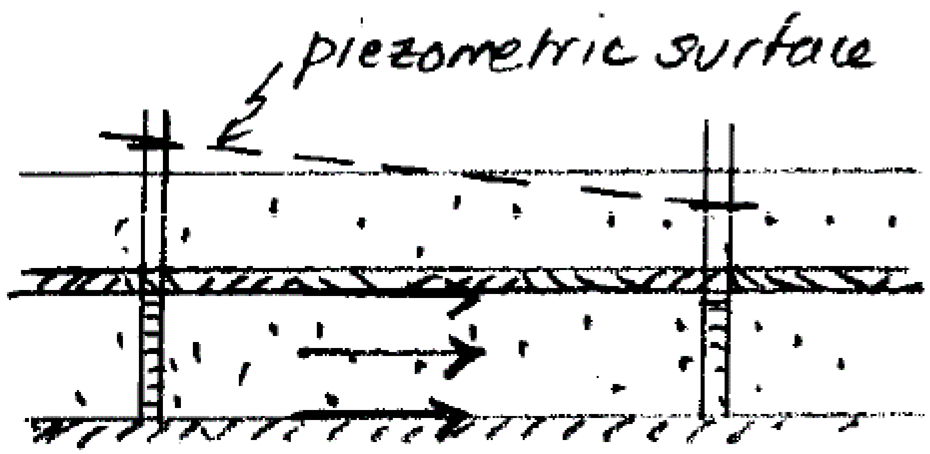


Figure 4. Confined Aquifer

A confined aquifer has a geologic unit as its upper flow boundary (think of a pipe with sand). In Figure 4 the confining layer is the thin horizontal layer (double cross-hatched) above the horizontal flow arrows. The height water would rise in a well is called the piezometric head. A line joining these elevations is the hydraulic grade line, and this line is called the piezometric surface. In a confined aquifer, the piezometric surface is above the elevation of the confining layer (otherwise the aquifer is not a confined aquifer!).

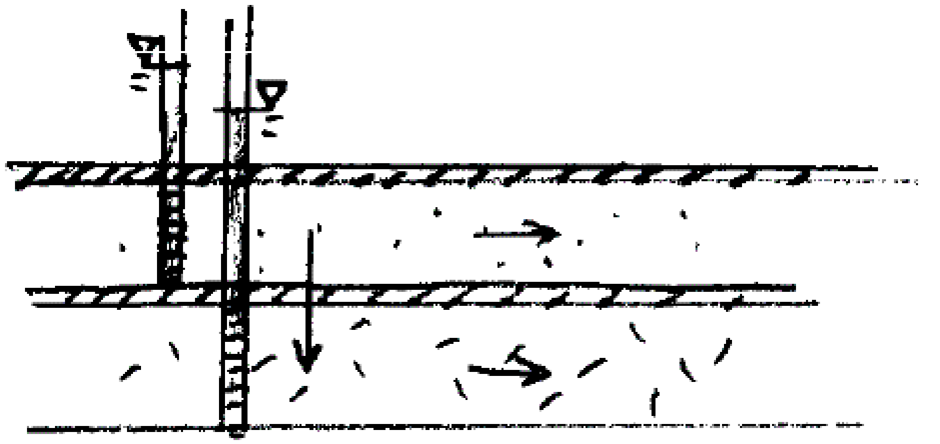


Figure 5. Leaky Aquifer System

A leaky aquifer is like a layer cake of aquifers with flow between layers through a thin boundary unit. In Figure 5, the lower aquifer receives some leakage from the aquifer above it through the confining layer (usually called an aquitard). The leakage rate is usually quite small compared to the horizontal flow but large enough to matter.

In the unconfined aquifer the hydraulic balance equation is a non-linear partial differential equation; while in the other two cases it is a linear partial differential equation. In steady flow, with some assumptions about geometry, the equations can be solved using techniques already discussed.

Porosity

Another property of aquifers is that they can store water. The storage is accomplished because the geologic material is porous – that is the volume it occupies is made up of solids (rock) and void (air). Water can fill this void space and that is how water is stored in an aquifer.

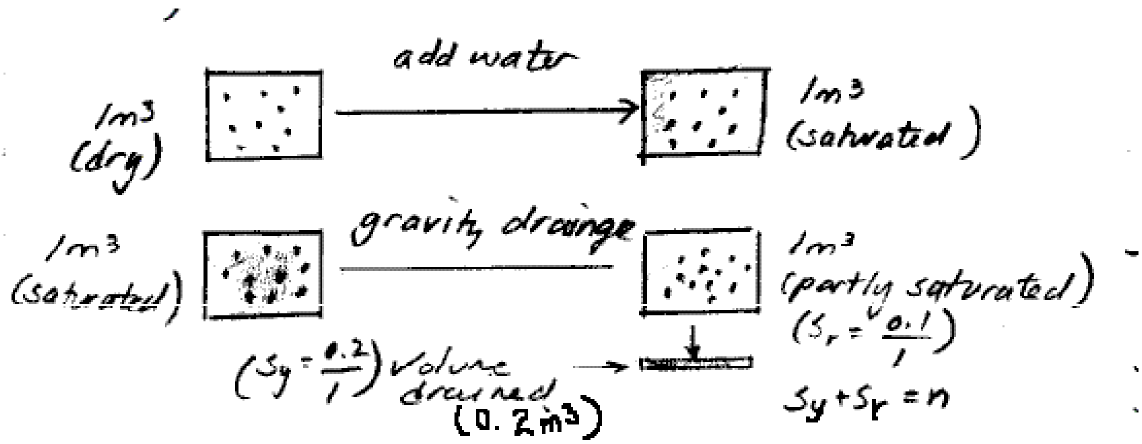


Figure 6. Porosity; Specific Yeild; Specific Retention

In Figure 6, suppose we added 0.3 m^3 to the 1 m^3 of dry aquifer. The water occupies void space $n = \frac{0.3}{1.0} = 0.3$ or 30%. Next suppose that $0.2m^3$ drains, leaving behind $0.1m^3$. The amount left behind (per unit volume of aquifer) is called the specific retention S_r, and the amount drained is called the specific yield S_y.

Hydraulic Gradient

The groundwater equivalent of the hydraulic grade line form hydraulics is called the hydraulic gradient. It is the slope of the piezometric surface in the direction of flow.

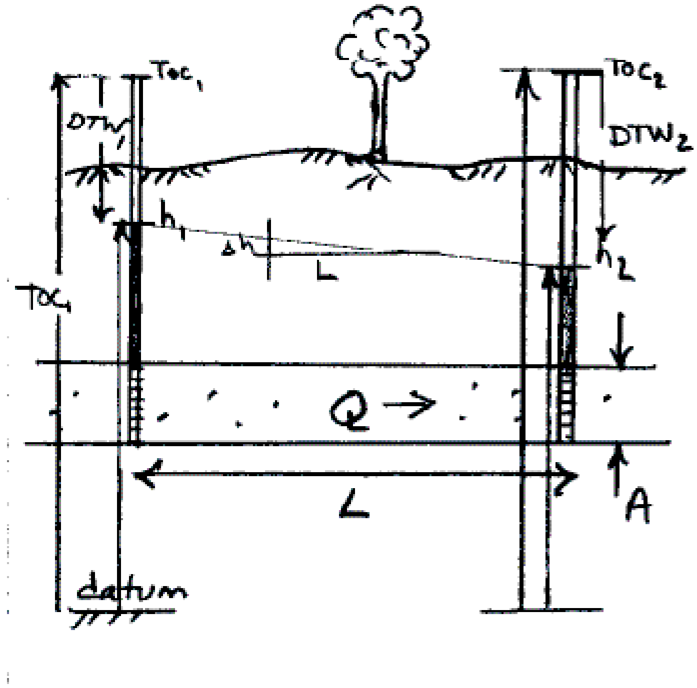


Figure 7. Hydraulic Gradient Sketch

Figure 7 is a sketch explaining how hydraulic gradients are conceptualized. The change in head from well 1 to well 2 is $\Delta h = h_1 - h_2$. The gradient is this value divided by the distance through which the change occurs: $slope = \frac{\Delta h}{L} = \frac{h_1 - h_2}{L}$. Observe that the direction used is the direction of flow (opposite to how derivatives are computed). In practice we usually survey the top of casing elevation (TOC in the picture) and measure depth to water (DTW in the picture). Using these concepts the hydraulic gradient is computed as:

$$h_1 = TOC_1 - DTW_1$$
$$h_2 = TOC_2 - DTW_2$$
$$\frac{\Delta h}{L} = \frac{(TOC_1 - DTW_1) - (TOC_2 - DTW_2)}{L}$$

It is important to remember that groundwater flows from HIGH HEAD to LOW HEAD where $h = \frac{p}{g} + z$.

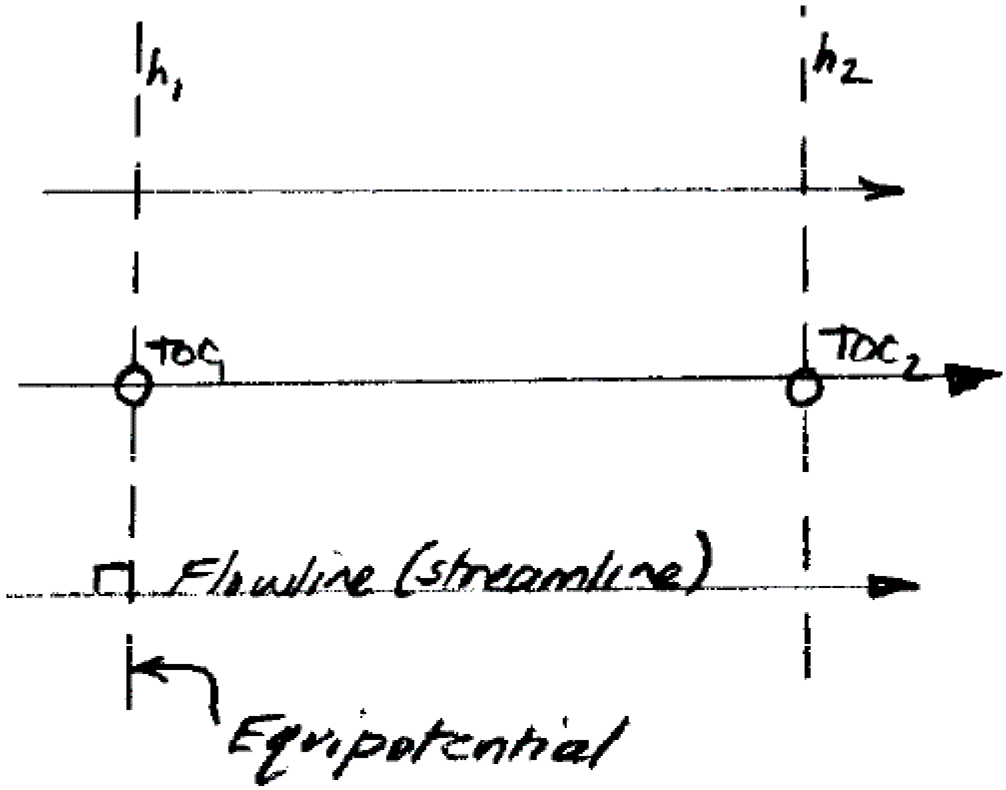


Figure 8. Plan View of Aquifer Hydraulics

In plan view the wells above are arranged as two wells on the same flowline. Equipotentials are contour lines of constant (and same) head. If the system is isotropic, the flowlines and the hydraulic gradient are collinear. Often the flow direction is unknown, and triangulation from three nearby wells is used to find the flow direction.

The triangulation procedure is illustrated in the next three figures. It essentially is a graphical approach to solving the equation of the plane of the piezometric surface that passes through the three wells .

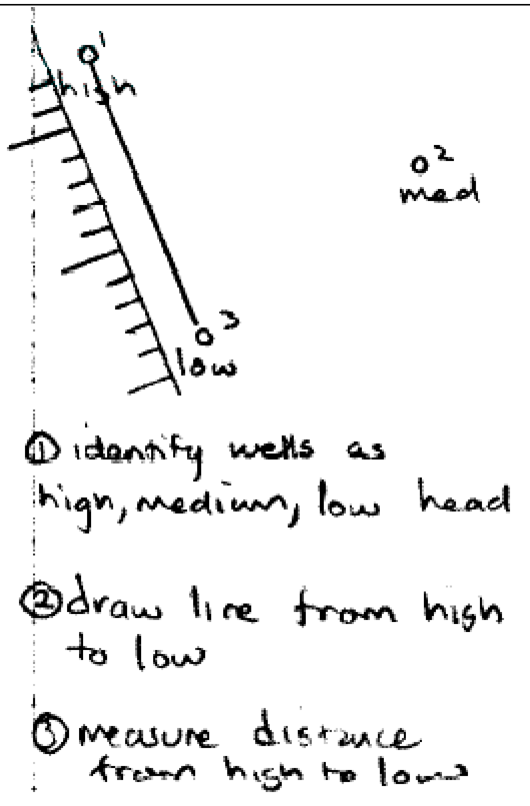


Figure 9. Triangulation Procedure (Picture 1 of 3)

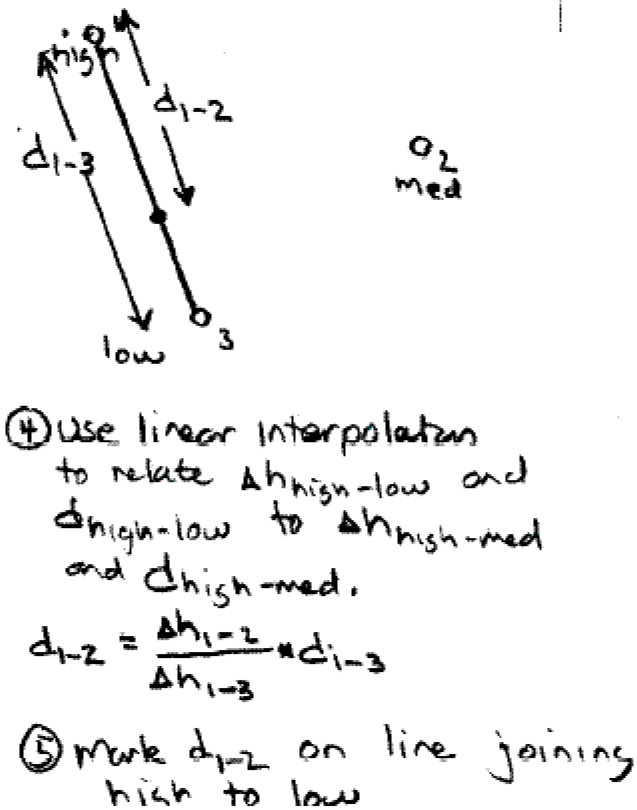


Figure 10. Triangulation Procedure (Picture 2 of 3)

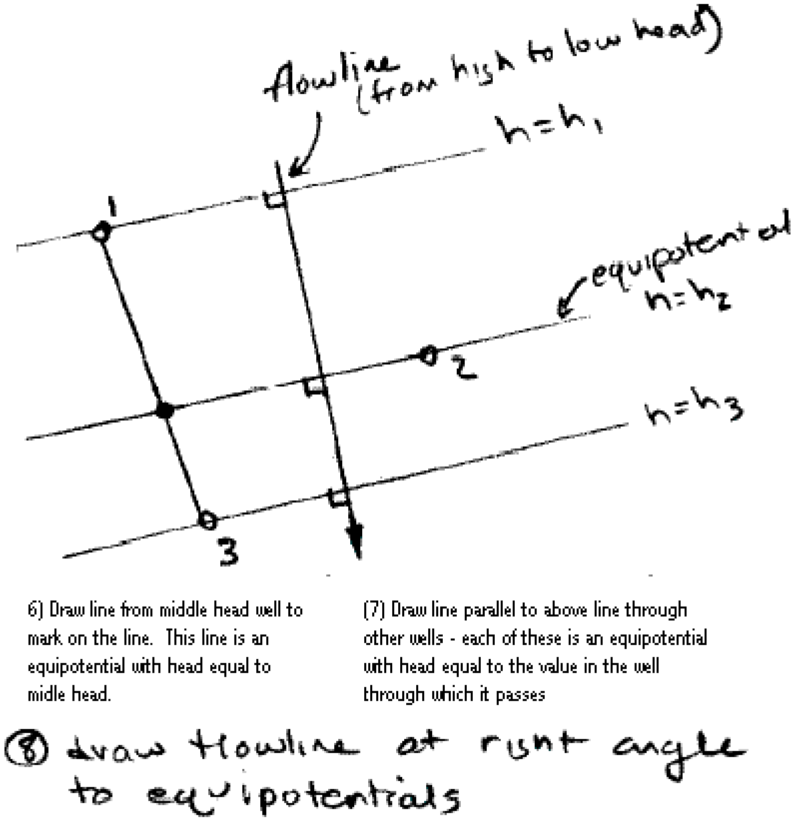


Figure 11. Triangulation Procedure (Picture 3 of 3)

Darcy's Law

Darcy's law is the equation of motion for groundwater flow. Darcy's law equates the hydraulic gradient to flow rate through a constant of proportionality called the hydraulic conductivity.

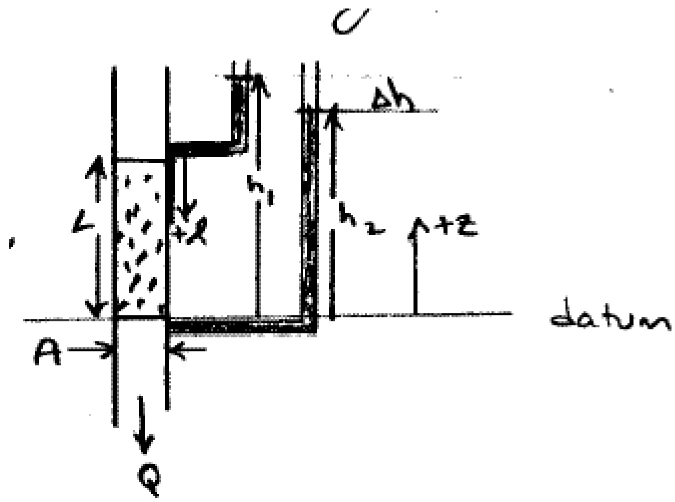


Figure 12. Darcy's Experiment (Permeameter)

The result of Darcy's experiment is

$$Q = KA\frac{h_1 - h_2}{L}$$

Where K is the hydraulic conductivity. Figure 13 examines some additional considerations about Darcy's law.

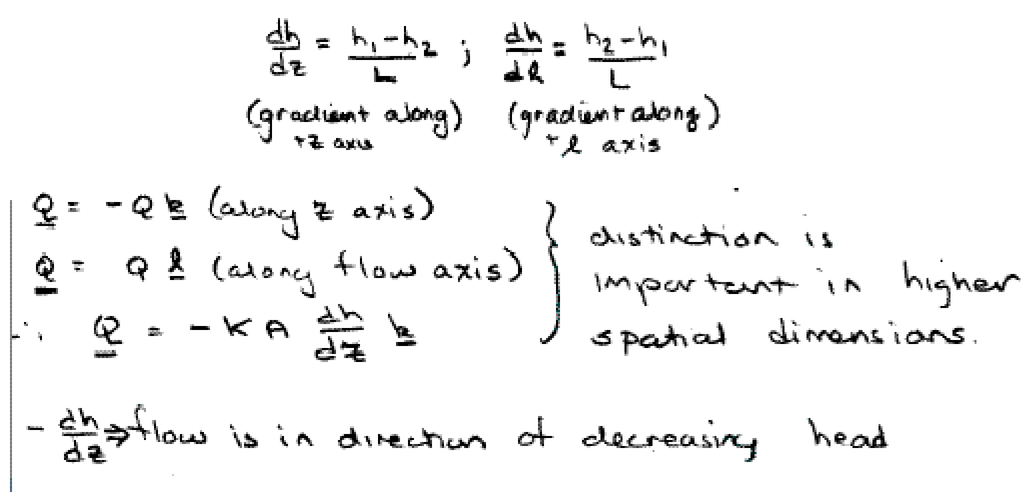


Figure 13. Gradients and Flow Concepts

An example of application of Darcy's law is the following. Suppose two wells 500 meters apart in a 20 foot thick aquifer on the same flowline have a decline in head of 2 meters. If the hydraulic conductivity is 50 m/day, what is the flow through a 1 meter wide portion of aquifer?

Sketch:

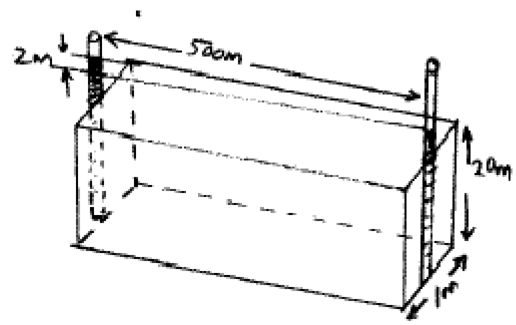


Figure 14. Darcy's Law Application

Darcy's Law:

$$Q = KA\frac{dh}{dl} = (50m/d)(20m)(1m)\frac{(2m)}{(500m)} = 4.0\ m^3/day$$

Average Linear (Pore) Velocity

Recall the definition of fluid velocity from earlier lectures. The velocity that water moves in an aquifer (pore space) is larger by a factor of n than the superficial velocity that we would determine from the ration of discharge to area. In contaminant transport calculation this distinction is critical. The mathematical statement is

$$\frac{Q}{A} = \text{specific discharge}$$

$$\frac{Q}{nA} = \text{average linear velocity; pore velocity}$$

(An alternative way to view this argument is that in pipe flow and surface flows, the porosity is one – of course Darcy's law is not the equation of motion).

The figures below are an attempt to illustrate the differences in the flows:

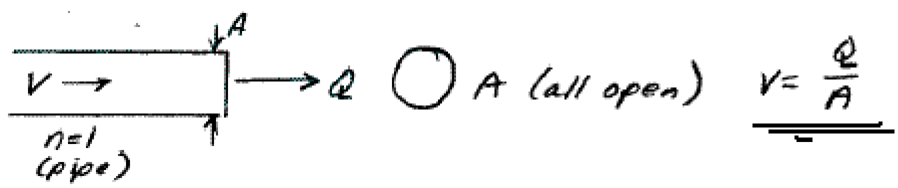


Figure 15. Pipe flow "section" velocity.

In Figure 15, the entire flow area contains water – thus a reasonable value for velocity is the ratio of discharge to area.

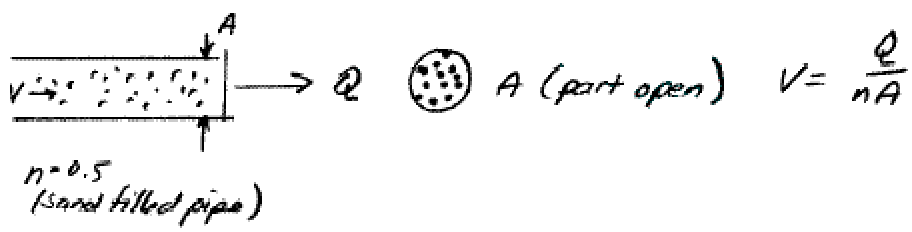


Figure 16. Pipe filled with sand (confined aquifer) velocity

In Figure 16, the flow area contains water and solids. The fraction of area actually available for water to flow is smaller by a factor of *n*, the porosity. Thus the section velocity is the ratio of discharge to actual flow section (*nA*).

The impact of this relationship is illustrated by considering the previous example. If contaminant is detected at the upgradient well, how long will it take for contaminant to reach the downgradient well? Assume n=0.5, neglect dispersion effects.

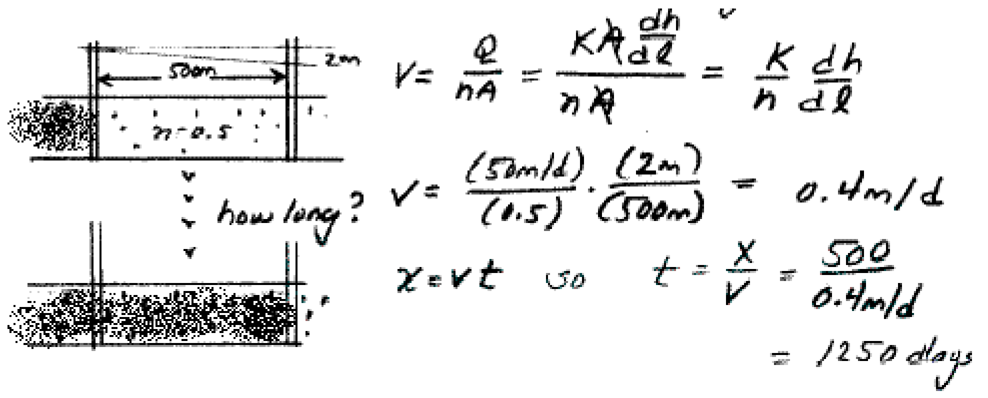


Figure 17. Transport Calculation

Observe that if we accidently used Q/A as the velocity we would calculate a 7 year travel time (not the ~3.5 year) – twice the expected time, a significant error.

Dispersion

Recall the lecture on transport mechanisms, one of the "spreading" mechanisms discussed was dispersion. In real plumes we do not observe sharp fronts. Variations fo velocity at the pore scale, braided flow path, spatial changes in K cause the front to "spread out." This spreading is called dispersion.

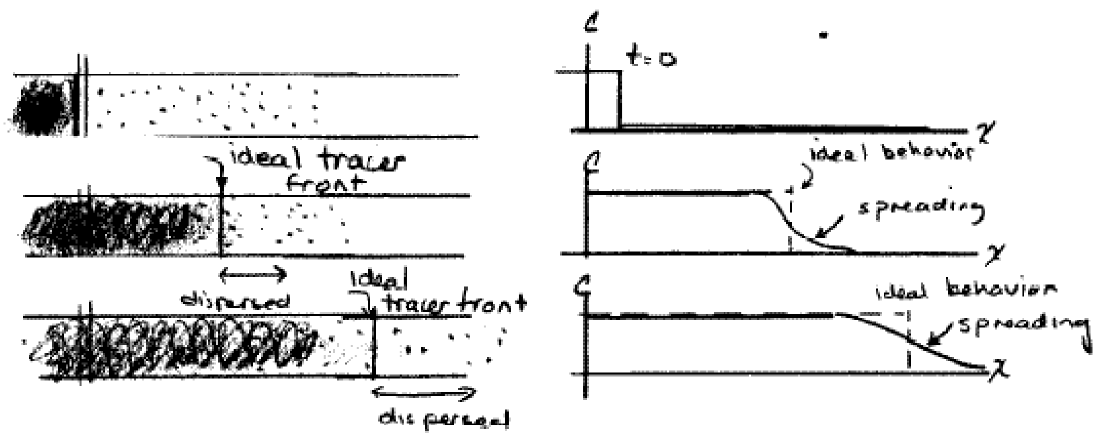


Figure 18. Dispersion Phenomenon

A mass balance of the above one-dimensional system leads to a partial differential equation with the associate initial an boundary conditons,

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} - V\frac{\partial C}{\partial x}$$
$$C(0,t) = C_0 \quad \text{boundary condition}$$
$$C(x > 0,0) = 0 \quad \text{initial condition}$$
$$C(\infty,t) = 0 \quad \text{boundary condition}$$

A solution to the above situation is

$$C(x,t) = \frac{C_0}{2}[erfc(\frac{x - Vt}{\sqrt{2Dt}}) - \exp(\frac{xV}{D})erfc(\frac{x + Vt}{\sqrt{2Dt}})]$$

The term “erfc” is the complimentary error function – it is built into Excel, MathCad, and similar tools – it is also tabulated (it can also be evaluated using tables of normal distributions). The value “D” is called the dispersion coefficient. The error function is just another special function (like ln, log, etc.) but most calculators don’t have the error function key. In this course, the ability to evaluate the function is less important than understanding that it is just another solution to a differential equation.

Retardation

Some pollutants interact with the aquifer solids in a process called adsorption. The simplest model is the linear-instantaneous model. When adsorption is significant the concentration (species) velocity is different (slower) than the water velocity. The ratio of velocities is called the retardation factor.

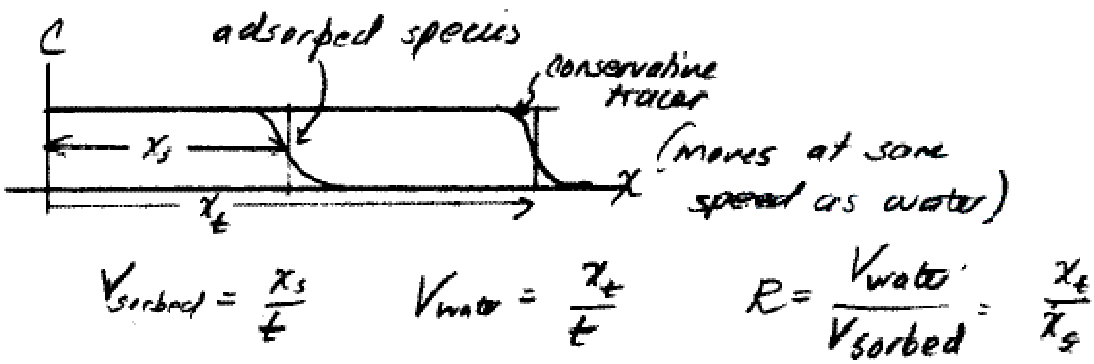


Figure 19. Retardation Concepts

The above pollutant transport model becomes

$$C(x,t) = \frac{C_0}{2}[erfc(\frac{x-\frac{V}{R}t}{\sqrt{2\frac{D}{R}t}}) - \exp(\frac{x\frac{V}{R}}{\frac{D}{R}})erfc(\frac{x+\frac{V}{R}t}{\sqrt{2\frac{D}{R}t}})]$$

Groundwater Hydraulics

Cone of Depression

When wells remove water from an aquifer the hydraulic gradient is affected as is the shape of the water table or piezometric surface. The shape of the surface in the vicinity of the well is a conical solid of revolution called the cone of depression.

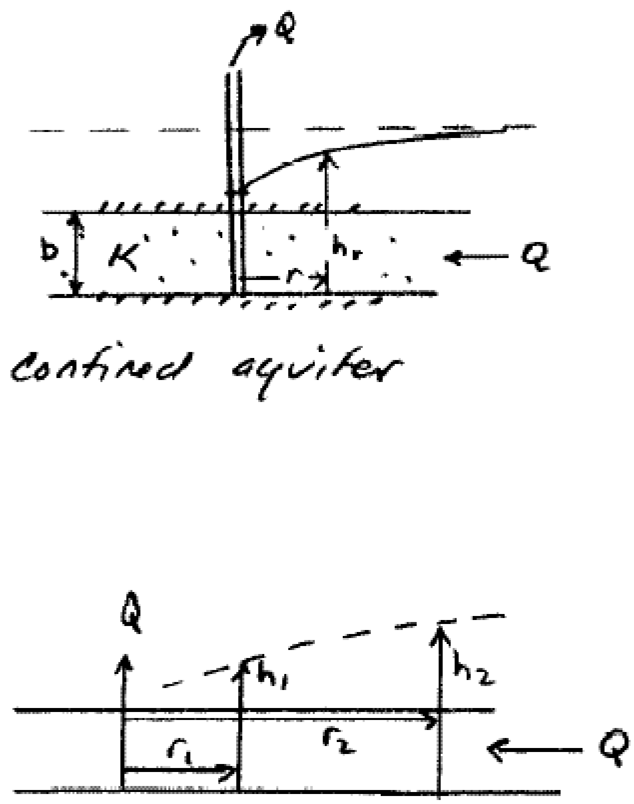
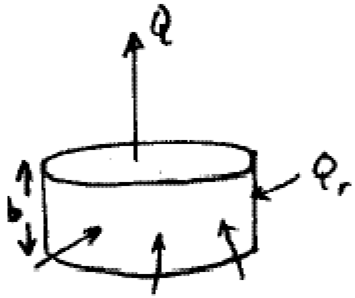


Figure 20. Steady Flow to Well - Confined Aquifer - Profile Sketch



Darcys law

$$Q_r = KA\frac{dh}{dr}$$

$$Q_r = K2\pi r b\frac{dh}{dr}$$

$$Q_r\int_{r_1}^{r_2}\frac{dr}{r} = 2\pi Kb\int_{h_1}^{h_2} dh$$

$$Q_r \ln\left|\frac{r_2}{r_1}\right| = 2\pi Kb(h_2-h_1)$$

mass balance $\Rightarrow Q_r = Q$

$$\therefore\ Q = \frac{2\pi Kb(h_2-h_1)}{\ln(r_2/r_1)}$$

Figure 21. Steady Flow to a Well - Confined Aquifer - Analysis

An unconfined aquifer is analyzed in the same fashion, except the “area” component varies with position from the well.

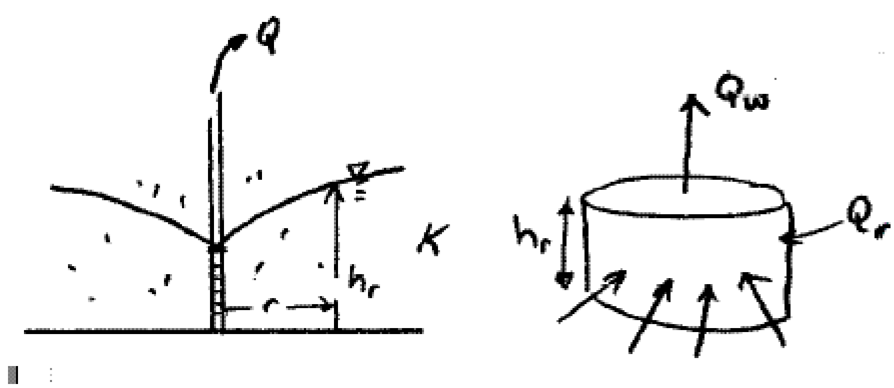


Figure 22. Steady flow to a well - unconfined aquifer - definition sketch

$$Q = KA\frac{dh}{dr} = Kh\,2\pi r\frac{dh}{dr} = K\pi r\frac{dh^2}{dr}$$

$$Q\int_{r_1}^{r_2}\frac{dr}{r} = K\pi\int_{h_1}^{h_2} dh^2$$

$$Q\ln(r_1/r_2) = K\pi(h_2^2 - h_1^2) \Rightarrow Q = \frac{K\pi(h_2^2 - h_1^2)}{\ln(r_2/r_1)}$$

Figure 23. Steady flow to well -unconfined aquifer - analysis

These formulas can be used to estimate *K* if the drawdowns at two locations (drawdown is the difference between the pre-pumping head and the observed head after pumping at a location). From these equations the cone of depression is defined as in Figure 24.

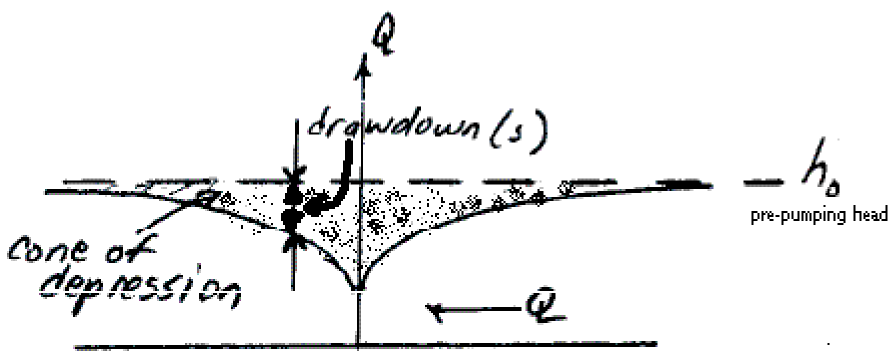


Figure 24. Cone of depression

Pollution Control

One common method to control polutted groundwater is with pump-and-treat strategies. The idea is to capture, extract, treat, and serve the polutted water. The design problem is to locate extraction wells and what pumping rates to choose.

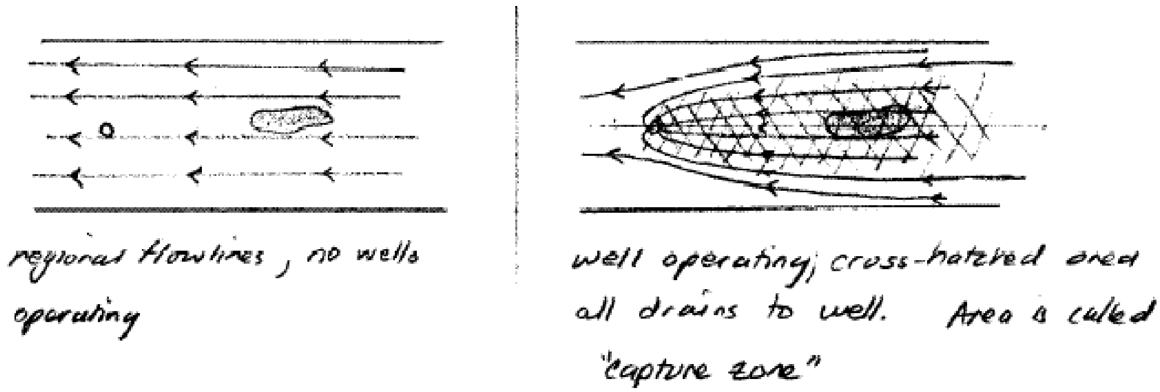


Figure 25. Capture zone concept

A simple model to predict capture zones is

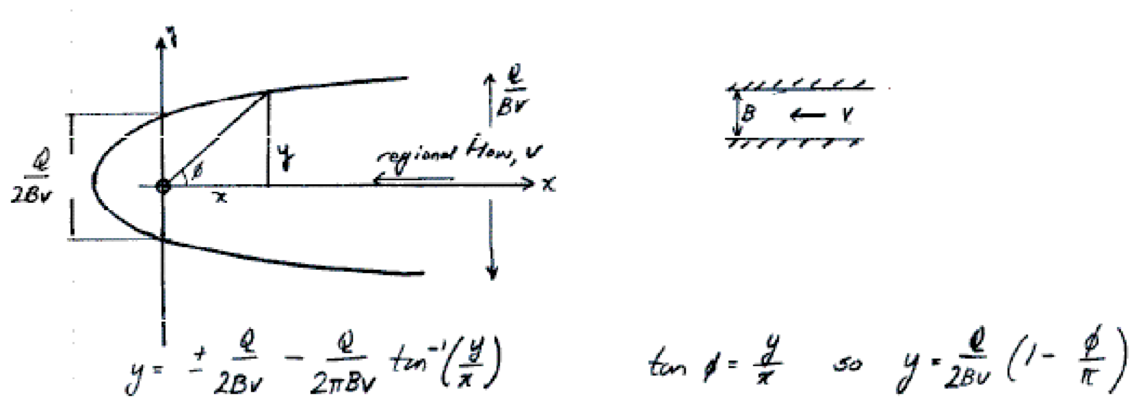


Figure 26. Capture-zone model

We can use these relationships to specify Q and the location of the well relative to the plume. As a rule of thumb, the wells are located so that the computed capture zone just encloses the plume, and the wells are located as close to the edge of the plume as possible without violating the first part of the rule.

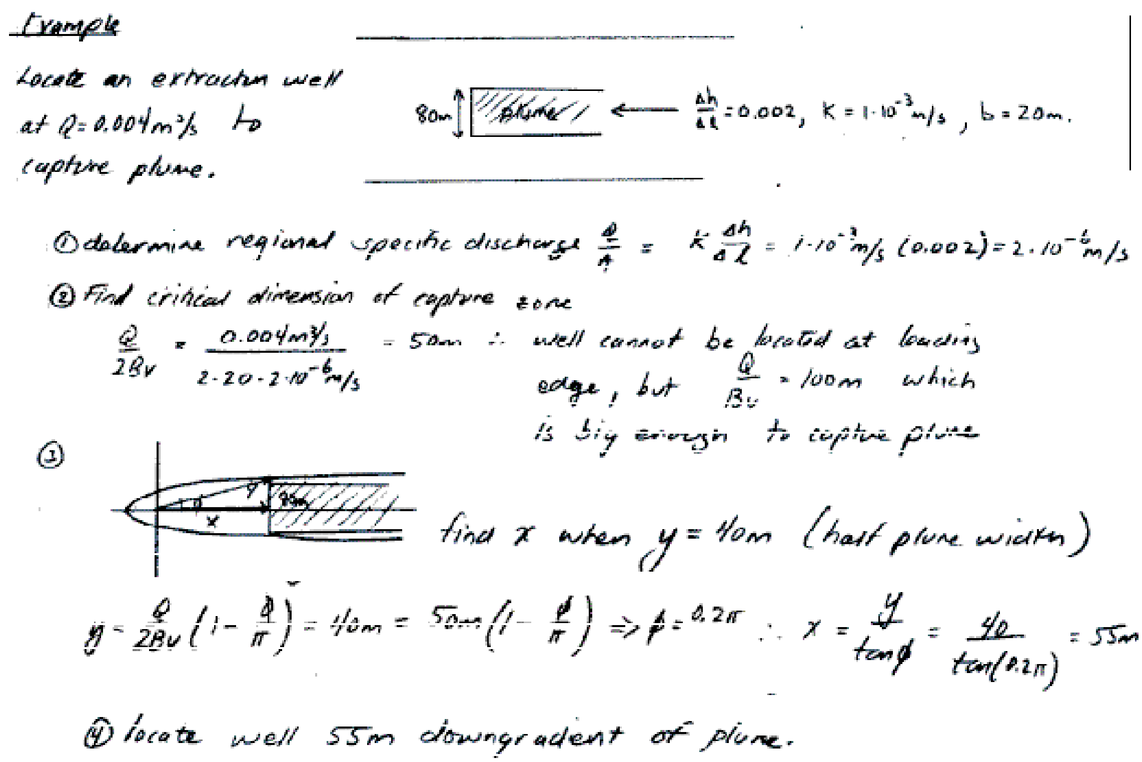


Figure 27. Capture Zone Example

Usually a single well is inadequate because it must pump a lot of unpolluted water to capture the plume, alternatives include a line of wells that widen the capture zone so the wells can be closer to the leading edge of the plume.

For multiple wells the following equations are used

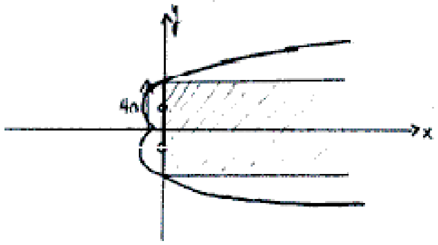
$$y = \frac{Q}{2Bv}\left(n - \frac{1}{\pi}\sum_{i=1}^{n}\phi_i\right)$$

and optimal well spacing is

$n=2$, $\frac{Q}{\pi B v}$

$n=3$, $\frac{(2^{\frac{1}{3}})Q}{\pi B v}$

Example – 2 wells, same problem



$$\frac{2Q}{2vB} = 80 = \frac{Q}{vB} \quad \therefore Q = 80(20)(2\cdot10^{-6}\,m/s)$$

$= 0.0032\,m^3/s$ each well

Optimal spacing $= \frac{80}{\pi} = 25.5m$

Figure 28. Multiple Well Capture Calculations

In addition to pumping other methods include: injection to stabilize the plume (hydrodynamic control), physical barriers (sheet piles, slurry walls, grout curtains, etc.), in-situ reactions (biodegradation, chemical oxidization), vapor extraction ("air stripping" in the aquifer medium).

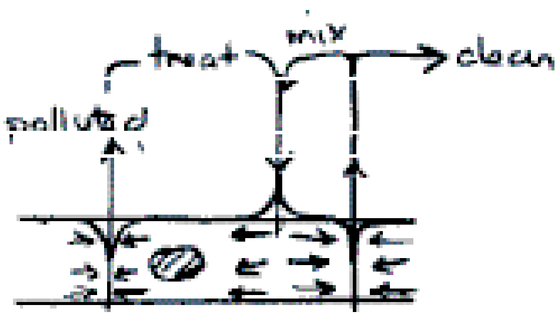


Figure 29. Hydrodynamic plume control schematic

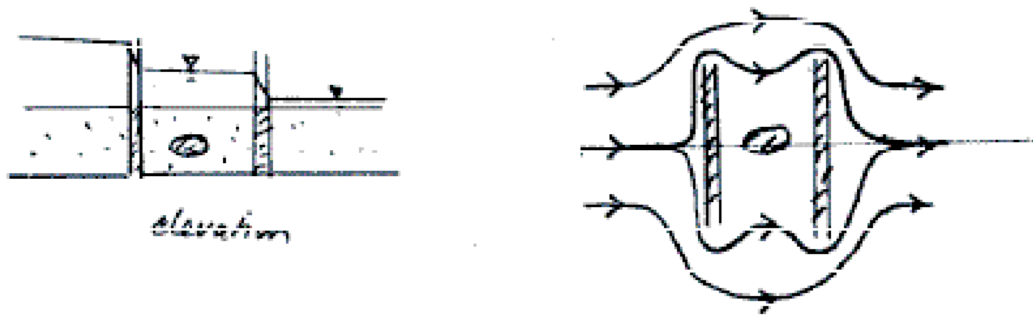


Figure 30. Barrier plume control schematic

Pollutant types

Inorganic – metals, some quite toxic. Mobility depends on pH and DO.

Organic – VOC; NAPL (Non-aqueous phase liquids).

Conventional – BOD exerting wastes, not usually considered a big problem.

NAPLS – very low solubility, thus need tomove a lot of water to remove. DNAPLs will sink to locations were flows are small. Groundwater flow cannot transfer momentum, so DNAPL (and LNAPL) pools are not easily mobilized. The example in the figure illustrates the enormous volumes involved in a best-case-scenario.

Example – illustrates difficulty of NAPL clean-up by pump & treat

$1 m^3$ aquifer, $n = 0.3$, $V = \frac{Q}{nA} = 0.03 m/d$, 30 L TCE, dissolves at 0.1 solubility

a) find mass TCE & dissolved TCE   b) estimate time to flush all TCE

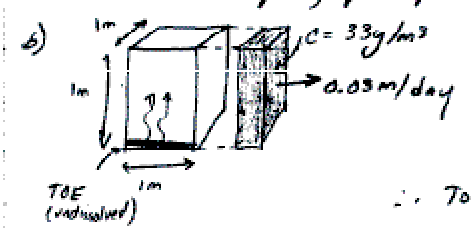
Solubility of TCE = 1100 mg/L, but only 10% dissolves (at a time)

$\therefore$ TCE $= 110 mg/L \times 0.3 m^3 \times 10^3 L/m^3 = 33 \cdot 10^3 mg = 33 g$

Total TCE $30 L \cdot \frac{1.47 kg}{L} = 44.1 kg = 44,100 g$

dissolved: 33 g, pure product: 44,067 g

b)



mass flow TCE out of box is

$(0.03 m/d)(1 m^3)(33 g/m^3) = 0.99 g/day$

$\therefore$ To flush entire 44,100 g

$\frac{44,100 g \; day}{0.99 g} = 44,545 \text{ days} \cdot \frac{1 yr}{365 \text{ day}} \approx 122 \text{ years}$

Figure 31. Contaminant flushing calculations

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