CIVE 3331

Plume Modeling

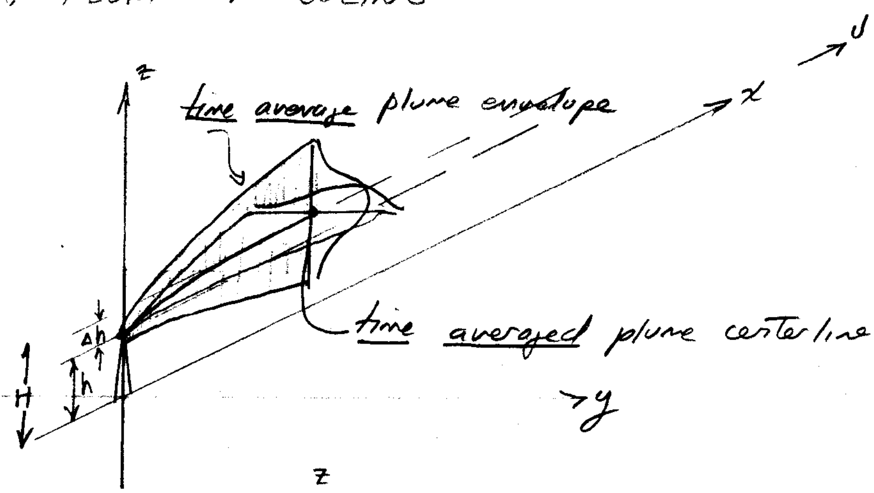
<Scan and append handwritten notes>

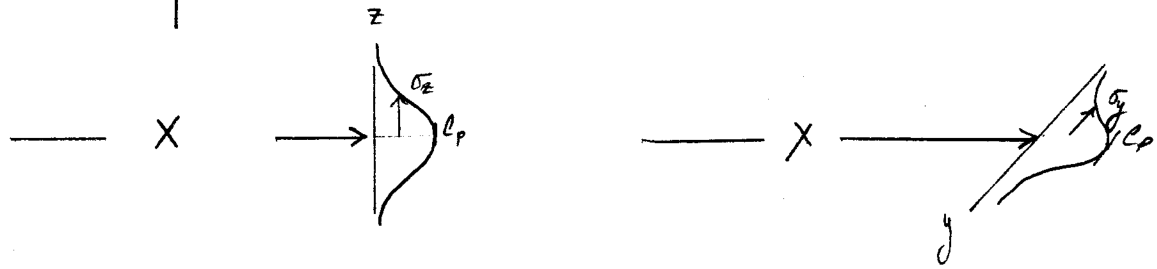
PLUME MODELING

POINT SOURCES ARE A MAJOR CONTRIBUITOR TO AIR POLLUTION ($\sim \frac{1}{2}$)
PLUME MODELS ARE USED TO ESTIMATE THE IMPACT OF EMMISSIONS ON SURROUNDING ENVIRONMENT.

THE MOST ADVANCED MODELS ACCOUNT FOR VARIOUS ATMOSPHERIC PROCESSES, CHEMICAL REACTIONS, AND DETAILED FLOW PHYSICS. — THESE MODELS REQUIRE IMMENSE EXPERTISE TO USE AND ARE BEYOND THE SCOPE OF THIS CLASS.

A SIMPLE APPROACH THAT CAPTURES A LOT OF THE OBSERVED BEHAVIOR OF CERTAIN CONTAMINANTS IS GAUSSIAN-PLUME MODELING





H - Effective stack height
h - actual stack height
Δh - average plume rise

Assume: constant emissions rate
constant windspeed, direction; Uniform in elevation
conservative pollutant
terrain can be approximated by a plane.

Resulting single source equation is

$$c(x,y,z) = \frac{\dot{M}}{\pi U_H \sigma_y \sigma_z} \exp\left(\frac{-(H-z)^2}{2\sigma_z^2}\right) \exp\left(\frac{-(y)^2}{2\sigma_y^2}\right)$$

$\dot{M}$ - emission rate of pollutants ($\mu g/s$)
x - distance downwind
y - distance cross wind
z - elevation
σ_y - crosswind (transverse-y) dispersion coefficient
σ_z - elevation (transverse-z) dispersion coefficient
U_H - effective stack height windspeed.

$\left(\frac{U_H}{U_a}\right) = \left(\frac{H}{Z_a}\right)^p$ p depends on terrain and atmospheric stability classifications

σ_y, σ_z are from table lookup or curve fit - also based on atmospheric stability classifications

Determining peak concentrations is easiest using a computer spreadsheet to plot values based on the equation (profiles).

Alternatively the dimensionless graph of $c(x,y,z)$ can be used to estimate the peak concentration and location (pg 417)

(prob 7-20 as example)

The EPA Support Center for Regulatory Air Models contains many downloadable versions of air emissions models - some are gaussian plume models, others are more complex

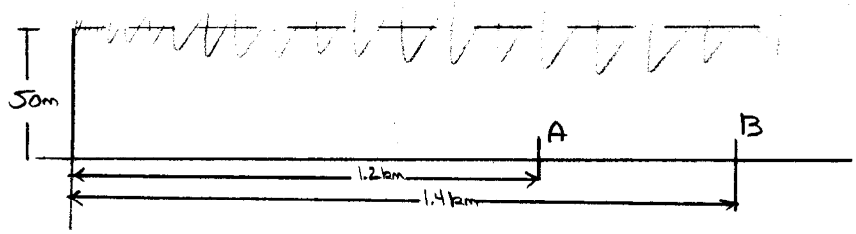
Additional modifications to the simple gaussian plume model include

plume rise corrections

temperature inversion corrections

Gaussian models can also be integrated in space to produce line-source models (such as a highway)

7.20



Step ① determine atmospheric stability classification
Class D — overcast conditions

a) Use chart pg 417.
$x = 1.0$ km. $\quad C_{max}\left(\frac{UH}{Q}\right) = 5 \cdot 10^{-5}$
location Ⓐ will have greater pollution level.

b) Clear sky, night class F (or maybe E) $U < 5$ m/s
x moves away from stack
$x \approx 2.0 - 3.5$ km

c) Location Ⓑ will have greater pollution level than location Ⓐ under conditions in part (b).

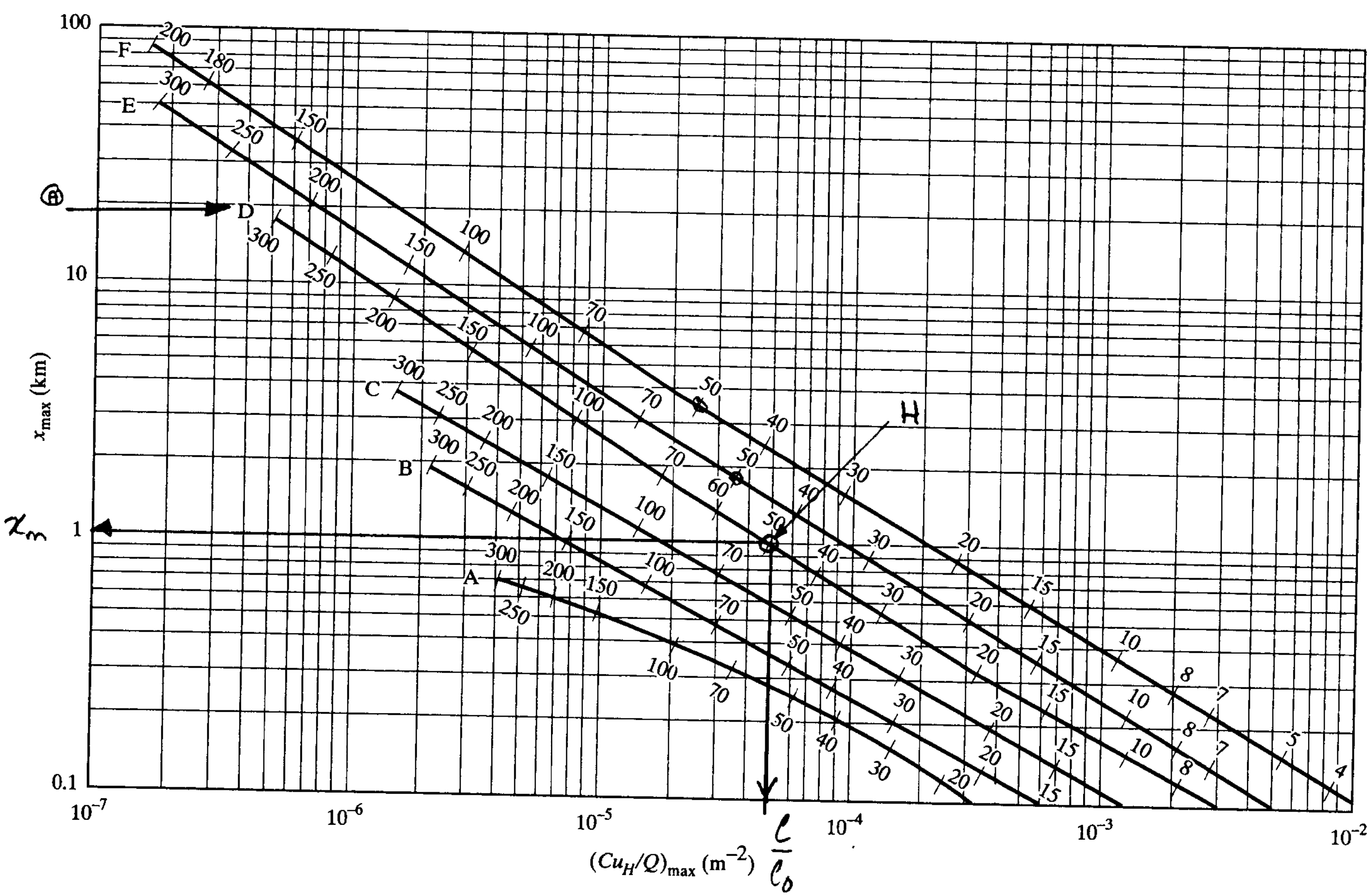
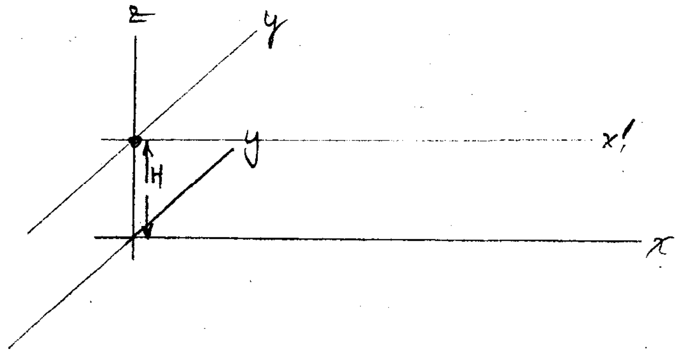


FIGURE 7.50 To determine the downwind concentration peak, enter the graph at the appropriate stability classification and effective stack height (numbers on the graph in meters) and then move across to find the distance to the peak, and down to find a parameter from which the peak concentration can be found (Turner, 1970).

Gaussian Dispersion Model – Steady State, Usual Air assumptions



Point source in x', y, z

$$c(x', y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right) \exp\left(-\frac{1}{2}\frac{z^2}{\sigma_z^2}\right)$$

Shift axis to x, y, z

$$c(x, y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right) \exp\left(-\frac{1}{2}\frac{(z-H)^2}{\sigma_z^2}\right)$$

Now add ground level reflection

$$c(x, y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right)\left[\exp\left(-\frac{1}{2}\frac{(z-H)^2}{\sigma_z^2}\right) + \exp\left(-\frac{1}{2}\frac{(z+H)^2}{\sigma_z^2}\right)\right]$$

Now add mixing-layer reflection(s)

$$c(x, y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right)\left\{\left[\exp\left(-\frac{1}{2}\frac{(z-H)^2}{\sigma_z^2}\right) + \exp\left(-\frac{1}{2}\frac{(z+H)^2}{\sigma_z^2}\right)\right] + \sum_{\substack{i=-\infty \\ i\neq 0}}^{\infty}\left(\exp\left(-\frac{1}{2}\frac{(z-H+2iL)^2}{\sigma_z^2}\right) + \exp\left(-\frac{1}{2}\frac{(z+H+2iL)^2}{\sigma_z^2}\right)\right)\right)$$

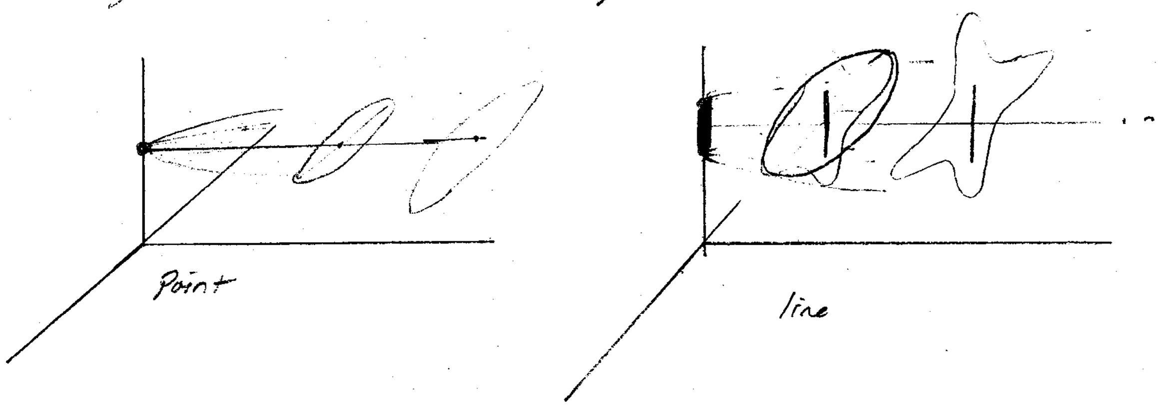
or more compactly:

mixing height (layer)

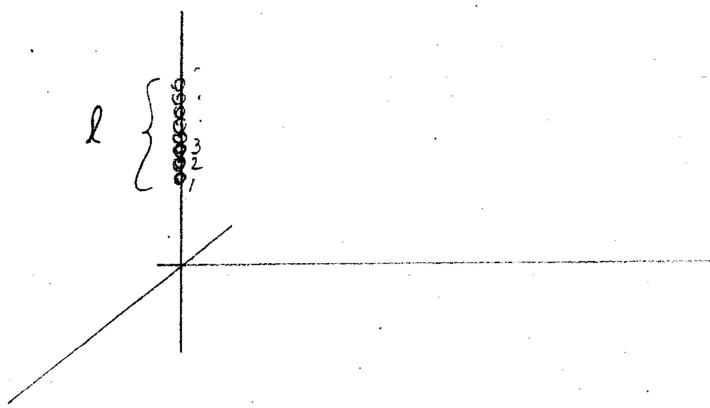
$$c(x, y, z) = \frac{Q}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{1}{2}\frac{y^2}{\sigma_y^2}\right) * \sum_{i=-\infty}^{\infty}\left[\exp\left(-\frac{1}{2}\frac{(z-H+2iL)^2}{\sigma_z^2}\right) + \exp\left(-\frac{1}{2}\frac{(z+H+2iL)^2}{\sigma_z^2}\right)\right]$$

Line Source concept –

assume (for your thesis) that source behaves like a finite line segment instead of a point

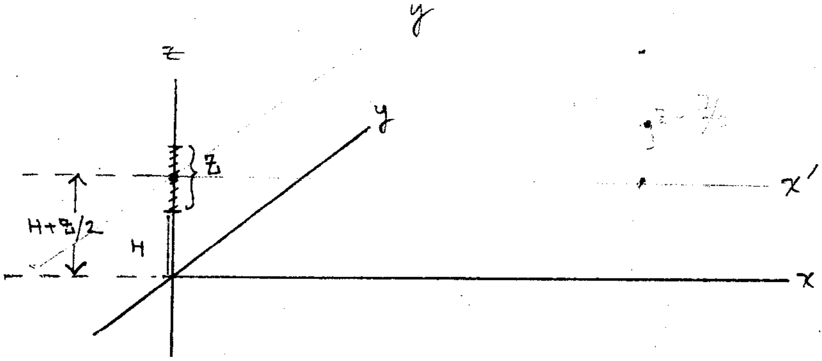


mathematically we "sum up" the contribution of an infinite number of point sources, each with contribution dC



$$C(x,y,z) = C_1(\ ) + C_2(\ ) + C_3(\ ) + \cdots$$
$$= \int_l dC(x,y,z)$$

Line Source Model



line source in x', y, z system

$$dc = \frac{dQ\, dz}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{y^2}{2\sigma_y^2}\right)\left\{\exp\left(-\frac{z^2}{2\sigma_z^2}\right)\right\}$$

$$dQ = \left(\frac{Q}{Z}\right) \qquad K = \frac{dQ}{2\pi u \sigma_y \sigma_z} \exp\left(-\frac{y^2}{2\sigma_y^2}\right)$$

$$c = \int_{-Z/2}^{Z/2} dc = \underbrace{K \int_{z-Z/2}^{z+Z/2} \exp\left(-\frac{z^2}{2\sigma_z^2}\right) dz}$$

examine this integral

$$= \int_0^{z+Z/2} \exp\left(-\frac{z^2}{2\sigma_z^2}\right) dz - \int_0^{z-Z/2} \exp\left(-\frac{z^2}{2\sigma_z^2}\right) dz$$

let $\beta = \frac{z}{\sqrt{2}\sigma_z}$ $\quad \frac{d\beta}{dz} = \frac{1}{\sqrt{2}\sigma_z}$

$\therefore \sqrt{2}\sigma_z\, d\beta = dz$

$z=0, \ \beta = 0$

$z = \frac{Z}{2}, \ \beta = \frac{z+Z}{2\sqrt{2}\sigma_z}$

$$\int_0^{\frac{z+Z}{2\sqrt{2}\sigma_z}} \exp(-\beta^2)\sqrt{2}\sigma_z\, d\beta$$

$$\left[\sqrt{2}\sigma_z \cdot \frac{\sqrt{\pi}}{2} \cdot \underbrace{\frac{2}{\sqrt{\pi}} \int_0^{\frac{Z}{2\sqrt{2}\sigma_z}} \exp(-\beta^2)\, d\beta}_{\mathrm{erf}\left(\frac{z+Z}{2\sqrt{2}\sigma_z}\right)}\right] - \left[\sqrt{2}\sigma_z \cdot \frac{\sqrt{\pi}}{2} \cdot \underbrace{\frac{2}{\sqrt{\pi}} \int_0^{z-\frac{Z}{2\sqrt{2}\sigma_z}} \exp(-\beta^2)\, d\beta}_{\cancel{-}\,+\ \mathrm{erf}\left(\frac{z-Z}{2\sqrt{2}\sigma_z}\right)}\right]$$

1 mg/sec

$Z = 10\,m$ $\quad dQ = \frac{1\,mg/s}{10\,m} = .1\,mg/s/m$

4

Collect terms & clean up model

$$C = \frac{Q}{Z} \cdot \frac{1}{\cancel{\sqrt{2\pi}} \cdot \sqrt{2\pi}\, N \sigma_y \cancel{\sigma_z}} \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \left[\frac{\cancel{\sqrt{2}\sigma_z} \cdot \cancel{\sqrt{\pi}}}{2} \operatorname{erf}\left(\frac{z + \frac{Z}{2}}{\sqrt{2}\,\sigma_z}\right) \cancel{+}\, - \frac{\cancel{\sqrt{2}\sigma_z \sqrt{\pi}}}{2} \operatorname{erf}\left(\frac{z - \frac{Z}{2}}{\sqrt{2}\,\sigma_z}\right) \right]$$

$$C = \frac{Q}{Z} \cdot \frac{1}{\sqrt{2\pi}\, U \sigma_y} \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \left[\frac{1}{2} \operatorname{erf}\left(\frac{z + \frac{Z}{2}}{\sqrt{2}\,\sigma_z}\right) \cancel{+}\, - \frac{1}{2} \operatorname{erf}\left(\frac{z - \frac{Z}{2}}{\sqrt{2}\,\sigma_z}\right)\right]$$

Now shift back to x, y, z system

$$C(x,y,z) = \frac{Q}{Z} \cdot \frac{1}{\sqrt{2\pi}\, U \sigma_y} \exp\left(-\frac{y^2}{2\sigma_y^2}\right) \left(\frac{1}{2}\right) \left(\operatorname{erf}\left(\frac{z + \frac{Z}{2} - H}{\sqrt{2}\,\sigma_z}\right) \cancel{+}\, - \operatorname{erf}\left(\frac{z - \frac{Z}{2} - H}{\sqrt{2}\,\sigma_z}\right)\right)$$

Then will need to add reflections, each plane will produce 4 terms.

Something like:

$$\sum_{i=-\infty}^{\infty} \frac{1}{2}\left[\operatorname{erf}\left(\frac{z + \frac{Z}{2} - H + 2iL}{\sqrt{2}\,\sigma_z}\right) \cancel{+}\, - \operatorname{erf}\left(\frac{z - \frac{Z}{2} - H + 2iL}{\sqrt{2}\,\sigma_z}\right)\right] + \frac{1}{2}\left[\operatorname{erf}\left(\frac{z + \frac{Z}{2} + H + 2iL}{\sqrt{2}\,\sigma_z}\right) \cancel{+}\, - \operatorname{erf}\left(\frac{z - \frac{Z}{2} + H + 2iL}{\sqrt{2}\,\sigma_z}\right)\right]$$

→ You will need to check the mathematics, best tool is Mathematica – it can handle the symbol manipulation –

Also test that with $\frac{Q}{ZU} = 1$ then $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C(x,y,z)\, dz\, dy = 1$

i from −2 to 2 should be equivalent to the 4 terms used in your model.

Excel has erf(z) built in, but it does not handle negative arguments correctly or large arguments correctly.

Check with Dr. Rafai, get her opinion on adding this component – I think it is relatively easy ~~– if 7 has to be less. to fit~~