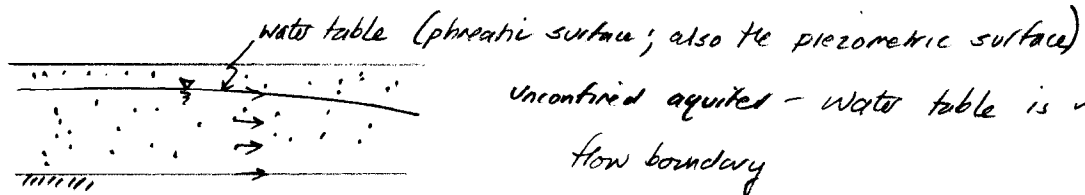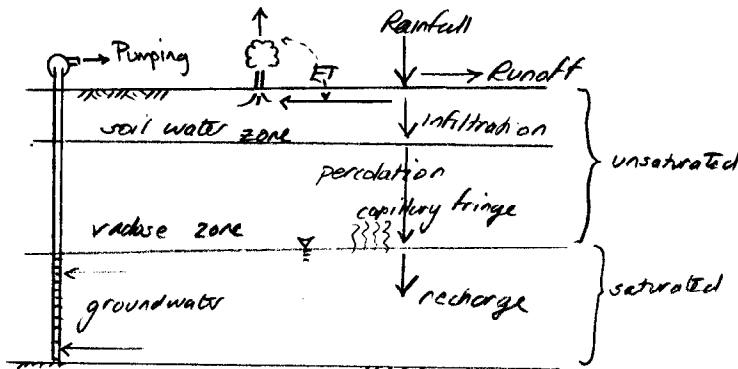Groundwater

Groundwater supplies 1/3-2/3 of world's drinking water

Typically high quality; natural filtration; chemical contaminants not filtered
   Once contaminated - difficult to restore

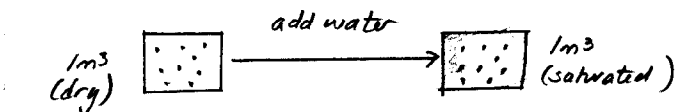aquifers - geologic unit that can store and transmit water.

water table (phreatic surface; also the piezometric surface)

Unconfined aquifer - water table is upper
                           flow boundary

piezometric surface
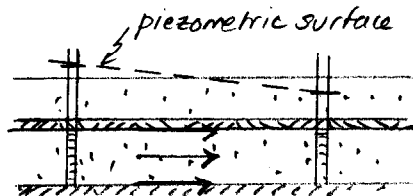
confined aquifer - geologic unit is upper
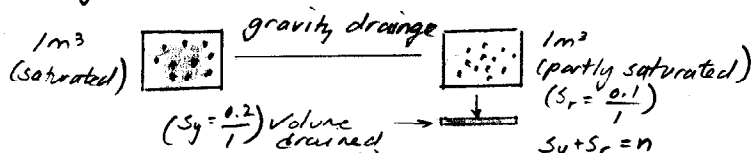(no water table in   flow boundary
   a confined aquifer)
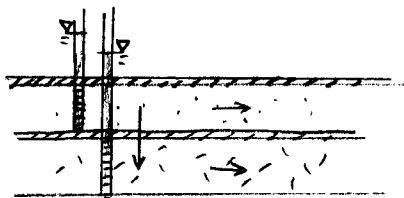
leaky aquifer - flow between layers
                    through a thin boundary unit

porosity - ratio of void volume to bulk volume
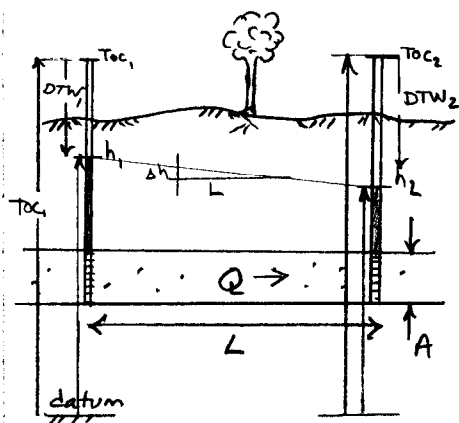

add water

$1m^3$
(dry) $1m^3$
                                   (saturated)

Suppose add $0.3m^3$ water. Water
occupies void space. $n = \frac{0.3}{1.0} = 0.3$

gravity drainage

$1m^3$
(saturated) $1m^3$
                                   (partly saturated)
                                   $(S_r = \frac{0.1}{1})$

$(S_y = \frac{0.2}{1})$ Volume
          drained

$S_y + S_r = n$

Suppose $0.2m^3$ drains, $0.1m^3$ left
behind. Amount left behind is
specific retention (S_r) amount drained is
specific yield (S_y)

Hydraulic gradient (groundwater equivalent of hydraulic grade line) is the _slope_ of the _piezometric_ surface in the _direction of flow_.

$$\Delta h = h_1 - h_2 \; ; \; slope = \frac{\Delta h}{L}$$
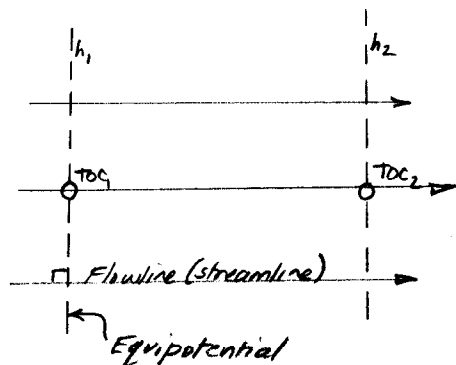
In practice one usually measures depth to water from the top of well (Toc, top of casing).

$$\therefore h_1 = Toc_1 - DTW_1$$
$$h_2 = Toc_2 - DTW_2$$
$$\frac{\Delta h}{L} = \frac{(Toc_1 - DTW_1) - (Toc_2 - DTW_2)}{L}$$

Groundwater _flows_ from _high head_ to _low head_ where: $h = \frac{p}{\gamma} + z$
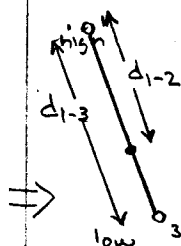

In plan view the wells above are arranged as two wells on the same flowline. Equipotentials are contour lines of constant head. If the system is isotropic, the flowlines and hydraulic gradient are collinear.
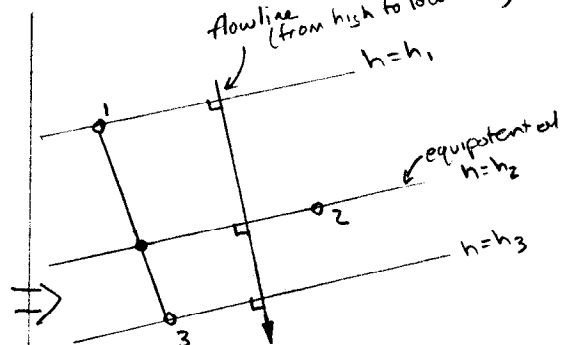
Often the flow direction is unknown, and triangulation from 3 nearby wells is used to find direction
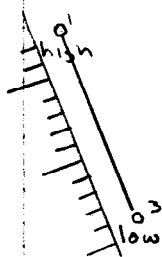
① identify wells as high, medium, low head

② draw line from high to low

③ measure distance from high to low

④ Use linear interpolation to relate $\Delta h_{high-low}$ and $d_{high-low}$ to $\Delta h_{high-med}$ and $d_{high-med}$.

$$d_{1-2} = \frac{\Delta h_{1-2}}{\Delta h_{1-3}} \times d_{1-3}$$

⑤ Mark d_{1-2} on line joining high to low

⑥ Draw line from middle head well to mark on line. — This line is an equipotential with head equal to middle head.

⑦ Draw line parallel to above through other wells
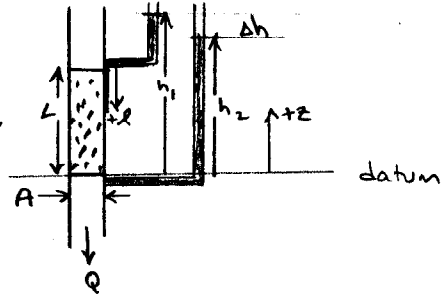
⑧ draw flowline at right angle to equipotentials

Darcy's law — is the equation of motion for groundwater flow

$$Q = KA \frac{h_1 - h_2}{L}$$

K is called the hydraulic conductivity

$$\frac{dh}{dz} = \frac{h_1 - h_2}{L} \quad ; \quad \frac{dh}{d\ell} = \frac{h_2 - h_1}{L}$$

(gradient along)   (gradient along)
+z axis   +ℓ axis

$\underline{Q} = -Q\,\hat{z}$ (along z axis)

$\underline{Q} = Q\,\hat{\ell}$ (along flow axis)

$\therefore \quad \underline{Q} = -KA \frac{dh}{dz}\,\hat{z}$

} distinction is important in higher spatial dimensions.

$-\frac{dh}{dz} \Rightarrow$ flow is in direction of decreasing head
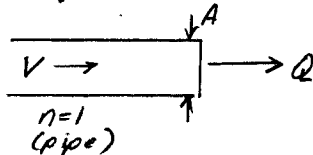
Confined aquifer example
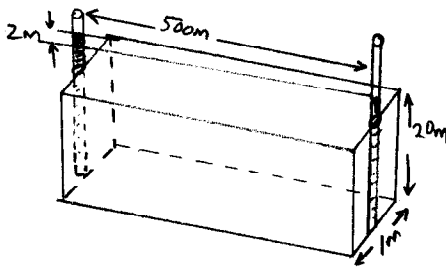
$\Delta h = 2m$ between wells

$L = 500m$
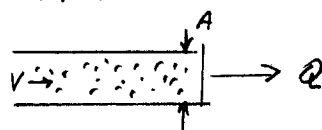
$K = 50 m/d$ find Q through 1m wide portion of aquifer

$$Q = KA \frac{dh}{d\ell} = (50m/d)(20m)(1m)\frac{(2m)}{(500m)} = 4.0 \, m^3/day$$

average linear (pore) velocity

$V \rightarrow$ $\rightarrow Q$ $V = \frac{Q}{A}$ $\bigcirc$ A (all open)

$n=1$
(pipe)

$V \rightarrow$ $\rightarrow Q$ $V = \frac{Q}{nA}$ A (part open)
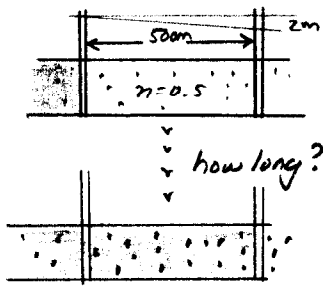
$n = 0.5$
(sand filled pipe)

The internal velocity that water moves with in pore space is larger by a factor of n than the superficial velocity determined by the ratio of discharge to area. In contaminant transport calculations this distinction is critical!

$\frac{Q}{A}$ = specific discharge $\frac{Q}{nA}$ = average linear velocity

Consider the previous example. If contaminant is detected at upgradient (upstream) well how long will it take for contaminant to reach the downgradient well? Assume $n = 0.5$, Neglect any dispersion effects

$$V = \frac{Q}{nA} = \frac{KA\frac{dh}{d\ell}}{nA} = \frac{K}{n}\frac{dh}{d\ell}$$

$$\therefore V = \frac{(50m/d)}{(0.5)} \cdot \frac{(2m)}{(500m)} = 0.4 m/d$$

$$x = Vt \quad \text{so} \quad t = \frac{x}{V} = \frac{500}{0.4m/d} = 1250 \, days \left(3.4 \, years\right)$$

Observe that if one accidently used $\frac{Q}{A}$ as the velocity would calculate a 7 year travel time — twice the "correct" time; a significant error.

Dispersion

Real groundwater contaminant plumes do not have sharp fronts. Variations in velocity at the pore scale, twisted flow path, spatial changes in K cause the front to spread. This spreading is called dispersion
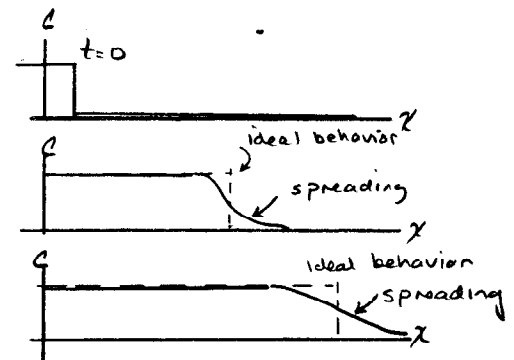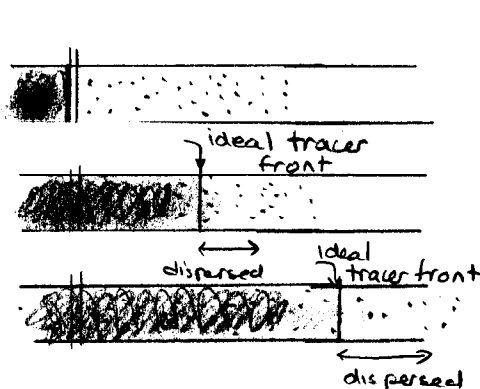
A model for the case depicted above to predict the concentration of pollutant is

$$C(x,t) = \frac{C_0}{2}\left[erfc\left(\frac{x-Vt}{\sqrt{2Dt}}\right) - exp\left(\frac{xV}{D}\right)erfc\left(\frac{x+Vt}{\sqrt{2Dt}}\right)\right]$$

$erfc(\cdot)$ is the complimentary error function (built into Excel)

D is called the dispersion coefficient

Retardation

Some pollutants interact with the aquifer solids in a process called adsorption. The simplest adsorption model is linear-instantaneous. When adsorption is significant the concentration velocity is different (slower) than the water velocity. The ratio is called the retardation factor
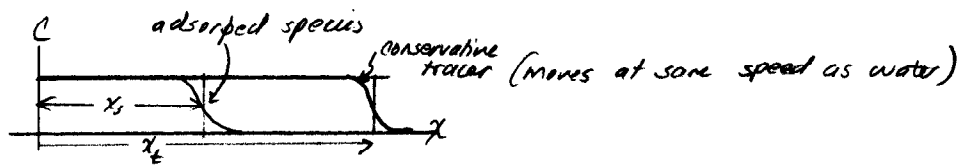
$$V_{sorbed} = \frac{x_s}{t} \qquad V_{water} = \frac{x_t}{t} \qquad R = \frac{V_{water}}{V_{sorbed}} = \frac{x_t}{x_s}$$

The simple ogata-banks solution becomes: $C(x,t) = \frac{C_0}{2}\left[erfc\left(\frac{x - \frac{V}{R}t}{\sqrt{2\frac{D}{R}t}}\right) - exp\left(\frac{xV}{D}\right)erfc\left(\frac{x + \frac{V}{R}t}{2\sqrt{\frac{D}{R}t}}\right)\right]$
(For Linear Equilibrium Adsorption)

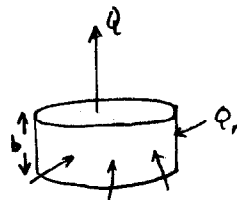A plot of pollutant C vs time is called a _history_ plot.
A plot of pollutant C vs distance is called a _profile_ plot

Cone of depression

When wells remove water from an aquifer the hydraulic gradient is affected as is the shape of the piezometric surface
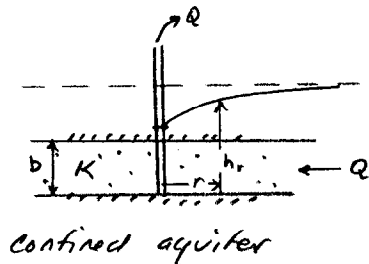
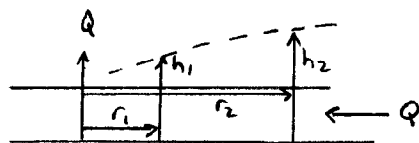

Confined aquifer

Darcy's law
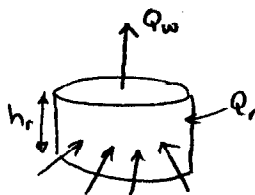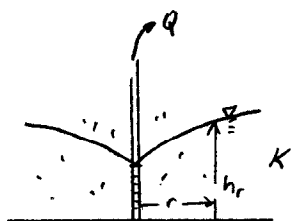$$Q_r = KA\frac{dh}{dr}$$
$$Q_r = K2\pi rb\frac{dh}{dr}$$
$$Q_r \int_{r_1}^{r_2}\frac{dr}{r} = 2\pi Kb\int_{h_1}^{h_2}dh$$

$$Q_r \ln\left|\frac{r_2}{r_1}\right| = 2\pi Kb(h_2 - h_1)$$

mass balance $\Rightarrow Q_r = Q$

$$\therefore Q = \frac{2\pi Kb(h_2 - h_1)}{\ln(r_2/r_1)}$$

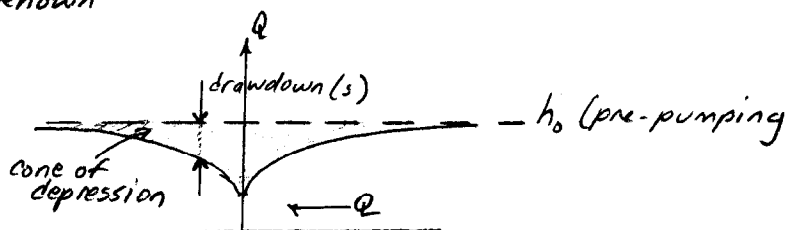$$Q = KA\frac{dh}{dr} = Kh \, 2\pi r \frac{dh}{dr} = K\pi r \frac{dh^2}{dr}$$

$$Q \int_{r_1}^{r_2} \frac{dr}{r} = K\pi \int_{h_1}^{h_2} dh^2$$

$$Q \ln(r_1/r_2) = K\pi (h_2^2 - h_1^2) \qquad \Rightarrow \qquad Q = \frac{K\pi (h_2^2 - h_1^2)}{\ln(r_2/r_1)}$$

These formulas can be used to estimate K if drawdown at two locations are known

Capture Zone / Hydraulic Plume Control

One common method to control polluted groundwater is with pump-and-trea strategies. The goal is to capture, extract, treat & serve the polluted groundwater. The design problem is where to locate extraction wells and what pumping rates to use.
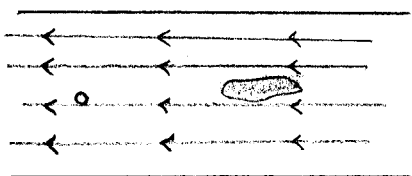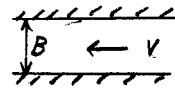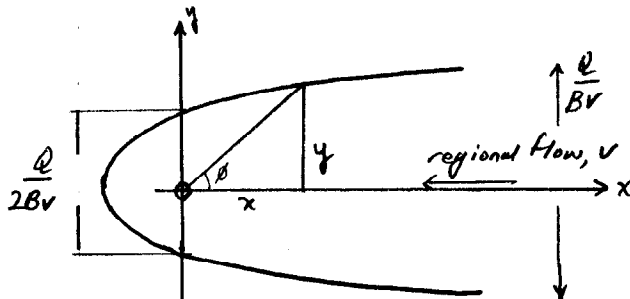

regional flowlines, no wells operating
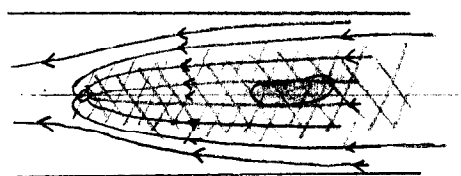
well operating; cross-hatched area all drains to well. Area is called "capture zone"

Simplest model to construct capture zone curves is

$$y = \pm \frac{Q}{2Bv} - \frac{Q}{2\pi Bv} \tan^{-1}\left(\frac{y}{x}\right) \qquad \tan \phi = \frac{y}{x} \quad so \quad y = \frac{Q}{2Bv}\left(1 - \frac{\phi}{\pi}\right)$$

We can use these relationships to specify Q and location of well relative to plume. (1) Well(s) is located so that computed capture zone just encloses plume. (2) Well is located as close as possible to plume, without violating rule (1).

Example _____

Locate an extraction well at $Q = 0.004 \, m^3/s$ to capture plume.

$\frac{\Delta h}{\Delta l} = 0.002$, $K = 1\cdot10^{-3} \, m/s$, $b = 20m$.
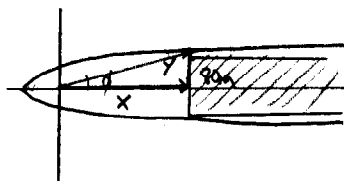
_____
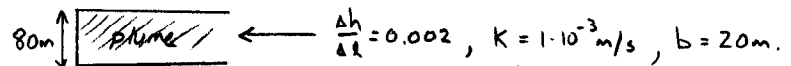
① determine regional specific discharge

$$\frac{Q}{A} = K\frac{\Delta h}{\Delta l} = 1\cdot10^{-3} \, m/s \, (0.002) = 2\cdot10^{-6} \, m/s$$

② Find critical dimension of capture zone

$$\frac{Q}{2Bv} = \frac{0.004 \, m^3/s}{2\cdot20\cdot2\cdot10^{-6} \, m/s} = 50m$$

∴ well cannot be located at leading edge, but $\frac{Q}{Bv} = 100m$ which is big enough to capture plume

③

find x when $y = 40m$ (half plume width)

$$y = \frac{Q}{2Bv}\left(1 - \frac{\phi}{\pi}\right) = 40m = 50m\left(1 - \frac{\phi}{\pi}\right) \Rightarrow \phi = 0.2\pi$$

∴ $x = \frac{y}{\tan\phi} = \frac{40}{\tan(0.2\pi)} = 55m$

④ locate well 55m downgradient of plume.

Usually a single well is inadequate because it must pump a lot of unpolluted water to capture the plume. Alternatives include a line of wells that widen the capture zone so the wells can be closer to the leading edge of the plume.
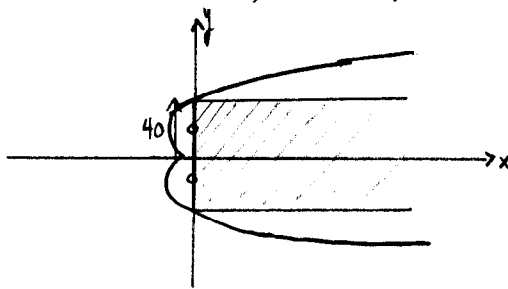
For multiple wells the following equations are used

$$y = \frac{Q}{2BV}\left(n - \frac{1}{\pi}\sum_{i=1}^{n}\phi_i\right)$$

and optimal well spacing is

$$n=2, \quad \frac{Q}{\pi B v}$$

$$n=3, \quad \frac{(2^{\frac{1}{3}})Q}{\pi B v}$$

Example - 2 wells, same problem

$$\frac{2Q}{2VB} = 80 = \frac{Q}{VB} \quad \therefore Q = 80(20)(2 \cdot 10^{-6} \text{m/s})$$
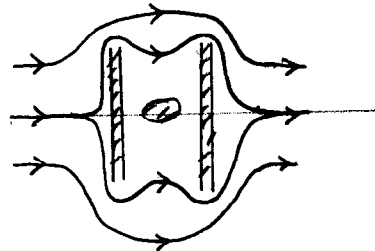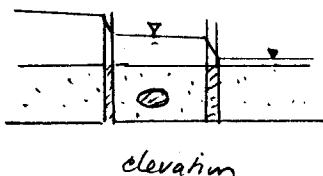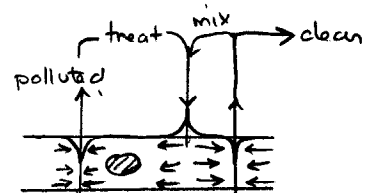
$$= 0.0032 \text{m}^3/\text{s} \text{ each well}$$

Optimal spacing $= \frac{80}{\pi} = 25.5\text{m}$

Other control approaches

In addition to pumping other methods include
- injection to stabilize plume (hydrodynamic control)
- physical barriers - slurry walls, sheet pile etc.

elevation

- in place reaction - biodegredation; chemical oxidization (rare)
- vapor extraction - "air stripping" of VOC in aquifer

Types of pollutants

    inorganic   — metals; some quite toxic — mobility depends on pH and D

    organic — VOC (volatile organic compounds); NAPL (nonaqueous phase liquids)

"conventional" — BOD exterting wastes; not considered much of a problem

NAPLs — very low solubility, thus must move lots of water to
        remove.
           — DNAPLs will tend to sink to locations where flows are
           small.
           — Groundwater flow cannot transfer momentum, so DNAPL (and LNAPL)
           "pools" cannot be mobilized.

Example - illustrates difficulty of NAPL clean-up by pump & treat

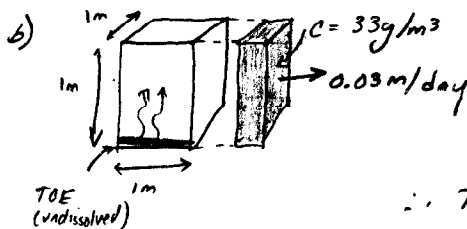 $1 m^3$ aquifer, $n = 0.3$, $V = \frac{d}{nA} = 0.8 m/d$, 30 L TCE, dissolves at 0.1 solubility

  a) find mass TCE & dissolved TCE   b) estimate time to flush all TCE

(Solubility of TCE = 1100 mg/L, but only 10% dissolves (at a time)

    ∴ TCE = 110 mg/L × $0.3 m^3$ × $10^3 L/m^3$ = $33 \cdot 10^3 mg$ = 33g

Total TCE   30 L · $\frac{1.47 kg}{L}$ = 44.1 kg = 44,100 g

    dissolved: 33 g , pure product: 44,067 g

b)

c = $33 g/m^3$

0.03 m/day

mass flow TCE out of box is
    $(0.03 m/d)(1 m^3)(33 g/m^3)$ = 0.99 g/day

TCE (undissolved)

∴ To flush entire 44,100 g

    $\frac{44,100 g}{0.99 g}$ day = 44,545 days $\frac{1 yr}{365 day}$ = 122 years

This is a best-case scenario. In practice, other methods are sought
to mobilize the undissolved TCE, destroy it in place, or encapsulate the TCE.
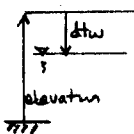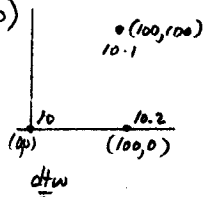
5-37; 5-40, 5-46, 5-48, 5.50, 5.53

5.37)
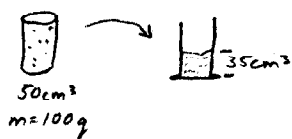
a) porosity?

$$n = \frac{V_{void}}{V_{bulk}} = \frac{V_{bulk} - V_{solids}}{V_{bulk}} = \frac{15 cm^3}{50 cm^3} = 0.3$$

50 cm³
m = 100 g

35 cm³

b) Solids density

$$\rho_s = \frac{m_{solids}}{V_{solids}} = \frac{100 g}{35 cm^3} = 2.85 g/cm^3 = \frac{2.85 g}{mL} \cdot \frac{1000 mL}{L} \cdot \frac{1000 L}{m^3}$$

$$= 2850 \, kg/m^3$$

5-40)

• (100,100)
10.1

10   10.2
(0,0)   (100,0)

dtw
===

dtw
elevation

let elevation = 20 then

$\alpha = 63.4°$

$\Delta L = 50 \sin 63.4°$

9.9

10   α 9.8
head

$d_{n-1} = 100m$

$d_{n-m} = \frac{0.1}{0.2} \cdot 100 = 50$

hyd. gradient $= \frac{0.1}{50 \sin 64°} = 0.00$

5-46)

Q = 5000 m³/d
3m
0.3m   find K
30m
15m   150m
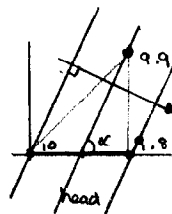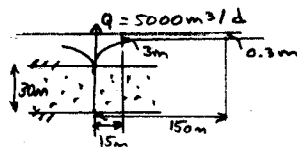
$Q = \frac{2 K b \pi (h_2 - h_1)}{\ln(r_2/r_1)}$ solve for K

$$K = \frac{Q \ln(r_2/r_1)}{2 \pi b (h_2 - h_1)} = \frac{(5000)\ln(150/15)}{2\pi(30)[(h_0 - 0.3) - (h_0 - 3.0)]}$$

h_0 - drawdown = h   (see defn. of ddn)   → Substitute any value for h_0 ($h_0 > 3.0$) and solve

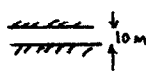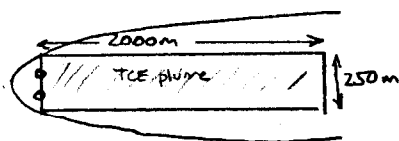$(h_0 - 0.3) - (h_0 - 3.0) = 3.0 - 0.3$ ⟵ or   to obtain   K = 22.6 m/d

5-48) $t = \frac{\pi B n}{Q}(R^2 - r_w^2)$ B = 30m, n = 0.3, r_w = 0.2m, Q = 5000 m³/d

$$t = \frac{\pi(0.3)(30m)(15^2 - 0.2^2)}{5000 m^3/d} = 1.27 \, days$$

5-50)

2000m
TCE plume
250m

10m

0.1 m³ TCE, n = 0.4, $\frac{\Delta h}{\Delta \ell} = 0.001$

K = 0.001 m/s

a) 0.1 m³ TCE · 1470 kg/m³ = 147 kg TCE

$$\frac{147 kg \, TCE}{(2000)(250)(10)(.4)} = 0.0735 \frac{g \, TCE}{m^3} = 73.5 \frac{mg \, TCE}{m^3} \cdot \frac{1 m^3}{1000 L} = 0.0735 \, mg/L$$

well below solubility
∴ can all dissolve

b) Single well solution $\frac{Q}{VB} = 250m$ $V = K\frac{dh}{d\ell} = (0.001)(0.001) = 1 \cdot 10^{-6} m/s$

∴ Q = 250 · 2 · 10 · 1·10⁻⁶ m/s = 0.005 m³/s > 0.003 m³/s ∴ one well has too high Q

2-wells

Q = 250 · 10 · 1·10⁻⁶ m/s = 0.0025 m³/s < 0.003 m³/s   ∴ two wells will work

c) Optimal spacing = $\frac{Q}{\pi B V} = \frac{0.0025}{\pi(10)(1·10^{-6})} = 79.6m \approx 80m$

5-53) Same as example in text; numbers changed, use table 5-12: t ≈ 7.3 years.