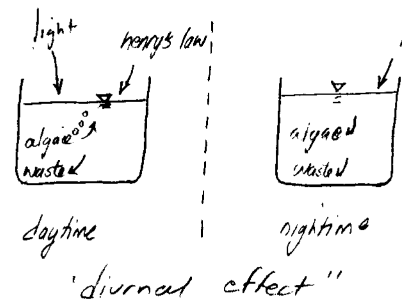
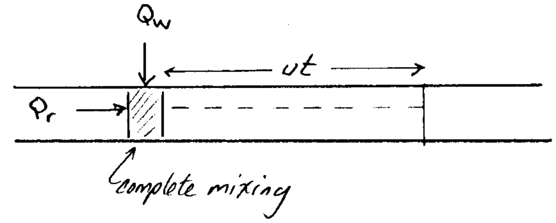
Lecture 009.PDF
CIVE3331

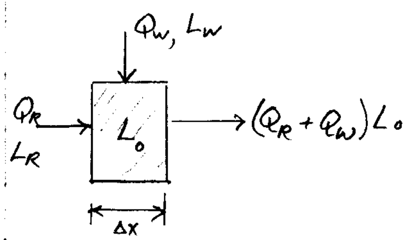


Many factors affect DO in water

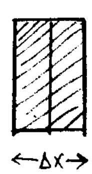



Simple modeling approach (Streeter & Phelps)
- Addition by reaeration
- Removal by degradation




$$L_o = \frac{Q_R L_R + Q_W L_W}{(Q_R + Q_W)} \quad ; \quad DO_o = \frac{Q_W DO_W + Q_R DO_R}{(Q_W + Q_R)}$$




Now study behavior of mixed parcel

$$L_t = L_o e^{-k_d t} \quad \text{(BOD remaining)}$$

- Removal: Assume the rate of DO removed is proportional to the BOD remaining

$$\frac{dDO}{dt} = -k_d L_o e^{-k_d t}$$

- Reaeration. Assume the rate of DO addition is proportional to the difference between $DO_{sat}$ and current DO (linear driving force model)

$$\frac{dDO}{dt} = k_r (DO_{sat} - DO) \qquad DO_{sat} = K_H P_{O_2}$$

$k_r$ is related to depth, flow speed and relative turbulence

One common model for $k_r$ is the O'Connor & Dobbins model

$$k_r = \frac{3.9U^{1/2}}{H^{3/2}}$$ $U$ is stream velocity (m/s)
$H$ is flow depth (m)

$\frac{dDO}{dt} = k_r(DO_{sat} - DO)$ (reaeration)

$\frac{dDO}{dt} = -k_d L_0 e^{-k_d t}$ (degradation)

Net: $$\frac{dDO}{dt} = k_r(DO_{sat} - DO) - k_d L_0 e^{-k_d t}$$

The term $(DO_{sat} - DO)$ is called the oxygen deficit

$$D = (DO_{sat} - DO)$$

$$\therefore \frac{dD}{dt} = \underbrace{\frac{dDO_{sat}}{dt}}_{=0} - \frac{dDO}{dt} = k_d L_0 e^{-k_d t} - k_r D$$

So the equation describing time evolution of the deficit in a moving water parcel is

$$\frac{dD}{dt} = k_d L_0 e^{-k_d t} - L_r D$$ by separation & integration

$$D(t) = \frac{k_d L_0}{k_r - k_d}\left(e^{-k_d t} - e^{-k_r t}\right) + D_0 e^{-k_r t}$$

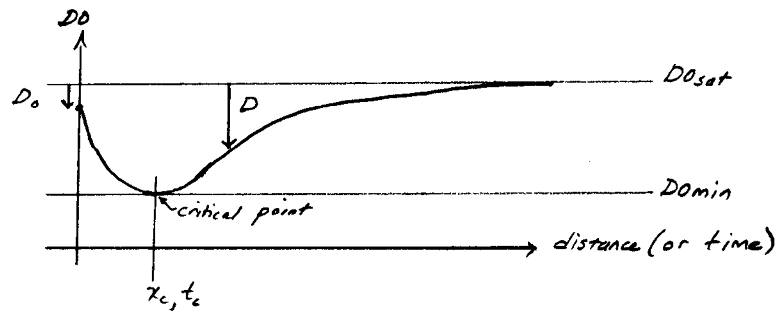
Where $D_0 = DO_{sat} - DO_0$

Thus $$DO(t) = DO_{sat} - \left[\frac{k_d L_0}{k_r - k_d}\left(e^{-k_d t} - e^{-k_r t}\right) + D_0 e^{-k_r t}\right]$$

for a stream of relatively constant cross section $x = Ut$ or $t = \frac{x}{U}$
Using this model (Streeter-Phelps Equation) profiles of DO downstream of a waste input are plotted and studied

Such a model is used to estimate: distance downstream to minimum DO; value of minimum DO; distance downstream to acceptable DO etc.

A typical DO-profile is illustrated below




The critical point is located from

$$\frac{d\,DO(t)}{dt} = 0 \quad \text{(sufficient condition for a minimum)}$$

solve for $t_c$

$$t_c = \frac{1}{k_r - k_d} \ln\left(\frac{k_r}{k_d}\left[1 - \frac{D_o(k_r - k_d)}{k_d L_o}\right]\right)$$

Example

1.10 m³/s of effluent with BOD = 50 mg/L, DO = 2 mg/L is discharged into a stream with flow 8.7 m³/s with BOD = 6 mg/L, DO = 8.3 mg/L. Deoxygenation constant is $k_d = 0.2$/day at 20°C. Average flow depth is 3.0 m, Average flow width is 10.85 m. Plot the DO in the stream as a function of distance downstream. Locate the critical point (min DO).

① Downstream BOD & DO + $D_o$ after mixing

$$L_o = \frac{(1.1\,m^3/s)(50\,mg/L) + (8.7\,m^3/s)(6\,mg/L)}{(1.1 + 8.7\,m^3/s)} = 10.9\,mg/L$$

$$DO_o = \frac{(1.1\,m^3/s)(2\,mg/L) + (8.7\,m^3/s)(8.3\,mg/L)}{(1.1 + 8.7\,m^3/s)} = 7.6\,mg/L$$

$DO_{sat}$ (@20°C) = 9.09 mg/L $\therefore$ $D_o = DO_{sat} - D_o = 9.09 - 7.6 = 1.5\,mg/L$

$$k_r \text{ (O'Connor \& Dobbins)} = \frac{3.9\,v^{1/2}}{H^{3/2}}$$

$$v = \frac{Q}{A} = \frac{1.1 + 8.7\,m^3/s}{(3.0)(10.85)} = 0.301\,m^3/s$$

$$k_r = \frac{(3.9)(0.3)^{1/2}}{(3.0)^{3/2}} = 0.41/day$$

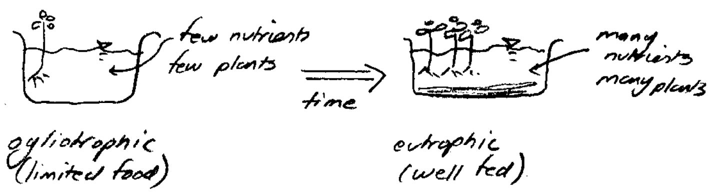
Now one has required parameters to determine $t_c$ & plot DO vs $x$ (or $t$)
(Attached spreadsheet implements formulae)

$$t_c = \frac{1}{(0.41-0.20)} \ln\left(\frac{0.41}{0.20}\left[1 - \frac{1.5(0.41-0.20)}{0.20 \times 10.9}\right]\right) = 2.67 \text{ days} * \frac{86400 \text{ sec}}{\text{day}} = 230{,}688 \text{ sec.}$$

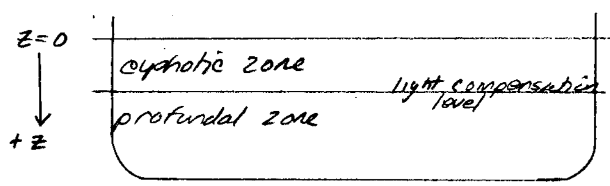
$$x_c = Ut_c = (0.30 \text{ m/sec})(230{,}688 \text{ sec}) = 69300 \text{ meters}$$

## Lakes & Reservoirs

Natural Aging called eutrophication:
Human activity accelerates the process.
wastes, runoff from agriculture etc.
supplies nutrients at a rate
greater than naturally occurs. – arthopegenic eutrophication




Factors that control eutrophication
– light penetration, related to clarity (turbidity)




euphotic zone: $O_2$ by photosynthesis is greater than $O_2$ used during respiration
profundal zone: $O_2$ used by respiration greater than $O_2$ produced by photosyn.
– nutrient supply

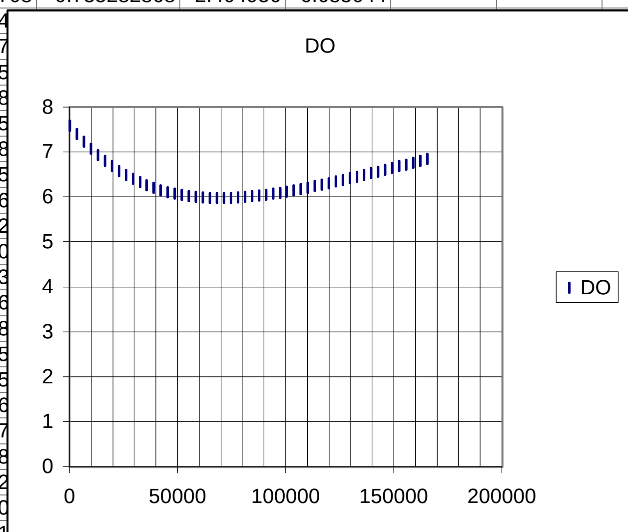
growth limited by least available nutrient. Control of limiting nutrient can help control eutrophication. Control of non-limiting nutrient only effective if it can be reduced enough to become the limiting nutrient.

Photosynthesis (approximation)

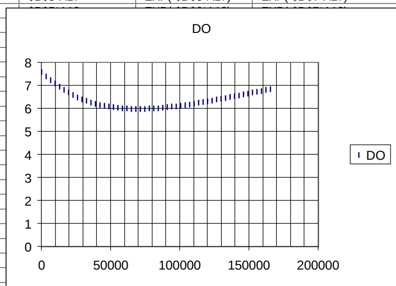
$$\underbrace{106\,CO_2}_{\text{carbon source}} + \underbrace{16NO_3^- + HPO_4^{2-}}_{\text{nutrients}} + 122H_2O + 18H^+ \xrightarrow{\text{light}} \underbrace{C_{106}H_{263}O_{110}N_{16}P}_{\text{cell mass (algae)}} + 138\,O_2$$

Calculate mass of N versus P in algae. It takes 7 times more N than P (by mass) to produce algae.

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Lo | 10.9 | mg/L | | | | | | | |
| 2 | Do | 1.5 | mg/L | | | | | | | |
| 3 | DOsat | 9.09 | mg/L | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | u | 0.3 | m/sec | | | | | | | |
| 6 | kd | 0.2 | /day | 2.31481E-06 | /sec | | | | | |
| 7 | kr | 0.41 | /day | 4.74537E-06 | /sec | | | | | |
| 8 | | | | | | | | | | |
| 9 | kdLo/(kr-kd | 10.38095 | | | | | | | | |
| 10 | Δt | 10800 | | | | | | | | |
| 11 | | | | | | | | | | |
| 12 | t(sec) | x(meters) | exp(-kdt) | exp(-krt) | D | DO | | | | |
| 13 | 0 | 0 | 1 | 1 | 1.5 | 7.59 | | | | |
| 14 | 10800 | 3240 | 0.97531 | 0.950041131 | 1.687376 | 7.402624 | | | | |
| 15 | 21600 | 6480 | 0.951229 | 0.90257815 | 1.858914 | 7.231086 | | | | |
| 16 | 32400 | 9720 | 0.927743 | 0.857486366 | 2.015565 | 7.074435 | | | | |
| 17 | 43200 | 12960 | 0.904837 | 0.814647316 | 2.15823 | 6.93177 | | | | |
| 18 | 54000 | 16200 | 0.882497 | 0.773948457 | 2.287759 | 6.802241 | | | | |
| 19 | 64800 | 19440 | 0.860708 | 0.735282868 | 2.404956 | 6.685044 | | | | |
| 20 | 75600 | 22680 | 0.8394 | | | | | | | |
| 21 | 86400 | 25920 | 0.8187 | | | | | | | |
| 22 | 97200 | 29160 | 0.7985 | | | | | | | |
| 23 | 108000 | 32400 | 0.7788 | | | | | | | |
| 24 | 118800 | 35640 | 0.7595 | | | | | | | |
| 25 | 129600 | 38880 | 0.7408 | | | | | | | 230688 |
| 26 | 140400 | 42120 | 0.7225 | | | | | | | |
| 27 | 151200 | 45360 | 0.7046 | | | | | | | |
| 28 | 162000 | 48600 | 0.6872 | | | | | | | |
| 29 | 172800 | 51840 | 0.670 | | | | | | | |
| 30 | 183600 | 55080 | 0.653 | | | | | | | |
| 31 | 194400 | 58320 | 0.6376 | | | | | | | |
| 32 | 205200 | 61560 | 0.6218 | | | | | | | |
| 33 | 216000 | 64800 | 0.6065 | | | | | | | |
| 34 | 226800 | 68040 | 0.5915 | | | | | | | |
| 35 | 237600 | 71280 | 0.576 | | | | | | | |
| 36 | 248400 | 74520 | 0.5627 | | | | | | | |
| 37 | 259200 | 77760 | 0.5488 | | | | | | | |
| 38 | 270000 | 81000 | 0.5352 | | | | | | | |
| 39 | 280800 | 84240 | 0.5220 | | | | | | | |
| 40 | 291600 | 87480 | 0.5091 | [illegible] | [illegible] | [illegible] | | | | |
| 41 | 302400 | 90720 | 0.496585 | 0.238115364 | 3.040337 | 6.049663 | | | | |
| 42 | 313200 | 93960 | 0.484325 | 0.22621939 | 3.018707 | 6.071293 | | | | |
| 43 | 324000 | 97200 | 0.472367 | 0.214917725 | 2.994941 | 6.095059 | | | | |
| 44 | 334800 | 100440 | 0.460704 | 0.204180678 | 2.969225 | 6.120775 | | | | |
| 45 | 345600 | 103680 | 0.449329 | 0.193980042 | 2.941735 | 6.148265 | | | | |
| 46 | 356400 | 106920 | 0.438235 | 0.184289019 | 2.912635 | 6.177365 | | | | |
| 47 | 367200 | 110160 | 0.427415 | 0.175082148 | 2.882078 | 6.207922 | | | | |
| 48 | 378000 | 113400 | 0.416862 | 0.166335242 | 2.850209 | 6.239791 | | | | |
| 49 | 388800 | 116640 | 0.40657 | 0.158025321 | 2.817165 | 6.272835 | | | | |
| 50 | 399600 | 119880 | 0.396531 | 0.150130555 | 2.783071 | 6.306929 | | | | |
| 51 | 410400 | 123120 | 0.386741 | 0.142630202 | 2.748048 | 6.341952 | | | | |
| 52 | 421200 | 126360 | 0.377192 | 0.135504558 | 2.712206 | 6.377794 | | | | |
| 53 | 432000 | 129600 | 0.367879 | 0.128734904 | 2.67565 | 6.41435 | | | | |
| 54 | 442800 | 132840 | 0.358796 | 0.122303453 | 2.638478 | 6.451522 | | | | |
| 55 | 453600 | 136080 | 0.349938 | 0.116193311 | 2.60078 | 6.48922 | | | | |
| 56 | 464400 | 139320 | 0.341298 | 0.110388425 | 2.562641 | 6.527359 | | | | |
| 57 | 475200 | 142560 | 0.332871 | 0.104873544 | 2.524142 | 6.565858 | | | | |
| 58 | 486000 | 145800 | 0.324652 | 0.09963418 | 2.485355 | 6.604645 | | | | |
| 59 | 496800 | 149040 | 0.316637 | 0.094656569 | 2.446351 | 6.643649 | | | | |
| 60 | 507600 | 152280 | 0.308819 | 0.089927634 | 2.407192 | 6.682808 | | | | |
| 61 | 518400 | 155520 | 0.301194 | 0.085434951 | 2.367939 | 6.722061 | | | | |
| 62 | 529200 | 158760 | 0.293758 | 0.081166717 | 2.328647 | 6.761353 | | | | |
| 63 | 540000 | 162000 | 0.286505 | 0.07711172 | 2.289367 | 6.800633 | | | | |
| 64 | 550800 | 165240 | 0.279431 | 0.073259306 | 2.250147 | 6.839853 | | | | |



| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Lo | 10.9 | mg/L | | | | | | | |
| 2 | Do | 1.5 | mg/L | | | | | | | |
| 3 | DOsat | 9.09 | mg/L | | | | | | | |
| 4 | | | | | | | | | | |
| 5 | u | 0.3 | m/sec | | | | | | | |
| 6 | kd | 0.2 | /day | =B6/86400 | /sec | | | | | |
| 7 | kr | 0.41 | /day | =B7/86400 | /sec | | | | | |
| 8 | | | | | | | | | | |
| 9 | kdLo/(kr-kd) | =D6*B1/(D7-D6) | | | | | | | | |
| 10 | Δt | =3600*3 | | | | | | | | |
| 11 | | | | | | | | | | |
| 12 | t(sec) | x(meters) | exp(-kdt) | exp(-krt) | D | DO | | | | |
| 13 | 0 | =$B$5*A13 | =EXP(-$D$6*A13) | =EXP(-$D$7*A13) | =$B$9*(C13-D13)+$B$2*D13 | =$B$3-E13 | | | | |
| 14 | =A13+$B$10 | =$B$5*A14 | =EXP(-$D$6*A14) | =EXP(-$D$7*A14) | =$B$9*(C14-D14)+$B$2*D14 | =$B$3-E14 | | | | |
| 15 | =A14+$B$10 | =$B$5*A15 | =EXP(-$D$6*A15) | =EXP(-$D$7*A15) | =$B$9*(C15-D15)+$B$2*D15 | =$B$3-E15 | | | | |
| 16 | =A15+$B$10 | =$B$5*A16 | =EXP(-$D$6*A16) | =EXP(-$D$7*A16) | =$B$9*(C16-D16)+$B$2*D16 | =$B$3-E16 | | | | |
| 17 | =A16+$B$10 | =$B$5*A17 | =EXP(-$D$6*A17) | =EXP(-$D$7*A17) | =$B$9*(C17-D17)+$B$2*D17 | =$B$3-E17 | | | | |
| 18 | =A17+$B$10 | | | | =$B$9*(C18-D18)+$B$2*D18 | =$B$3-E18 | | | | |
| 19 | =A18+$B$10 | | | | =$B$9*(C19-D19)+$B$2*D19 | =$B$3-E19 | | | | |
| 20 | =A19+$B$10 | | | | =$B$9*(C20-D20)+$B$2*D20 | =$B$3-E20 | | | | |
| 21 | =A20+$B$10 | | | | =$B$9*(C21-D21)+$B$2*D21 | =$B$3-E21 | | | | |
| 22 | =A21+$B$10 | | | | =$B$9*(C22-D22)+$B$2*D22 | =$B$3-E22 | | | | |
| 23 | =A22+$B$10 | | | | =$B$9*(C23-D23)+$B$2*D23 | =$B$3-E23 | | | | |
| 24 | =A23+$B$10 | | | | =$B$9*(C24-D24)+$B$2*D24 | =$B$3-E24 | | | | |
| 25 | =A24+$B$10 | | | | =$B$9*(C25-D25)+$B$2*D25 | =$B$3-E25 | | | | =3600*2.67*24 |
| 26 | =A25+$B$10 | | | | =$B$9*(C26-D26)+$B$2*D26 | =$B$3-E26 | | | | |
| 27 | =A26+$B$10 | | | | =$B$9*(C27-D27)+$B$2*D27 | =$B$3-E27 | | | | |
| 28 | =A27+$B$10 | | | | =$B$9*(C28-D28)+$B$2*D28 | =$B$3-E28 | | | | |
| 29 | =A28+$B$10 | | | | =$B$9*(C29-D29)+$B$2*D29 | =$B$3-E29 | | | | |
| 30 | =A29+$B$10 | | | | =$B$9*(C30-D30)+$B$2*D30 | =$B$3-E30 | | | | |
| 31 | =A30+$B$10 | | | | =$B$9*(C31-D31)+$B$2*D31 | =$B$3-E31 | | | | |
| 32 | =A31+$B$10 | | | | =$B$9*(C32-D32)+$B$2*D32 | =$B$3-E32 | | | | |
| 33 | =A32+$B$10 | | | | =$B$9*(C33-D33)+$B$2*D33 | =$B$3-E33 | | | | |
| 34 | =A33+$B$10 | | | | =$B$9*(C34-D34)+$B$2*D34 | =$B$3-E34 | | | | |
| 35 | =A34+$B$10 | | | | =$B$9*(C35-D35)+$B$2*D35 | =$B$3-E35 | | | | |
| 36 | =A35+$B$10 | | | | =$B$9*(C36-D36)+$B$2*D36 | =$B$3-E36 | | | | |
| 37 | =A36+$B$10 | =$B$5*A37 | =EXP(-$D$6*A37) | =EXP(-$D$7*A37) | =$B$9*(C37-D37)+$B$2*D37 | =$B$3-E37 | | | | |
| 38 | =A37+$B$10 | =$B$5*A38 | =EXP(-$D$6*A38) | =EXP(-$D$7*A38) | =$B$9*(C38-D38)+$B$2*D38 | =$B$3-E38 | | | | |
| 39 | =A38+$B$10 | =$B$5*A39 | =EXP(-$D$6*A39) | =EXP(-$D$7*A39) | =$B$9*(C39-D39)+$B$2*D39 | =$B$3-E39 | | | | |
| 40 | =A39+$B$10 | =$B$5*A40 | =EXP(-$D$6*A40) | =EXP(-$D$7*A40) | =$B$9*(C40-D40)+$B$2*D40 | =$B$3-E40 | | | | |
| 41 | =A40+$B$10 | =$B$5*A41 | =EXP(-$D$6*A41) | =EXP(-$D$7*A41) | =$B$9*(C41-D41)+$B$2*D41 | =$B$3-E41 | | | | |
| 42 | =A41+$B$10 | =$B$5*A42 | =EXP(-$D$6*A42) | =EXP(-$D$7*A42) | =$B$9*(C42-D42)+$B$2*D42 | =$B$3-E42 | | | | |
| 43 | =A42+$B$10 | =$B$5*A43 | =EXP(-$D$6*A43) | =EXP(-$D$7*A43) | =$B$9*(C43-D43)+$B$2*D43 | =$B$3-E43 | | | | |
| 44 | =A43+$B$10 | =$B$5*A44 | =EXP(-$D$6*A44) | =EXP(-$D$7*A44) | =$B$9*(C44-D44)+$B$2*D44 | =$B$3-E44 | | | | |
| 45 | =A44+$B$10 | =$B$5*A45 | =EXP(-$D$6*A45) | =EXP(-$D$7*A45) | =$B$9*(C45-D45)+$B$2*D45 | =$B$3-E45 | | | | |
| 46 | =A45+$B$10 | =$B$5*A46 | =EXP(-$D$6*A46) | =EXP(-$D$7*A46) | =$B$9*(C46-D46)+$B$2*D46 | =$B$3-E46 | | | | |
| 47 | =A46+$B$10 | =$B$5*A47 | =EXP(-$D$6*A47) | =EXP(-$D$7*A47) | =$B$9*(C47-D47)+$B$2*D47 | =$B$3-E47 | | | | |
| 48 | =A47+$B$10 | =$B$5*A48 | =EXP(-$D$6*A48) | =EXP(-$D$7*A48) | =$B$9*(C48-D48)+$B$2*D48 | =$B$3-E48 | | | | |
| 49 | =A48+$B$10 | =$B$5*A49 | =EXP(-$D$6*A49) | =EXP(-$D$7*A49) | =$B$9*(C49-D49)+$B$2*D49 | =$B$3-E49 | | | | |
| 50 | =A49+$B$10 | =$B$5*A50 | =EXP(-$D$6*A50) | =EXP(-$D$7*A50) | =$B$9*(C50-D50)+$B$2*D50 | =$B$3-E50 | | | | |
| 51 | =A50+$B$10 | =$B$5*A51 | =EXP(-$D$6*A51) | =EXP(-$D$7*A51) | =$B$9*(C51-D51)+$B$2*D51 | =$B$3-E51 | | | | |
| 52 | =A51+$B$10 | =$B$5*A52 | =EXP(-$D$6*A52) | =EXP(-$D$7*A52) | =$B$9*(C52-D52)+$B$2*D52 | =$B$3-E52 | | | | |
| 53 | =A52+$B$10 | =$B$5*A53 | =EXP(-$D$6*A53) | =EXP(-$D$7*A53) | =$B$9*(C53-D53)+$B$2*D53 | =$B$3-E53 | | | | |
| 54 | =A53+$B$10 | =$B$5*A54 | =EXP(-$D$6*A54) | =EXP(-$D$7*A54) | =$B$9*(C54-D54)+$B$2*D54 | =$B$3-E54 | | | | |
| 55 | =A54+$B$10 | =$B$5*A55 | =EXP(-$D$6*A55) | =EXP(-$D$7*A55) | =$B$9*(C55-D55)+$B$2*D55 | =$B$3-E55 | | | | |
| 56 | =A55+$B$10 | =$B$5*A56 | =EXP(-$D$6*A56) | =EXP(-$D$7*A56) | =$B$9*(C56-D56)+$B$2*D56 | =$B$3-E56 | | | | |
| 57 | =A56+$B$10 | =$B$5*A57 | =EXP(-$D$6*A57) | =EXP(-$D$7*A57) | =$B$9*(C57-D57)+$B$2*D57 | =$B$3-E57 | | | | |
| 58 | =A57+$B$10 | =$B$5*A58 | =EXP(-$D$6*A58) | =EXP(-$D$7*A58) | =$B$9*(C58-D58)+$B$2*D58 | =$B$3-E58 | | | | |
| 59 | =A58+$B$10 | =$B$5*A59 | =EXP(-$D$6*A59) | =EXP(-$D$7*A59) | =$B$9*(C59-D59)+$B$2*D59 | =$B$3-E59 | | | | |
| 60 | =A59+$B$10 | =$B$5*A60 | =EXP(-$D$6*A60) | =EXP(-$D$7*A60) | =$B$9*(C60-D60)+$B$2*D60 | =$B$3-E60 | | | | |
| 61 | =A60+$B$10 | =$B$5*A61 | =EXP(-$D$6*A61) | =EXP(-$D$7*A61) | =$B$9*(C61-D61)+$B$2*D61 | =$B$3-E61 | | | | |
| 62 | =A61+$B$10 | =$B$5*A62 | =EXP(-$D$6*A62) | =EXP(-$D$7*A62) | =$B$9*(C62-D62)+$B$2*D62 | =$B$3-E62 | | | | |
| 63 | =A62+$B$10 | =$B$5*A63 | =EXP(-$D$6*A63) | =EXP(-$D$7*A63) | =$B$9*(C63-D63)+$B$2*D63 | =$B$3-E63 | | | | |
| 64 | =A63+$B$10 | =$B$5*A64 | =EXP(-$D$6*A64) | =EXP(-$D$7*A64) | =$B$9*(C64-D64)+$B$2*D64 | =$B$3-E64 | | | | |

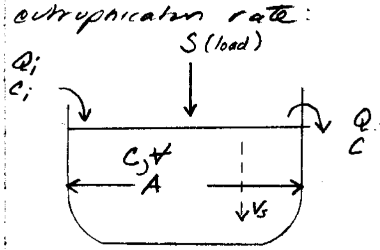


∴ when $N \simeq 10 * P$ , then $P$ is limiting nutrient
$N < 10 * P$ , then $N$ is limiting nutrient

In addition to relative abundance, if absolute abundance exceeds $0.3\,mg/L-N$ & $0.015\,mg/L-P$ one expects algal blooms.

## Simple Phosphorous Model

Modeling is used to estimate amount of control to apply to reduce eutrophication rate: $P \leq 0.01\,mg/L$ : "OK" ; $P \geq 0.02\,mg/L$ "too much"




Inflow: $S + Q_i C_i$ Accumulation: $\frac{dVC}{dt}$

Outflow: $QC + \underbrace{V_s A C}$

fraction that "settles" – absorve it is an internal mass transfer term

Mass Balance:

$$\frac{dVC}{dt} = S + Q_i C_i - QC - V_s A C$$

at equilibrium $\left(\frac{dVC}{dt} = 0\right)$ ∴ $S + Q_i C_i = QC + V_s A C$ Solve for $C$

$$C = \frac{S + Q_i C_i}{Q + V_s A}$$

## Example

Lake with $A = 80 \cdot 10^6 m^2$, $15 m^3/s$ inflow; $C = 0.01\,mg/L$. Load of $1g/s$, settling rate is $10m/yr$. Find $P$ at equilibrium; % removal at load to keep lake at $0.01\,mg/L$.

① $$C = \frac{S + Q_i C_i}{Q + V_s A} = \frac{1000\,mg/s + 0.01\frac{mg}{L} \cdot \frac{1000L}{m^3} \cdot \frac{15m^3}{s}}{*\left\{\frac{15m^3}{s} + \frac{10m}{yr} \cdot \frac{1yr}{365d} \cdot \frac{1d}{86400sec} \cdot 80 \cdot 10^6 m^2\right.} = 28.48\,mg/m^3 \cdot \frac{1m^3}{1000L} = 0.0284\,\frac{mg}{L}$$

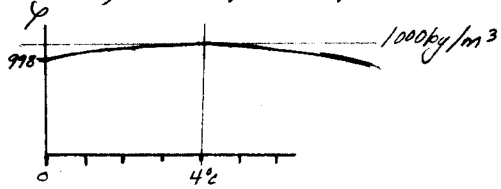
② $$\frac{x\,mg/s + 150\,mg/s}{*\{40.367\,m^3/s} = 10.00\,mg/m^3$$ Solve for $x$

$x = 253.6\,mg/s$ ∴ $\%\ removal = \frac{1000 - 253.6\,mg/s}{1000\,mg/s} \times 100 = 74.6\%$

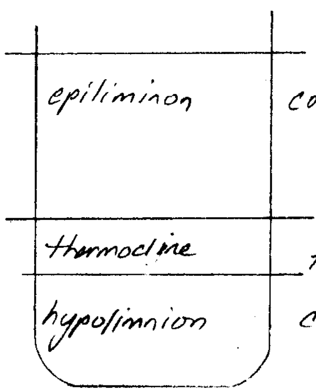
This simple model neglects stratification (incomplete mixing) and seasonal fluctuation in inputs. Such model features are beyond scope of course, but one should be aware of thermal stratification effects.

## Thermal Stratification

$H_2O$ density - temp. dependence is not monotonic — has peak value near 4°C



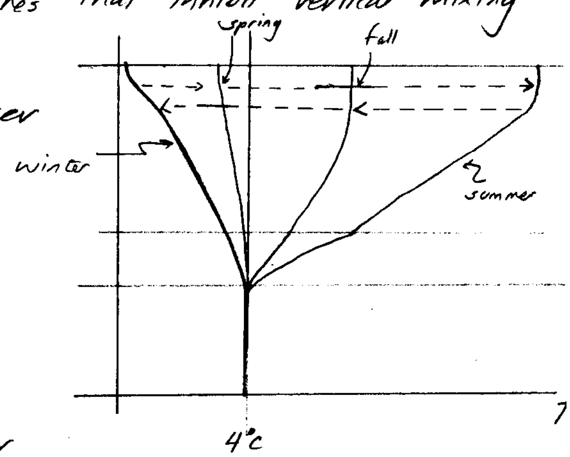

∴ 4°c water "sinks" relative to colder or warmer water. Creates distinct temp. layers in lakes that inhibit vertical mixing



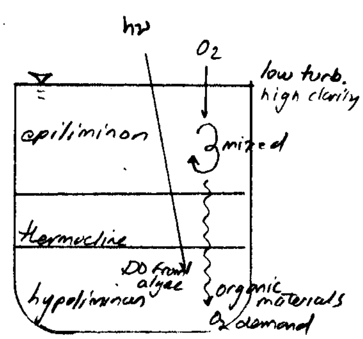

completly mixed upper layer

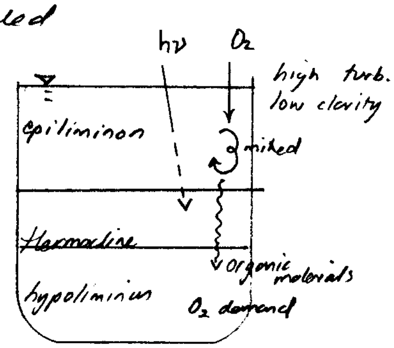
transition layer (zone)

cool bottom zone




between spring & summer and summer & fall vertical profile makes "overturn" possible. During overturn materials in sediment are resuspended




oglliotrophic

eutrophic — light cant reach as deep. $O_2$ demand in hypoliminon not helped by algal $O_2$ production

If turnover is rare then $H_2S$, $CH_4$ & $CO_2$ build up in hypoliminon (anoxic) region. Lake can "burp" with catastrophic results (to living organisms).

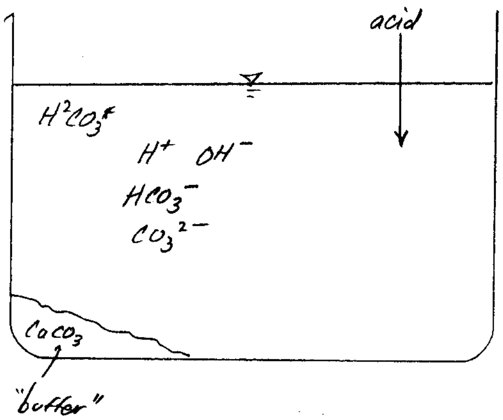
## Acid deposition

Natural rain (in equilibrium with atmospheric $CO_2$) has $pH \approx 5.6$

Some rain has $pH < 4.0$ caused by $SO_4$ & $NO_x$ in atmosphere.

If lake is poorly buffered, it will experience a drop in pH.

A buffered system resists change in pH when acid is added
- carbonate system is a natural buffer.




$H^+$ is increased, but it combines with $HCO_3^-$ to make $H_2CO_3^*$. Reduction shifts all equilibria so $CaCO_3$ dissolves to supply more $HCO_3^-$

$$H_2CO_3^* \rightleftharpoons H^+ + HCO_3^-$$
$$HCO_3^- \rightleftharpoons H^+ + CO_3^{2-}$$
$$CaCO_{3(s)} \rightleftharpoons Ca^{2+} + CO_3^{2-}$$

At very low pH, certain metals play important buffering role

$$Al_2Si_2O_5(OH)_4 + 6H^+ \rightleftharpoons 2Al^{3+} + 2Si(OH)_4 + H_2O$$
(kaolinite)

$$Al(OH)_3 + 3H^+ \rightleftharpoons Al^{3+} + 3H_2O$$
(gibbsite)

(Clays)