# Bond Market Making Simulation

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## 1 Objective

This report presents a simulation-based market-making framework for a 10-year zero-coupon bond. The model is driven by short-term interest rate dynamics under the CIR model, with synthetic bid/ask pricing and trade arrival modeling.

## 2 Short Rate Model (CIR)

The short rate $r(t)$ follows the Cox–Ingersoll–Ross (CIR) process:

$$dr = a(b - r)dt + \sigma\sqrt{r}dW_t \tag{1}$$

Bond prices are computed using the closed-form CIR solution:

$$P(t,T) = A(t,T) \cdot e^{-B(t,T)\cdot r(t)} \tag{2}$$

where $A(t,T)$ and $B(t,T)$ are model-specific functions of time to maturity and the CIR parameters, as outlined in John C. Hull's *Options, Futures and Other Derivatives*.

## 3 Simulation Framework

### 3.1 Calibrating Parameters

Using the most recent Yield on Zero Coupon Bonds from FRED, we compute observed market bond prices via:

$$P(t,T) = e^{-r(T-t)} \tag{3}$$

We then compute the corresponding model price using the CIR bond pricing function. Calibration is performed by minimizing the squared error:

$$\text{Loss} = \sum (P_{\text{model}} - P_{\text{market}})^2 \tag{4}$$

with respect to the parameters $a$, $b$, $\sigma$, and $r_0$.

### 3.2 Simulating Midpoint Bond Prices

Using the calibrated parameters, we simulate the short rate using Euler discretization at 1-second resolution across 252 trading days (539 minutes per day):

$$r_{t+\Delta t} = r_t + a(b - r_t)\Delta t + \sigma\sqrt{r_t}\sqrt{\Delta t} \cdot z_t \tag{5}$$

where $z_t$ is drawn from a standard normal distribution.

### 3.3 Bid/Ask Spread

We simulate bid-ask spreads from a uniform distribution:

$$\text{spread} \sim U(0.01, 0.04) \tag{6}$$

Bid and ask prices are then calculated as:

$$\text{bid} = \text{mid} - \frac{\text{spread}}{2} \tag{7}$$

$$\text{ask} = \text{mid} + \frac{\text{spread}}{2} \tag{8}$$

### 3.4 Simulating Fills

Liquidity sensitivity $k$ is drawn from a normal distribution:

$$k \sim \mathcal{N}(60, 2^2) \tag{9}$$

We fix the baseline activity level $A = 2$ trades per side per second. For each quote, we compute the arrival rates as:

$$\lambda_{\text{bid}} = A \cdot e^{-k\cdot(\text{mid}-\text{bid})} \tag{10}$$

$$\lambda_{\text{ask}} = A \cdot e^{-k\cdot(\text{ask}-\text{mid})} \tag{11}$$

We then simulate the number of trades per side using Poisson random draws:

$$N \sim \text{Poisson}(\lambda) \tag{12}$$

### 3.5 Final Simulated Data

The resulting dataset includes the following fields for each simulated second:

**Timestamp, Midpoint, Bid, Ask, Bid Volume, Ask Volume**

## 4 Market Making Strategy

We simulate a market-making strategy using the above synthetic market environment.

### 4.1 Estimating $k$ and $A$ from Simulated Data

Every 1200 seconds (20 minutes), we estimate separate liquidity parameters $k$ and $A$ for the bid and ask sides.

We fit the model:

$$\log(\text{volume}) = \log(A) - k \cdot \delta \tag{13}$$

where $\delta$ is the distance from the quote to the midprice. Volume is aggregated by binning spreads and summing counts in each bin. This rolling regression allows us to infer time-varying liquidity sensitivity.

### 4.2 Setting Bid and Ask Prices

We use a base spread of 0.04 and adjust quotes based on inventory using a skew parameter $\gamma$:

$$\text{bid} = \text{mid} - \frac{\text{base\_spread}}{2} - \gamma \cdot \text{inventory} \tag{14}$$

$$\text{ask} = \text{mid} + \frac{\text{base\_spread}}{2} - \gamma \cdot \text{inventory} \tag{15}$$

When inventory is positive, the bid is reduced to encourage selling, and the ask is lowered to discourage further buying. The opposite is applied when inventory is negative.

### 4.3 Simulating Fills

At each second, we use the estimated $k_{\text{bid}}, A_{\text{bid}}$ and $k_{\text{ask}}, A_{\text{ask}}$ to compute fill probabilities:

$$\lambda_{\text{ask}} = A_{\text{ask}} \cdot e^{-k_{\text{ask}} \cdot (\text{ask} - \text{mid})} \tag{16}$$

$$\lambda_{\text{bid}} = A_{\text{bid}} \cdot e^{-k_{\text{bid}} \cdot (\text{mid} - \text{bid})} \tag{17}$$

These are used to draw the number of fills from a Poisson distribution.

### 4.4 Tracking Cash and Inventory

Each executed trade updates inventory and cash:

- Selling at the ask: inventory decreases, cash increases
- Buying at the bid: inventory increases, cash decreases

We log both inventory and cash at each time step to evaluate performance over the simulation horizon.