Supplementary material of "A biogeography-based dual-strategy particle swarm algorithm for numerical and engineering design optimization"

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A. *Complexity analysis of DBPSO*

The computational complexity of an NMA primarily comprises two components: the computational cost associated with evaluating the fitness function and the computational complexity inherent in the algorithm. To ensure a fair comparison, all compared algorithms share the same maximum number of fitness function evaluations, and the associated cost is uniform. Consequently, we focus solely on discussing the computational complexity intrinsic to each algorithm. Let N, D, and T represent the population size, the number of dimensions, and the total iteration number, respectively. Therefore, the computational complexity of DBPSO encompasses two aspects: (1) population initialization; (2) updating the positions of each agent, including the updating methods in both the IBBO and DSS phases. According to Algorithm 3, particles in the first phase undergo updates using the IBBO method, while particles in the second phase undergo updates using the DSS strategy. Assuming that the time units for population initialization, velocity updates with IBBO, and velocity updates with DSS are denoted as t_0, t_1, and t_2, respectively. The computational complexity of DBPSO is expressed as $O_{DBPSO} = O(N \times D \times t_0) + O((N \times T \times D + N \times T \times D) \times t_1) + O((N \times T \times D + N \times logN \times T \times D) \times t_2)$. If t_0, t_1, and t_2 are all approximately equal to 1 time unit, then the overall computational complexity of DBPSO is $O_{DBPSO} = O(N \times D) + O(N \times T \times D + N \times T \times D) + O(N \times T \times D + N \times logN \times T \times D) \approx O(N \times logN \times T \times D)$. Therefore, the overall computational complexity order of DBPSO is approximately $O(N \times logN \times T \times D)$.

A comparison of the computational complexity of DBPSO with PSO and other MAs variants is also given in Table S.1.

TABLE S.1
COMPLEXITY ANALYSIS OF SOME ALGORITHMS

Algorithms	Computational complexity
ACLPSO	$O(N * D)$
TLS-PSO	$O(N * T * D)$
GL-PSO	$O((D + N)T)$
HCLPSO	$O(ND)$
MSSCS	$O(TN(D + f(D)))$
TUS-HBO	$O(N * T * D)$
MPA	$O(T(N * D + MaxFEs * N))$
DBPSO	$O(N * logN * T * D)$

B. *Time complexity analysis of DBPSO*

Following IEEE CEC2013 guidelines [1], this subsection analyzes the time complexity of PSO and DBPSO, defining the parameters $T0$, $T1$, and $T2$ as in IEEE CEC2013. $T0$ represents the execution time of a specific test program from CEC2013. $T1$ represents the time for 2×10^5 evaluations of Function 14 at a certain dimension D. $T2$ is the total computation time for one algorithm to perform 2×10^5 evaluations of Function 14 in a D-dimensional space. The average of five $T2$ values is denoted as $\hat{T}2$. Table S.2 displays the time complexities of PSO and DBPSO for 30- and 50-dimensional functions. For the 30-dimensional case, the computing time $\hat{T}2$ of PSO is about 2.0943 times that of DBPSO. Additionally, $\hat{T}2$ of PSO is roughly 67.07% higher than that of DBPSO for the 50-dimensional case. This difference can be attributed to the effective combination of the dual search strategy and the improved migration operator IBBO employed in DBPSO. These enhancements improve the optimization performance of the algorithm and result in a better balance between exploration and exploitation. As a result, DBPSO outperforms PSO in terms of time complexity.

TABLE S.2
TIME COMPLEXITY FOR PSO AND DBPSO

Dimension	$T0$	$T1$	$\hat{T}2$		$(\hat{T}2 - T1)/T0$	
			PSO	DBPSO	PSO	DBPSO
$D = 30$	0.08	0.78	2.22	1.06	17.37	3.42
$D = 50$	0.08	1.33	2.79	1.67	17.50	4.04

C. *Parameter settings*

TABLE S.3
SPECIFIC PARAMETER SETTINGS OF THE COMPARISON ALGORITHMS

No.	Algorithm	Year	Parameter settings
1	RQ-PSO	2022	$N = 100, \omega = 0 \sim 1, q = 0.955$
2	HCLPSO	2015	$N = 40, \omega = 0.99 \sim 0.2, c_1 = 2.5 \sim 0.5, c_2 = 0.5 \sim 2.5, c = 3 \sim 1.5, g_1 = 15, g_2 = 25$
3	DSPSO	2019	$N = 40, c_1 = 2, F_{min} = 0.7$
4	BLPSO	2017	$N = 40, \omega = 0.9 \sim 0.2, c_1 = 1.496, I = 1, E = 1$
5	TLS-PSO	2022	$N = 100, c_2 = 2$
6	GLPSO	2016	$N = 50, \omega = 0.7298, c_1 = 1.49618, pm = 0.01, sg = 7$
7	ACLPSO	2019	$N = 40, \omega = 0.99 \sim 0.2, c_1 = 2.5 \sim 0.5, c_2 = 0.5 \sim 2.5, m = 5, \mu = 0.3$
8	DMSDL-PSO	2018	$N = 40, \omega = 0.7298, c = 1.49618, CR = 0.025, R = 20$
9	HGWOP	2021	$N = 100, a_{max} = 2, a_{min} = 0, Cr_{max} = 1, Cr_{min} = 0$
10	HCLDMS-PSO	2020	$N = 40, \omega = [0.29, 0.99], c_1 = [0.5, 2.5], c_2 = [0.5, 2.5], P_m = 0.1, V_{max} = 0.5Range$
11	MPSO	2020	$N = 50, \omega = 1, \varphi_1 = \varphi_2 = 2.5$
12	MPA	2020	$N = 50, FADs = 0.2, P = 0.5$
13	IS-EO	2022	$N = 40, V = 1, a_1 = 2, a_2 = 1, G_P = 0.5$
14	PaDE	2019	$N = 25\log(D)\sqrt{D} \sim 4, \mu_F = 0.5, F \sim C(\mu_F, 0.1), \mu_{Cr} = 0.5, Cr \sim N(\mu_{Cr}, 0.1), k = 4, p = 0.11, r^{arc} = 1.6, T_0 = 70, r^d = 0.04$
15	TUS-HBO	2022	$N = 40, degree = 3$
16	MSSCS	2021	$N = 50, \alpha = 0.01, \beta = 1.5, P_a = 0.25, c = 0.2, PA_{max} = 0.35, PA_{min} = 0.25, L = 1$
17	TDBBO	2019	$N = 50, c = 0.3, m_{max} = 0.01$
18	CMM-DE/BBO	2016	$N = 100, P_e = 0.5$
19	HBBOS	2019	$N = 40, p_{cmin} = 0.97, m = 5, I = 1, n = 8$
20	CGO	2021	$N = 100, \beta = [0, 1], \gamma = [0, 1], \delta = [0, 1], \epsilon = [0, 1]$
21	JS	2021	$N = 100, \beta = 3, \gamma = 0.1$
22	HBO	2020	$N = 40, C = \lfloor T/25 \rfloor, p_1 = 1 - t/T, p_2 = p_1 + (1 - p_1)/2$
23	EO	2020	$N = 100, V = 1, a_1 = 2, a_2 = 1, G_P = 0.5$
-	DBPSO	-	$N = 100, pm = 0.935, c_1 = c_2 = 1$

D. Comparison results

TABLE S.4
COMPARISON RESULTS OF DBPSO AND THE INCOMPLETE ALGORITHMS ON THE 30-DIMENSIONAL CEC2013 TEST SET

	PSO		DBPSO (w/o B)		DBPSO (w/o R)		DBPSO (w/o Q)		DBPSO	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std	Mean	Std
F1	4.1908E-13	1.3922E-13	**0.0000E+00**	**0.0000E+00**	2.7196E-13	9.1172E-14	**0.0000E+00**	**0.0000E+00**	**0.0000E+00**	**0.0000E+00**
F2	1.2414E+07	6.0793E+06	1.6126E+01	3.0849E+01	**1.1978E+01**	**1.8854E+01**	3.2014E+06	8.6997E+05	2.2499E+01	6.2954E+01
F3	1.6660E+08	2.7005E+08	6.1871E+03	2.2211E+04	2.8231E+08	5.1691E+08	3.3332E+03	8.4073E+03	**1.3263E+03**	**4.0024E+03**
F4	5.7296E+03	1.4348E+03	6.8023E+00	8.7901E+00	7.9918E+00	1.1843E+01	1.6408E+04	2.6359E+03	**2.3409E+00**	**4.2568E+00**
F5	4.3914E-13	1.3262E-13	3.3437E-14	5.2316E-14	1.9527E-12	8.8213E-12	1.3375E-14	3.6993E-14	**4.4583E-15**	**2.2287E-14**
F6	8.0745E+01	2.8908E+01	3.6137E-11	2.4998E-11	7.0352E-01	1.5349E+00	2.5886E+01	9.7445E+00	**2.0265E-11**	**1.8126E-11**
F7	3.2940E+01	1.0393E+01	3.7511E-02	5.1134E-02	9.1930E+01	2.4686E+01	3.5724E-02	5.0119E-02	**3.2740E-02**	**1.0591E-01**
F8	2.0942E+01	5.1409E-02	2.0927E+01	5.4928E-02	2.0926E+01	5.0272E-02	2.0943E+01	5.1345E-02	**2.0913E+01**	**5.4244E-02**
F9	2.1960E+01	3.8274E+00	**6.1260E+00**	**1.9076E+00**	2.8847E+01	3.2776E+00	6.4522E+00	1.9455E+00	6.4976E+00	1.8845E+00
F10	3.1946E-01	3.2094E-01	1.4834E-02	1.3097E-02	1.1625E-01	8.6914E-02	1.1646E-02	9.9427E-03	**1.0436E-02**	**1.0210E-02**
F11	1.9612E+01	5.7079E+00	6.0868E+00	2.0646E+00	**7.4677E-14**	**2.8963E-14**	1.4604E-05	5.8313E-05	3.7407E-10	6.3686E-10
F12	7.5430E+01	3.3527E+01	**7.8231E+00**	**1.9801E+00**	2.1128E+02	5.7823E+01	1.5148E+02	1.0555E+01	1.0125E+01	1.8742E+00
F13	1.4821E+02	3.6338E+01	**1.9164E+01**	**8.2030E+00**	2.3573E+02	6.1336E+01	1.4965E+02	8.1497E+00	2.5330E+01	8.7939E+00
F14	8.1641E+02	2.5821E+02	1.4282E+03	2.7067E+02	**3.5235E+00**	**2.4654E+00**	5.6406E+00	2.9482E+00	3.6702E+00	2.4636E+00
F15	6.6444E+03	6.9454E+02	**1.5248E+03**	**3.6132E+02**	3.7718E+03	5.0404E+02	2.3482E+03	1.3821E+03	1.5303E+03	4.1394E+02
F16	2.2718E+00	2.9664E-01	1.2067E+00	3.6105E-01	**9.8288E-01**	**3.0378E-01**	1.9125E+00	3.1792E-01	1.0538E+00	3.0826E-01
F17	5.7387E+01	1.9410E+01	3.8344E+01	1.8030E+00	**3.0924E+01**	**2.1265E-01**	3.1300E+01	2.8389E-01	3.1066E+01	2.2570E-01
F18	2.2360E+02	4.1843E+01	**3.9530E+01**	**2.2368E+00**	1.8041E+02	6.9081E+01	1.8564E+02	1.0656E+01	4.0752E+01	2.1798E+00
F19	3.2289E+00	8.5344E-01	1.9072E+00	3.7742E-01	**1.0740E+00**	**3.7794E-01**	2.3410E+00	4.5088E-01	1.1726E+00	3.8229E-01
F20	1.4164E+01	1.3205E+00	**8.7776E+00**	**1.0862E+00**	1.3249E+01	1.6534E+00	1.1095E+01	8.4318E-01	9.2179E+00	1.3351E+00
F21	2.9873E+02	8.4376E+01	3.1689E+02	4.6709E+01	2.3811E+02	5.9203E+01	2.6165E+02	3.6842E+01	**2.3726E+02**	**4.8827E+01**
F22	8.9535E+02	2.7309E+02	1.1152E+03	2.6496E+02	**1.2228E+02**	**3.3135E+01**	1.6593E+02	3.4029E+01	1.2637E+02	3.0373E+01
F23	6.3989E+03	1.0590E+03	**1.6569E+03**	**3.4285E+02**	4.6832E+03	6.3842E+02	2.0720E+03	1.0733E+03	1.8105E+03	3.4274E+02
F24	2.6449E+02	9.1713E+00	2.0005E+02	3.3148E-02	2.8515E+02	1.2846E+01	2.0005E+02	3.7881E-02	**2.0003E+02**	**2.5169E-02**
F25	2.7972E+02	1.0372E+01	2.0001E+02	7.9369E-03	3.1090E+02	6.9970E+00	**2.0001E+02**	**6.7669E-03**	2.0119E+02	8.4572E+00
F26	3.0689E+02	7.2927E+01	2.8467E+02	3.7595E+01	2.0343E+02	2.4249E+01	2.0273E+02	1.0486E+00	**2.0027E+02**	**6.7456E-01**
F27	8.7513E+02	7.9304E+01	3.0101E+02	8.2584E-01	1.1036E+03	7.7546E+01	3.0111E+02	8.5342E-01	**3.0060E+02**	**4.8386E-01**
F28	3.9625E+02	3.3399E+02	2.6471E+02	7.7003E+01	2.6863E+02	7.3458E+01	**2.4045E+02**	**9.1616E+01**	2.5686E+02	8.3078E+01
Count	0		8		7		3		12	
Ave. Rank	4.43		2.57		3.29		2.93		1.68	
Total Rank	5		2		4		3		1	

TABLE S.5

EXPERIMENTAL RESULTS OF DBPSO AND THE PSO VARIANTS ON CEC 2013 TEST SUITE WITH 30-DIMENSIONAL

Function	Value	RQ-PSO	HCLPSO	DSPSO	BLPSO	TLS-PSO	GLPSO	ACLPSO	DMSDL-PSO	HGWOP	DBPSO
F1	Mean	**0.0000E+00**	3.2100E-13	1.3821E-13	**0.0000E+00**	**0.0000E+00**	1.3375E-13	2.2292E-13	8.9166E-15	**0.0000E+00**	**0.0000E+00**
	Std	**0.0000E+00**	1.1302E-13	1.1212E-13	**0.0000E+00**	**0.0000E+00**	1.1302E-13	3.1839E-14	4.4574E-14	**0.0000E+00**	**0.0000E+00**
F2	Mean	2.3907E+01	2.7877E+06	7.7717E+05	6.3069E+06	5.2724E+05	3.7722E+05	3.8057E+05	7.8480E+01	5.4624E+05	**2.2499E+01**
	Std	4.3249E+01	3.0112E+06	9.1771E+05	1.5689E+06	1.6145E+05	1.5066E+05	1.7445E+05	9.5146E+01	2.5214E+05	**6.2954E+01**
F3	Mean	6.5910E+03	2.8240E+07	1.2156E+06	1.2865E+06	6.3585E+03	3.8823E+06	4.2188E+06	1.5461E+07	5.9223E+05	**1.3263E+03**
	Std	1.3708E+04	7.4404E+07	2.1989E+06	1.2720E+06	2.4801E+04	6.5429E+06	8.1332E+06	1.7833E+07	1.2312E+06	**4.0024E+03**
F4	Mean	7.7721E+00	1.8539E+03	1.5002E+01	1.2147E+03	1.7989E+03	7.6666E+02	2.5722E+02	2.7850E+01	1.1995E+03	**2.3409E+00**
	Std	1.0801E+01	9.3602E+02	1.8456E+01	3.6696E+02	5.9359E+02	6.2524E+02	1.5366E+02	4.6731E+01	5.5954E+02	**4.2568E+00**
F5	Mean	6.4645E-14	3.9233E-13	1.6719E-13	1.0254E-13	1.0031E-13	1.2483E-13	1.8502E-13	1.1146E-13	1.1592E-13	**4.4583E-15**
	Std	6.9171E-14	1.0979E-13	7.3160E-14	3.4143E-14	3.6993E-14	3.4143E-14	5.5513E-14	1.5919E-14	2.7756E-14	**2.2287E-14**
F6	Mean	3.9804E-11	4.2529E+01	6.5861E+01	1.4709E+01	1.6080E+01	1.4170E+01	1.6899E+01	7.8169E-02	1.7963E+01	**2.0265E-11**
	Std	3.6572E-11	2.3725E+01	2.8140E+01	5.4940E+00	2.8195E-01	2.5022E+00	1.4065E+01	5.5824E-01	8.3588E+00	**1.8126E-11**
F7	Mean	4.2304E-02	2.1310E+01	2.3161E+00	9.6286E-01	5.6208E-02	4.7027E+00	4.9196E+00	6.1045E+01	1.6186E+00	**3.2740E-02**
	Std	4.4422E-02	9.1386E+00	1.7084E+00	7.2968E-01	1.0365E-01	3.2901E+00	2.5564E+00	1.2773E+01	1.5593E+00	**1.0591E-01**
F8	Mean	2.0915E+01	2.0950E+01	2.0945E+01	2.0942E+01	2.0913E+01	2.0938E+01	2.0946E+01	2.0930E+01	**2.0896E+01**	2.0913E+01
	Std	5.0679E-02	5.2805E-02	5.0626E-02	5.6721E-02	5.7819E-02	5.7904E-02	5.0573E-02	6.4354E-02	**6.6425E-02**	5.4244E-02
F9	Mean	6.0334E+00	2.0861E+01	9.7796E+00	1.7297E+01	**4.8636E+00**	1.1217E+01	8.6899E+00	2.7327E+01	9.2560E+00	6.4976E+00
	Std	1.6616E+00	3.6098E+00	2.2110E+00	5.2354E+00	**1.5545E+00**	2.5897E+00	2.3753E+00	1.8690E+00	2.1958E+00	1.8845E+00
F10	Mean	1.5897E-02	2.1713E-01	6.1507E-02	8.7545E-02	1.7007E-02	8.2828E-02	1.0281E-01	1.1127E-01	7.6542E-02	**1.0436E-02**
	Std	1.3606E-02	1.2852E-01	3.0524E-02	3.8635E-02	1.7573E-02	4.0273E-02	4.2624E-02	4.6893E-02	3.9869E-02	**1.0210E-02**
F11	Mean	5.2284E+00	1.9509E-02	1.6544E+01	1.3656E-01	8.7010E+00	3.1214E-01	2.3372E+01	**1.1146E-15**	1.4768E+01	3.7407E-10
	Std	1.6749E+00	1.3932E-01	4.4716E+00	3.9896E-01	3.1137E+00	4.6626E-01	5.7342E+00	**7.9597E-15**	5.3631E+00	6.3686E-10
F12	Mean	**6.9842E+00**	5.8640E+01	1.7812E+01	4.7246E+01	1.4866E+01	3.7351E+01	4.0481E+01	6.6838E+01	2.2045E+01	1.0125E+01
	Std	**2.4248E+00**	1.7098E+01	4.3014E+00	8.0865E+00	5.1523E+00	1.2319E+01	1.2165E+01	1.2096E+01	9.6204E+00	1.8742E+00
F13	Mean	**1.4725E+01**	1.1537E+02	4.3918E+01	6.6813E+01	3.2082E+01	7.9523E+01	1.0341E+02	1.2504E+02	5.5793E+01	2.5330E+01
	Std	**8.7783E+00**	2.5423E+01	1.5265E+01	1.2260E+01	1.6795E+01	2.3468E+01	3.7683E+01	1.6989E+01	2.2352E+01	8.7939E+00
F14	Mean	1.4379E+03	3.5324E+01	1.4698E+03	6.9768E+02	1.3890E+03	3.5402E+00	9.5771E+02	**1.0576E+00**	1.5264E+03	3.6702E+00
	Std	2.6058E+02	6.0944E+01	3.9684E+02	2.1812E+02	2.2628E+02	3.3859E+00	3.5413E+02	**2.1617E+00**	3.6865E+02	2.4636E+00
F15	Mean	1.5002E+03	3.4143E+03	2.2352E+03	3.8795E+03	**1.4005E+03**	3.0847E+03	3.4753E+03	3.3104E+03	2.4047E+03	1.5303E+03
	Std	4.6740E+02	4.9377E+02	6.2543E+02	4.9256E+02	**4.3124E+02**	7.2220E+02	7.7547E+02	4.0533E+02	6.6458E+02	4.1394E+02
F16	Mean	1.2199E+00	1.6469E+00	2.3152E+00	2.4808E+00	**8.1078E-01**	1.3732E+00	2.4826E+00	1.2353E+00	9.7916E-01	1.0538E+00
	Std	4.5298E-01	2.2332E-01	3.3988E-01	2.8597E-01	**4.8585E-01**	9.8584E-01	2.7564E-01	3.5738E-01	4.5420E-01	3.0826E-01
F17	Mean	3.7534E+01	3.1123E+01	4.4313E+01	7.0979E+01	4.8580E+01	3.3533E+01	6.1800E+01	**3.0434E+01**	4.1503E+01	3.1066E+01
	Std	1.7563E+00	2.3423E-01	4.5003E+00	4.9054E+00	6.6158E+00	1.1533E+00	8.5555E+00	**4.7642E-04**	5.7160E+00	2.2570E-01
F18	Mean	**3.9016E+01**	8.5728E+01	6.8770E+01	1.8169E+02	5.5998E+01	6.6800E+01	9.6461E+01	7.9899E+01	4.6295E+01	4.0752E+01
	Std	**2.8562E+00**	2.2319E+01	3.2766E+01	1.5268E+01	8.2701E+00	1.6993E+01	4.0926E+01	1.1569E+01	7.2970E+00	2.1798E+00
F19	Mean	1.8962E+00	1.5168E+00	3.0843E+00	3.8153E+00	2.8070E+00	1.4981E+00	2.6667E+00	**8.5016E-01**	3.3720E+00	1.1726E+00
	Std	3.5225E-01	2.6002E-01	7.1275E-01	4.7234E-01	7.3816E-01	2.7458E-01	5.8405E-01	**2.1690E-01**	6.7114E-01	3.8229E-01
F20	Mean	**8.7442E+00**	1.0708E+01	9.5607E+00	1.0548E+01	8.7750E+00	9.9337E+00	9.8293E+00	1.1024E+01	1.0045E+01	9.2179E+00
	Std	**9.2421E-01**	7.8924E-01	9.3345E-01	3.8968E-01	4.7015E-01	9.1242E-01	8.2833E-01	8.3771E-01	1.0138E+00	1.3351E+00
F21	Mean	3.3659E+02	2.6867E+02	3.3352E+02	2.9864E+02	3.2619E+02	3.2875E+02	3.2312E+02	2.8884E+02	3.3096E+02	**2.3726E+02**
	Std	6.3180E+01	4.2841E+01	7.2272E+01	4.7270E+01	6.0180E+01	7.2900E+01	6.9195E+01	5.5760E+01	5.9627E+01	**4.8827E+01**
F22	Mean	1.0181E+03	1.1874E+02	1.0574E+03	2.8815E+02	6.5350E+02	1.1222E+02	7.7038E+02	**1.1003E+02**	1.0693E+03	1.2637E+02
	Std	2.8582E+02	2.3873E+01	3.2941E+02	7.9594E+01	1.9910E+02	1.9837E+01	2.5406E+02	**1.5539E+01**	3.3529E+02	3.0373E+01
F23	Mean	1.6325E+03	3.8036E+03	2.2911E+03	3.4108E+03	**1.3396E+03**	3.2333E+03	3.2990E+03	4.2421E+03	2.5688E+03	1.8105E+03
	Std	4.0718E+02	5.9050E+02	4.7358E+02	4.4766E+02	**4.0195E+02**	5.8954E+02	7.1027E+02	4.4384E+02	6.5479E+02	3.4274E+02
F24	Mean	2.0006E+02	2.2826E+02	2.0914E+02	2.0026E+02	2.0005E+02	2.0926E+02	2.0569E+02	2.6271E+02	2.0466E+02	**2.0003E+02**
	Std	4.8845E-02	8.5696E+00	1.0664E+01	2.8111E-01	5.5636E-02	5.1627E+00	5.8188E+00	8.3222E+00	8.6737E+00	**2.5169E-02**
F25	Mean	**2.0002E+02**	2.6346E+02	2.4693E+02	2.5088E+02	2.0107E+02	2.4464E+02	2.4844E+02	2.9131E+02	2.4813E+02	2.0119E+02
	Std	**1.0706E-02**	2.9800E+01	3.5507E+01	2.4817E+01	7.5578E+00	1.9296E+01	6.1671E+00	5.0216E+00	2.4597E+01	8.4572E+00
F26	Mean	2.9391E+02	2.0005E+02	2.7086E+02	2.3221E+02	2.6910E+02	2.2892E+02	2.5415E+02	2.0009E+02	2.8350E+02	2.0027E+02
	Std	2.4719E+01	2.5306E-02	5.5428E+01	4.7278E+01	4.7200E+01	5.0002E+01	5.3701E+01	1.3639E-01	4.7294E+01	6.7456E-01
F27	Mean	3.0108E+02	5.7442E+02	3.9904E+02	3.0451E+02	3.0087E+02	4.1868E+02	3.6297E+02	9.9342E+02	3.7878E+02	**3.0060E+02**
	Std	7.8856E-01	9.5420E+01	9.5229E+01	4.6986E+00	8.2840E-01	7.6097E+01	6.9835E+01	7.2944E+01	8.1674E+01	**4.8386E-01**
F28	Mean	2.6078E+02	2.9710E+02	3.0442E+02	3.0000E+02	2.8039E+02	3.0000E+02	3.2197E+02	3.0000E+02	2.7255E+02	**2.5686E+02**
	Std	8.0196E+01	2.0738E+01	2.1960E+02	0.0000E+00	6.0065E+01	2.7427E-13	1.5691E+02	2.6271E-13	6.9508E+01	**8.3078E+01**
Count		6	1	0	1	5	0	0	5	2	12
Ave. Rank		3.48	7.39	6.68	6.57	3.93	5.57	7.29	6.04	5.75	1.93
Total Rank		2	10	8	7	3	4	9	6	5	1

E. Tension/compression spring design problem

The first engineering problem is the Tension/compression spring design problem, which is shown in Fig. S.1. The main objective of this problem is to optimize the weight of a tension or compression spring. This problem contains four constraints and three variables are utilized to calculate the weight: the diameter of the wire $d(x_1)$, the mean of the diameter of coil $D(x_2)$, and the number of active coils $P(x_3)$. The mathematical model of this problem can be defined as Eq. (S.1). Fig. S.2(a) gives the convergence curves of the DBPSO, HGWOP, and DSPSO on the tension/compression spring design problem.

TABLE S.6
EXPERIMENTAL RESULTS OF DBPSO AND THE PSO VARIANTS ON CEC 2013 TEST SUITE WITH 50-DIMENSIONAL

Function	Value	RQ-PSO	HCLPSO	DSPSO	BLPSO	TLS-PSO	GLPSO	ACLPSO	DMSDL-PSO	HGWOP	DBPSO
F1	Mean	4.4583E-15	5.8850E-13	3.2546E-13	2.0954E-13	1.6050E-13	2.3629E-13	3.7450E-13	2.1846E-13	9.3624E-14	**0.0000E+00**
	Std	3.1839E-14	1.2182E-13	1.3066E-13	6.1737E-14	1.0463E-13	4.4574E-14	1.2720E-13	4.4574E-14	1.1302E-13	**0.0000E+00**
F2	Mean	5.5838E+01	1.1395E+07	7.6035E+05	1.3833E+07	8.1521E+05	1.1208E+06	9.1678E+05	5.2152E+01	9.1369E+05	**2.5922E+01**
	Std	1.0616E+02	1.0173E+07	3.1195E+05	3.7790E+06	1.9023E+05	3.6709E+05	3.6006E+05	4.6093E+01	1.8998E+05	**5.1733E+01**
F3	Mean	2.7122E+05	2.0752E+08	1.2127E+07	1.0523E+06	**3.3393E+04**	9.5476E+06	2.5250E+07	1.9961E+08	6.9620E+06	3.8944E+04
	Std	8.3094E+05	1.7151E+08	2.4328E+07	1.6287E+06	**9.7790E+04**	9.5362E+06	3.1605E+07	1.6078E+08	2.0160E+07	1.3368E+05
F4	Mean	3.1827E+01	1.9357E+03	6.7816E+01	7.6424E+02	7.5229E+02	1.0851E+03	3.6452E+02	2.3742E+01	9.1418E+02	**6.4866E-01**
	Std	2.1632E+02	6.3848E+02	6.5420E+01	1.7832E+02	2.8038E+02	5.8915E+02	1.5726E+02	4.3467E+01	4.0349E+02	**2.5768E+00**
F5	Mean	2.3183E-13	8.1141E-13	3.7896E-13	**1.1369E-13**	1.7164E-13	3.2991E-13	5.0602E-13	**1.1369E-13**	1.9839E-13	**1.1369E-13**
	Std	1.2446E-13	1.2664E-13	1.0083E-13	**0.0000+00**	6.5791E-14	6.9171E-14	1.0248E-13	**0.0000E+00**	5.4962E-14	**0.0000E+00**
F6	Mean	3.0294E-11	4.5323E+01	4.3586E+01	4.3447E+01	4.3590E+01	4.3647E+01	4.3450E+01	6.5506E-11	4.5187E+01	**1.9719E-11**
	Std	2.8969E-11	8.2767E-01	5.9635E-01	1.6510E-10	1.6493E-01	7.4160E-01	8.0591E-03	8.2977E-11	5.9924E-01	**1.3861E-11**
F7	Mean	3.4919E-01	3.8223E+01	9.1784E+00	4.6430E+00	**1.6095E-01**	1.1045E+01	1.3750E+01	7.2225E+01	4.6733E+00	2.5917E-01
	Std	3.7801E-01	1.0102E+01	3.6357E+00	3.2962E+00	**1.5987E-01**	4.9479E+00	5.6694E+00	9.9040E+00	2.3658E+00	3.2474E-01
F8	Mean	2.1125E+01	2.1126E+01	2.1137E+01	2.1130E+01	2.1119E+01	2.1132E+01	2.1136E+01	2.1122E+01	**2.1109E+01**	2.1124E+01
	Std	3.3804E-02	4.5239E-02	3.3040E-02	3.8992E-02	4.2881E-02	3.9772E-02	3.2794E-02	3.6819E-02	**3.9706E-02**	3.8650E-02
F9	Mean	1.2320E+01	4.1692E+01	1.8520E+01	4.0637E+01	**1.1149E+01**	1.7224E+01	2.1716E+01	5.4488E+01	1.6776E+01	1.2388E+01
	Std	2.6993E+00	5.2985E+00	2.6716E+00	4.7986E+00	**2.5129E+00**	3.0446E+00	3.7571E+00	2.1891E+00	3.5416E+00	2.1447E+00
F10	Mean	9.7330E-02	2.2727E-01	1.2830E-01	1.6878E-01	**5.9756E-02**	1.3194E-01	1.7200E-01	2.1311E-01	1.1806E-01	6.9221E-02
	Std	4.3163E-02	1.2342E-01	6.9555E-02	9.2006E-02	**3.9950E-02**	6.6956E-02	8.9223E-02	9.9109E-02	5.8973E-02	3.9003E-02
F11	Mean	1.8085E+01	3.9018E-02	3.8238E+01	6.8118E-01	1.6251E+01	9.7545E-02	4.9963E+01	3.9018E-02	2.7625E+01	**2.6066E-10**
	Std	4.1727E+00	1.9505E-01	9.4584E+00	1.1367E+00	4.2009E+00	2.9881E-01	9.5439E+00	1.9505E-01	9.6886E+00	**5.5532E-10**
F12	Mean	**1.9353E+01**	1.3062E+02	3.7087E+01	1.0613E+02	2.7547E+01	6.8195E+01	8.5313E+01	1.5650E+02	4.6529E+01	2.1733E+01
	Std	**3.4997E+00**	3.1956E+01	7.8219E+00	1.6512E+01	6.5983E+00	1.4889E+01	2.2521E+01	2.3258E+01	1.3365E+01	3.1986E+00
F13	Mean	**5.1332E+01**	2.6214E+02	1.0316E+02	1.5589E+02	6.4077E+01	1.7067E+02	1.9794E+02	2.8131E+02	1.2171E+02	6.9981E+01
	Std	**1.7290E+01**	5.7142E+01	2.5709E+01	2.3842E+01	2.4441E+01	3.5848E+01	3.9204E+01	3.1818E+01	3.2211E+01	1.6021E+01
F14	Mean	2.2418E+03	4.6508E+01	2.6973E+03	1.1708E+03	2.1342E+03	**4.1948E+00**	2.2095E+03	5.8940E+00	2.6150E+03	9.2474E+00
	Std	5.9922E+02	5.9582E+01	8.0278E+02	2.6264E+02	4.3813E+02	**2.1746E+00**	6.1187E+02	7.0354E+00	6.1321E+02	3.8064E+00
F15	Mean	4.1085E+03	7.4096E+03	5.3371E+03	7.9186E+03	**3.9101E+03**	6.2467E+03	7.1275E+03	6.8980E+03	5.8599E+03	4.1807E+03
	Std	7.4923E+02	8.6419E+02	9.2270E+02	5.8557E+02	**8.6362E+02**	9.6979E+02	1.2102E+03	7.7205E+02	1.0972E+03	6.3311E+02
F16	Mean	1.7937E+00	1.9999E+00	3.3388E+00	3.3326E+00	**9.2664E-01**	1.9217E+00	3.2228E+00	1.6128E+00	1.3672E+00	1.5441E+00
	Std	4.8414E-01	2.7583E-01	2.8268E-01	2.8719E-01	**4.7574E-01**	1.2069E+00	3.9652E-01	4.9571E-01	5.8721E-01	4.8465E-01
F17	Mean	6.5216E+01	5.2276E+01	8.0263E+01	1.2860E+02	8.3593E+01	5.4466E+01	1.2028E+02	**5.0786E+01**	7.7407E+01	5.2084E+01
	Std	2.8397E+00	4.2443E-01	5.1206E+00	8.2674E+00	1.1866E+01	1.3855E+00	1.3253E+01	**1.9686E-03**	1.3814E+01	3.7228E-01
F18	Mean	**6.7333E+01**	1.7601E+02	1.1329E+02	3.3607E+02	9.0630E+01	1.1888E+02	1.5282E+02	1.3800E+02	8.0117E+01	6.9621E+01
	Std	**3.1543E+00**	3.8968E+01	4.5735E+01	1.9910E+01	1.1106E+01	1.4629E+01	6.0853E+01	1.6653E+01	1.6881E+01	3.7247E+00
F19	Mean	4.5813E+00	2.9660E+00	5.7516E+00	7.3768E+00	5.0952E+00	2.4649E+00	5.2683E+00	**1.8490E+00**	6.5615E+00	2.6239E+00
	Std	7.2137E-01	4.2764E-01	9.7114E-01	6.3576E-01	8.8273E-01	3.3702E-01	1.0149E+00	**3.2153E-01**	1.0284E+00	4.7052E-01
F20	Mean	**1.6190E+01**	1.9203E+01	1.8192E+01	1.9706E+01	1.6277E+01	1.8582E+01	1.8720E+01	2.0418E+01	1.8006E+01	1.6222E+01
	Std	**9.6730E-01**	8.3631E-01	1.3298E+00	5.3109E-01	1.1420E+00	8.6937E-01	1.0926E+00	8.9996E-01	1.0149E+00	7.7155E-01
F21	Mean	9.0801E+02	2.9389E+02	9.0546E+02	8.5042E+02	8.7999E+02	8.9553E+02	8.8228E+02	7.8372E+02	9.3730E+02	**2.6240E+02**
	Std	1.6545E+02	1.9190E+02	2.2467E+02	3.5925E+02	1.4999E+02	1.9271E+02	3.2423E+02	3.9733E+02	1.3792E+02	**1.9114E+02**
F22	Mean	2.1541E+03	3.3240E+01	2.6600E+03	5.8059E+02	1.5285E+03	3.1686E+01	1.9460E+03	**2.4784E+01**	3.0521E+03	8.6253E+01
	Std	4.6090E+02	2.6960E+01	6.5084E+02	2.1850E+02	3.7670E+02	3.8130E+01	5.2737E+02	**2.4048E+01**	9.3757E+02	5.1966E+01
F23	Mean	3.9161E+03	8.1926E+03	5.6657E+03	7.5104E+03	**3.3445E+03**	6.5148E+03	6.8414E+03	8.5669E+03	6.6893E+03	4.8859E+03
	Std	8.9280E+02	9.5399E+02	1.0449E+03	8.0351E+02	**8.2919E+02**	1.0215E+03	1.1652E+03	8.3338E+02	1.5041E+03	1.3630E+03
F24	Mean	2.0041E+02	2.6880E+02	2.5529E+02	2.0292E+02	2.0039E+02	2.3210E+02	2.2733E+02	3.4192E+02	2.2184E+02	**2.0026E+02**
	Std	2.1788E-01	1.2321E+01	2.2500E+01	2.7905E+00	2.3209E-01	1.5596E+01	8.6389E+00	9.6833E+00	1.9178E+01	**1.5714E-01**
F25	Mean	3.0161E+02	3.8795E+02	3.2223E+02	3.2117E+02	2.8840E+02	2.8815E+02	2.9986E+02	3.7909E+02	3.1367E+02	**2.7827E+02**
	Std	4.2313E+01	1.2860E+01	1.4165E+01	2.4861E+01	7.3635E+00	8.6369E+00	9.5327E+00	6.4444E+00	1.1132E+01	**5.8944E+01**
F26	Mean	3.0246E+02	2.0033E+02	3.2105E+02	2.9511E+02	2.7808E+02	3.1452E+02	3.0231E+02	**2.0006E+02**	3.3774E+02	2.0205E+02
	Std	3.6781E+01	4.3501E-01	6.1126E+01	4.2218E+01	4.6680E+01	5.1111E+01	6.7520E+01	**2.7210E-01**	2.3078E+01	1.4095E+01
F27	Mean	4.1190E+02	1.2072E+03	8.5206E+02	6.0387E+02	3.3786E+02	7.4861E+02	7.7251E+02	1.7099E+03	7.7974E+02	**3.3700E+02**
	Std	1.8380E+02	1.6182E+02	1.5124E+02	2.9590E+02	8.9372E+01	1.3230E+02	1.7125E+02	8.4598E+01	1.7760E+02	**8.7597E+01**
F28	Mean	5.1447E+02	4.0000E+02	6.9228E+02	4.0000E+02	4.0000E+02	4.5913E+02	4.5909E+02	**4.0000E+02**	6.3302E+02	4.0000E+02
	Std	5.7224E+02	6.8515E-13	8.9536E+02	2.8705E-13	3.1782E-13	4.2225E+02	4.2200E+02	**2.6123E-13**	8.0671E+02	2.7320E-13
Count		4	0	0	1	7	1	0	6	1	10
Ave. Rank		4.04	7.36	6.93	6.39	3.54	5.79	7.21	5.50	6.04	2.07
Total. Rank		3	10	8	7	2	5	9	4	6	1

$$
\begin{aligned}
&\text{minimize}: \\
&f(\mathbf{x}) = f\left(x_1, x_2, x_3\right) = \left(x_3 + 2\right) x_2 x_1^2 \\
&\text{subject to}: \\
&g_1(\mathbf{x}) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \leq 0 \\
&g_2(\mathbf{x}) = \frac{4x_2^2 - x_1 x_2}{12566\left(x_2 x_1^3 - x_1^4\right)} + \frac{1}{5108 x_1^2} - 1 \leq 0 \\
&g_3(\mathbf{x}) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \leq 0 \\
&g_4(\mathbf{x}) = x_2 + \frac{x_1}{1.5} - 1 \leq 0 \\
&\text{variable range:} \\
&0.05 \leq x_1 \leq 2,\ 0.25 \leq x_2 \leq 1.3,\ 2 \leq x_3 \leq 15
\end{aligned}
\tag{S.1}
$$

TABLE S.7

EXPERIMENTAL RESULTS OF DBPSO AND THE OTHER NMAS ON CEC 2013 TEST SUITE WITH 50-DIMENSIONAL

Function	Value	MPA	ISEO	PaDE	TUSHBO	MSSCS	TDBBO	CMMDEBBO	HBBOS	DBPSO
F1	Mean	4.4681E-04	2.2737E-13	4.0125E-14	2.1846E-13	**0.0000E+00**	3.5221E-13	4.0125E-14	1.7387E-13	**0.0000E+00**
	Std	4.5871E-04	0.0000E+00	8.7542E-14	4.4574E-14	**0.0000E+00**	1.9461E-13	8.7542E-14	9.7408E-14	**0.0000E+00**
F2	Mean	1.2955E+06	3.9144E+06	1.7648E+03	3.2722E+05	2.8511E+06	5.8308E+05	1.1500E+06	5.1532E+05	**2.5922E+01**
	Std	6.4186E+05	1.5496E+06	2.2100E+03	1.1642E+05	1.1985E+06	2.1890E+05	3.9248E+05	2.1913E+05	**5.1733E+01**
F3	Mean	3.5540E+08	1.0886E+08	**3.3930E+03**	9.0914E+06	8.7818E+06	1.7667E+07	1.0606E+05	2.1059E+07	3.8944E+04
	Std	2.5559E+08	9.7457E+07	**1.0943E+04**	1.7910E+07	1.0188E+07	1.3259E+07	3.3947E+05	2.2359E+07	1.3368E+05
F4	Mean	9.7696E+01	3.7000E+01	**3.8863E-11**	1.6041E-01	9.2050E+03	1.3065E+02	1.3576E+00	2.7367E+00	6.4866E-01
	Std	8.5491E+01	1.5355E+01	**8.2639E-11**	2.0461E-01	3.4265E+03	6.4122E+01	2.5516E+00	7.1697E+00	2.5768E+00
F5	Mean	1.0049E-02	1.3375E-13	1.8056E-13	1.9394E-13	**1.1369E-13**	5.2960E-04	1.1592E-13	**1.1369E-13**	**1.1369E-13**
	Std	3.6172E-03	4.3771E-14	5.6508E-14	5.7044E-14	**0.0000E+00**	3.1065E-03	1.5919E-14	**0.0000E+00**	**0.0000E+00**
F6	Mean	7.1804E+01	4.6599E+01	4.3447E+01	4.3976E+01	4.4123E+01	6.5421E+01	4.3447E+01	4.4552E+01	**1.9719E-11**
	Std	2.9337E+01	9.9428E+00	1.6078E-14	8.0923E+00	1.6067E+00	2.4683E+01	1.7681E-11	7.1035E+00	**1.3861E-11**
F7	Mean	5.5734E+01	2.9605E+01	7.3659E-01	2.5424E+01	7.5800E+01	2.3192E+01	5.3488E-01	1.8471E+01	**2.5917E-01**
	Std	9.1703E+00	8.8895E+00	6.2929E-01	1.0386E+01	1.4765E+01	7.9407E+00	5.4811E-01	7.3225E+00	**3.2474E-01**
F8	Mean	2.1124E+01	2.1135E+01	**2.1072E+01**	2.1128E+01	2.1124E+01	2.1130E+01	2.1120E+01	2.1136E+01	2.1124E+01
	Std	4.1315E-02	3.4509E-02	**9.5595E-02**	4.8656E-02	3.6768E-02	3.2811E-02	3.3490E-02	3.7502E-02	3.8650E-02
F9	Mean	3.7861E+01	5.7050E+01	5.0370E+01	3.5511E+01	4.4151E+01	5.3669E+01	5.5775E+01	4.9827E+01	**1.2388E+01**
	Std	4.4844E+00	3.4591E+00	3.8302E+00	6.9896E+00	6.3830E+00	6.3054E+00	2.3505E+00	4.0189E+00	**2.1447E+00**
F10	Mean	1.1589E+00	1.4537E-01	**1.2463E-02**	1.4206E-01	5.9438E-02	2.1756E-01	6.0744E-02	1.6510E-01	6.9221E-02
	Std	3.9181E-01	8.7936E-02	**1.1448E-02**	1.0410E-01	3.9427E-02	1.1586E-01	3.4392E-02	9.5030E-02	3.9003E-02
F11	Mean	4.0279E+01	3.4221E+01	**1.7387E-13**	1.5822E+01	2.0094E+00	6.6046E+01	1.0021E+01	3.5116E-01	2.6066E-10
	Std	1.3192E+01	1.5423E+01	**7.7451E-14**	4.6805E+00	2.0047E+00	1.5139E+01	2.0804E+00	7.3985E-01	5.5532E-10
F12	Mean	1.6065E+02	9.6970E+01	**1.8369E+01**	9.3038E+01	2.2945E+02	1.0443E+02	1.3131E+02	9.5496E+01	2.1733E+01
	Std	2.9596E+01	1.9491E+01	**2.6643E+00**	2.1752E+01	4.5106E+01	2.3160E+01	1.4151E+01	2.2029E+01	3.1986E+00
F13	Mean	2.9092E+02	2.2775E+02	**3.3484E+01**	2.0499E+02	3.3224E+02	2.1070E+02	2.0058E+02	1.8175E+02	6.9981E+01
	Std	5.2703E+01	4.2006E+01	**9.1013E+00**	4.7928E+01	4.3256E+01	2.9393E+01	1.7901E+01	3.6414E+01	1.6021E+01
F14	Mean	2.3506E+03	9.3824E+02	**5.7285E-02**	7.3883E+01	2.3695E+00	1.4151E+03	4.9462E+02	2.8947E+01	9.2474E+00
	Std	4.7153E+02	3.6686E+02	**2.4083E-02**	7.4951E+01	1.6079E+00	4.8963E+02	9.8784E+01	3.2372E+01	3.8064E+00
F15	Mean	6.7073E+03	7.9659E+03	6.4379E+03	9.4909E+03	8.2007E+03	7.2418E+03	1.0171E+04	7.1701E+03	**4.1807E+03**
	Std	6.2941E+02	7.6239E+02	3.8493E+02	2.0064E+03	7.9179E+02	5.0477E+02	6.1257E+02	8.4406E+02	**6.3311E+02**
F16	Mean	**1.5560E-01**	1.8352E+00	1.0755E+00	2.4719E+00	2.0105E+00	1.5102E+00	2.7779E+00	1.4572E+00	1.5441E+00
	Std	**5.1875E-02**	4.2003E-01	5.2145E-01	7.6760E-01	2.9499E-01	2.7077E-01	3.5012E-01	9.3156E-01	4.8465E-01
F17	Mean	1.3917E+02	8.1888E+01	**5.0786E+01**	5.5501E+01	5.1191E+01	1.3460E+02	7.3947E+01	5.2941E+01	5.2084E+01
	Std	1.9211E+01	2.2622E+01	**1.4453E-09**	2.5860E+00	6.1552E-01	1.6435E+01	1.9517E+00	1.3591E+00	3.7228E-01
F18	Mean	2.2444E+02	1.6224E+02	1.0782E+02	2.4408E+02	2.8765E+02	1.7997E+02	2.9743E+02	1.1273E+02	**6.9621E+01**
	Std	3.4585E+01	2.4284E+01	6.3508E+00	8.1412E+01	4.0541E+01	1.8354E+01	1.9181E+01	1.8028E+01	**3.7247E+00**
F19	Mean	9.5837E+00	4.7829E+00	2.3474E+00	3.7282E+00	3.2391E+00	1.3362E+01	6.0471E+00	**1.9755E+00**	2.6239E+00
	Std	2.1451E+00	9.5955E-01	1.3134E-01	8.8765E-01	3.2932E-01	3.5581E+00	3.8870E-01	**3.2853E-01**	4.7052E-01
F20	Mean	1.9417E+01	1.9555E+01	1.7733E+01	2.0764E+01	2.0882E+01	1.8893E+01	2.0914E+01	1.9325E+01	**1.6222E+01**
	Std	8.3224E-01	8.0838E-01	5.1861E-01	7.6067E-01	8.9230E-01	7.1906E-01	3.6914E-01	8.0452E-01	**7.7155E-01**
F21	Mean	9.0802E+02	9.1106E+02	9.3016E+02	8.5605E+02	3.1792E+02	9.4345E+02	3.7089E+02	7.6437E+02	**2.6240E+02**
	Std	1.6542E+02	2.2646E+02	3.6843E+02	3.6667E+02	2.6007E+02	2.5783E+02	3.3581E+02	4.1623E+02	**1.9114E+02**
F22	Mean	2.9160E+03	1.1323E+03	**1.1694E+01**	1.7520E+02	5.0288E+01	2.2581E+03	5.1492E+02	4.2714E+01	8.6253E+01
	Std	6.4576E+02	3.9313E+02	**5.8364E-01**	1.4658E+02	3.6567E+01	4.3835E+02	8.3599E+01	5.7879E+01	5.1966E+01
F23	Mean	6.9813E+03	7.7247E+03	5.8957E+03	1.0346E+04	9.3854E+03	7.9231E+03	9.9210E+03	7.2057E+03	**4.8859E+03**
	Std	5.7081E+02	8.5325E+02	4.1928E+02	1.9267E+03	1.0416E+03	5.5429E+02	6.2775E+02	9.4809E+02	**1.3630E+03**
F24	Mean	2.8855E+02	3.1421E+02	2.0210E+02	2.3879E+02	3.2495E+02	2.4712E+02	2.0111E+02	2.3511E+02	**2.0026E+02**
	Std	1.1492E+01	1.6375E+01	1.7131E+00	1.1308E+01	1.7751E+01	8.7820E+00	4.9617E+00	1.1137E+01	**1.5714E-01**
F25	Mean	3.3917E+02	3.4987E+02	2.8512E+02	3.2516E+02	3.6124E+02	3.4190E+02	**2.7293E+02**	3.4790E+02	2.7827E+02
	Std	1.1886E+01	1.4267E+01	7.6779E+00	1.3896E+01	1.4762E+01	3.1981E+01	**5.9802E+00**	1.7951E+01	5.8944E+01
F26	Mean	**2.0013E+02**	3.3839E+02	2.5741E+02	2.3598E+02	2.0023E+02	2.6070E+02	2.7164E+02	3.0792E+02	2.0205E+02
	Std	**5.1098E-02**	1.1697E+02	5.2590E+01	6.5925E+01	8.5444E-02	7.3483E+01	8.4091E+01	5.5886E+01	1.4095E+01
F27	Mean	1.2349E+03	1.6288E+03	3.4147E+02	9.7152E+02	1.4884E+03	8.5398E+02	4.1564E+02	8.8365E+02	**3.3700E+02**
	Std	1.1407E+02	1.2634E+02	2.7684E+01	1.5961E+02	1.7265E+02	8.8938E+01	1.5239E+02	2.4494E+02	**8.7597E+01**
F28	Mean	4.0015E+02	7.0045E+02	4.0000E+02	4.5949E+02	4.6155E+02	4.0000E+02	4.0000E+02	4.5689E+02	**4.0000E+02**
	Std	4.8618E-02	9.2041E+02	2.7882E-13	4.2482E+02	4.3954E+02	3.9742E-13	3.6769E-13	4.0627E+02	**2.7320E-13**
Count		2	0	10	0	2	0	1	2	14
Ave. Rank		6.46	6.96	2.39	5.36	5.57	6.50	5.04	4.50	2.00
Total. Rank		7	9	2	5	6	8	4	3	1

TABLE S.8
EXPERIMENTAL RESULTS OF DBPSO AND SEVEN ALGORITHMS ON CEC2017 TEST SUITE

Function	HCLDMS-PSO	MPSO	CGO	JS	EO	HBO	TLS-PSO	DBPSO
f_1	1.61E+03	4.75E+07	2.08E+03	2.19E+03	3.32E+03	2.92E+02	2.24E+03	**7.60E+01**
f_3	2.72E+01	1.68E-01	**1.26E-09**	3.89E+03	2.35E+01	2.82E+03	9.20E+00	6.78E-06
f_4	7.81E+01	6.63E+01	4.54E+01	8.79E+01	6.96E+01	9.08E+01	1.13E+02	**1.25E+00**
f_5	2.94E+01	6.92E+01	1.02E+02	1.09E+02	5.77E+01	1.22E+02	1.13E+02	**1.79E+01**
f_6	1.45E-03	7.28E-02	2.48E+01	2.97E+00	5.66E-02	**1.14E-13**	1.51E-07	8.05E-09
f_7	5.60E+01	9.92E+01	2.09E+02	1.43E+02	8.83E+01	1.59E+02	4.69E+01	**3.99E+01**
f_8	2.75E+01	6.35E+01	9.05E+01	9.59E+01	5.71E+01	1.28E+02	**1.03E+01**	1.79E+01
f_9	1.07E-02	1.13E+02	1.16E+03	1.26E+02	3.21E+01	9.49E-01	**0.00E+00**	**0.00E+00**
f_{10}	2.30E+03	3.28E+03	4.21E+03	4.46E+03	3.16E+03	4.44E+03	**1.39E+03**	1.65E+03
f_{11}	3.27E+01	1.08E+02	1.53E+02	6.73E+01	5.95E+01	5.33E+01	5.93E+01	**3.17E+01**
f_{12}	7.37E+04	5.16E+05	1.81E+04	3.81E+04	6.64E+04	2.10E+06	1.60E+04	**1.38E+03**
f_{13}	4.83E+03	1.29E+04	1.63E+04	3.02E+03	2.17E+04	1.03E+04	7.83E+03	**4.29E+02**
f_{14}	3.39E+03	5.32E+02	2.46E+02	2.25E+03	5.68E+03	4.08E+04	1.14E+03	**1.29E+02**
f_{15}	2.84E+03	1.34E+03	1.08E+04	1.71E+03	5.52E+03	1.52E+03	2.05E+03	**6.20E+02**
f_{16}	3.13E+02	8.79E+02	8.83E+02	6.77E+02	6.84E+02	7.47E+02	**3.01E+02**	4.81E+02
f_{17}	**7.61E+01**	3.79E+02	4.89E+02	1.57E+02	2.22E+02	1.73E+02	8.30E+01	1.43E+02
f_{18}	1.05E+05	2.72E+04	1.70E+04	1.23E+05	1.24E+05	5.06E+05	6.15E+04	**2.13E+03**
f_{19}	3.22E+03	5.15E+03	2.00E+03	3.53E+03	7.55E+03	1.20E+03	3.57E+03	**1.38E+02**
f_{20}	**1.40E+02**	3.03E+02	4.16E+02	2.74E+02	2.10E+02	2.05E+02	1.95E+02	2.00E+02
f_{21}	2.27E+02	2.58E+02	2.93E+02	2.78E+02	2.49E+02	3.31E+02	**2.15E+02**	2.23E+02
f_{22}	**1.00E+02**	1.91E+02	3.63E+02	1.01E+02	1.12E+03	2.01E+03	**1.00E+02**	**1.00E+02**
f_{23}	3.77E+02	4.34E+02	4.93E+02	4.28E+02	4.07E+02	4.72E+02	**3.58E+02**	3.67E+02
f_{24}	4.45E+02	4.93E+02	5.47E+02	4.95E+02	4.70E+02	6.03E+02	4.23E+02	**4.20E+02**
f_{25}	3.86E+02	4.30E+02	4.00E+02	3.91E+02	3.88E+02	3.87E+02	3.88E+02	**3.79E+02**
f_{26}	1.15E+03	1.40E+03	2.83E+03	1.47E+03	1.55E+03	2.19E+03	4.12E+02	**2.51E+02**
f_{27}	5.14E+02	5.37E+02	5.80E+02	5.24E+02	5.17E+02	5.09E+02	5.26E+02	**4.88E+02**
f_{28}	3.62E+02	5.21E+02	3.46E+02	4.09E+02	3.57E+02	3.73E+02	3.69E+02	**3.04E+02**
f_{29}	4.88E+02	6.83E+02	1.17E+03	6.60E+02	6.36E+02	7.73E+02	**4.65E+02**	5.01E+02
f_{30}	3.98E+03	5.78E+03	**3.89E+03**	3.92E+03	1.19E+04	1.83E+04	5.73E+03	5.44E+03
$w/t/l$	23/1/5	29/0/0	27/0/2	28/0/1	29/0/0	28/0/1	19/2/8	-

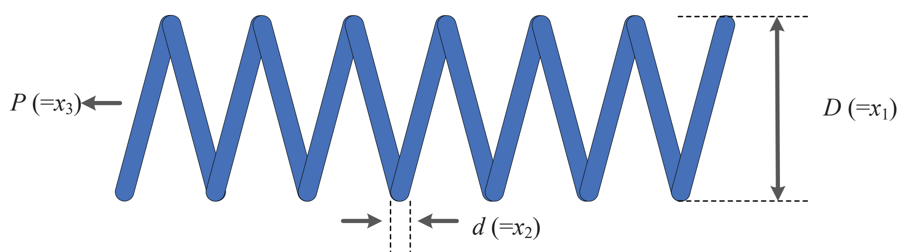
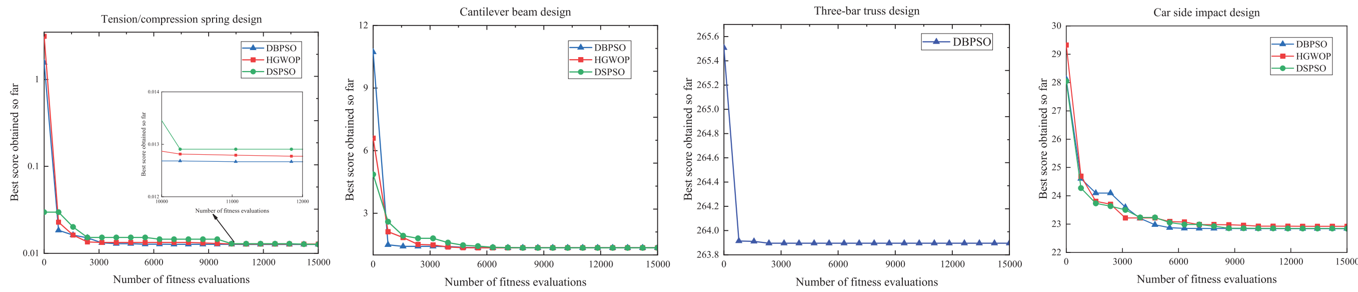


Fig. S.1. Structure of the tension/compression spring design problem

F. Cantilever beam design problem

The second engineering problem is the Cantilever beam design problem, which is shown in Fig. S.3. This is a case of a structural engineering design problem that aims to optimize the weight of a cantilever beam with a square cross-section. The beam is firmly secured at one end and is subject to a vertical force at the other end, as depicted in Fig. S.3. The beam is composed of five square blocks that have a uniform thickness, and their heights (or widths) are the critical parameters to be optimized. Notably, the thickness of each block is consistently fixed at $2/3$. The problem can be represented mathematically as Eq. (S.2):



(a) (b) (c) (d)

Fig. S.2. Convergence curves on the four engineering design problem

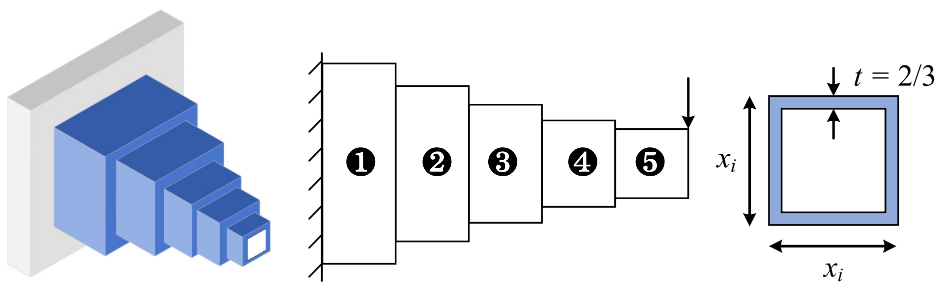


Fig. S.3. Structure of the cantilever beam design problem

$$
\begin{aligned}
&\text{minimize}: \\
&f(\mathbf{x}) = 0.0624\,(x_1 + x_2 + x_3 + x_4 + x_5) \\
&\text{subject to}: \\
&g(\mathbf{x}) = \frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \le 0 \\
&\text{ variable range:} \\
&0.01 \le x_i \le 100, \quad i = 1, \ldots, 5
\end{aligned}
\tag{S.2}
$$

TABLE S.9
COMPARISON RESULTS OF DBPSO WITH OTHER ALGORITHMS ON THE CBD PROBLEM

Variable	CSO	DSPSO	HGWOP	DBPSO
x_1	6.762800	6.013784	6.042949	6.016981
x_2	5.158300	5.307666	5.143976	5.307830
x_3	5.653700	4.499189	4.596050	4.493848
x_4	2.927900	3.501157	3.537155	3.503550
x_5	1.885400	2.151879	2.170036	2.151456
Optimal Cost	1.3970239	1.3399574	1.3399845	**1.3399567**

* Bold represents the optimal value.

Table S.9 presents the optimal solutions of DBPSO and three compared meta-heuristic algorithms including CSO, DSPSO, and HGWOP in solving the Cantilever beam design problem. From Table S.9, DBPSO obtains the optimal value, and its optimal cost is better than the results of other compared algorithms. Fig. S.2(b) gives the convergence curves of the DBPSO, HGWOP, and DSPSO on the Cantilever beam design problem. Therefore, it is not difficult to draw the conclusion that DBPSO can better solve the Cantilever beam design problem.

G. Three-bar truss design problem

The third practical engineering design problem is the three-bar truss design problem [2], which is shown in Fig. S.4. The main objective of this problem is to minimize the relevant weight. This case includes two optimization variables $(A_1(= x_1), A_2(=$

x_2)) with three optimization constraints: stress constraint, deflection constraint, and buckling constraint. The mathematical model of this problem can be defined as Eq. (S.3).

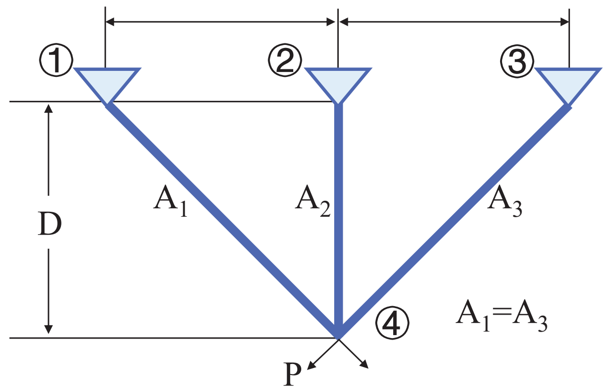


Fig. S.4. Structure of the three-bar truss design problem

$$
\begin{aligned}
&\text{minimize}: \\
&f(\vec{x}) = \left(2\sqrt{2}x_1 + x_2\right) * l \\
&\text{subject to}: \\
&g_1(\vec{x}) = \frac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1x_2}P - \sigma \leq 0 \\
&g_2(\vec{x}) = \frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2}P - \sigma \leq 0 \\
&g_3(\vec{x}) = \frac{1}{\sqrt{2}x_2 + x_1}P - \sigma \leq 0 \\
&\text{variable range:} \\
&0 \leq x_1, x_2 \leq 1. \\
&where \quad l = 100cm, P = 2KN/cm^2, \sigma = 2KN/cm^2.
\end{aligned}
\tag{S.3}
$$

Table S.10 presents the optimal solutions of DBPSO and three compared meta-heuristic algorithms including ICHIMP-SHO [2], Hybrid GWO-Simulated Annealing (hGWO-SA) [3], Hybrid Harris Hawks-Sine Cosine Algorithm (hHHO-SCA) [4] and Cooperation Search Algorithm (CSA) [5] in solving the three-bar truss design problem. From Table S.10, DBPSO obtains the optimal value, and its optimal weight is better than the results of other compared algorithms. Fig. S.2(c) shows the convergence curves of DBPSO for the three-bar truss design problem. Therefore, it is not difficult to draw the conclusion that DBPSO can better solve the three-bar truss design problem.

TABLE S.10
COMPARISON RESULTS OF DBPSO WITH OTHER ALGORITHMS ON THE TBTD PROBLEM

	ICHIMP-SHO	hGWO-SA	hHHO-SCA	CSA	DBPSO
x_1	0.788595	0.789	0.788498	0.788638976	0.7886767
x_2	0.408486	0.408	0.40875	0.408350573	0.4082440
Optimal Weight	263.89701	263.896	263.8958665	263.8958443	**263.8958434**

* Bold represents the optimal value.

H. Car side impact design problem

The fourth practical engineering design problem is the car side impact design problem [6], which is shown in Fig. S.5. The main objective of this problem is to minimize the total weight. This problem contains ten constraints and eleven design variables are utilized to calculate the weight. The design variables consist of the thicknesses of the B-Pillar inner (x_1), B-Pillar reinforcement (x_2), floor side inner (x_3), cross members (x_4), door beam (x_5), door beltline reinforcement (x_6), and roof rail (x_7). Additionally, the materials of the B-Pillar inner (x_8) and floor side inner (x_9), as well as the barrier height (x_{10}) and hitting position (x_{11}), are also considered as design variables. The mathematical model of this problem can be defined as Eq. (S.4). Fig. S.2(d) gives the convergence curves of the DBPSO, HGWOP, and DSPSO on the Car side impact design problem.

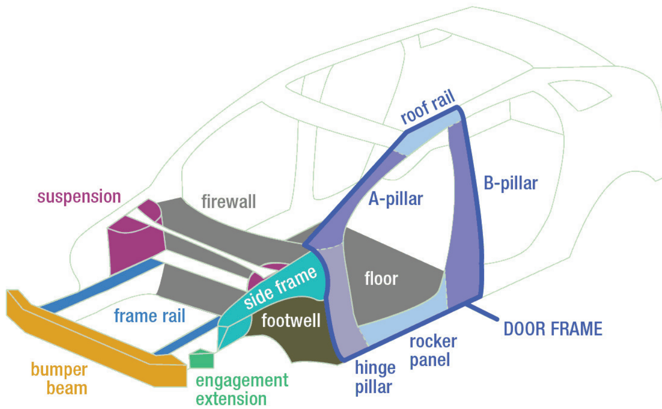


Fig. S.5. Structure of the car side impact design problem

$$
\begin{aligned}
&\text{minimize}: \\
&f(\mathbf{x}) = 1.98 + 4.90x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 2.73x_7 \\
&\text{subject} \quad \text{to}: \\
&g_1(\mathbf{x}) = 1.16 - 0.3717x_2x_4 - 0.00931x_2x_{10} - 0.484x_3x_9 + 0.01343x_6x_{10} - 1 \le 0 \\
&g_2(\mathbf{x}) = 46.36 - 9.9x_2 - 12.9x_1x_2 + 0.1107x_3x_{10} - 32 \le 0, \\
&g_3(\mathbf{x}) = 33.86 + 2.95x_3 + 0.1792x_3 - 5.057x_1x_2 - 11.0x_2x_8 - 0.0215x_5x_{10} - 9.98x_7x_8 \\
&\qquad\qquad + 22.0x_8x_9 - 32 \le 0 \\
&g_4(\mathbf{x}) = 28.98 + 3.818x_3 - 4.2x_1x_2 + 0.0207x_5x_{10} + 6.63x_6x_9 - 7.7x_7x_8 + 0.32x_9x_{10} - 32 \le 0 \\
&g_5(\mathbf{x}) = 0.261 - 0.0159x_1x_2 - 0.188x_1x_8 - 0.019x_2x_7 + 0.0144x_3x_5 + 0.0008757x_5x_{10} \\
&\qquad\qquad + 0.08045x_6x_9 + 0.00139x_8x_{11} + 0.00001575x_{10}x_{11} - 0.32 \le 0 \\
&g_6(\mathbf{x}) = 0.214 + 0.00817x_5 - 0.131x_1x_8 - 0.0704x_1x_9 + 0.03099x_2x_6 - 0.018x_2x_7 \\
&\qquad\qquad + 0.0208x_3x_8 + 0.121x_3x_9 - 0.00364x_5x_6 + 0.0007715x_5x_{10} - 0.0005354x_6x_{10} \\
&\qquad\qquad + 0.00121x_8x_{11} + 0.00184x_9x_{10} - 0.02x_2^2 - 0.32 \le 0 \\
&g_7(\mathbf{x}) = 0.74 - 0.61x_2 - 0.163x_3x_8 + 0.001232x_3x_{10} - 0.166x_7x_9 + 0.227x_2^2 - 0.32 \le 0 \\
&g_8(\mathbf{x}) = 4.72 - 0.5x_4 - 0.19x_2x_3 - 0.0122x_4x_{10} + 0.009325x_6x_{10} + 0.000191x_{11}^2 - 4 \le 0 \\
&g_9(\mathbf{x}) = 10.58 - 0.674x_1x_2 - 1.95x_2x_8 + 0.02054x_3x_{10} - 0.0198x_4x_{10} + 0.028x_6x_{10} - 9.9 \le 0 \\
&g_{10}(\mathbf{x}) = 16.45 - 0.489x_3x_7 - 0.843x_5x_6 + 0.0432x_9x_{10} - 0.0556x_9x_{11} - 0.000786x_{11}^2 - 15.7 \le 0, \\
&\ \text{variable range:} \\
&0.5 \le x_1, x_2, x_3, x_4, x_5, x_6, x_7 \le 1.5, x_8, x_9 \in \{0.192, 0.345\}, -30 \le x_{10}, x_{11} \le 30
\end{aligned}
\tag{S.4}
$$

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