Final Exam
Principles of Economics with Calculus
Caltech/edX
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Question 1

- Consider the problem of a consumer with a utility function given by $\ln x + m$, where x and m denote the amount of the goods consumed.

- The consumer faces a non-linear pricing function for the first good given by $p(x) = 2\sqrt{x}$, where $p(x)$ denotes the total cost (in units of the $ good m) of purchasing x units.

- QUESTION: What is the amount of good x bought by the consumer?

Question 2

- Consider a market with 3 types of goods, a, b, and m. There are two types of consumers: 100 consumers of type X and 100 consumers of type Y. All consumers of the same type are identical to ech other.

- Let x_a denote the amount of good a consumed by each type X consumer, y_b denote the amount of good b consumed by each type b consumer, etc.

- Consumers of type X derive a benefit of $\ln x_a$ from their personal consumption of good a, but they do not care about how much of good b they consume. However, they also derive a benefit of $\ln y_b$ from the amount of good b consumed by EACH consumer of type Y.

- Consumers of type Y have analagous preferences. They derive a benefit of $\ln y_b$ from their personal consumption of good b, but they do not care about how much of good a they consume. However, they also derive a benefit of $\ln x_a$ from the amount of good a consumed by EACH consumer of type X.

- Goods a and b are sold in a competitive market. The marginal cost of producing good a is 1. The marginal cost of producing good b is $\frac{1}{2}$.

- Individuals are not allowed to give gifts to each other.

- QUESTION: What is the amount of good a produced in equilibrium?

- QUESTION: What is the amount of good b produced in equilibrium?

- QUESTION: What is the amount of good a produced at the Pareto optimal allocation?

- QUESTION: What is the amount of good b produced at the Pareto optimal allocation?

Question 3

- Consider the same setting as in Question 2.

- In order to address the inefficiency in the market, the government introduces a subsidy of α per unit of good a purchased, and a subsidy of β per unit of good b purchased. The subsidies are given to the consumers making those purchases, and are financed using a lump-sum tax of equal size on all consumers, regardless of their type.

- QUESTION: What value of α is required to restore optimality?

- QUESTION: What value of β is required to restore optimality?

Question 4

- Consider the decision of an individual that chooses how much of an addictive good to consume. Each unit of the good sells for \$1. There is also a government regulation that limits the purchases of each individual to a maximum of 4 units.

- The true experienced benefit that the consumer gets from consuming q units of the good is given by $2q^{\frac{1}{2}}$.

- QUESTION: If the individual is rational, how many units does he buy?

- Suppose, however, that the individual is not rational, and that he makes decisions by maximizing the wrong benefit function, given by $\frac{2}{3}q^{\frac{3}{2}} + Bq$.

- QUESTION: If $B > 1$, how many units does the individual buy?

- QUESTION: What is the maximum value of B at which the individual buys nothing?

Question 5

- Consider the setting of Question 4.

- QUESTION: What is the consumer surplus for the rational individual?

- QUESTION: What is the consumer surplus for an irrational individual with $B > 1$?

- QUESTION: What is the consumer surplus for an irrational individual with $B = -10$?

Question 6

- Consider the setting of the previous problem for the case of an irrational consumer with $B = 1$.

- Suppose that the market consists of 100 identical consumers. In order to help the consumers, the government is considering the introduction of a per-unit tax of size τ on each unit purchased. Any revenue raised by the tax is returned to consumers using identical lump-sum taxes.

- QUESTION: What is the minimal size of τ that induces the consumer to purchase nothing?

- QUESTION: What is the consumer surplus for values of τ exceeding your answer to the previous question?

- QUESTION: What is the consumer surplus for values of τ below your answer to the first question?

Question 7

- Consider a market with 3 identical firms. Aggregate demand in the market is given by $\frac{9000}{p}$, where p denotes the price in the market. For each firm, the cost of producing q units is given by $\frac{q^2}{20}$.

- QUESTION: What is the total amount produced in equilibrium?

- QUESTION: What is the equilibrium price?

Question 8

- Consider the same market as in Question 7.

- QUESTION: What is the deadweight loss in equilibrium?