Unit 9: Externalities

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1 Public bads

1.1 What is an externality?

- An *externality* arises when the actions of one economic actor affect DIRECTLY the utility function or production function of another economic actor (i.e. not through a market transaction)

- Examples:

	Effect on others?	Direct/Market?
Roommate plays loud rock music	✓	Direct
Consumption of pharmaceuticals contaminates fish stocks	✓	Direct
Microsoft hires 10,000 new software engineers	✓	Market
Company invents drug that makes people 10x smarter	✓	Market

- First two examples are externalities since they directly affect others.

- Last two examples are not externalities since they impact others take place through the market

1.2 Public bads

- Public bads are externalities that are:

  - Negative
  - Non-targeted (i.e. affect either all consumers, or all firms, or both)

- Examples:

  - CO_2 emmissions and global warming
  - Noise pollution
  - Poverty, when people care about others' well-being

- Simple model:

  - 3 goods: q, m, e, where e denotes the level of the externality (e.g. pollution)
  - Each unit of q consumed generates 1 unit of e
  - N identical consumers, with utility

$$U(q, m, e) = B(q) + m - \gamma D(e),$$

    with $D' > 0, D'' \geq 0$, and $\gamma \geq 0$
  - F identical firms, with CRS production function, so that $c(q) = \mu q$
  - Market is competitive

- Market equilibrium:

  - Consumers maximize utility and firms maximize profit, taking as given the level of externality
  - Prices adjust so that q-market clears

- Consumer's problem:

$$\max_{q \geq 0} B(q) - pq - \gamma D(e)$$

- Since e is taken as a constant by the consumer, the demand function for good q is as it was without externalities

- Firm's problem:

$$\max_{q \geq 0} pq - \mu q$$

- As before:

$$q^S(p) = \begin{cases} 0 & \text{if } p < \mu \\ \text{anything} & \text{if } p = \mu \\ \infty & \text{if } p > \mu \end{cases}$$

- Optimal allocations:
  - Optimal allocation given by solution to:

$$\max_{q_1 \ldots q_N \geq 0} \sum_i \left[B(q_i) - \gamma D \left(\sum_i q_i \right) \right] - \sum_i \mu q_i$$

  - FOCs:

$$\underbrace{B'\left(\frac{q^{opt}}{N}\right) - \gamma N D'(q^{opt})}_{\text{Marginal social benefit}} = \underbrace{\mu}_{\text{Marginal social cost}}$$

    (note that, given symmetry, everyone consumes the same amount of q)

- RESULT:
  - $q^* > q^{opt}$
  - $DWL > 0$
- Accounting recipe/convention for where to place the externalities:
  - Externalities that affect utility go into MSB
  - Externalities that affect production costs go into MSC

1.3 A more complex example

- Model:
  - All externalities are firm-to-firm
  - Each unit of q produced generates one unit of pollution e
  - Aggregate demand function: $q^D_{mkt}(p) = 1000 - p$

- 10 identical firms with cost function $c(q_j) = 5q_j^2 + eq_j$, so that $MC(q_j|e) = 10q_j + e$

- Firm's problem:
  - $\max_{q\geq 0} pq - (5q^2 + eq)$, with e taken as given
  - $MR = p, MC = 10q + e$
    $\implies q_j^S(p) = \frac{p-e}{10}$
    $\implies q_{mkt}^S(p) = p - e$

- Market equilibrium:
  - Equilibrium conditions:
    1. Prices adjust so that $q_{mkt}^D(p^*) = q_{mkt}^S(p^*)$
    2. $e^* = q_{mkt}^S(p^*)$
  - Supply function plus (2) imply that in equilibrium $q_{mkt}^S(p) = \frac{p}{2}$
  - Then the market clearing condition is given by $1000 - p = \frac{p}{2}$
  - It follows that
$$p^* = \frac{2000}{3}, q^* = \frac{1000}{3}$$

- Optimality:
  - q^{opt} given by $MSB = MSC$
  - MSB = MPB = $p_{mkt}^D = 1000 - q$ (where MPB = marginal private benefit)
  - MSC:
    * cost function convex, so optimal to split production equally across firms
    * Total social cost: $TSC(q) = 10c\left(\frac{q}{10}\right) = 10\left(5\frac{q^2}{100} + q\frac{q}{10}\right) = \frac{3}{2}q^2$
    * $\implies MSC = 3q$
  - FOCs:
$$MSB = MSC \implies 1000 - q = 3q \implies q^{opt} = 250$$

- Marginal social cost vs. marginal private cost
  - MSC $= 3q$
  - MPC $= p^S_{mkt}(q) = q + e = q + \frac{1000}{3}$
- See equilibrium and DWL diagram in the video lectures

1.4 Why does the FWT fail?

- Market forces lead to an allocation at which $MPB = p = MPC$
- At optimal allocation we have that $MSB = MSC$
- Without externalities we have that $MSB = MPB$ and $MSC = MPC$.
- Thus, the invisible hand of the market leads to an allocation at which $MSB = MPB = p = MPC = MSC$, which is optimal
- With externalities, $MPB > MSB$ and/or $MPC < MSC$
- Given this, market induces an equilibrium allocation at which $MSB < MSC$, which implies that $q^* > q^{opt}$

2 Corrective policies for public bads

2.1 Pigouvian taxation

- Policy:
  - Tax per unit imposed on every consumer or firm generating an externality
  - Tax equal to total marginal externality at optimum
  - Tax revenue returned to consumers in lump-sum transfer
- RESULT: Pigouvian tax system leads to an optimal equilibrium allocation
- Why?

- Look at basic model of public bads
  - Externality from consumers to consumers
  - N consumers with $U(q, m, e) = B(q) + m - \gamma D(e)$
  - F firms with $MC = \mu$
  - N consumers
- Pigouvian tax: $\tau^* = \gamma N D'(q^{opt})$ imposed on consumers (per unit of good q)
- Consumer's problem:

$$\max_{q \geq 0} B(q) - pq - \tau^* q - \gamma D(e)$$

  with $D(e)$ taken as given.
- FOC: $B' = p + \tau^*$
- In equilibrium, $p^* = \mu$
  $\implies B'\left(\frac{q^{tot}}{N}\right) = \mu + \gamma N D'(q^{opt})$
- This is the same as optimality condition, so $\frac{q^{tot}}{N} = \frac{q^{opt}}{N}$

- Intuition:
  - Optimality requires an allocation at which:

$$\begin{array}{ccc} MSB & = & MSC \\ \shortparallel & & \shortparallel \\ MPB - \gamma N D' & & MPC = \mu \end{array}$$

  - Utility maximization leads consumers to set $B' - \tau^* = p$
  - Profit maximization leads firms to set $p = \mu$
  - $B' = MPB$ and τ^* chosen to be $\gamma N D'(q^{opt})$, which implies that the invisibile hand of the market with the tax induces an equilibrium allocation at which the optimality condition is satisfied

- Remarks:
  1. Optimal Pigouvian tax restores efficiency BUT feasible only if the goverment has all of the information needed to calculate $\gamma N D'(q^{opt})$

2. Model assumes everyone produces the same marginal damage, otherwise more complicated tax system is required.
3. Policy works if tax τ^* is imposed either on firms or consumers
4. Policy doesn't work if τ^* imposed on both consumers and firms

2.2 Permit markets

- Policy:
  - Government creates $\Pi^* = q^{opt}$ units of permits
  - Each permit allows owner to produce/consume 1 unit of q (which is the good generating the externality)
  - Individuals/firms who exceed allocated permits pay ∞ fine (so no cheating in equilibrium)
  - Consumers/firms allowed to trade permits freely
- Two versions of the policy:
  1. Permits sold by government
  2. Permits are freely allocated
- RESULT: If $\Pi^* = q^{opt}$, then the permit policy leads to an efficient allocation. Furthermore, in equilibrium permits trade at a price equal to the optimal Pigouvian tax (i.e., $p^*_\Pi = \tau^*$)
- Why?
  - Again, consider simple model of public bads
  - Look at case in which permits freely allocated
  - Consumer's problem:

$$\max_{q \geq 0, r} B(q) - pq - p_\Pi(r - \Pi_i^{endowed})$$

    where r is the number of permits bought, sold

- This is equivalent

$$\max_{q \geq 0} B(q) - (p + p_\Pi)q,$$

  where p_Π is price of permits.

- FOCs:

$$B' = p + p_\Pi$$

- In equilibrium, $p = \mu$ (by CRS cost function of firms)
- q-market: demand curve with permits is demand curve without permits shifted down by p_Π.
- Then, if $p_\Pi = \gamma N D'(q^{opt})$, the inverse demand function with permits = MSB, and thus in equilibrium $q^* = q^{opt}$
- r-market: inverse demand curve for permits is equal to demand curve for q without permits, shifted down by $p = \mu$. Supply fixed at $\Pi^* = q^{opt}$.
- From the FOCs of the utility maximization problem we get that $p^*_\Pi = \gamma N D'(q^{opt})$.
- Intuition: cost of permits acts as a per-unit tax, but now the size of the tax is determined endogenously in the permit market

• Why?

  - Now consider the case in which permits are sold by government
  - Government sells Π^* permits in the permit market
  - Permit endownment affects the wealth of consumers (since they can sell the permits in the market), but it has no effect on their demand functions
  - So equilibrium is same as before: p^*_Π, p^*, q^*
  - Only difference is that now policy raises revenue

• EQUIVALENCE RESULT: Optimal Pigouvian tax system (with revenue returned by lump-sum transfers to consumers) is equivalent to the optimal permit market with permits sold (with revenue returned by same lump-sum transfers)

- Why?
  - In Pigouvian taxation the government sets the "price" of the externality (i.e., $\tau^* = \gamma N D'(q^{opt})$) and the market finds q^{opt}
  - In the permit market the government sets the quantity to $\Pi^* = q^{opt}$, and the market finds price of externality $p_\Pi = \gamma N D'(q^{opt})$
  - For both, total revenue raised $= q^{opt}\gamma N D'(q^{opt})$, and thus can support identical lump-sum transfers to consumers
- Remarks:
  1. Permit markets can restore efficiency, but government needs to know q^{opt}
  2. Permit markets vs. command-and-control/direct regulation: Command-and-control requires knowing what everyone should consume and produce, not just optimal total quantity. Thus, permit markets require less information to be able to design optimal policy.
  3. System optimal only if $\Pi^* = q^{opt}$

3 Public goods

- Key features:
  - Positive externalities to other consumers/firms
  - Non-rivalry in consumption (everyone can benefit from positive externality)
- Examples:
  - Basic research & development
  - LoJack anti-theft system
  - Network externalities in consumption (HBO, facebook)
- Simple model:

- N consumers, each with utility

$$U(q, m, e) = B(q) + m + \gamma G(e),$$

  $\gamma \geq 0$ measures the strength of externality, $G' > 0, G'' \leq 0$

- good q produced by firms with constant marginal cost of production μ

- RESULT: $q^* < q^{opt}$; i.e., there is underproduction of the public good.

- See video lecture for graphical analysis of equilibrium and DWL

- Corrective policy: Pigouvian subsidies
  - Policy:
    1. Subsidize sale/purchase of good q with subsidy $\sigma^* = \gamma N G'(q^{opt})$
    2. Subsidy financed with lump-sum taxes
  - RESULT: Optimal Pigouvian subsidy system restores optimality

- Corrective policy: Government provision
  - Consider extreme case in which individuals derive no private benefit from consuming the good generating the externality: $U(q, m, e) = 0 + m + \gamma G(e)$
  - Market equilibrium: $q^* = 0$
  - Policy: Buy q^{opt} and finance w/ lump-sum taxes.
  - This restores optimality
  - Goverment provision can't restore full optimality if lump-sum taxes are not feasible

4 Summary

- With externalities, first welfare theorem fails:
  - With public bads, overprovision of good generating the externality
  - With public goods, underprovision of the good generating the externality

- Corrective policy for public bads:
  - Pigouvian taxes: τ^* = marginal damage at q^{opt}
  - Permit market: $\Pi^* = q^{opt}$
- Corrective policy for public goods:
  - Pigouvian subsidies: σ^* = marginal positive externality at q^{opt}
  - Direct government provision (financed with taxes)