Lecture 3

Outline

Expanding $a(t)$

- Expanding $a(t)$ in a Taylor series
  - $z = H_0(t_0 - t_1) = H_0 t_{lookback}$
  - Definition of lookback time (see above)
  - Going beyond, definition of q_0

Distances in Cosmology

Comoving radial distance

$$d\chi = \frac{cdt}{a(t)}$$

or

$$\chi = c\int \frac{dt}{a(t)}$$

or

$$\chi = c\int \frac{dz}{H(z)}$$

From the above, we see that measuring $\chi(z)$ allows us to infer $H(z)$ (or the reverse). And then one can get $a(t)$.

Also note

$$d_M = S_k(\chi)$$

Luminosity Distance

$$F = \frac{F}{4\pi d_L^2}$$

where

$$d_L = (1+z)d_M$$

This is the basis of the standard candle method to measuring the expansion of the Universe (Cepheids, SNe 1a, standard sirens)

Angular Diameter Distance

$$D = d_A\theta$$

where

$$d_A = \frac{d_M}{1+z}$$

Note that

$$d_L = d_A(1+z)^2$$

Dynamics

Friedmann and continuity equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{R_0^2 a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

$$\dot{\rho} + 3H(\rho + P) = 0$$

These equations are not independent, one can follow from the others.

Define the critical density

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

and define

$$\Omega = \frac{\rho}{\rho_{crit}}$$

Then, the first Friedmann equation is given by

$$1 - \Omega = \frac{\kappa}{R_0^2 a^2 H^2}$$

Note that Ω can be used to determine the geometry of the Universe. Finally, note that the physical density parameter is

$$\omega = \Omega h^2$$

Reading

Huterer, Chaps 2 & 3 : The discussion of distances is in Chap. 3 towards the end, after the dynamics discussion.

Baumann: Chap. 2. This is also a good reference if you want to look at the full derivation of the Friedmann equations.

A reference that I return to time and time again for distance measures in the "Distance Measures in Cosmology" paper by David Hogg listed in the class bibliography.