

In[175]:= (*Ordinary Ideal gas*)

$$\mathbf{A} = \left\{ \{0, 1, 0\}, \left\{ -\frac{1}{2} (3 - \gamma) \left(\frac{u_2}{u_1} \right)^2, (3 - \gamma) \left(\frac{u_2}{u_1} \right), \gamma - 1 \right\}, \left\{ -\frac{\gamma u_2 u_3}{u_1^2} + (\gamma - 1) \left(\frac{u_2}{u_1} \right)^3, \frac{\gamma u_3}{u_1} - \frac{3}{2} (\gamma - 1) \left(\frac{u_2}{u_1} \right)^2, \frac{\gamma u_2}{u_1} \right\} \right\};$$

MatrixForm[A]

Out[176]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 \\ -\frac{(3-\gamma) u_2^2}{2 u_1^2} & \frac{(3-\gamma) u_2}{u_1} & -1+\gamma \\ \frac{(-1+\gamma) u_2^3}{u_1^3} - \frac{\gamma u_2 u_3}{u_1^2} & -\frac{3 (-1+\gamma) u_2^2}{2 u_1^2} + \frac{\gamma u_3}{u_1} & \frac{\gamma u_2}{u_1} \end{pmatrix}$$

In[177]:= FullSimplify[MatrixForm[A] /. {u1 -> rho} /. {u2 -> rho u} /. {u3 -> 1/2 rho u^2 + P/(gamma - 1)}]

Out[177]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} u^2 (-3 + \gamma) & -u (-3 + \gamma) & -1 + \gamma \\ \frac{1}{2} u^3 (-2 + \gamma) + \frac{P u \gamma}{\rho - \gamma \rho} u^2 \left(\frac{3}{2} - \gamma \right) + \frac{P \gamma}{(-1 + \gamma) \rho} & u \gamma & \end{pmatrix}$$

In[178]:= eigenvals = Eigenvalues[A] /. {u1 -> rho} /. {u2 -> rho u} /. {u3 -> 1/2 rho u^2 + P/(gamma - 1)};

$$\text{FullSimplify}\left[\text{eigenvals} /. \left\{ P \rightarrow \frac{\rho c_s^2}{\gamma} \right\} /. \left\{ \rho \rightarrow \frac{\gamma P}{c_s^2} \right\}, \{c_s > 0, \gamma > 0, P > 0, \rho > 0\} \right]$$

eigenvectors = Eigenvectors[A] /. {u1 -> rho} /. {u2 -> rho u} /. {u3 -> 1/2 rho u^2 + P/(gamma - 1)};

$$\text{MatrixForm}\left[\text{Transpose}\left[\text{FullSimplify}\left[\text{eigenvectors} /. \left\{ P \rightarrow \frac{\rho c_s^2}{\gamma} \right\} /. \left\{ \rho \rightarrow \frac{\gamma P}{c_s^2} \right\}, \{c_s > 0, \gamma > 0, P > 0, \rho > 0\} \right]\right]\right]$$

Out[179]= {u, u - c_s, u + c_s}

Out[181]//MatrixForm=

$$\begin{pmatrix} \frac{2}{u^2} & \frac{2 (-1+\gamma)}{u^2 (-1+\gamma)+2 c_s (u-u \gamma+c_s)} & \frac{2 (-1+\gamma)}{u^2 (-1+\gamma)+2 c_s (u (-1+\gamma)+c_s)} \\ \frac{2}{u} & \frac{2 (-1+\gamma) (u-c_s)}{u^2 (-1+\gamma)+2 c_s (u-u \gamma+c_s)} & \frac{2 (-1+\gamma) (u+c_s)}{u^2 (-1+\gamma)+2 c_s (u (-1+\gamma)+c_s)} \\ 1 & 1 & 1 \end{pmatrix}$$

In[182]:=

(***2 fluid Magnetic field***)

$$\text{Fluxp} = \left\{ \{\rho \, u\}, \{\rho \, u^2 + P\}, \left\{ u \left(\frac{\rho \, u^2}{2} + \frac{\gamma \, (P - C \, \rho^{\gamma_m})}{\gamma - 1} + \frac{\gamma_m \, C \, \rho^{\gamma_m}}{\gamma_m - 1} \right) \right\} \right\};$$

MatrixForm[Fluxp]

$$\text{Flux} = \text{FullSimplify}\left[\text{MatrixForm}[\text{Fluxp}] \ /. \ \left\{ \rho \rightarrow u_1, \ u \rightarrow \frac{u_2}{u_1}, \ P \rightarrow -\frac{\gamma - 1}{2} \frac{u_2^2}{u_1} + C \, u_1^{\gamma_m} \left(1 - \frac{\gamma - 1}{\gamma_m - 1} \right) + (\gamma - 1) \, u_3 \right\}, \ \{\gamma > 0, \ \gamma_m > 0, \ C > 0, \ P > 0, \ \rho > 0\} \right];$$

MatrixForm[Flux]

Out[183]//MatrixForm=

$$\begin{pmatrix} u \, \rho \\ P + u^2 \, \rho \\ u \left(\frac{u^2 \, \rho}{2} + \frac{\gamma \, (P - C \, \rho^{\gamma_m})}{-1 + \gamma} + \frac{C \, \rho^{\gamma_m} \, \gamma_m}{-1 + \gamma_m} \right) \end{pmatrix}$$

Out[185]//MatrixForm=

$$\begin{pmatrix} u_2 \\ -\frac{(-3 + \gamma) \, u_2^2}{2 \, u_1} + (-1 + \gamma) \, u_3 + \frac{C \, u_1^{\gamma_m} \, (-\gamma + \gamma_m)}{-1 + \gamma_m} \\ \frac{u_2 \left(-(-1 + \gamma) \, u_2^2 + 2 \, u_1 \left(\gamma \, u_3 + \frac{C \, u_1^{\gamma_m} \, (-\gamma + \gamma_m)}{-1 + \gamma_m} \right) \right)}{2 \, u_1^2} \end{pmatrix}$$

In[186]:= (*** Some checks with manually done algebra ... Phew! That was tiring!***)

$$\text{Fluxc} = \left\{ \{u_2\}, \left\{ \frac{3 - \gamma}{2} \left(\frac{u_2^2}{u_1} \right) + \left(1 - \frac{\gamma - 1}{\gamma_m - 1} \right) C \, u_1^{\gamma_m} + (\gamma - 1) \, u_3 \right\}, \left\{ -\frac{\gamma - 1}{2} \left(\frac{u_2^3}{u_1^2} \right) + \frac{\gamma_m - \gamma}{\gamma_m - 1} C \, u_1^{\gamma_m - 1} \, u_2 + \gamma \frac{u_2 \, u_3}{u_1} \right\} \right\};$$

MatrixForm[Fluxc]

$$\text{Fluxorg} = \text{FullSimplify}\left[\text{MatrixForm}[\text{Fluxc}] \ /. \ \left\{ u_1 \rightarrow \rho, \ u_2 \rightarrow \rho \, u, \ u_3 \rightarrow \frac{1}{2} \, \rho \, u^2 + \frac{P - C \, \rho^{\gamma_m}}{\gamma - 1} + \frac{C \, \rho^{\gamma_m}}{\gamma_m - 1} \right\} \right];$$

MatrixForm[Fluxorg]

Out[187]//MatrixForm=

$$\begin{pmatrix} u_2 \\ \frac{(3 - \gamma) \, u_2^2}{2 \, u_1} + (-1 + \gamma) \, u_3 + C \, u_1^{\gamma_m} \left(1 - \frac{-1 + \gamma}{-1 + \gamma_m} \right) \\ \frac{(1 - \gamma) \, u_2^3}{2 \, u_1^2} + \frac{\gamma \, u_2 \, u_3}{u_1} + \frac{C \, u_1^{-1 + \gamma_m} \, u_2 \, (-\gamma + \gamma_m)}{-1 + \gamma_m} \end{pmatrix}$$

Out[189]//MatrixForm=

$$\begin{pmatrix} u \, \rho \\ P + u^2 \, \rho \\ u \left(\frac{2 \, P \, \gamma + u^2 \, (-1 + \gamma) \, \rho + \frac{2 \, C \, \rho^{\gamma_m} \, (\gamma - \gamma_m)}{-1 + \gamma_m}}{2 \, (-1 + \gamma)} \right) \end{pmatrix}$$

In[190]:=

(*Creating the Jacobian of the flux matrix*)

$$f1 = u_2; f2 = -\frac{(-3+\gamma) u_2^2}{2 u_1} + (-1+\gamma) u_3 + \frac{C u_1^{\gamma_m} (-\gamma+\gamma_m)}{-1+\gamma_m}; f3 = \frac{u_2 \left(-(-1+\gamma) u_2^2 + 2 u_1 \left(\gamma u_3 + \frac{C u_1^{\gamma_m} (-\gamma+\gamma_m)}{-1+\gamma_m} \right) \right)}{2 u_1^2};$$

A = {{D[f1, u1], D[f1, u2], D[f1, u3]}, {D[f2, u1], D[f2, u2], D[f2, u3]}, {D[f3, u1], D[f3, u2], D[f3, u3]}} ;
MatrixForm[A]

$$\text{MatrixForm[A]} /. \{u_1 \rightarrow \rho, u_2 \rightarrow \rho u, u_3 \rightarrow \frac{1}{2} \rho u^2 + \frac{P - C \rho^{\gamma_m}}{\gamma - 1} + \frac{C \rho^{\gamma_m}}{\gamma_m - 1}\}$$

Out[192]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 \\ \frac{(-3+\gamma) u_2^2}{2 u_1^2} + \frac{C u_1^{-1+\gamma_m} \gamma_m (-\gamma+\gamma_m)}{-1+\gamma_m} & -\frac{(-3+\gamma) u_2}{u_1} & -1+\gamma \\ u_2 \left(\frac{2 C u_1^{\gamma_m} \gamma_m (-\gamma+\gamma_m)}{-1+\gamma_m} + 2 \left(\gamma u_3 + \frac{C u_1^{\gamma_m} (-\gamma+\gamma_m)}{-1+\gamma_m} \right) \right) & u_2 \left((1-\gamma) u_2^2 + 2 u_1 \left(\gamma u_3 + \frac{C u_1^{\gamma_m} (-\gamma+\gamma_m)}{-1+\gamma_m} \right) \right) & \frac{(1-\gamma) u_2^2}{u_1^2} + \frac{(1-\gamma) u_2^2 + 2 u_1 \left(\gamma u_3 + \frac{C u_1^{\gamma_m} (-\gamma+\gamma_m)}{-1+\gamma_m} \right)}{2 u_1^2} \end{pmatrix}$$

Out[193]//MatrixForm=

$$\begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} u^2 (-3+\gamma) + \frac{C \rho^{-1+\gamma_m} \gamma_m (-\gamma+\gamma_m)}{-1+\gamma_m} & -u (-3+\gamma) & -1+\gamma \\ u \left(\frac{2 C \rho^{\gamma_m} \gamma_m (-\gamma+\gamma_m)}{-1+\gamma_m} + 2 \left(\gamma \left(\frac{u^2 \rho}{2} + \frac{P - C \rho^{\gamma_m}}{-1+\gamma} + \frac{C \rho^{\gamma_m}}{-1+\gamma_m} \right) + \frac{C \rho^{\gamma_m} (-\gamma+\gamma_m)}{-1+\gamma_m} \right) \right) & u \left(u^2 (1-\gamma) \rho^2 + 2 \rho \left(\gamma \left(\frac{u^2 \rho}{2} + \frac{P - C \rho^{\gamma_m}}{-1+\gamma} + \frac{C \rho^{\gamma_m}}{-1+\gamma_m} \right) + \frac{C \rho^{\gamma_m} (-\gamma+\gamma_m)}{-1+\gamma_m} \right) \right) & u^2 (1-\gamma) + \frac{u^2 (1-\gamma) \rho^2 + 2 \rho \left(\gamma \left(\frac{u^2 \rho}{2} + \frac{P - C \rho^{\gamma_m}}{-1+\gamma} + \frac{C \rho^{\gamma_m}}{-1+\gamma_m} \right) + \frac{C \rho^{\gamma_m} (-\gamma+\gamma_m)}{-1+\gamma_m} \right)}{2 \rho^2} \end{pmatrix}$$

In[204]:=

$$\text{eigenvals} = \text{Eigenvalues[A]} /. \{u_1 \rightarrow \rho, u_2 \rightarrow \rho u, u_3 \rightarrow \frac{1}{2} \rho u^2 + \frac{P - C \rho^{\gamma_m}}{\gamma - 1} + \frac{C \rho^{\gamma_m}}{\gamma_m - 1}\};$$

$$\text{eigenvals} = \text{FullSimplify}[\text{eigenvals} /. \{P \rightarrow \rho \left(\frac{c_s^2}{\gamma} + \frac{c_m^2}{\gamma_m} \right), C \rightarrow \frac{c_m^2}{\gamma_m \rho^{\gamma_m - 1}}\}, \{c_s > 0, c_m > 0, \gamma > 0, P > 0, \rho > 0, \theta < \gamma_m < 1\}];$$

$$\text{eigenvals} = \text{FullSimplify}[\text{eigenvals} /. \{c_s^2 + c_m^2 \rightarrow c_t^2\}, \{c_s > 0, c_m > 0, c_t > 0\}]$$

$$\text{eigenvectors} = \text{Eigenvectors[A]} /. \{u_1 \rightarrow \rho, u_2 \rightarrow \rho u, u_3 \rightarrow \frac{1}{2} \rho u^2 + \frac{P - C \rho^{\gamma_m}}{\gamma - 1} + \frac{C \rho^{\gamma_m}}{\gamma_m - 1}\};$$

$$\text{eigenvectors} = \text{FullSimplify}[\text{eigenvectors} /. \{P \rightarrow \rho \left(\frac{c_s^2}{\gamma} + \frac{c_m^2}{\gamma_m} \right), C \rightarrow \frac{c_m^2}{\gamma_m \rho^{\gamma_m - 1}}\}, \{c_s > 0, c_m > 0, \gamma > 0, P > 0, \rho > 0, \theta < \gamma_m < 1\}];$$

$$\text{eigenvectors} = \text{MatrixForm}[\text{Transpose}[\text{FullSimplify}[\text{eigenvectors} /. \{c_s^2 + c_m^2 \rightarrow c_t^2\}, \{c_s > 0, c_m > 0, c_t > 0\}]]]$$

Out[206]= {u, u + C_t, u - C_t}

Out[209]//MatrixForm=

$$\begin{pmatrix} \frac{2 (-1+\gamma) (-1+\gamma_m)}{2 c_m^2 (\gamma-\gamma_m) + u^2 (-1+\gamma) (-1+\gamma_m)} & \frac{1}{\frac{u^2}{2} + \frac{c_s^2}{-1+\gamma} + u c_t + \frac{c_m^2}{-1+\gamma_m}} & \frac{1}{\frac{c_s^2}{-1+\gamma} + \frac{1}{2} u (u - 2 c_t) + \frac{c_m^2}{-1+\gamma_m}} \\ \frac{2 u (-1+\gamma) (-1+\gamma_m)}{2 c_m^2 (\gamma-\gamma_m) + u^2 (-1+\gamma) (-1+\gamma_m)} & -\frac{2 (-1+\gamma) (-c_m^2 - c_s^2 + u (u (-1+\gamma) + (-2+\gamma) c_t)) (-1+\gamma_m)}{(2 c_s^2 c_t - u^2 (-1+\gamma) (u (-1+\gamma) + (-3+2 \gamma) c_t)) (-1+\gamma_m) + 2 (-1+\gamma) c_m^2 (c_t + u (-\gamma+\gamma_m))} & -\frac{2 (-1+\gamma) \left(u \left(u - u \gamma + (-2+\gamma) \sqrt{c_m^2 + c_s^2} \right) + c_t^2 \right) (-1+\gamma_m)}{2 (-1+\gamma) c_m^2 (c_t + u (\gamma-\gamma_m)) + (2 c_s^2 c_t + u^2 (-1+\gamma) (u (-1+\gamma) + (-3-2 \gamma) c_t)) (-1+\gamma_m)} \end{pmatrix}$$

In[210]:= (*Just some more checks*)

$$\text{eigenvals} = \text{Eigenvalues}[A] /. \left\{ u_1 \rightarrow \rho, u_2 \rightarrow \rho u, u_3 \rightarrow \frac{1}{2} \rho u^2 + \frac{P - C \rho^{\gamma_m}}{\gamma - 1} + \frac{C \rho^{\gamma_m}}{\gamma_m - 1} \right\};$$

$$\text{FullSimplify}\left[\text{eigenvals} /. \left\{ P \rightarrow \rho \left(\frac{c_s^2}{\gamma} + \frac{c_m^2}{\gamma_m} \right), C \rightarrow \frac{c_m^2}{\gamma_m \rho^{\gamma_m - 1}} \right\}, \{c_s > 0, c_m > 0, \gamma > 0, P > 0, \rho > 0, \gamma_m > 1\} \right]$$

$$\text{eigenvectors} = \text{Eigenvectors}[A] /. \left\{ u_1 \rightarrow \rho, u_2 \rightarrow \rho u, u_3 \rightarrow \frac{1}{2} \rho u^2 + \frac{P - C \rho^{\gamma_m}}{\gamma - 1} + \frac{C \rho^{\gamma_m}}{\gamma_m - 1} \right\};$$

$$\text{FullSimplify}\left[\text{eigenvectors} /. \left\{ P \rightarrow \rho \left(\frac{c_s^2}{\gamma} + \frac{c_m^2}{\gamma_m} \right), C \rightarrow \frac{c_m^2}{\gamma_m \rho^{\gamma_m - 1}} \right\}, \{c_s > 0, c_m > 0, \gamma > 0, P > 0, \rho > 0, \gamma_m > 1\} \right]$$

$$\begin{aligned} \text{Out[213]} = & \left\{ \left\{ \frac{2(-1+\gamma)(-1+\gamma_m)}{2c_m^2(\gamma-\gamma_m)+u^2(-1+\gamma)(-1+\gamma_m)}, \frac{2u(-1+\gamma)(-1+\gamma_m)}{2c_m^2(\gamma-\gamma_m)+u^2(-1+\gamma)(-1+\gamma_m)}, 1 \right\}, \right. \\ & \left\{ \frac{1}{\frac{c_s^2}{-1+\gamma} + \frac{1}{2}u(u-2\sqrt{c_m^2+c_s^2}) + \frac{c_m^2}{-1+\gamma_m}}, -\frac{2(-1+\gamma)(c_m^2+c_s^2+u(u-u\gamma+(-2+\gamma)\sqrt{c_m^2+c_s^2}))(-1+\gamma_m)}{(2c_s^2\sqrt{c_m^2+c_s^2}+u^2(-1+\gamma)(u(-1+\gamma)+(3-2\gamma)\sqrt{c_m^2+c_s^2}))(-1+\gamma_m)+2(-1+\gamma)c_m^2(u\gamma+\sqrt{c_m^2+c_s^2}-u\gamma_m)} \right\}, \\ & \left. \left\{ \frac{1}{\frac{c_s^2}{-1+\gamma} + \frac{1}{2}u(u+2\sqrt{c_m^2+c_s^2}) + \frac{c_m^2}{-1+\gamma_m}}, -\frac{2(-1+\gamma)(-c_m^2-c_s^2+u(u(-1+\gamma)+(-2+\gamma)\sqrt{c_m^2+c_s^2}))(-1+\gamma_m)}{(2c_s^2\sqrt{c_m^2+c_s^2}-u^2(-1+\gamma)(u(-1+\gamma)+(-3+2\gamma)\sqrt{c_m^2+c_s^2}))(-1+\gamma_m)+2(-1+\gamma)c_m^2(-u\gamma+\sqrt{c_m^2+c_s^2}+u\gamma_m)} \right\} \right\} \end{aligned}$$

$$\text{In[214]} = \left\{ u, u - \sqrt{c_m^2 + c_s^2}, u + \sqrt{c_m^2 + c_s^2} \right\}$$

$$\text{Out[214]} = \left\{ u, u - \sqrt{c_m^2 + c_s^2}, u + \sqrt{c_m^2 + c_s^2} \right\}$$

$$\begin{aligned} \text{In[215]} = & \left\{ \frac{1}{\frac{c_s^2}{-1+\gamma} + \frac{1}{2}u(u+2\sqrt{c_m^2+c_s^2}) + \frac{c_m^2}{-1+\gamma_m}}, \right. \\ & - \left(\left(2(-1+\gamma)(-c_m^2-c_s^2+u(u(-1+\gamma)+(-2+\gamma)\sqrt{c_m^2+c_s^2}))(-1+\gamma_m) \right) / \left((2c_s^2\sqrt{c_m^2+c_s^2}-u^2(-1+\gamma)(u(-1+\gamma)+(-3+2\gamma)\sqrt{c_m^2+c_s^2}))(-1+\gamma_m)+2(-1+\gamma)c_m^2(-u\gamma+\sqrt{c_m^2+c_s^2}+u\gamma_m) \right) \right), \\ & 1 \} == \left\{ \frac{1}{\frac{c_s^2}{-1+\gamma} + \frac{1}{2}u(u+2\sqrt{c_m^2+c_s^2}) + \frac{c_m^2}{-1+\gamma_m}}, \right. \\ & - \left(\left(2(-1+\gamma)(-c_m^2-c_s^2+u(u(-1+\gamma)+(-2+\gamma)\sqrt{c_m^2+c_s^2}))(-1+\gamma_m) \right) / \left((2c_s^2\sqrt{c_m^2+c_s^2}-u^2(-1+\gamma)(u(-1+\gamma)+(-3+2\gamma)\sqrt{c_m^2+c_s^2}))(-1+\gamma_m)+2(-1+\gamma)c_m^2(-u\gamma+\sqrt{c_m^2+c_s^2}+u\gamma_m) \right) \right), 1 \} \end{aligned}$$

Out[215]= True

$$\begin{aligned} \text{In[216]} = & \left\{ \frac{1}{\frac{c_s^2}{-1+\gamma} + \frac{1}{2}u(u-2\sqrt{c_m^2+c_s^2}) + \frac{c_m^2}{-1+\gamma_m}}, - \left(\left(2(-1+\gamma)(c_m^2+c_s^2+u(u-u\gamma+(-2+\gamma)\sqrt{c_m^2+c_s^2}))(-1+\gamma_m) \right) / \left((2c_s^2\sqrt{c_m^2+c_s^2}+u^2(-1+\gamma)(u(-1+\gamma)+(3-2\gamma)\sqrt{c_m^2+c_s^2}))(-1+\gamma_m)+2(-1+\gamma)c_m^2(u\gamma+\sqrt{c_m^2+c_s^2}-u\gamma_m) \right) \right), \right. \\ & 1 \} == \left\{ \frac{1}{\frac{c_s^2}{-1+\gamma} + \frac{1}{2}u(u-2\sqrt{c_m^2+c_s^2}) + \frac{c_m^2}{-1+\gamma_m}}, \right. \\ & - \left(\left(2(-1+\gamma)(c_m^2+c_s^2+u(u-u\gamma+(-2+\gamma)\sqrt{c_m^2+c_s^2}))(-1+\gamma_m) \right) / \left((2c_s^2\sqrt{c_m^2+c_s^2}+u^2(-1+\gamma)(u(-1+\gamma)+(3-2\gamma)\sqrt{c_m^2+c_s^2}))(-1+\gamma_m)+2(-1+\gamma)c_m^2(u\gamma+\sqrt{c_m^2+c_s^2}-u\gamma_m) \right) \right), 1 \} \end{aligned}$$

Out[216]= True

In[217]:= ?? TeXForm

Symbol ⓘ

TeXForm[*expr*] prints as a TeX version of *expr*.

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Attributes {Protected}

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Out[217]=