Problems on Trains

1. **km/hr to m/s conversion:**

   a km/hr = $\left(a \times \frac{5}{18} \right)$ m/s.

2. **m/s to km/hr conversion:**

   a m/s = $\left(a \times \frac{18}{5} \right)$ km/hr.

3. [Formulas for finding Speed, Time and Distance]
4. Time taken by a train of length l metres to pass a pole or standing man or a signal post is equal to the time taken by the train to cover l metres.
5. Time taken by a train of length l metres to pass a stationery object of length b metres is the time taken by the train to cover (l + b) metres.
6. Suppose two trains or two objects bodies are moving in the same direction at u m/s and v m/s, where $u > v$, then their relative speed is = (u - v) m/s.
7. Suppose two trains or two objects bodies are moving in opposite directions at u m/s and v m/s, then their relative speed is = (u + v) m/s.
8. If two trains of length a metres and b metres are moving in opposite directions at u m/s and v m/s, then:

   The time taken by the trains to cross each other = $\frac{(a + b)}{(u + v)}$ sec.

9. If two trains of length a metres and b metres are moving in the same direction at u m/s and v m/s, then:

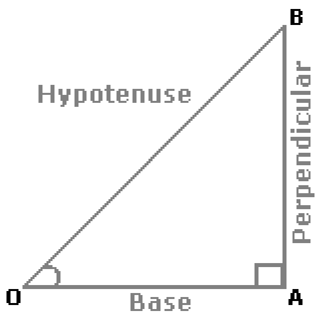
   The time taken by the faster train to cross the slower train = $\frac{(a + b)}{(u - v)}$ sec.

10. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively, then:
    (A's speed) : (B's speed) = (b : a)

Height and Distance

1. **Trigonometry**:

   In a right angled Δ OAB, where $\angle$BOA = θ,



   i. $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{OB}$;

   ii. $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{OA}{OB}$;

   iii. $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{OA}$;

   iv. $\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{OB}{AB}$;

v. $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{OB}{OA}$;

vi. $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{OA}{AB}$;

2. **Trigonometrical Identities:**

i. $\sin^2\theta + \cos^2\theta = 1$.

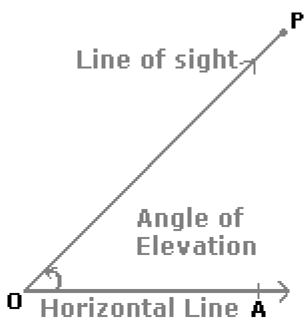
ii. $1 + \tan^2\theta = \sec^2\theta$.

iii. $1 + \cot^2\theta = \mathrm{cosec}^2\theta$.

3. **Values of T-ratios:**

θ	0°	(π/6) 30°	(π/4) 45°	(π/3) 60°	(π/2) 90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	1
$\cos\theta$	1	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{3}$	1	3	not defined

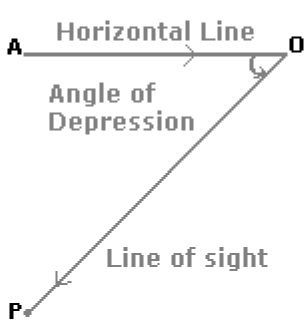
4. **Angle of Elevation:**



Suppose a man from a point O looks up at an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the **angle of elevation** of P as seen from O.

$\therefore$ Angle of elevation of P from O = $\angle$AOP.

5. **Angle of Depression:**



Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the **angle of depression** of P as seen from O.

Simple Interest

1. **Principal:**
   The money borrowed or lent out for a certain period is called the **principal** or the **sum**.
2. **Interest:**
   Extra money paid for using other's money is called **interest**.

3. Simple Interest (S.I.):
   If the interest on a sum borrowed for certain period is reckoned uniformly, then it is called **simple interest**.

   Let Principal = P, Rate = R% per annum (p.a.) and Time = T years. Then

   (i). Simple Intereest = $\left(\frac{P \times R \times T}{100}\right)$

   (ii). P = $\left(\frac{100 \times S.I.}{R \times T}\right)$; R = $\left(\frac{100 \times S.I.}{P \times T}\right)$ and T = $\left(\frac{100 \times S.I.}{P \times R}\right)$.

Profit and Loss

MPORTANT FACTS
Cost Price:
The price, at which an article is purchased, is called its **cost price**, abbreviated as **C.P.**
Selling Price:
The price, at which an article is sold, is called its **selling prices**, abbreviated as **S.P.**
Profit or Gain:
If S.P. is greater than C.P., the seller is said to have a **profit** or **gain**.
Loss:
If S.P. is less than C.P., the seller is said to have incurred a **loss**.
IMPORTANT FORMULAE

1. Gain = (S.P.) - (C.P.)

2. Loss = (C.P.) - (S.P.)

3. Loss or gain is always reckoned on C.P.

4. Gain Percentage: (Gain %)

   Gain % = $\left(\frac{\text{Gain} \times 100}{\text{C.P.}}\right)$

5. Loss Percentage: (Loss %)

   Loss % = $\left(\frac{\text{Loss} \times 100}{\text{C.P.}}\right)$

6. Selling Price: (S.P.)

   SP = $\left[\frac{(100 + \text{Gain }\%)}{100} \times \text{C.P}\right]$

7. Selling Price: (S.P.)

$$SP = \left[\frac{(100 - \text{Loss }\%)}{100} \times \text{C.P.}\right]$$

8. Cost Price: (C.P.)

$$C.P. = \left[\frac{100}{(100 + \text{Gain }\%)} \times \text{S.P.}\right]$$

9. Cost Price: (C.P.)

$$C.P. = \left[\frac{100}{(100 - \text{Loss }\%)} \times \text{S.P.}\right]$$

10. If an article is sold at a gain of say 35%, then S.P. = 135% of C.P.

11. If an article is sold at a loss of say, 35% then S.P. = 65% of C.P.
12. When a person sells two similar items, one at a gain of say x%, and the other at a loss of x%, then the seller always incurs a loss given by:

$$\text{Loss }\% = \left(\frac{\text{Common Loss and Gain }\%}{10}\right)^2 = \left(\frac{x}{10}\right)^2.$$

13. If a trader professes to sell his goods at cost price, but uses false weights, then

$$\text{Gain }\% = \left[\frac{\text{Error}}{(\text{True Value}) - (\text{Error})} \times 100\right]\%.$$

Percentage

1. **Concept of Percentage:**

   By a certain **percent**, we mean that many hundredths.
   Thus, x percent means x hundredths, written as x%.

   To express x% as a fraction: We have, $x\% = \frac{x}{100}$.

   Thus, $20\% = \frac{20}{100} = \frac{1}{5}$.

   To express $\frac{a}{b}$ as a percent: We have, $\frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%$.

   Thus, $\frac{1}{4} = \left(\frac{1}{4} \times 100\right)\% = 25\%$.

2. **Percentage Increase/Decrease:**

   If the price of a commodity increases by R%, then the reduction in consumption so as not to increase the expenditure is:

$$\left[\frac{R}{(100 + R)} \times 100\right]\%$$

   If the price of a commodity decreases by R%, then the increase in consumption so as not to decrease the expenditure is:

$$\left[\frac{R}{(100 - R)} \times 100\right]\%$$

3. **Results on Population:**

Let the population of a town be P now and suppose it increases at the rate of R% per annum, then:

1. Population after *n* years = $P\left(1+\frac{R}{100}\right)^n$

2. Population *n* years ago = $\frac{P}{\left(1+\frac{R}{100}\right)^n}$

4. **Results on Depreciation:**

   Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum. Then:

   1. Value of the machine after *n* years = $P\left(1-\frac{R}{100}\right)^n$

   2. Value of the machine *n* years ago = $\frac{P}{\left(1-\frac{R}{100}\right)^n}$

   3. If A is R% more than B, then B is less than A by $\left[\frac{R}{(100+R)} \times 100\right]\%$.

   4. If A is R% less than B, then B is more than A by $\left[\frac{R}{(100-R)} \times 100\right]\%$.

Calendar

1. **Odd Days:**

   We are supposed to find the day of the week on a given date.

   For this, we use the concept of 'odd days'.
   In a given period, the number of days more than the complete weeks are called **odd days**.

2. **Leap Year:**

   (i). Every year divisible by 4 is a leap year, if it is not a century.
   (ii). Every 4^{th} century is a leap year and no other century is a leap year.
   Note: **A leap year has 366 days.**
   Examples:

   i. Each of the years 1948, 2004, 1676 etc. is a leap year.

   ii. Each of the years 400, 800, 1200, 1600, 2000 etc. is a leap year.

   iii. None of the years 2001, 2002, 2003, 2005, 1800, 2100 is a leap year.

3. **Ordinary Year:**
   The year which is not a leap year is called an **ordinary years**. An ordinary year has 365 days.

4. **Counting of Odd Days:**

   1. 1 ordinary year = 365 days = (52 weeks + 1 day.)

      $\therefore$ 1 ordinary year has 1 odd day.

   2. 1 leap year = 366 days = (52 weeks + 2 days)

      $\therefore$ 1 leap year has 2 odd days.

3. 100 years = 76 ordinary years + 24 leap years

 = (76 x 1 + 24 x 2) odd days = 124 odd days.

 = (17 weeks + days) $\equiv$ 5 odd days.

$\therefore$ Number of odd days in 100 years = 5.

Number of odd days in 200 years = (5 x 2) $\equiv$ 3 odd days.

Number of odd days in 300 years = (5 x 3) $\equiv$ 1 odd day.

Number of odd days in 400 years = (5 x 4 + 1) $\equiv$ 0 odd day.

Similarly, each one of 800 years, 1200 years, 1600 years, 2000 years etc. has 0 odd days.

5. **Day of the Week Related to Odd Days:**

No. of days:	0	1	2	3	4	5	6
Day:	Sun.	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.

Average

1. **Average:**

 Average = $\left(\dfrac{\text{Sum of observations}}{\text{Number of observations}}\right)$

2. **Average Speed:**

 Suppose a man covers a certain distance at x kmph and an equal distance at y kmph.

 Then, the average speed druing the whole journey is $\left(\dfrac{2xy}{x+y}\right)$ kmph.

Volume and Surface Area

1. **CUBOID**

 Let length = l, breadth = b and height = h units. Then

 i. **Volume** = (l x b x h) cubic units.
 ii. **Surface area** = 2(lb + bh + lh) sq. units.
 iii. **Diagonal** = $l^2 + b^2 + h^2$ units.

2. **CUBE**

 Let each edge of a cube be of length a. Then,

 i. **Volume** = a^3 cubic units.
 ii. **Surface area** = $6a^2$ sq. units.
 iii. **Diagonal** = 3a units.

3. **CYLINDER**

 Let radius of base = r and Height (or length) = h. Then,

 i. **Volume** = ($\pi r^2 h$) cubic units.
 ii. **Curved surface area** = ($2\pi rh$) sq. units.
 iii. **Total surface area** = $2\pi r(h + r)$ sq. units.

4. **CONE**

 Let radius of base = r and Height = h. Then,

 i. **Slant height,** $l = h^2 + r^2$ units.
 ii. **Volume** = $\left(\frac{1}{3}\pi r^2 h\right)$ cubic units.
 iii. **Curved surface area** = (πrl) sq. units.

iv. **Total surface area** = ($\pi rl + \pi r^2$) sq. units.

5. **SPHERE**

Let the radius of the sphere be r. Then,

i. **Volume** = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.

ii. **Surface area** = ($4\pi r^2$) sq. units.

6. **HEMISPHERE**

Let the radius of a hemisphere be r. Then,

i. **Volume** = $\left(\frac{2}{3}\pi r^3\right)$ cubic units.

ii. **Curved surface area** = ($2\pi r^2$) sq. units.

iii. **Total surface area** = ($3\pi r^2$) sq. units.

Note: 1 litre = 1000 cm^3.

Numbers

Some Basic Formulae:

i. $(a + b)(a - b) = (a^2 - b^2)$

ii. $(a + b)^2 = (a^2 + b^2 + 2ab)$

iii. $(a - b)^2 = (a^2 + b^2 - 2ab)$

iv. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

v. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

vi. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$

vii. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$

viii. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

Problems on H.C.F and L.C.M

1. **Factors and Multiples:**

If number a divided another number b exactly, we say that a is a **factor** of b.
In this case, b is called a **multiple** of a.

2. **Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.):**

The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.

There are two methods of finding the H.C.F. of a given set of numbers:

I. **Factorization Method:** Express the each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.

II. **Division Method:** Suppose we have to find the H.C.F. of two given numbers, divide the larger by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is required H.C.F.

Finding the H.C.F. of more than two numbers: Suppose we have to find the H.C.F. of three numbers, then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given number.

Similarly, the H.C.F. of more than three numbers may be obtained.

3. **Least Common Multiple (L.C.M.):**

The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

There are two methods of finding the L.C.M. of a given set of numbers:

I. **Factorization Method:** Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors.

II. **Division Method (short-cut):** Arrange the given numbers in a rwo in any order. Divide by a number which divided exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.

4. **Product of two numbers = Product of their H.C.F. and L.C.M.**
5. **Co-primes:** Two numbers are said to be co-primes if their H.C.F. is 1.
6. **H.C.F. and L.C.M. of Fractions:**

1. H.C.F. = $\dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$

2. L.C.M. = $\dfrac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$

8. **H.C.F. and L.C.M. of Decimal Fractions:**

In a given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.

9. **Comparison of Fractions:**

Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

Surds and Indices

1. **Laws of Indices:**

i. $a^m \times a^n = a^{m+n}$

ii. $\dfrac{a^m}{a^n} = a^{m-n}$

iii. $(a^m)^n = a^{mn}$

iv. $(ab)^n = a^n b^n$

v. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

vi. $a^0 = 1$

2. **Surds:**
Let a be rational number and n be a positive integer such that $a^{(1/n)} = a$
Then, a is called a surd of order n.

3. **Laws of Surds:**

i. $a = a^{(1/n)}$

ii. $ab = a \times b$

iii. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{a}{b}$

iv. $(a)^n = a$

v. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

vi. $(a)^m = a^m$

Chain Rule

1. **Direct Proportion:**

   Two quantities are said to be directly proportional, if on the increase (or decrease) of the one, the other increases (or decreases) to the same extent.
   Eg. Cost is directly proportional to the number of articles.
   (More Articles, More Cost)
2. **Indirect Proportion:**

   Two quantities are said to be indirectly proportional, if on the increase of the one, the orther decreases to the same extent and vice-versa.

   Eg. The time taken by a car is covering a certain distance is inversely proportional to the speed of the car. (More speed, Less is the time taken to cover a distance.)
   Note: In solving problems by chain rule, we compare every item with the term to be found out.

Boats and Streams

1. **Downstream/Upstream:**
   In water, the direction along the stream is called **downstream**. And, the direction against the stream is called **upstream**.
2. If the speed of a boat in still water is u km/hr and the speed of the stream is v km/hr, then:
   Speed downstream = $(u + v)$ km/hr.
   Speed upstream = $(u - v)$ km/hr.
3. If the speed downstream is a km/hr and the speed upstream is b km/hr, then:

   Speed in still water = $\frac{1}{2}(a + b)$ km/hr.

   Rate of stream = $\frac{1}{2}(a - b)$ km/hr.

Odd Man Out and Series

Time and Distance

1. **Speed, Time and Distance:**
   Speed = $\left(\frac{\text{Distance}}{\text{Time}}\right)$, Time = $\left(\frac{\text{Distance}}{\text{Speed}}\right)$, Distance = (Speed x Time).
2. **km/hr to m/sec conversion:**
   x km/hr = $\left(x \times \frac{5}{18}\right)$ m/sec.
3. **m/sec to km/hr conversion:**
   x m/sec = $\left(x \times \frac{18}{5}\right)$ km/hr.
4. If the ratio of the speeds of A and B is $a : b$, then the ratio of the
   the times taken by then to cover the same distance is $\frac{1}{a} : \frac{1}{b}$ or $b : a$.
5. Suppose a man covers a certain distance at x km/hr and an equal distance at y km/hr. Then,

the average speed during the whole journey is $\left(\frac{2xy}{x+y}\right)$ km/hr.

Time and Work

1. **Work from Days:**

   If A can do a piece of work in *n* days, then A's 1 day's work = $\frac{1}{n}$.

2. **Days from Work:**

   If A's 1 day's work = $\frac{1}{n}$, then A can finish the work in *n* days.

3. **Ratio:**

   If A is thrice as good a workman as B, then:

   Ratio of work done by A and B = 3 : 1.

   Ratio of times taken by A and B to finish a work = 1 : 3.

Compound Interest

1. Let Principal = P, Rate = R% per annum, Time = *n* years.
2. **When interest is compound Annually:**

   $$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

3. **When interest is compounded Half-yearly:**

   $$\text{Amount} = P\left[1+\frac{(R/2)}{100}\right]^{2n}$$

4. **When interest is compounded Quarterly:**

   $$\text{Amount} = P\left[1+\frac{(R/4)}{100}\right]^{4n}$$

5. **When interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years.**

   $$\text{Amount} = P\left(1+\frac{R}{100}\right)^3 \times \left(1+\frac{\frac{2}{5}R}{100}\right)$$

6. **When Rates are different for different years**, say R_1%, R_2%, R_3% for 1st, 2nd and 3rd year respectively.

   $$\text{Then, Amount} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right).$$

7. **Present worth of Rs. *x* due *n* years hence is given by:**

   $$\text{Present Worth} = \frac{x}{\left(1+\frac{R}{100}\right)^n}$$

Partnership

1. **Partnership:**
   When two or more than two persons run a business jointly, they are called **partners** and the deal is known as **partnership**.
2. **Ratio of Divisions of Gains:**

I. When investments of all the partners are for the same time, the gain or loss is distributed among the partners in the ratio of their investments.
Suppose A and B invest Rs. x and Rs. y respectively for a year in a business, then at the end of the year:
(A's share of profit) : (B's share of profit) = x : y.

II. When investments are for different time periods, then equivalent capitals are calculated for a unit of time by taking (capital x number of units of time). Now gain or loss is divided in the ratio of these capitals.
Suppose A invests Rs. x for p months and B invests Rs. y for q months then,
(A's share of profit) : (B's share of profit)= xp : yq.

3. **Working and Sleeping Partners:**
A partner who manages the the business is known as a **working partner** and the one who simply invests the money is a **sleeping partner**.

Problems on Ages

Important Formulas on "Problems on Ages" :

1. If the current age is x, then n times the age is nx.
2. If the current age is x, then age n years later/hence = $x + n$.
3. If the current age is x, then age n years ago = $x - n$.
4. The ages in a ratio $a : b$ will be ax and bx.
5. If the current age is x, then $\frac{1}{n}$ of the age is $\frac{x}{n}$.

Clock

1. **Minute Spaces:**

The face or dial of watch is a circle whose circumference is divided into 60 equal parts, called minute spaces.
Hour Hand and Minute Hand:
A clock has two hands, the smaller one is called the **hour hand** or **short hand** while the larger one is called **minute hand** or **long hand**.

2.

i. In 60 minutes, the minute hand gains 55 minutes on the hour on the hour hand.

ii. In every hour, both the hands coincide once.

iii. The hands are in the same straight line when they are coincident or opposite to each other.

iv. When the two hands are at right angles, they are 15 minute spaces apart.

v. When the hands are in opposite directions, they are 30 minute spaces apart.

vi. Angle traced by hour hand in 12 hrs = 360°

vii. Angle traced by minute hand in 60 min. = 360°.

viii. If a watch or a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes **too fast**.
On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes **too slow**

Permutation and Combination

1. **Factorial Notation:**

Let n be a positive integer. Then, factorial n, denoted $n!$ is defined as:

$$n! = n(n - 1)(n - 2) \ldots 3.2.1.$$

Examples:

i. We define **0! = 1**.

ii. 4! = (4 x 3 x 2 x 1) = 24.

iii. 5! = (5 x 4 x 3 x 2 x 1) = 120.

2. **Permutations:**

The different arrangements of a given number of things by taking some or all at a time, are called permutations.

Examples:

i. All permutations (or arrangements) made with the letters *a*, *b*, *c* by taking two at a time are (***ab***, ***ba***, ***ac***, ***ca***, ***bc***, ***cb***).

ii. All permutations made with the letters *a*, *b*, *c* taking all at a time are: (***abc***, ***acb***, ***bac***, ***bca***, ***cab***, ***cba***)

3. **Number of Permutations:**

Number of all permutations of n things, taken r at a time, is given by:

$$^nP_r = n(n - 1)(n - 2) \ldots (n - r + 1) = \frac{n!}{(n - r)!}$$

Examples:

i. $^6P_2 = (6 \times 5) = 30$.

ii. $^7P_3 = (7 \times 6 \times 5) = 210$.

iii. **Cor. number of all permutations of *n* things, taken all at a time = *n*!.**

4. **An Important Result:**

If there are n subjects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of r^{th} kind,
such that $(p_1 + p_2 + \ldots p_r) = n$.

Then, number of permutations of these n objects is = $\dfrac{n!}{(p_1!).(p_2)!.....(p_r!)}$

5. **Combinations:**

Each of the different groups or selections which can be formed by taking some or all of a number of objects is called a **combination**.

Examples:

1. Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.

   Note: AB and BA represent the same selection.
2. All the combinations formed by *a*, *b*, *c* taking ***ab***, ***bc***, ***ca***.
3. The only combination that can be formed of three letters *a*, *b*, *c* taken all at a time is ***abc***.

4. Various groups of 2 out of four persons A, B, C, D are:

   AB, AC, AD, BC, BD, CD.
5. Note that *ab ba* are two different permutations but they represent the same combination.
6. **Number of Combinations:**

   The number of all combinations of n things, taken r at a time is:

$$^nC_r = \frac{n!}{(r!)(n - r)!} = \frac{n(n - 1)(n - 2) \ldots \text{to } r \text{ factors}}{r!}.$$

Note:

. $^nC_n = 1$ and $^nC_0 = 1$.

i. $^nC_r = {^nC_{(n - r)}}$

Examples:

i. $^{11}C_4 = \dfrac{(11 \times 10 \times 9 \times 8)}{(4 \times 3 \times 2 \times 1)} = 330.$

ii. $^{16}C_{13} = {}^{16}C_{(16-13)} = {}^{16}C_3 = \dfrac{16 \times 15 \times 14}{3!} = \dfrac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560.$

Problems on Numbers

i. $(a + b)(a - b) = (a^2 - b^2)$
ii. $(a + b)^2 = (a^2 + b^2 + 2ab)$
iii. $(a - b)^2 = (a^2 + b^2 - 2ab)$
iv. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
v. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
vi. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
vii. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
viii. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

Probability

1. **Experiment:**

   An operation which can produce some well-defined outcomes is called an experiment.
2. **Random Experiment:**

   An experiment in which all possible outcomes are know and the exact output cannot be predicted in advance, is called a random experiment.
   Examples:

i. Rolling an unbiased dice.

ii. Tossing a fair coin.

iii. Drawing a card from a pack of well-shuffled cards.

iv. Picking up a ball of certain colour from a bag containing balls of different colours.
Details:

v. When we throw a coin, then either a Head (H) or a Tail (T) appears.

vi. A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively. When we throw a die, the outcome is the number that appears on its upper face.

vii. A pack of cards has 52 cards.
It has 13 cards of each suit, name **Spades, Clubs, Hearts and Diamonds**.
Cards of spades and clubs are **black cards**.
Cards of hearts and diamonds are **red cards**.

There are 4 honours of each unit.
There are **Kings, Queens and Jacks**. These are all called **face cards**.

3. **Sample Space:**
   When we perform an experiment, then the set S of all possible outcomes is called the **sample space**.
   Examples:

1. In tossing a coin, S = {H, T}

2. If two coins are tossed, the S = {HH, HT, TH, TT}.
3. In rolling a dice, we have, S = {1, 2, 3, 4, 5, 6}.
4. **Event:**
   Any subset of a sample space is called an **event**.
5. **Probability of Occurrence of an Event:**

   Let S be the sample and let E be an event.

   Then, $E \subseteq S$.
   $\therefore$ $P(E) = \frac{n(E)}{n(S)}$.
6. **Results on Probability:**

. P(S) = 1

i. $0 \leq P(E) \leq 1$

ii. $P(\phi) = 0$

iii. For any events A and B we have : $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
iv. If A denotes (not-A), then P(A) = 1 - P(A).

Alligation or Mixture

1. **Alligation:**

   It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of desired price.
2. **Mean Price:**

   The cost of a unit quantity of the mixture is called the mean price.
3. **Rule of Alligation:**

   If two ingredients are mixed, then
   $$\left(\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}}\right) = \left(\frac{\text{C.P. of dearer - Mean Price}}{\text{Mean price - C.P. of cheaper}}\right)$$

   We present as under:
   C.P. of a unit quantity
   of cheaperC.P. of a unit quantity
   of dearer

(c)	Mean Price	(d)
(d - m)	(m)	(m - c)

   $\therefore$ (Cheaper quantity) : (Dearer quantity) = (d - m) : (m - c).
4. Suppose a container contains x of liquid from which y units are taken out and replaced by water.

   After n operations, the quantity of pure liquid = $\left[x\left(1 - \frac{y}{x}\right)^n\right]$ units.

Pipes and Cistern

1. **Inlet:**

   A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.

Outlet:

A pipe connected with a tank or cistern or reservoir, emptying it, is known as an outlet.

2. If a pipe can fill a tank in *x* hours, then:

   part filled in 1 hour = $\frac{1}{x}$.

3. If a pipe can empty a tank in *y* hours, then:

   part emptied in 1 hour = $\frac{1}{y}$.

4. If a pipe can fill a tank in *x* hours and another pipe can empty the full tank in *y* hours (where $y > x$), then on opening both the pipes, then

   the net part filled in 1 hour = $\left(\frac{1}{x} - \frac{1}{y}\right)$.

5. If a pipe can fill a tank in *x* hours and another pipe can empty the full tank in *y* hours (where $x > y$), then on opening both the pipes, then

   the net part emptied in 1 hour = $\left(\frac{1}{y} - \frac{1}{x}\right)$.