SURE SUCCESS INSIGHT MCA CLASSES
Exculsive Coaching Classes for MCA & IIT-JAM Entrance
Regd. No. 1014
INSIGHT MCA CLASSES
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1. Let A(t) denote the area bounded by the curve $y = e^{-|x|}$, the x-axis and the straight lines x = -t and x = t. Then $\lim_{t\to\infty} A(t)$ is equal to **[IIT JAM 2007]**
(a) 1* (b) 1 (c) 1/2 (d) 0

2. If k is a constant such that $xy + k = e^{(x-1)^{2/2}}$ satisfies the differential equation $x\frac{dy}{dx} = (x^2 - x - 1)y + (x-1)$, then k is equal to **[IIT JAM 2007]**
(a) 1* (b) 0 (c) -1 (d) -2

3. Which of the following functions is uniformly continuous on the domain as stated ?
(a) $f(x) = x^2,\ x \in \mathbb{R}$
(b) $f(x) = \frac{1}{x},\ x \in [1, \infty)$ *
(c) $f(x) = \tan x,\ x \in (-\pi/2, \pi/2)$
(d) $f(x) = [x],\ x \in [0,1]$ **[IIT JAM 2007]**
([x] is the greatest integer less than or equal to x)

4. Let R be the ringh of polynomials over Z_2 and let I be the ideal of R generated by the polynomial $x^3 + x + 1$. Then the number of elements in the quotient ring R/I is **[IIT JAM 2007]**
(a) 2 (b) 4 (c) 8* (d) 16

5. Which of the following sets is a basis for the subspace **[IIT JAM 2007]**
$$W = \left\{ \begin{bmatrix} x & y \\ 0 & t \end{bmatrix} : x + 2y + t = 0, y + t = 0 \right\}$$
of the vector space all real 2 × 2 matrices ? **[IIT JAM 2007]**
(a) $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$
(b) $\left\{ \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \right\}$
(c) $\left\{ \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix} \right\}$ (d) $\left\{ \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \right\}$ *

6. Let G be an Abelian group of order 10. Let $S\{g \in G : g^{-1} = g\}$. Then the number of non-identity elements in S is **[IIT JAM 2007]**
(a) 5 (b) 2 (c) 1* (d) 0

7. Let (a_n) be an increasing sequence of positive real numbers such that the series $\sum_{k=1}^{\infty} a_k$ is divergent. Let $s_n = \sum_{k=1}^{n} a_k$ for n = 1, 2,... and $l_n = \sum_{k=2}^{n} \frac{a_b}{s_{k-1}s_k}$ for n = 2, 3,... Then $\lim_{n\to\infty} t_n$ is equal to **[IIT JAM 2007]**
(a) $1/a_1$* (b) 0 (c) $1/(a_1 + a_2)$ (d) $a_1 + a_2$

8. For every function $f : [0,1] \to \mathbb{R}$ which is twice differentiable and satisfies $f'(x) \ge 1$ for all $x \in [0,1]$, we must have **[IIT JAM 2007]**
(a) $f''(x) \ge 0$ for all $x \in [0,1]$
(b) $f(x) \ge x$ for all $x \in [0,1]$
(c) $f(x_2) - x_2 \le f(x_1) - x_1$ for all $x_1, x_2 \in [0,1]$ with $x_2 \ge x_1$
*(d) $f(x_2) - x_2 \ge f(x_1) - x_1$ for al with

$x_2 \geq x_1$

9. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by **[IIT JAM 2007]**

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2} & if\ (x,y) \neq (0,0), \\ 0 & if\ (x,y) = (0,0). \end{cases}$$

Which of the following statements holds regarding the continuity and the existence of partial derivatives of f at (0, 0) ?

(a) Both partial derivatives of f exist at (0,0) and f is continuous at (0, 0)
(b) Bothe partial derivatives of f exist at (0, 0) and f is NOT continuous at (0, 0)*
(c) One partial derivative of f does NOT exist at (0, 0) and f is continuous at (0, 0)
(d) One partial derivative of f does NOT exist at (0, 0) and f is NOT continuous at (0, 0)

10. Suppose (c_n) is a sequence of real numbers such that $\lim\limits_{n\to\infty} |c_n|^{1/n}$ exists and is non-zero. If the radius of convergence of the power series $\sum\limits_{n=0}^{\infty} c_n x^n$ is equal to r, then the radius of convergence of the power series $\sum\limits_{n=0}^{\infty} n^2 c_n x^n$ is **[IIT JAM 2007]**

(a) less than r (b) greater than r
(c) equal to r* (d) equal to 0

11. The rank of the matrix $\begin{bmatrix} 1 & 4 & 8 \\ 2 & 10 & 22 \\ 0 & 4 & 12 \end{bmatrix}$ is

[IIT JAM 2007]

(a) 3 (b) 2* (c) 1 (d) 0

12. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. If $\int_0^x f(2t)dt = \frac{x}{\pi}\sin(\pi x)$ for all $x \in \mathbb{R}$, then f(2) is qual to **[IIT JAM 2007]**

(a) -1* (b) 0 (c) 1 (d) 2

13. Let $\vec{u} = (ae^x \sin y - 4x)\hat{i} + (2y + e^x \cos y)\hat{j} + az\hat{k}$, where a is a constant. If the line integral $\oint_C \vec{u}.d\vec{r}$ over every closed curve C is zero, then a is equal to **[IIT JAM 2007]**

(a) -2 (b) -1 (c) 0 (d) 1*

14. One of the integrating factors of the differential equation $(y^2 - 3xy)dx + (x^2 - xy)dy = 0$ is

(a) $1/(x^2y^2)$ (b) $1/(x^2y)$*
(c) $1/(xy^2)$ (d) 1/(xy) **[IIT JAM 2007]**

15. Let C denote the boundary of the semi-circular disk $D = \left\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1, y \geq 0\right\}$ and let $\varphi(x,y) = x^2 + y$ for $(x,y) \in D$. If $\hat{n}$ is the outward unit normal to C, then the integral $\oint_C (\vec{\nabla}\varphi).\hat{n}ds$, evaluated counter-clockwise over C, is equal to **[IIT JAM 2007]**

(a) 0 (b) $\pi - 2$* (c) π (d) $\pi + 2$

16. (a) Let $M = \begin{bmatrix} 1+i & 2i & i+3 \\ 0 & 1-i & 3i \\ 0 & 0 & i \end{bmatrix}$. Determine the eigenvalues of the matrix $B = M^2 - 2M + I$.

ANS. -1, -1 , -2i

(b) Let N be a square matrix of order 2. If the determinant of N is equal to 9 and the sum of the diagonal entries of N is equal to 10, then determine the eigenvalues of N.(9+6) **[IIT JAM 2007]**

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17. (a) Using the method of variation of parameters, solve the differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2,$$ **[IIT JAM 2007]**

given that x and 1/x are two solutions of the corresponding homogeneous equation. (9)

(b) Find the real number α such that the differential equation

$$\frac{d^2y}{dx^2} + 2(\alpha - 1)(\alpha - 3)\frac{dy}{dx} + (\alpha - 2)y = 0$$

has a solution $y(x) = \alpha\cos(\beta x) + b\sin(\beta x)$ for some non-zero real number a, b, β. (6)

18. (a) Let a, b, c be non-zero real numbers such that $(a - b)^2 = 4ac$. Solve the differential equation

$$a\left(x + \sqrt{2}\right)^2 \frac{d^2y}{dx^2} + b\left(x + \sqrt{2}\right)\frac{dy}{dx} + cy = 0$$.(9)

(b) Solve the differential equation

$$dx+(e^{y\sin y}-x)(y\cos y+\sin y)dy=0.(6)$$

[IIT JAM 2007]

19. Let f(x, y) = x(x - 2y²) for $(x, y) \in \mathbb{R}^2$. Show that f has a local minimum at (0, 0) on every straight line through (0, 0). Is (0, 0) a critical point of f ? Find the discriminant of f at (0, 0). Does f have a local minimum at (0, 0)? Justify your answers.(15) **[IIT JAM 2007]**

20. (a) Find the finite volume enclosed by the paraboloids $z = 2 - x^2 - y^2$ and $z = x^2 + y^2$.(9)

(b) Let $f:[0,3]\to\mathbb{R}$ be a continuous function with $\int_0^3 f(x)dx = 3$. Evaluate $\int_0^3 [xf(x)+\int_0^x f(t)dt]dx$. (6) **[IIT JAM 2007]**

21. (a) Let S be the surface $\{(x,y,z)\in\mathbb{R}^3 : x^2+y^2+2z=2,\ z\ge 0\}$, and let $\hat{n}$ be the outward unit normal to S. If $\vec{F} = y\hat{i} + xz\hat{j} + (x^2+y^2)\hat{k}$, then evaluate the integral $\iint_S \vec{F}.\hat{n}dS$. (9)

(b) Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. If a scalar field φ and a vector field $\vec{u}$ satisfy $\vec{\nabla}\varphi = \vec{\nabla}\times\vec{u} + f(r)\vec{r}$, where f is an arbitrary differentiable function, then show that $\nabla^2\varphi = rf'(r)+3f(r)$.(6) **[IIT JAM 2007]**

22. (a) Let D be the region bounded by the concentric spheres $S_1 : x^2+y^2+z^2 = a^2$ and $S_2 : x^2+y^2+z^2 = b^2$, where a < b. Let $\hat{n}$ be the unit normal to S_1 directed away from the origin. If $\nabla^2\varphi = 0$ in D and $\varphi = 0$ on S_2, then show that

$$\iiint_D \left|\vec{\nabla}\varphi\right|^2 dV + \iint_{S_1} \varphi(\vec{\nabla}\varphi).\hat{n}dS = 0 \qquad (9)$$

(b) Let C be the curve In R^3 given by $x^2+y^2=a^2$, z = 0 traced counter-clockwise, and let $\vec{F} = x^2y^3\hat{i} + \hat{j} + z\hat{k}$. Using Stoke's theorem, evaluate $\oint_C \vec{F}.d\vec{r}$.(6)**[IIT JAM 2007]**

23. Let V be the subspace of R^4 spanned by the vectors (1, 0, 1, 2), (2, 1, 3, 4) and (3, 1, 4, 6). Let $T: V\to R^2$ be a linear transformation given by T (x, y, z, t) = (x - y, z - t) for all $(x,y,z,t)\in V$. Find a basis for the null space of T and also a basis for the range space of T. (15)**[IIT JAM 2007]**

24. (a) Compute the double integral $\iint_D (x+2y)dx\,dy$, where D is the region in the xy-plane bounded by the straight lines y = x + 3, y = x - 3, y = -2x + 4 and y = - 2x - 2.(9)**[IIT JAM 2007]**

(b) $\int_0^{\pi/2}\left[\int_{\pi/2}^{\pi}\frac{\sin x}{x}dx\right]dy + \int_{\pi/2}^{\pi}\left[\int_y^{\pi}\frac{\sin x}{x}dx\right]dy$ (6)

25. (a) Deos the series $\sum_{k=1}^{\infty}\frac{(-1)^k k + x^k}{k^2}$ converge uniformly for $x\in[-1,1]$? Justify. (9)

(b) Suppose (f_n) is a sequence of real-valued functions defined on R and f is a real-valued function defined on R such that $|f_n(x)-f(x)|\le|a_n|$ for all $n\in N$ and $a_n\to 0$ as $n\to\infty$. Must the sequence (f_n) be uniformly convergent on **R** ? Justify. (6) **[IIT JAM 2007]**

26. (a) Suppose f is a real-valued thrice differentiable function defined on **R** such that f'''(x) > 0 for all $x\in R$.Using Taylor's formula, show that $f(x_2)-f(x_1) > (x_2-x_1)f'\left(\frac{x_1+x_2}{2}\right)$ for all x_1 and x_2 in **R** with $x_2 > x_1$. (9)

(b) Let (a_n) and (b_n) be sequences of real numbers such that $a_n \le a_{n+1} \le b_{n+1} \le b_n$ for all $n\in\mathbf{N}$. Must there exist a real number x such that $a_n \le x \le b_n$ for all $n\in\mathbf{N}$? Justify your answer. (6)**[IIT JAM 2007]**

27. Let G be the group of all 2×2 matrices with real entries with respect to matrix multiplication. Let

G_1 be the smallest subgroup of G containing $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, and G_2 be the smallest of G containing $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Determine all elements of G_1 and find their orders. Determine all elements of G_2 and find their orders. Does there exist a one-to-one homomorphism from G_1 onto G_2 ? Justify. (15)**[IIT JAM 2007]**

28. (a) Let p be a prime number and let **Z** be the ring of integers. If an ideal J and **Z** contains the set p**Z** properly, then show that J = **Z**. (Here $p\mathbf{Z} = \{px : x \in \mathbf{Z}\}$). (9)

(b) Consider the ring $R = \{a + ib : a, b \in \mathbf{Z}\}$ with usual addition and multiplication. Find all invertible elements of **R**.(6)**[IIT JAM 2007]**

29. (a) Suppose E is a non-empty subset of **R** which is bounded above, and let α =sup E. (9)

(b) Find all limit points of the set $E = \left\{ n + \frac{1}{2m} : n,\, m \in \mathbf{N} \right\}$.(6)**[IIT JAM 2007]**