J.N.U.(MCA) ENTRANCE EXAMINTION (2000)

(BASED ON MEMORY)

Time Allowed : 3 Hours **Max. Marks : 100**

INSTRUCTIONS FOR CANDIDATES

(i) All questions are compulsory.
(ii) Paper consists of two parts-A and B.
(iii) Part A, again, consists of two section-I and II :
- **(a)** Section-I consists of twenty-five multiple choice question of 1 mark each and question is to be answered by putting a circle around exactly one of the letters a, b, c and d indicating the only correct choice. 0.25 mark would be deducted for each wrong answer in section-I only.
- **(b)** Section-II consists of twenty-five question of fill in the blanks type. Answer should be writen only in the space provided. Each question carries 1 mark.

(iv) Part B consists of twenty-five question for each of which a short answer is to be written utilizing only the space provided. Each question carries 2 marks.
(v) Calculators and log tables may be used.
(vi) Sheets are attached at the end of the question paper for rough work.

PART - A

1. Find the digit in the unit's position of the integer denoting the sum
$1! + 2! + 3! + \ldots\ldots\ldots\ldots + 99!$.

2. A point is picked uniformly at random from the perimeter of a unit circle. Find the probability density of X, the x-coordinate of the point, given that the equation of the circle is $x^2 + y^2 = 1$.

3. Solve the equation $x^3 - 3x + 2 = 0$, given that two of its roots are equal.

4. Let n be the number of ordered quadruples (x_1, x_2, x_3, x_4) of positive odd integers that satisfy .

$$\sum_{i=1}^{4} x_i = 98.$$

Find n /100.

5. For how many values of k is 12^{12} the least common multiple of the positive in tegers $6^6, 8^8$ and k ?

6. Obtain the only real value of λ for which the following system of linear equations has nonzero solution :

$$x + 2y + 3z = \lambda x$$
$$3x + y + 2z = \lambda y$$
$$2x_1 + 3y + z = \lambda z.$$

7. Find the solution of

$$y_k + {}_2y^2_k = y^3_{k+1}$$

With initial values $y_1 = 2$ and $y_2 = 1$.

8. In a factory cafeteria the customers (employees) have to pass through three counters. The customers buy coupons at the first counter, select and collect the snacks at the second counter and collect

19. Let [x] stand for the greatest integer function Obtain the value of

$$\int_0^{11} [x]^3 \, dx.$$

20. Given $\vec{a} = \vec{i} + \vec{k}$ and $\vec{b} = \vec{i} - 2\vec{j} + \vec{k}$, find $\vec{c}$ such that $\vec{a}.\vec{c} = 2$ and $\vec{a} \times \vec{c} = b$.

21. Fifty pairs of measurement (X_i, Y_i) are such that

$$\overline{x} = \overline{y} = 0, \sum_{i=1}^{50} X_i = 10, \sum_{i=1}^{50} x_i^{\ 2} = 15 \text{ and}$$

$$\sum_{i=1}^{50} X_i Y_i = 8$$

A straight line $Y = mX + k$ fitted to 50 points. representing the 50 pairs of measurement by minimizing the sum of squares of the vertical deviations of these points from the fitted line. What is the minimum sum of squares ?

22. How many different prime numbers are factors of N if $\log_2 (\log_3 (\log_5 (\log_7 N) = 11$?

23. Let H be a hexagon. How many circles in the plane of H have a diameter both of whose end points are vertices of H ?

24. Evaluate the double integral

$$\iint_A xy \, dx\, dy.$$

Over the triangle A formed by the axes and the line $x + y = 1$.

25. Let Q be the field of rational numbers. A new field F is formed whose elements are ordered pairs of numbers belonging to Q. In F, we define equality, addition and multiplication as follows : (a, b , c, d in Q)

(1) (a, b) = (c, d) if and only if a = c, b= d

(2) (a, b) + (c, d) = (a + c, b + d)

(3) (a, b). (c, d) = (ac + 5bd, ad + bd)

If (a, b) is not the identity element for addition in F, find the multiplicative inverse of (a, b) in F.

tea at the third. The server at each counter takes on an average 1.5 minutes although the distribution of service time is approxi mately poisson at an average rate of 6 per hour. Calculate the average time a customer spends waiting in the cafeteria.

9. Find the modulus and argument of

$$\left(\frac{1 + \text{Cos}x + i\text{Sin}x}{1 + \text{Cos}x - i\text{Sin}x}\right)^n$$

Where n is a positive integer and $x \neq (2k+1)\pi$

10. If $2x + 5y = 3$, find the maximum value of x^3y^4.

11. Find the value of

$$\lim_{n\to\infty} \frac{[(n+1)(n+2).....(2n)]^{1/n}}{n}$$

12. Determine the centre of gravity of a uniform lamina bounded by the coordinate axes and the arc of the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant.

13. Evaluate

$$\int_0^{\sqrt{1/2}} \frac{\text{Sin}^{-1}x}{(1-x^2)^{3/2}} dx.$$

14. The viscosity η of a gas depends on the mass, the effective diameter and the mean speed of the molecules. Use dimensional analysis to find η as a function of these variables.

15. Determine the order of bubble sort method to sort a file of size n.

16. Let $B = \{v_1, v_2\}$ be an ordered basis of R^2 with $v_1 = (\text{Cos}\theta, \text{Sin}\theta)$. Find the coordinates of the point (a, b) in the ordered basis.

17. The straight line $4x + 3y = p$ is a tangent to the circle $x^2 + y^2 - 4x - 6y + 4\sqrt{3} = 0$. Obtain p.

18. A coin of 10 gm rolls along a horizontal table with a velocity of 6cm s^{-1}. What is its kinetic energy.

PART - B

SECTION - I

2. Stored program computer concept was first proposed by (a) Turing (b) Pascal
(c) Von-Neumann (d) None of these C(JNU2000)

3. How many bits does one need to encode all twenty-six letters, ten symbols and ten numerals?
(a) 2 (b) 5
(c) 6 (d) 46 B (JNU2000)

4. The total time a prepare a disk drive mechanism for a block of data to be read from it is
(a) Seek time
(b) Transmission time
(c) Latency plus seek time
(d) Latency plus seek time plus transmission time D (JNU2000)

5. Two persons X and Y start walking from point A along the two adjacent sides AB and AD of a rectangular field ABCD 50m × 40m . If AB is 50m and X walks twice as fast as Y, on which side shall X cross Y if they move round the field ?
(a) BD (b) BC
(c) AB (d) CD (JNU2000) D

7. In a dinner attended by 432 people, 300 chose non-vegetarian and 132 chose vegetarian dishes. If 20 chose both, then how many chose exactly one kind of food ?
(a) 412 (b) 392
(c) 402 (d) 372 A (JNU2000)

8. If $2^{1998} - 2^{1997} + 2^{1995} = k \cdot 2^{1995}$, what is the value of k ?
(a) 1 (b) 2
(c) 3 (d) 4 (JNU2000) C

9. Following is the distribution of marks in statistics obtained by 50 students :

Marks more than :	0	10	20	30	40	50
No of Students :	50	46	40	20	10	3

If 60 % of the students pass the test what, on an average, is the minimum mark obtained by a candidate who has passed ?
(a) 25 (b) 28
(c) 35 (d) 38 A (JNU2000)

10. The smallest positive root of the equation $\tan x - x = 0$ lies in (JNU2000) D
(a) $(0, \pi/2)$ (b) $(\pi, 3\pi/2)$
(c) $(\pi/2, \pi)$ (d) None of these

11. If $\alpha_1, \alpha_2, \ldots\ldots\ldots\ldots\alpha_v$ are the roots of equation $x^n - nax - b = 0$ and $(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)\ldots\ldots(\alpha_1 - \alpha_n) = A$, the value of $A - n\,\alpha_1^{n-1}$ is............

12. The modulus and the principal argument of the complex number

$$\frac{1+2i}{1-(1-i)^2}$$ are....................... respectively

13. The volume of the largest rectangular box, with faces parallel to the coordinate planes, which can be inscribed in the ellipsoid

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$ is...................

14. In a quadrilateral ABCD, it is given that $\angle A = 120^0$, angles B and D are right angles, AB=13 and AD=46. Then AC =...............

15. If x_i (i = 1, 2 , 3) are uncorrelated variables with equal mean M and variances v_1^2, v_2^2, v_3^2 respectively. Obtain the correlation coefficient between $\frac{x_1}{x_3}$ and $\frac{x_2}{x_3}$

16. Find the number of zeros at the ends if 100! is fully expanded and written out.

17. Find the interval of convergence for the series

$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$

18. Find the common ordinate of the inflection points of the curve $y = \exp[-(x/a)^2]$, where a is a positive constant.

19. Consider the program segment ;

```
void function (int* px, int*py)
{ int temp ;
      temp = *px ;
      * px = * py ;
       * py = temp ;
 }
```

What does function do ?

20. The binary equivalent of $(0.5625)_{10}$ is......

21. Let $x_1, x_2..........x_{100}$ be positive integers such that $x_i + x_{i+1} = k$ for all i, where k is a constant. If $x_{10} = 1$, then the value of x_1 is..

22. Two trains of equal length L, travelling at speeds V_1 and V_2 miles per hour in opposite directions, take T seconds to cross each other. Then L in feet (1mile = 5280 feet) is..

23. If x and y are integers such that $(x+2y)^2 + (x+4y) = 710$, then x equals........

24. The postfix form of the following expression (A + B) * (C - D) is...................

25. A person has to cross on foot a river along a narrow paved path. Which statistic or datum about the depth of water is of relevance to him?

PART - A

EXPLANATORY ANSWER

1. 1! = 1, 2! = 2, 3! = 6; 4! = 24, 5! = 120, 6!= 720, ...

From 5! onwards every number h[as 0 in] its unit position

But 1! + 2! + 3! + 4! = 33.

Thus, the number at its unit position is 3.

2. Probability density $= \int_{-\infty}^{\infty} f(x)dx$

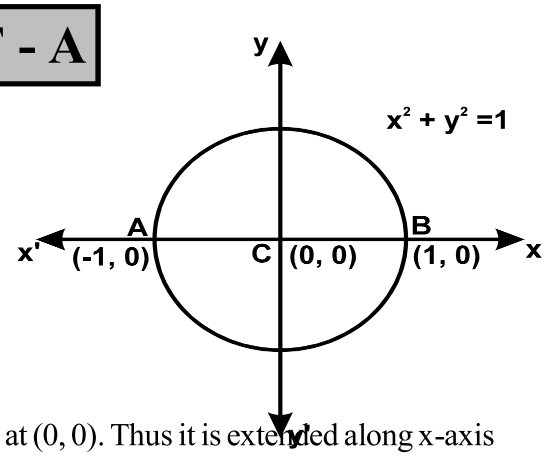
$= \int_{-\infty}^{\infty} y dx$

$= \int_{-\infty}^{\infty} \sqrt{(1-x)^2}\, dx \quad [\because x^2 + y^2 = 1]$

$= \int_{-\infty}^{-1} \sqrt{(1-x^2)}dx + \int_{-1}^{1} \sqrt{(1-x^2)}dx + \int_{1}^{\infty} \sqrt{(1-x^2)}dx$

$= \int_{-\infty}^{-1} \sqrt{(1-x^2)}dx$

[Radius of the given circle is 1 with centre



at (0, 0). Thus it is extended along x-axis such that $-1 \le x \le 1$]

$= 2\int_0^1 \sqrt{1-x^2}\, dx = 2\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x\right]_0^1$

$= 2 [\, 1/2 \sin^{-1} 1\,] = \sin^{-1} 1 = \pi/2$

3. Let the roots be α, β, γ

Then $\alpha + \beta + \gamma = 0$

$\alpha\beta + \beta\gamma + \gamma\alpha = -3$

$\alpha\beta\gamma = -2$

But, as two roots are equal, let $\alpha = \beta$. Then equations take the form

$2\alpha + \gamma = 0$ --------(i)

$\alpha^2 + 2\alpha\gamma = -3$ -------(ii)

$\alpha^2\gamma = -2$ --------(iii)

From equation (i) and (ii), we get

$\alpha^2 . (-2\alpha) = -2$

or $\alpha^3 = 1$

$\Rightarrow$ $\alpha = 1$

Pitting value of a in (i), we get

$\gamma = -2$

Thus, the roots are 1, 1 and -2.

4. $n = 4 \times {}^{16}p_3$

$\therefore$ $\frac{n}{100} = \frac{1}{25}{}^{16}p_3$

5. Here, $6^6 = (3 \times 2)^6 = 3^6 \times 2^6$

$8^8 = (2^3)^8 = 2^{24}$

Let, $K = 3^x . 2^y$

3^6 $3^6 \times 2^6$, 2^{24}, 3^x, 2^y

2^6 2^6 , 2^{24}, $3^{x-6} . 2^y$

1 , 2^{18}, $3^{x-6} . 2^{y-6}$

$\therefore$ Least common factor

$= 3^6 \times 3^6 \times 2^{18} \times 3^{x-6} \times 2^{y-6}$

$= 3^x . 2^{y+18}$

But the given least common factor is

$12^{12} = 3^{12} . 2^{24}$

Equation, we get $3^x . 2^{y+18} = 3^{12} . 2^{24}$

$\Rightarrow$ $x = 12$ and $y = 6$

$\therefore$ $K = 3^{12} . 2^6$.

6. Arranged form of the equation is

$(1 - \lambda)x + 2y + 3z = 0$

$(3 - \lambda)x + y + 2z = 0$

$(2 - \lambda)x + 3y + z = 0$

Above equations shows thar $D_1 = D_2 = D_3 = 0$. For having non-zero solution D must be equal to zero.

$$\text{Thus, } D = \begin{vmatrix} 1-\lambda & 2 & 3 \\ 3-\lambda & 1 & 2 \\ 2-\lambda & 3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (1-\lambda)\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} - 2\begin{vmatrix} 3-\lambda & 2 \\ 2-\lambda & 1 \end{vmatrix} + 3\begin{vmatrix} 3-\lambda & 1 \\ 2-\lambda & 3 \end{vmatrix} = 0$$

$\Rightarrow (1 - \lambda)(1 - 6) - 2(3 - \lambda - 4 + 2\lambda) + 3(9 - 3\lambda - 2 + \lambda) = 0$

$\Rightarrow -5(1 - \lambda) - 2(\lambda - 1) + 3(7 - 2\lambda) = 0$

$\Rightarrow -5 + 5\lambda - 2\lambda + 2 + 21 - 6\lambda = 0$

$\Rightarrow -3\lambda + 18 = 0$

$\Rightarrow 3\lambda = 18$

$\Rightarrow \lambda = 6$

Thus required value of λ is 6.

7. $y_{k+2} = \frac{y_{k+1}^3}{y_k^2}$

$y_3 = \frac{y_2^3}{y_1^2} = \frac{1}{4}$

$y_4 = \frac{y_3^3}{y_2^2} = \left(\frac{1}{4}\right)^3$ and so on.

8. From the data of the problem

$\lambda = 6$ customers/hour

service time per phase = 15 minutes

$\mu = 4.5$ (1.5 x 3) customer/ minute or 13.34/h; k = 3

Average time a customer spends waiting in the cafeteria

$$Wq = \frac{k+1}{2k}.\frac{\lambda}{\mu(\mu-\lambda)}$$

$$= \frac{3+1}{2 \times 3}.\frac{6}{13.34(13.34-6)}$$

$$= \frac{9}{220} \text{ hour or 2.43 minutes.}$$

9. $1 + \text{Cos}\theta + i\,\text{Sin}\theta = (\text{Cos}^2\theta + \text{Sin}^2\theta + \text{Cos}\theta + i\,\text{Sin}\theta)$

$= (\text{Cos}^2\theta - i^2\,\text{Sin}^2\theta) + (\text{Cos}\theta + i\,\text{Sin}\theta)$

$= (\text{Cos}\theta + i\,\text{Sin}\theta)(\text{Cos}\theta - i\,\text{Sin}\theta) + (\text{Cos}\theta + i\,\text{Sin}\theta)$

$= (\text{Cos}\theta + i\,\text{Sin}\theta)(\text{Cos}\theta - i\,\text{Sin}\theta + 1)$

$$\therefore \left(\frac{1+\cos\theta+i\sin\theta}{1+\cos\theta-i\sin\theta}\right)^n$$

$$= \left(\frac{(\cos\theta+i\sin\theta)(1+\cos\theta-i\sin\theta)}{1+\cos\theta-i\sin\theta}\right)^2$$

$= (\cos\theta + i\sin\theta)^n$ $\therefore \theta \neq (2k+1)\pi$

If $\theta = (2k+1)\pi$, then

$= \cos n\theta + i \sin n\theta$ $1 + \cos\theta - i\sin\theta = 0$

(By De Moivre's theorem)

Thus, modulus is 1 and argument = $n\theta$.

10. According to the theorem, if $a_1, a_2, \ldots a_n$ be n positive number and $p_1, p_2, \ldots p_n$ be n positive rationals, then

$$\frac{p_1a_1 + p_2a_2 + \ldots\ldots\ldots + p_na_n}{p_1 + p_2 + \ldots\ldots + p_n}$$

$$\geq \left(a_1^{p_1}.a_2^{p_2}\ldots\ldots\ldots a_n^{p_n}\right)^{1/(p_1+p_2+p_n)}$$

Here let $p_1 = 3$, $p_2 = 4$, $a_1 = 2x/3$ and $b_2 = 5y/4$

Then, $$\frac{3.\frac{2x}{3}+4.\frac{5y}{4}}{3+4} \geq \left[\left(\frac{2x}{3}\right)^3.\left(\frac{5y}{4}\right)^4\right]^{\frac{1}{3+4}}$$

$$\Rightarrow \frac{2x+5y}{7} \geq \left[\left(\frac{2x}{3}\right)^3.\left(\frac{5y}{4}\right)^4\right]^{\frac{1}{7}}$$

$$\Rightarrow \left(\frac{3}{7}\right)^7 \geq x^3.y^4.\frac{2^3}{3^3}.\frac{5^4}{2^8}$$

$$\Rightarrow x^3.y^4 \leq \frac{3^7}{7^7}.\frac{3^3}{5^4}.2^5$$

$$\Rightarrow x^3.y^4 \leq \frac{3^{10}.2^5}{7^7.5^4}$$

Thus, the maximum value of $x^3.y^4$ is $3^{10}.2^5/7^7.5^4$

11. Let $$u = \lim_{n\to\infty} \frac{[(n+1)(n+2)\ldots\ldots(2n)]^{1/n}}{n}$$

$$= \lim_{n\to\infty}\left[\frac{(n+1)(n+2)\ldots\ldots\ldots(2n)}{n^n}\right]^{1/n}$$

$$\lim_{n\to\infty}\left[\frac{(n+1)}{n}.\frac{(n+2)}{n}\ldots\ldots\left(\frac{n+r}{n}\right)\ldots\ldots\left(\frac{n+n}{n}\right)\right]^{1/n}$$

Taking log both the sides, we get

$$\log u = \log_{n\to\infty}\frac{1}{n}\left[\log_e\left(1+\frac{1}{n}\right)+\log_e\left(1+\frac{2}{n}\right)+\ldots + \log_e\left(1+\frac{r}{n}\right)+\ldots\right]$$

$$= \int_0^1 \log_e(1+x)dx$$

$$= \left[x\log_e(1+x) - \int\frac{xdx}{1+x}\right]_0^1$$

$$= \left[x\log_e(1+x) - \int dx + \int\frac{xdx}{1+x}\right]_0^1$$

$$= \left[x\log_e(1+x) - x + \log_e(1+x)\right]_0^1$$

$$= \left[(1+x)\log_e(1+x) - x\right]_0^1$$

$= 2 \ln 2 - 1$

$$\therefore u = e^{\ln 4 - 1} = \frac{e^{\ln 4}}{e} = 4/e$$

12. Parametric form of the ellipse is $x = a\cos\theta$, $y = b\sin\theta$

Dividing the area into stripes of breadth δx parallel to the axis of y, we have,

$$\bar{x} = \frac{\int_0^a xydx}{\int_0^a ydy} = \frac{\int_0^{\pi/2}(a\cos\theta)(b\sin\theta)(-a\sin\theta d\theta)}{\int_0^{\pi/2}(b\sin\theta)(-a\sin\theta d\theta)}$$

$$= a\frac{\int_0^{\pi/2}\sin^2\theta.\cos\theta.d\theta}{\int_0^{\pi/2}\sin^2\theta d\theta}$$

$= a. \frac{1/3}{\pi/4} = \frac{4a}{3\pi}$

As x/a and y/b are symmetrically placed in the equation of the curve,therefore, $\bar{y}$ =4b/3p.

Hence, centre of gravity $(\bar{x}, \bar{y}) = \left(\frac{4a}{3\pi}, \frac{4b}{3\pi}\right)$

13. Let $Sin^{-1}x = z$ Also when x=0, $z=Sin^{-1}0=0$

$\Rightarrow x = Sinz \quad x = 1/\sqrt{2}, z = Sin^{-1}1/\sqrt{2} = \pi/4$

$$\int_0^{1/\sqrt{2}} \frac{\sin^{-1} x.dx}{(1-x^2)^{3/2}} = \int_0^{\pi/4} \frac{z\cos zdz}{(1-\sin^2 z)^{3/2}} = \int_0^{\frac{\pi}{4}} \frac{zCosz}{Cos^3 z}dz$$

$$= \int_0^{\pi/4} z\sec^2 z.dz$$

$$= \left[z.\int Sec^2 zdz - \int\left\{\int Sec^2 zdz\right\}\frac{d}{dz}(z).dz\right]_0^{\pi/4}$$

$$= \left[z\tan z - \int \tan zdz\right]_0^{\pi/4} = \left[z\tan z + \log\cos z\right]_0^{\pi/4}$$

$= \pi/4. \tan \pi/4 + \log Cos\, \pi/4 = \pi/4 + \log 1/\sqrt{2}$

$= \pi/4 - 1/2\log 2.$

14. Let $\eta = K\, m^a d^b\, \nu^c$

Where η= viscosity of gas

m= mass of the molecule

d = diameter of the molecule

ν = mean speed of the molecule

K= constant

Here K is dimensionless constants of pro--portionality and a, b, c are the powers of m, d and ν respectively to represent η.

Writing the dimensions of various quantities in (i),we get $[ML^{-1}T^{-1}] = [M]^{a.} [L]^b [LT^{-1}]^c$

$= M^a L^{b+c} T^{-c}$

Applying the principal of homogeneity of dimensions, we get

a = 1

b + c = -1 gives a = 1, b = -2, c = 1.

c = 1

Putting in equation (i), we get $\eta = K\, md^{-2}\nu$

15.

```
# include < io stream. h >
# include < conio . h >
# include < process .h >
# include < string . h >
# include < f stream . h >
void sort (char[]);
void main (int arge, char*arg v[])
{clrsc r ( ) ;
f stream ∏
it . (arge ! = 2)
{ cout << "In valid arguments";
exit (0);
   }
fp . open (argv [1], i as :: in );
char ch [r0];
cout << " unsorted list in is \t";
While (fp)
{ fp. getline (ch, r0);
cout << ch ;
if (strlen (ch) > 0)
sort (ch) ;
}
  fp . close ( );
}
void sort (char chl [])
{ cout << "sorted list is";
for (int i = 0; chl [i] ! = \0;i+f)
for (int j = 0; chl [y] ! = \0; f+t)
if (chl [i] < chl [y])
{ char temp = chl [i];
chl [i] = chl (y);
chl [y] = chl [i]
}
cout << chl;
```

output

unsorted list is dbace

sorted list is abcde.

16. $(a, b) = x\, v_1 + x\, v_2$

$= x(Cos\theta , Sin\theta) + y(-Sin\theta, Cos\theta)$

$= (x\, Cos\theta - x\, Sin\theta, x\, Sin\theta + y\, Cos\theta)$

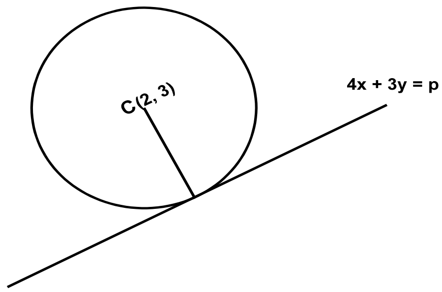
Where for different values of x and y, we get different points.

17. Equation of the circle is

$x^2 + y^2 - 4x - 6y + 4\sqrt{3} = 0$

or, $(x^2 - 4x + 4) + (y^2 - 6y + 9) = 13 - 4\sqrt{3}$

or, $(x - 2)^2 + (y - 3)^2 = 13 - 4\sqrt{3}$



Thus, centre = C (2, 3), Radius = $\sqrt{13-4\sqrt{3}}$

As the line $4x + 3y = p$ is a tangent to the circle $x^2 + y^2 - 4x - 6y + 4\sqrt{3} = 0$.
then the perpendicular distance from the center (2, 3) of the circle to the line must be equal to the radius of the circle.
Now perpendicular distance from (2, 3) to $4x + 3y - p = 0$ is

$$\left|\frac{4.2+3.3-p}{\sqrt{4^2+3^2}}\right| = \left|\frac{17-p}{5}\right|$$

$$\therefore \quad \left|\frac{17-p}{5}\right| = \sqrt{13-4\sqrt{3}}$$

or, $\quad 17 - p = \pm\, 5\sqrt{13-4\sqrt{3}}$

or, $\quad p = 17 \pm \sqrt{13-4\sqrt{3}}$

18. Let the radius of the coin be R cm

Mass of the disc, M = 10gm

Liner velocity of the disc, $v = 6\ cms^{-1}$

Also, moment of inertia of disc about an axis through its centre, $I = 1/2\ MR^2$

Here,kinetic energy of translation = $1/2Mv^2$

and kinetic energy of rotation = $1/2\ I\omega^2$

$$= \frac{1}{2}.\left(\frac{1}{2}MR^2\right).\left(\frac{v}{R}\right)^2 = \frac{1}{4}Mv^2$$

Thus total kinetic energy = kinetic energy of translation + kinetic energy of rotational.

$= 1/2\ Mv^2 + 1/4\ Mv^2$

$= 3/4\ Mv^2$

$= 3/4 \times 10 \times 6 \times 6$ erg = 270 erg

19. $\int_0^{11}[x]^3 dx = \int_0^1 [x]^3 dx + \int_1^2 [x]^3 dx$

$+\int_2^3 [x]^3 dx = \int_3^4 [x]^3 dx + ...\int_{10}^{11} [x]^3 dx$

$= 0\ .\int_0^1 dx + 1^3.\int_1^2 dx + 2^3.\int_2^3 dx........10^3.\int_{10}^{11} dx$

$= 1^3\ .\ (2 - 1) + 2^3\ .(3 - 2) + 3^3.\ (4 - 3) +$ ....$+\ 10^3\ .\ (11 - 10)$

$= 1^3 + 2^3 + 3^3 + ...+ 10^3$

$$= \left[\frac{10(10+1)}{2}\right]^2 = 55^2 = 3025$$

20. Let a $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore \vec{a}.\vec{c} = (\hat{i}+\hat{j}+\hat{k}).(x\hat{i}+y\hat{j}+z\hat{k}) = 2$

$\Rightarrow x + y + z = 2$ ....(i)

Also $\vec{a}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{i}\begin{vmatrix} 1 & 1 \\ y & z \end{vmatrix}$

$-\hat{j}\begin{vmatrix} 1 & 1 \\ x & z \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 1 \\ x & y \end{vmatrix}$

$= \hat{i}(z-y) - \hat{j}(z-x) + \hat{k}(y-x)$

But given that $\vec{a}\times\vec{c} = \hat{b}$

$\Rightarrow (z-y)\hat{i} - \hat{j}(z-x) + (y-x)\hat{k} = \hat{i} - 2\hat{j} + \hat{k}$

$\Rightarrow z - y = 1$ ....(ii)

$\Rightarrow z - x = 2$ ....(iii)

$\Rightarrow y - x = 1$ ....(iv)

Replacing value of x from equation (iv) in (i) we get

$y - 1 + y + z = 2$

or, $zy + z = 3$ ......(v)

From equation (ii) and (v)

$$2y + z = 3$$
$$-y + z = 1$$

$$3y = 2$$

or, $y = 2/3$

Putting the value of y in equation (ii) and (iii), we get $x = -1/3$, $z = 5/3$

Thus, putting the respective values of x, y, z in vector, we get

$$\vec{c} = \frac{-\hat{i} + 2\hat{j} + 5\hat{k}}{3}$$

21. Equation of the line $y = mx + K$

Here, $m\sum x_i^2 + k\sum x_i = \sum x_i y_i$ ....(i)

and $m\sum x_i + nk = \sum y_i$ ....(ii)

$i = 1, 2, \ldots\ldots\ldots\ldots n$

Condition for minimum sum of square is

$E = \sum[y_i - (mx_i + k)]^2$ ....(iii)

Putting respectives values in (ii), we get

$$m\bar{x} + K = \bar{y}$$

or $m \times 0 + K = 0$

or $K = 0$

Thus, equation (i) takes the form

$$m\sum_{i-1}^{50} x_i = \sum_{i-1}^{50} x_i y_i$$

or $m \times 10 = 8$

or $m = 4/5$

$$\therefore E = \sum_{i=1}^{50}\left(y_i - \frac{4}{5}x_i\right)^2$$

$$= \sum_{i=1}^{50} y_i^2 - \frac{8}{5}\sum_{i=1}^{50} x_i y_i + \frac{16}{25}\sum x_i^2$$

$= 15 - 8/5 \times 8 + 16/25 \times 10 = 43/5 = 8.6$

22. According to logarithmic law

$\log_a b = x$

$\Rightarrow b = a^x$

Thus, $\log_2 (\log_3 (\log_5(\log_7))) = 11$

$\Rightarrow \log_3 (\log_5(\log_7 N)) = 2^{11}$

$\Rightarrow \log_5 (\log_7 N) = 3^{(2^{11})}$

$\Rightarrow \log 7\ N = 5^{(3^{2^{11}})}$

$\Rightarrow N = 7^{5^{3^{2^{11}}}}$

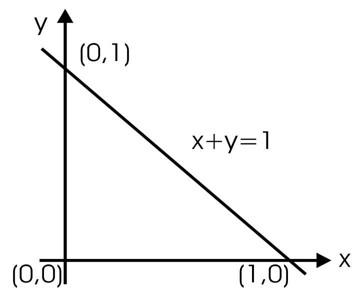
Thus, N is $5^{3^{2^{11}}}$ times multiple of 7 only.

Prime number 7 is the only factor of N.

23. Total number of vertices of hexagon is 6. For single diameter of circle we use 2 ver--tices. Thus, number of circles in the plane of H which have a diameter both of whose and points are vertices of H is C_2^6.

$$\therefore \frac{6!}{4!2!} = \frac{6.5}{2.1} = 15 \text{ circles.}$$

24.



$$\iint_A xy\,dx\,dy$$

$$= \int_0^1\left[\int_0^{1-x} xy\,dy\right]dx$$

$$= \int_0^1\left[\frac{xy^2}{2}\right]_0^{1-x} dx = \int_0^1 \frac{x(1-x)^2}{2}\,dx$$

$$= \frac{1}{2}\int_0^1 x(1-2x+x^2)\,dx = \frac{1}{2}\int_0^1 (x-2x^2+x^3)\,dx$$

$$= \frac{1}{2}\left[\frac{x^2}{2} - \frac{2x^3}{3} + \frac{x^4}{4}\right]_0^1 = \frac{1}{2}\left[\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right]$$

$$= \frac{1}{2} \times \frac{1}{12} = \frac{1}{24}$$

25. Let (a, b)

Where $(a, b)^{-1}$ is the multiplicative inverse of (a, b)

Thus, $(a, b) . (a, b)^{-1} = (a, b) . (x, y)$

$= (ax + 5by, ay + by)$ ....(i)

$[\because (a, b) . (c, d) = (ac + 5bd, ad + bd)]$

1. $f(x) . f(y) - \frac{1}{2}\left[f\left(\frac{x}{y}\right) + f(xy)\right]$

$= Cos (lnx) . Cos (ln y)$

$-\frac{1}{2}\left[Cos\left(ln\frac{x}{y}\right) + Cos(ln\, xy)\right]$

$= Cos (ln x) . Cos (ln y)$

$-\frac{1}{2}\left[Cos(ln\, x - ln\, y) + Cos(ln\, x + ln\, y)\right]$

$= Cos(ln\, x).Cos(ln\, y) - 1/2[2.Cos(ln\, x).Cos(ln\, y)]$

$= 0$

2. (c) Von-Neumann
3. (b) 5
4. (d) latency plus seek time plus transmission time.
5.

When Y covers x distance

X covers 2x dixtance

Suppose they meet together when y has covered already a distance x .

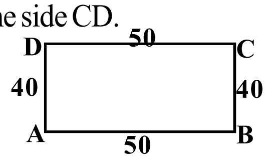
So, $2x + x = 2 (40 + 50)$

or, $3x = 180$

$x = 60$ m

Thus, X covered a distance 120 m

Thus, X and Y cross at the side CD.



6. $xdx - dy = 0$

or, $xdx = dy$

Integrating, we get

But, $(a, b) . (a, b)^{-1} = (1, 0)$. ....(ii)

[$\because$ They are inverse of each other]

From (i) and (ii),

$(ax + 5by, ay + by) = (1, 0)$

$\therefore\ ax + 5by = 1$ ....(iii)

and $ay + by = 0$ ....(iv)

$[\because (a, b) = (c, d)$ if $a = c, b = d)$

Equation (iv) gives

$y (a + b) = 0$

$a + b \neq 0$ as (a,b) is not the identity element for additionm i.e., $(a, b) \neq (0,0)$

Thus, $y = 0$ ....(v)

Putting value of y from (v) in (iii), we get,

$ax = 1 \Rightarrow a^{-1} (ax) = a^{-1}.1$

$\Rightarrow (a^{-1}a)x = a^{-1}$

$\Rightarrow 1. x = a^{-1}$ $(\because a^{-1}a = 1)$

$\Rightarrow x = a^{-1}$

Thus, $(a, b)^{-1} = (x, y)$

$= (a^{-1}, 0)$

PART - B

SECTION - I

$$\int xdx = \int dy$$

$\Rightarrow$ $x^2/2 = y + c$

$\Rightarrow$ $x^2 = 2 (y + c)$

It implies a parabola with vertex at point (0, -c).

7. n(A or B) = n(A) + n(B) - n(A and B)

$= 300 + 132 - 20 = 412$

8. $K . 2^{1995} = 2^{1998} - 2^{1997} - 2^{1996} + 2^{1995}$

$\Rightarrow K . 2^{1995} = 2^{1995} (2^3 - 2^2 - 2 + 1)$

$\Rightarrow K .2^{1995} = 2^{1995} . (8 - 4 - 2 + 1)$

$\Rightarrow K .2^{1995} = 2^{1995} . 3$

$\Rightarrow K = 3.$

9. (a) 25
10. (d) None of these
11. Let, $f(x) = x^n - nax - b$.........................(i)

If α_1, α_2,..................α_n are the roots of equation

xn - nax -b = 0

Then, $f(x) = (x - \alpha_1) (x - \alpha_2)$ ....$(x - \alpha_n)$

$\therefore f'(x) = (x - \alpha_2) (x - \alpha_3)$ ....$(x - \alpha_n) +$

$(x - \alpha_1)(x - \alpha_3)\ldots\ldots\ldots(x - \alpha_n) + \ldots\ldots\ldots\ldots + (x - \alpha_1)$
$(x - \alpha_2)\ldots\ldots\ldots\ldots\ldots\ldots(x - \alpha_{n-1})$

Thus, $f'(\alpha_1) = (\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)\ldots\ldots\ldots\ldots(\alpha_1 - \alpha_n)$
$= A$ (given) ....(ii)

Differentiating equation (i), we get
$f(x) = nx^{n-1} - na$
$(\alpha_1) = -n\alpha_1^{n-1} - na$...............................(iii)

From equation (ii) and (iii), we get
$A = n\alpha_1^{n-1} - na$
or $A - n\alpha_1^{n-1} = -na$, is the required solution.

12. $$\frac{1+2i}{1-(1-i)^2} = \frac{1+2i}{1-(1-2i+i^2)}$$

$$= \frac{1+2i}{1+2i} = 1$$

Let , $1 = r(\text{Cos}\theta + i\,\text{Sin}\theta)$
This gives
$r\,\text{Cos}\theta = 1, r\,\text{Sin} = 0$
$\therefore r2 = 1 \Rightarrow r = 1$
$\therefore \text{Cos}\theta = 1, \text{Sin}\theta = 0$
These determine $\theta = 2n\pi$, where $n \in I$
But principal argument is defined to be the unique angle θ which satisfy the inequality $-\pi < \theta < \pi$.
Thus, in this case $\theta = 0$ for $n = 0$.
Also modulus, $r = 1$.
Ans. = 1, 0 .

13. Let (x, y, z) be the corner of the rectangular box in the first quadrant. Then the lenghts of three mutually perpendicular edges are 2x, 2ym 2z.
If V is the volume, $V = 8\,xyz$ ....(i)
$\log V = \log 8 + \log x + \log y + \log z$

$$\text{or, } \frac{1}{V}dv = 0 + \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

For maximum and minimum values of V, $dv = 0$

$$\therefore \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0 \text{ ....(ii)}$$

Equation of the ellipaoid is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \text{ ....(iii)}$$

Differentiating, we get

$$\frac{2xdx}{a^2} + \frac{2ydy}{b^2} + \frac{2zdz}{c^2} = 0$$

$$\text{or, } \frac{xdx}{a^2} + \frac{ydy}{b^2} + \frac{zdz}{c^2} = 0 \text{ ....(iv)}$$

From (ii) + l x (iv), we get

$$\left(\frac{1}{x} + \frac{\lambda x}{a^2}\right)dx + \left(\frac{1}{y} + \frac{\lambda y}{b^2}\right)dy + \left(\frac{1}{z} + \frac{\lambda z}{c^2}\right)dz = 0$$

Using Lagrange's method of undeterminded mul tiplies, and equating the coefficints of dx, dy, dz to zero, we get

$$\frac{1}{x} + \frac{\lambda x}{a^2} = 0 \text{ ....(v)}$$

$$\frac{1}{y} + \frac{\lambda y}{b^2} = 0 \text{ ....(vi)}$$

$$\frac{1}{z} + \frac{\lambda z}{c^2} = 0 \text{ ....(vii)}$$

From $x \times$ (iv) + $y \times$(iv) + $z \times$ (vii), we get

$$3 + \lambda\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = 0$$

or $3 + \lambda = 0$ using (iii)
$\therefore \quad \lambda = -3$
$\therefore \quad$ Fro m (v), (vi), (vii), We get

$$\frac{1}{x} - \frac{3x}{a^2} = 0 : \text{or}, 3x^2 = a^2 \therefore x = \frac{a}{\sqrt{3}}$$

$$\frac{1}{y} - \frac{3y}{b^2} = 0 : \text{or}, 3y^2 = b^2 \therefore y = \frac{b}{\sqrt{3}}$$

$$\text{and } \frac{1}{z} - \frac{3z}{c^2} = 0 : \text{or}, 3z^2 = c^2 \therefore z = \frac{z}{\sqrt{3}}$$

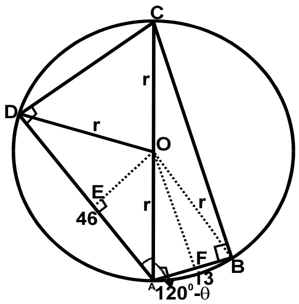
$\therefore$ The volume of the rectangular box is the maxi -mum of minimum if the corner in the first quadrant is $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$

As the question of minimum does not aries ($\therefore$ the minimum volume of the rectangular box is zero)

$\therefore$ The maximum volume = 8 xyz

$$= 8.\frac{a}{\sqrt{3}}.\frac{b}{\sqrt{3}}.\frac{c}{\sqrt{3}} = \frac{8}{3\sqrt{3}}abc.$$

14. Δ AOD and Δ AOB are isosceles triangles as OA = OB = OD are the radius of the circles.



As OE and OF are altitudes of isosceles tringle AOD and AOB bisect the sides AD and AB.
Thus, AE = 23 and AF = 13/2
New let $\angle OAE = q$ $\therefore$ $\angle OAF = 120^0 - \theta$
In the right tringle OEA
$\cos\theta = AE/OA = 23/r$..............(i)
In the right tringle OAF
$\cos(120^0 - \theta)$ AF / OA = 13/2r = ...(ii)
But
$\cos(120^0 - \theta) = \cos 120^0.\cos\theta + \sin 120^0.\sin\theta$
$= \cos(180^0 - 60^0).\cos\theta + \sin(180^0 - 60^0)\sin\theta$
$= -\cos 60^0.\cos\theta + \sin 60^0.\sin\theta$............(iii)
From (i), (ii) and (iii), we get
$\sin 60^0.\sin\theta - \cos 60^0.\cos\theta = 13/2r$

or $$\frac{\sqrt{3}}{2}.\frac{\sqrt{r^2-23^2}}{r} - \frac{1}{2}.\frac{23}{r} = \frac{13}{2r}$$

or $$\frac{\sqrt{3}\sqrt{(r^2-529)}}{2r} = \frac{36}{2r}$$

$$\sqrt{r^2-529} = \frac{36}{\sqrt{3}} = 12\sqrt{3}$$

or $r^2 - 529 = 432$
or $r^2 = 961$
$\therefore$ $r = \sqrt{961} = 31$
But AC = 2r (Diameter of the circle)
$\therefore$ AC = 2 × 31 = 62.

15. Neglecting the cubes and higher powers of x_i/M, x_i being the deviation of x_i from M and letting the means and s.d's of x_1/x_3 and x_2/x_3 to be I_1, I_2 and S_1, S_2 respectively, we get

$$I_1 = \frac{1}{N}\sum\frac{X_1}{X_3} = \frac{1}{N}\sum_i (x_{1i}+M)(x_{3i}+M)^{-1}$$

$$= \frac{1}{N}\sum\left(1+\frac{x_{1i}}{M}\right)\left(1+\frac{x_{3i}}{M}\right)^{-1}$$

$$= \frac{1}{N}\sum\left[\left(1-\frac{x_{3i}}{M}+\frac{x_{3i}^2}{M^2}..\right)+\frac{x_{1i}}{M}-\frac{x_{1i}x_{3i}}{M^2}+..\right]$$

$$= 1+\frac{v_3^2}{M^2}$$

Similarly $I_2 = 1+\dfrac{v_3^2}{M^2}$

$I_1 = I_2$

or $S_1^2 + I_1^2 = 1 + \dfrac{3v_3^2}{M^2} + \dfrac{v_1^2}{M^2}$ and so we

have $S_1^2 = \dfrac{v_3^2}{M^2} + \dfrac{v_1^2}{M^2}$

Similarly $S_2^2 = \dfrac{v_3^2}{M^2} + \dfrac{v_2^2}{M^2}$ v

Now $N_\delta S_1 S_2 = \sum\left(\dfrac{X_1}{X_2} - I_1\right)\left(\dfrac{X_2}{X_3} - I_2\right)$

$= \dfrac{v_3^2}{M^2}$ (on simplication)

Hence, $\delta = \dfrac{N_\delta S_1 S_2}{S_1 S_2} = \dfrac{v_3^2}{\sqrt{(v_3^2+v_3^2)(v_3^2+v_2^2)}}$

Where δ is the requried correlation coefficient.

16. 100! contains 2 as a factor

$$\left[\frac{100}{2}\right]+\left[\frac{100}{2^2}\right]+\left[\frac{100}{2^3}\right]+\left[\frac{100}{2^4}\right]+$$

$\left[\frac{100}{2^5}\right]+\left[\frac{100}{2^6}\right]$ times or, the highest power of 2 in 100! is 50 + 25 + 12 + 6 + 3 + 1 = 97

Similarly, the highest power of 5 in 100! is

$$\left[\frac{100}{5}\right]+\left[\frac{100}{5^2}\right]+\left[\frac{100}{5^3}\right]=20+4+0=24$$

Now, each pair of 5 and 2 will give rise to a 10 or a zero at the end.

Hence, the number of zero in 100! is equal to 24.

17. We Know

$$\log(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots\ldots$$

But, log (1+x) is defined only when

$1+x > 0$

or, $x > -1$

But, when $x > 1$, clearly the function is deversing Thus, when $-1 < x < 1$ the function is converging .

18. Point of inflection is that point where it is nei ther maximum nor minimum.

To get the point of inflection we have to solve $d^2y/dx^2 = 0$

But, $y=e^{-\left(\frac{x}{a}\right)^2}$ ....(i)

$\Rightarrow$ $\log y = -x^2/a^2$

$\Rightarrow$ $1/y.dy/dx = -2x/a^2$

$\Rightarrow$ $dy/dx = -y.\ 2x/a^2$

$$\Rightarrow \frac{d^2y}{dx^2}=-\frac{2}{a^2}\left[y+x.\frac{dy}{dx}\right]$$

$$\Rightarrow \quad =-\frac{2}{a^2}\left[y-\frac{2}{a^2}y.x^2\right]$$

But $\frac{d^2y}{dx^2}=0$

$$\therefore \quad -\frac{2}{a^2}\left[y-\frac{2}{a^2}y.x^2\right]=0$$

$$\Rightarrow \quad \left[1-\frac{2x^2}{a^2}\right]=0$$

$$\Rightarrow \quad 1-\frac{2x^2}{a^2}=0$$

$\Rightarrow$ $x^2 = a^2/2$

$\Rightarrow$ $x^2/a^2 = 1/2$ ....(ii)

Putting (ii) in (i), we get

$y = e^{-1/2} = 1/\sqrt{e}$

Thus, common coordinate of inflection point is $1/\sqrt{e}$.

19. The function interchange the values of two variables.

20. $0.5625 \times 2 = 1.125$

$0.1250 \times 2 = 0.25$

$0.25 \times 2 = 0.50$

$0.50 \times 2 = 1.00$

$\therefore$ $(0.5625)_{10} = 0.1001_2$

21. $x_i + x_{i+1} = K$

We put, $i = 9$

So, $x_9 + x_{10} = K$

or $x_9 + 1 = K$

or $x_9 = K - 1$

We put, $i = 8$

So, $x_8 + x_9 = K$

or $x_8 + (K - 1) = K$

or $x_8 = K - (K - 1)$

or $x_8 = 1$

Similarly, $x_7 = K-1$, $x_6 = 1$, $x_5 = K-1$, $x_4 = 1$

$x_3 = K - 1$, $x_2 = 1$

and $x_1 = K - 1$

Thus, $x_1 = K - 1$

22. Relative speed = $(V_1 + V_2)$ miles

Total length covered = 2L

Requried time to cross each other

= Total Length/Relative speed

$$\therefore \quad T=\frac{2L}{V_1+V_2}$$

or $L = 1/2.\ (V_1 + V_2)$.T.miles/hour. second

$= 1/2.\ (V_1 + V_2).\ T.\ 5280/(60 \times 60)$ feet

$= 11/15\ (V_1 + V_2)$. T feet

23. $(x + 2y)^2 + (x + 9) = 710$

or, $x^2 + 4xy + 4y^2 + x + 4y = 710$

or, $x^2 + (4y+1)x + (4y^2 + 4y - 710) = 0$

$$x = \frac{-(4y+1) \pm \sqrt{(4y+1)^2 - 4(4y^2+4y-710)}}{2}$$

or $x = -(4y+1) \pm \sqrt{16y^2+8y+1-16y^2-16y+2840}$

or $2x = -(4y+1) \pm \sqrt{2841-8y}$

If x is to be interer then the term inside the square root, i.e., 2841 - 8y must be a perfect square for some integer y.

By trail method see that for $y = 4$, $\sqrt{2841-8y}$ is a perfect square

Thus, $2x = -(4 \times 4 + 1) \pm \sqrt{2841 - 8 \times 4}$

$= -17 \pm \sqrt{2809}$

$= -17 \pm 53$

$= 36$ or -70

$\therefore \quad x = 18$ or -35

24. (A + B) * (C - D) = AB + CD - *.

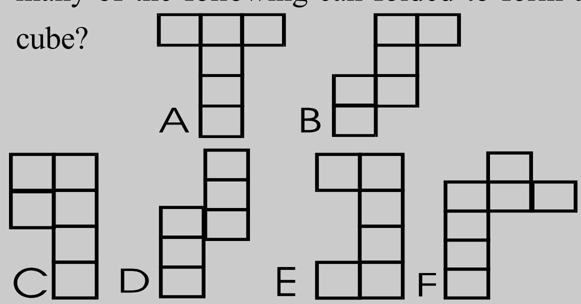
25. The point at which his height is equal to or greater than the depth of the river is useful for him.

PUZZLES

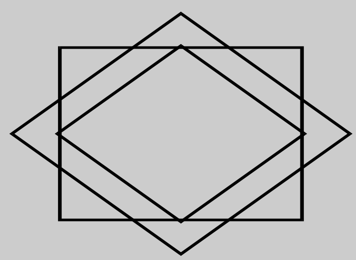
[CHECK YOUR CALIBER]

(1) There is a number, the second digit of which is smaller than its first digit by 4, and if the number was divided by the digit's sum , the quotient would be 7. Can you find the number?

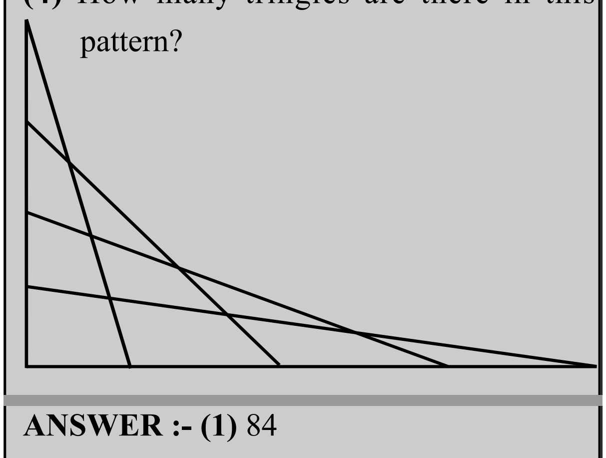
(2) Can you tell, just by studying them, how many of the following can folded to form a cube?



(3) Draw this figure in one continuous line without anywhere crossing a line or going over any part of a line twice.



(4) How many tringles are there in this pattern?



ANSWER :- (1) 84

(2) A,B & D canbe folded to form a cube. **(4)** 20