JNU M.C.A. ENTRANCE EXAMINATION - 2001
COMPUTER SCIENCE

(2) A polygon has 10 diagonals. Then it has
(a) 15 sides (b) 10 sides
(c) 18 sides (d) 25 sides
(a)(JNU-2001)

(3) Computing $\sqrt{1}+\sqrt{2}+......+\sqrt{100}$ with all the roots correct to two decimal places, the maximum possible error in the result is
(a) 0.1 (b) 0.01
(c) 0.5 (d) 5.0 (JNU-2001)

(4) On simplification $(5^2+1)(5^{2^2}+1).....(5^{2^n}+1)$ yields
(a) $5^{2^{n+1}}$ (b) $5^{2^{n+1}}-1$
(c) $(5^{2^{n+1}}-1)/12$ (d) none of these
(d)(JNU-2001)

(6) The expressions $A\times(B\cap C)$ is the same as
(a) $(A\times C)\cup(B\times C)$ (b) $(A\times C)\cap(B\times C)$
(c) $(A\times B)\cup(A\times C)$ (d) $(A\times B)\cap(A\times C)$ (d)(JNU-2001)

(8) The number 3 can be written as 3, 2 + 1, 1 + 2 or 1 + 1 + 1 in four ways. In how many ways can the number n be written ?
(a) n ! (b) 2 (n - 1)
(c) $(n - 1)^2$ (d) 2^{n-1} (d)(JNU-2001)

(17) A die is loaded in such a way that each odd number is twice as likely to occur as each even number. If E is the event that a number greater than or equal to 4 occurs on a single toss of the die then P(E) is
(a) 1/2 (b) 2/3
(c) 4/9 (d) 1/3 (c)(JNU-2001)

(18) The three vectors $i+j, j+k, k+i$ taken two at a time form three planes. The three unit vectors drawn perpendicular to these three planes form a parallelopiped of volume
(a) $4/3\sqrt{3}$ (b) 4
(c) $3\sqrt{3}/4$ (d) 1/3 (a)(JNU-2001)

(19) Six boys and six girls sit along a line alternately in x ways and along a circle (again alternately) in y ways ; then
(a) x = y (b) y = 12 x
(c) x = 10 y (d) x = 12 y (d)(JNU-2001)

(20) By using the digits 0, 1, 2, 3, 4 and 5 (repetitions not allowed) any number of digits being used , the number of numbers that can be formed is
(a) 1630 (b) 1030
(c) 1200 (d) 1530 (a)(JNU-2001)

(21) The number of committees consisting of eight or more that can be selected from 15 persons is
(a) 32,678 (b) 32,768
(c) 16,384 (d) 16,834 (c)(JNU-2001)

(22) Let $D_r = \begin{vmatrix} a & 2^r & 2^{16}-1 \\ b & 3(4^r) & 2(4^{16}-1) \\ c & 7(8^r) & 4(8^{16}-1) \end{vmatrix}$ then the value of $\sum_{k=1}^{16} D_k$ is
(a) 0 (b) a + b + c (b)(JNU-2001)
(c) ab + bc + ca (d) none of these

(23) Consider the following C program :

```
# include < stdio.h >
main ( )
{ int x, y ;
void swap  (int*, int);
x = 10; y = 15; printf (“%d %d \ n”, x , y);
swap (& x, y); printf (“%d %d \ n”, x , y);
swap (& x, y);printf (“%d %d \ n”, x , y);
return ;
}
void swap (int *a, int b)
{ int temp ;
temp = *a ;
*a = b ;
b = temp;
printf (“%d %d \ n”, *a, b)
}
```

What would be the output of the above program ?

(a) 10 15
15 10
10 15
15 10
10 15

(b) 10 15
15 10
15 15
15 15
15 15

(c) 10 15
15 10
10 10
10 10
10 10

(d) none of these

(b)(JNU-2001)

(24) Which of the following is an advantage to using fibre optic data transmission ?

(a) Fast data transmission rate
(b) Low noise level
(c) Few transmission errors
(d) All of the above (a) (JNU-2001)

(25) A darea dictionary does not provide infor mation about

(a) where data is located
(b) the size of the disk storage
(c) who owns the data (d)
(d) security and privacy limitations (JNU-2001)

(26) In an absolute loading scheme, the func tion accomplished by an assembler is

(a) reallocation (b) allocation
(c) loading (d) linking

(a)(JNU-2001)

(27) You can read input that consists of multiple lines of text using

(a) the normal cont << combination
(b) the cin. get ( ) function with two argument
(c) the cin.get ( ) function with two arguments
(d) the cin.get ( ) function with three arguments

(JNU-2001)

(28) With round-robin CPU scheduling in a time-shared system

(a) using very large time slices degenerates into FCFS algorithm
(b) using very small time slices degenerates into LIFO algorithm
(c) using extremly small time slices improves performances
(d) none of these (JNU-2001)

(29) The language of the Grammar

$S \to 0A1$

$A \to 00A11$

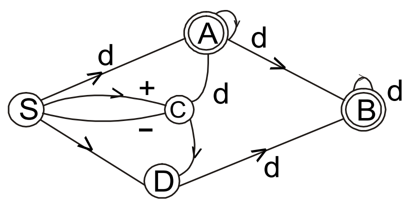
$A \to d(empty\ \ string)$

(a) {01, 0101, 010101, ...}
(b) {01, 000111, 0000011111, ...}
(c) {0011, 00110011, 001100110011,..}
(d) none of these (b) (JNU-2001)

(30) Suppose that you are asked to sort a file which is so large that it cannot fit in the RAM available to you. The sorting algo rithm that you will use is

(a) Merge sort (b) Bubble sort
(c) Quick sort (d) Insertion sort (JNU-2001)

(31) Consider the following DFA :



What is the equivalent RE (Regulation Ex pression) ?

(a) dd* . d* (b) (+ | -) dd* . d*

(c) (+ | -). dd* (d) none of these(JNU-2001)

(32) Which of the following WFF (Well Formed Formula) is logically equivalent to the sen tence ‘ Every persons has a mother’?

(a) $x(person(x) \rightarrow \forall y\, mother\, of\,(x, y))$

(b) $\forall x(\forall y\, mother\, of\,(x, y) \rightarrow person(x))$

(c) $\forall x\, persons\,(x) \rightarrow y\, mother\, of\,(x, y)$

(d) none of these (c) (JNU-2001)

(33) A speaks the truth in 70% cases and B in 80% cases. In what percentage of cases are they likely to contradict each other while narrating the same incident?

(a) 42 (b) 30

(c) 38 (d) 34 (c)(JNU-2001)

(39) If p is positive, then the sum to infinity of the series $\frac{1}{1+p} - \frac{1-p}{(1+p)^2} + \frac{(1-p)^2}{(1+p)^3} - \cdots$ is

(a) 1/2 (b) 3/4 (a)

(c) 1 (d) none of these(JNU-2001)

(40) Following is the distribution of marks is Statistics obtained by 50 students :

Marks more than :	0	10	20	30	40	50
No. of students :	50	46	40	20	10	3

If 60% of students passes the test, then what is the minimum marks obtained by a candi date who has passed ?

(a) 28 (b) 35

(c) 25 (d) 38 (JNU-2001)

PART - B

SECTION - I

(1) Consider the following truth table for a Boolean expression ϕ :

x	y	z	ϕ
1	1	1	1
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

The Karnaugh map for the above expres sion ϕ is

....

....

....

....

(2) An equilateral triangle and a regular hexa gon have the same perimeter. What is the ratio of their areas ?

....

(3) If 1, a_1, a_2,........, a_{n-1} are the roots of unity, then $(1-a_1)(1-a_2)....(1-a_{n-1})$ equals..........

(8) If P(A) = 0.59, P(B) = 0.30, $P(A \cap B) = 0.21$, then $P(A` \cap B`)$ is ....

(9) If a,b,c are in AP; b,c,d are in GP and c,d,e are in HP, then a,c,e are in ....

(10) The number of solutions of the equation $Cos^4 x + Sin^4 x = Sinx\, Cosx (0 \le x \le 2\pi)$ is.......

(11) If the ordered pair (x, y) of real numbers satisfies the equation $x^2 - xy + y^2 = 4(x + y - 4)$ then (x, y) = ...

(12) A recursive function for computing the sum of integers from 0 to n is

```
int sum (int n)
{
if (n = 0) return (0) :
```

else return (?)
}
What would be the missing expression (marked ?) in the above program ? ....

(13) How many different binary numbers can be stored in a register consisting of six switches ? ....

(14) Consider the recursive function :

```
int fun (int n)
{
if (n = 1) return (1) :
else return (1 + fun (n/2))
}
```

The value of fun (2000) is ....

(15) Octal equivalent of 1011.1111_2 is ....

(16) If binary search algorithm takes 5 ms for searching an item in a file of size = 1000, then how much time will it take to search a file of size = 1000000000 ?

(17) $\left(\hat{e}_k\right)_{k=1}^{n}$ is a system of n orthogonal vectors in IR^n, $\sum_{k=1}^{n}\alpha_k\hat{e}_k = \vec{0}.$ with $\alpha_k \in IR \forall k = 1,2,....,n,$ then $\sum_{k=1}^{n}\alpha_k =$ ....

(18) The equation of the circle cutting orthogonal the circles

$$x^2 + y^2 - 2x - 4y + 2 = 0$$

$$x^2 + y^2 - 4x + 2y - 6 = 0$$

$x^2 + y^2 + 8x + 4y - 4 = 0$ is ....

(19) The equation of the circle whose radius is 5 and which touches the circle

$x^2 + y^2 - 2x - 4y - 20 = 0$ at point (5, 5) is

....

(20) The set {2, 4, 6, 8, 10, 12} is a group under multiplication mod 14 with identity element ....

SECTION - II

(1) f is a differentiable function on IR such that $f(x+h) = f(x) + hg(x)$ with $g(x) = x + Sinx$, h being any real parameter. Then, find f(x).

(2) In a multiple choice question there are four alternative answers of which one or more are correct. A candidate will get marks in the question only if he ticks all the correct answer. The candidate decides to tick answers at random. If he is allowed up to three chances to answer the question, what is the probability that he will get marks in the question ?

(3) If $1 \le i_m \le i_{m-1} \le ...... \le i_1 \le n,$ then find the value of k after execution of the following pseudo code :

```
k : = 0
for i1 : = 1 to n
for i2 : = 1 to i1
:
for im : = 1 to im-1
    k : = k + 1
```

(4) If r numbers are selected at random from the ten numbers 1, 2, 3, ...,10, repetitions being allowed, what is the probability that the product of the number is of the form 3n- 1 or 3n + 1,n being a non-negative integer ?

(5) Solve the equations :

$$x + y + z = 9$$

$$x^2 + y^2 + z^2 = 35$$

$$xyz = 15$$

One obvious solution is x = 1, y = 3, z = 5. Write down all other solutions.

(6) Find the point within a triangle for which the sum of the squares of the perpendicular distances to the sides is least.

(7) Evaluate $\int_0^{\pi}\frac{Sin2kx}{Sinx}$, k being any integer.

(8) Consider the following grammar :

(i) $E \to E + T$

(ii) $E \to T$

(iii) $T \to T * F$

(iv) $T \to F$

(v) $F \to (E)$

(vi) $F \to id$

Give the sequence of rules under bottom-up parsing for the input string

(id + id) * id)

(9) Construct a Binary Tree whose nodes in two orders are :

Postorder : M J K H F D N L I G E C B

Inorder : D J M H K F C I N L G E B

(10) Consider the following functions in C :

```
unsigned try (unsigned x, int p, int n)
{
return (x^(~(~ 0 << n) << (p + 1 - n)))
}
```

What would be the output of try (x, 8, 6) if x = 1101101101101101 ?

SOLUTION

PART - A

(1) **(a)** $(x-7)^2+(y+1)^2=25$

$x^2+y^2-14x+2y+25=0$

equation of pair of tangent from (0, 0)

$(-7x+y+25)^2=(x^2+y^2-14x+2y+25)(25)$

$\Rightarrow \quad 49x^2+y^2+625-14xy+50y-350x$

$=25x^2+25y^2-350x+50y+625$

$\Rightarrow \quad 24x^2-24y^2-14xy=0$

$\because$ coeff. of x^2 + coeff. of y^2 = 0

$\Rightarrow$ Angle between tangents = 90^0

(2) **(a)** ${}^nC_2=90 \Rightarrow \dfrac{n(n-1)}{2}-n=90$

$n(n-3)=180 \Rightarrow n^2-3n-180=0$

$n^2-15n+12n-180=0$

$(n-15)(n+12)=0 \quad \Rightarrow n=15,-12$

(3)

(4) **(d)** $\dfrac{(5^2-1)(5^2+1)(5^{2^2}+1)......(5^{2n}+1)}{(5^2-1)}$

$=\dfrac{(5^{2^2}-1)(5^{2^2}-1)....(5^{2^n}+1)}{(5^2-1)}$

$=\dfrac{(5^{2^n}-1)(5^{2^n}+1)}{24}=\dfrac{(5^{2^n})^2-1}{24}=\dfrac{5^{2^{n+1}}-1}{24}$

(5) **(a)**

(6) **(d)** $A\times(B\cap C)=(A\times B)\cap(A\times C)$

(7) **(d)** $\dfrac{dy}{dx}=-\dfrac{x}{y} \quad \Rightarrow \quad x\,dx+y\,dy=0$

$\int x\,dx+\int y\,dy=0 \Rightarrow \dfrac{x^2}{2}+\dfrac{y^2}{2}=C^2$

$\Rightarrow \quad x^2+y^2=2C^2$

which is a circle.

(8) **(d)** $3=3,2+1,1+2,1+1+1=total\ 4\ ways.$

$4=4,1+3,3+1,1+1+2,1+2+1,2+1+1,$

$1+1+1+1,2+2=total\ 8\ ways$

$2=2,1+1=total\ 2\ ways$

so, in general no. n can be written in 2^{n-1} ways.

(9) **(a)** $f(x)=x^n \ \Rightarrow \ f`(x)=n\,x^{n-1}$

$\therefore \quad f`(a+b)=f`(a)+f`(b)$

$n(a+b)^{n-1}=n\,a^{n-1}+n\,b^{n-1}$

$(a+b)^{n-1}=a^{n-1}+b^{n-1}$

This equation is valid for n = 2.

If $\quad n=0 \ \Rightarrow \ f(x)=1 \ \Rightarrow \ f`(x)=0$

$\Rightarrow \quad f`(a+b)=f`(a)+f`(b)=0$

so the value of n are 0, 2

(10) **(d)** $4x^2-9y^2=36 \quad \Rightarrow \quad \dfrac{x^2}{9}-\dfrac{y^2}{4}=1$

$\Rightarrow$ eq. of tangent is, $y=mx\pm\sqrt{9m^2-4}$

But, $\quad m=2/5$

$y=\dfrac{2}{5}x\pm\sqrt{9\times\dfrac{4}{25}-4}$

$y=\dfrac{2}{5}x\pm\sqrt{\dfrac{36-100}{25}}$

$y=(2/5)x\pm(8/5)i$

so equation of tangent parallel to line 2y + 5x = 10 is not possible.

(11) **(a)** $x^4+y^4=a^4$

$\Rightarrow 4x^3+4y^3\dfrac{dy}{dx}=0 \ \Rightarrow \ \dfrac{dy}{dx}=-\dfrac{x^3}{y^3}$

tangent at (x_1, y_1)

$(y-y_1)=-\dfrac{x_1^3}{y_1^3}(x-x_1)$

$yy_1^3-y_1^4+xx_1^3-x_1^4=0$

$\Rightarrow \quad xx_1^3+yy_1^3=x_1^4+y_1^4=a^4$

$\Rightarrow \quad xx_1^3+yy_1^3=a^4 \qquad (1)$

This equation cuts the intercepts p & q on the co-ordinate axis.

$\therefore \quad y=0 \ \Rightarrow \ x=a^4/x_1^3=p$

& $x=0 \Rightarrow y = a^4/y_1^3 = q$

$\therefore \quad p^{-4/3}+q^{-4/3} = (a^4/x_1^3)^{-4/3} + (a^4/y_1^3)^{-4/3}$

$= a^{-16/3}(x_1^4+y_1^4)$

$= a^{-16/3}.a^4 = a^{-\frac{16}{3}+4}$

$p^{-4/3}+q^{-4/3} = a^{-4/3}$

(12) (d) $I = \int \frac{Cos4x-1}{Cotx - \tan x}dx$

$= \int \frac{(1-2Sin^2 2x)-1}{(Cos^2 x - Sin^2 x)/Cosx\,Sinx}dx$

$= -2\int \frac{Sin^2 2x\,Sinx\,Cosx}{Cos2x}dx$

$I = -\int \frac{Sin2x(1-Cos^2 2x)}{Cos2x}dx$

Let $Cos2x = t \Rightarrow -2Sin2xdx = dt$

$I = \frac{1}{2}\int \frac{(1-t^2)}{t}dt = \frac{1}{2}\int\left(\frac{1}{t}-t\right)dt$

$I = \frac{1}{2}\left[\log t - \frac{t^2}{2}\right]+c$

$I = \frac{1}{2}\left[\log Cos2x - \frac{1}{2}Cos^2 2x\right]+c$

(13) $x = t^2+3t-8$ & $y = 2t^2-2t-5 \quad M=(2,-1)$

$\therefore \quad 2 = t^2+3t-8$ & $-1 = 2t^2-2t-5$

$\Rightarrow \quad t^2+3t-10=0 \qquad 2t^2-2t-4=0$

$t^2+5t-2t-10=0 \qquad t^2-t-2=0$

$(t+5)(t-2)=0 \qquad t^2-2t+t-2=0$

$\Rightarrow \quad t = 2,-5 \qquad (t-2)(t+1)=0$

$t=-1,\ 2$

$\Rightarrow \quad t = 2$

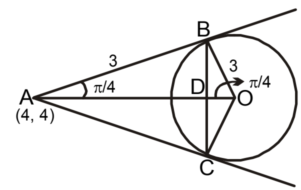
$\therefore \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t-2}{2t+3}$

$\frac{dy}{dx} = \frac{8-2}{4+3} = \frac{6}{7}$

(14) (b) $x^2+y^2-2x-2y-7=0$

O(1, 1) r = BO = 3

$AB = \sqrt{16+16-8-8-7} = 3$



In ΔOAB

$\tan\angle OAB = \frac{3}{3} = 1$

$\Rightarrow \quad \angle OAB = \pi/4 \Rightarrow \angle BOA = \pi/4$

In ΔOBD,

$Sin\frac{\pi}{4} = \frac{BD}{OB} \Rightarrow \frac{1}{\sqrt{2}} = \frac{BD}{3}$

$\Rightarrow \quad BD = 3/\sqrt{2}$

$\Rightarrow \quad BC = 2BD = 2\times 3/\sqrt{2} = 3\sqrt{2} = BC$

(15) (b) Let e_1 be the ecce. of hy.

$x^2 - y^2 Sec^2\alpha = 5$

$\frac{x^2}{5} - \frac{y^2}{5Cos^2\alpha} = 1$

$\Rightarrow \quad e_1 = \sqrt{1+\frac{5Cos^2\alpha}{5}} = \sqrt{1+Cos^2\alpha}$

Let e_2 be the ecce. of ellipse $x^2Sec^2\alpha + y^2 = 25$

$\frac{x^2}{25Cos^2\alpha} + \frac{y^2}{25} = 1$

$e_2 = \sqrt{1-\frac{25Cos^2\alpha}{25}} = \sqrt{1-Cos^2\alpha} = Sin\alpha$

but $e_1 = \sqrt{3}e_2 \Rightarrow \sqrt{1+Cos^2\alpha} = \sqrt{3Sin\alpha}$

$\Rightarrow \quad 1+Cos^2\alpha = 3Sin^2\alpha$

$\Rightarrow \quad 1+Cos^2\alpha = 3(1-Cos^2\alpha)$

$\Rightarrow \quad 4Cos^2\alpha = 2 \Rightarrow Cos^2\alpha = 1/2$

$\Rightarrow \quad Cos^2\alpha = (1/\sqrt{2})^2 \Rightarrow \alpha = \pi/4$

(16) (c) $f(x)=\dfrac{ax+b}{cx+d}$ & $f(f^{-1}(x))=\dfrac{af^{-1}(x)+b}{c(f^{-1}(x))+d}$

$\Rightarrow \quad x(cf^{-1}(x)+d)=af^{-1}(x)+b$

$\Rightarrow \quad f^{-1}(x)=\dfrac{dx-b}{a-cx}$

$\because \quad f(x)=f^{-1}(x) \Rightarrow \dfrac{ax+b}{cx+d}=\dfrac{dx-b}{a-cx}$

$(ax+b)(a-cx)=(cx+d)(dx-b)$

$ax^2+ab-acx^2-bcx=cdx^2+d^2x-bcx-bd$

$(a^2-d^2)x^2-(ac+dc)x+(ab+bd)=0$

$(a-d)(a+d)x^2-c(a+d)x+b(a+d)=0$

$\Rightarrow \quad (a+d)[(a-d)x^2-cx+b]=0$

$\Rightarrow \quad (a+d)=0$

(17) (c) Let P(E)=prob. of occouring an even no.
P(E) = P
& P(O) = prob. of accouring an odd no.
P(O) = 2P
$\therefore$ 2P + P + 2P + P + 2P + P = 1
$\Rightarrow$ 9P = 1 $\Rightarrow$ P = 1/9
\ P(E) = 1/9 & P(O) = 2/9
P(no. $\geq$ 4) = P(no. is 4) + P(no. is 5) + P(no. is 6)
= 1/9 + 2/9 + 1/9 = 4/9

(18) (a) Let $\vec{a}=i+j,\ \vec{b}=j+k,\ \vec{c}=k+i$

& $\vec{\alpha}=\dfrac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|},\ \vec{\beta}=\dfrac{\vec{b}\times\vec{c}}{|\vec{b}\times\vec{c}|},\ \vec{\gamma}=\dfrac{\vec{c}\times\vec{a}}{|\vec{c}\times\vec{a}|}$

value of para. $=[\vec{\alpha}\ \ \vec{\beta}\ \ \vec{\gamma}]$

$$=\frac{[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]}{|\vec{a}\times\vec{b}||\vec{b}\times\vec{c}||\vec{c}\times\vec{a}|}$$

$$=\frac{[\vec{a}\ \ \vec{b}\ \ \vec{c}]^2}{|\vec{a}\times\vec{b}||\vec{b}\times\vec{c}||\vec{c}\times\vec{a}|}$$

$$\therefore [\vec{a}\ \vec{b}\ \vec{c}]=\begin{vmatrix}1&1&0\\0&1&1\\1&0&1\end{vmatrix}=1(1-0)-1(0-1)=2$$

$\vec{a}\times\vec{b}=(i+j)\times(j+k)=k-j+i=i-j+k$

$\vec{b}\times\vec{c}=(j+k)\times(k+i)=i-k+j=i+j-k$

$\vec{c}\times\vec{a}=(k+i)\times(i+j)=j-i+k=-i+j+k$

$\therefore \quad |\vec{a}\times\vec{b}|=|\vec{b}\times\vec{c}|=|\vec{c}\times\vec{a}|=\sqrt{3}$

$\therefore$ Req. Vol. $=4/(\sqrt{3})^3=4/3\sqrt{3}$

(19) (d) x = 2 × 6 ! × 6 !
y = 5 ! × 6 !
$\Rightarrow$ x/y = 2 × 6 $\Rightarrow$ x = 12 y

(20) (a) 1 digit no. = 5
2 digit no. = 5 × 5
3 digit no. = 5 × 5 × 4 = 100
4 digit no. = 5 × 5 × 4 × 3 = 300
5 digit no. = 5 × 5 × 4 × 3 × 2 = 600
6 digit no. = 5 × 5 × 4 × 3 × 2 × 1 = 600
sum of all these no. = 1630

(21) (c) ${}^{15}C_8+{}^{15}C_9+{}^{15}C_{10}+{}^{15}C_{11}+{}^{15}C_{12}$
$+{}^{15}C_{13}+{}^{15}C_{14}+{}^{15}C_{15}$
$=6435+5005+3003+1365+455+105+15+1$
$=16384$

(22) (a) $Dr=\begin{vmatrix}a&2^r&2^{16}-1\\b&3(4^r)&2(4^{16}-1)\\c&7(8^r)&4(8^{16}-1)\end{vmatrix}$

$$\sum_{k=1}^{16}Dr=\begin{vmatrix}a&\sum 2^r&2^{16}-1\\b&\sum 3(4^r)&2(4^{16}-1)\\c&\sum 7(8^r)&4(8^{16}-1)\end{vmatrix}$$

$$=\begin{vmatrix}a&2(2^{16}-1)/(2-1)&2^{16}-1\\b&3.4(4^{16}-1)/(4-1)&2(4^{16}-1)\\c&7.8(8^{16}-1)/(8-1)&4(8^{16}-1)\end{vmatrix}$$

$$=\begin{vmatrix}a&2(2^{16}-1)&2^{16}-1\\b&4(4^{16}-1)&2(4^{16}-1)\\c&8(8^{16}-1)&4(8^{16}-1)\end{vmatrix}$$

$$=2\begin{vmatrix}a&2^{16}-1&2^{16}-1\\b&2(4^{16}-1)&2(4^{16}-1)\\c&4(8^{16}-1)&4(8^{16}-1)\end{vmatrix}=\sum_{k=1}^{16}Dr=0$$

(23) b (24) a (25) d (26) a (27) (28) (29) b (30) (31) (32) c

(33) (c) No. of ways = (A speaks truth) (B tells lie) + (B speaks truth) (A tells lie)

$$= \frac{70}{100}\times\frac{20}{100}+\frac{30}{100}\times\frac{80}{100}$$

$$= \frac{14}{100}+\frac{24}{100}=\frac{38}{100}=38\%$$

(34) (b) $I=\int_{-1/2}^{1/2} Cosx\log\left(\frac{1+x}{1-x}\right)dx$

Let $f(x)=Cosx.\log\left(\frac{1+x}{1-x}\right)$

$$f(-x)=Cos(-x).\log\left(\frac{1-x}{1+x}\right)=Cosx\log\left(\frac{1+x}{1-x}\right)^{-1}$$

$$f(-x)=-Cosx\log\left(\frac{1+x}{1-x}\right)=-f(x)$$

$\because \quad f(-x)=-f(x)$

$\therefore \quad I=\int_{-1/2}^{1/2} Cosx\log\left(\frac{1+x}{1-x}\right)dx=0$

(35) (d) $\frac{x^2}{4}+\frac{y^2}{b^2}=1 \quad \& \quad y^2=4x$

Equation of tangent $y=mx\pm\sqrt{4m^2+b^2}$ (1)

& $x-2y+4=0 \Rightarrow 2y=x+4$

$\Rightarrow \quad y=x/2+2$ (2)

equation (2) $\Rightarrow \quad m=1/2 \quad \& \quad \sqrt{4m^2+b^2}=2$

$\Rightarrow \quad 4m^2+b^2=4$

$4(1/4)+b^2=4 \quad \Rightarrow \quad b=\sqrt{3}$

& equation of tangent is

$y=x/2-2 \Rightarrow 2y=x-4$

$\Rightarrow \quad x-2y-4=0$

(36) (b) $f(x)=2x^3-3x^2-12x+1$

$f`(x)=6x^2-6x-12=6(x^2-x-2)$

$f`(x)=0 \quad \Rightarrow \quad x^2-x-2=0$

$(x^2-2x+x-2)=0$

$\Rightarrow \quad (x-2)(x+1)=0 \Rightarrow x=-1/2$

but $\quad -1\le x\le 3/2 \quad \because \quad x=-1$

Now, $\quad f``(x)=6(2x-1)$

$f``(-1)=6(-2-1)=-18$

$\Rightarrow$ x = -1 is a point of maxima

& x = 3/2 is a point of minima.

$\therefore$ f(-1) = -2-3+12+1 = 8 = M

$$f(3/2)=\frac{27}{4}-\frac{27}{4}-\frac{12\times 3}{2}+1\Rightarrow -18+1=-17=m$$

$\therefore \quad (M,m)=(8,-17)$

(37) (c) y = -3x² - | x | + 2

y = -3x² - x + 2, x > 0

- y + 2 = 3x² + x

(- y + 2) / 3 = x² + x/3

$$\frac{-y+2}{3}+\frac{1}{36}=x^2+\frac{x}{3}+\frac{1}{36}=\left(x+\frac{1}{6}\right)^2$$

$$\left(x+\frac{1}{6}\right)^2=-\frac{1}{3}\left(y-\frac{25}{12}\right), \quad x>0$$

& $\quad y=-3x^2+x+2, \quad x<0$

$3x^2-x=-y+2$

$$x^2-\frac{x}{3}+\frac{1}{36}=\frac{-y+2}{3}+\frac{1}{36}$$

$(x-1/6)^2=-1/3(y-25/12), \quad x<0$

$$y=\begin{cases}-3x^2-x+2, & x>0\\ -3x^2+x+2, & x<0\end{cases}$$

$$y=\begin{cases}-(3x-2)(x+1), & x>0\\ -(3x+2)(x-1), & x<0\end{cases}$$

$$A=2\int_0^{2/3}(-3x^2-x+2)dx=2\left[\frac{-3x^3}{3}-\frac{x^2}{2}+2x\right]_0^{2/3}$$

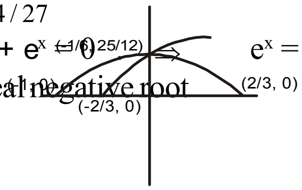
$$A = 2\left[-x^3 - \frac{x^2}{2} + 2x\right]_0^{2/3} = 2\left[-\frac{8}{27} - \frac{2}{9} + \frac{4}{3}\right]$$

$$A = 2\left[\frac{-8-6+36}{27}\right] = 2\left[\frac{36-14}{27}\right] = \frac{2\times 22}{27}$$

$\Rightarrow \quad A = 44/27$

(38) (d) x + e^x = 0 $\Rightarrow$ e^x = -x

$\Rightarrow$ one real negative root



(39) (a)

$$\frac{1}{1+P} - \frac{1-P}{(1+P)^2} + \frac{(1-P)^2}{(1+P)^3} .... = \frac{1/(1+P)}{1+(1-P)/(1+P)}$$

$$= \frac{1}{(1+P)}\frac{(1+P)}{[(1+P)+(1-P)]} \quad = 1/2$$

(40)

PART - B

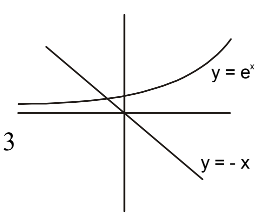
(1)

(2) Let the side of equilateral triangle & hexa gon be a_1 and a_2 then

$3a_1 = 6a_2 \quad \Rightarrow \quad a_1 = 2a_2$

then $\dfrac{A_1}{A_2} = \dfrac{Area\ of\ eq.\ \Delta}{Area\ of\ hexagon} = \dfrac{\sqrt{3}}{4}a_1^2.\dfrac{4}{6\sqrt{3}a_2^2}$

$$= \frac{a_1^2}{6a_2^2} = \frac{4a_2^2}{6a_2^2} = \frac{2}{3}$$



$\Rightarrow \qquad A_1 A_2 = 2:3$

(3) We know that,

$(x^n - 1) = (x-1)(x-\alpha)(x-\alpha^2)...(x-\alpha^{n-1})$

$\because$ 1, a_1, a_2....a_{n-1} are the roots of unity.

$\Rightarrow \quad (x^n - 1) = (x-1)(x-a_1)(x-a_2)...(x-a_{n-1})$

$\Rightarrow \quad (x-a_1)(x-a_2)........(x-a_{n-1}) = \dfrac{x^n - 1}{x-1}$

$(x-a_1)(x-a_2)..(x-a_{n-1}) = 1 + x + x^2 + ...x^{n-1}$

put x = 1

$\Rightarrow \quad (1-a_1)(1-a_2)......(1-a_{n-1}) = 1+1......+1$

= n

(4) $\displaystyle\lim_{x\to 1}\frac{x + x^2 + ... + x^n - n}{x-1}$

$$= \lim_{x\to 1}\frac{1 + 2x + 3x^2 + .. + nx^{n-1}}{1}$$

= 1 + 2 + 3 + ...+ n

= n(n + 1) / 2

(5) $y = \dfrac{2^{1/x} - 1}{2^{1/x} + 1}$

$$\lim_{x\to 0^+} y = \lim_{h\to 0}\frac{2^{1/(0+n)} - 1}{2^{1/(0+n)} + 1} = \lim_{h\to 0}\frac{2^{1/n} - 1}{2^{1/n} + 1}$$

$$= \lim_{h\to 0}\frac{1 - 2^{-1/n}}{1 + 2^{-1/n}} = \lim_{x\to 0^+} y = 1$$

(6) $f``(x) = x^{-3/2}$

$\Rightarrow \quad f`(x) = \dfrac{x^{-1/2}}{-1/2} = -2x^{-1/2} + C_1$

$\because \quad f`(4) = 2 \quad \Rightarrow \quad 2 = -2(4)^{-1/2} + C_1$

$\Rightarrow \quad C_1 = 3$

$\therefore \quad f`(x) = -2x^{-1/2} + 3$

$\Rightarrow \quad f(x) = -2\dfrac{x^{1/2}}{1/2} + 3x + C_2$

$f(x) = -4x^{1/2} + 3x + C_2$

$\because \quad f(0) = 0 \quad \Rightarrow \quad C_2 = 0$

$\therefore \quad f(x) = 3x - 4\sqrt{x}$

(7) f (x) = $(x - 2)^{2/3}$ (2x + 1)

$f`(x) = 2/3(x-2)^{-1/3}(2x+1) + (x-2)^{2/3}.2$

$$= \frac{2(2x+1)}{3(x-2)^{1/3}} + 2(x-2)^{2/3} = \frac{2(2x+1) + 2(x-2).3}{3(x-2)^{1/3}}$$

$$f`(x) = \frac{2(5x-5)}{3(x-2)^{1/3}} = \frac{10(x-1)}{3(x-2)^{1/3}}$$

$\Rightarrow \quad f`(x)=0 \quad \Rightarrow \quad x=1$

$\because$ x = 1 is the critical point of the function

(8) $P(A`\cap B`)=P(A\cup B)=1-P(A\cup B)$

$$=1-P(A)-P(B)+P(A\cap B)$$
$$=1-.59+.30+.21$$
$$=1.21-.89$$
$$=.32$$

(9) 2b = a + c , c^2 = bd , d = 2ce/e + c

$\because \quad b=\dfrac{a+c}{2} \quad \Rightarrow \quad c^2=\dfrac{(a+c)d}{2}$

$\therefore \quad d=\dfrac{2c^2}{a+c} \quad \& \quad d=\dfrac{2ce}{e+c}$

$\therefore \quad \dfrac{2c^2}{a+c}=d=\dfrac{2ce}{e+c}$

$\Rightarrow$ c(e + c) = e(a + c)

ce + c^2 = ea + ce

$\Rightarrow$ c^2 = ea

$\Rightarrow$ a, c, e are in G.P.

(10) $\cos^4$x + sm^4x = smx . cosx

$(sm^2x + \cos^2x)^2 = 2sm^2x \cos^2x + smx\ cosx$

1 = 1/2 $sm^2$2x + 1/2 sm2x

$\Rightarrow$ $sm^2$2x + sm2x - 2 = 0

$sm^2$2x + 2sm2x - sm2x - 2 = 0

(sm2x - 1) (sm2x + 2) = 0

$\Rightarrow$ sm2x = 1, -2

but sm2x = -2 is not possible

$sm2x=1 \quad \Rightarrow \quad 2x=\pi/2, 5\pi/2$

$\Rightarrow \quad x=\pi/4,\ 5\pi/4$ [no. of solutions = 2]

(11) $x^2-xy+y^2=4(x+y-4)$

$$x^2-x(y+4)+(y^2-4y+16)=0$$
$$x=\left[(y+4)\pm\sqrt{(y+4)^2-4(y^2-4y+16)}\right]/2$$
$$x=\left[(y+4)\pm\sqrt{y^2+16+8y-4y^2+16y-64}\right]/2$$
$$x=\left[(y+4)\pm\sqrt{-3y^2+24y-48}\right]/2$$
$$x=\left[(y+4)\pm\sqrt{-3(y^2-8y+16)}\right]/2$$
$$x=\left[(y+4)\pm\sqrt{-3(y-4)^2}\right]/2$$

$\because$ x & y are real no.

$\Rightarrow \quad \sqrt{-3(y-4)^2}=0 \quad \Rightarrow \quad y=4$

$\& \quad y=4 \quad \Rightarrow \quad x=(4+4)/2 \quad \Rightarrow \quad x=4$

$\because$ ordered pair (x, y) = (4, 4)

(12)

(13)

(14) 1001

(15) $(13.74)_8$

(16)

(17)

(18) Let the equation of circle be

$x^2 + y^2$ + 2gx + 2fy + c = 0

$\therefore$ 2(-g - 2f) = c + 2 (1)

2(-2g + f) = c - 6 (2)

2(4g + 2f) = c - 4 (3)

(1)-(2) $\Rightarrow$ 2(g - 3f) = 8 $\Rightarrow$ 9 - 3f - 4 = 0 (4)

(2)-(3) $\Rightarrow$ 2(-6g - f) = -2 $\Rightarrow$ 6g + f -1 = 0 (5)

$\Rightarrow \quad \dfrac{9}{+7}=\dfrac{f}{-23}=\dfrac{+1}{19}$

$\therefore \quad 2\left(\dfrac{-7}{19}+\dfrac{2\times 23}{19}\right)=c+2$

$$2(-7+46)=19(c+2)\Rightarrow 2\times 39=19(c+2)$$

$$c=\frac{78}{19}-2 \quad \Rightarrow \quad c=\frac{40}{19}$$

$\therefore$ equation of circle is

$$x^2+y^2+\frac{14}{19}x-\frac{46}{19}y+\frac{40}{19}=0$$

$$19(x^2+y^2)+14x-46y+40=0$$

(19)

$\Rightarrow \quad 5=\dfrac{1+h}{2} \quad \Rightarrow \quad h=9$

$5=\dfrac{1+k}{2} \quad \Rightarrow \quad k=8$

∴ equation of circle

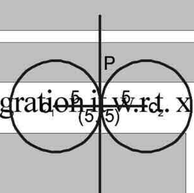
(20)

x_{14}	2	4	6	8	10	12
2	4	8	12	2	6	10
4	8	2	10	4	12	6
6	12	10	8	6	4	2
8	2	4	6	8	10	12
10	6	12	4	10	2	8
12	10	6	2	12	8	4

identity element = 8

SECTION - B

(1) f(x + h) = f(x) + hg(x) & g(x) = x + Sinx



Integration i w.r.t. x

(2) The total no. of ways

= 4 + 6 + 4 + 1

= 15

The prob. of ticking all the answers correctly in the first trail = 1/15

The prob. of ticking all the answers correctly in the second trail = (14/15)(1/14) = 1/15

The prob. of ticking all the answer correctly in the third trail = (14/15)(13/14)(1/13) = 1/15

∴ Req. prob.

(3)

(4) If the product is of the form (3n - 1) or (3n + 1) then no. should not be multiple of 3.

so the no. of ways of secefing r no. from (0, 1, 2, ...10) is = T^r

but if no. 1 cannot be expressed in the from 3n - 1 or 3n + 1 for n > 0

∴ No. of ways of selecting r no. from (1, 2...10) of the form (3n - 1) or (3n + 1) = T^r - 1

& Total no. of ways = 10^r

∴ Req. prob. = 7^r - 1/10^r

(5) $x + y + z = 9$ (1)

$x^2 + y^2 + z^2 = 35$ (2)

$xyz = 15$ (3)

$\because \quad (x+y+z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

$(9)^2 = 35 + 2(xy + yz + zx)$

$2(xy + yz + zx) = 81 - 35 = 46$

$xy + y^2 + zx = 23$ (4)

$x(y+z) + yz = 23$

$x(9-x) + 15/x = 23$

$x^2(9-x) + 15 = 23x \Rightarrow 9x^2 - x^3 + 15 - 23x = 0$

$x^3 - 9x^2 + 23x - 15 = 0$

$x^3 - x^2 - 8x^2 + 8x + 15x - 15 = 0$

$x^2(x-1) - 8x(x-1) + 15(x-1) = 0$

$(x-1)(x^2 - 8x + 15) = 0$

$(x-1)(x^2 - 3x - 5x + 15) = 0$

$(x-1)(x-3)(x-5) = 0$

$\Rightarrow \quad x = 1, 3, 5$

If $x = 1$

$\Rightarrow \quad y + z = 8 \quad \& \quad yz = 15$

$(y - z)^2 = 64 - 60 = 4$

y - z = I 2

$\therefore \quad y + z = 8 \quad \& \quad y + z = 8$

$y - z = 2 \qquad y - z = -2$

$\Rightarrow \quad y = 5 \ \& \ z = 3 \qquad y = 3 \ \& \ z = 5$

If $x = 3$

$y + z = 6 \quad \& \quad yz = 5$

$(y - z)^2 = 36 - 20 = 16$

$y - z = \pm 4$

$\therefore \quad y + z = 6 \quad \& \quad y + z = 6$

$y - z = 4 \qquad y - z = -4$

$\Rightarrow \quad y = 5 \ \& \ z = 1 \qquad y = 1, z = 5$

If $x = 5 \Rightarrow$

$y + z = 4 \quad \& \quad yz = 3$

$(y - z)^2 = 16 - 12 = 4$

$y - z = \pm 2$

$\therefore \quad y + z = 4 \quad \& \quad y + z = 4$

$y - z = 2 \qquad y - z = -2$

$\Rightarrow \quad y = 3, z = 1 \qquad y = 1, z = 3$

$\therefore \quad x = 1, y = 5, z = 3$

$x = 1, y = 3, z = 5$

$x = 3, y = 5, z = 1$

$x = 3, y = 1, z = 5$

$x = 5, y = 3, z = 1$

$x = 5, y = 1, z = 3$

(6)

(7) $I = \int_0^\pi \frac{\sin 2kx}{Sinx} dx$

we know that

$$\frac{Sin\,2k\pi}{Sinx} = 2[Cosx + Cos3x + ... + Cos(2k-1)x]$$

$$R.H.S. == \frac{2}{Sinx}(SinxCosx + SinxCos3x +$$

$$...SinxCos(2k-1)x]$$

$$= \frac{1}{Sinx}[Sin2x + (Sin4x - Sin2x) +$$

$$... + (Sin2kx - Sin(2k-2)x]$$

$$= \frac{Sin\,2kx}{Sinx}$$

$$\therefore \quad I = \int_0^\pi 2(Cosx + Cos3x + ... + Cos(2k-1)x)dx$$

$$I = 2\left[Sinx + \frac{Sin3x}{3} + ... + \frac{Sin(2k-1)x}{(2k-1)} \right]_0^\pi$$

$\Rightarrow \quad I = 0$