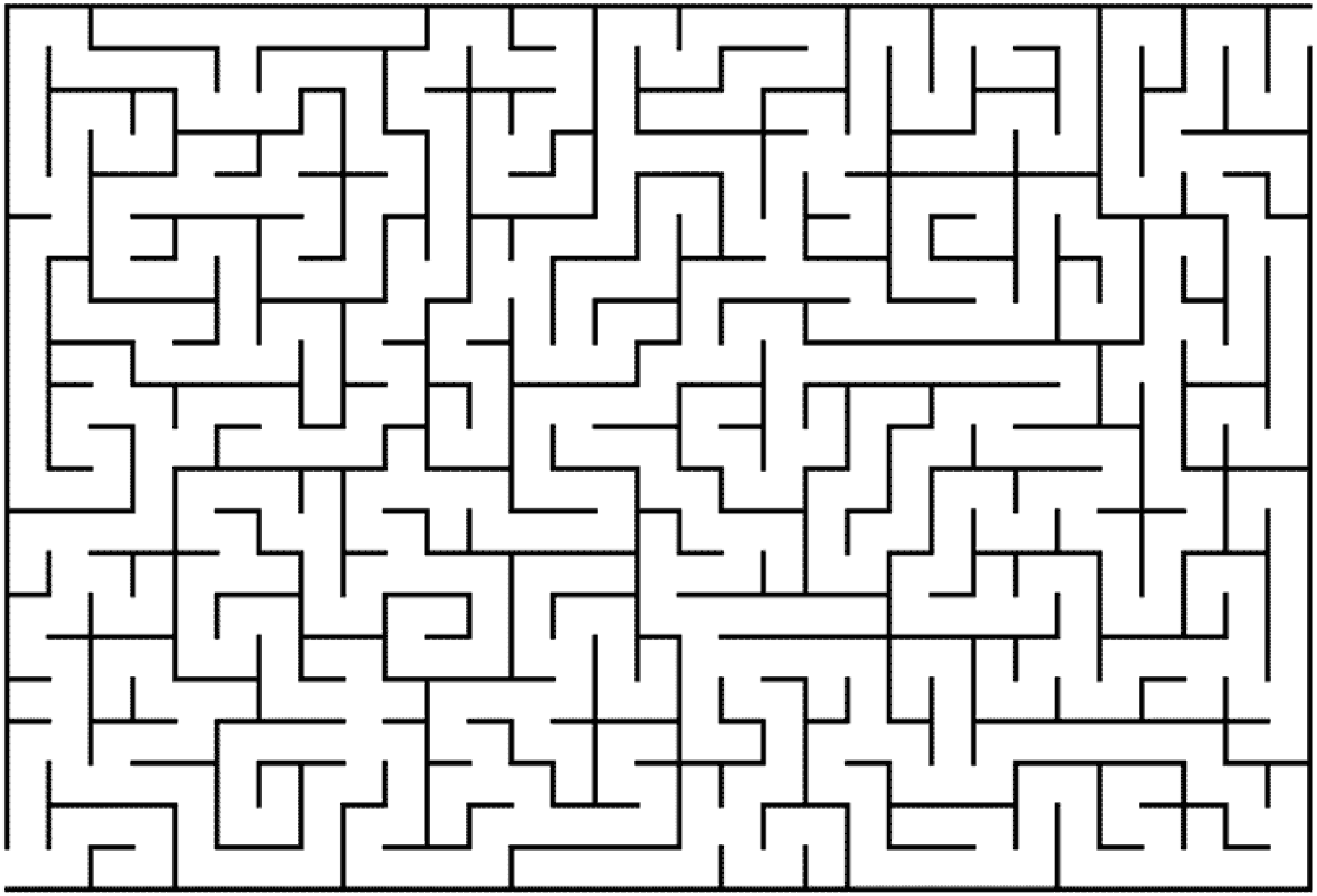


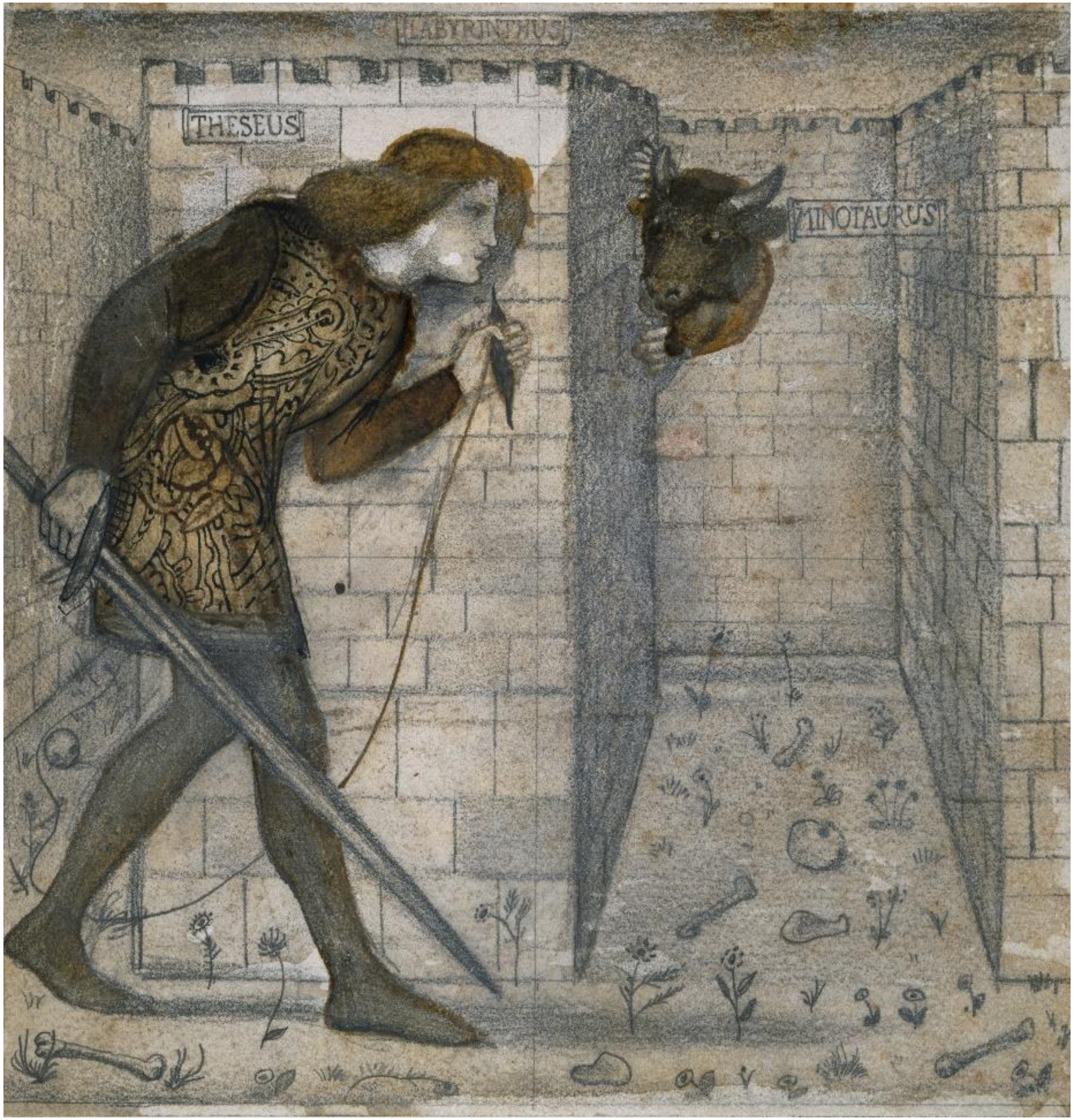
MAZES TO AMAZE

Duttatrey Nath Srivastava

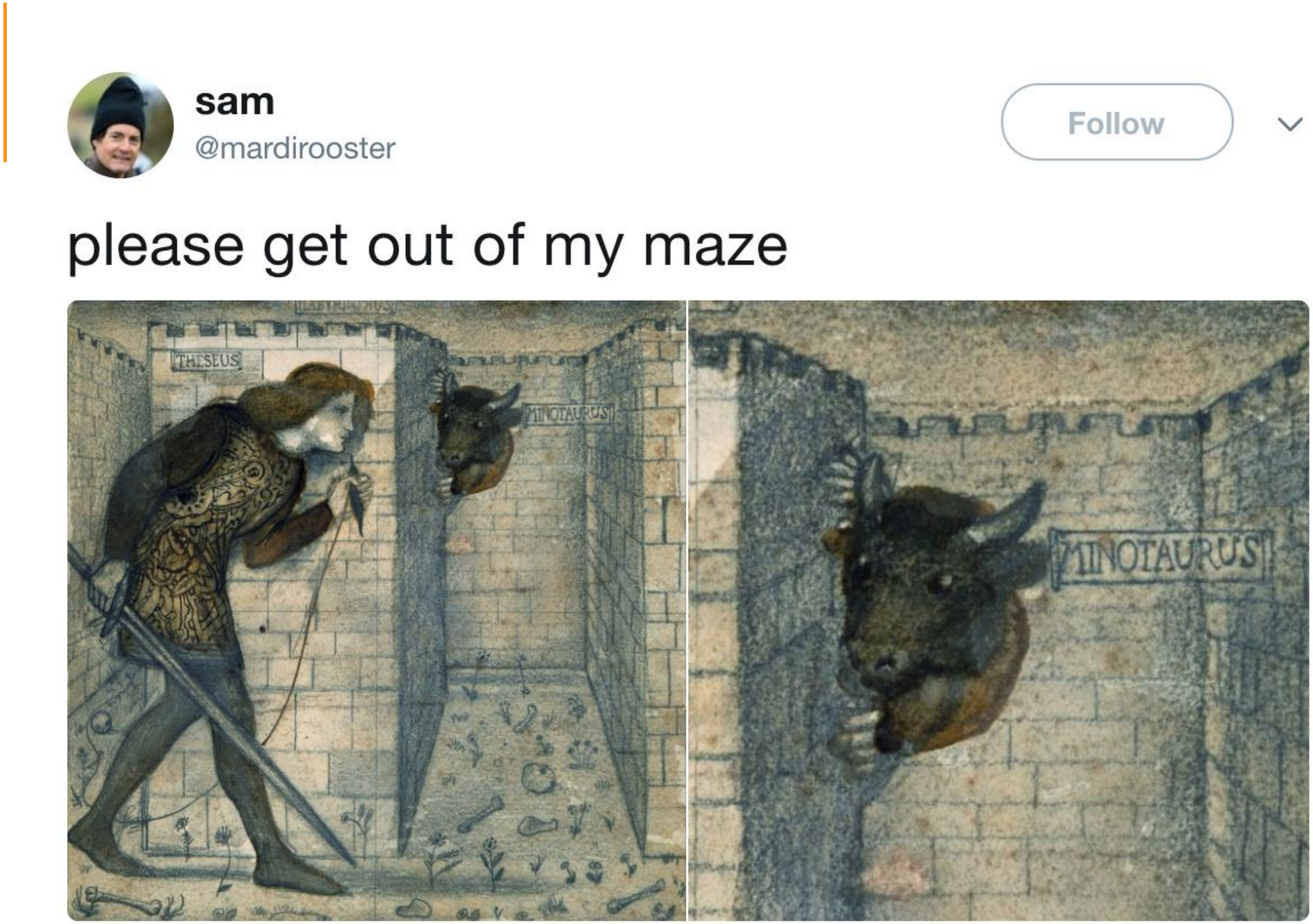
MINOTAURUS AND THESEUS (FEAT. ARIADNE)

Clew

© www.maicar.com



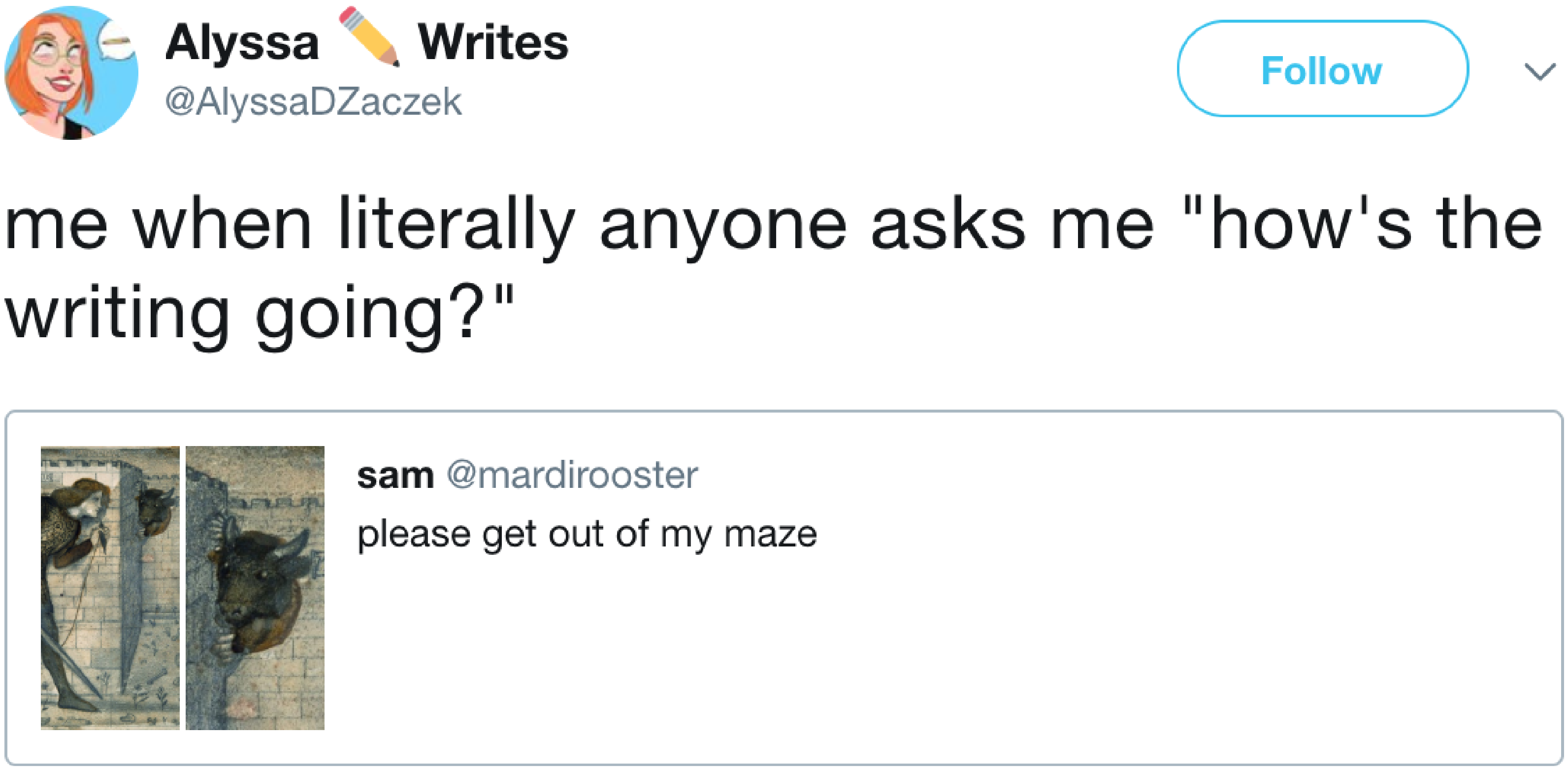
Theseus and the Minotaur in the Labyrinth, Edward Burne-Jones(1861)



sam

@mardirooster

please get out of my maze



Alyssa ✏️ Writes
@AlyssaDZaczek

me when literally anyone asks me "how's the writing going?"

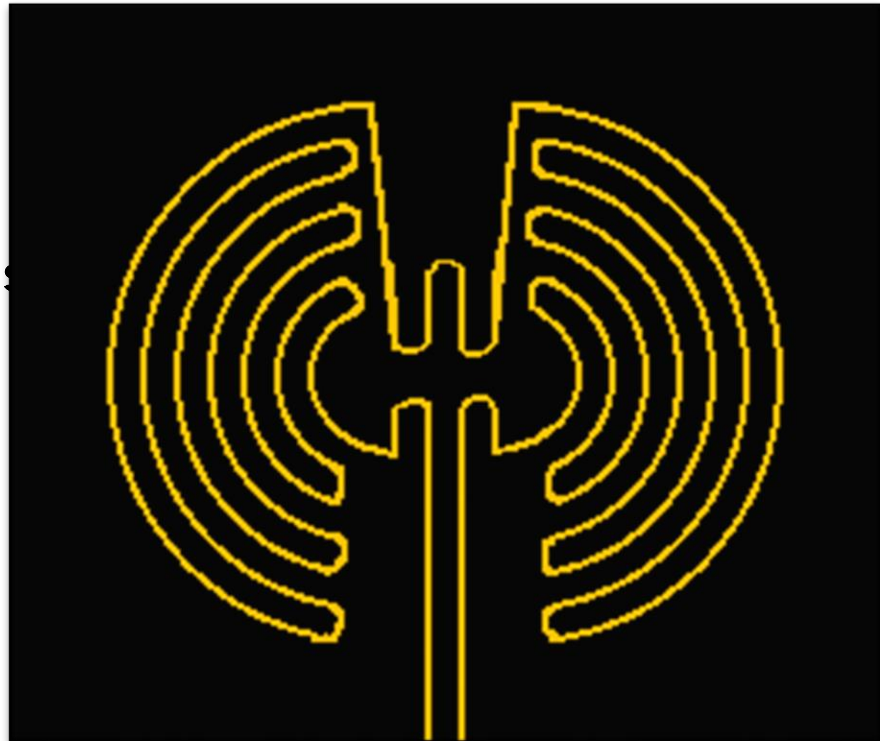
sam @mardirooster
please get out of my maze

MAZES: AN INTRODUCTION

- A maze is a complex structure of interconnected passageways.
- Must have: (at least) one way to get from a designated START location to a designated END.
- Difficult to solve with *Eagle eye view* from above with all information exposed.
- Even more difficult when you're inside one.

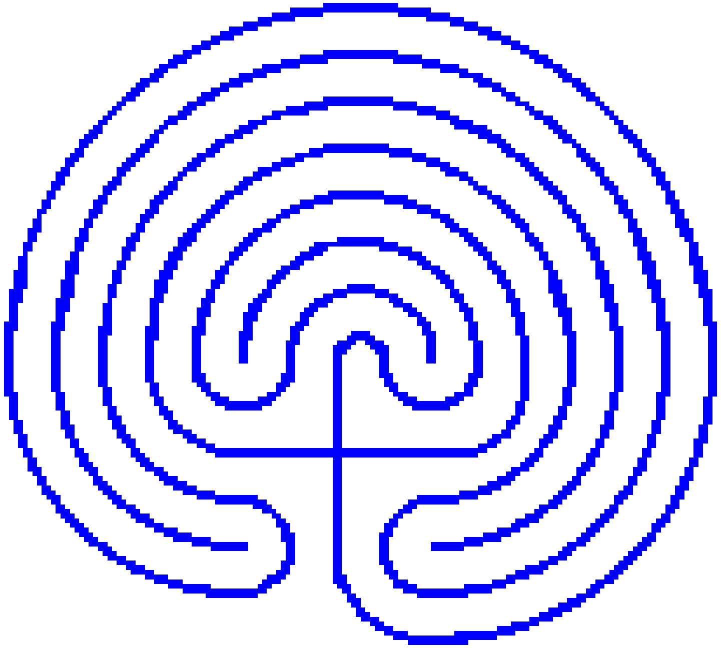
MAZES AND LABYRINTHS

- The word Labyrinth probably comes from the word *Labrys*, a double headed axe.
- This was a religious symbol of ancient Crete.
- A labyrinth is usually unicursal(one long winding path), while a maze is branched.



THE CRETAN MAZE

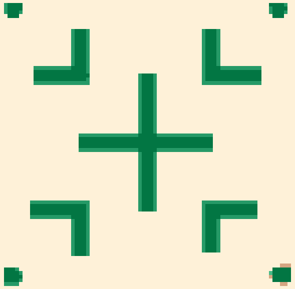
- The Cretan maze is the oldest known maze design.
- It is a unicursal maze, a single path without choices or branches.



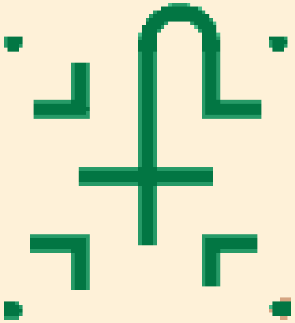
INTERESTING QUESTION 1:

HOW DO I **BUILD** A MAZE?

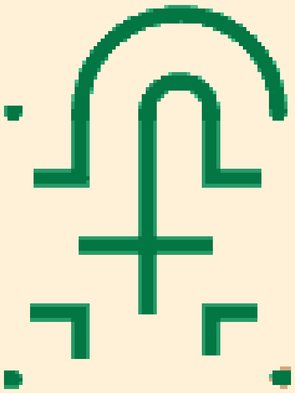
BUILDING THE CRETAN MAZE: METHOD I



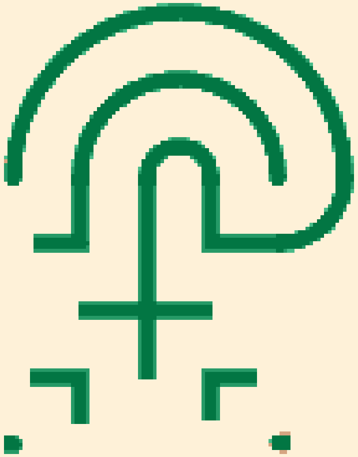
BUILDING THE CRETAN MAZE: METHOD I



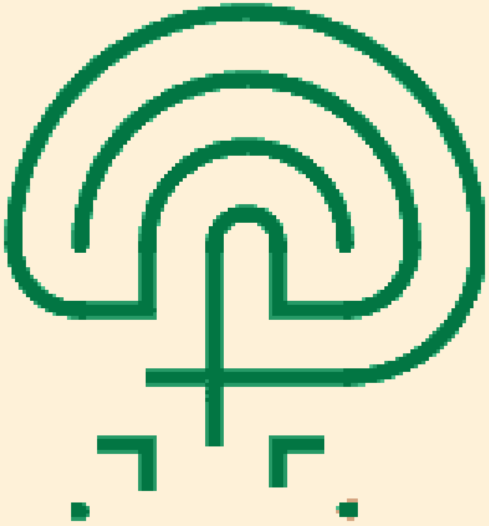
BUILDING THE CRETAN MAZE: METHOD I



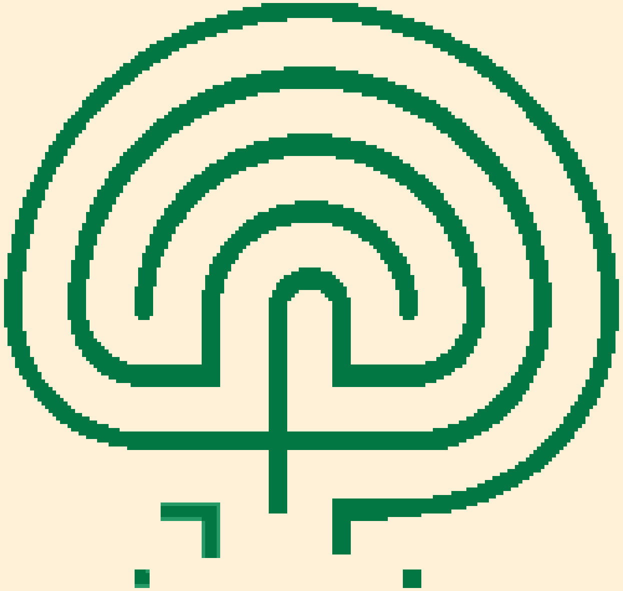
BUILDING THE CRETAN MAZE: METHOD I



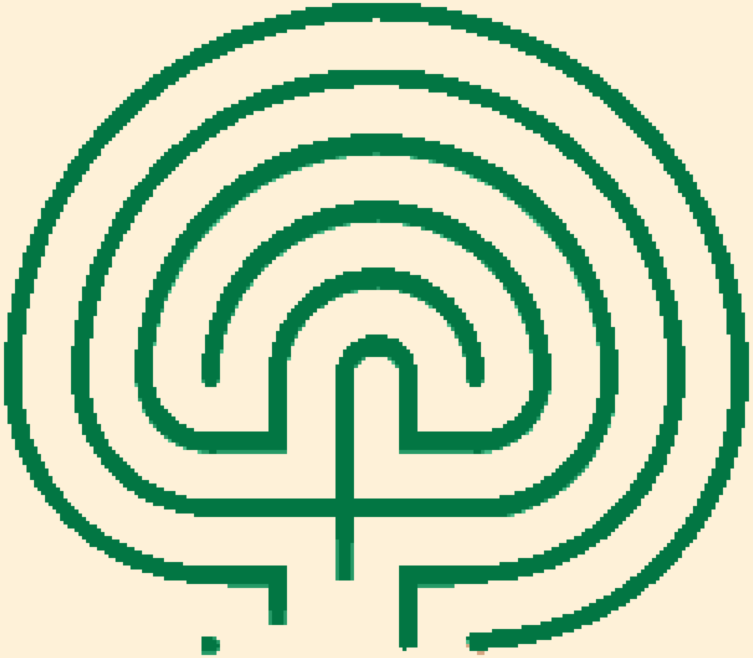
BUILDING THE CRETAN MAZE: METHOD I



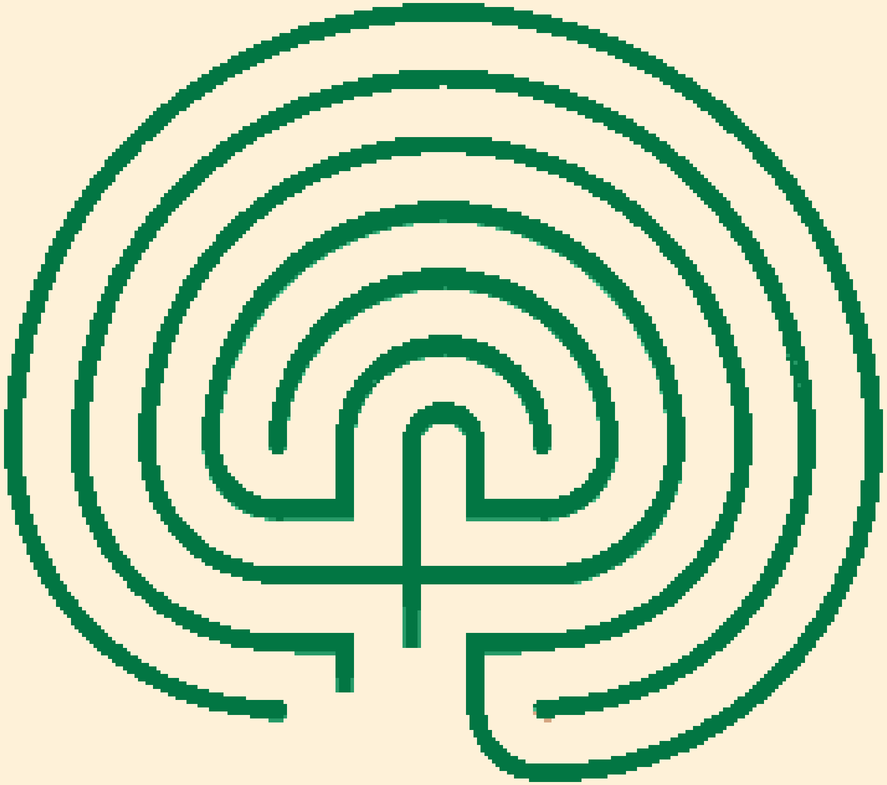
BUILDING THE CRETAN MAZE: METHOD I



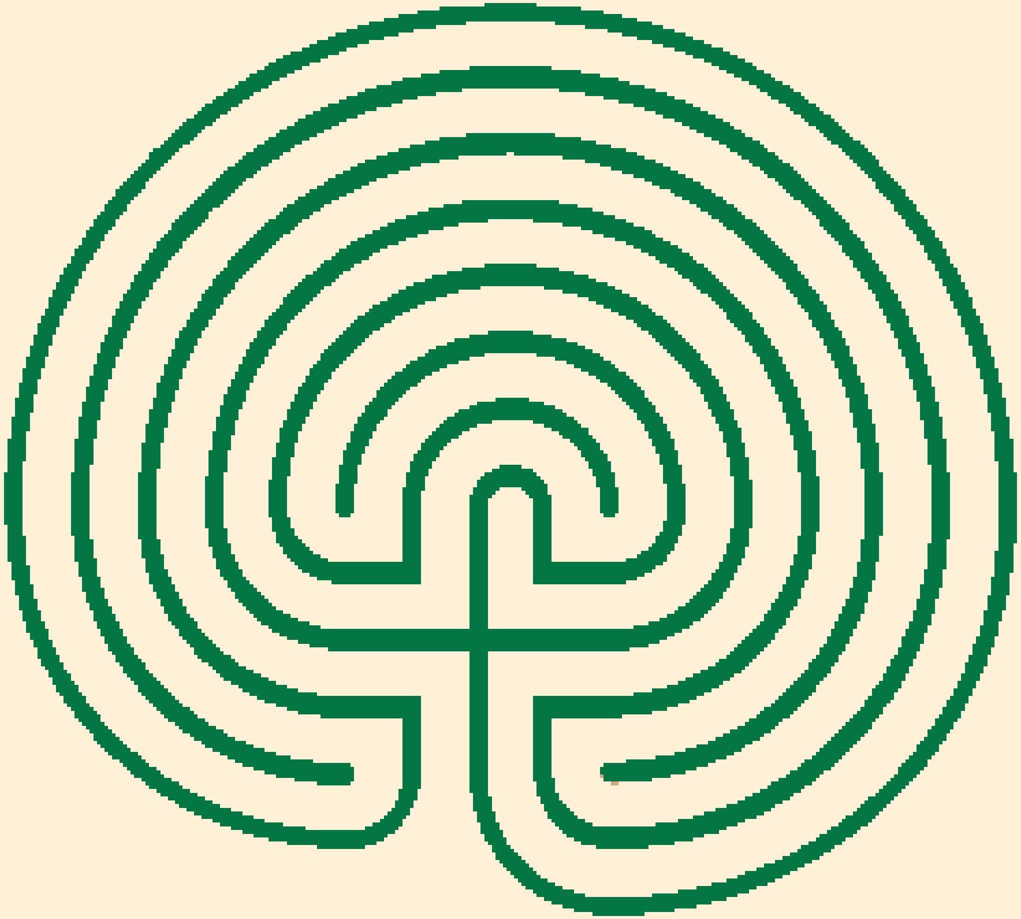
BUILDING THE CRETAN MAZE: METHOD I



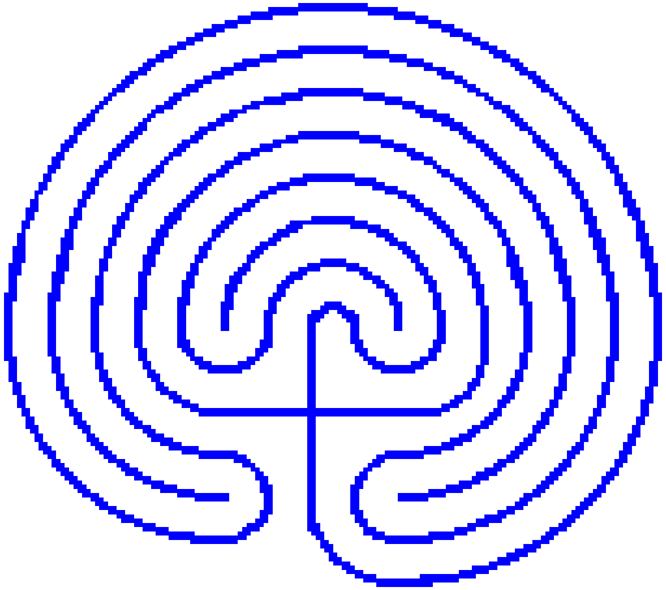
BUILDING THE CRETAN MAZE: METHOD I

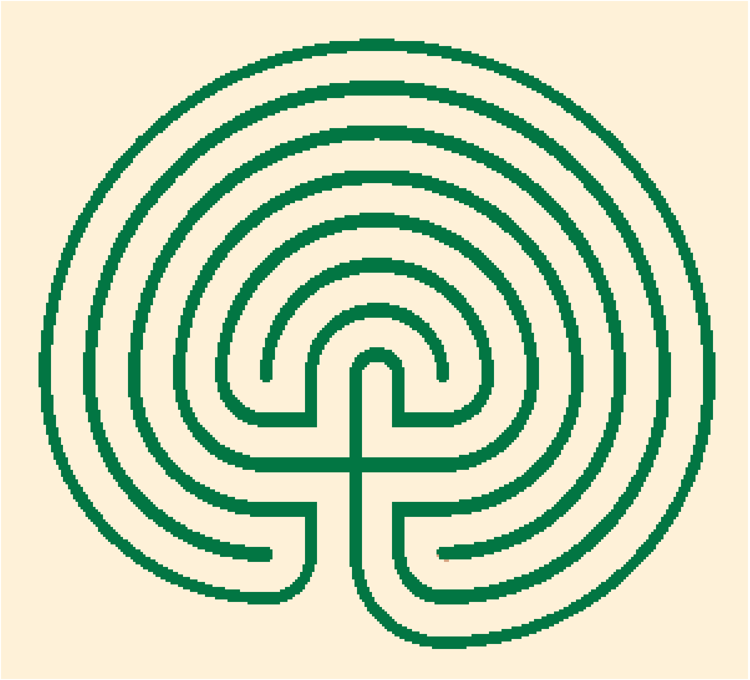


BUILDING THE CRETAN MAZE: METHOD I

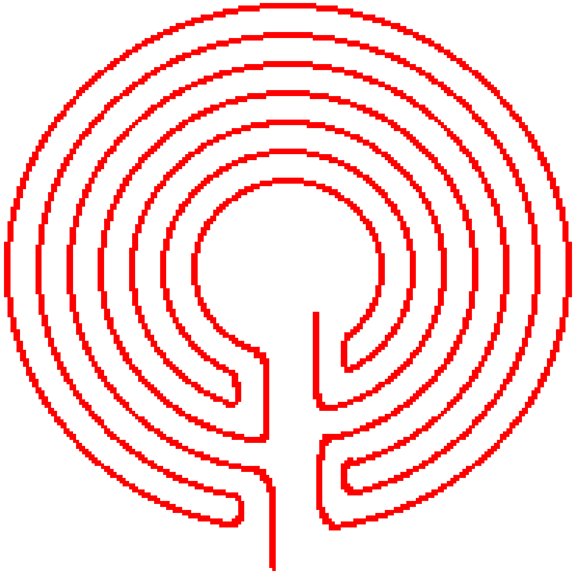


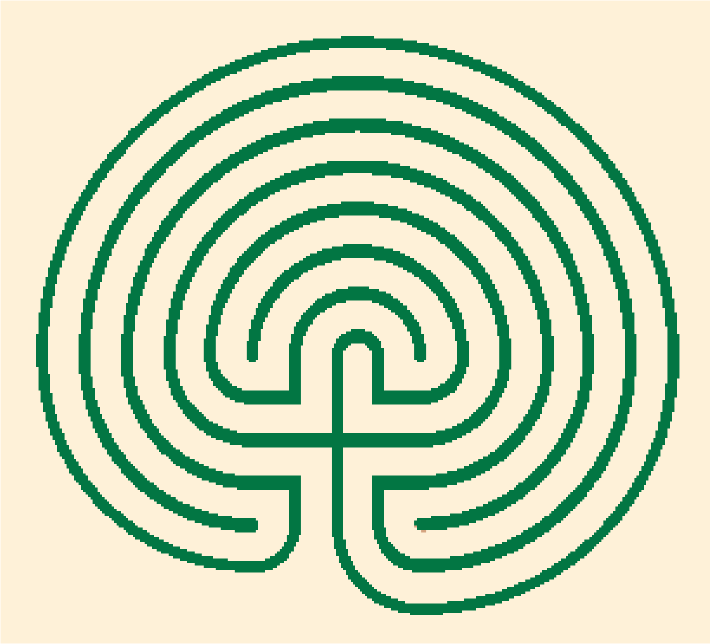
BUILDING THE CRETAN MAZE: METHOD I





METHOD II: WALK THE CRETAN MAZE

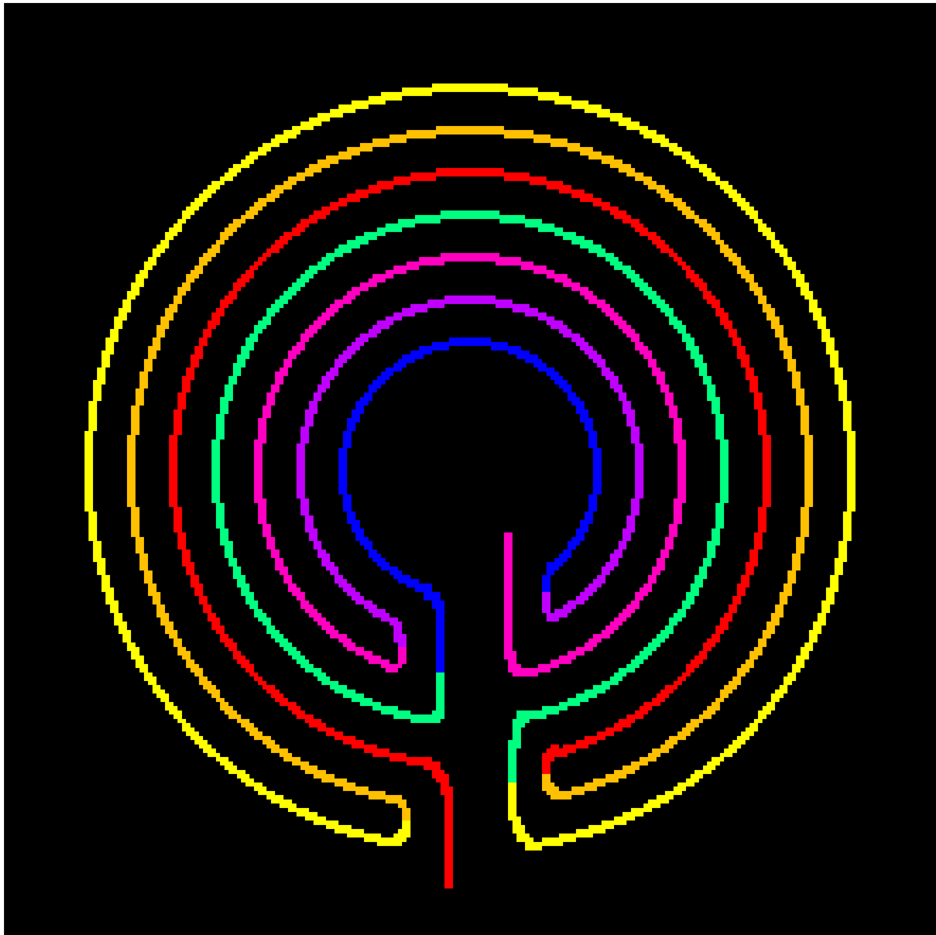




- Alternatively: Design the path to be walked instead.
- The figure on left shows the walk for the Cretan maze

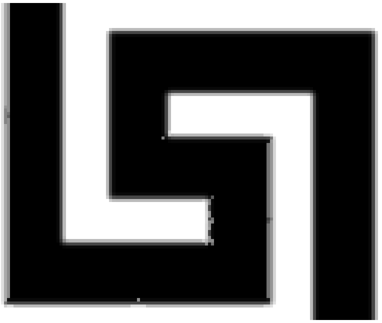
METHOD II: WALKING THE CRETAN MAZE

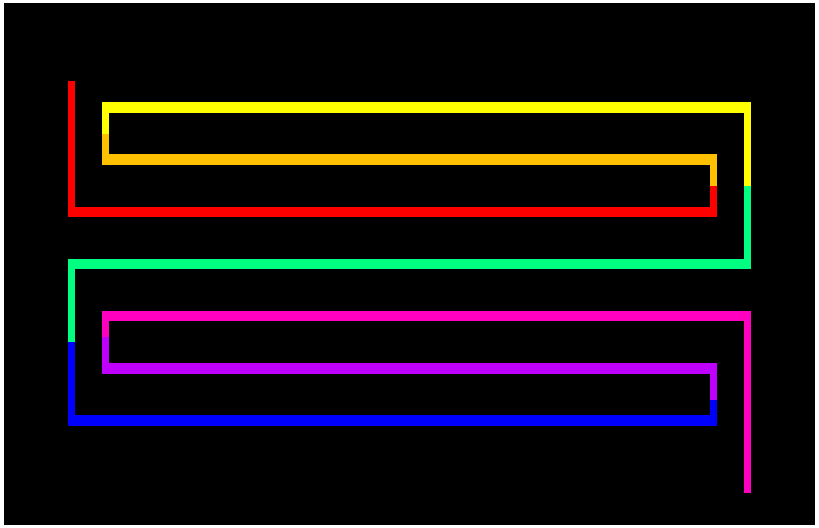
- Start on red, which turns to orange, then yellow, and so on through the rainbow.



THE GREEK KEY

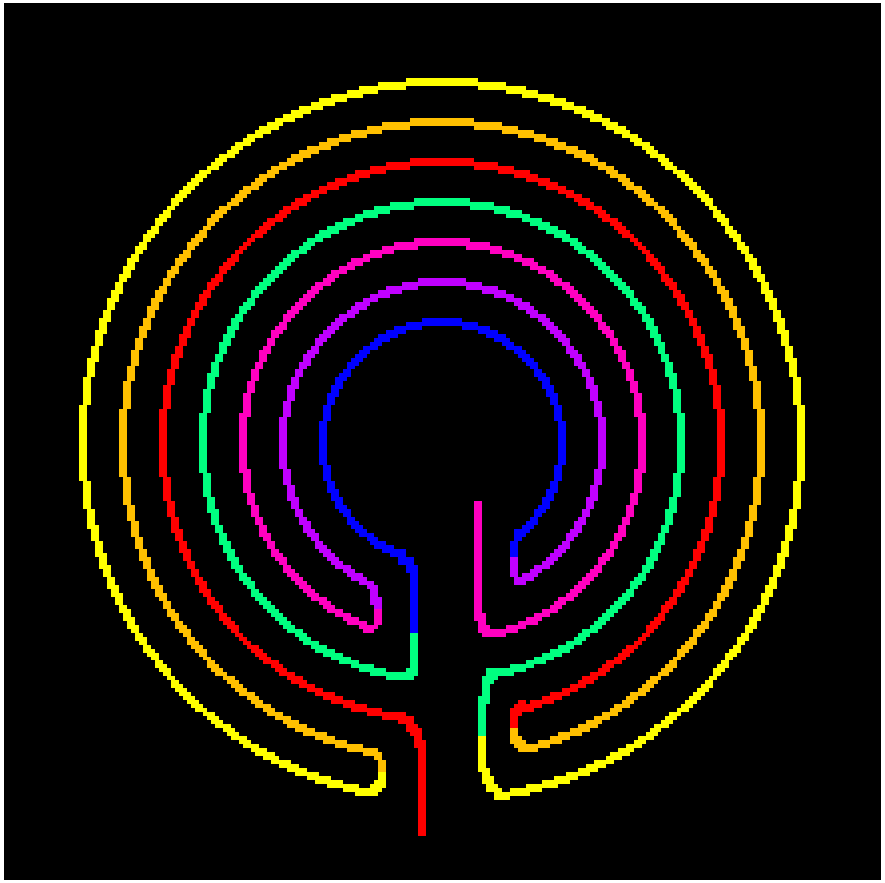
- The Greek key is a pattern often found in Pottery found from ancient Greek archaeology sites

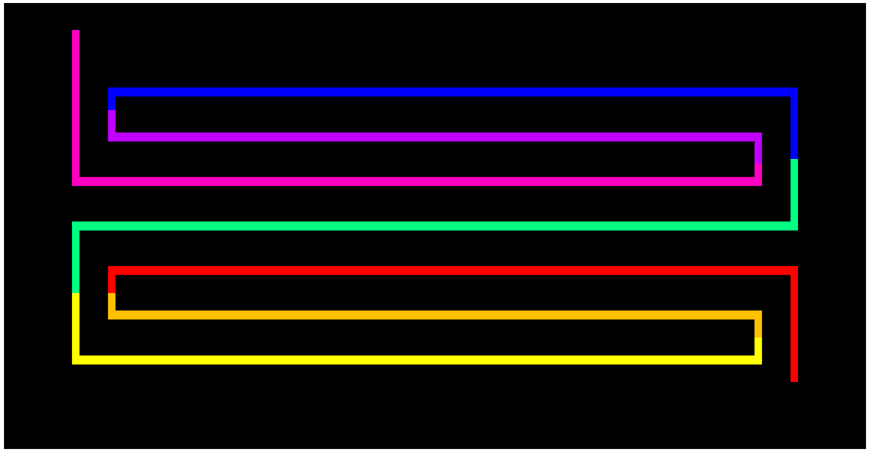




METHOD III: THE GREEK KEY, BUT CURVED

- Cut from the bottom of the maze to the middle, being careful not to cut through a path.



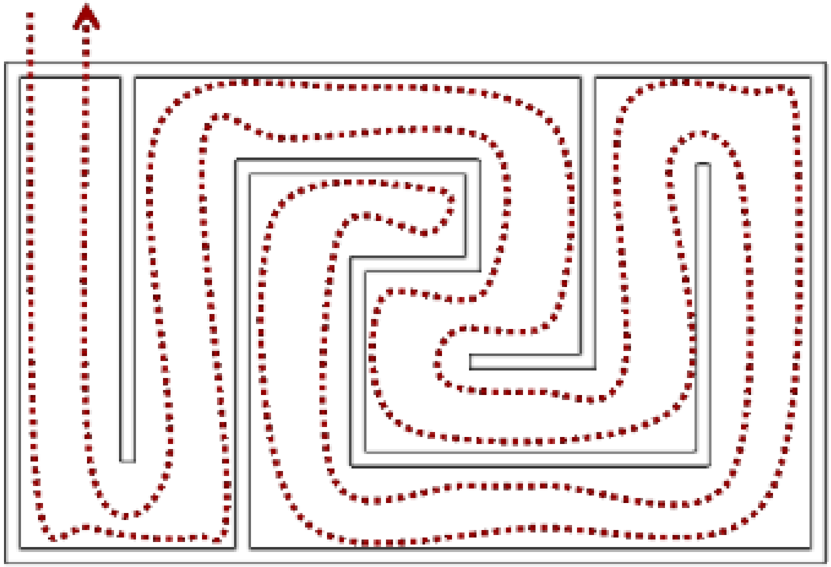


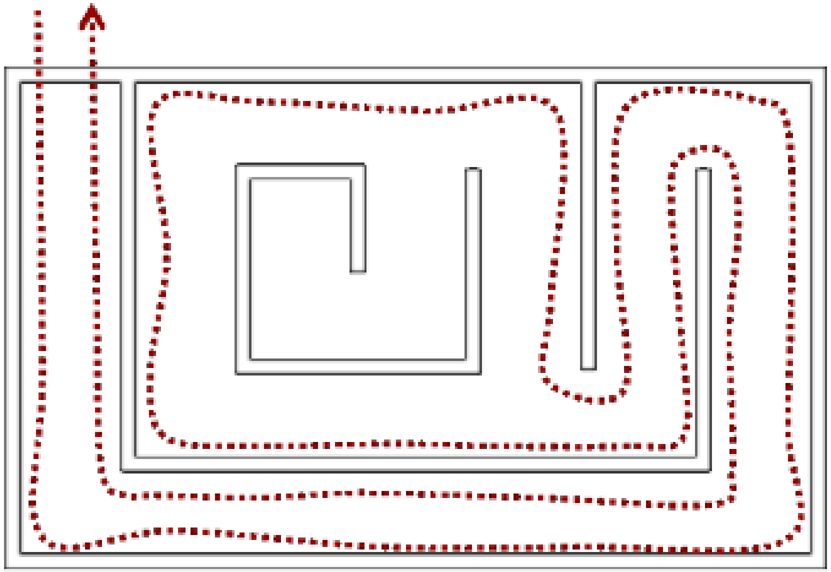
RECAP: BUILDING CRETAN MAZE

1. Build the walls
2. Build a walk, then a wall
3. Elongate a small piece of repetitive design, and build the walls!

PERFECT, SIMPLE, OR BOTH?

- A maze is called **Perfect** or **Simply Connected** (or **Simple**) if it does not contain loops.





1. The maze on the left above is a **Perfect** maze.
2. The maze on the right is not simple; it has a loop (island).

BUILDING RECTANGULAR MAZES

I will now describe two algorithms for building rectangular mazes:

- Recursive Division
- Recursive backtracker

The list is by no means exhaustive!

BUILD WALLS: RECURSIVE DIVISION

Step 1: Take an empty box.

Step 2: Build at random point a wall perpendicular to one side.

Step 3: Choose at random a cell on the wall, and open a one cell-wide hole.

Continue in this manner recursively, until every chamber has a width of one cell in either of the two directions.

MAKE PATHS, NOT WALLS: RECURSIVE BACKTRACKER

Step 1: Build a Grid of walls.

Step 2: Pick a random cell

Step 3: Pick any one of the neighbours at random, and make a path between these two.

Step 4: Look at the unvisited neighbours of this new cell, Repeat Step 3 till you reach a cell where you can't find a unvisited cell.

Step 5(Backtrack):start moving back from this cell, till you find a cell that has unvisited members. Repeat Step 3.

Repeat Step 4 and 5 till all the cells are visited.

OTHER PROPERTIES OF A MAZE

- **Simply connected?**
- **Number of dead-ends** – Measures the number of locations in the maze that have only one way in.
- **Length of longest path** - This is often measured as fraction of (No.of cells required for the shortest path)/(total number of cells).
- **Convolution** - Some metric to measure how often in the maze that the path exits a cell on the opposite side of the way it came in, and how often it turns a corner in a cell. The human brain has an easier job following paths that are straight
- **Distribution of Valency** – No. of ways to enter or exit a cell
- **Complexity** - Related to all the above, this is a measure of the average number of 'decisions' a solver will have make to get from the start to the end. What is the average number of junctions you'll encounter when solving a maze?

THE MAHABHARATA CONNECTION: CHAKRAVYUHA

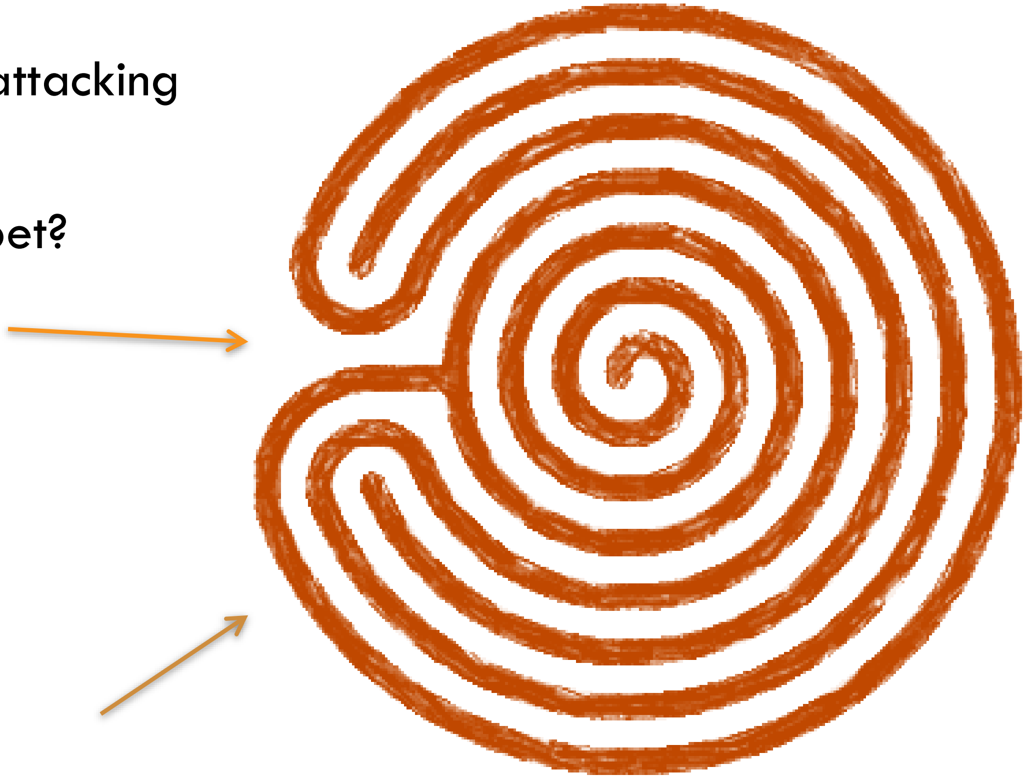
- The Chakravyuha refers to a military formation in the Hindu epic Mahabhara
- Formed on the 13th day of Mahabharc Dronacharya to capture the eldest of Pandava clan, Yudhisthira.

THE MAHABHARATA CONNECTION: CHAKRAVYUHA

- The Chakravyuha was an enormous structure formed entirely of Soldiers.
- It charges towards the enemy army, and traps the soldiers.

SOLVING THE CHAKRAVYUHA

- Imagine, walls made of living, attacking soldiers.
- Which one would be the safe bet?



INTERESTING QUESTION 2:

HOW DO I **ESCAPE** A MAZE?

ESCAPING A SIMPLE MAZE

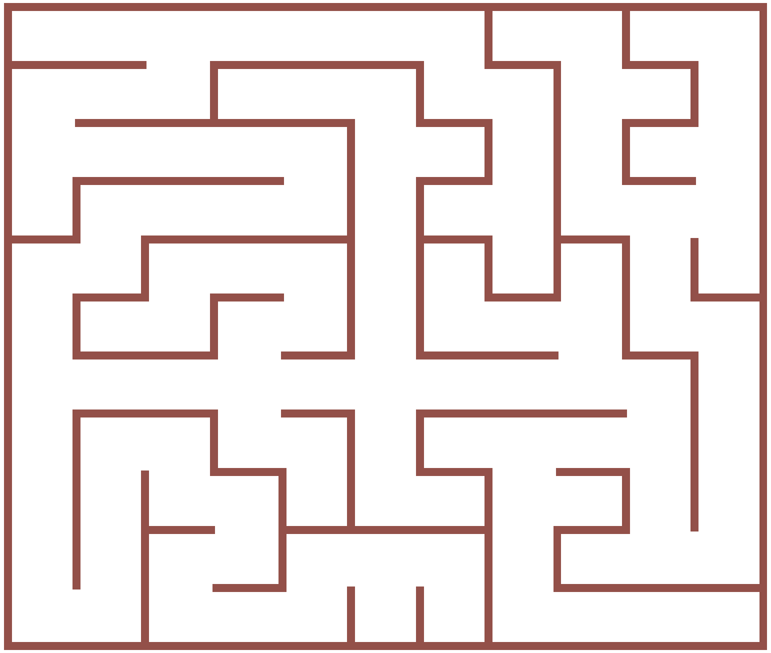
If you're trapped in a Simple maze, the surest way out is the Wall Following:

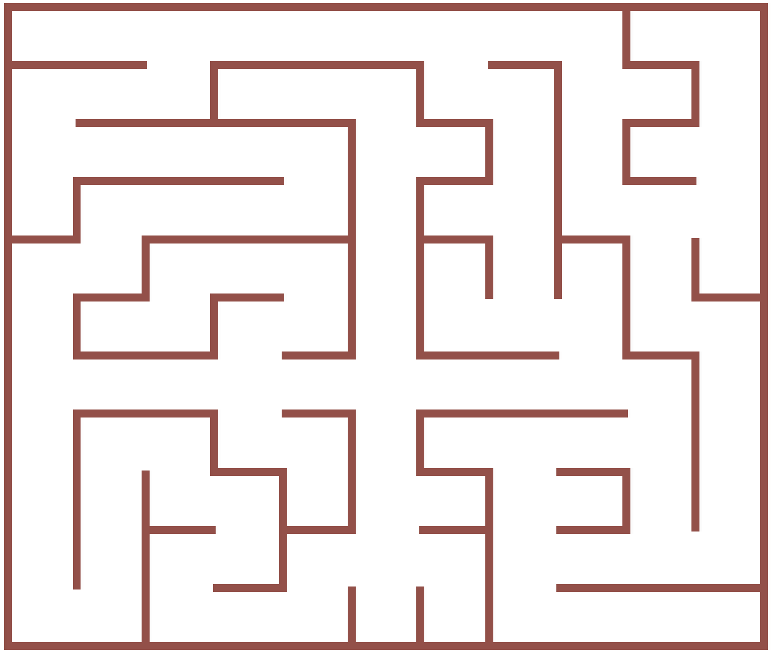
1. Put one arm on the wall next to you.
2. While maintaining this contact, walk in the maze
3. You're guaranteed to stumble upon the exit or the entrance.

However, you may have been trapped in a not simple maze.

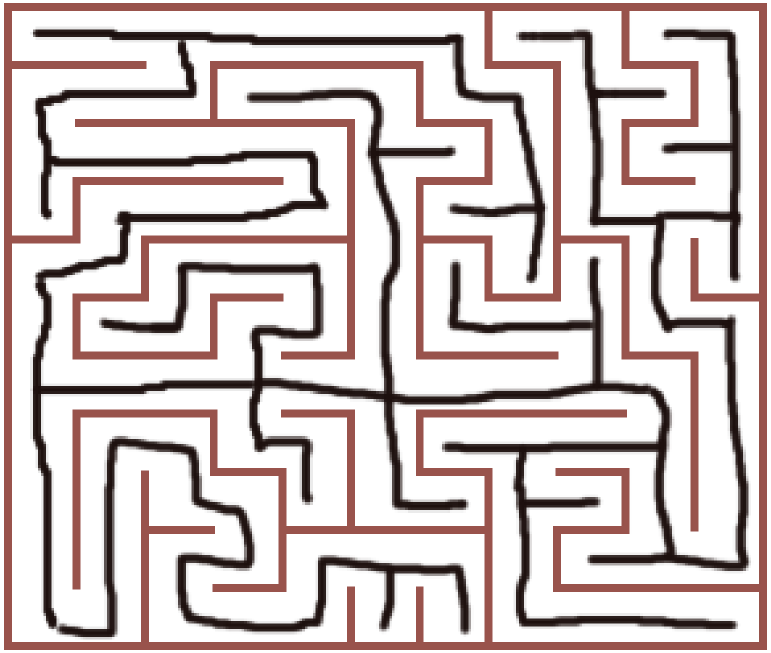
THE WALKS GRAPH

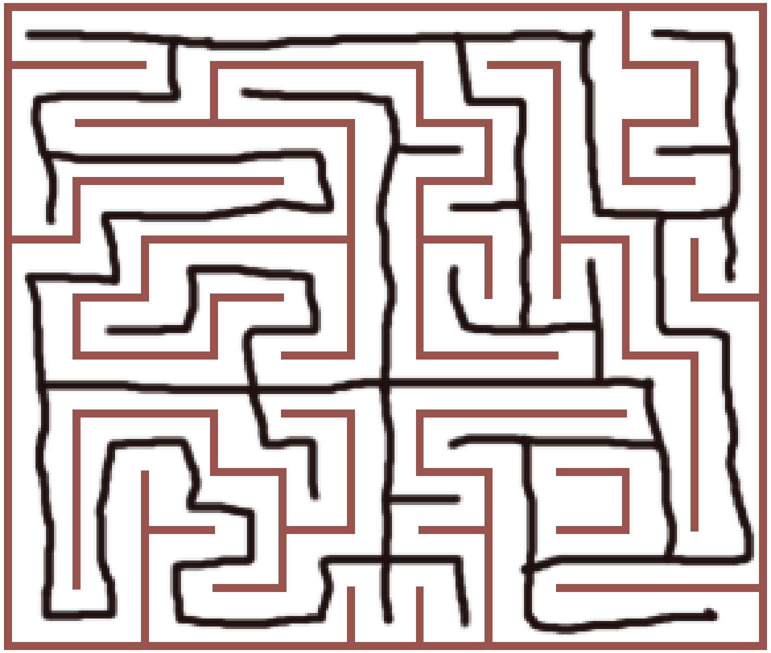
Consider these two mazes



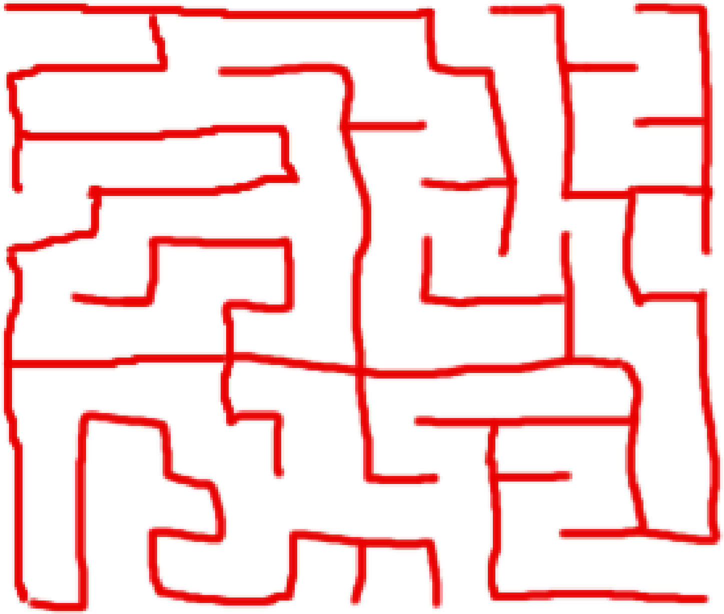


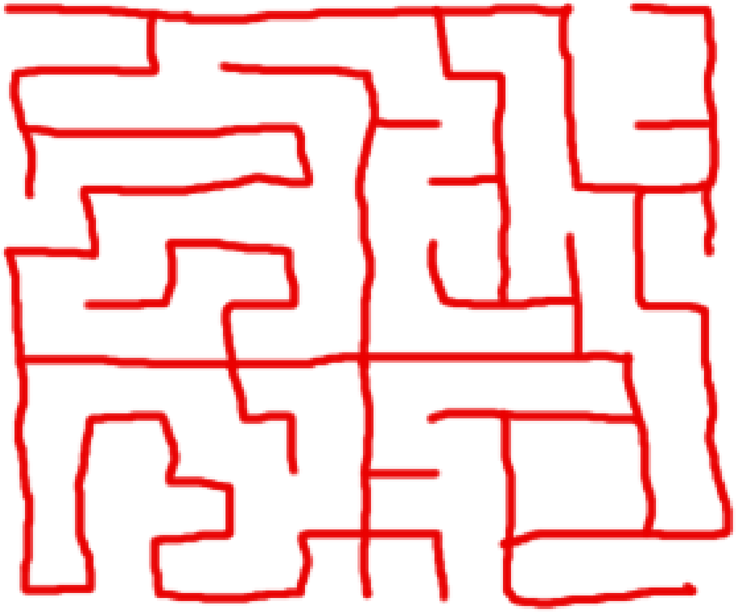
The Walks Graph





THE WALKS GRAPH





Consider each rectangular cell block a point, with a line connecting neighbouring cells if and only if one can walk from one cell to other.

The path to exit, now, is an exercise in Graph Theory!

PROPERTIES OF THE WALK GRAPH

- If there is more than one graph the maze is **not** solvable; this is because there would be no path between the separate trees.
- Note that in general, this graph can have a loop.(commonly called a Tree in Graph Theory)
- For a perfect maze, there will be no loops in this Walks Graph.
- The Walks graph for a Perfect/Simple maze that can be solved is a Spanning Tree.

TREMAUX'S ALGORITHM

- Found by Charles Tremaux around 1899
- This is an efficient method to find the way out of a maze
- Requires: Drawing lines on the floor to mark a path
- Guaranteed to work for all mazes that have well-defined passages.

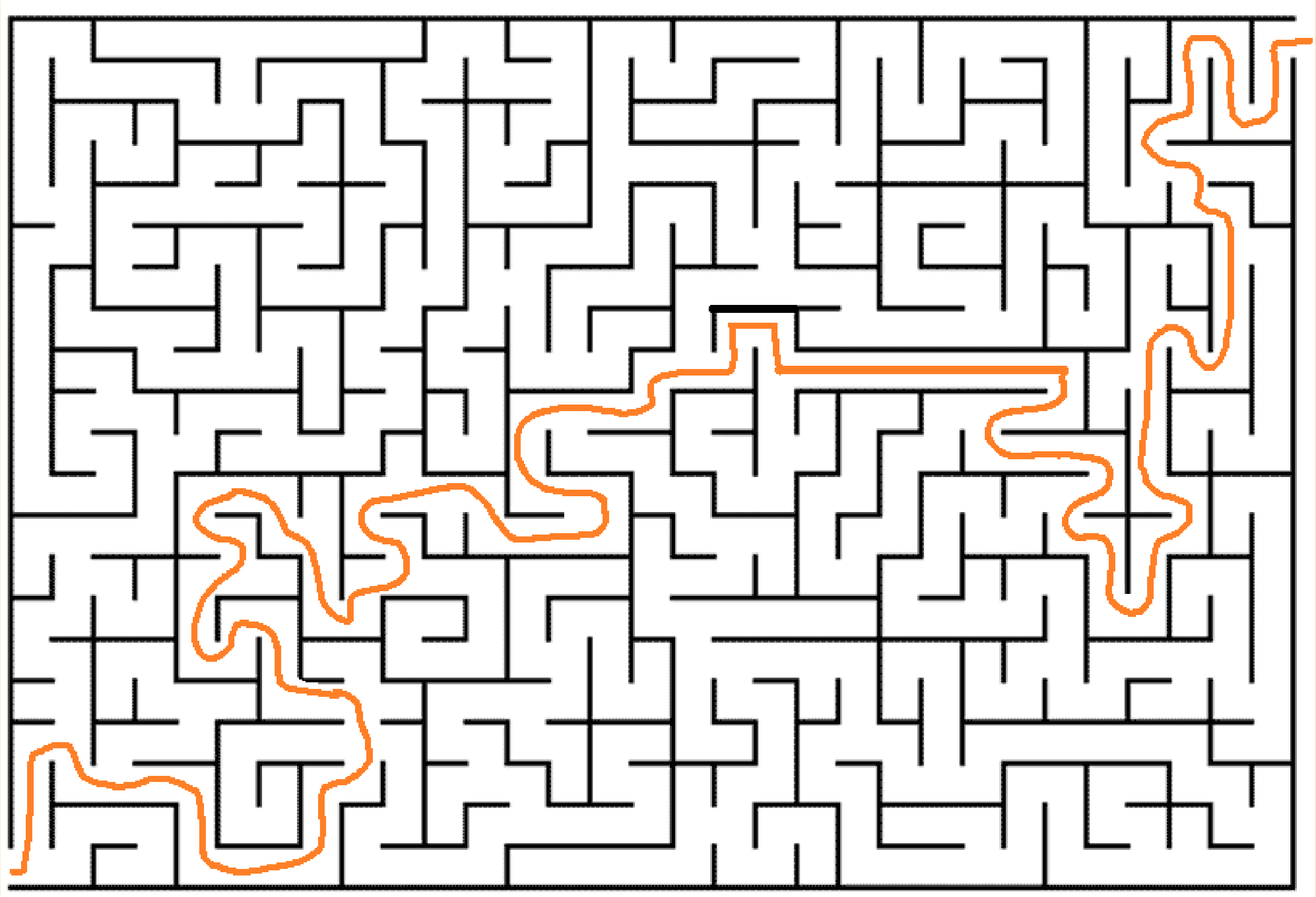
TREMAUX'S ALGORITHM

1. A path is either unvisited, marked once or marked twice.
2. Every time a direction is chosen it is marked by drawing a line on the floor (from junction to junction).
3. In the beginning a random direction is chosen (if there is more than one).
4. On arriving at a junction that has not been visited before (no other marks), pick a random direction that is not marked (and mark the path).

TREMAUX'S ALGORITHM (CONTINUED)

1. When arriving at a marked junction:
    1. If your current path is marked only once then turn around and walk back (and mark the path a second time).
    2. If this is not the case, pick the direction with the fewest marks (and mark it, as always).
2. When you finally reach the end, paths marked exactly once will indicate a direct way back to the start.
3. If there is no exit, this method takes you back to the start where all paths are marked twice. In this case each path is walked down exactly twice, once in each direction.

SLIME-Y WAY OUT



THANK YOU!

A UC Students' Mathematics Colloquium Talk

Questions?

SOLVING THE CHAKRAVYUHA

You hit the person directly in front of you, the Vyuha recovers:

Instead, you hit the two people on either side of this as fast as possible.

The recovery process makes sure you get a big region to move ahead, for a very small period of time.

This formation was supposed to be nearly invincible as only Krishna and Arjuna could defeat the formation, both entering it and leaving it, and they were not present at this battle.

But Arjuna's son, Abhimanyu, knew how to enter the formation, but not leave it. He died in the centre of the formation.