Lecture 8.

PID control

1. The role of P, I, and D action

2. PID tuning

Industrial process control (1920 . . . today)

Feedback control is used to improve the process performance:

- static performance: for constant reference, the output asymptotically converges to the desired value, regardless of load disturbance.

- dynamic performance: ensure proper transient behavior during step changes

Proportional control is not sufficient: feedback gain must be frequency dependent !

$\Rightarrow$ controller is itself a dynamical system $C(s)$

PID control

The magical three-term controller (99% of industrial controllers):

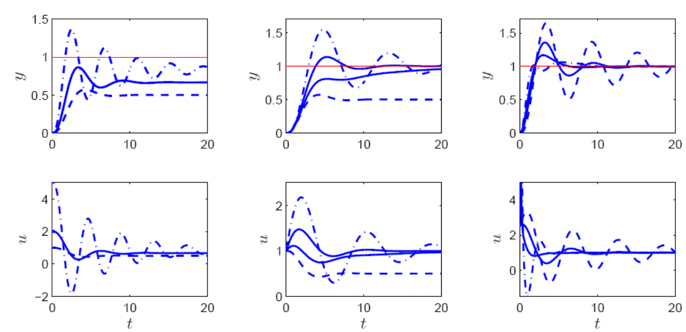
$$u = \underbrace{K_P e}_{\text{proportional}} + \underbrace{K_I \int e}_{\text{integral}} + \underbrace{K_D \dot{e}}_{\text{derivative}}$$

Frequency interpretation:

$$C(s) = K_P + \frac{K_I}{s} + K_D s$$

K_I dominates at low frequencies while K_D dominates at high frequencies

Insights about PID actions



P I D

Proportional control

Suppose $P(s) = \frac{1}{\tau s+1}$ and $C(s) = K$.

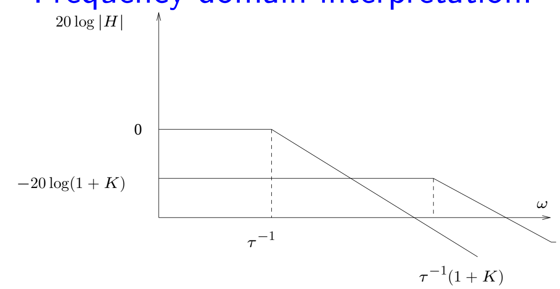
Closed-loop transfer function: $\frac{Y}{R} = \frac{K}{1+K}\frac{1}{1+\frac{\tau}{1+K}s}$

The static error and the time constant are divided by $(1+K)$

Feedback modifies the location of the poles.

More generally:
rlocus shows how closed-loop poles move with K.

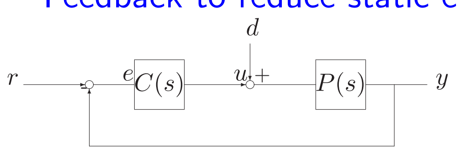
Frequency domain interpretation:



Feedback distributes the loop transfer from u to y: trading gain for bandwidth ...

Increasing the bandwidth typically reduces the phase margin. $\Rightarrow$ Proportional control usually increases oscillations and overshoot in step response.

Feedback to reduce static errors



$$Y = \frac{CP}{1+CP}R + \frac{P}{1+CP}D$$

Asymptotic value of step response is given by $\lim_{t\to\infty} y(t) = \lim_{s\to\infty} sY(s)$ if ROC($sY(s)$) includes the imaginary axis.

For step inputs $R(s) = \frac{r_0}{s}$ and $D(s) = \frac{d_0}{s}$, we obtain

$$\begin{aligned} y(\infty) &= \frac{C(0)G(0)}{1+C(0)G(0)}r_0 + \frac{G(0)}{1+C(0)G(0)}d_0 \\ &\approx r_0 \text{ if } |1+C(0)G(0)| = |S(0)|^{-1} >> 0 \end{aligned}$$

The role of integral feedback

- Integral action $\Rightarrow C(s)$ has a pole at $s = 0$.

$$\Rightarrow y(\infty) = 1r_0 + 0d_0 \;\; !!$$

- a general feature in feedback loops with constant external signals: modulo stability (static regime may not exist!), blocks that contain a pole at $s = 0$ asymptotically force their input to zero.

- the property is independent of system parameters and linearity assumption!

Integral feedback

- Internal model principle: for perfect tracking (rejection), the controller must include a model of the reference (disturbance) signal. For step signals, the model is $\frac{1}{s}$.

- the controller pole at $s = 0$ causes a phase lag of $90\deg$ in the loop transfer $\Rightarrow$ integral action usually reduces the phase margin and increases overshoot

Integral feedback as automatic bias adjustment

$$u(t) = Ke(t) + u_{ff}$$

compensates for static error if

$$u_{ff} = \frac{1}{1 + KP(0)}$$

With a PI control $u(t) = Ke(t) + K_I \int e$, the bias is automatically adjusted.

Derivative feedback

- PD control anticipates future errors: $u(t) = ke(t) + k_d\dot{e}(t) = k(e(t) + T_d\dot{e}(t)) \approx ke(t + T_d)$

- Think of pendulum model: u is a torque, y is a position $\Rightarrow$ derivative feedback acts as velocity feedback, i.e. as a friction force

- derivative control adds a phase lead of $+90\deg$ in the loop transfer $\Rightarrow$ usually increases the phase margin and damps the oscillations of the step response

Derivative feedback amplifies noise

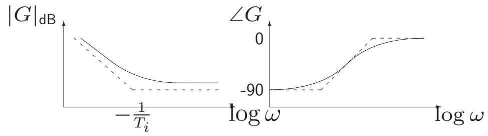
- the derivative action acts as a noise amplifier $\Rightarrow$ requires good sensors

- the derivative is usually filtered: sT_d is replaced by $\frac{sT_d}{1+sT_d/N}$ to limit the high-frequency gain amplification

- in a digital implementation, discretizing the derivative feedback can be interpreted as filtering

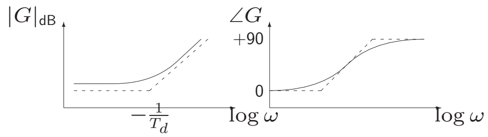
- the derivative action is often not used in process control

PI controller : $1 + \frac{1}{sT_i}$



Good for static performance but phase lag $\Rightarrow$ choose $\frac{1}{T_i} \ll \omega_B$

PD controller : $1 + sT_d$



Phase lead is beneficial to stability margins but the closed-loop bandwidth is increased $\Rightarrow$ limit $\frac{1}{T_d}$

Set-point weighting

PID control is often implemented in the form

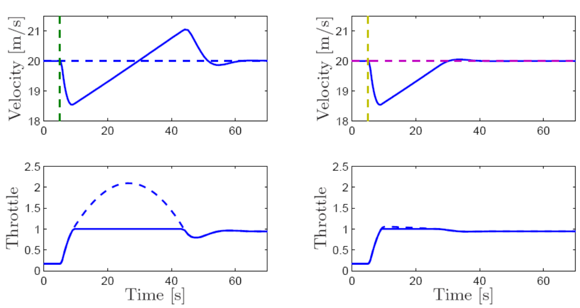
$$u(t) = k(\beta r(t) - y(t)) + k_i \int_0^t (r - y)(\tau) d\tau + k_d(\gamma \dot{r}(t) - \dot{y}(t))$$

which corresponds to a controller with two degrees of freedom: $U(s) = C(s)(R - Y) + F(s)R$

$C(s)$ is tuned for disturbance rejection and $F(s)$ is adjusted for reference tracking

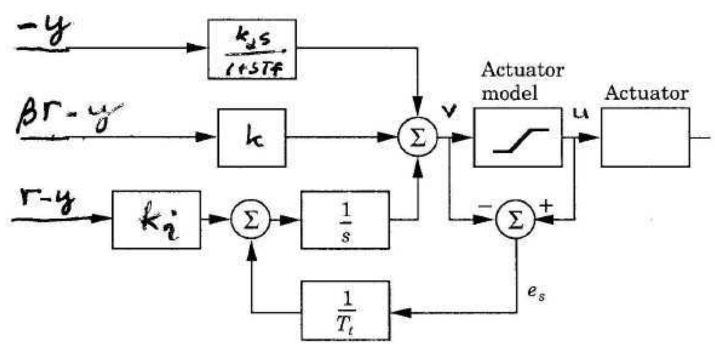
Antiwindup

Due to actuator limitations, integral action is never implemented without an antiwindup mechanism.



The antiwindup resets the integral action when the input saturates.

A general structure for PID control



PID tuning

- control parameter tuning can be interpreted as pole placement for low-order models
- control parameter tuning can be interpreted as loop shaping
- Special tuning methods are model-free: the tuning is instead based on a few open-loop experiments
- Many industrial controllers are mistuned because they are tuned once forever $\Rightarrow$ adaptive control or auto-tuning methods

PI control of a first order system

Process transfer function

$$P(s) = \frac{K}{\tau s + 1}$$

Controller transfer function

$$C(s) = k_P + \frac{k_I}{s}$$

Closed loop transfer function from reference r to output y:

$$\frac{Y(s)}{R(s)} = \frac{CP}{1+CP} = \frac{K(k_P s + k_I)}{\tau s^2 + (1 + K k_P)s + K k_I}$$

Closed loop system of second order; two parameters allow to 'place' the poles of the closed loop system

PI control of a first order system

Example of design: cancel the (slow) process pole, i.e. $k_I = \tau k_P$ and place the closed loop pole $\Rightarrow$ no overshoot, assignment of time constant, no static error.

Perhaps not the best choice: one should also pay attention to the closed loop transfer function from the disturbance d to the output y:

$$\frac{Y(s)}{D(s)} = \frac{P}{1+CP} = \frac{K}{\tau s^2 + (1 + K k_P)s + K k_I}$$

PD control of a second order system

Process transfer function

$$P(s) = \frac{b}{s^2 + a_1 s + a_2}$$

Controller transfer function

$$C(s) = k_P + k_D s$$

Closed loop transfer function from reference r to output y:

$$\frac{Y(s)}{R(s)} = \frac{CP}{1+CP} = \frac{b(k_D s + k_P)}{s^2 + (a_1 + b k_D)s + a_2 + b k_P}$$

Closed loop system of second order; two parameters allow to 'place' the poles of the closed loop system.

PD control of a second order system

The derivative action will damp the oscillations ($\to \zeta$)

The closed loop static gain is $\frac{bk_P}{bk_P+a_2}$. But increasing k_P may be harmful to bandwidth.

The zero $s = -k_P/k_D$ may cause considerable overshoot if it is closer to the origin than the dominant poles.

PID control of a second order system

Process transfer function

$$G(s) = \frac{b}{s^2 + a_1 s + a_2}$$

Controller transfer function

$$C(s) = k_P + k_D s + \frac{k_I}{s}$$

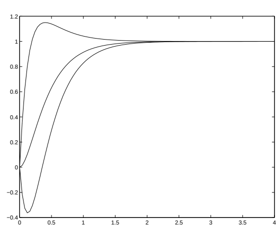
Closed loop transfer function from reference r to output y:

$$\frac{Y(s)}{R(s)} = \frac{CG}{1+CG} = \frac{b(k_D s^2 + k_P s + k_I)}{s^3 + (a_1 + bk_D)s^2 + (a_2 + bk_P)s + bk_I}$$

Closed loop system of third order; three parameters allow to 'place' the poles of the closed loop system. Instability may occur for bad choice of control parameters !

A zero in the transfer function affects the step response

$H_{new} = (z^{-1}s + 1)H_{old}$



$s_{new}(t) = s_{old}(t) + \frac{1}{z}\frac{ds_{old}}{dt}(t)$
zero in left half plane: increases the overshoot as it approaches imaginary axis.
zero in right-half plane: responsible for inverse response phenomenon.

The location of a zero is NOT affected by feedback. In contrast, feedforward action changes the location of zeros. Interest of controllers with two degrees of freedom.

Loop shaping with PI control

Two parameters allow to fix one point on the Nyquist curve.

$$C(s) = K \frac{s + \frac{1}{T_i}}{s}$$

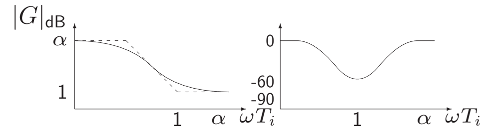
Choose K to assign the crossover frequency ω_c with sufficient phase margin.

Maximize $\frac{1}{T_i}$ under the constraint $\phi_m \geq \phi_m^{min}$.

Note: A larger T_i slows down the disturbance rejection.

Phase-lag compensation $K\frac{T_i s+1}{T_i s+\frac{1}{\alpha}}, \quad \alpha > 1$

$$= \alpha K \frac{T_i s + 1}{\alpha T_i s + 1} = \frac{K'(s+z)}{s+p}, \quad z \ll p$$

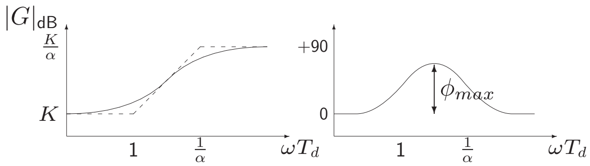


Static gain is increased by factor α. Phase lag will not affect the phase margin if $\frac{1}{T_i} \ll \omega_C$.

Synthesis guidelines: adjust K to assign ω_C with sufficient phase margin. Evaluate the necessary reduction of static error to choose α; Maximize T_i without degrading the phase margin.

Loop shaping with phase-lead compensator

$$\begin{aligned} C(s) &= K(T_d s + 1) \times \tfrac{1}{\alpha T_d s + 1}, \quad \alpha < 1 \\ &= \text{PD control} \times \text{Low-pass filter} \\ &= K' \tfrac{s+z}{s+p}, \quad p \ll z \end{aligned}$$



Limitation of derivative action at high frequency.

Phase-lead compensation

Useful formulas:

$$\log \omega_{max} = \frac{1}{2}\left(\log \frac{1}{T_d} + \log\left(\frac{1}{\alpha T_d}\right)\right)$$

$$\phi_{max} = \arcsin\left(\frac{1-\alpha}{1+\alpha}\right)$$

$$\omega_{max} = \sqrt{|z||p|}$$

Synthesis guidelines: Choose ω_c to assign closed-loop bandwidth. Evaluate the necessary phase lead at ω_c (not more than $60^\circ \Rightarrow \frac{1}{\alpha} = 16$) and adjust T_d to place the maximal phase lead at ω_C. Choose K to have $|L(j\omega_c)| = 1$.

Summary of lecture

- PID is widely used in the industry.
- Three parameters roughly correspond to three performance criteria: static performance (I), closed-loop bandwidth (P), and stability margins (D).
- Implementation of PID control includes antiwindup, filtering of the derivative action, and set-point weighting.