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Proof: $\sum_r \delta[r]x[n-r] \stackrel{\text{(i)}}{=} x[n]$ (i) all terms zero except $r = 0$.

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Proof $(1) \Rightarrow (2)$:

Define $x[n] = \begin{cases} 1 & h[-n] \geq 0 \\ -1 & h[-n] < 0 \end{cases}$

then $y[0] = \sum x[0-n]h[n] = \sum |h[n]|$.

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(2) $h[n]$ is absolutely summable, i.e. $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$

(3) $H(z)$ region of absolute convergence includes $|z| = 1$.

Proof $(1) \Rightarrow (2)$:

Define $x[n] = \begin{cases} 1 & h[-n] \geq 0 \\ -1 & h[-n] < 0 \end{cases}$

then $y[0] = \sum x[0-n]h[n] = \sum |h[n]|$.

But $|x[n]| \leq 1 \forall n$ so BIBO $\Rightarrow y[0] = \sum |h[n]| < \infty$.

Proof $(2) \Rightarrow (1)$:

Suppose $\sum |h[n]| = S < \infty$ and $|x[n]| \leq B$ is bounded.

Then $|y[n]| = \left|\sum_{r=-\infty}^{\infty} x[n-r]h[r]\right|$

$\leq \sum_{r=-\infty}^{\infty} |x[n-r]| \, |h[r]|$

BIBO Stability

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BIBO Stability: Bounded Input, $x[n] \Rightarrow$ Bounded Output, $y[n]$

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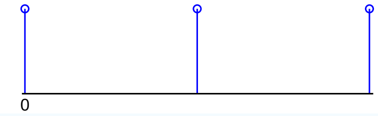

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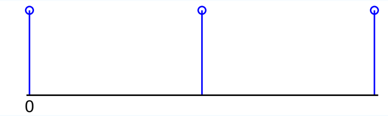

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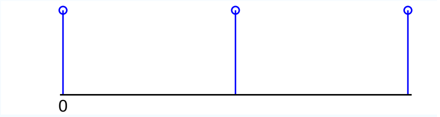

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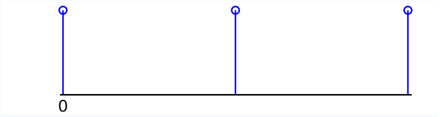

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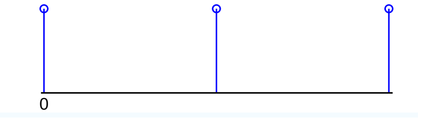

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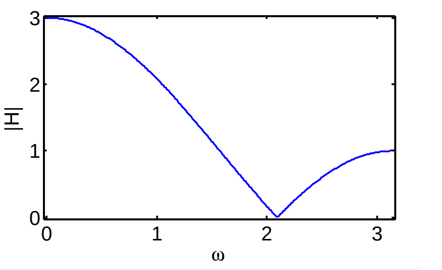

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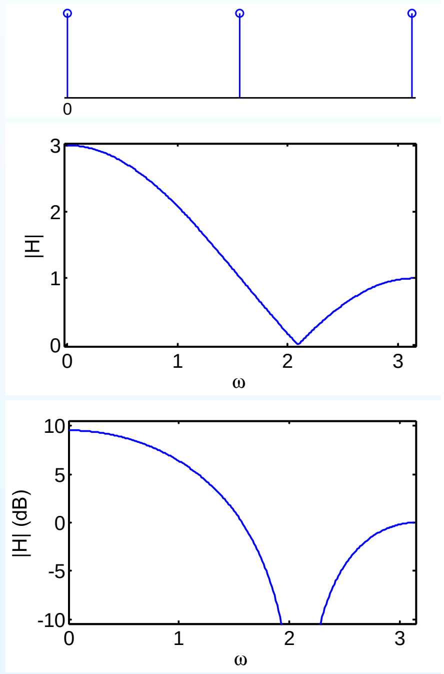

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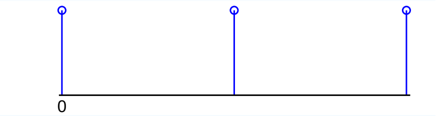

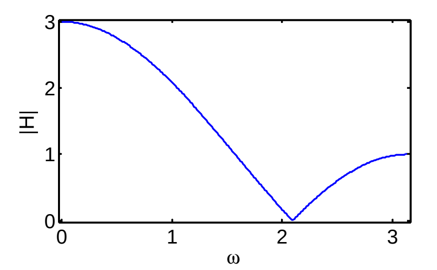

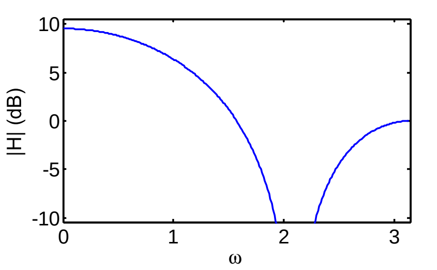

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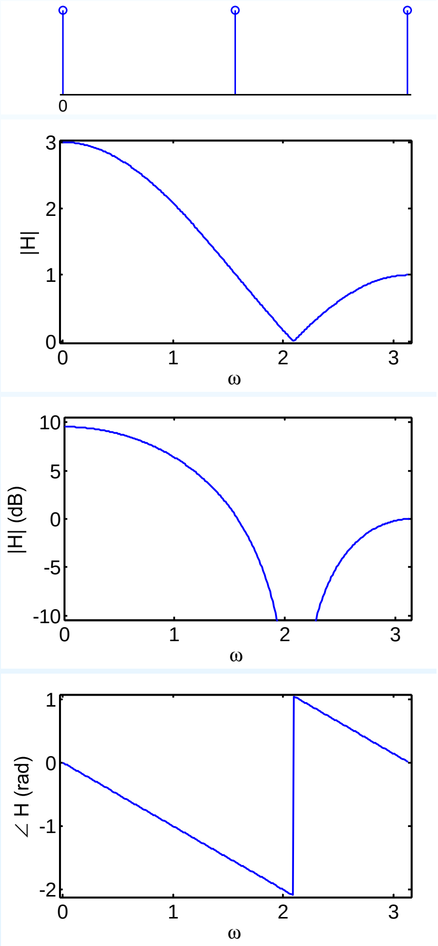

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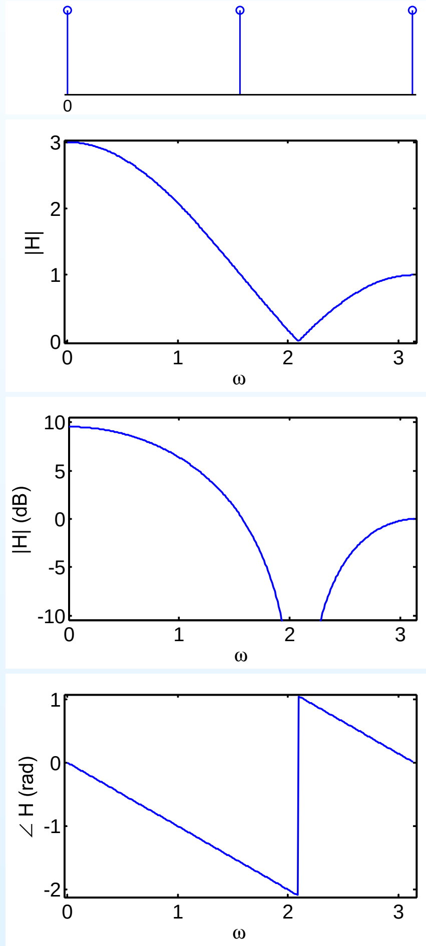

Sign change in $(1 + 2\cos\omega)$ at $\omega = 2.1$ gives

(a) gradient discontinuity in $|H(e^{j\omega})|$

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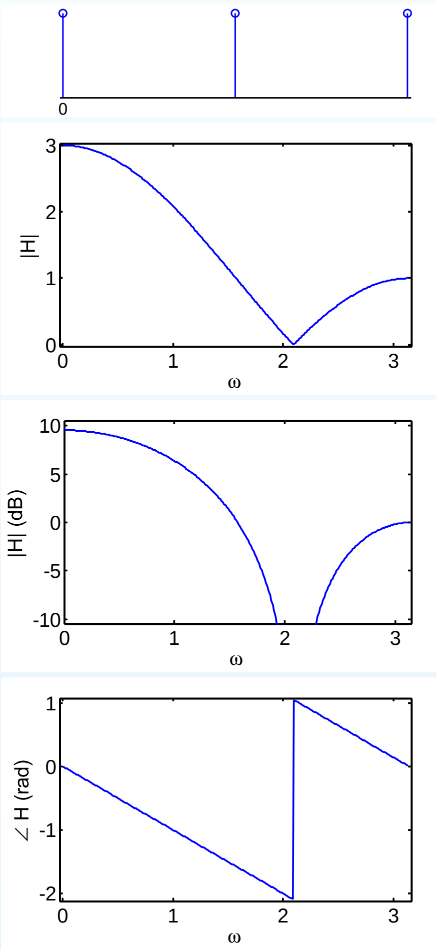

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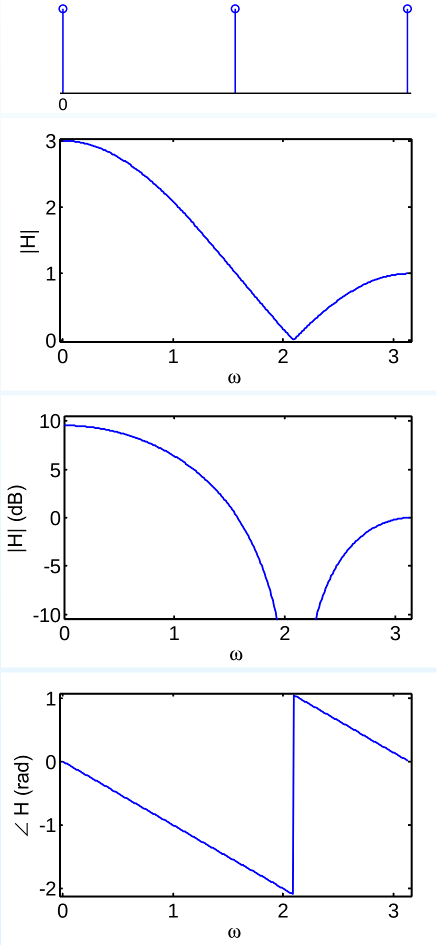

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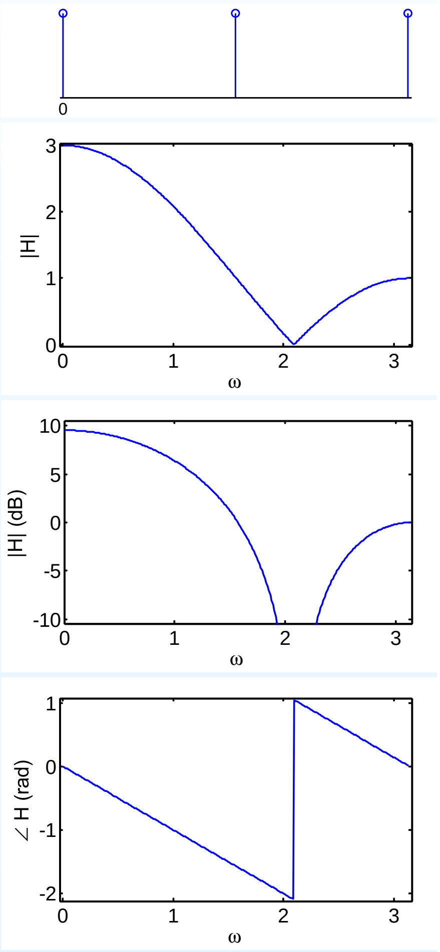

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Causality

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i.e. output at time n depends only on inputs up to time n.

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All the systems we will deal with are causal.

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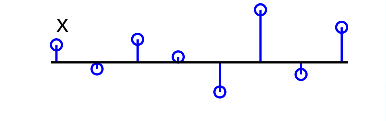

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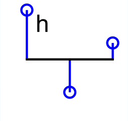

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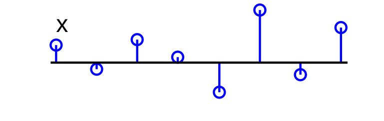

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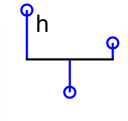

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Convolution Complexity

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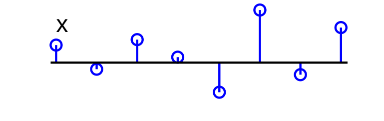

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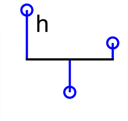

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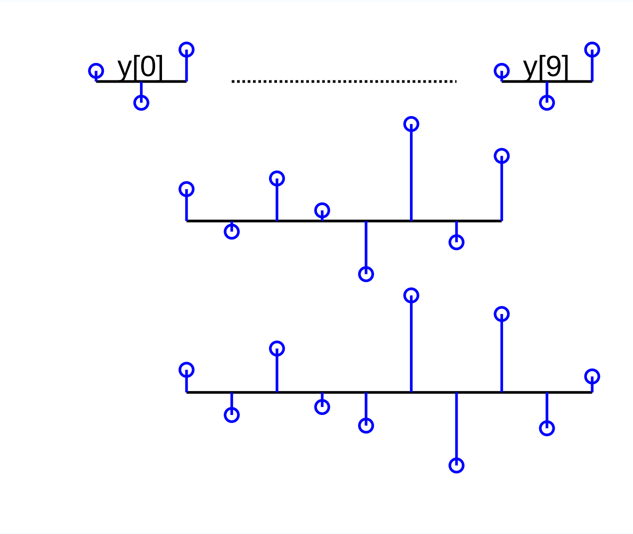

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$$N = 8, M = 3$$
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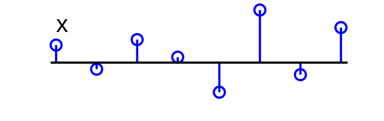

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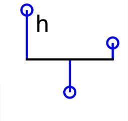

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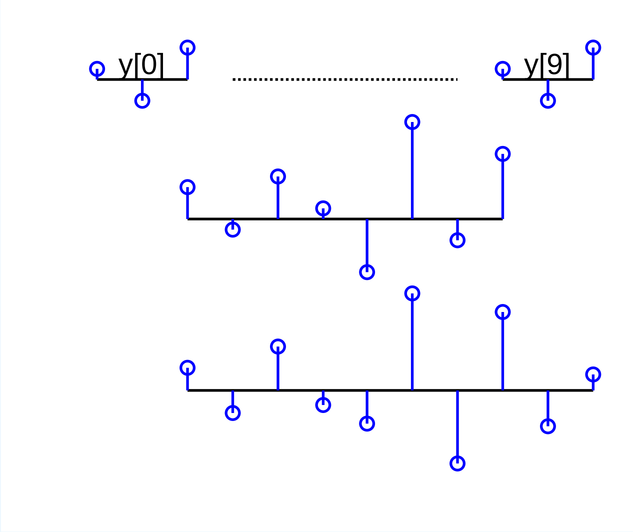

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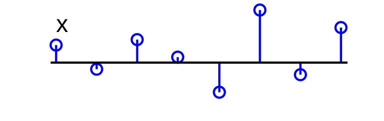

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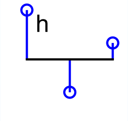

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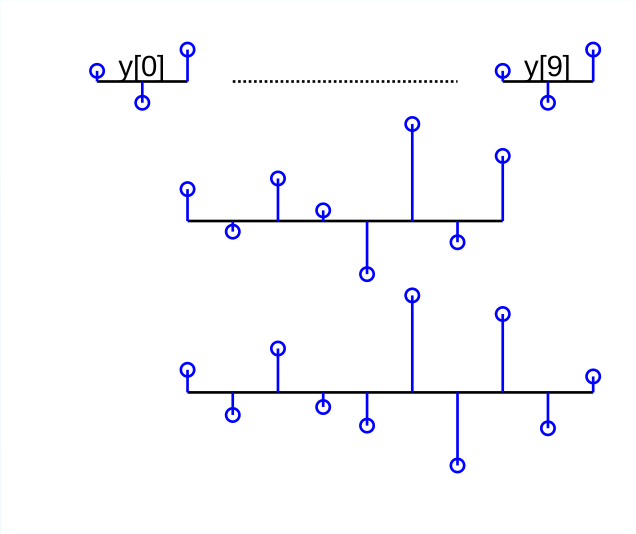

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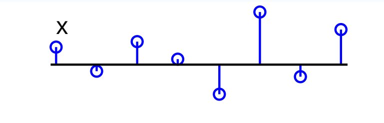

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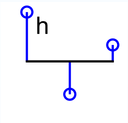

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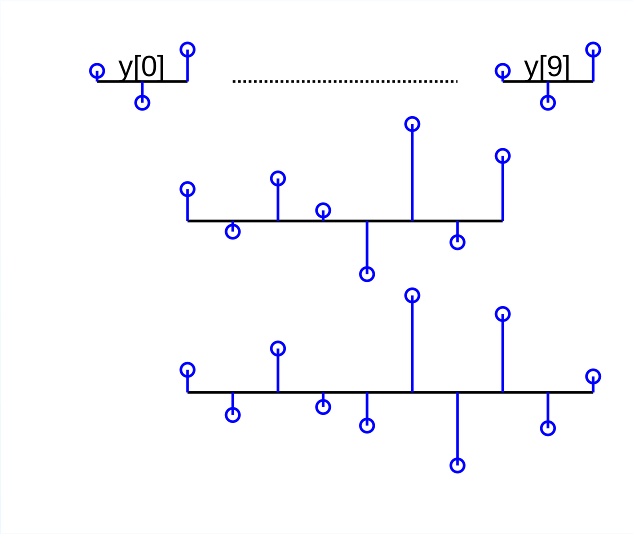

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Algebraically:

$$x[n-r] \neq 0 \Rightarrow 0 \le n-r \le N-1$$

Convolution Complexity

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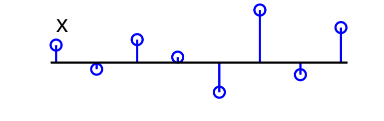

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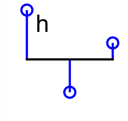

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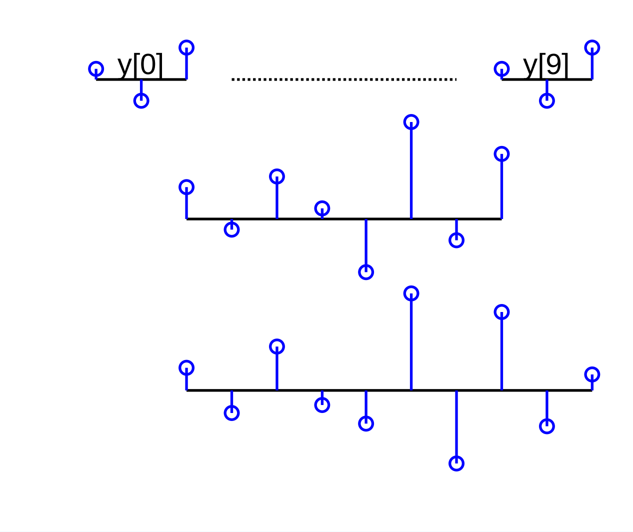

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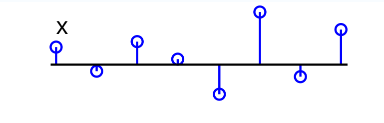

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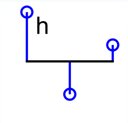

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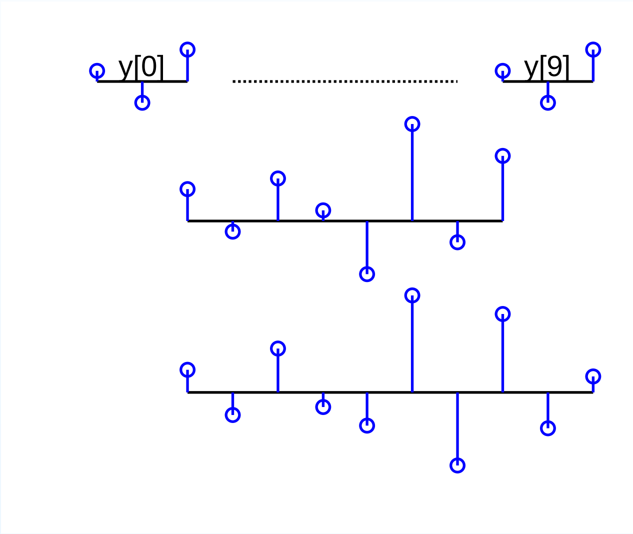

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Hence: $y[n] = \sum_{r=\max(0,n+1-N)}^{\min(M-1,n))} h[r]x[n-r]$

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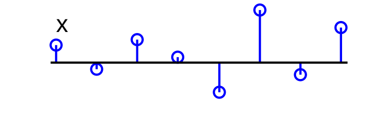

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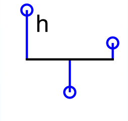

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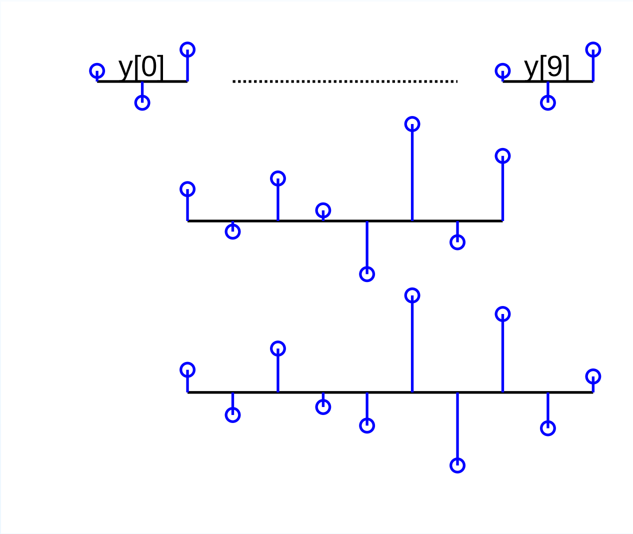

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Hence: $y[n] = \sum_{r=\max(0,n+1-N)}^{\min(M-1,n))} h[r]x[n-r]$

We must multiply each $h[n]$ by each $x[n]$ and add them to a total

$\Rightarrow$ total arithmetic complexity ($\times$ or $+$ operations) $\approx 2MN$

Circular Convolution

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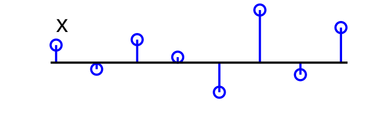
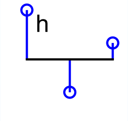
$y_{\circledast}[n] = x[n] \circledast_N h[n]$: circ convolve $x[0:N-1]$ with $h[0:M-1]$



$\circledast_N$



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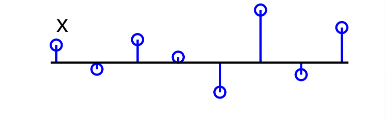

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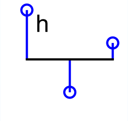

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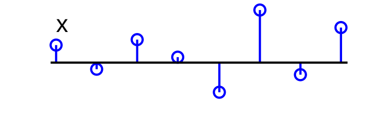

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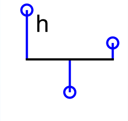

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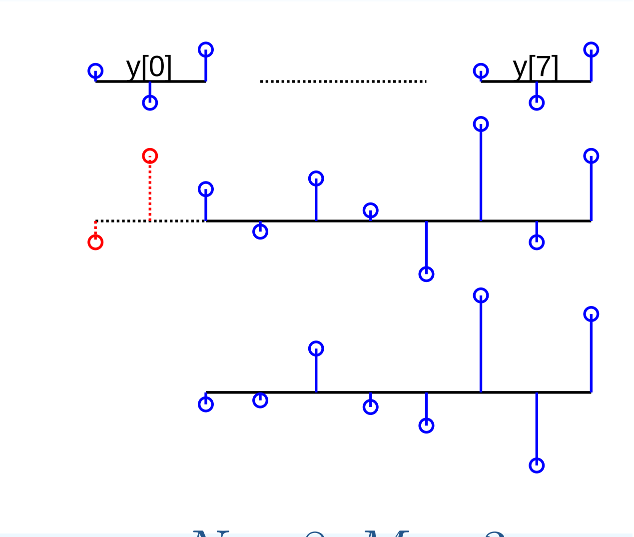

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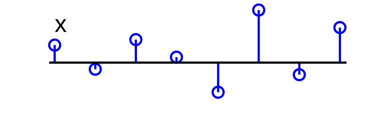

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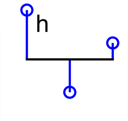

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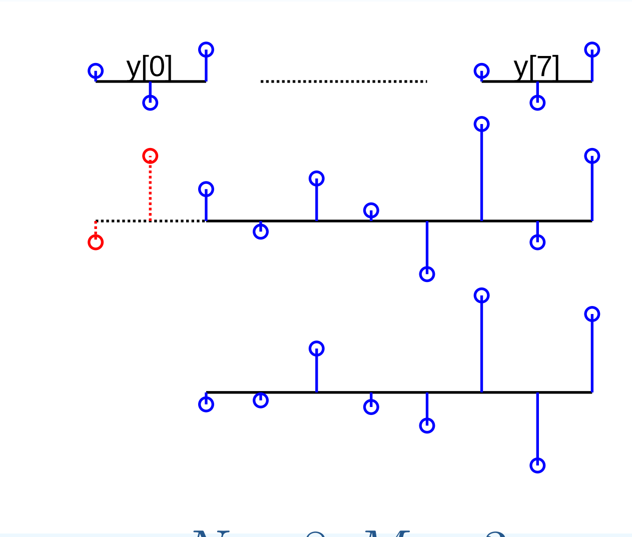

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$y_{\circledast_N}[n]$ has period N

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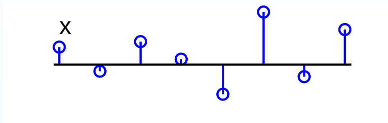

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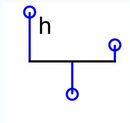

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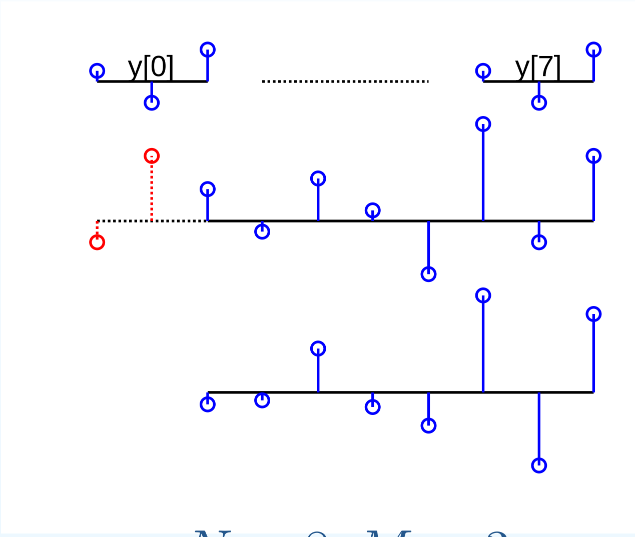

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$\Rightarrow y_{\circledast_N}[n]$ has N distinct values

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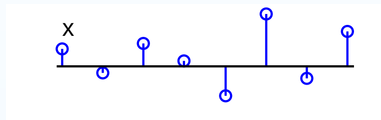

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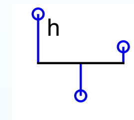

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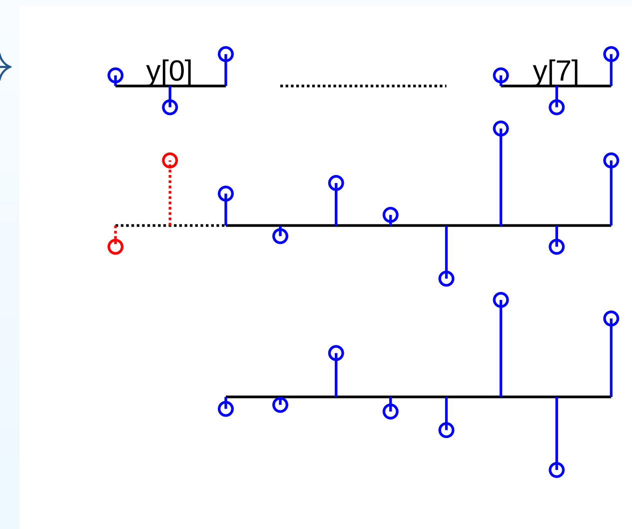

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$\Rightarrow y_{\circledast_N}[n]$ has N distinct values

- Only the first $M-1$ values are affected by the circular repetition:

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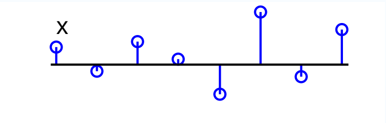

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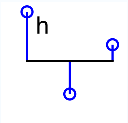

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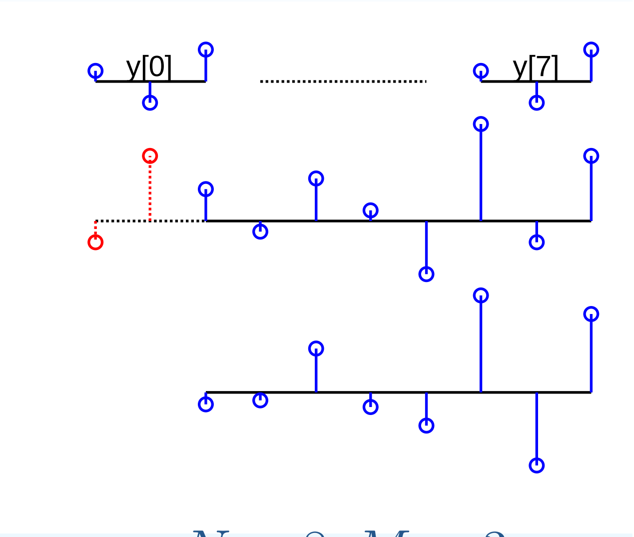

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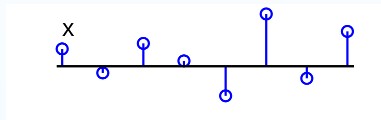
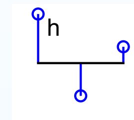
- Only the first $M-1$ values are affected by the circular repetition:
  $y_{\circledast_N}[n] = y[n]$ for $M-1 \le n \le N-1$

Circular Convolution

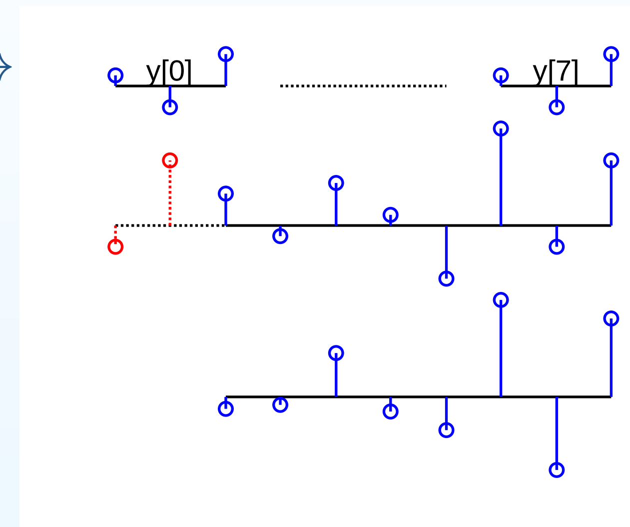
$y_{\circledast}[n] = x[n] \circledast_N h[n]$: circ convolve $x[0:N-1]$ with $h[0:M-1]$



$\circledast_N$



$\rightarrow$



Convolution sum:

$$y_{\circledast_N}[n] = \sum_{r=0}^{M-1} h[r]x[(n-r)_{\text{mod } N}]$$

$y_{\circledast_N}[n]$ has period N

$\Rightarrow y_{\circledast_N}[n]$ has N distinct values

$N = 8$, $M = 3$

- Only the first $M-1$ values are affected by the circular repetition:

  $$y_{\circledast_N}[n] = y[n] \text{ for } M-1 \le n \le N-1$$

- If we append $M-1$ zeros (or more) onto $x[n]$, then the circular repetition has no effect at all and:

  $$y_{\circledast_{N+M-1}}[n] = y[n] \text{ for } 0 \le n \le N+M-2$$

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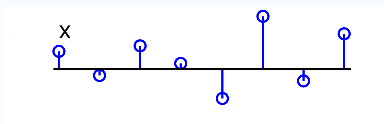
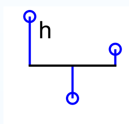
Circular Convolution

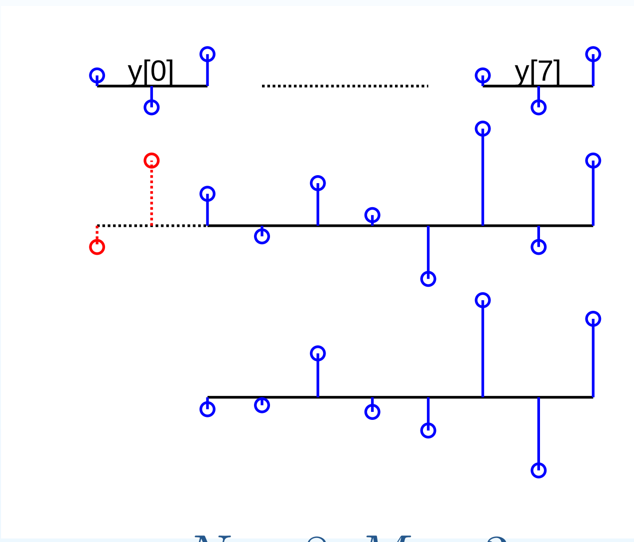
$y_{\circledast}[n] = x[n] \circledast_N h[n]$: circ convolve $x[0:N-1]$ with $h[0:M-1]$



$\circledast_N$



$\rightarrow$



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  $y_{\circledast_N}[n] = y[n]$ for $M-1 \le n \le N-1$
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Circular convolution is a necessary evil in exchange for using the DFT

Frequency-domain convolution

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Idea: Use DFT to perform circular convolution - less computation

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(1) Choose $L \geq M + N - 1$ (normally round up to a power of 2)

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(1) Choose $L \geq M + N - 1$ (normally round up to a power of 2)

(2) Zero pad $x[n]$ and $h[n]$ to give sequences of length L: $\tilde{x}[n]$ and $\tilde{h}[n]$

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Frequency-domain convolution

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(4) $y[n] = \tilde{y}[n]$ for $0 \leq n \leq M + N - 2$.

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Arithmetic Complexity:

DFT or IDFT take $4L\log_2 L$ operations if L is a power of 2
(or $16L\log_2 L$ if not).

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Arithmetic Complexity:

DFT or IDFT take $4L\log_2 L$ operations if L is a power of 2

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Total operations: $\approx 12L\log_2 L \approx 12\,(M+N)\log_2\,(M+N)$

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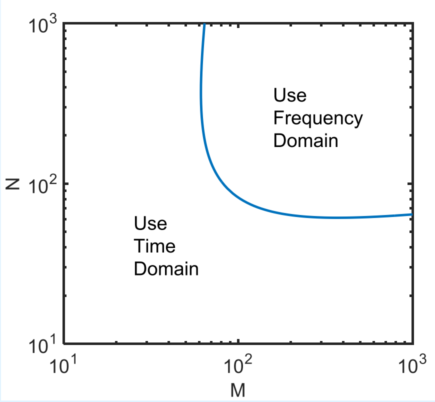
Total operations: $\approx 12L \log_2 L \approx 12\,(M+N) \log_2 (M+N)$

Beneficial if both M and N are $>\sim 70$.



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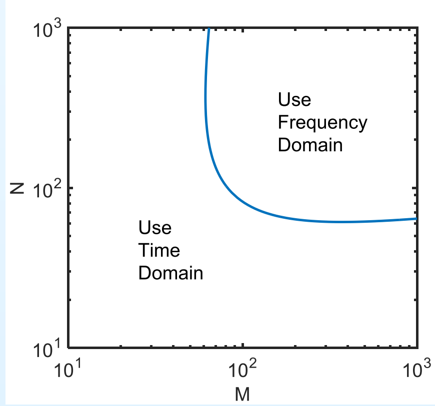

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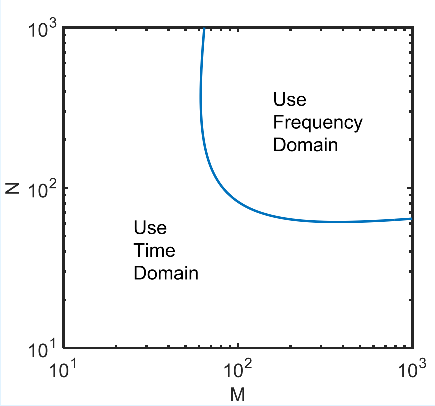
Example: $M = 10^3$, $N = 10^4$:

Direct: $2MN = 2 \times 10^7$



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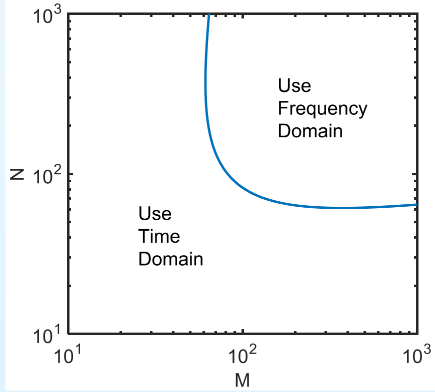
Example: $M = 10^3$, $N = 10^4$:
Direct: $2MN = 2 \times 10^7$
with DFT: $= 1.8 \times 10^6$ ☺



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Beneficial if both M and N are $> \sim 70$.

Example: $M = 10^3$, $N = 10^4$:

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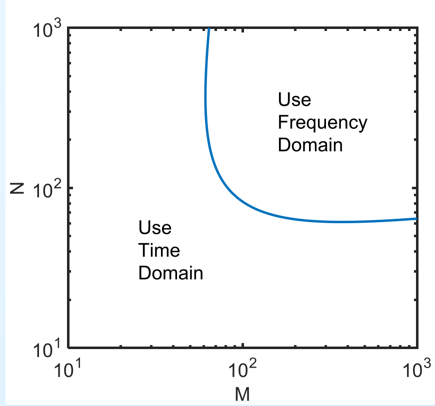

But: (a) DFT may be very long if N is large

(b) No outputs until all $x[n]$ has been input.

Overlap Add

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If N is very large:

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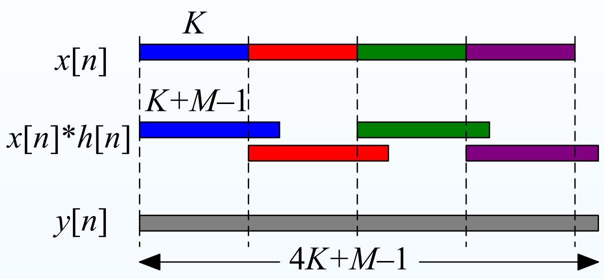
Overlap Add

If N is very large:

(1) chop $x[n]$ into $\frac{N}{K}$ chunks of length K



Overlap Add

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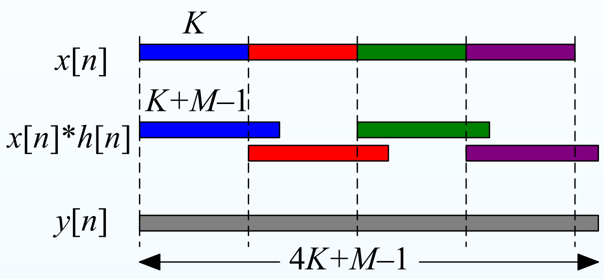
If N is very large:

(1) chop $x[n]$ into $\frac{N}{K}$ chunks of length K

(2) convolve each chunk with $h[n]$



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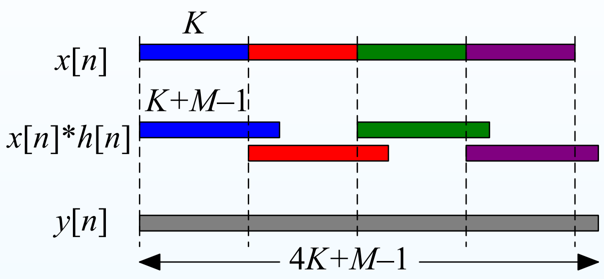

Each output chunk is of length $K + M - 1$ and overlaps the next chunk

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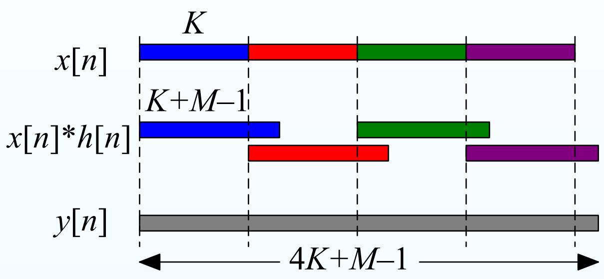
If N is very large:
(1) chop $x[n]$ into $\frac{N}{K}$ chunks of length K
(2) convolve each chunk with $h[n]$
(3) add up the results



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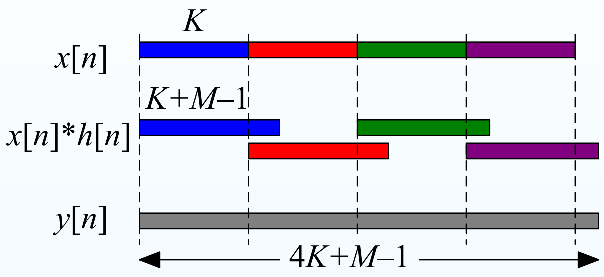

Each output chunk is of length $K+M-1$ and overlaps the next chunk

Operations: $\approx \frac{N}{K} \times 8\,(M+K)\log_2(M+K)$

Overlap Add

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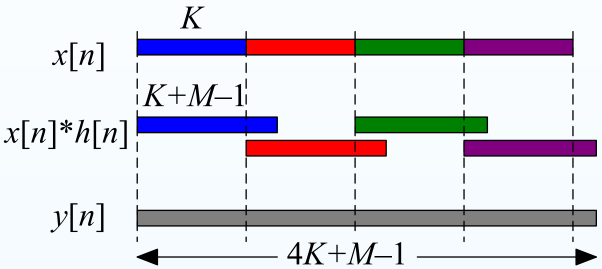

Each output chunk is of length $K + M - 1$ and overlaps the next chunk

Operations: $\approx \frac{N}{K} \times 8\,(M+K)\log_2(M+K)$

Computational saving if $\approx 100 < M \ll K \ll N$

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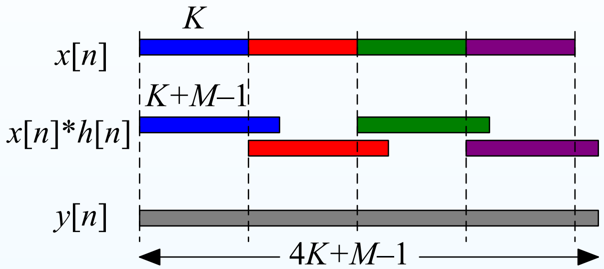

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Operations: $\approx \frac{N}{K} \times 8\,(M+K)\log_2(M+K)$

Computational saving if $\approx 100 < M \ll K \ll N$

Example: $M = 500$, $K = 10^4$, $N = 10^7$

Direct: $2MN = 10^{10}$

Overlap Add

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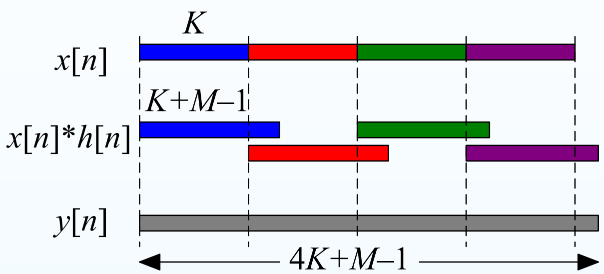

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Operations: $\approx \frac{N}{K} \times 8\,(M+K)\log_2(M+K)$

Computational saving if $\approx 100 < M \ll K \ll N$

Example: $M = 500$, $K = 10^4$, $N = 10^7$

Direct: $2MN = 10^{10}$

single DFT: $12\,(M+N)\log_2(M+N) = 2.8 \times 10^9$

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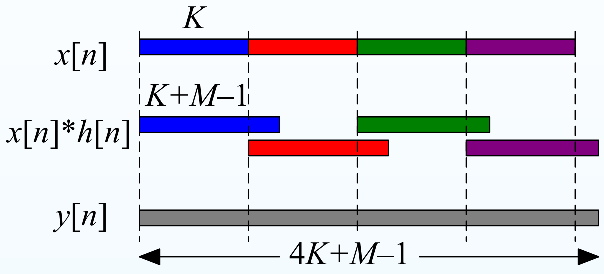

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overlap-add: $\frac{N}{K} \times 8\,(M + K)\log_2(M + K) = 1.1 \times 10^9$ ☺

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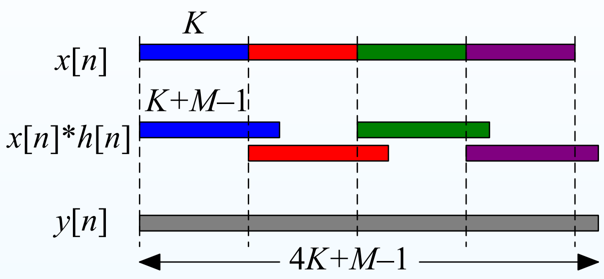
Overlap Add

If N is very large:
(1) chop $x[n]$ into $\frac{N}{K}$ chunks of length K
(2) convolve each chunk with $h[n]$
(3) add up the results



Each output chunk is of length $K+M-1$ and overlaps the next chunk

Operations: $\approx \frac{N}{K} \times 8\,(M+K)\log_2(M+K)$

Computational saving if $\approx 100 < M \ll K \ll N$

Example: $M = 500$, $K = 10^4$, $N = 10^7$

Direct: $2MN = 10^{10}$

single DFT: $12\,(M+N)\log_2(M+N) = 2.8 \times 10^9$

overlap-add: $\frac{N}{K} \times 8\,(M+K)\log_2(M+K) = 1.1 \times 10^9$ ☺

Other advantages:

(a) Shorter DFT

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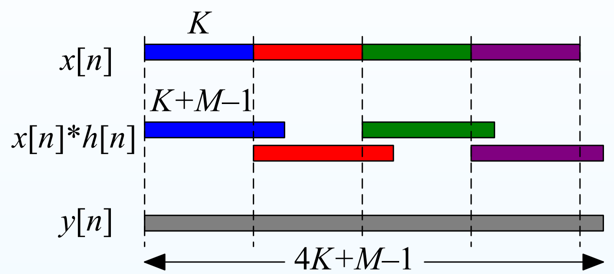

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Other advantages:

(a) Shorter DFT

(b) Can cope with $N = \infty$

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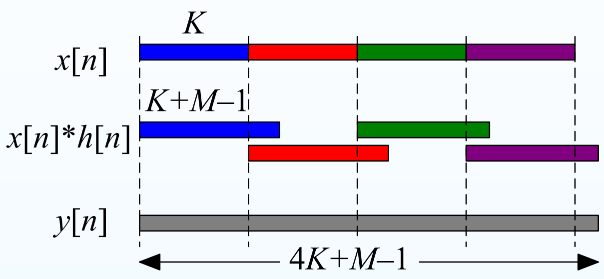

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Other advantages:
(a) Shorter DFT
(b) Can cope with $N = \infty$
(c) Can calculate $y[0]$ as soon as $x[K-1]$ has been read:
algorithmic delay = $K - 1$ samples

Overlap Save

Alternative method:

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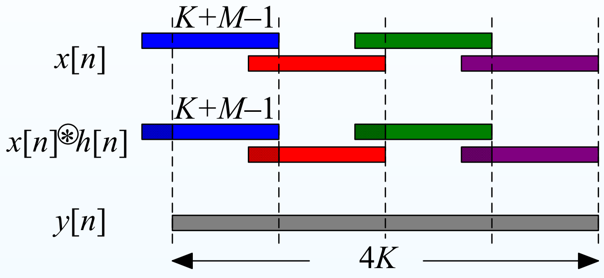

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Alternative method:

(1) chop $x[n]$ into $\frac{N}{K}$ overlapping chunks of length $K+M-1$

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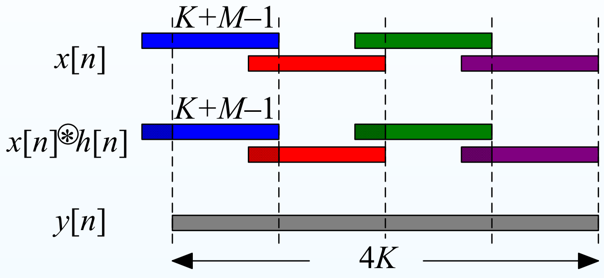

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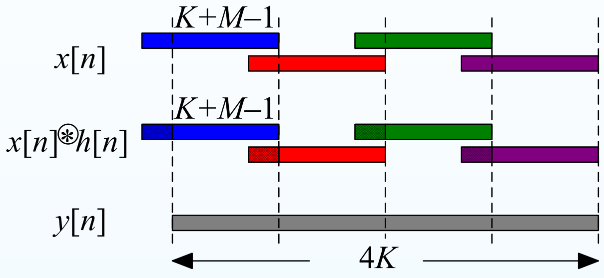

The first $M-1$ points of each output chunk are invalid

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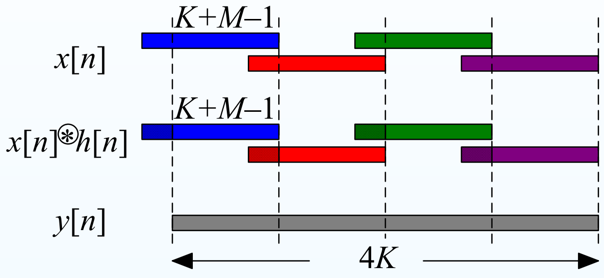

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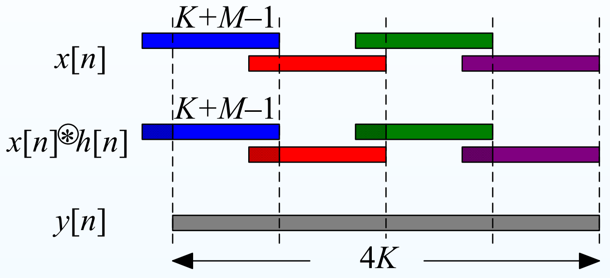

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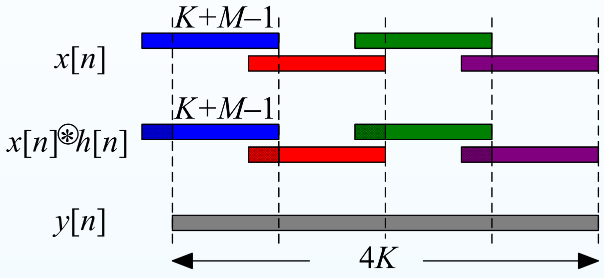

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Operations: slightly less than overlap-add because no addition needed to create $y[n]$

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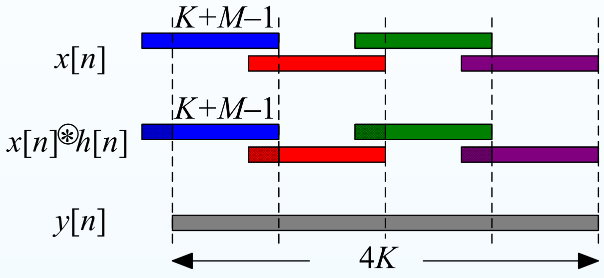

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Operations: slightly less than overlap-add because no addition needed to create $y[n]$

Advantages: same as overlap add

Strangely, rather less popular than overlap-add

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For further details see Mitra: 4 & 5.

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MATLAB routines

fftfilt	Convolution using overlap add
$x[n] \circledast y[n]$	real(ifft(fft(x).*fft(y)))