PID Controller Design

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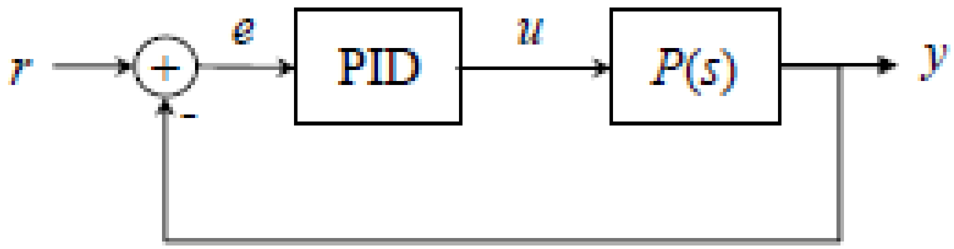
This Lecture Contains

- What is PID Control?
- Advantage of PID
- Advantage of PI and PD Controller
- Assignment

What is PID Control?

- **Dynamic Systems are often controlled with the help of a three term compensator known as PID; P – stands for Proportional Control, I stands for Integral Control and D stands for Derivative Control.**
- **Consider a closed loop system with unity feedback as follows:**



- **The plant *P*, is controlled by a control input *u(t),* which can be expressed as follows:**

$$u(t) = K_p e(t) + K_i \int e(t)\,dt + K_d \dot{e}(t)$$

Frequency Domain Representation of Controller

- **The control input could be represented in frequency domain as follows:**

$$U(s) = [K_p + \frac{K_i}{s} + K_d s] E(s)$$

- **The closed loop transfer function could be written as follows:**

$$\frac{Y(s)}{R(s)} = \frac{C(s)P(s)}{1 + C(s)P(s)}$$

- **This tells us that the poles of the closed loop transfer functions are actually the zeros of *1+C(s)P(s)*. By considering the three terms as three parameters, you can study the effect of changing each one of these parameters on the root locus.**

Advantage of Different Parameters of PID Controller

Parameters	Advantage	Limitation
K_p	Adjustment of Controller output	May cause instability
K_i	Produces zero steady state error	Slow dynamic Response and Instability
K_d	Provides rapid system response	Sensitive to Noise and non-zero offset

Application of PI Controller on a First Order System

- **Consider a first order plant as follows:**

$$P(s) = \frac{K}{1+\tau s}$$

- **If we apply a PI controller then *C(s)* becomes:**

$$C(s) = K_p + \frac{K_i}{s} = K_p \frac{s + \frac{K_i}{K_p}}{s}$$

- **The closed loop transfer function may be written as:**

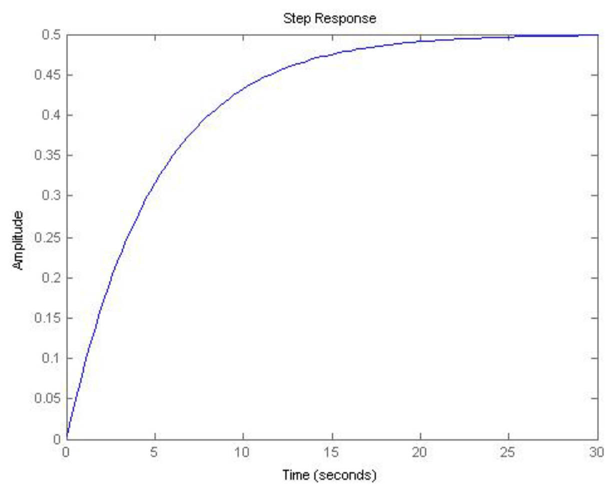
$$\frac{Y(s)}{R(s)} = (KK_p)\frac{s + K_i / K_p}{\tau s^2 + (1 + K K_p)s + KK_i}$$

Numerical Simulation of the system

- Let us consider the first order system with K=1 and τ = 10 s, hence the open loop system transfer function may be written as

$$T(s) = \frac{1}{1+10s}$$

- Let us look at the unit step response of this system – the controller has miserably failed to follow!



First Order System - Numerical Simulation

- **Now let us consider a PI controller with the following parameters:**

$$K = 1$$

$$K_p = 1$$

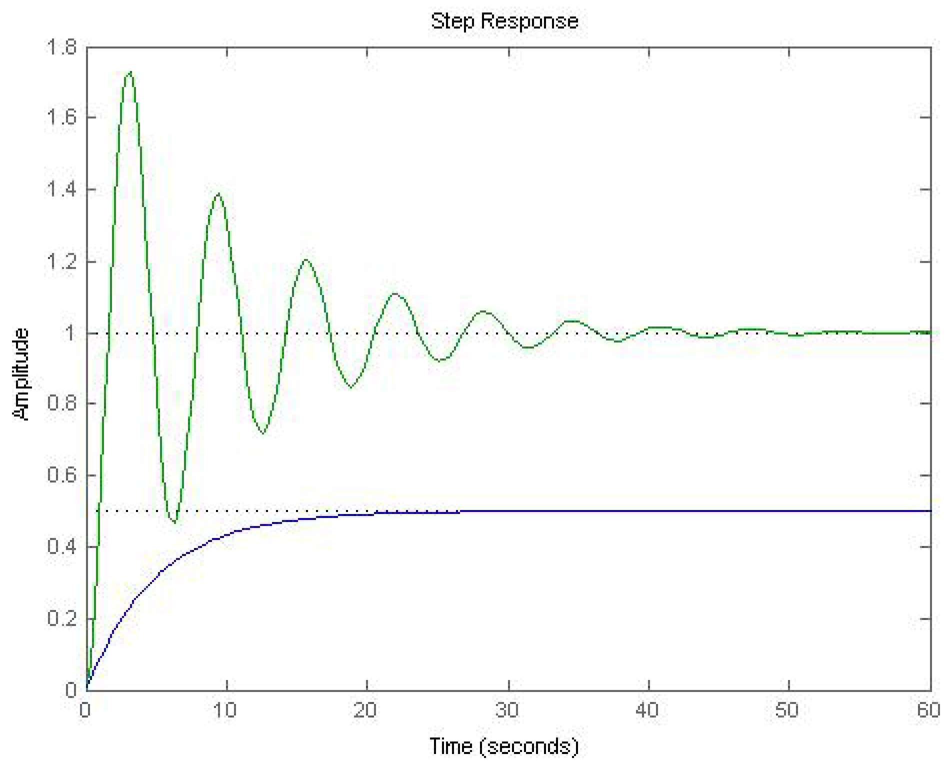
$$K_i = 10$$

- **The new closed loop transfer function may be written as:**

$$T = \frac{s + 10}{10s^2 + 2s + 10}$$

Response of the new system

- **The unit step response of the new system is shown below vis a vis the old system:**



Application of PD Controller on a First Order System

- **Consider a first order plant as follows:**

$$P(s) = \frac{K}{1+\tau s}$$

- **If we apply a PD controller then *C(s)* becomes:**

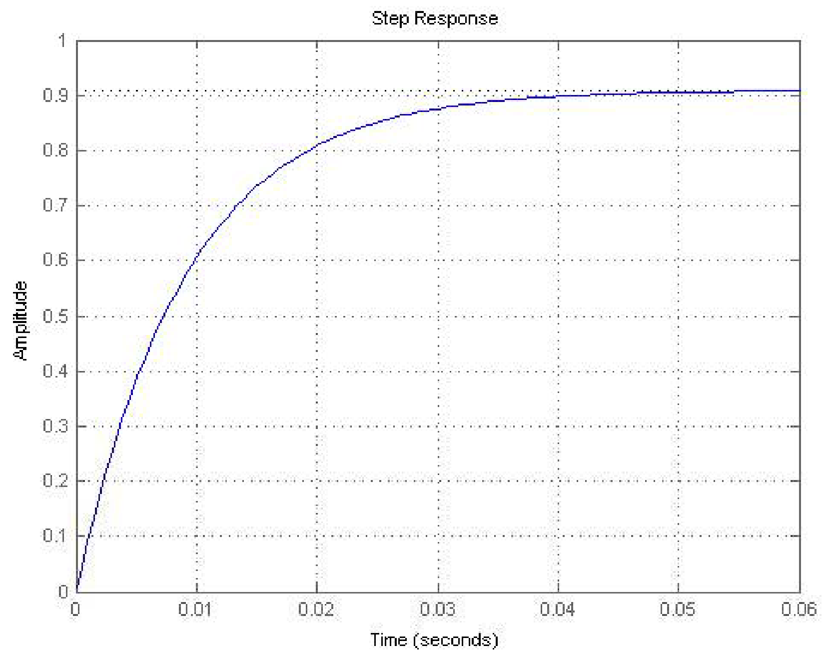
$$C(s) = K_p + K_d s = K_d \left(s + K_p / K_d\right)$$

- **The closed loop transfer function may be written as:**

$$\frac{Y(s)}{R(s)} = \frac{KK_d s + KK_p}{(\tau + KK_d)s + (1 + K K_p)}$$

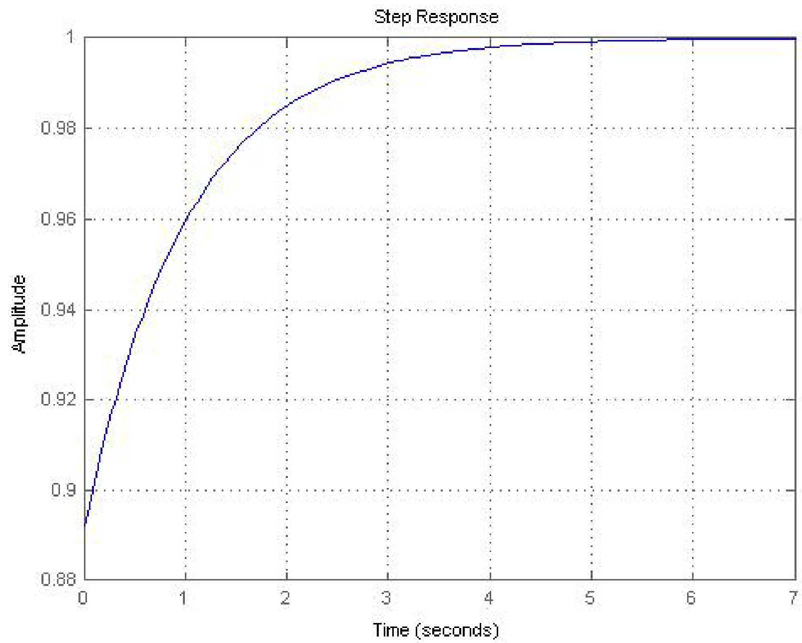
A numerical example

- **Let us consider a numerical example where K=10 and τ = 0.1. The response of the system without any compensator is as follows: Note that it took about 0.05 seconds for the system to reach the steady state which is 0.9.**



A Modified Response

- **If you now add a compensator with K_p = 9 and K_d = 1, you can see the change in step response as follows. You may observe that the same value (0.9) is obtained in less than 0.2 seconds.**



Assignment

Consider a second order system with natural frequency 10 Hz and Damping coefficient = 0.1. Find out a PID controller which will improve the steady state performance ten times and also improve the peak-time 5 times.

Hints: Consider the standard second order models discussed earlier and cascade it with a PID controller as shown in this lecture. Take trial values of $K_{p,}$ K_i and K_d and find out the response – continue till the performance is satisfactory.

Special References for this lecture

- **Feedback Control of Dynamic Systems, Frankline, Powell and Emami, Pearson**
- ***Control Systems Engineering* – Norman S Nise, John Wiley & Sons**
- ***Design of Feedback Control Systems* – Stefani, Shahian, Savant, Hostetter**

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