Punto 4

De esta manera se simula la serie y se obtiene:

```
library(knitr)
library(astsa)
library(forecast)

xt <- rbinom(11,size = 1,p=0.5)
xt_1 <- xt[-1]
xt <- xt[-11]
y=5+xt-0.65*xt_1
y
```

```
##  [1] 5.35 6.00 4.35 6.00 4.35 5.35 5.35 5.35 6.00 5.00
```

Luego, usando $\hat{\gamma}(h) = n^{-1}\sum_{t=1}^{n-h}(y_{t+h} - \bar{y})(y_t - \bar{y})$, dividiendo este por la varianza, y haciendolo para n = 10, 100, 200, 500 y 1000 se obtiene:

n=10:

```
xt <- rbinom(11,size = 1,p=0.5)
xt_1 <- xt[-1]
xt <- xt[-11]
y=5+xt-0.65*xt_1

s2 <- c()
var_1 <- var(y)
for (h in 1:6){
  s <- 0
  for (i in 1:(length(y)-h)){
    s1 <- (y[i+h]-mean(y))*(y[i]-mean(y))
    s <- s+s1
  }
  s <- s*(1/length(y))/var_1
  s2 <- c(s2,s)
}
s2
```

```
## [1] -0.19278065 -0.27342550  0.14416222 -0.01389098 -0.01736372 -0.22557879
```

n=100:

```
xt <- rbinom(101,size = 1,p=0.5)
xt_1 <- xt[-1]
xt <- xt[-101]
y=5+xt-0.65*xt_1

s2 <- c()
var_1 <- var(y)
for (h in 1:6){
  s <- 0
  for (i in 1:(length(y)-h)){
```

```
    s1 <- (y[i+h]-mean(y))*(y[i]-mean(y))
    s <- s+s1
  }
  s <- s*(1/length(y))/var_1
  s2 <- c(s2,s)
}
s2
```

```
## [1] -0.48087600  0.10842344 -0.07562534  0.02226791 -0.12739693  0.03522338
```

n=200:

```
xt <- rbinom(201,size = 1,p=0.5)
xt_1 <- xt[-1]
xt <- xt[-201]
y=5+xt-0.65*xt_1

s2 <- c()
var_1 <- var(y)
for (h in 1:6){
  s <- 0
  for (i in 1:(length(y)-h)){
    s1 <- (y[i+h]-mean(y))*(y[i]-mean(y))
    s <- s+s1
  }
  s <- s*(1/length(y))/var_1
  s2 <- c(s2,s)
}
s2
```

```
## [1] -0.423983388 -0.068123734  0.005044254  0.045404805 -0.046867154
## [6]  0.025482457
```

n=500:

```
xt <- rbinom(501,size = 1,p=0.5)
xt_1 <- xt[-1]
xt <- xt[-501]
y=5+xt-0.65*xt_1

s2 <- c()
var_1 <- var(y)
for (h in 1:6){
  s <- 0
  for (i in 1:(length(y)-h)){
    s1 <- (y[i+h]-mean(y))*(y[i]-mean(y))
    s <- s+s1
  }
  s <- s*(1/length(y))/var_1
  s2 <- c(s2,s)
}
s2
```

```
## [1] -0.438388365  0.007187857 -0.092431222  0.094333367 -0.019742824
## [6] -0.008599897
```

n=1000:

```
xt <- rbinom(1001,size = 1,p=0.5)
xt_1 <- xt[-1]
xt <- xt[-1001]
y=5+xt-0.65*xt_1

s2 <- c()
var_1 <- var(y)
for (h in 1:6){
  s <- 0
  for (i in 1:(length(y)-h)){
    s1 <- (y[i+h]-mean(y))*(y[i]-mean(y))
    s <- s+s1
  }
  s <- s*(1/length(y))/var_1
  s2 <- c(s2,s)
}
s2
```

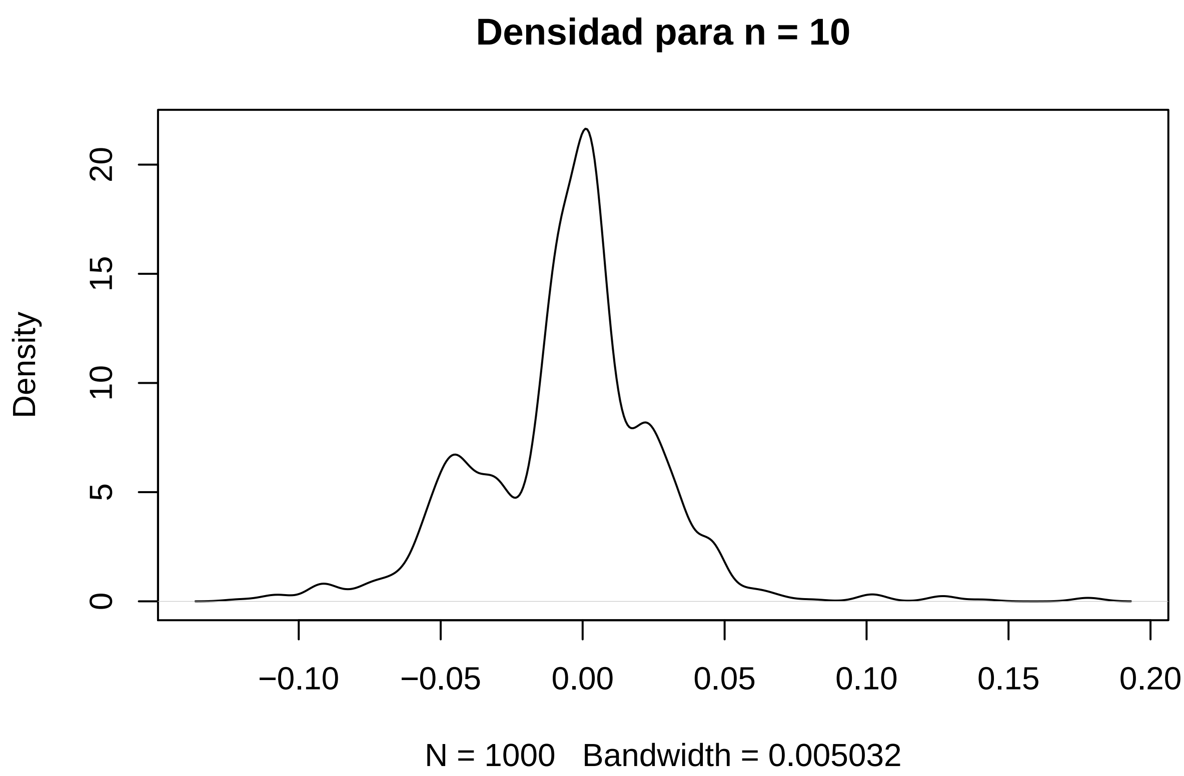
```
## [1] -0.410746349 -0.076491628  0.012053527  0.011051280  0.008794145
## [6]  0.021403615
```

b) Para cada n = 10, 100, 200, 500 y 1000, simule 1000 repeticiones, y calcule el ACF muestral para el rezago 10, verifique la distribución de muestras grandes del ACF.

Para esto, se hacen 1000 repeticiones de la simulación y para cada repetición se guardan los 10 rezagos calculados (para n=10 se hace con el rezago 9) y del vector con todos los rezagos de todas las repeticiones se hace el gráfico de densidad. De esta manera se obtiene:

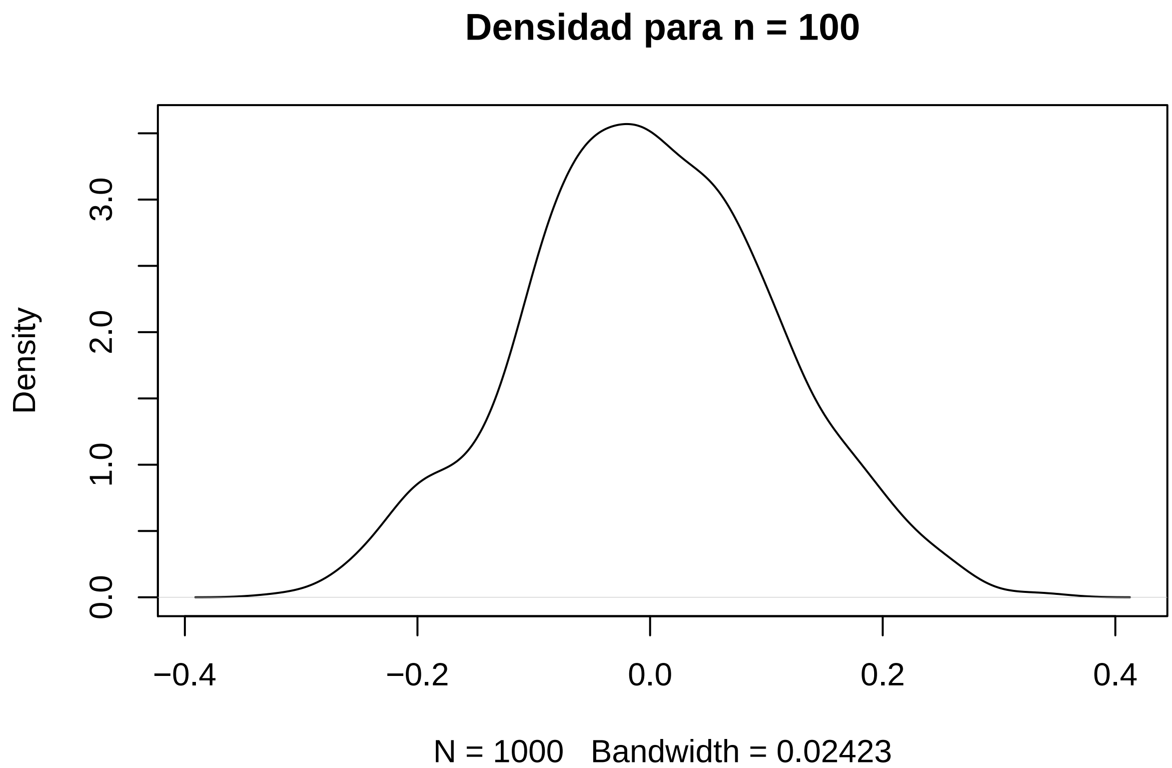
```
s3 <- c()
for (ii in 1:1000){
  n=10
  xt <- rbinom(n+1,size = 1,p=0.5)
  xt_1 <- xt[-1]
  xt <- xt[-n+1]
  y=5+xt-0.65*xt_1

  s2 <- c()
  var_1 <- var(y)
  for (h in 1:9){
    s <- 0
    for (i in 1:(length(y)-h)){
      s1 <- (y[i+h]-mean(y))*(y[i]-mean(y))
      s <- s+s1
    }
    s <- s*(1/length(y))/var_1
    s2 <- c(s2,s)
  }
  s3 <- c(s3,s2[9])
```

```
}
dens_1 <- density(na.omit(s3))
plot(dens_1,main = 'Densidad para n = 10')
```



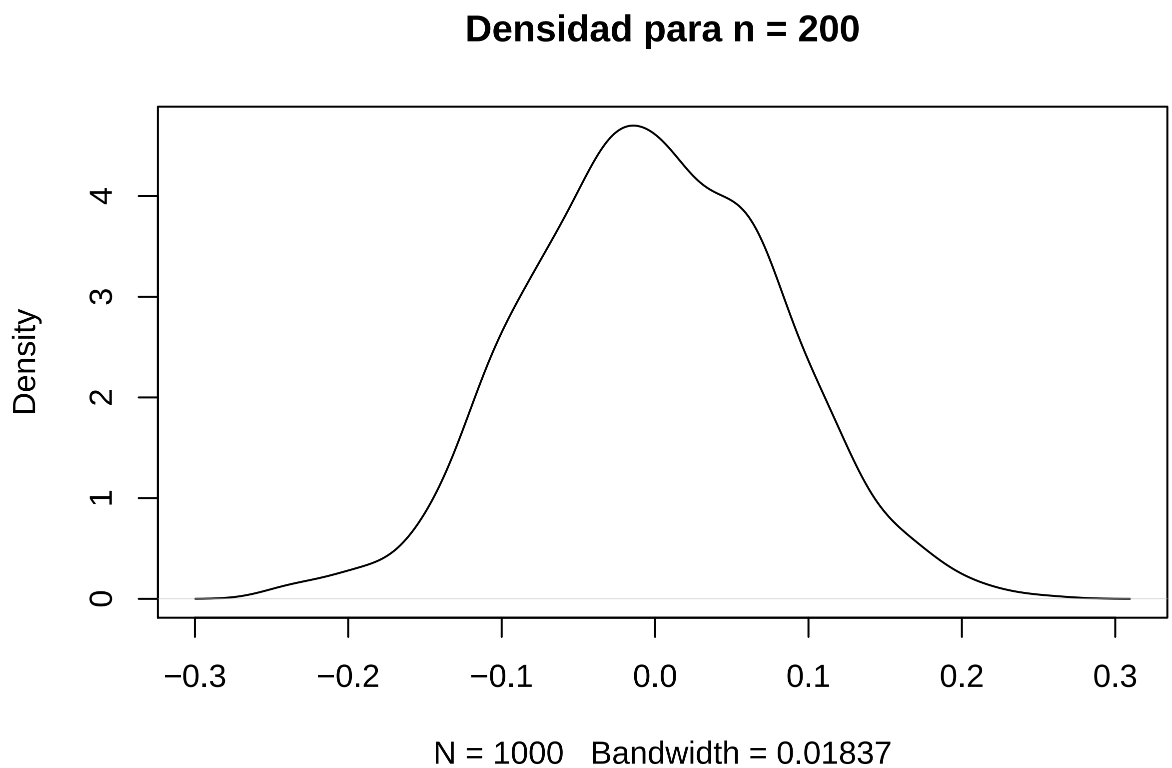

```
s3 <- c()
for (ii in 1:1000){
  n=100
  xt <- rbinom(n+1,size = 1,p=0.5)
  xt_1 <- xt[-1]
  xt <- xt[-n+1]
  y=5+xt-0.65*xt_1

  s2 <- c()
  var_1 <- var(y)
  for (h in 1:10){
    s <- 0
    for (i in 1:(length(y)-h)){
      s1 <- (y[i+h]-mean(y))*(y[i]-mean(y))
      s <- s+s1
    }
    s <- s*(1/length(y))/var_1
    s2 <- c(s2,s)
  }
  s3 <- c(s3,s2[10])
}
```

```r
dens_1 <- density(na.omit(s3))
plot(dens_1,main = 'Densidad para n = 100')
```

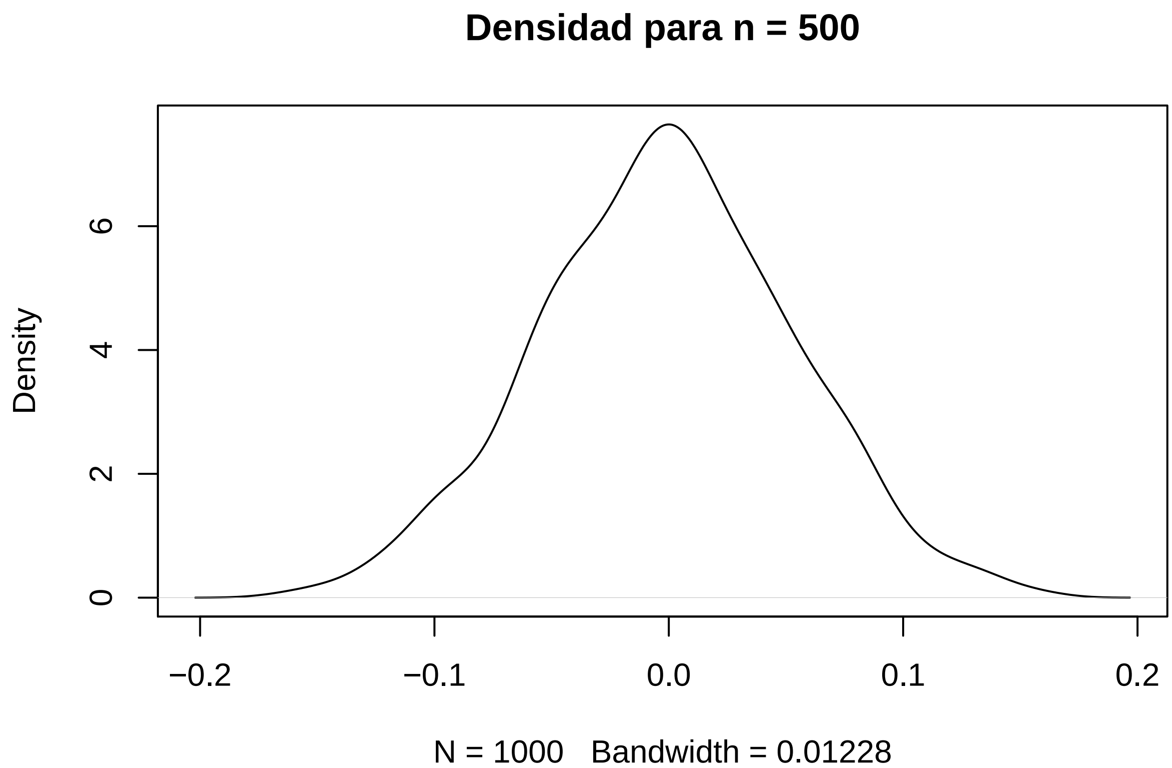



```r
s3 <- c()
for (ii in 1:1000){
  n=200
  xt <- rbinom(n+1,size = 1,p=0.5)
  xt_1 <- xt[-1]
  xt <- xt[-n+1]
  y=5+xt-0.65*xt_1

  s2 <- c()
  var_1 <- var(y)
  for (h in 1:10){
    s <- 0
    for (i in 1:(length(y)-h)){
      s1 <- (y[i+h]-mean(y))*(y[i]-mean(y))
      s <- s+s1
    }
    s <- s*(1/length(y))/var_1
    s2 <- c(s2,s)
  }
  s3 <- c(s3,s2[10])
}
dens_1 <- density(na.omit(s3))
```

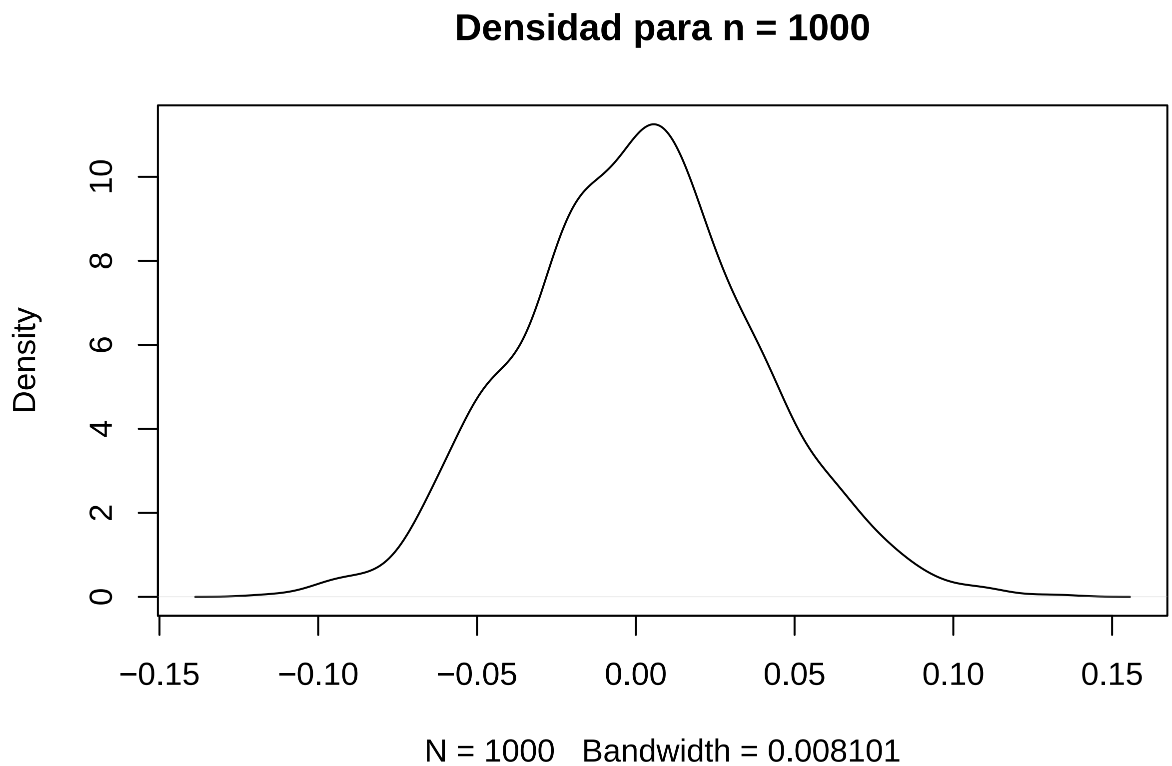
```r
plot(dens_1,main = 'Densidad para n = 200')
```




```r
s3 <- c()
for (ii in 1:1000){
  n=500
  xt <- rbinom(n+1,size = 1,p=0.5)
  xt_1 <- xt[-1]
  xt <- xt[-n+1]
  y=5+xt-0.65*xt_1

  s2 <- c()
  var_1 <- var(y)
  for (h in 1:10){
    s <- 0
    for (i in 1:(length(y)-h)){
      s1 <- (y[i+h]-mean(y))*(y[i]-mean(y))
      s <- s+s1
    }
    s <- s*(1/length(y))/var_1
    s2 <- c(s2,s)
  }
  s3 <- c(s3,s2[10])
}
dens_1 <- density(na.omit(s3))
plot(dens_1,main = 'Densidad para n = 500')
```




```
s3 <- c()
for (ii in 1:1000){
  n=1000
  xt <- rbinom(n+1,size = 1,p=0.5)
  xt_1 <- xt[-1]
  xt <- xt[-n+1]
  y=5+xt-0.65*xt_1

  s2 <- c()
  var_1 <- var(y)
  for (h in 1:10){
    s <- 0
    for (i in 1:(length(y)-h)){
      s1 <- (y[i+h]-mean(y))*(y[i]-mean(y))
      s <- s+s1
    }
    s <- s*(1/length(y))/var_1
    s2 <- c(s2,s)
  }
  s3 <- c(s3,s2[10])
}
dens_1 <- density(na.omit(s3))
plot(dens_1,main = 'Densidad para n = 1000')
```



Según la distribución de muestras grandes para el ACF, dice en (Shumway, 2011) que "bajo condiciones generales, si xt es ruido blanco, entonces para n grande, el ACF muestral está ditribuido aproximadamente normal con media cero y varianza específica (fórmula)...". Dice en la nota que "The general conditions are that xt is iid with finite fourth moment. A suficient condition for this to hold is that xt is white Gaussian noise. Precise details are given in Theorem A.7 in Appendix A."

En este caso xt no es ruido blanco, pero su comportamiento es muy similar, por lo que se espera que la distribución para muestras grandes del ACF sea también aproximadamente normal y esto se observó en los gráficos realizados.