UNIVERSIDAD DE ANTIOQUIA
Facultad de Ciencias Exactas y Naturales
Instituto de Matemáticas
Series de Tiempo II
Taller # 1

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1. For a moving average process of the form

$$x_t = w_{t-1} + 2w_t + w_{t+1},$$

where w_t are independent with zero means and variance σ_w^2, determine the autocovariance and autocorrelation functions as a function of lag $h = s - t$ and plot the ACF as a function of h.

2. For an $MA(1)$, $x_t = w_t + \theta w_{t-1}$, show that $|\rho_x(1)| \leq 1/2$. for any number θ. For which values of θ does $\rho_x(1)$ attain its maximum and minimum?

3. A real-valued function $g(t)$, defined on the integers, is non-negative definite if and only if

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_i g(t_i - t_j) a_j \geqslant 0,$$

for all positive integers n and for all vectors $a = (a_1, a_2, \ldots, a_n)'$ and $t = (t_1, t_2, \ldots, t_n)'$. For the matrix $G = \{g(t_i - t_j); i, j = 1, 2, \ldots, n\}$, this implies that $a'Ga \geqslant 0$ for all vectors a.

   a. Prove that $\gamma(h)$, the autocovariance function of a stationary process, is a non-negative definite function.

   b. Verify that the sample autocovariance $\hat{\gamma}(h)$, is a non-negative definite function.

4. Consider a contrived set of data generated by tossing a fair coin, letting $x_t = 1$ when a head obtained and $x_t = 0$ when a tail obtained. Construt y_t as

$$y_t = 5 + x_t - 0{,}65x_{t-1}.$$

   a. Compare the sample ACF you obtain to the actual ACF, $\rho(h)$, with $n = 10, 100, 200, 500$ and 1000.

   b. For each $n = 10, 100, 200, 500$ and 1000, simulate 1000 replications, and compute the sample ACF to lag 10, verify the Large Sample Distribution of the ACF.

5. Identify the following models as ARMA(p, q) models (watch out for parameter redundancy), and determine whether they are causal and/or invertible:

   a. $x_t = 0{,}80x_{t-1} - 0{,}15x_{t-2} + w_t - 0{,}30w_{t-1}$.

   b. $x_t = x_{t-1} - 0{,}50x_{t-2} + w_t - w_{t-1}$.

6. Crude oil prices in dollars per barrel are in oil; see Appendix R for more details. Fit an ARIMA(p, d, q) model to the growth rate performing all necessary diagnostics. Comment.

7. Fit an ARIMA(p, d, q) model to the global temperature data gtemp performing all of the necessary diagnostics. After deciding on an appropriate model, forecast (with limits) the next 10 years. Comment.