SIGGRAPH 2022
VANCOUVER+ 8-11 AUG

THE PREMIER CONFERENCE & EXHIBITION ON
COMPUTER GRAPHICS & INTERACTIVE TECHNIQUES

ROBUST COMPUTATION OF IMPLICIT SURFACE NETWORKS FOR PIECEWISE LINEAR FUNCTIONS

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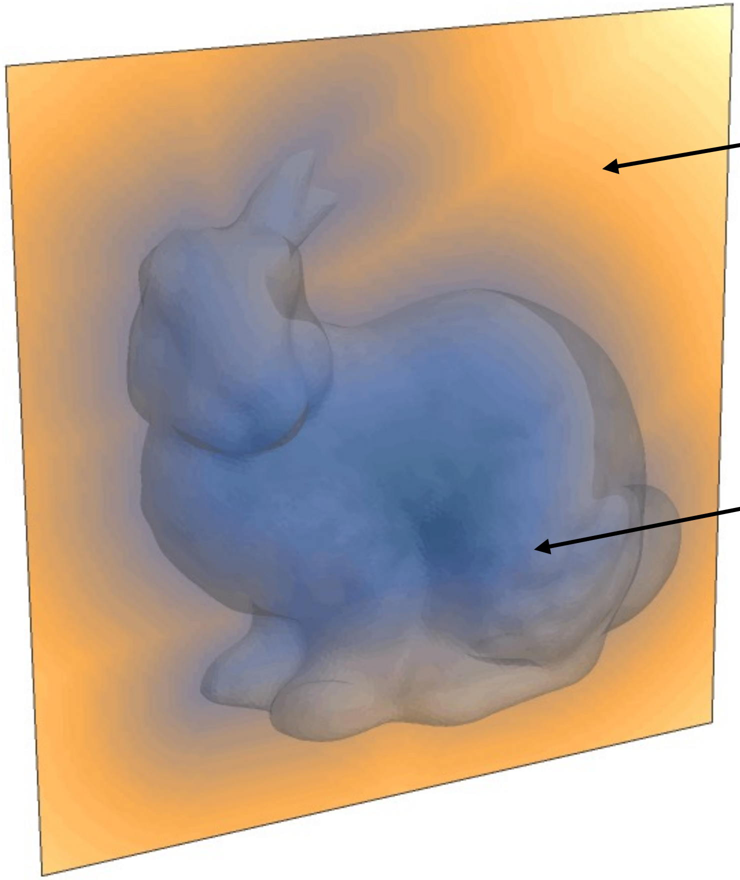
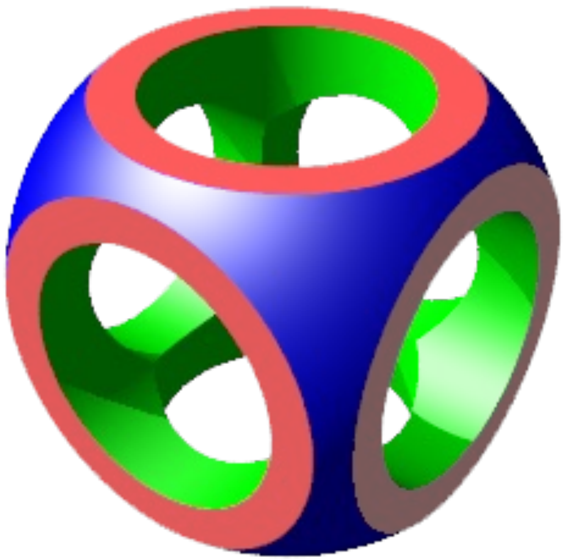
IMPLICIT SURFACES

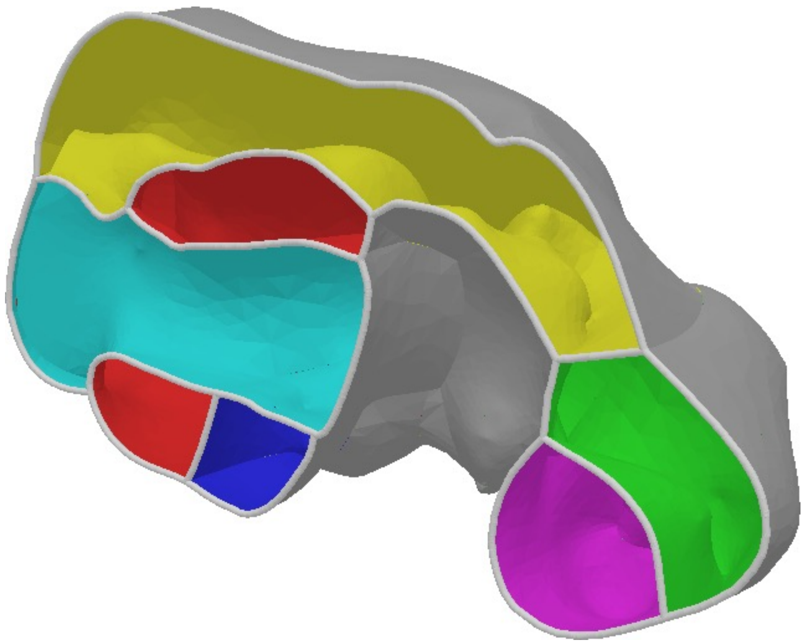


$f(x, y, z)$

$\{x, y, z\}\colon\ f(x, y, z) = 0$

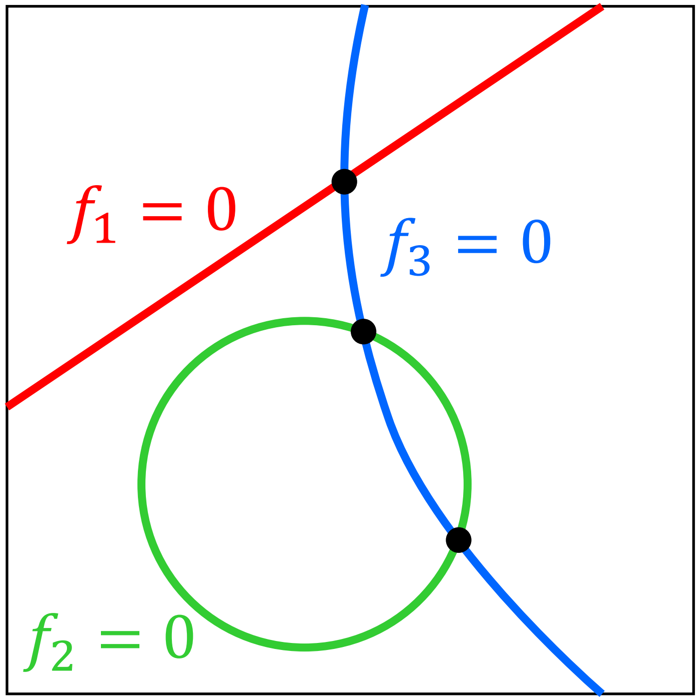
IMPLICIT SURFACE NETWORKS





IMPLICIT SURFACE NETWORKS

- Implicit Arrangement (IA): Intersecting multiple implicit surfaces



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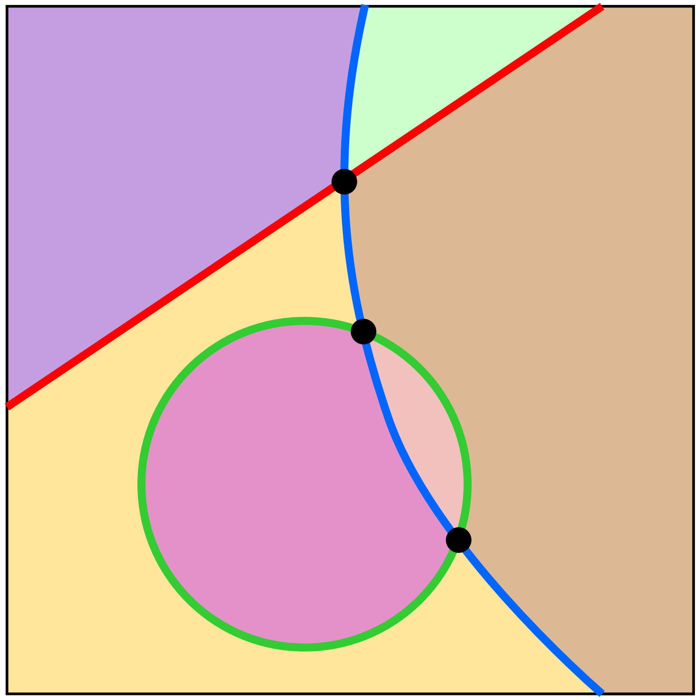

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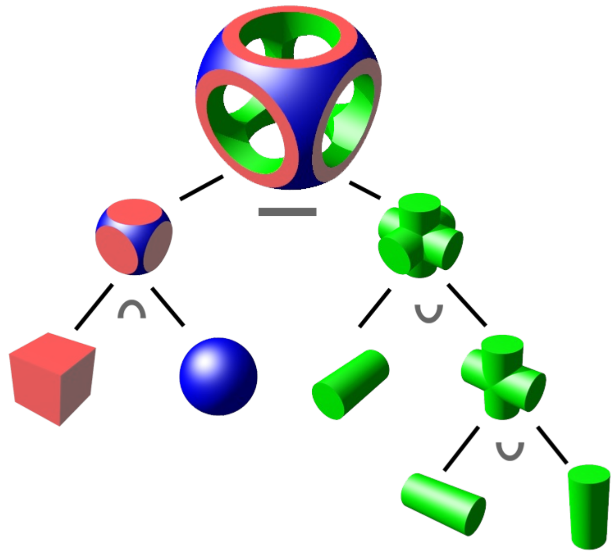
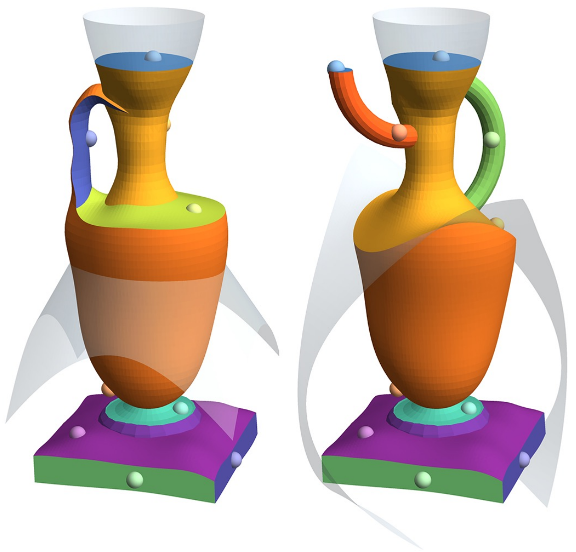
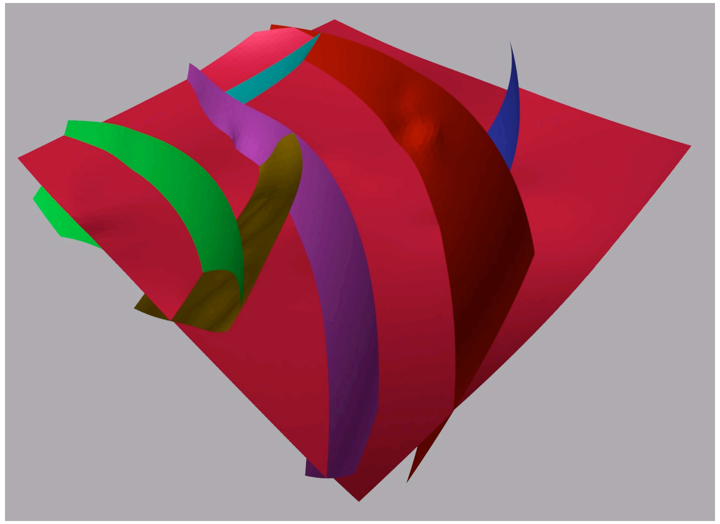


Constructive Solid Geometry (CSG)

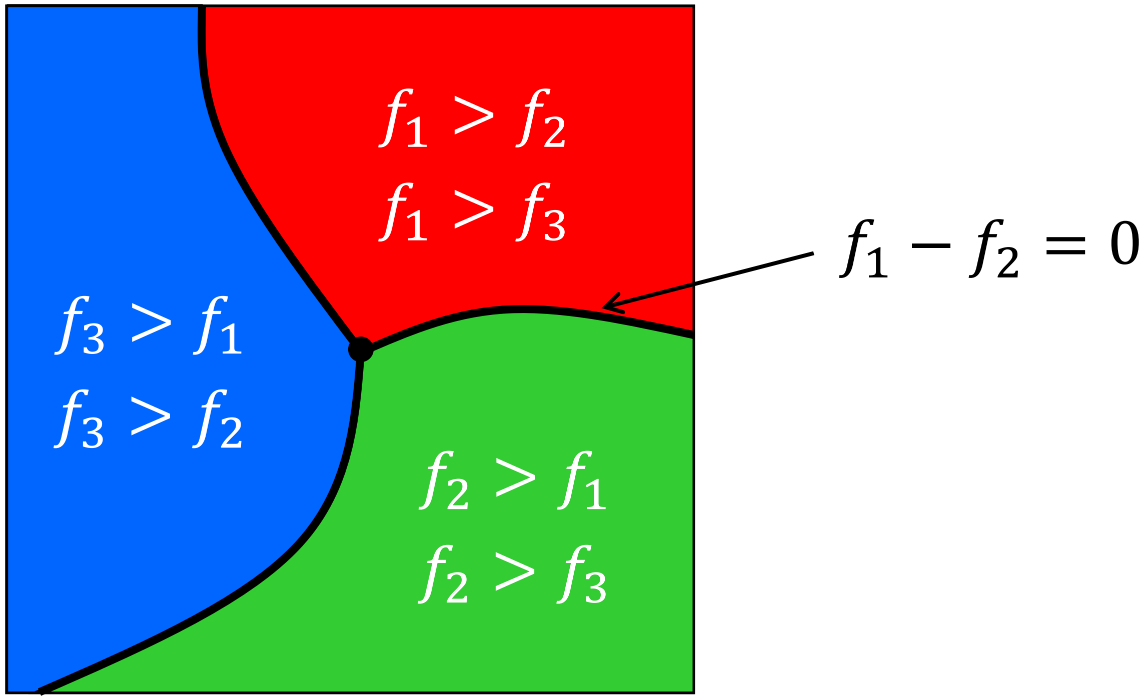


Boundary-sampled halfspaces [Du 21]



Geological structure [Bagley 16]

- Material Interfaces (MI): Boundary of regions dominated by one function



IMPLICIT SURFACE NETWORKS

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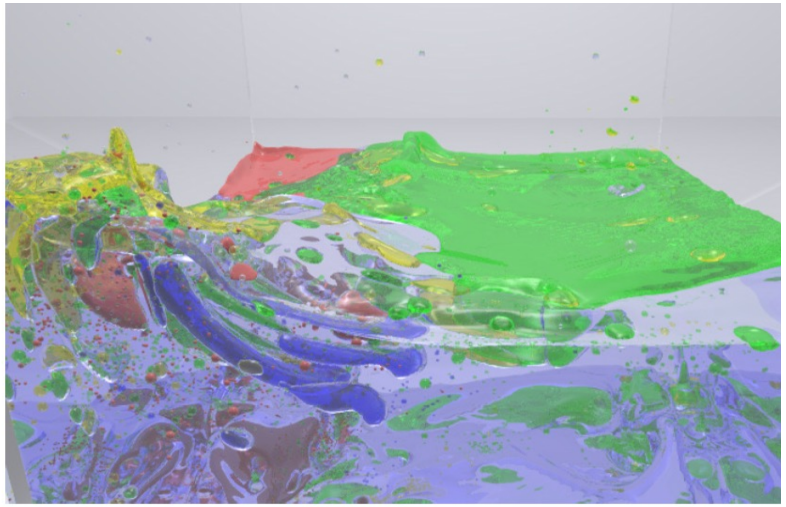
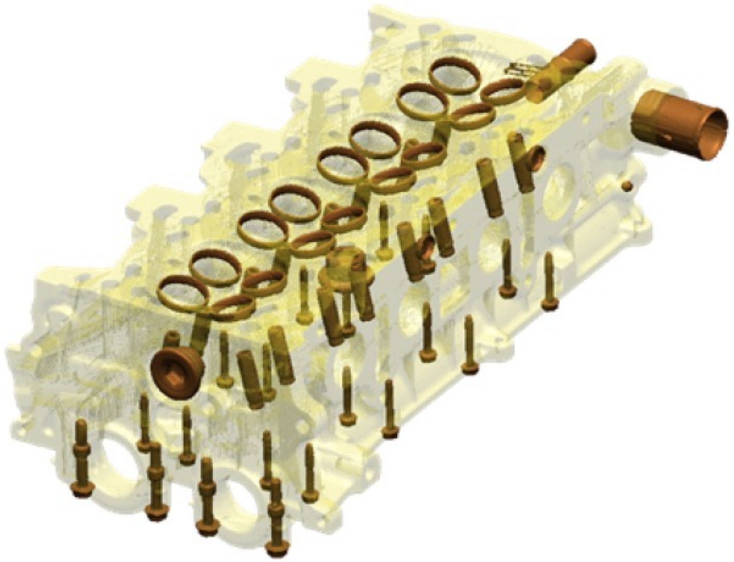
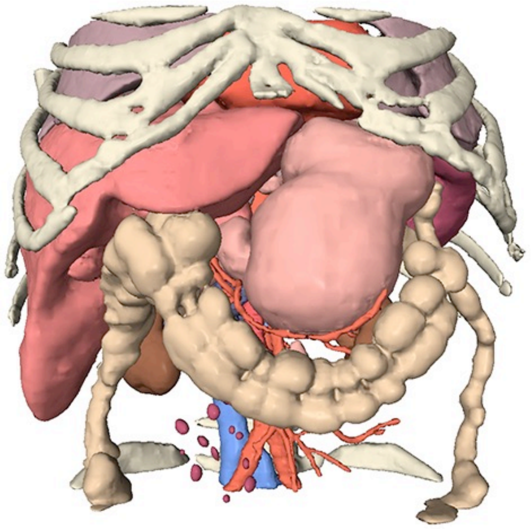


Multi-phase fluid
[Kim 10]



Mechanical parts
[Shammaa 08]

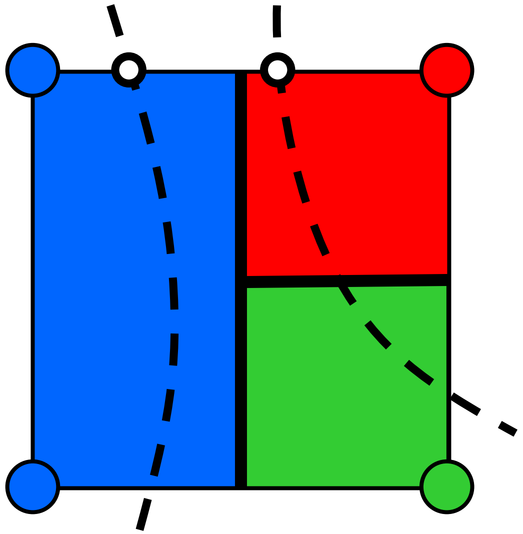


Anatomical structure
[Yuan 12]

DISCRETIZATION ON A GRID

DISCRETIZATION ON A GRID

- Separating grid vertices of different labels
  - [Nielson 97; Hege 97; Ju 02; Wu 03; Bertram 05; Reitinger 05; Dillard 07; Shammaa 08,10; Zhang 07,12]
  - Geometric and topological artifacts near thin features



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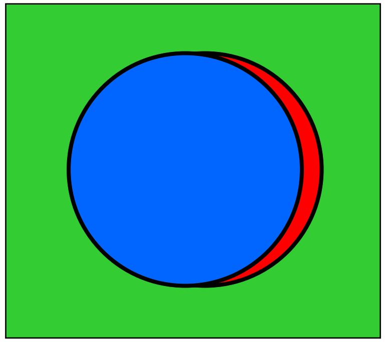
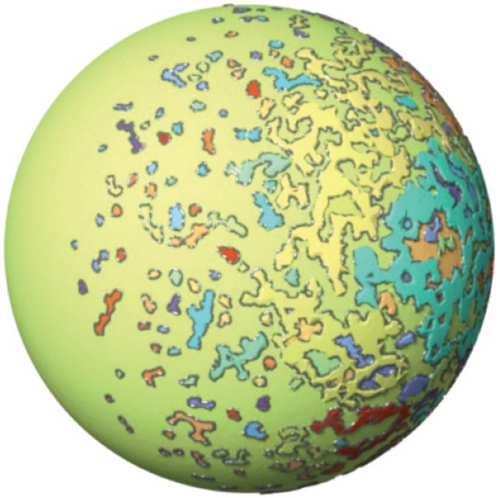
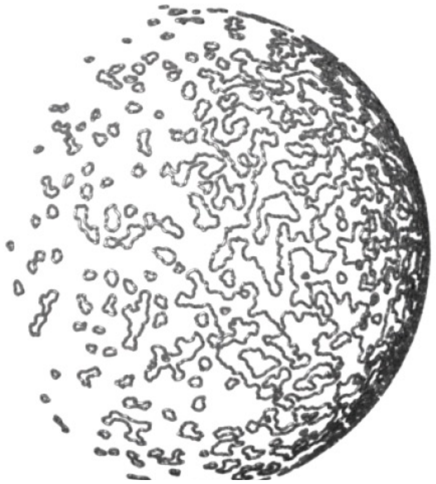


Material interface (cross-section)



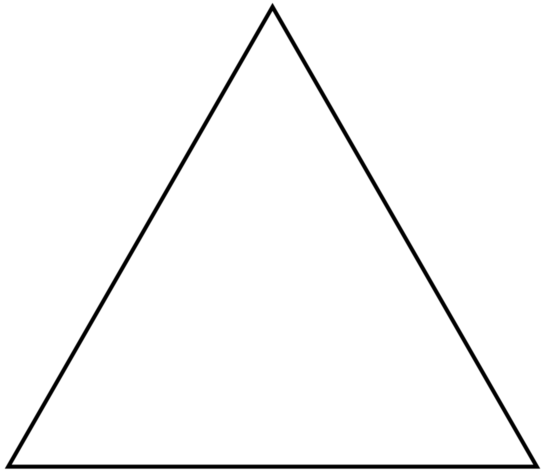
[Nielson 97]

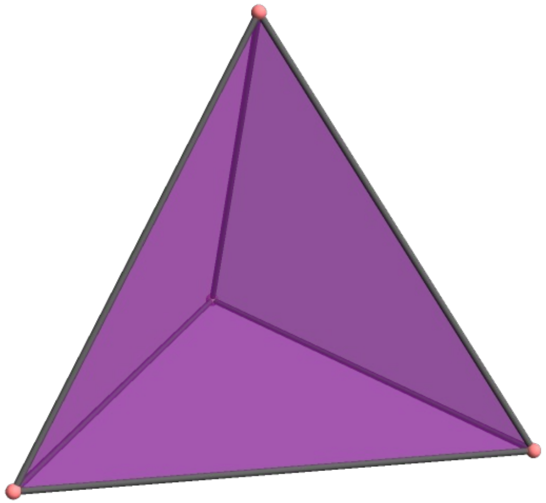


Non-manifold curves

DISCRETIZATION ON A GRID

- Implicit surface network of a piecewise linear (PL) function
  - Implicit arrangement: [Kim 00; Bagley 16; Guo 21]
  - Material interface: [Bloomenthal 95; Bonnell 03; Saye 12,15]





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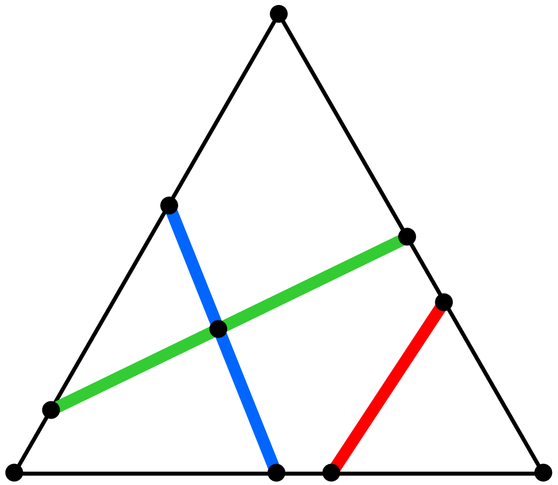
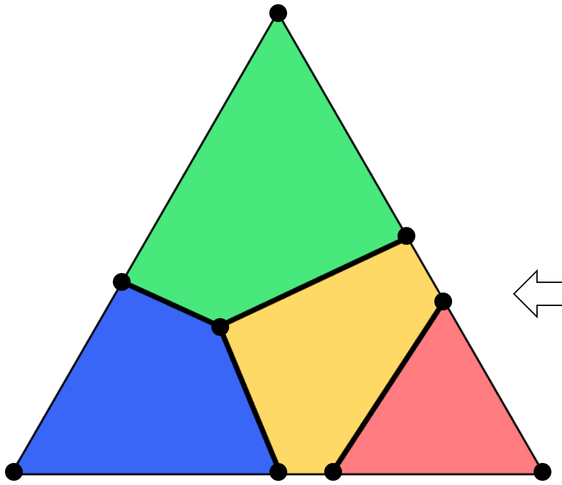
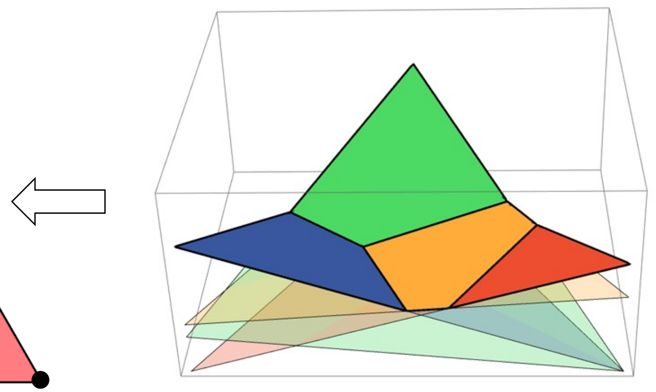


Implicit arrangement



Material interface



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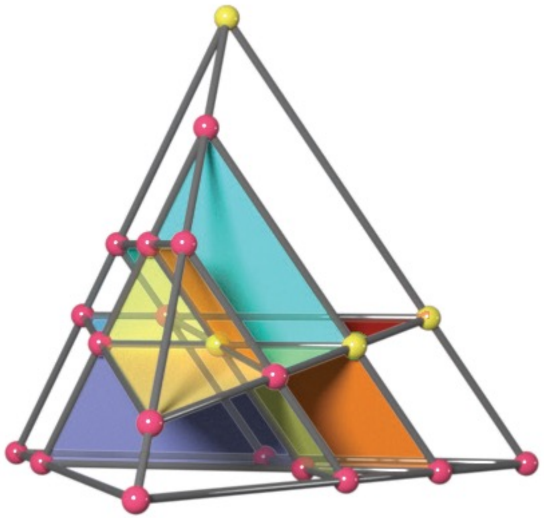
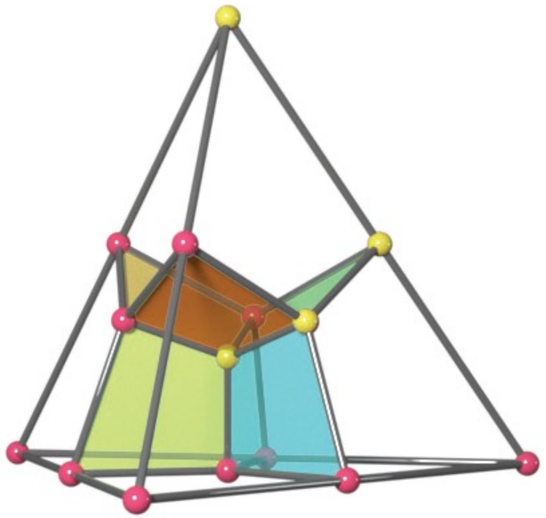


Implicit arrangement



Material interface

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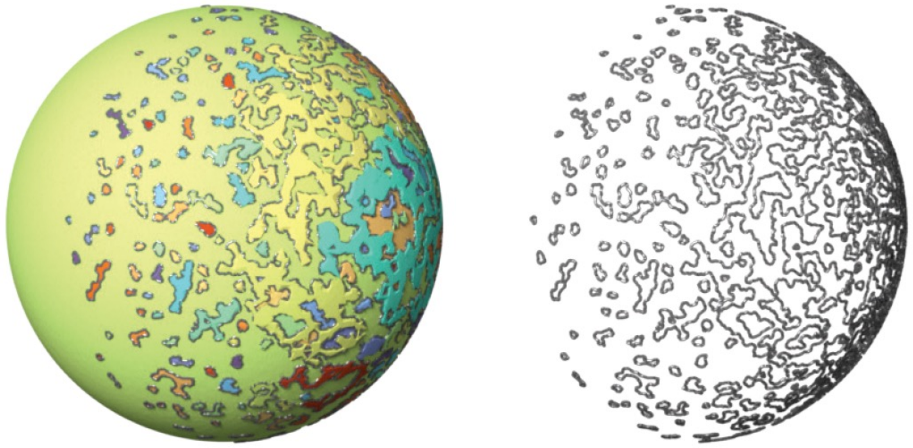
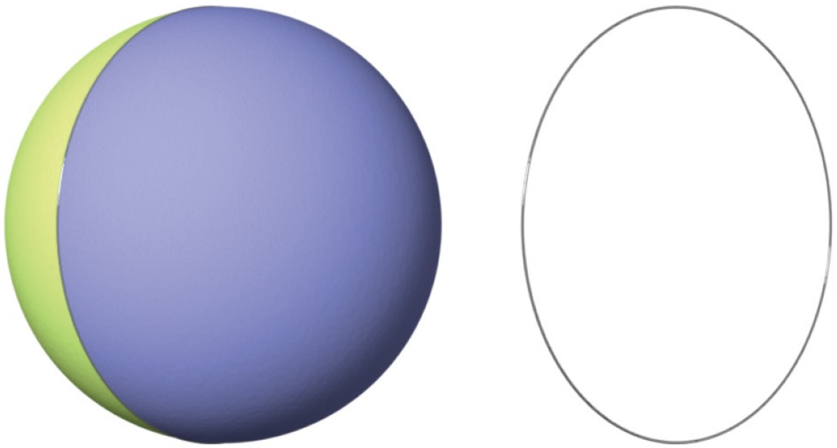


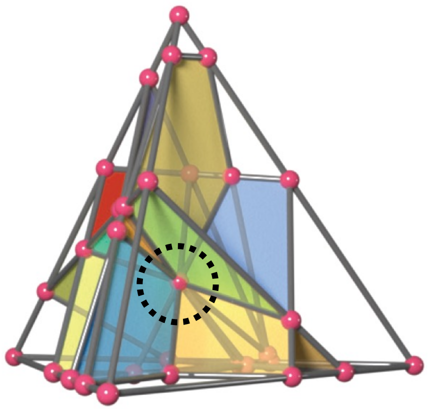
Label separation



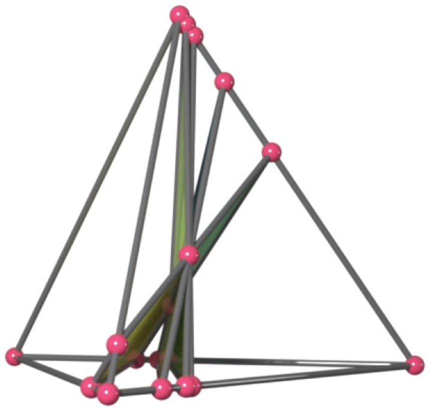
PL interpolation

DISCRETIZATION ON A GRID

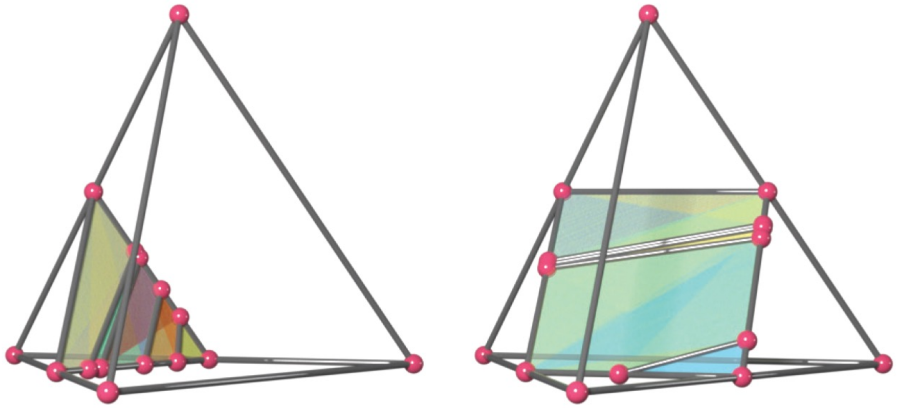
- Implicit surface network of a piecewise linear (PL) function
  - Implicit arrangement: [Kim 00; Bagley 16; Guo 21]
  - Material interface: [Bloomenthal 95; Bonnell 03; Saye 12,15]
  - Geometric intersections are prone to numerical errors



>3 planes at a point



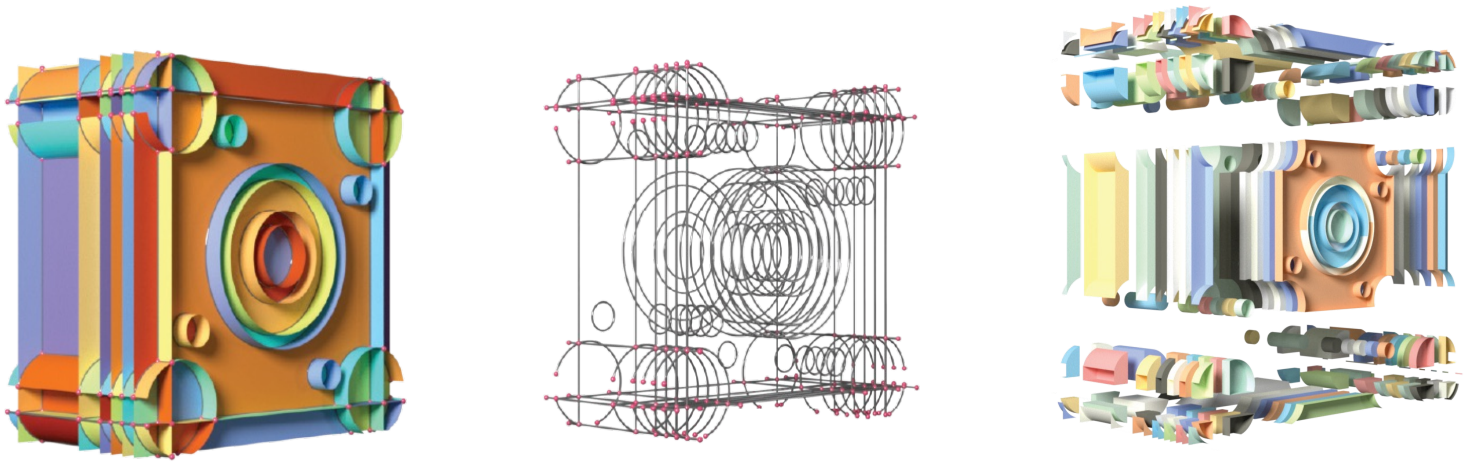
>2 planes at a line



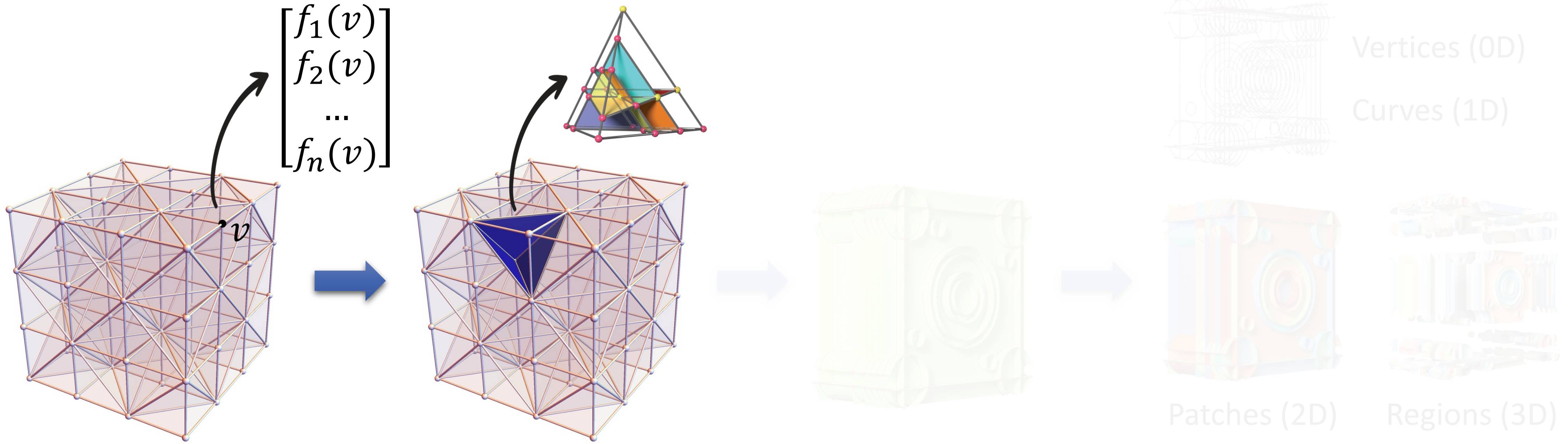
Almost co-planar

OUR METHOD

- Discretizing implicit surface networks using PL interpolation
  - Unified framework for both IA and MI
  - Guaranteed correct combinatorial structure
  - Scalable to complex inputs



PIPELINE



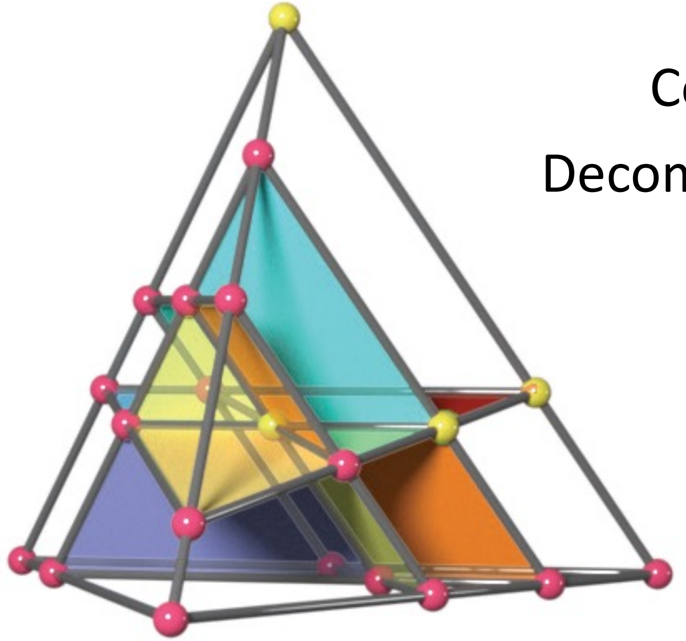
Input

Per-tet mesh

Complete mesh

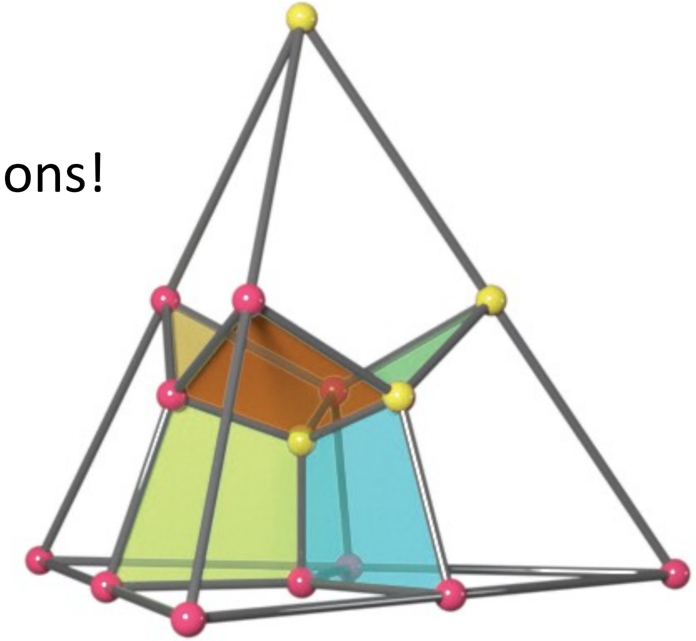
Space partitioning

TETRAHEDRON PROCESSING



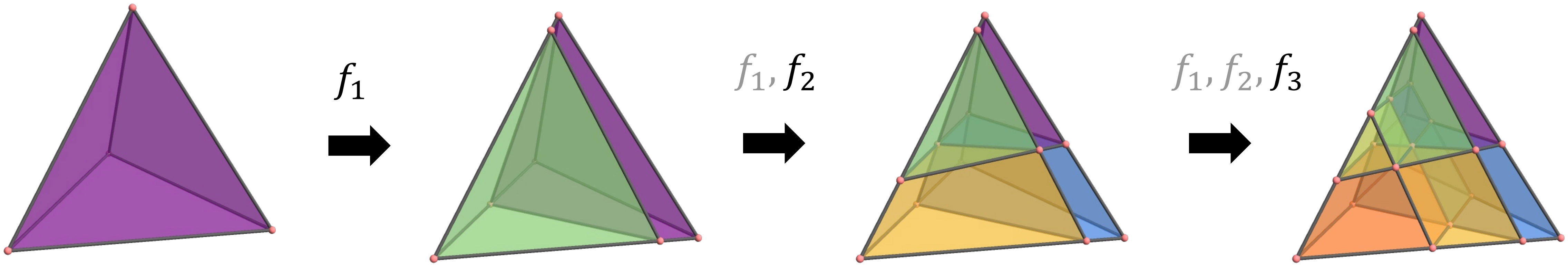
Implicit arrangement (IA)

Convex
Decompositions!



Material interface (MI)

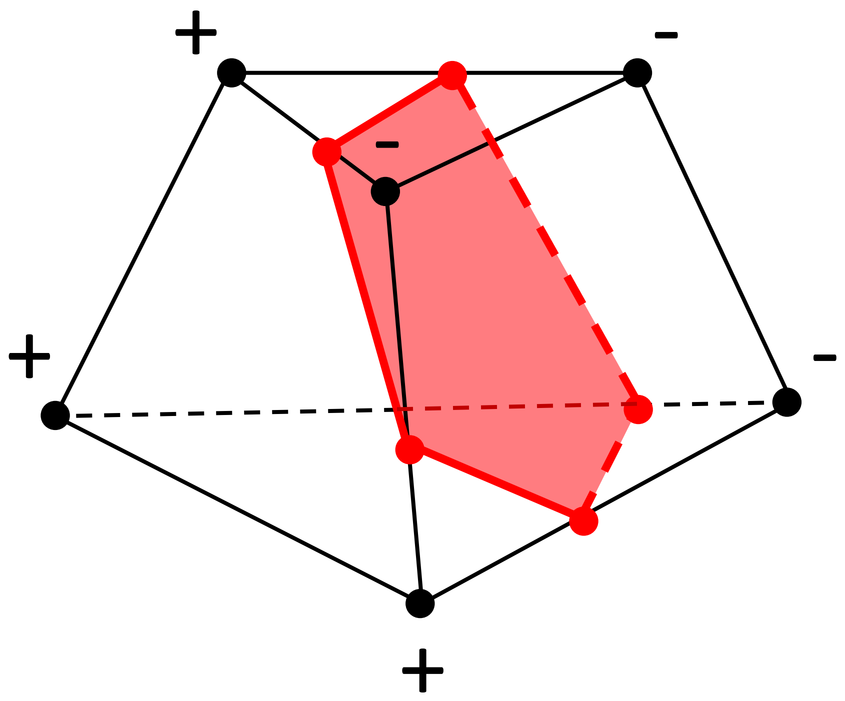
INCREMENTAL CONSTRUCTION



Implicit arrangement (IA)

CELL SPLITTING

- Given a function f_i:
  - Compute sign of f_i (for IA) or $f_i - f_j$ (for MI) at vertices
  - Split edges by cut vertices
  - Split faces by cut edges
  - Split cells by cut faces



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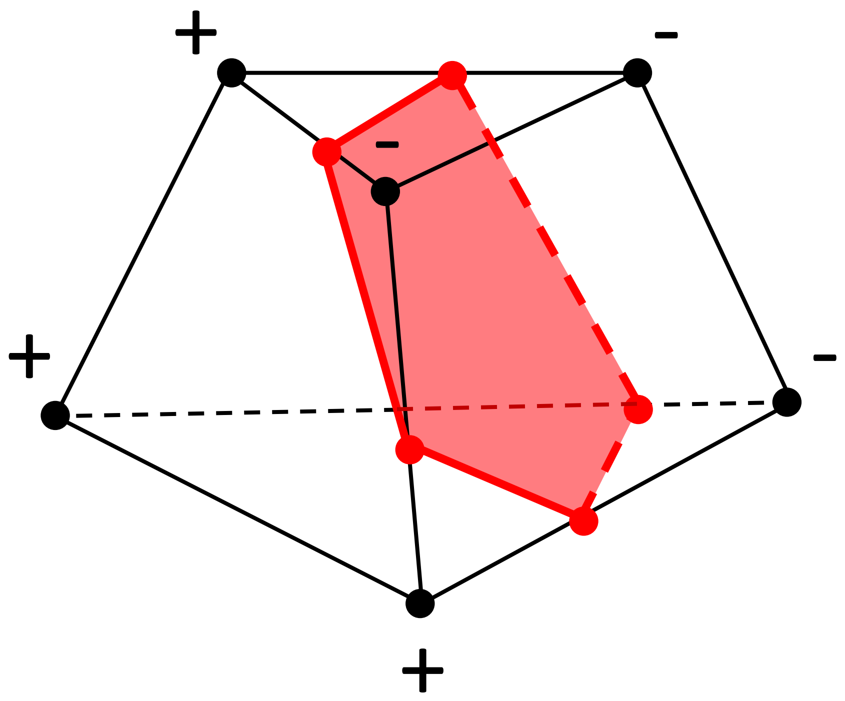

ROBUST SIGNING

- Exact intersection and predicates
  - Requires rational representations of coordinates
- Our approach: coordinate-free exact signing
  - Encodes vertices using functions and tet faces
  - Simple predicate using only function values at tet vertices
  - Inspired by plane-based representations [Sugihara 89; Bernstein 09; Campen 10; Attene 20; Cherchi 20; Nehring-Wirxel 21; Diazzi 21]

VERTEX ENCODING

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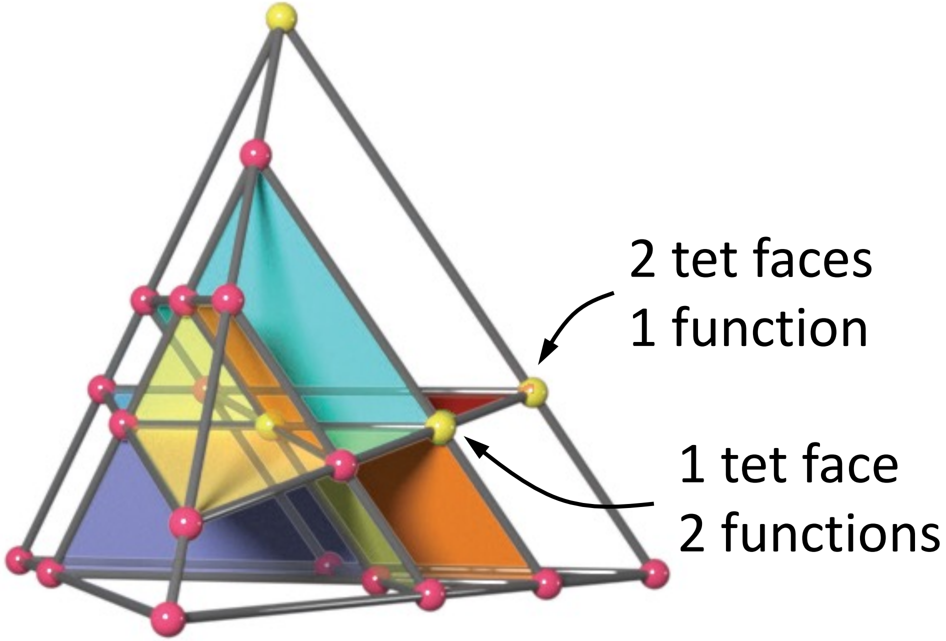
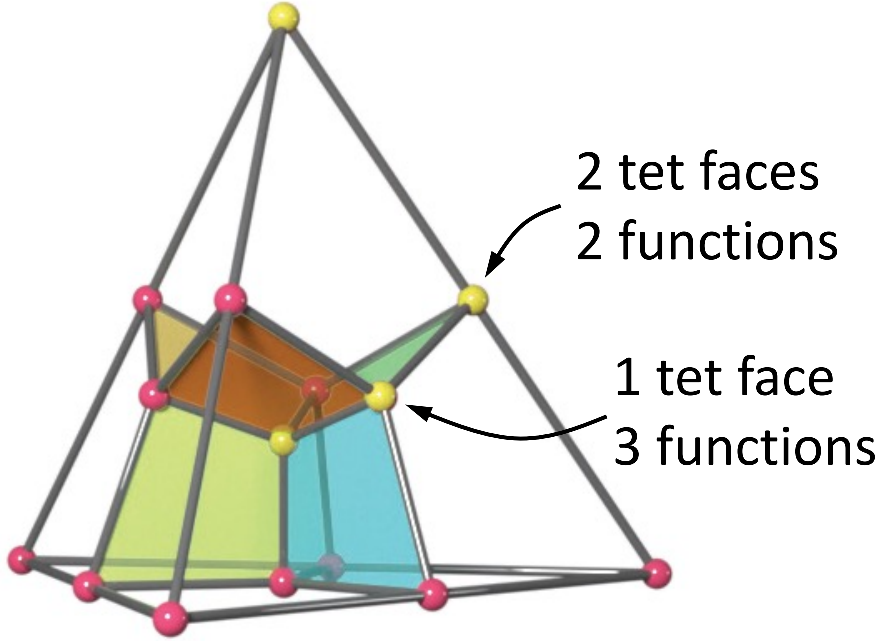
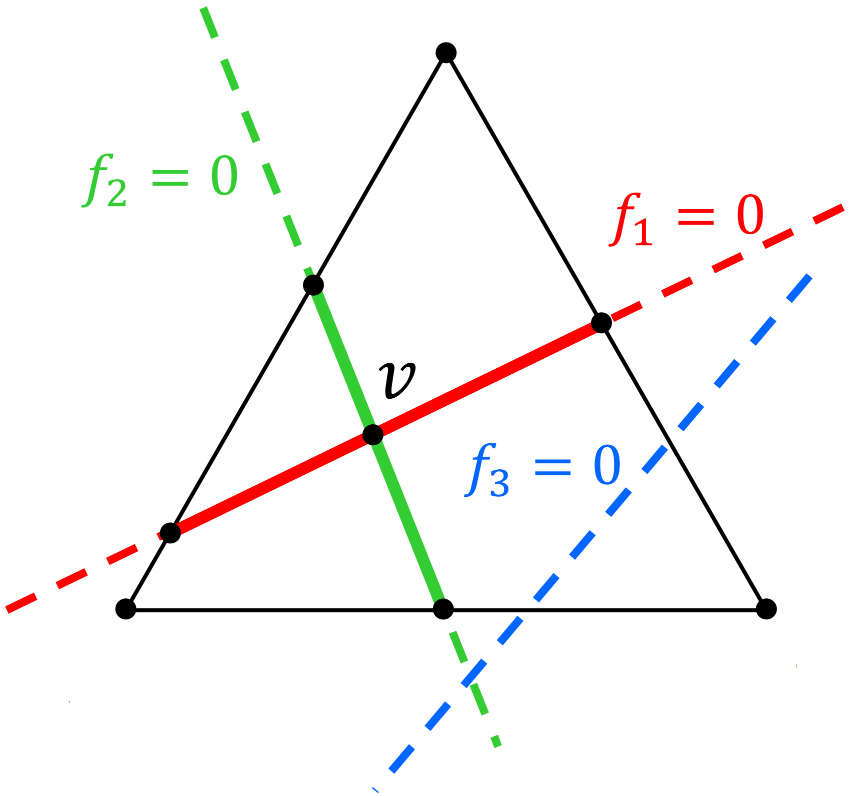


Implicit arrangement: **3** indices



Material interface: **4** indices

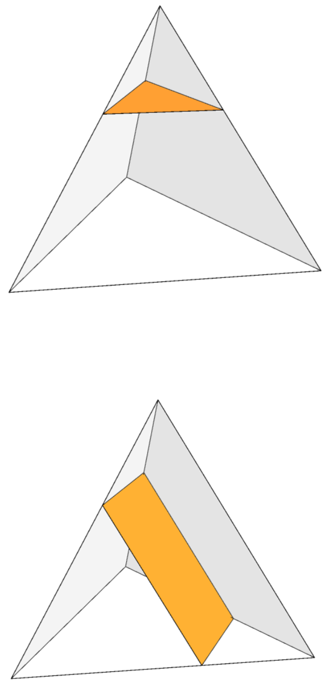
BARYCENTRIC PREDICATE



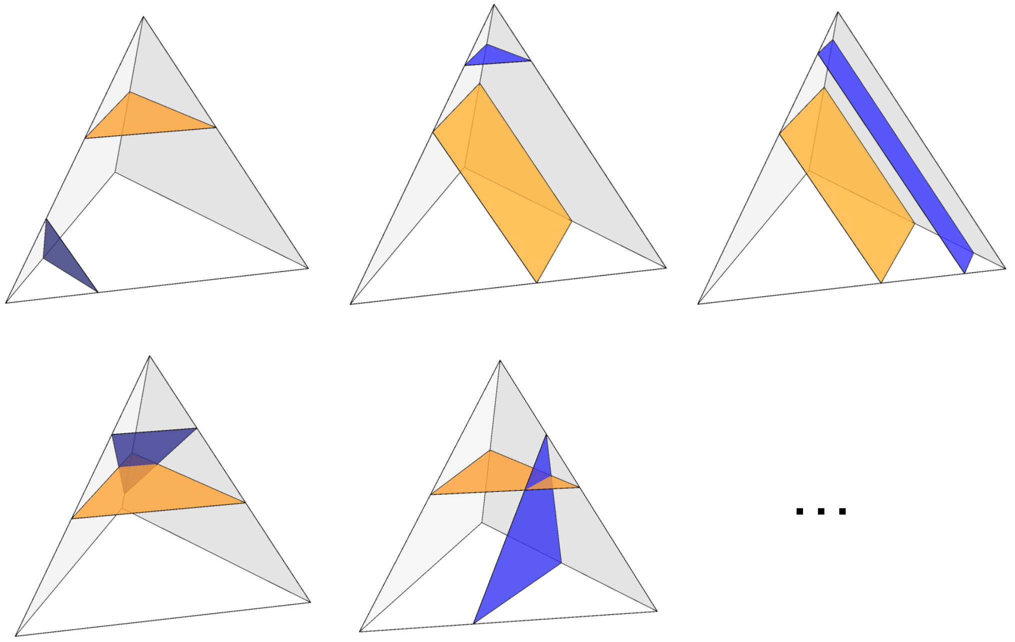
$$sign(f_3(v)) =$$

$$sign\left(det\begin{bmatrix} f_1^1 & f_1^2 & f_1^3 \\ f_2^1 & f_2^2 & f_2^3 \\ f_3^1 & f_3^2 & f_3^3 \end{bmatrix}\right) * sign\left(det\begin{bmatrix} f_1^1 & f_1^2 & f_1^3 \\ f_2^1 & f_2^2 & f_2^3 \\ 1 & 1 & 1 \end{bmatrix}\right)$$

LOOK-UP TABLES



1 function (MI)



2 functions (MI)

RESULTS

RESULTS: ROBUSTNESS

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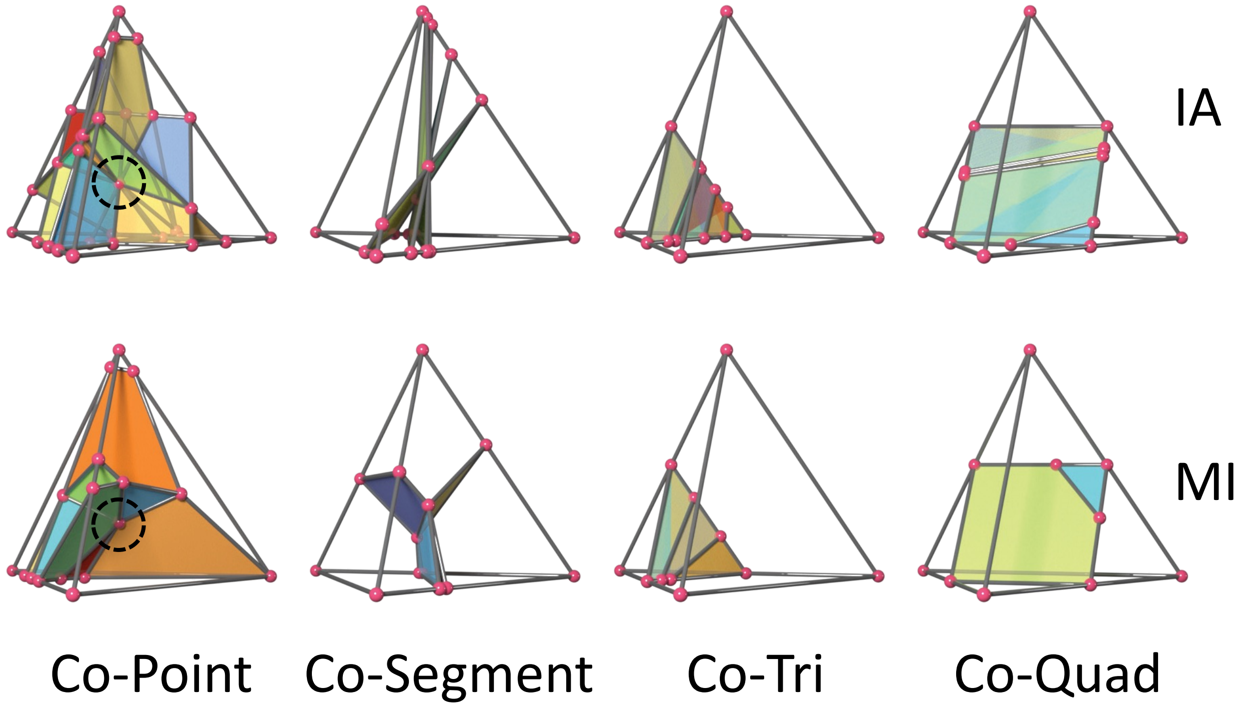
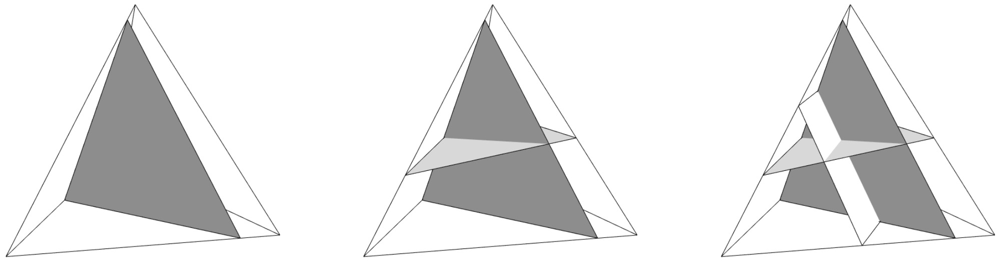
- Near-degeneracy test
  - 4 (IA) or 5 (MI) functions in a tetrahedron
  - 10 000 instances per degeneracy type
- 100% consistency
  - Same combinatorial structure after changing the order of functions



RESULTS: EFFICIENCY

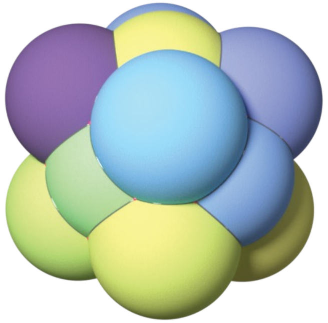
- Comparison with exact coordinate representations [CGAL]
  - IA of 1, 2, and 3 functions in a tetrahedron
  - 100 instances of each type
- Our method is ~100 faster



CGAL:	666.3	1754.8	3302.8
Ours:	6.6	13.7	26.3

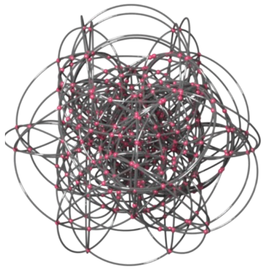
(average time in μs)

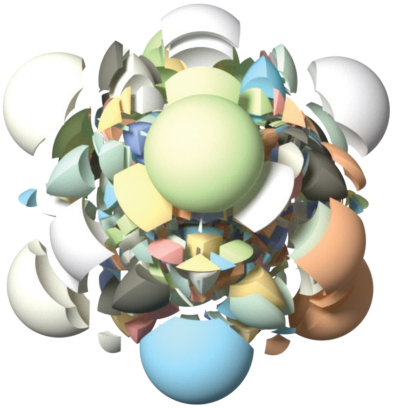
RESULTS: EFFICIENCY

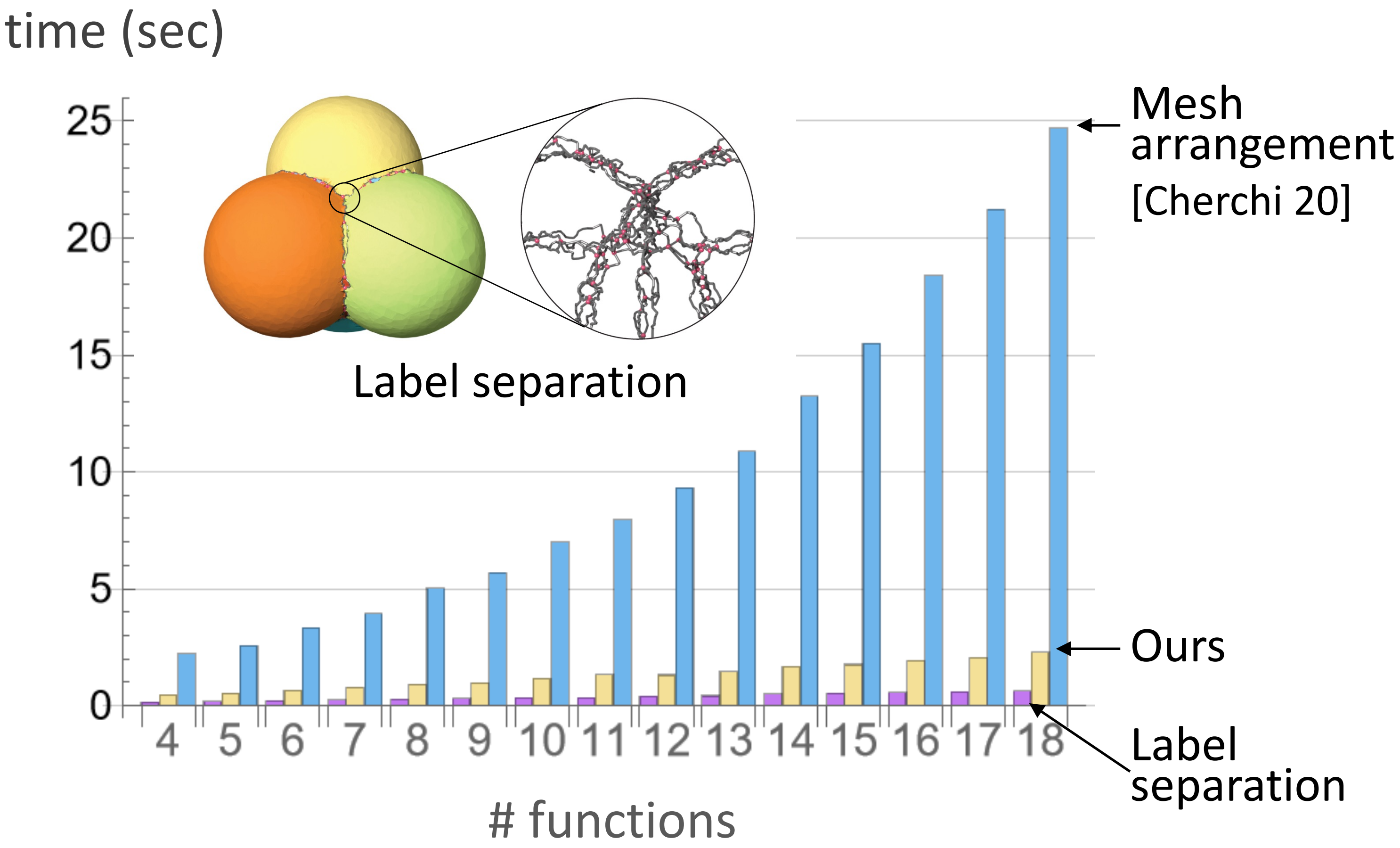


IA of spherical distance functions

(10^6 tet vertices)



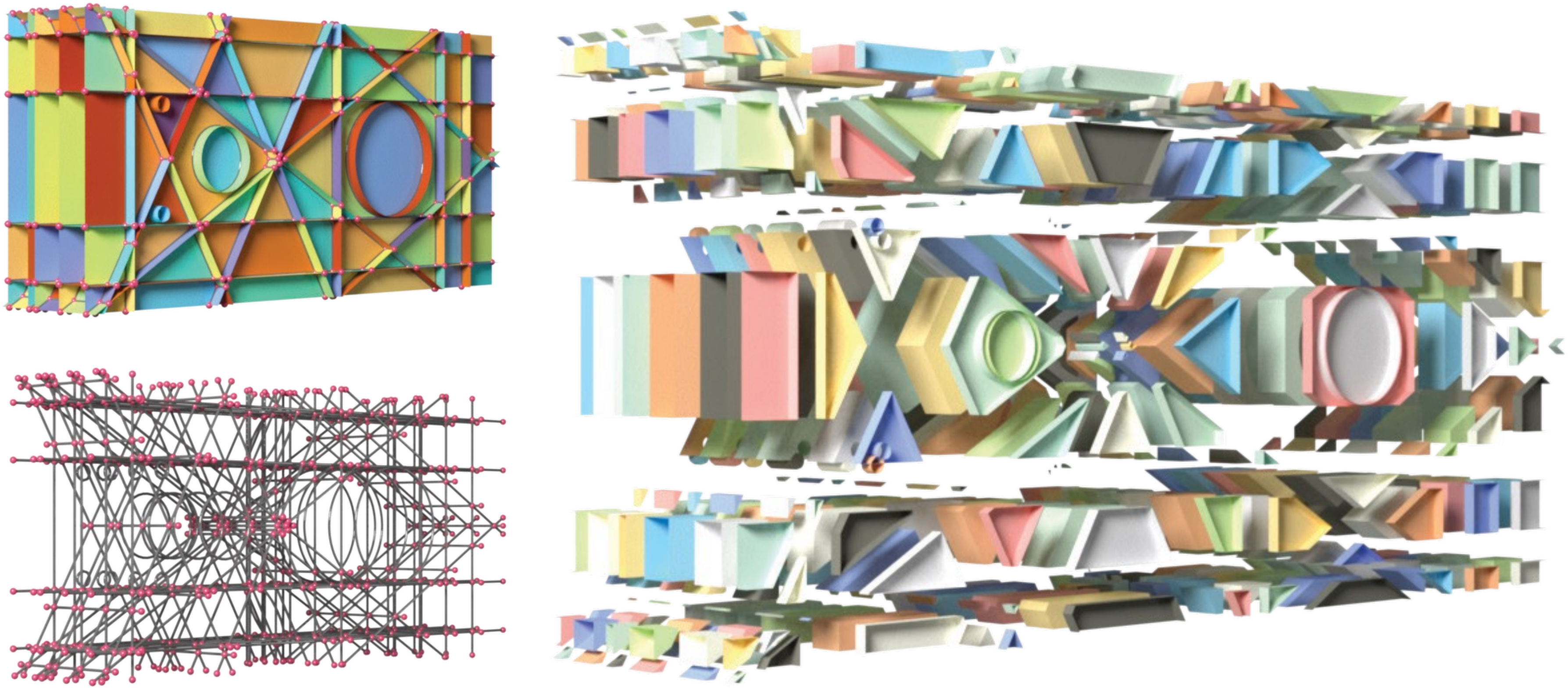




EXAMPLE: CSG

Arrangement of 26 CSG primitives

(10^6 tet vertices)

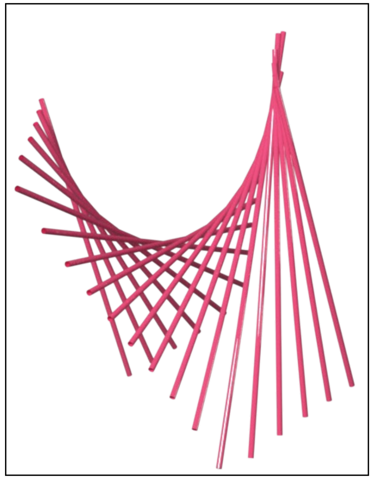


1543 patches, 590 cells

Time: 3.25 sec (ours), 153.4 sec (mesh arr.)

EXAMPLE: VORONOI DIAGRAM

Voronoi diagram of
20 rotating lines
(10^6 tet vertices)



345 patches, 20 cells
Time: 3.54 sec

CONCLUSION

- Robust and efficient algorithms for discretizing implicit arrangement and material interfaces
- Future directions:
  - Parallelization
  - Improving mesh quality
  - Grid generation

Code and data