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DATE

Physics of Wave and Oscillation

Assignment 06:

*) Fourier Series defined in $[-L \leq x \leq L]$ in sine/cosine form.

$$f(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left\{A_n \cos\frac{n\pi x}{L} + B_n \sin\frac{n\pi x}{L}\right\}$$

where: $A_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$

$$A_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$B_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

This is just Summary A but instead of the interval $[-\pi,\pi]$, we expand the Fourier Series on $[-L, L]$ by using $\frac{n\pi x}{L}$ insead of x.

*) More rigorously, to expand function $f(x)$ on interval $[-L, L]$, we use subsitution $x = \frac{Ly}{\pi}$: $F(y) = f\left(\frac{Ly}{\pi}\right)$

Then we can find the fourier series of $F(y)$ on $[-\pi, \pi]$

$$F(y) = f\left(\frac{Ly}{\pi}\right) = \frac{a_0}{2} + \sum_{1}^{\infty}(a_n \cos ny + b_n \sin ny)$$

$$\begin{cases} a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(y)\,dy = \frac{1}{L}\int_{-L}^{L} f(x)\,dx = A_0 \\ a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(y)\cos ny\,dy = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx = A_n \\ b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(y)\sin ny\,dy = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx = B_n \end{cases}$$

• $F(y) = f(x) = \frac{a_0}{2} + \sum_{1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$

*) Explaining the equations $\begin{cases} A_k = \frac{2}{L}\int_0^L \{f(x).\sin kx\}\,dx \\ f(x) = \sum_k A_k \sin kx \end{cases}$ (8_4 & 8_6) for $k = \frac{n\pi}{L}$

The function $f(x)$ has the interval $[0, L]$. To find the fourier series of $f(x)$ easier, we will extend it into the function $f_{odd}(x)$ that has the interval $[-L, L]$.

$$f_{odd} = \begin{cases} -f(-x) & \text{for } -L \leq x < 0 \\ f(x) & \text{for } 0 \leq x \leq L \end{cases}$$

Then because f_{odd} is an odd function, A_0 and A_n of the fourier series of f_{odd} is 0. On the other hand, the coefficient B_n is:

$$B_n = \frac{1}{L}\int_{-L}^{L} f_{odd} \cdot \sin\frac{n\pi x}{L}\,dx$$

$$= \frac{2}{L}\int_{0}^{L} f(x)\,\sin\frac{n\pi x}{L}\,dx$$

And: $$f_{odd} = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{L}$$

Because the value of $f(x)$ at every point on $[0, L]$ could be expressed as the sum $\sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{L}$:

$$f(x) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{L}$$

could be considered one Forrier series expansion of $f(x)$

This concludes the explantion of Eq. 8_4 and Eq. 8_6.

*) Note that: ① there is also an even expansion of the function $f(x)$, which results in :

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos\frac{n\pi x}{L}$$

as $B_n = 0$.

② In sead of using expansion we could find the fourier series of a function on an arbitrary interval $[a, b]$ by setting $L' = \frac{b-a}{2}$. Then the fourier series may have all the coefficients (which takes more time to find)