Calculus with Exercises A — Report 07

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Problem 7.1

(1)

$$\begin{aligned} f'(x) &= (xe^{-x})\cos(xe^{-x}) \\ &= (e^{-x} - xe^{-x})\cos(xe^{-x}) \end{aligned}$$

(2)

$$\begin{aligned} f'(x) &= 4(x^2+\cos^3 x)'(x^2+\cos^3 x)^3 \\ &= 4(2x - \sin x\cos^2 x)(x^2+\cos^3 x)^3 \end{aligned}$$

(3)

$$\begin{aligned} f'(x) &= \frac{-2x\sin(x^2)\sin^2(x)}{(1+x^2)^2} + \frac{2x\cos(x^2)\sin^2(x)}{(1+x^2)} + \frac{2\sin(x^2)\sin(x)\cos x}{(1+x^2)} \\ &= 2\sin x\left(-\frac{x\sin(x^2)\sin x}{(1+x^2)^2} + \frac{x\cos(x^2)\sin x}{(1+x^2)} + \frac{\sin(x^2)\cos x}{(1+x^2)}\right) \end{aligned}$$

Problem 7.2

(1)

$$\begin{aligned} f'(x) &= \lim_{x\to 0}\frac{x\sqrt{|x|} - 0\sqrt{|0|}}{x-0} \\ &= \lim_{x\to 0}\sqrt{|x|} = 0 \end{aligned}$$

(2)

$$\begin{aligned} f'(x) &= \lim_{x\to 0}\frac{x\cos\frac{1}{x} - 0}{x-0} \\ &= \lim_{x\to 0}\cos\frac{1}{x} \end{aligned}$$

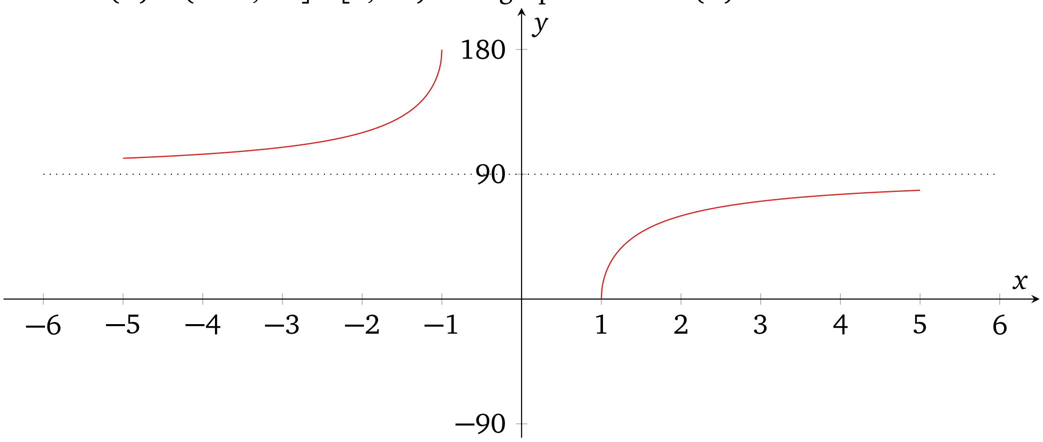
which does not exist. This could be proved using the two sequences $a_n = \dfrac{1}{2n\pi}$ and $b_n = \dfrac{1}{(2n+1)\pi}$.

(3)

$$\begin{aligned} f'(x) &= \lim_{x^3\to 0}\frac{x\cos\frac{1}{x} - 0}{x-0} \\ &= \lim_{x\to 0}x^2\cos\frac{1}{x} = 0 \text{ using Squeeze Theorem} \end{aligned}$$

Problem 7.3

The $\sec x$ function has no inverse. To determine the domain of $\operatorname{arcsec}(x)$, we first have to select a restricted domain for $\sec x$, that is creating a new function $\sec x$ with domain $[0, \pi]$. Then the domain of $\operatorname{arcsec}(x)$ is $(-\infty, -1] \cup [1, \infty)$. The graph for $\operatorname{arcsec}(x)$:



$$
\begin{aligned}
y = \operatorname{arcsec} x &\implies \sec y = x \\
&\implies \frac{dx}{dt} = \sec y \tan y \\
&\implies \frac{dy}{dx} = \frac{1}{\sec y \tan y} = \frac{1}{x \tan y} \text{ as } \tan y = \pm\sqrt{1 - x^2} \text{ we will square the equation.} \\
&\implies \left(\frac{dy}{dx}\right)^2 = \left(\frac{1}{(\sec y \tan y)^2}\right) = \left(\frac{1}{x^2(x^2 - 1)}\right) \\
&\implies \left|\frac{dy}{dx}\right| = \left(\frac{1}{|x|\sqrt{x^2 - 1}}\right)
\end{aligned}
$$

Since $\dfrac{dy}{dx} = \dfrac{1}{\sec y \tan y} = \dfrac{\cos^2 y}{\sin y}$, $\dfrac{dy}{dx}$ will have the same sign as $\sin y$, which is always positive. We conclude that:

$$
\frac{d(\operatorname{arcsec} x)}{dx} = \frac{1}{|x|\sqrt{x^2 - 1}} = \begin{cases} \dfrac{1}{x\sqrt{x^2 - 1}} & \text{for } x \geq 1 \\ \dfrac{-1}{x\sqrt{x^2 - 1}} & \text{for } x \leq -1 \end{cases}
$$

Problem 7.4

We have $p^{(k)}(x) = a_k k! + a_{k+1} \dfrac{(k+1)!}{1!} x + \cdots + a_n \dfrac{n!}{(n-k)!} x^{n-k}$

Therefore $p^{(k)}(0) = a_k k!$.