A dice game

You are suggested to play the following game. You pay 1 euro and can roll four dice. If they score less than 9, you win $r = 10$ euros, otherwise you lose 1 euro you paid.

a. Suppose that the dice are fair, i.e. the probability of obtaining any side of the die is $\frac{1}{6}$.

b. Suppose that the dice are unfair, the probabilities of obtaining sides of the die are as follows:

Side of the die	1	2	3	4	5	6
Probability	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$

Run $n = 10^4$ simulations to check whether you win or lose money in the long run, given that you have 10 euros initially?

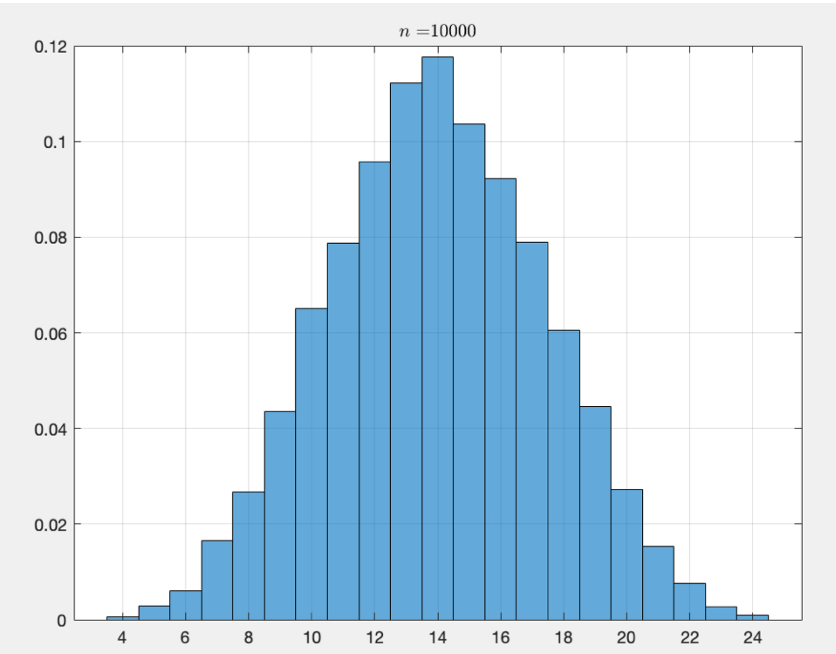
The problem is solved by **Matlab** programming language.

We use ***inverse Cumulative Distribution Function (CDF)*** to generate random number of 4 dices and iterate them with 10 000 times. If the sum of them is less than 9, we gain 10 euros, otherwise the sum is deducted 1 euro. We store all the sums of each iteration.

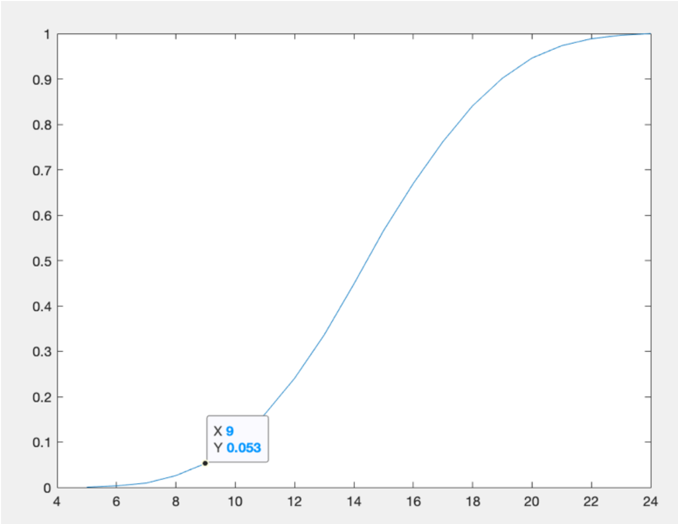
a/ With the equal probability of 1/6 for all sides of the dice:

The mean of all the sums is 13.98 (~ 14).

And total sum overall is -4160, which means that we will lose 4 160 euros if we decide to play this game in the long run. Let's check the distribution of the sums.

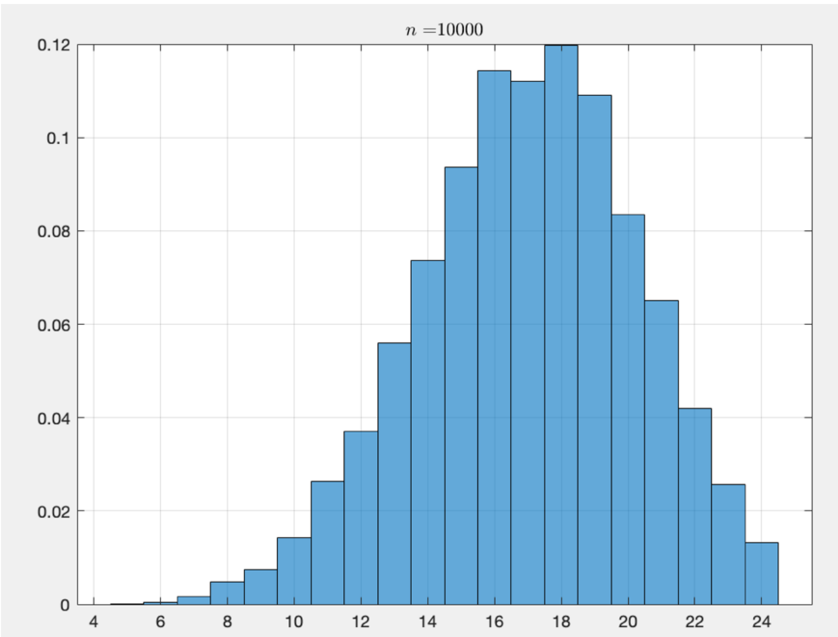


Since the probability is equal in all sides of a dice and we draw random numbers from a normal distribution, the distribution of sums is also normal and symmetric. The most frequent sum is around 14 with nearly 12% as in figure above.

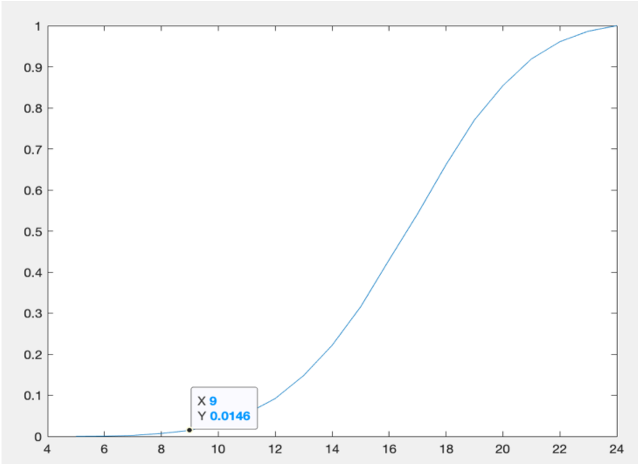


The graph above is the cumulative frequency of the sum in each iteration. So the total frequency of the sum from 4 to 9 is 5.3% in all 10 000 iteration of this simulation.

b/ The probability of sides are different as in table above:
The mean of all the sums in this case is 16.98 (~ 17).
The total sum is -9209, which means we may lose 9209 euros if we play this game in a long run. The distribution of the sums is as following:



Since the probability is not equal in all sides. The distribution is no longer symmetric but left-skewed. The most frequent sum is 18. Let's check cumulative frequency distribution:



So, since the probability of small number (1,2,3) of the sides is lower with 1/12 probability, the total frequency of sums from 4 to 9 is smaller than the previous case. It is only 1.46% in all 10 000 iterations of the simulation process.