Directed graphs

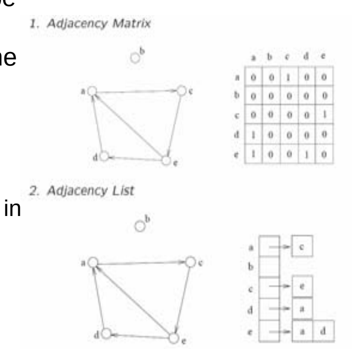
anhtt-fit@mail.hut.edu.vn

Terminology

- Connected graph
  - A graph is connected if and only if there exists a path between every pair of distinct vertices
- Sub-graph
  - A graph with the vertex and edge set being subsets of the original graph
- Connected Components
  - A connected component of a graph is a maximally connected subgraph of a graph
- Cycle
  - A path in a graph that starts and ends at the same vertex
- Tree
  - A graph G is a tree if and only if it is connected and acyclic
- Directed Graph
  - A graph whose the edges (arcs) are directional
- Directed Acyclic Graph
  - A directed graph with no directed cycles

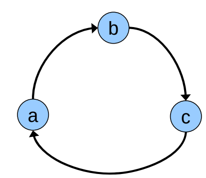
Directed Graphs

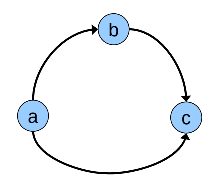
- A directed graph can be represented by an adjacency matrix/list the same way as in undirected graph, except:
  - An arc (u, v) only contributes to 1 entry in the adj. matrix or 1 node in the adj. list



Paths/Cycles

- A directed graph can also contain paths and cycles ("directed paths" and "directed cycles")
  - Graph on top has directed paths and directed cycle
  - Graph on bottom has directed paths but NO directed cycle (acyclic)





Graph traversal

- BFS and DFS can be used to traverse a directed graph, the same way as in undirected graph
- To check for connectivity of a graph
  - run BFS or DFS using an arbitrary vertex as the source. If all vertices have been visited, then the graph is connected; otherwise, the graph is disconnected

Finding Connected Components

- Run DFS or BFS from a vertex
  - the set of visited vertices form a connected component
- Find another vertex i which has not been visited before, run DFS or BFS from it
  - we have another connected component
- Repeat the steps until all vertices are visited
- Running time is

$$\sum_i O(n_i + m_i) = O(\sum_i n_i + \sum_i m_i) = O(n+m)$$

A complete graph API

- In the current graph API, only the edges are managed. Therefore we can not know how many vertices there are in the graph. Each vertex need also a name for identification.
- Redefine the graph structure in order the vertices data are stored in a tree as the following

```
typedef struct {
   JRB edges;
   JRB vertices;
} Graph;
```

Quiz 1

- Rewrite the (directed) graph API based the new data structure with the functions below

```
Graph createGraph();
void addVertex(Graph graph, int id, char* name);
char *getVertex(Graph graph, int id);
void addEdge(Graph graph, int v1, int v2);
void hasEdge(Graph graph, int v1, int v2);
int indegree(Graph graph, int v, int* output);
int outdegree(Graph graph, int v, int* output);
int getComponents(Graph graph);
void dropGraph(Graph graph);
```

Topological Sort

- One can make use of the direction in the directed graph to represent a dependent relationship
  - COMP104 is a pre-requisite of COMP171
  - Breakfast has to be taken before lunch
- A typical application is to schedule an order preserving the order-of-completion constraints following a topological sort algorithm
  - We let each vertex represents a task to be executed. Tasks are inter-dependent that some tasks cannot start before another task finishes
  - Given a directed acyclic graph, our goal is to output a linear order of the tasks so that the chronological constraints posed by the arcs are respected
  - The linear order may not be unique

Topological Sort Algorithm

1. Build an “indegree table” of the DAG
2. Output a vertex v with zero indegree
3. For vertex v, the arc (v, w) is no longer useful since the task (vertex) w does not need to wait for v to finish anymore
   - So after outputting the vertex v, we can remove v and all its outgoing arcs. The result graph is still a directed acyclic graph. So we can repeat from step 2 until no vertex is left

Demo

- demo-topological.ppt

Pseudocode

```
Algorithm TSort(G)
Input: a directed acyclic graph G
Output: a topological ordering of vertices
Initialize Q to be an empty queue;
For each vertex v
     do if indegree(v) = 0
             then enqueue(Q,v);
While Q is non-empty
     do v := dequeue(Q);
          output v;
          for each arc(v,w)
               do indegree(w) = indegree(w)-1;
                    if indegree(w) = 0
                         then enqueue(w);
```

Quiz 2

- Let a file describe the perquisites between classes as the following

```
CLASS CS140
  PREREQ CS102
CLASS CS160
  PREREQ CS102
CLASS CS302
  PREREQ CS140
CLASS CS311
  PREREQ MATH300
  PREREQ CS302
```

- Use the last graph API to write a program to give a topological order of these classes