한양대학교

Model Predictive Control

2019. 11. 01

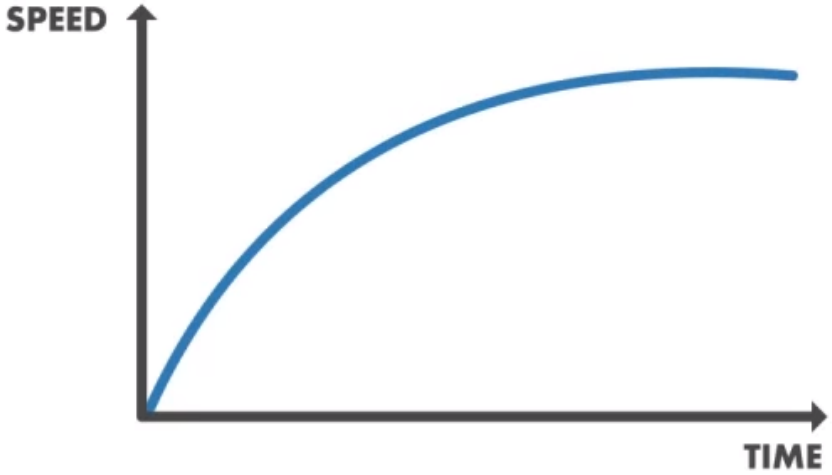
김정환

한양대학교 전자공학부

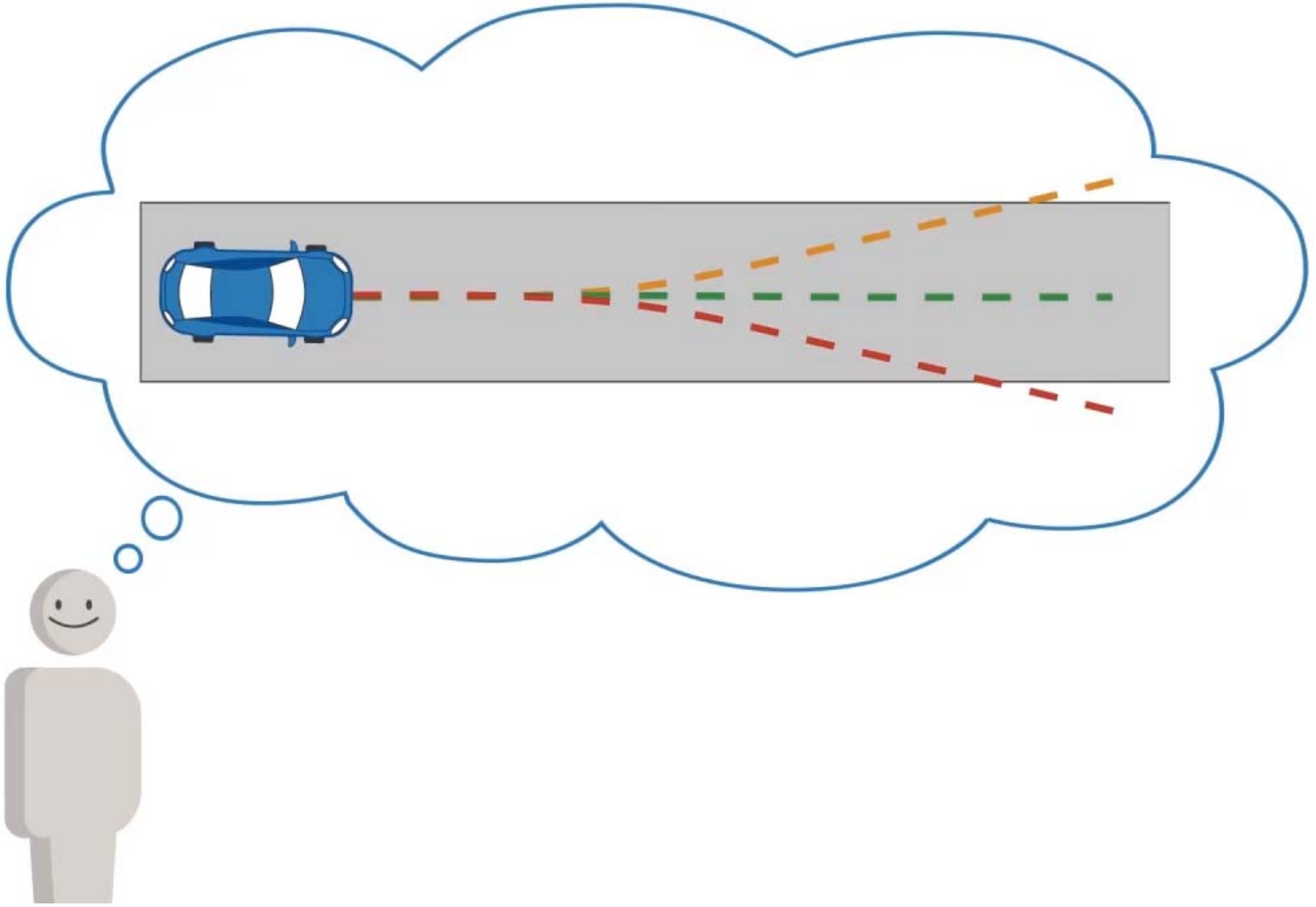
Contents

1. Why Use MPC?

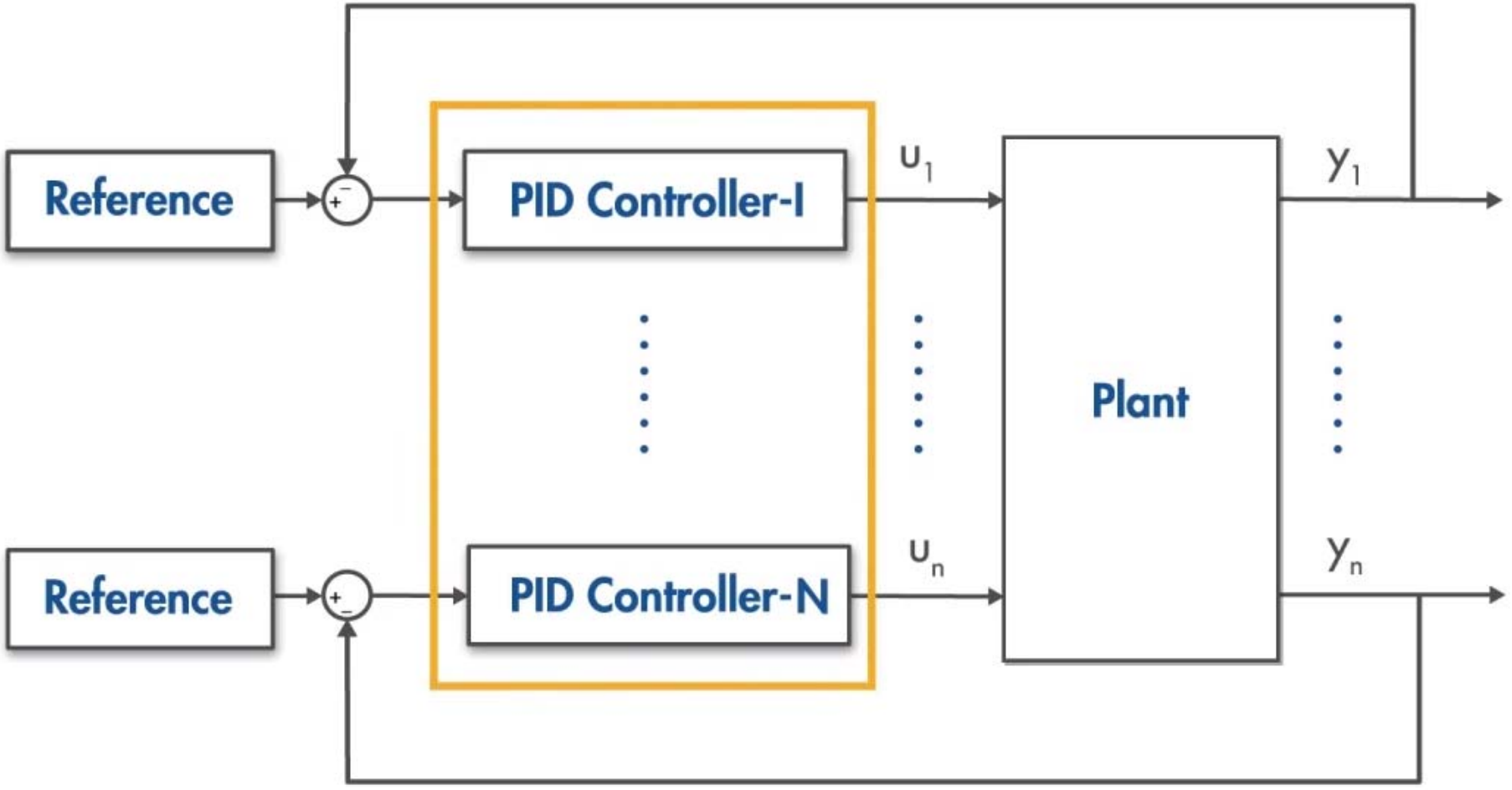
1. Why Use MPC?



1. Why Use MPC?

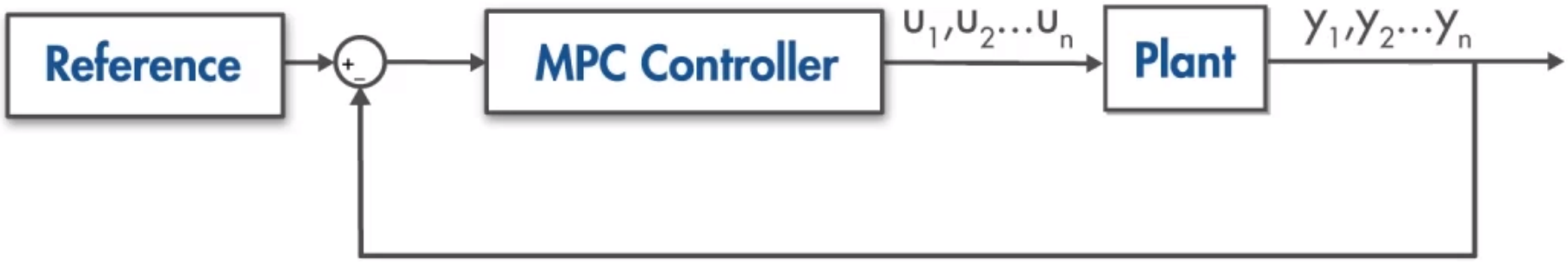


1. Why Use MPC?


Reference
PID Controller-I
u_1
y_1
Plant
Reference
PID Controller-N
u_n
y_n

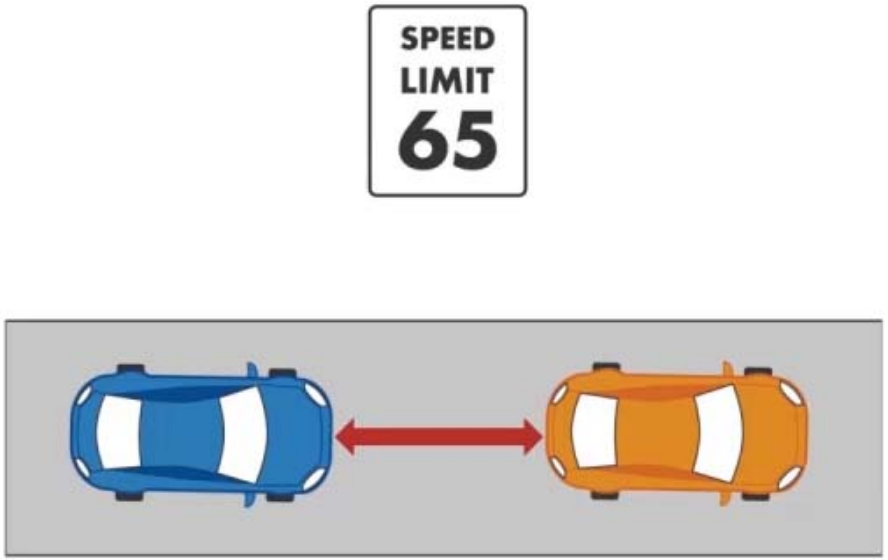
1. Why Use MPC?

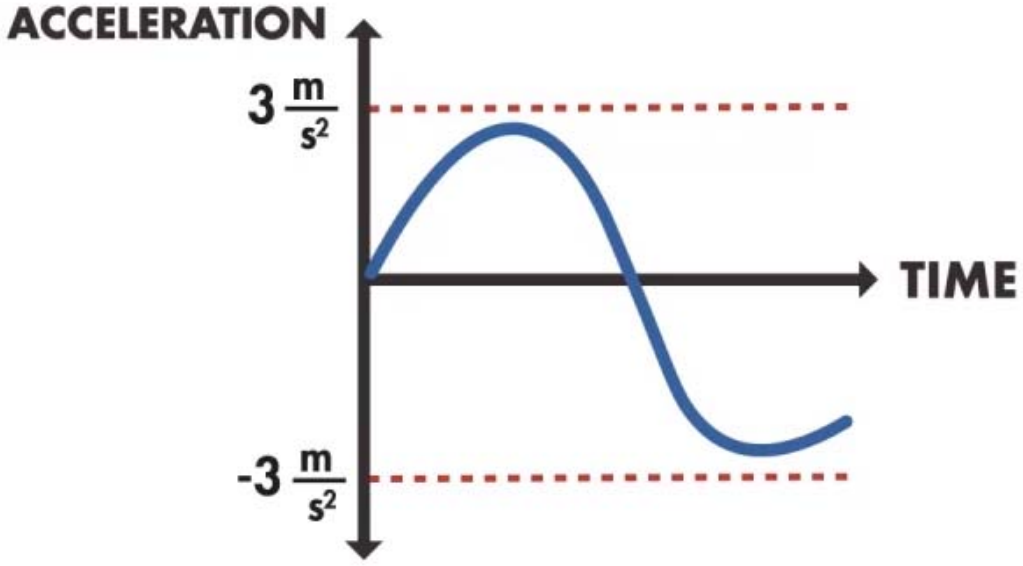
MPC can handle multi-input multi-output (MIMO) systems.



1. Why Use MPC?

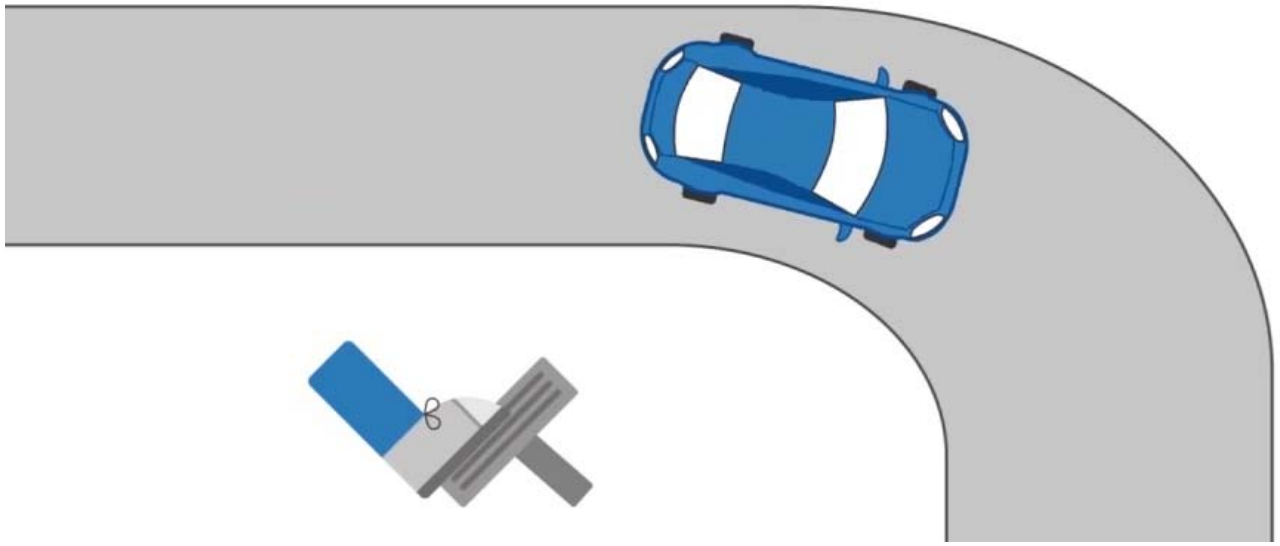
MPC can handle constraints.





1. Why Use MPC?

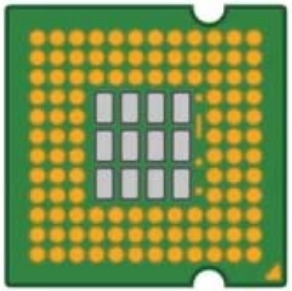
MPC has preview capability.



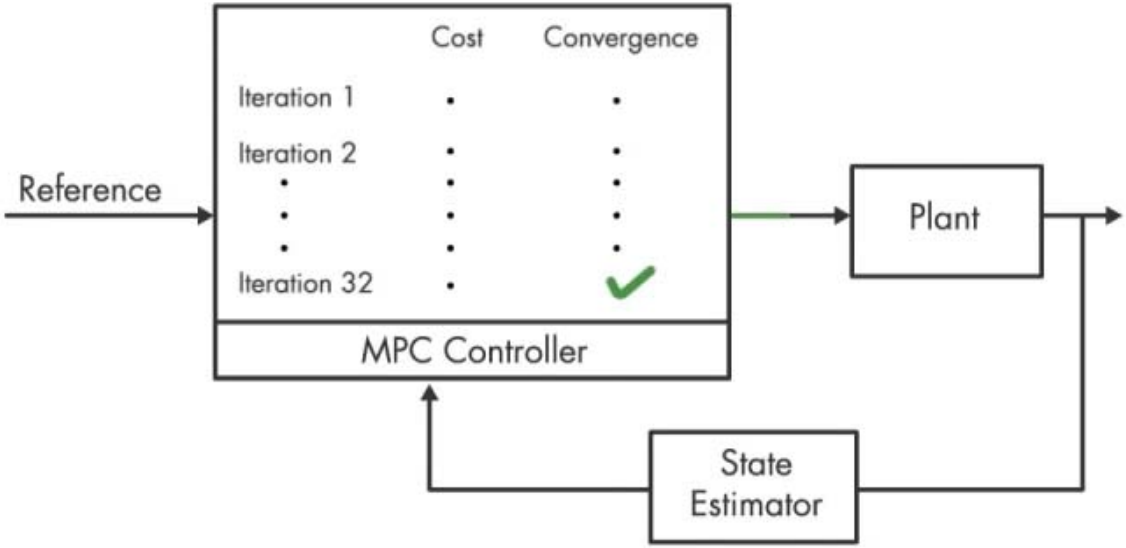
1. Why Use MPC?

Process Industry

Automative
Aerospace
Energy
Food Processing
Industrial Manufacturing
Metallurgy and Mining
Robotics



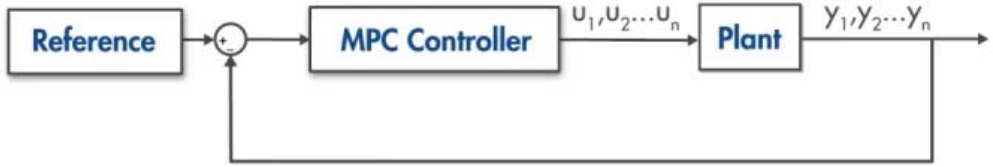
MPC requires a powerful, fast processor with a large memory.



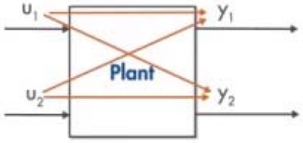
1. Why Use MPC?

Model Predictive Control (MPC)

- MIMO systems

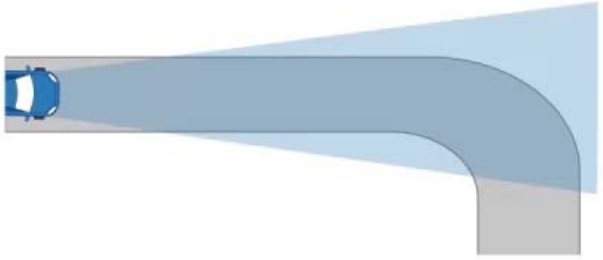


- Input-output interactions

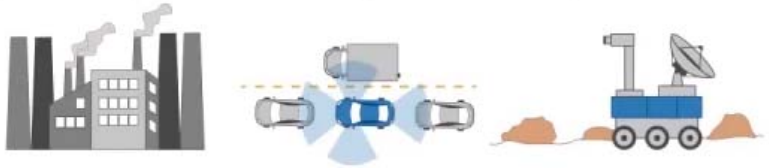


- Constraints

- Preview



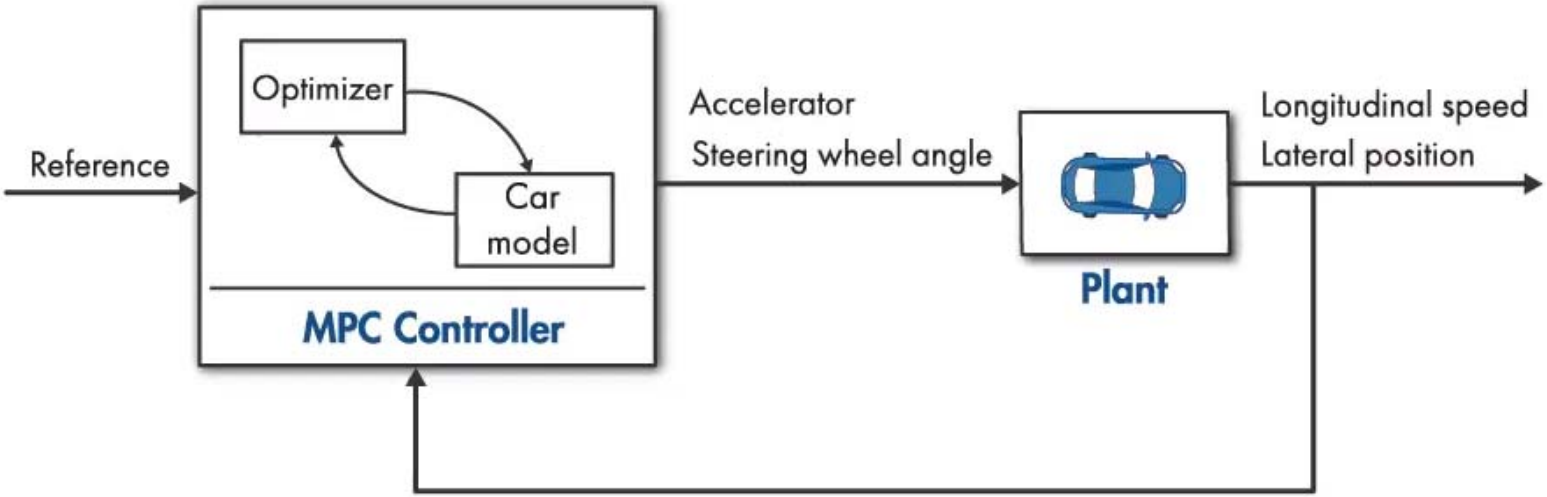
- Has been used in many industries such as process, automotive, and aerospace



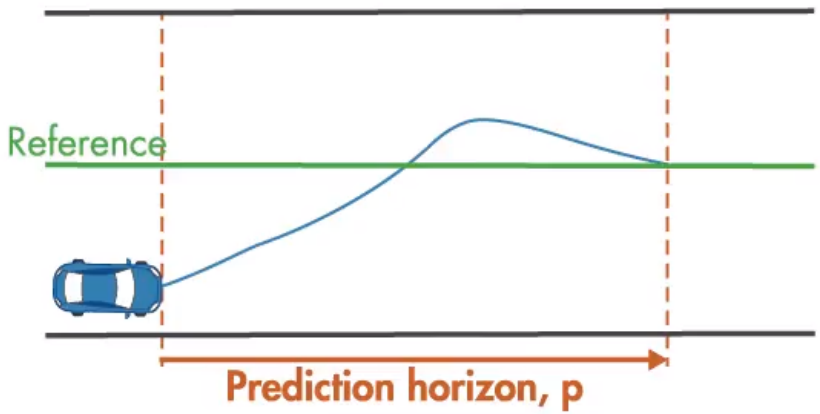
2. What is MPC?

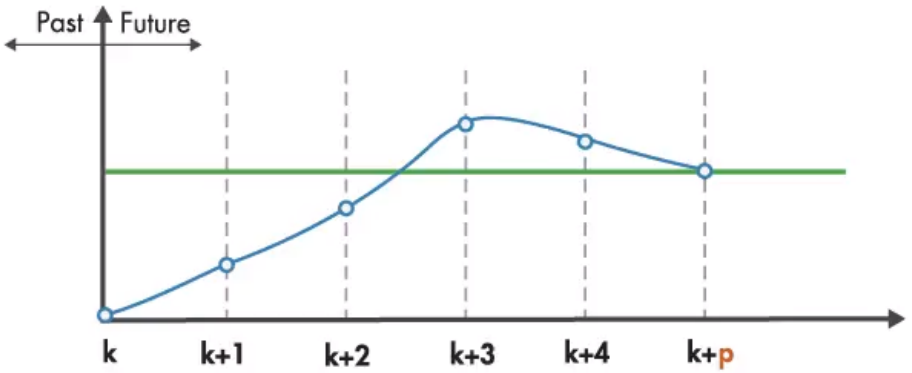

Reference

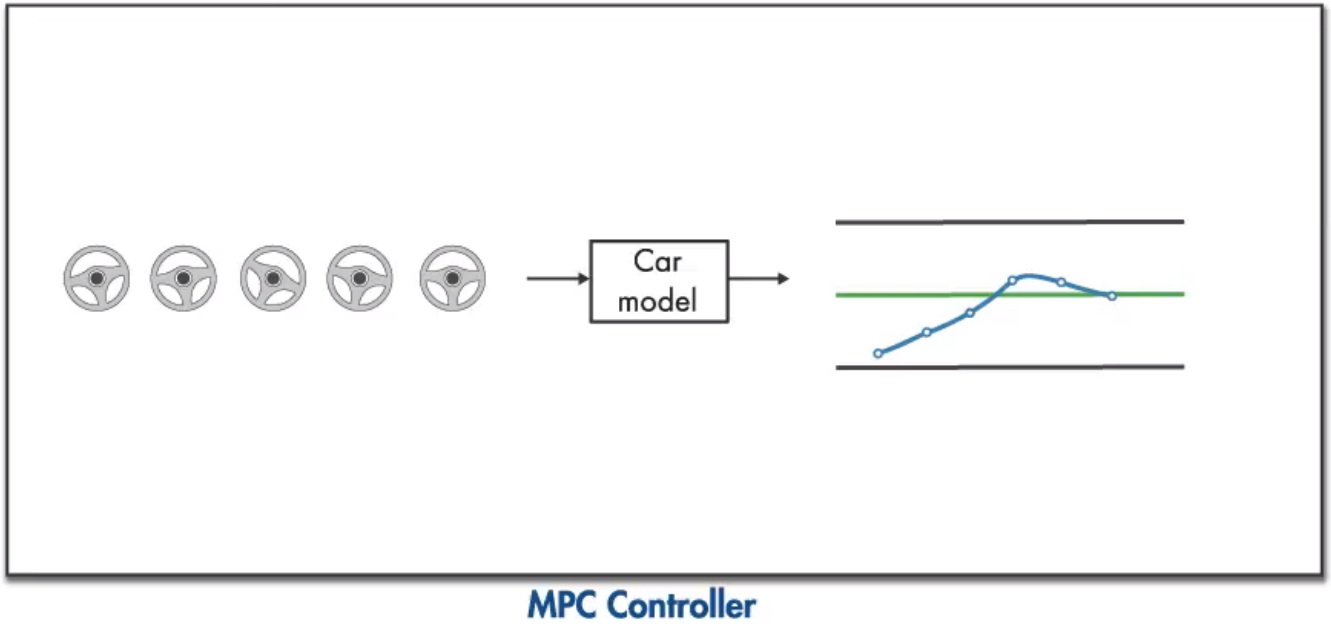

Past
Future
k


Optimizer
Car model
MPC Controller
Reference
Accelerator
Steering wheel angle
Plant
Longitudinal speed
Lateral position

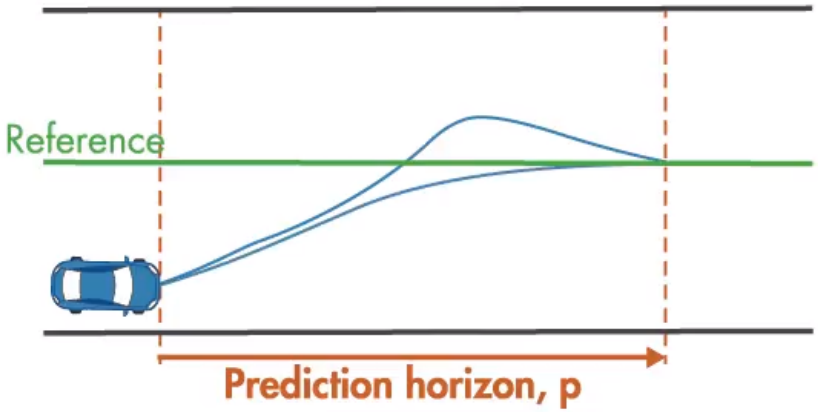
2. What is MPC?

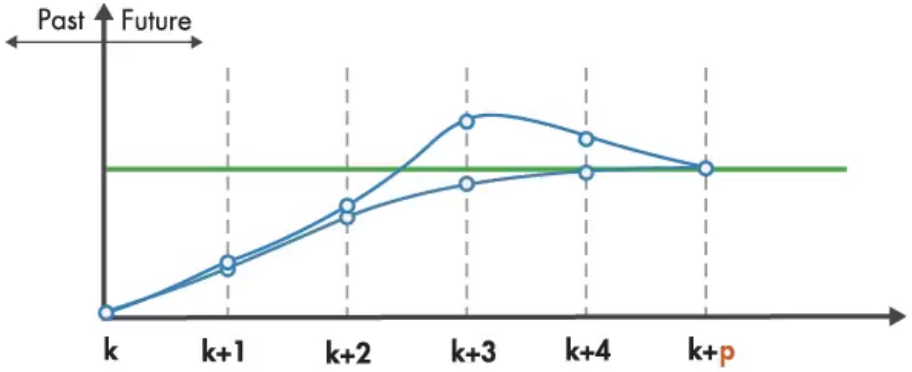

Reference
Prediction horizon, p

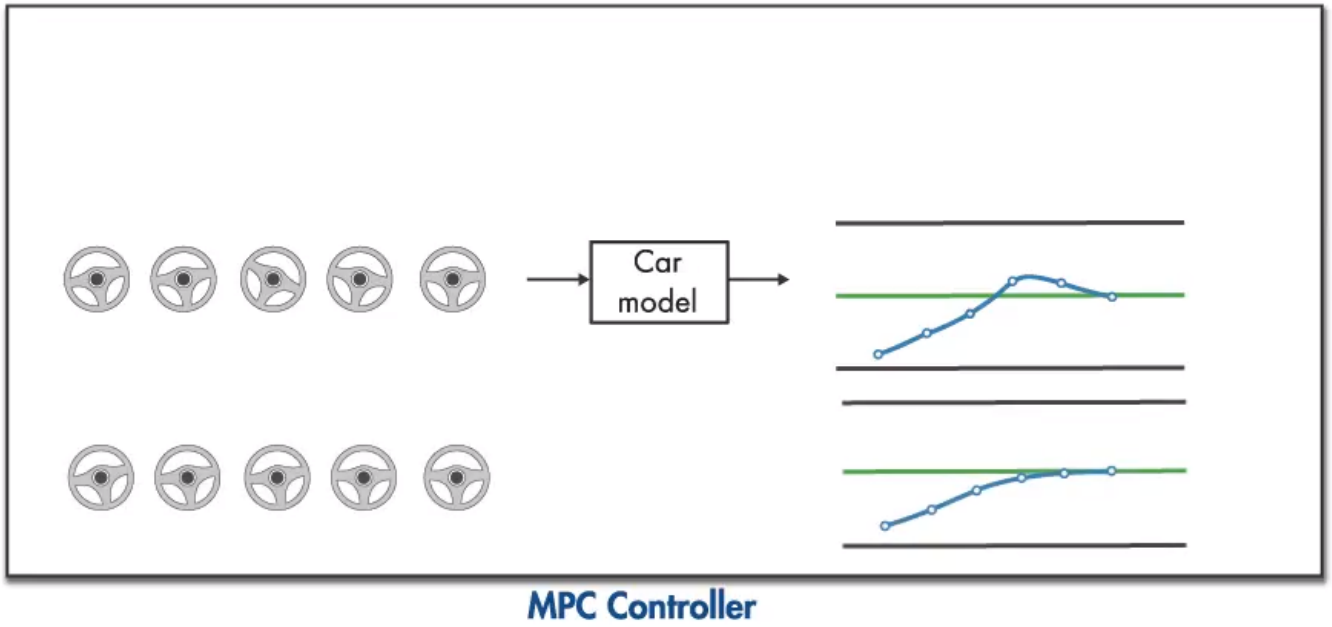

Past
Future
k
k+1
k+2
k+3
k+4
k+p


Car
model
MPC Controller

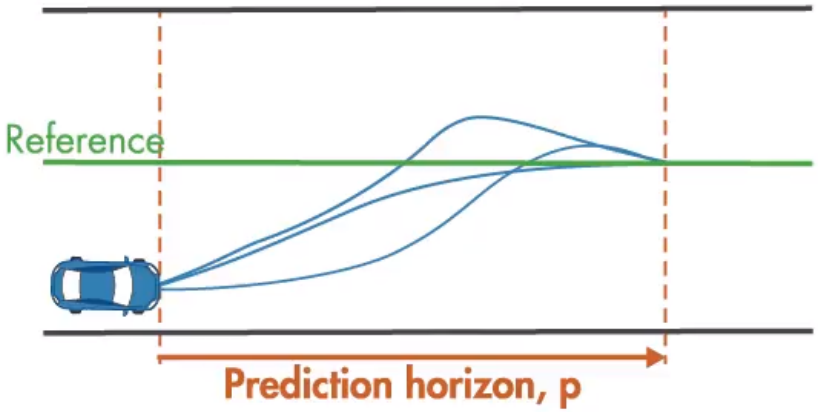
2. What is MPC?

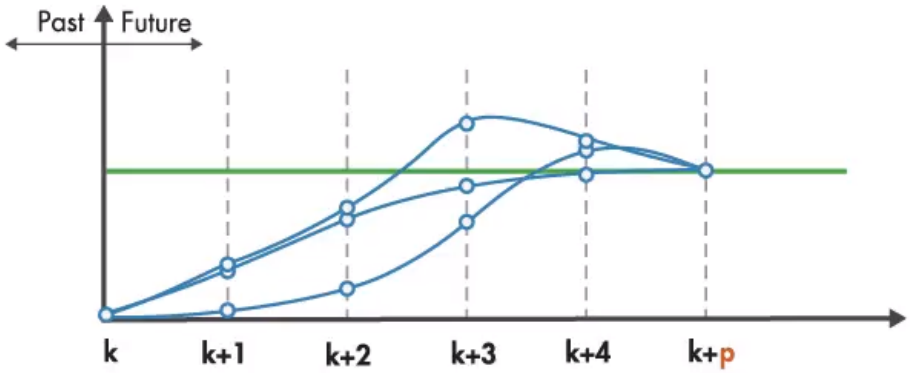

Reference
Prediction horizon, p

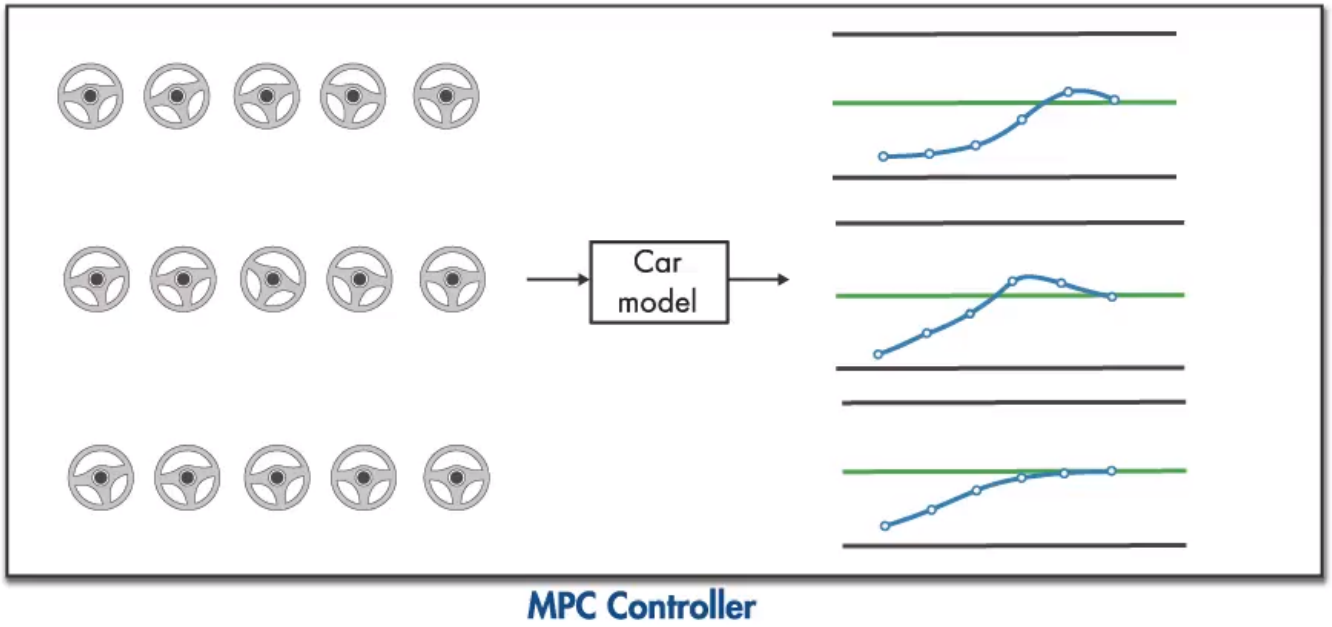

Past
Future
k
k+1
k+2
k+3
k+4
k+p


Car model
MPC Controller

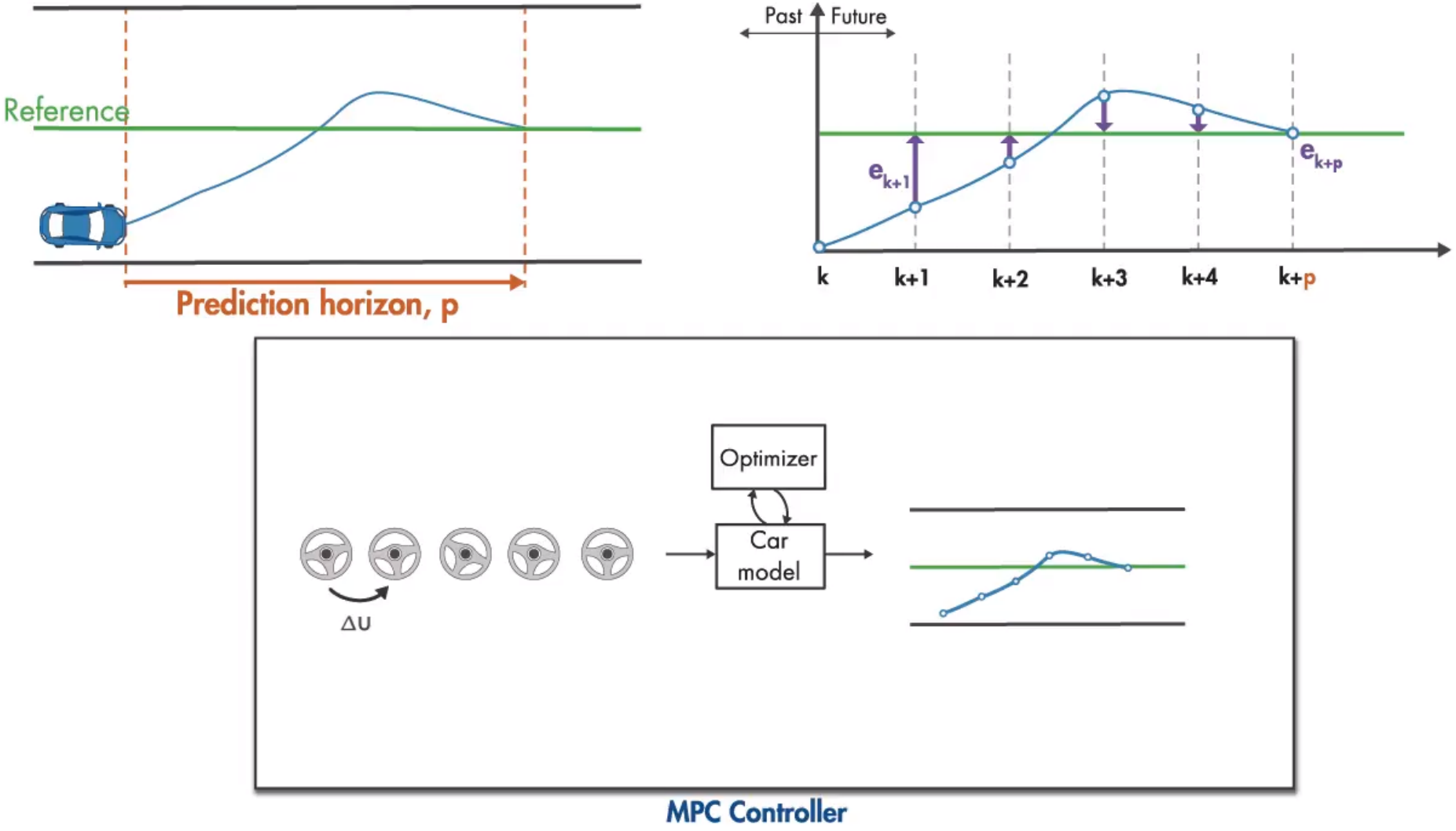
2. What is MPC?


Reference
Prediction horizon, p

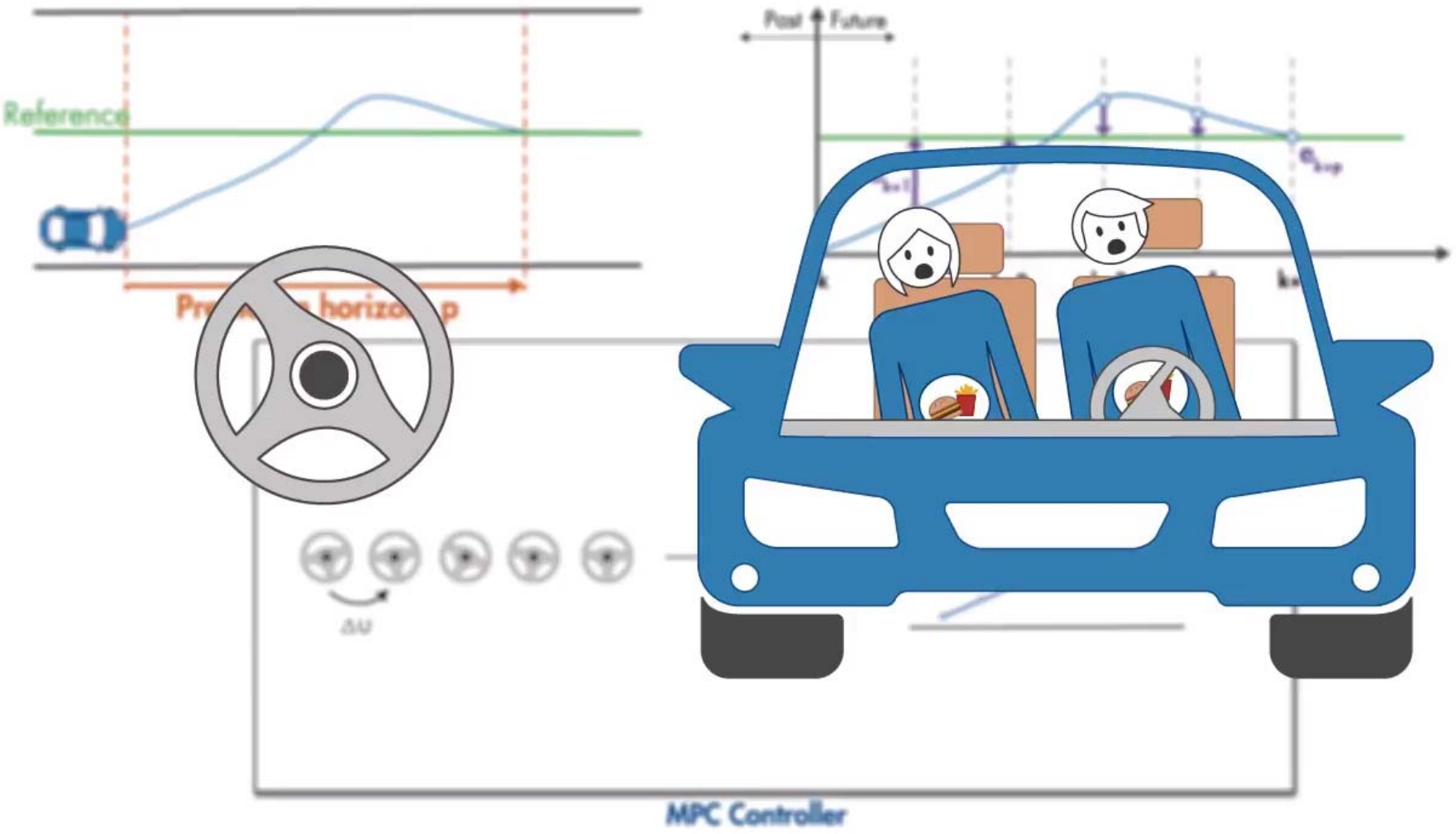

Past
Future
k
k+1
k+2
k+3
k+4
k+p


Car model
MPC Controller

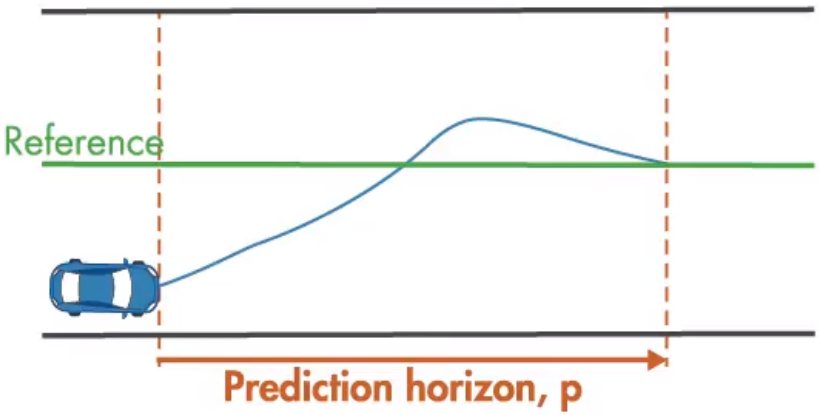
2. What is MPC?

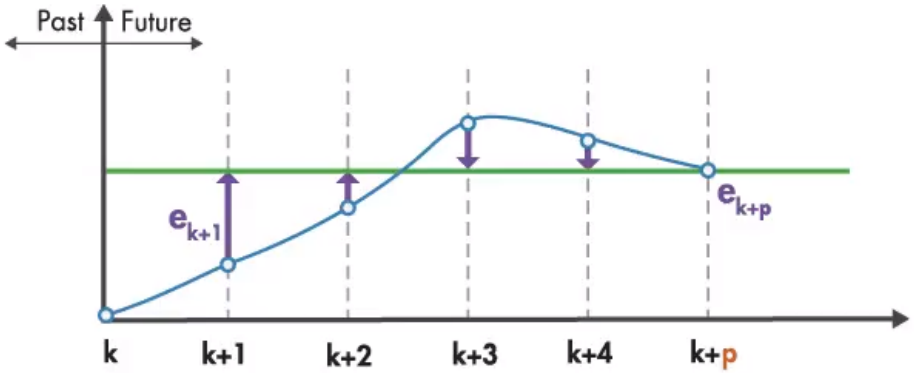

Reference
Prediction horizon, p
Past
Future
e_{k+1}
e_{k+p}
k
k+1
k+2
k+3
k+4
k+p
Optimizer
Car model
ΔU
MPC Controller

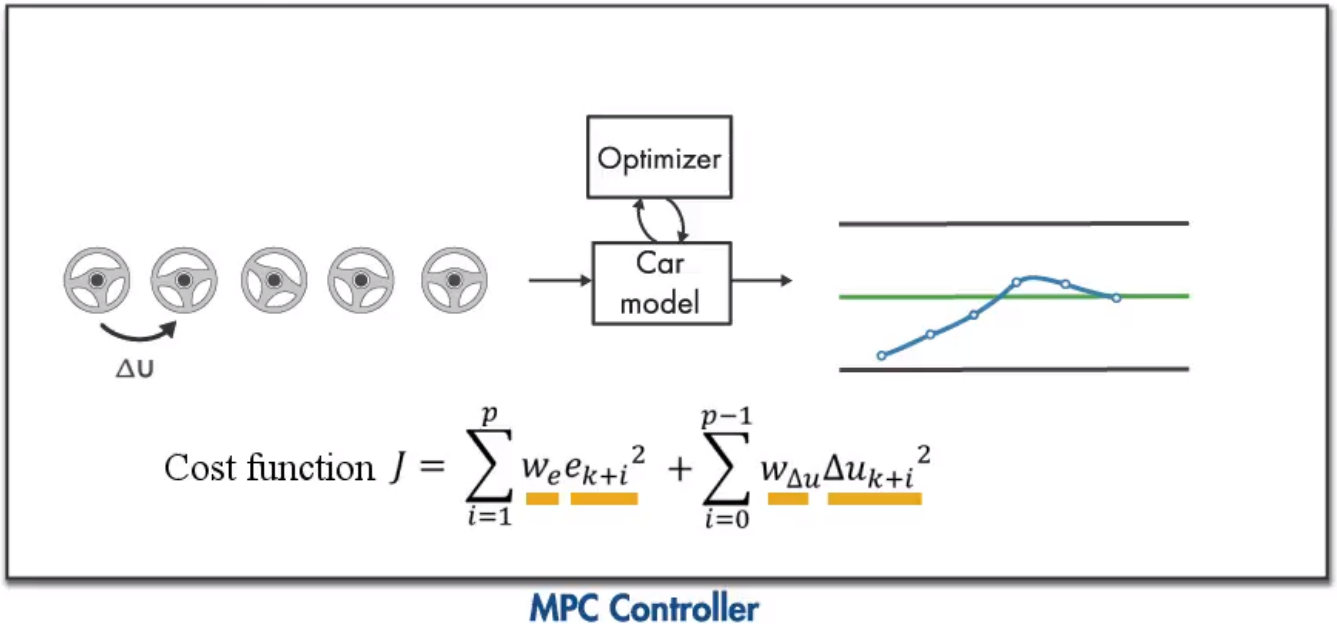
2. What is MPC?



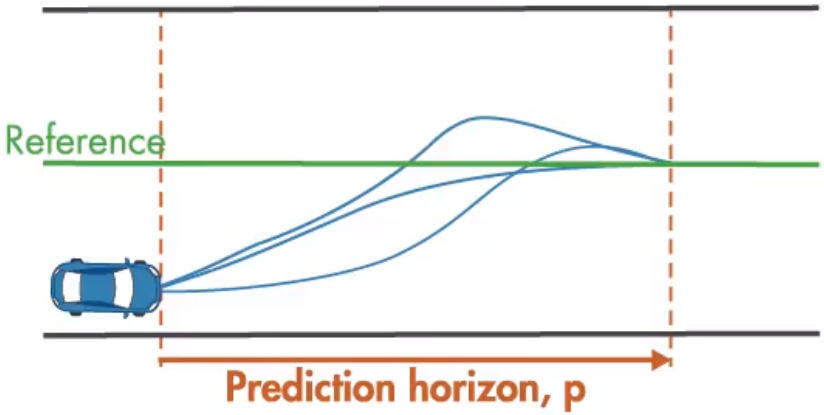
2. What is MPC?

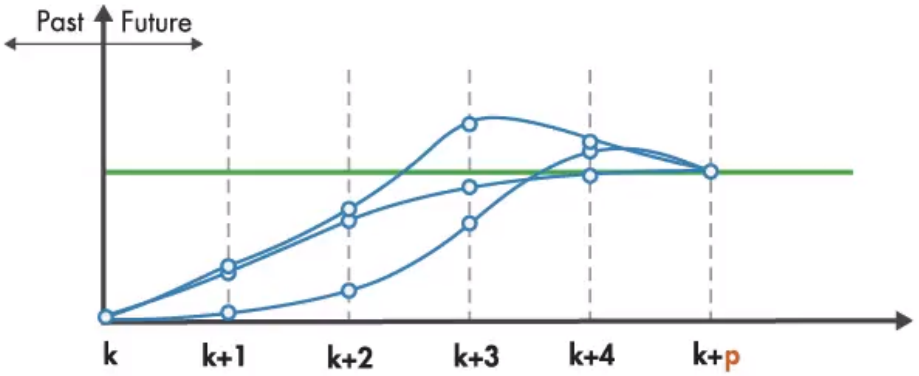

Reference
Prediction horizon, p

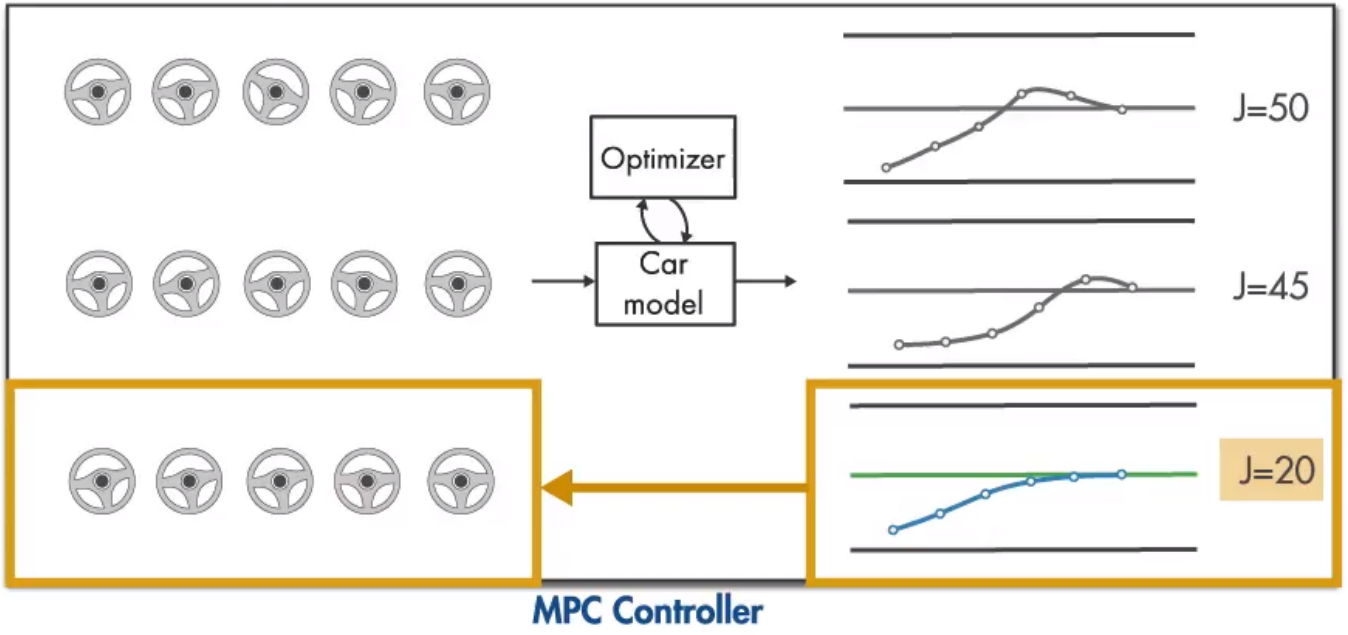

Past
Future
e_{k+1}
e_{k+p}
k
k+1
k+2
k+3
k+4
k+p


Optimizer
Car model
ΔU
Cost function $J = \sum_{i=1}^{p} w_e e_{k+i}^2 + \sum_{i=0}^{p-1} w_{\Delta u} \Delta u_{k+i}^2$
MPC Controller

2. What is MPC?

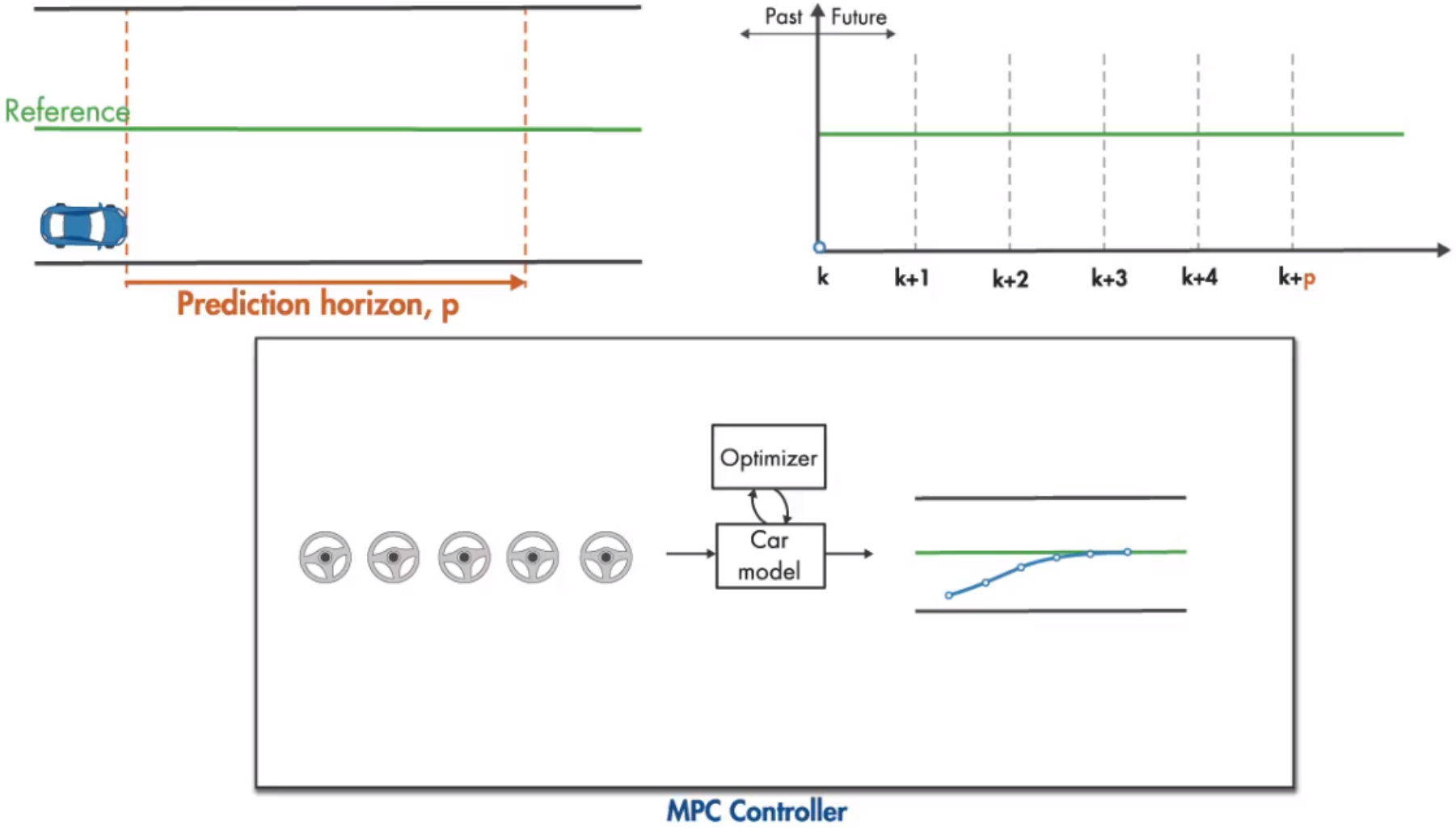




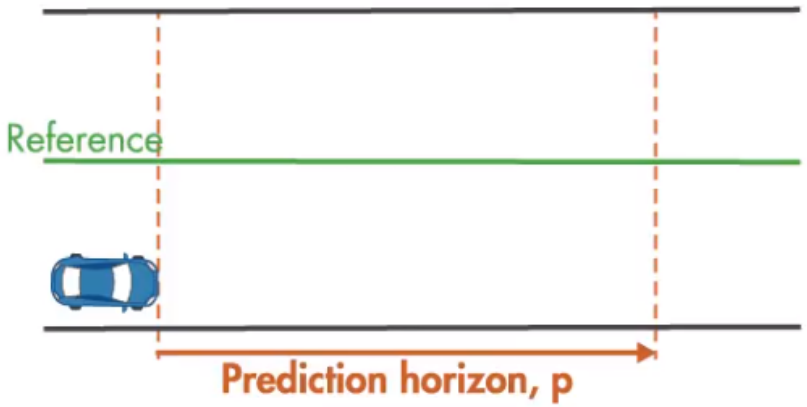


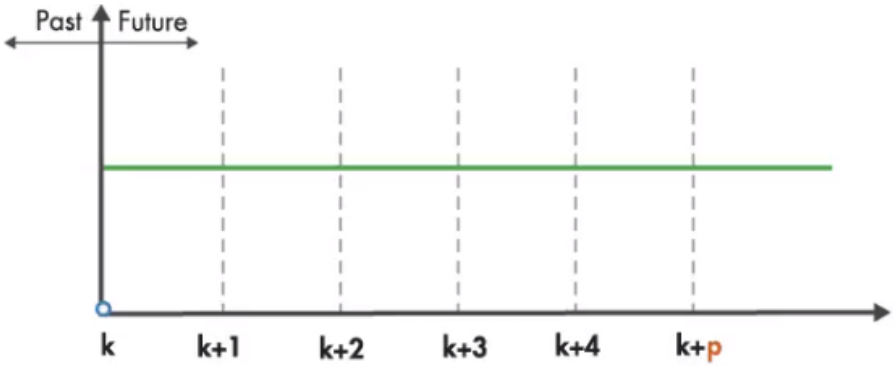
J = cost

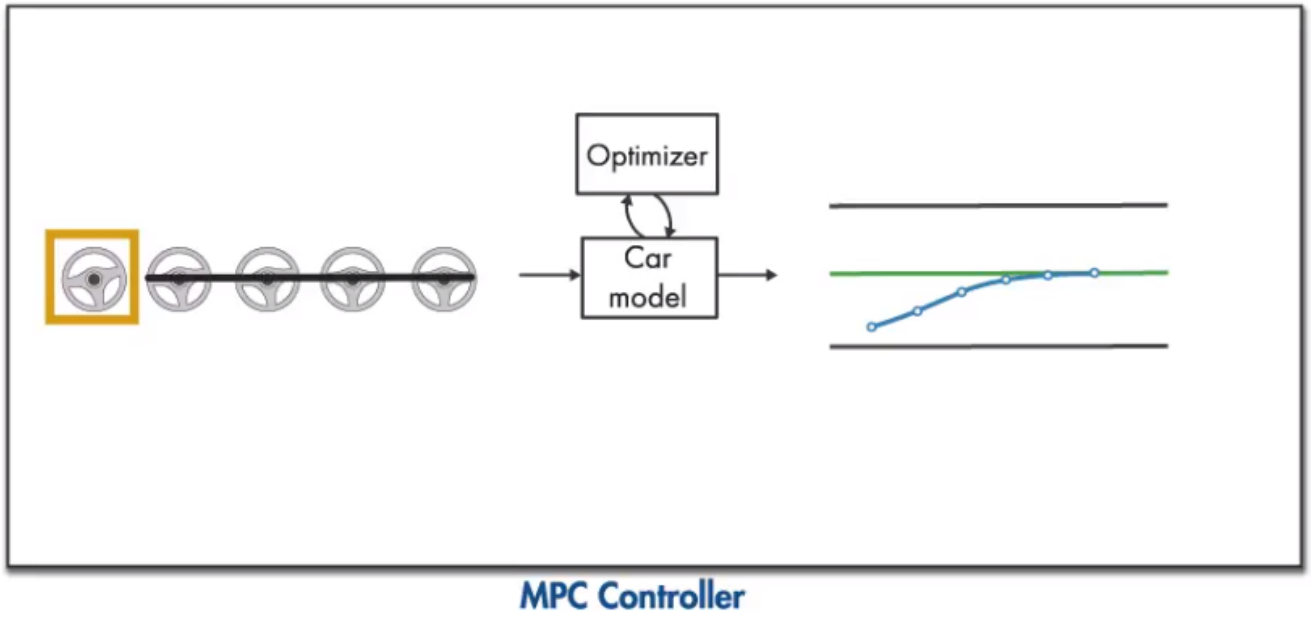
2. What is MPC?


Reference
Prediction horizon, p
Past
Future
k
k+1
k+2
k+3
k+4
k+p
Optimizer
Car
model
MPC Controller

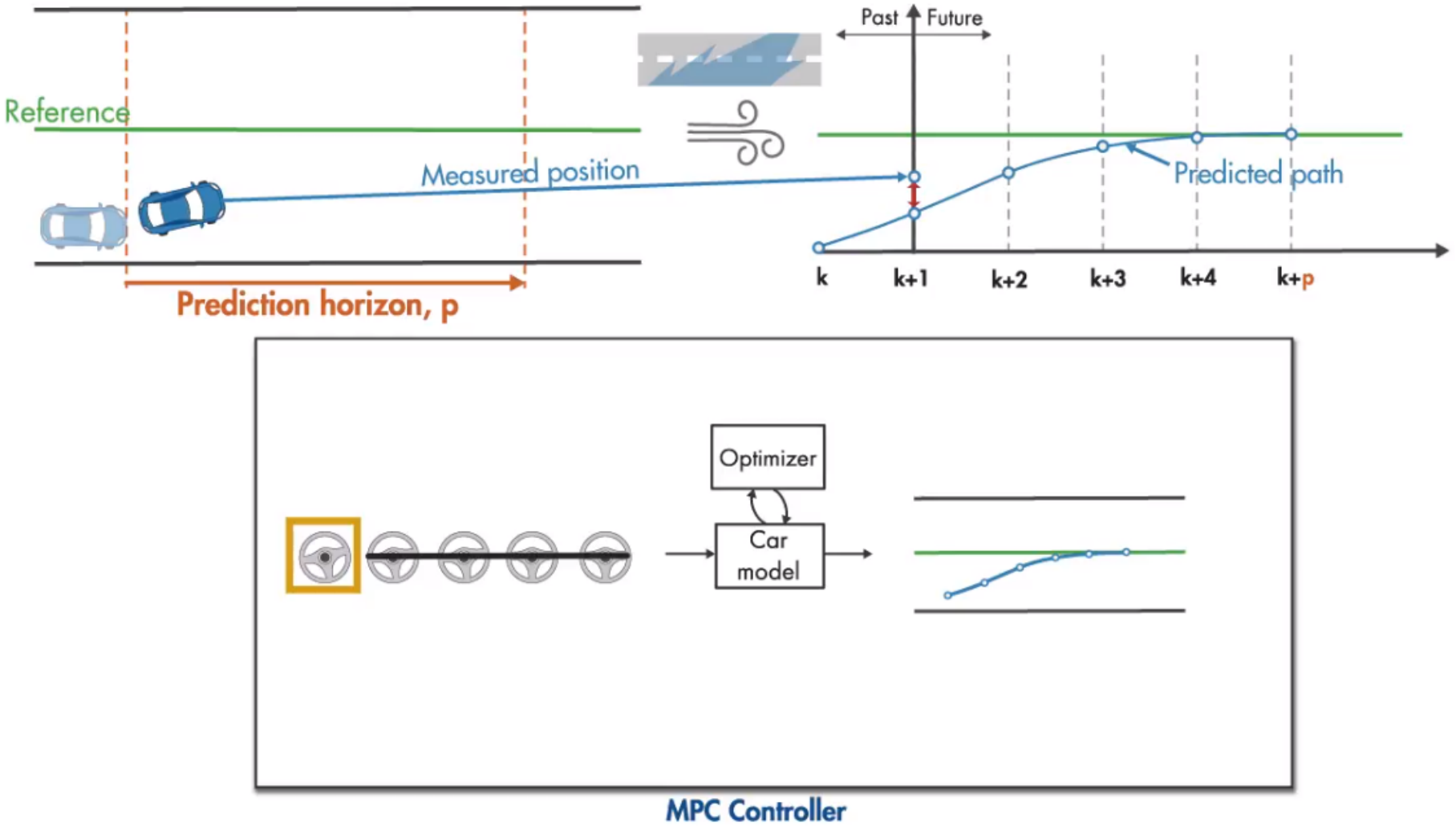
2. What is MPC?


Reference
Prediction horizon, p

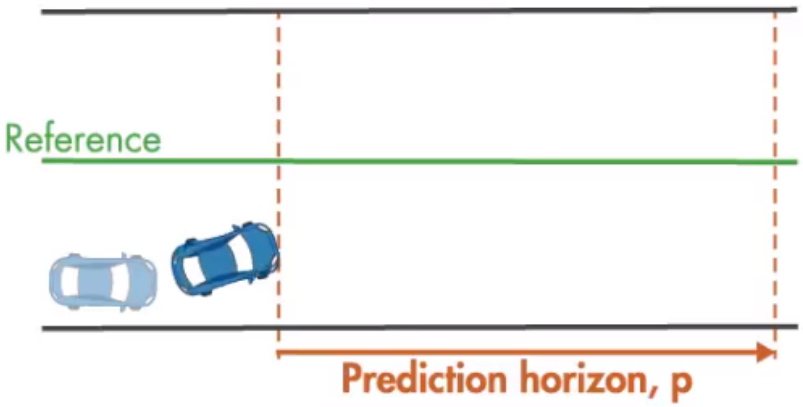

Past
Future
k
k+1
k+2
k+3
k+4
k+p

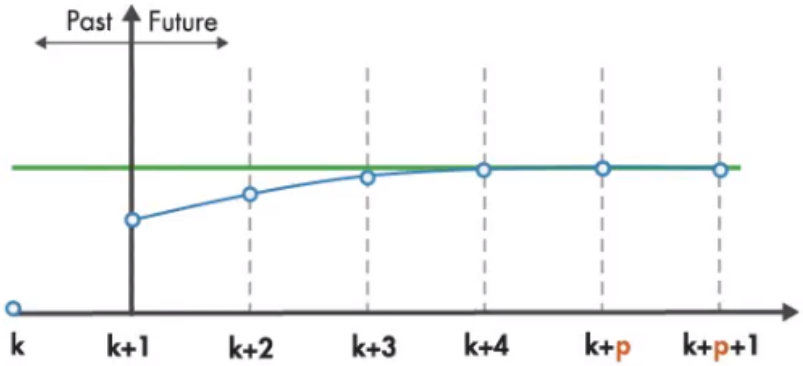

Optimizer
Car model
MPC Controller

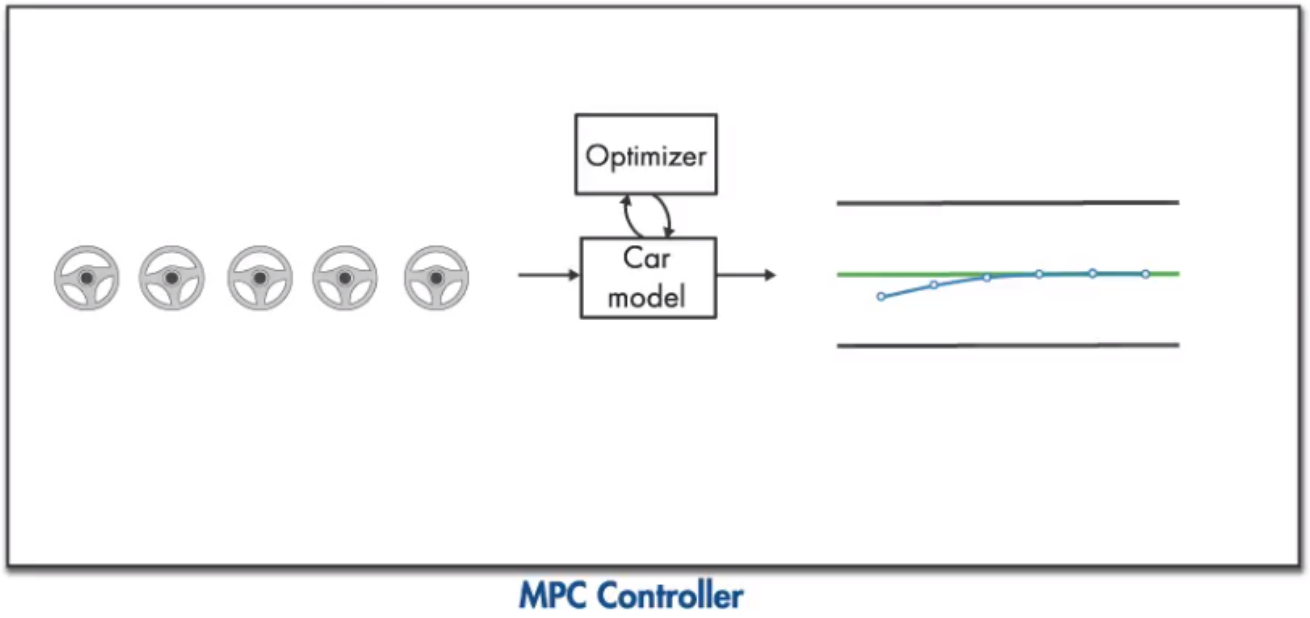
2. What is MPC?


Reference
Measured position
Prediction horizon, p
Past
Future
Predicted path
k
k+1
k+2
k+3
k+4
k+p
Optimizer
Car model
MPC Controller

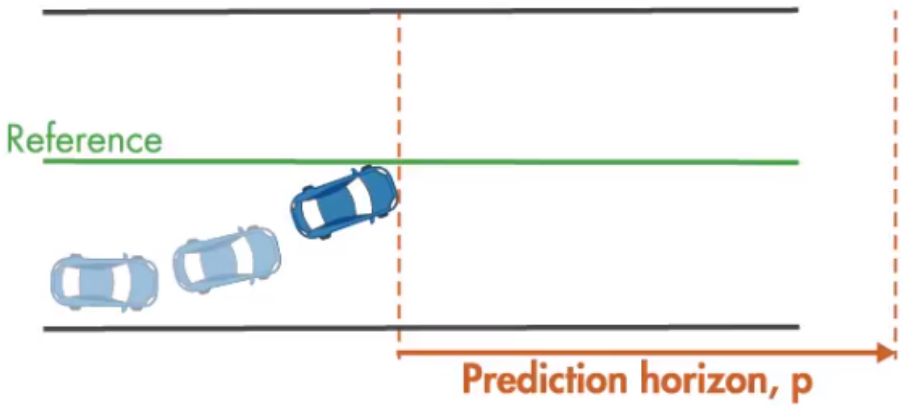
2. What is MPC?

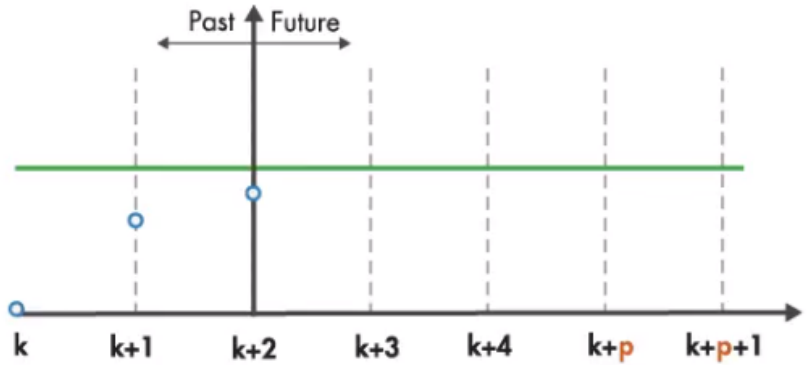

Reference
Prediction horizon, p

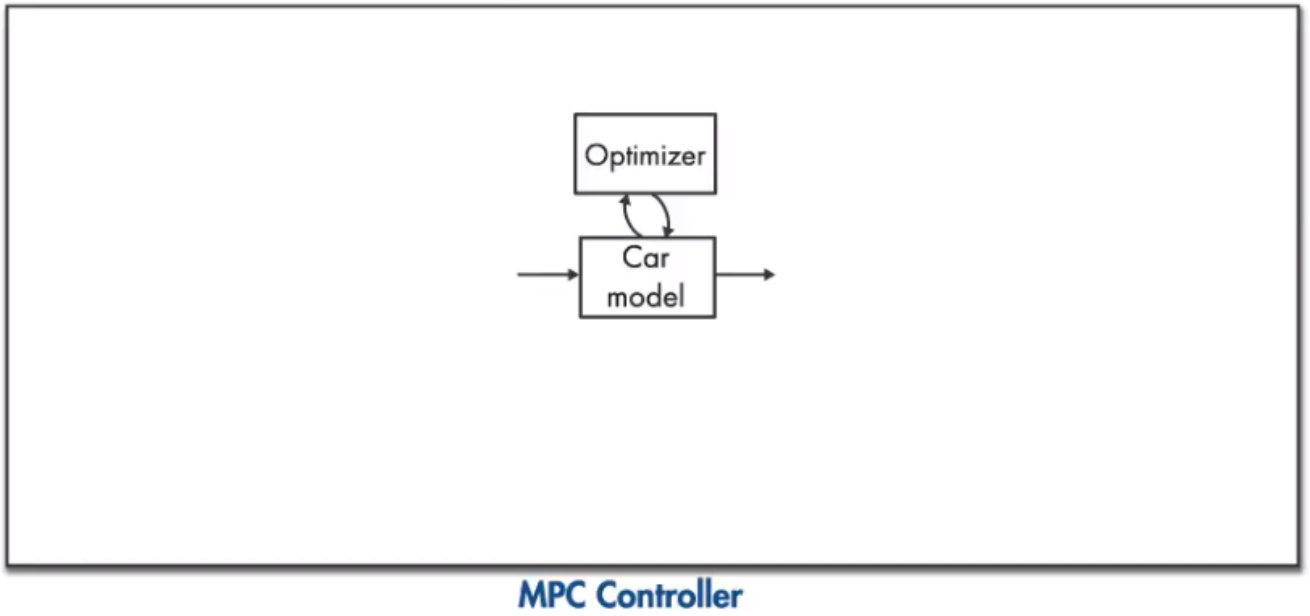

Past
Future
k
k+1
k+2
k+3
k+4
k+p
k+p+1


Optimizer
Car model
MPC Controller

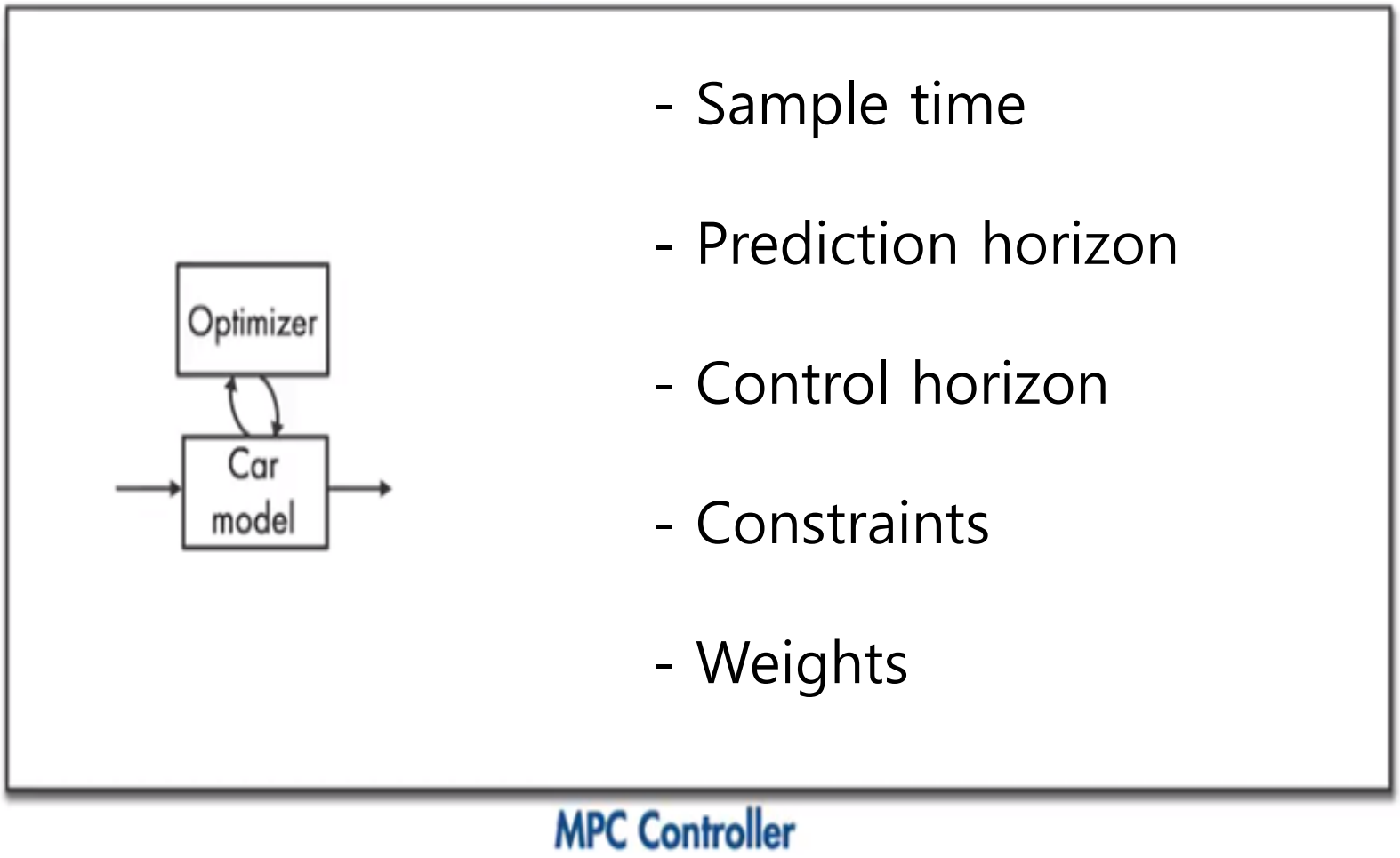
2. What is MPC?


Reference
Prediction horizon, p

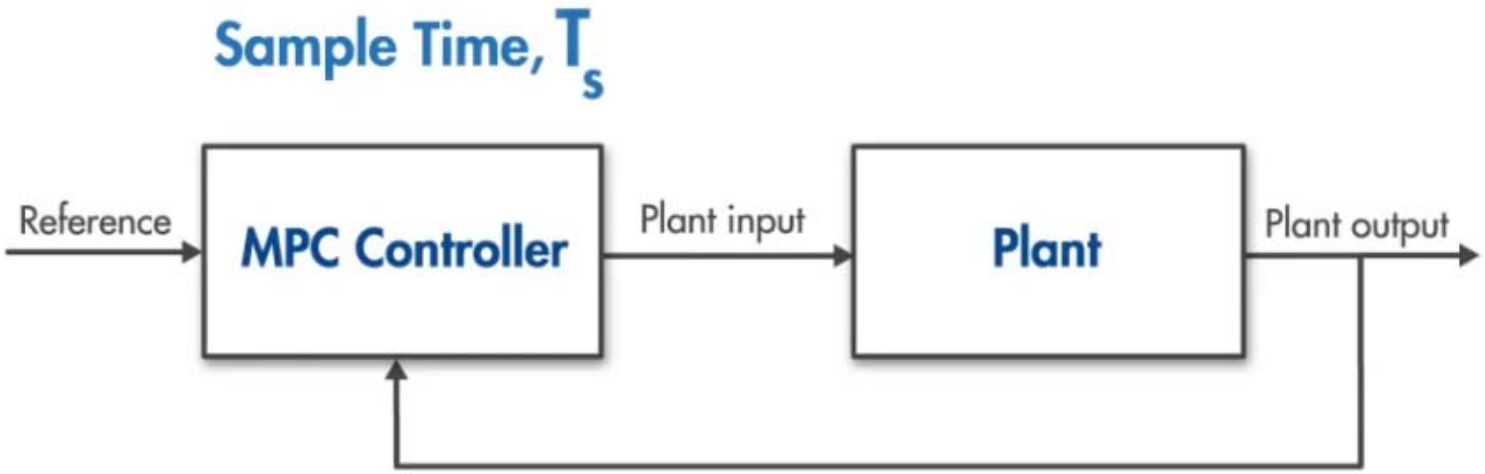

Past
Future
k
k+1
k+2
k+3
k+4
k+p
k+p+1


Optimizer
Car model
MPC Controller

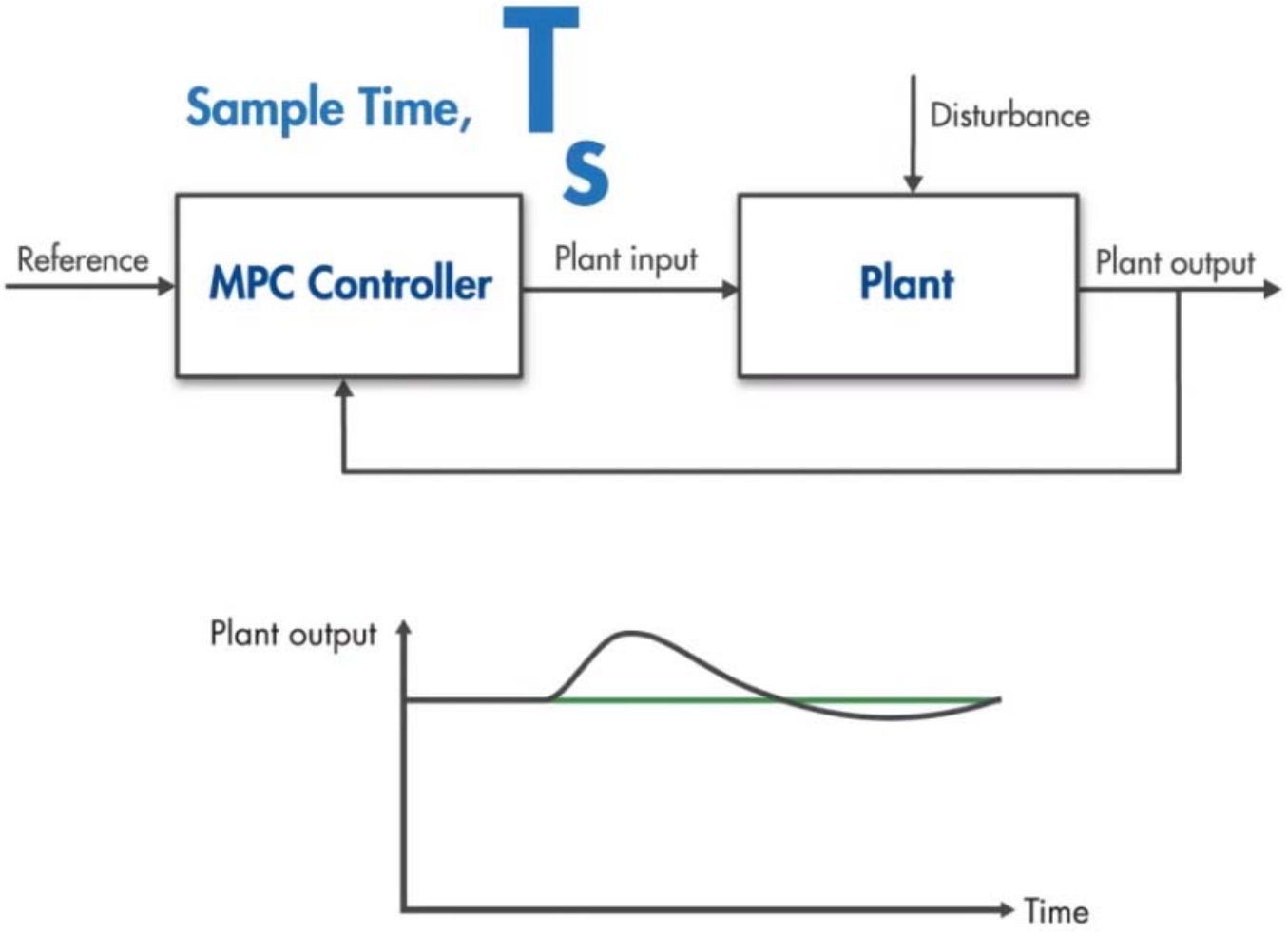
3. MPC Design Parameters



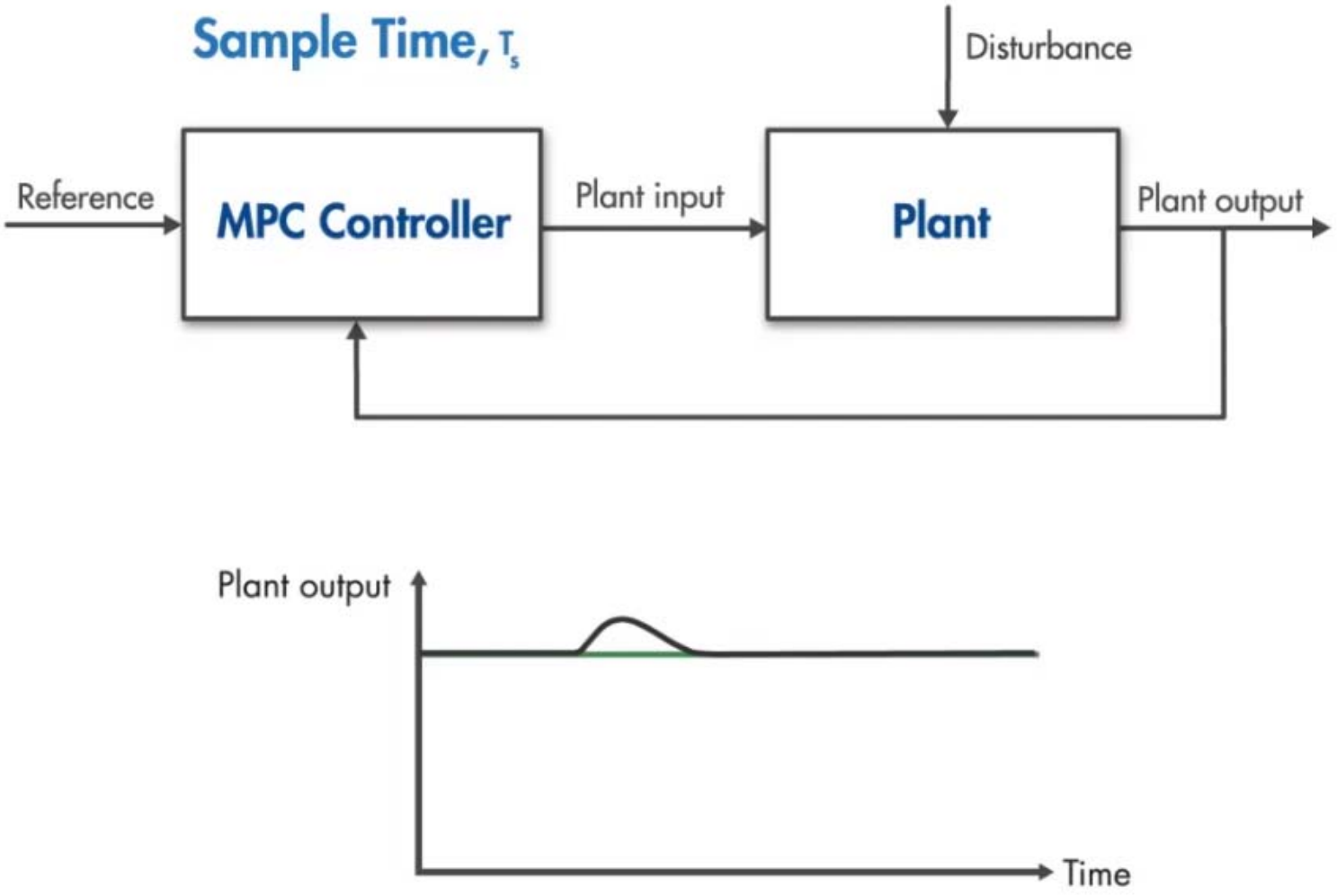
3. MPC Design Parameters [Sample time]



3. MPC Design Parameters [Sample time]

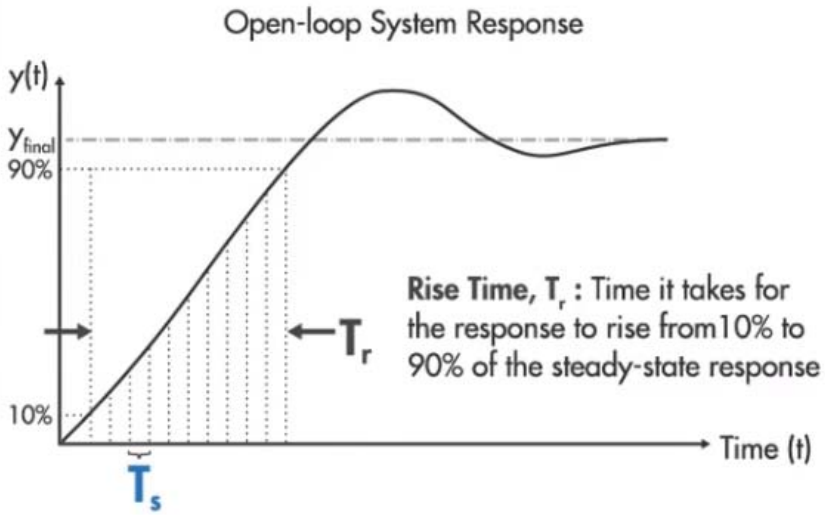


3. MPC Design Parameters [Sample time]



3. MPC Design Parameters [Sample time]

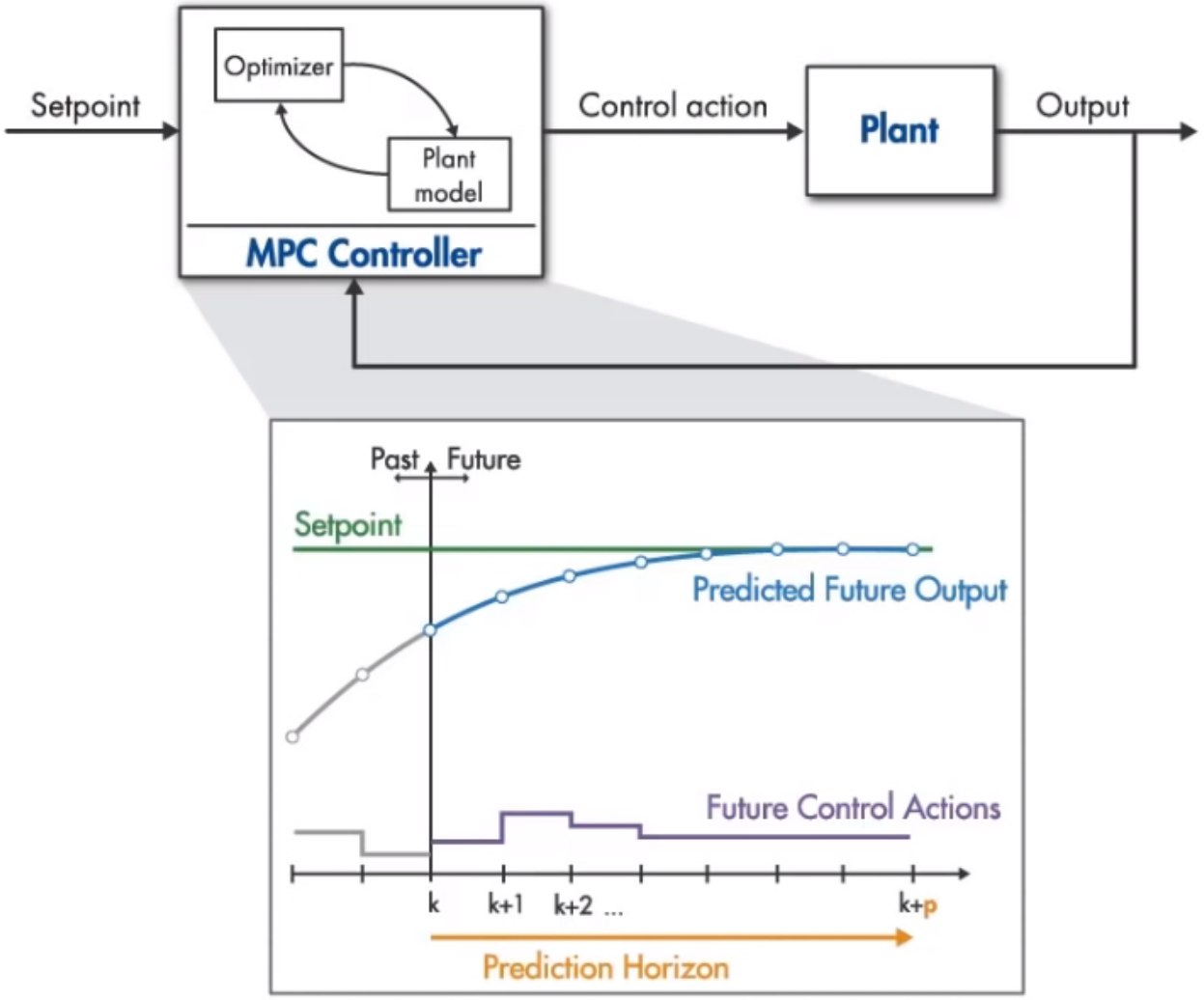
Recommendation:



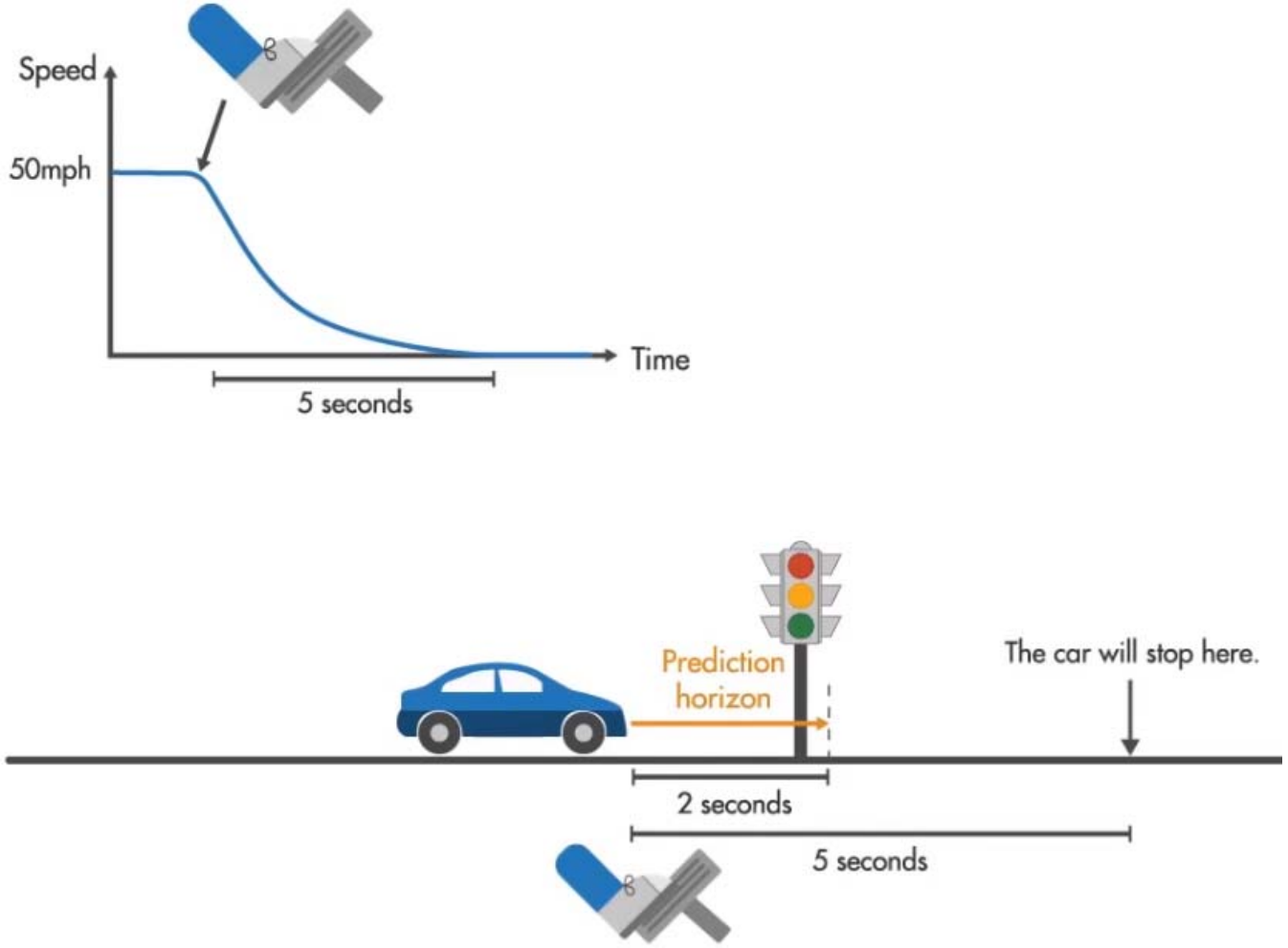
Sample Time, T_s: Controller execution rate

$$\frac{T_r}{20} \leqslant T_s \leqslant \frac{T_r}{10}$$

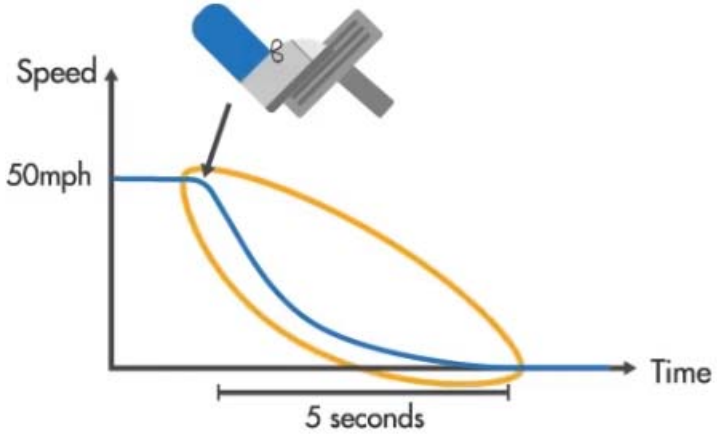
3. MPC Design Parameters [Prediction horizon]

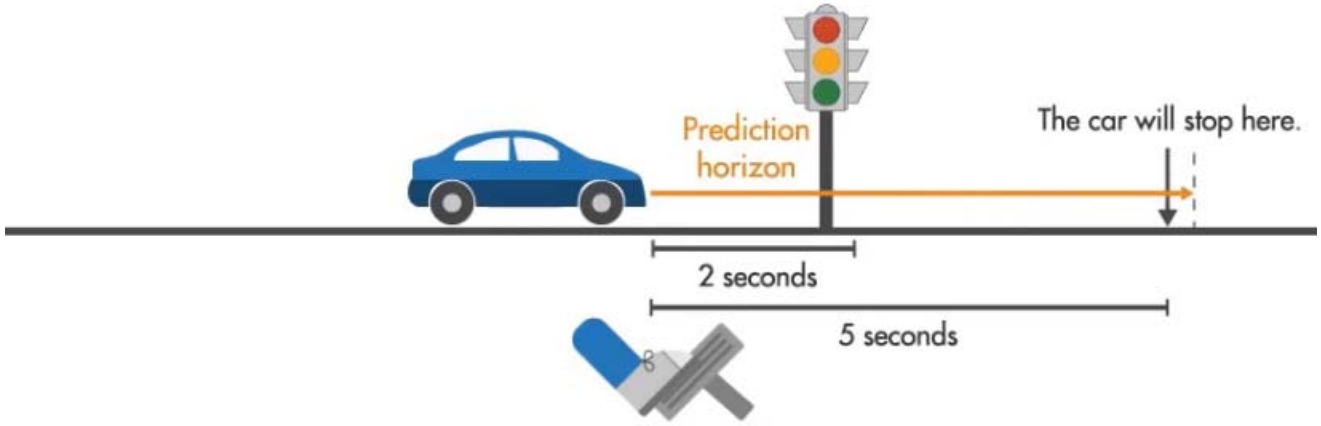


3. MPC Design Parameters [Prediction horizon]

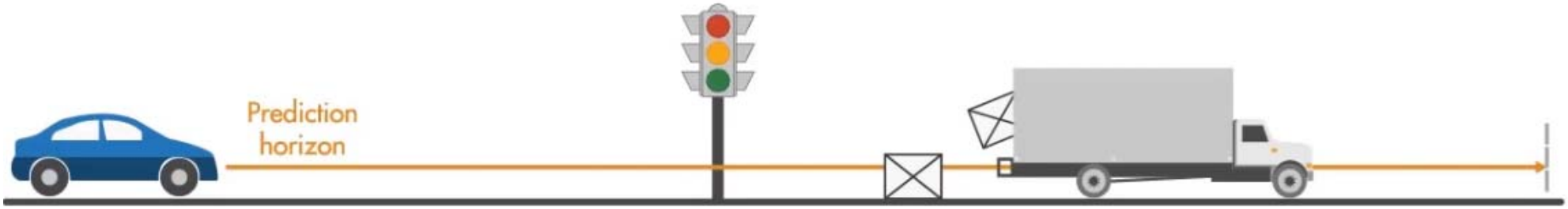


3. MPC Design Parameters [Prediction horizon]



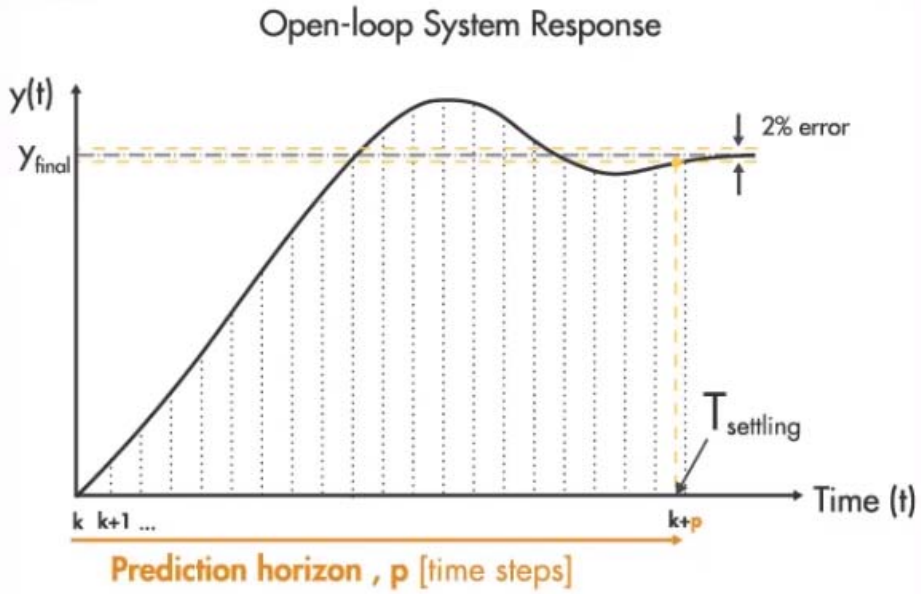


3. MPC Design Parameters [Prediction horizon]



3. MPC Design Parameters [Prediction horizon]

Recommendation:

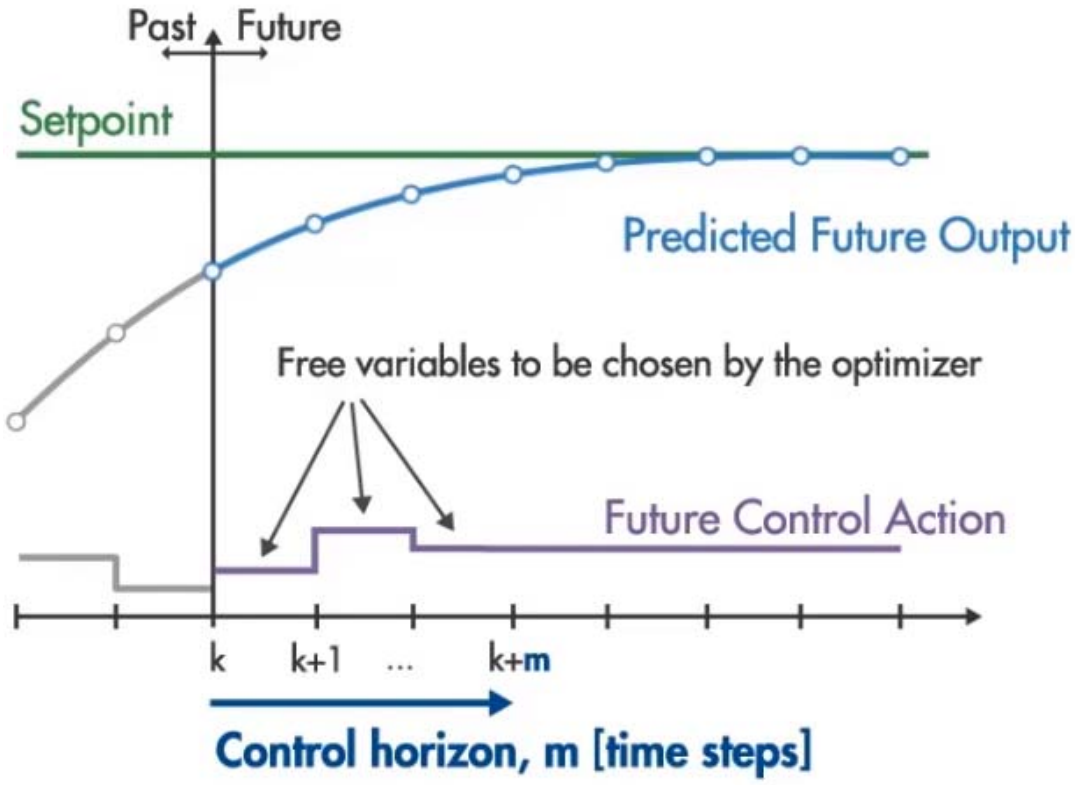


$T_{settling}$:Time it takes for the error $|y(t) - y_{final}|$ to fall to within 2% of y_{final}

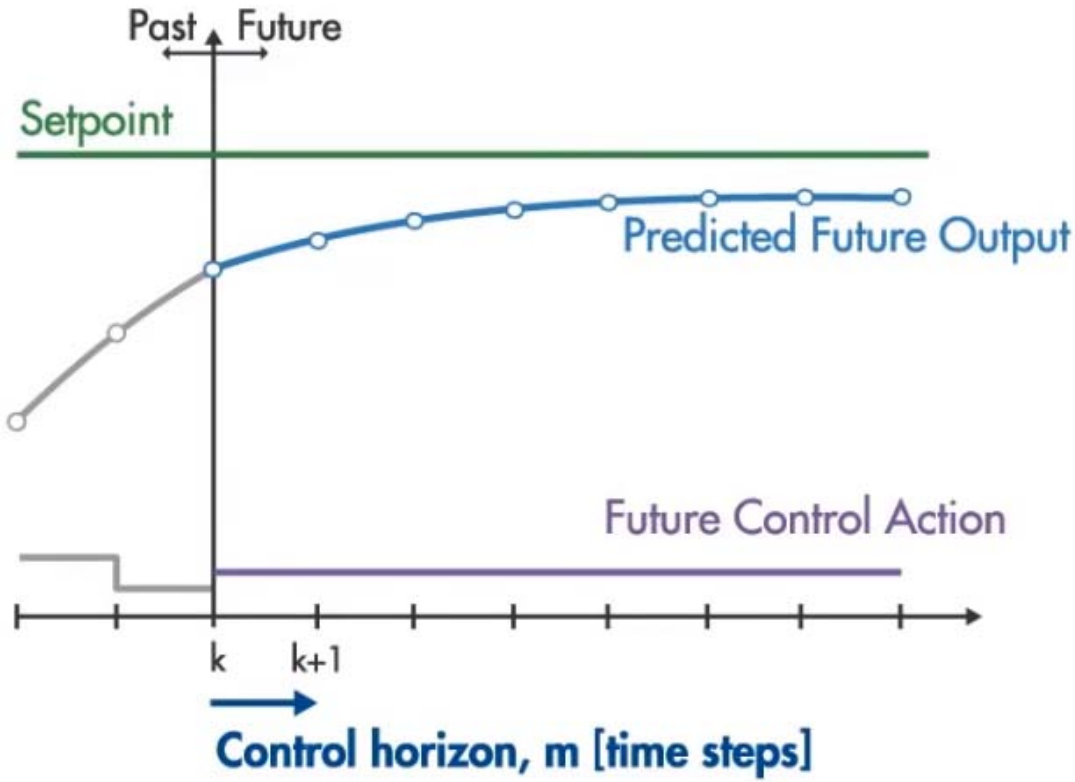
$$\frac{T_r}{20} \leqslant T_s \leqslant \frac{T_r}{10}$$, T_s: Sample time

$$p.T_s \geqslant T_{settling}$$

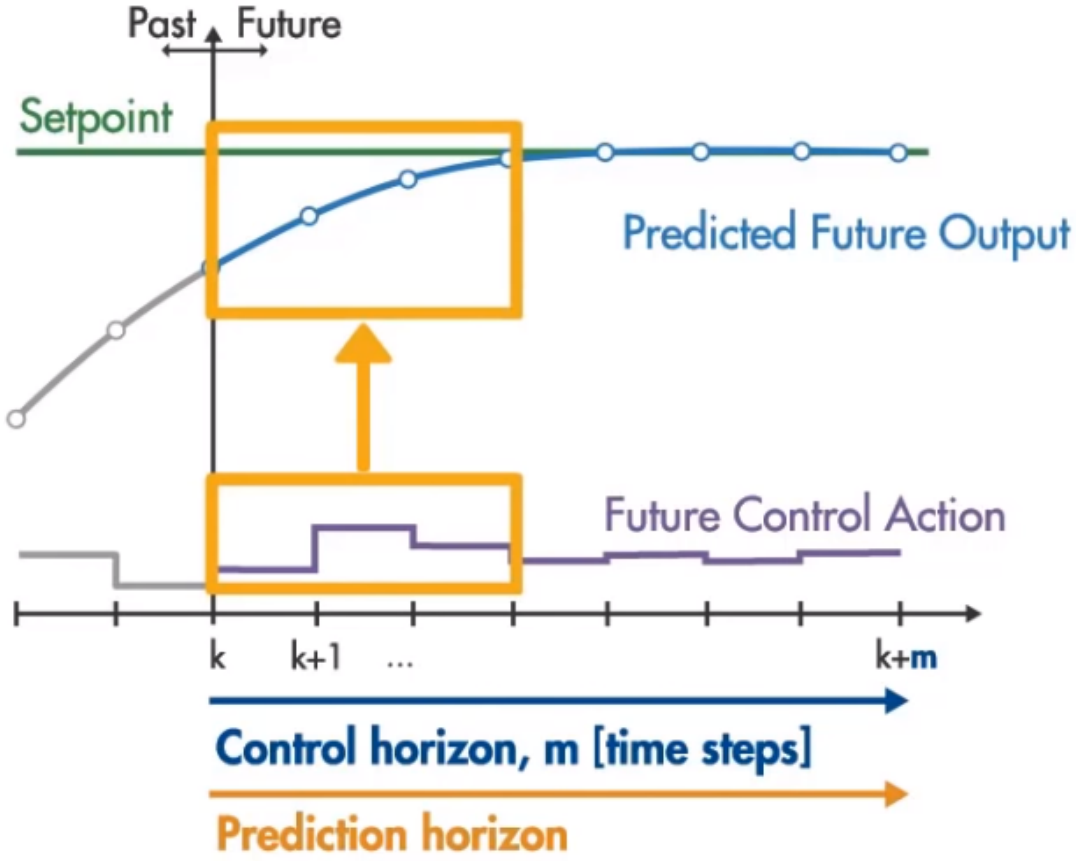
3. MPC Design Parameters [Control horizon]



3. MPC Design Parameters [Control horizon]

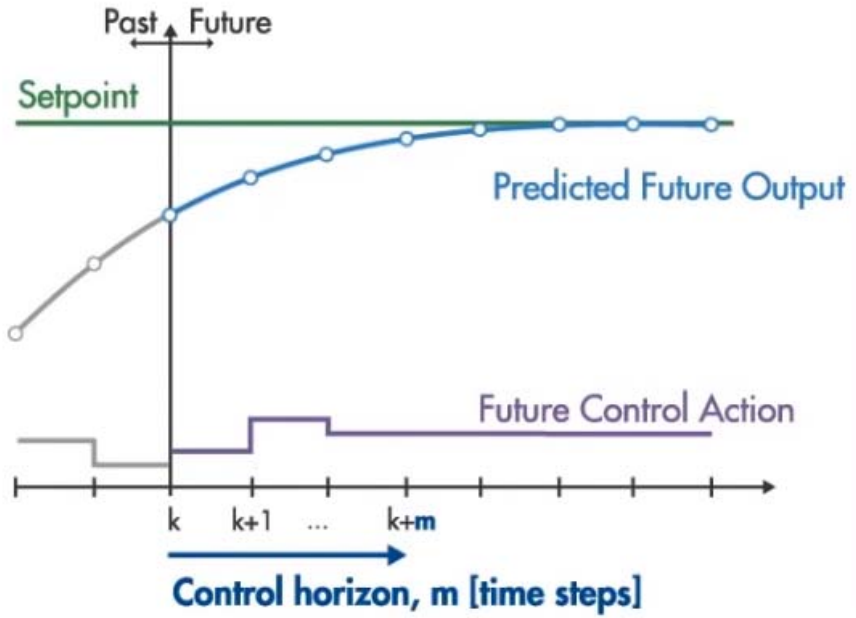


3. MPC Design Parameters [Control horizon]



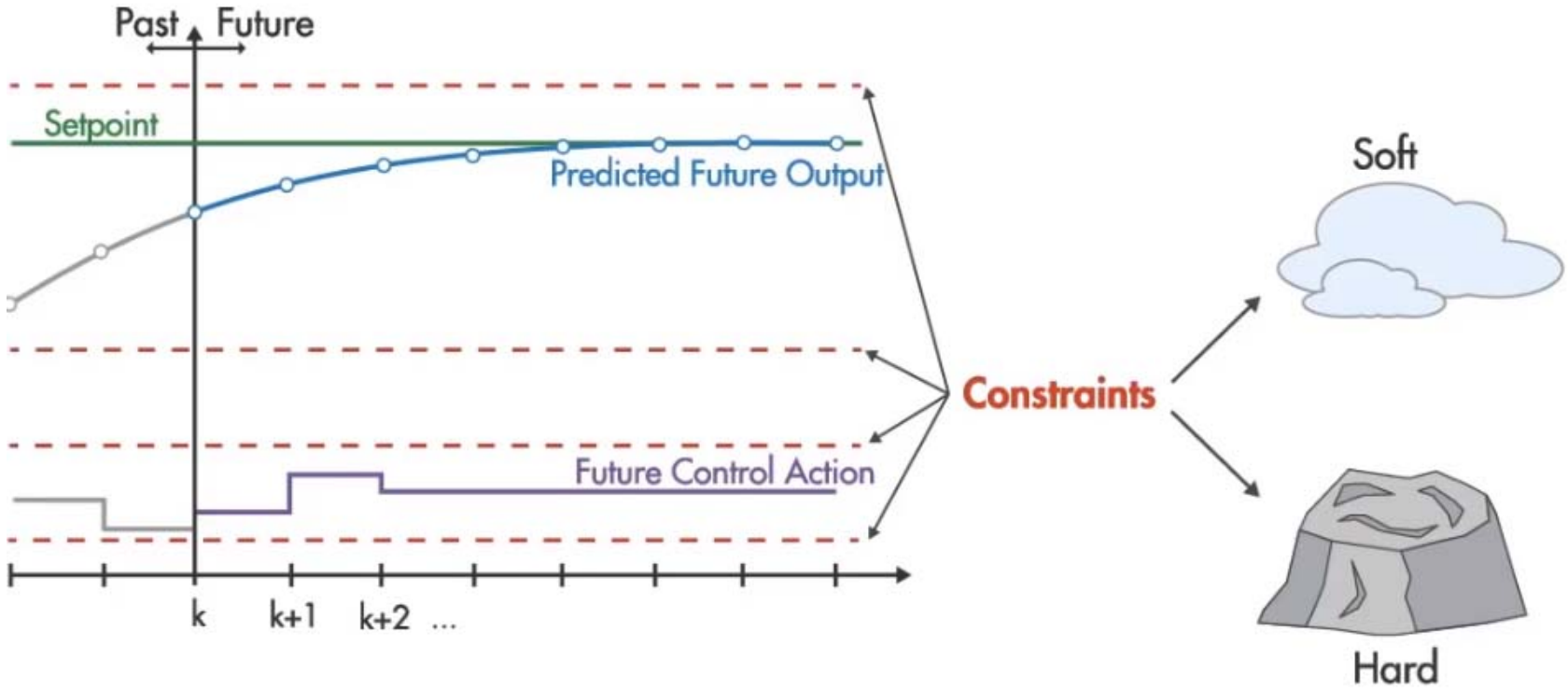
3. MPC Design Parameters [Control horizon]

Recommendation:

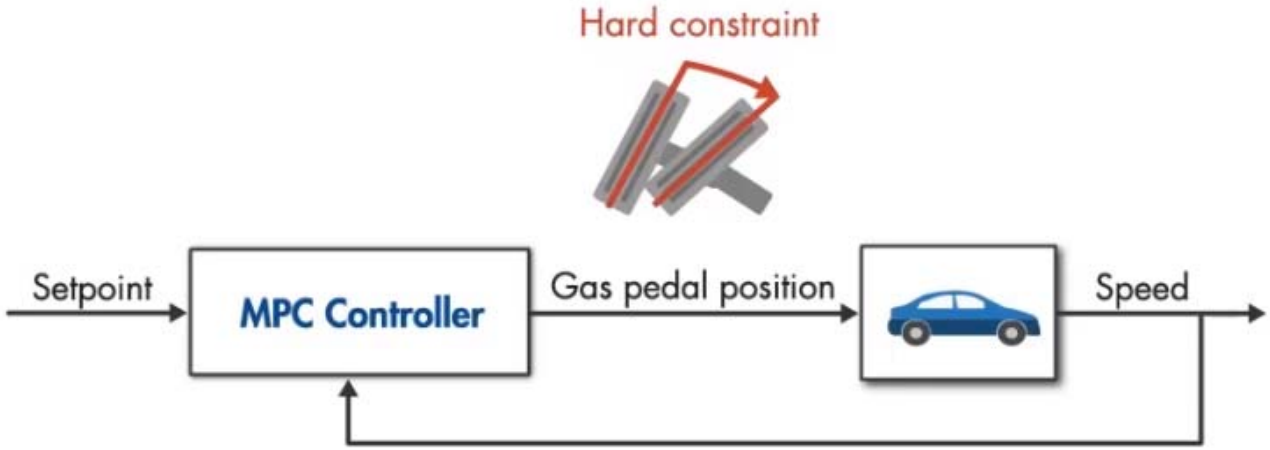


$0.1p \leq m \leq 0.2p$, p: Prediction horizon

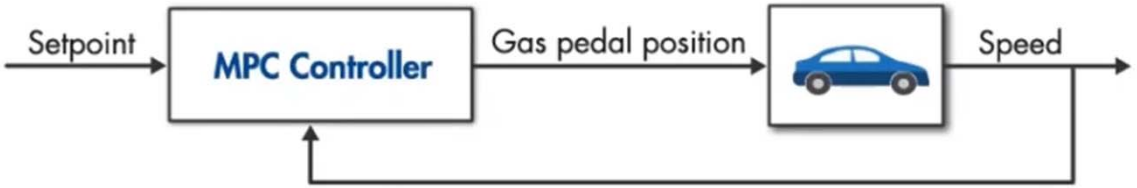
3. MPC Design Parameters [Constraints]



3. MPC Design Parameters [Constraints]

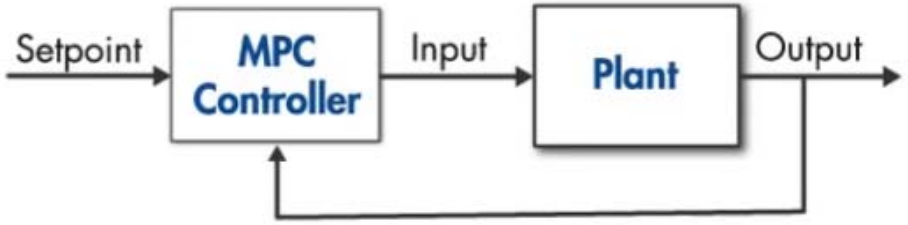


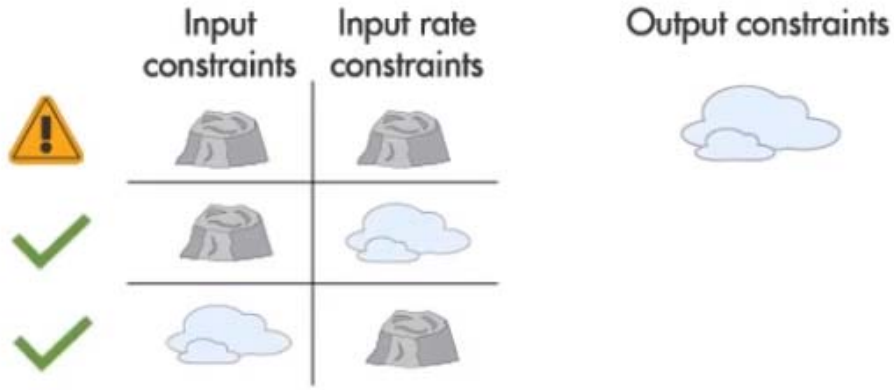
3. MPC Design Parameters [Constraints]



3. MPC Design Parameters [Constraints]

Recommendation:





3. MPC Design Parameters [Weights]

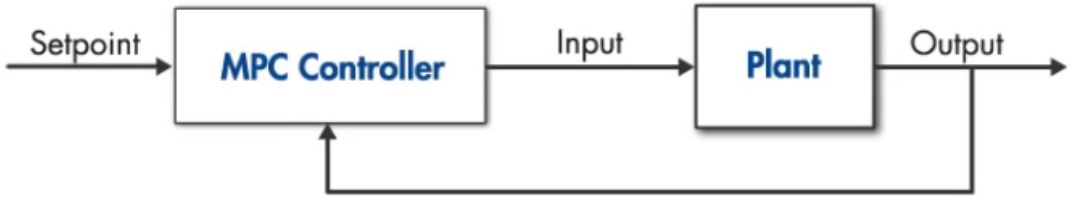
$W_{sleep} = 50$

$W_{eat} = 10$

$$\frac{W_{sleep}}{W_{eat}} = 5 > 1$$

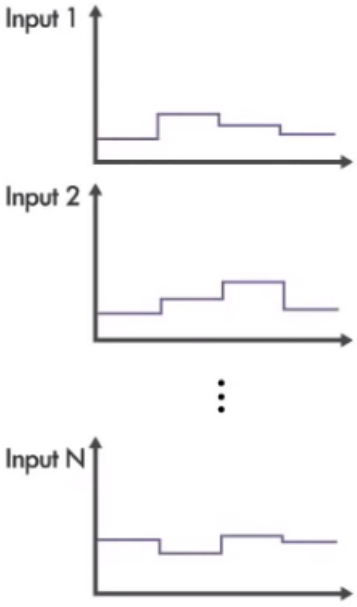
W: Weight

3. MPC Design Parameters [Weights]

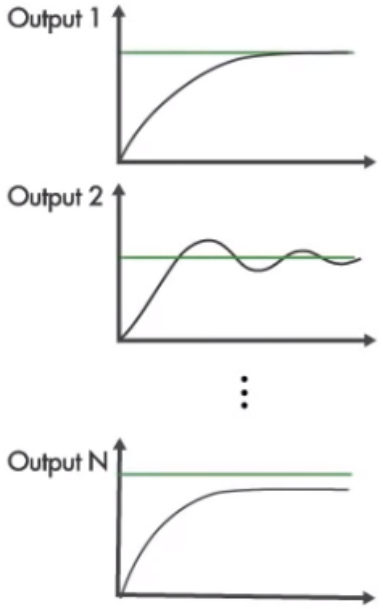


Goals

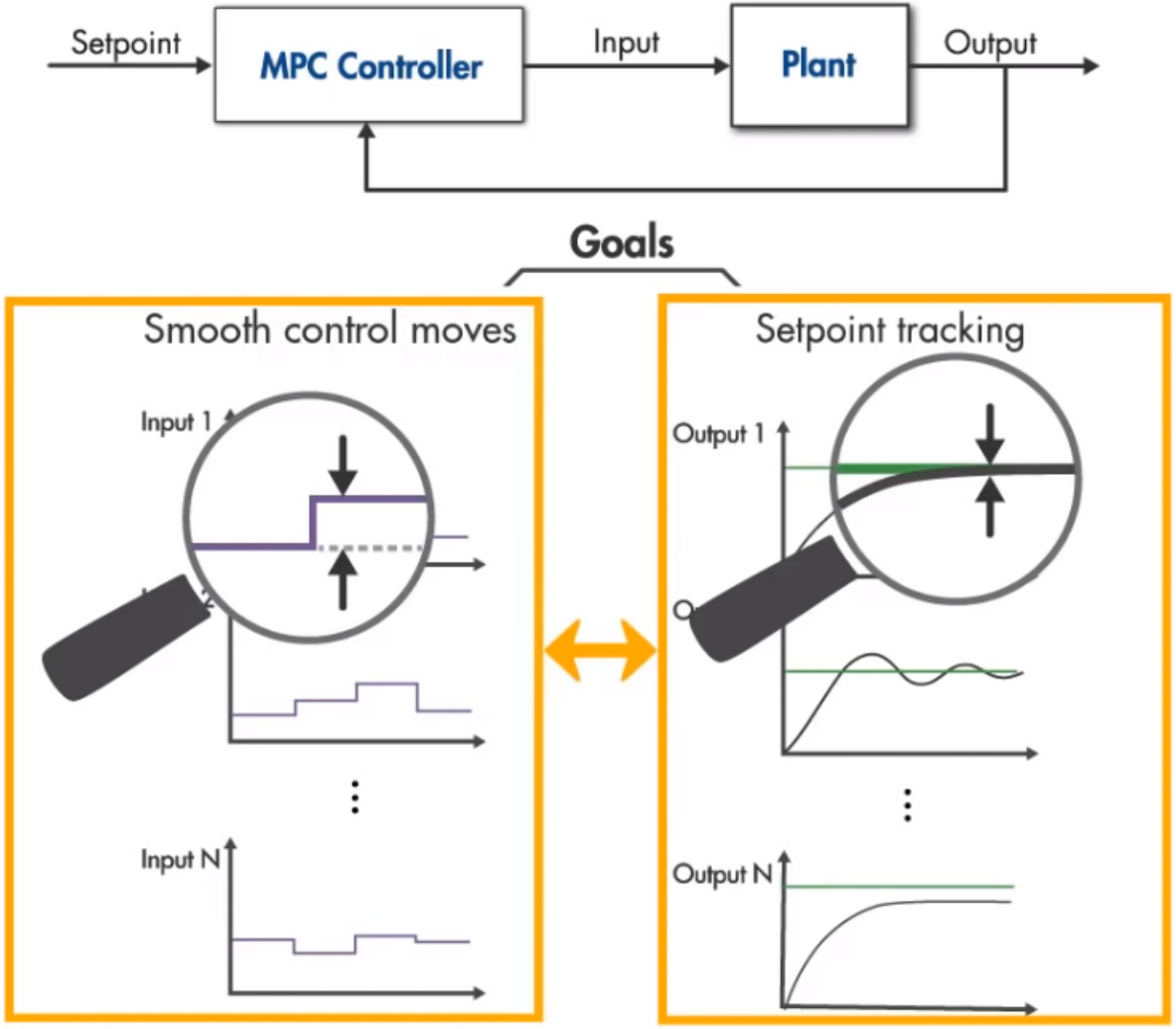
Smooth control moves



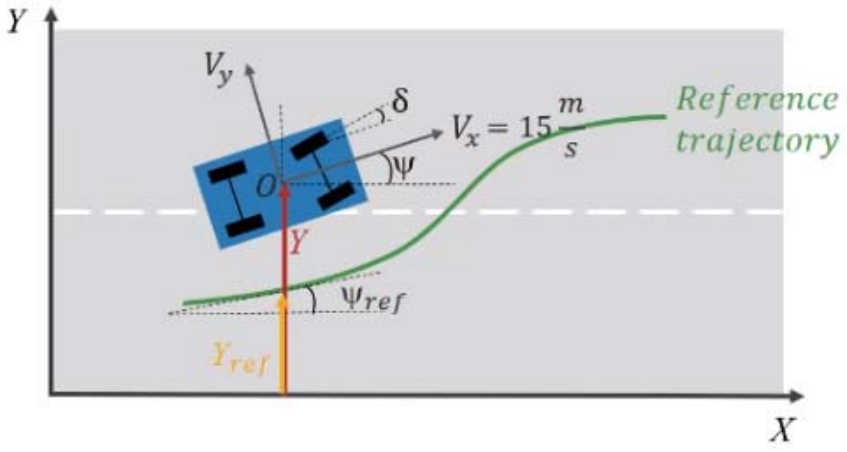
Setpoint tracking



3. MPC Design Parameters [Weights]



4. How to Design an MPC Controller with Simulink



V_y: *Lateral velocity*

V_x: *Longitudinal velocity*

(X, Y): *Vehicle's global position*

ψ: *Yaw angle*

δ: *Front steering angle*

Y_{ref}: *Reference lateral position*

ψ_{ref}: *Reference yaw angle*

- Lateral dynamics:

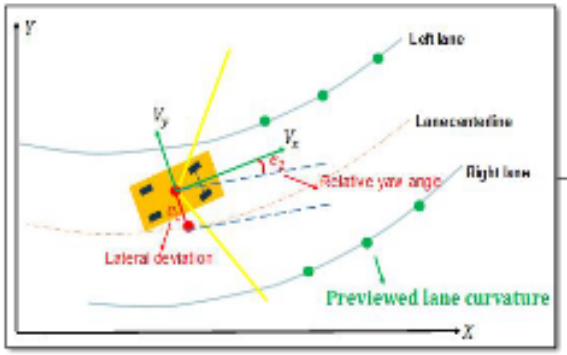
$$\frac{d}{dt}\begin{bmatrix}\dot{y}\\ \psi\\ \dot{\psi}\end{bmatrix} = \begin{bmatrix}-\frac{2C_{af}+2C_{ar}}{mV_x} & 0 & -V_x-\frac{2C_{af}\ell_f-2C_{ar}\ell_r}{mV_x}\\ 0 & 0 & 1\\ -\frac{2\ell_f C_{af}-2\ell_r C_{ar}}{I_z V_x} & 0 & -\frac{2\ell_f^2 C_{af}+2\ell_r^2 C_{ar}}{I_z V_x}\end{bmatrix}\begin{bmatrix}\dot{y}\\ \psi\\ \dot{\psi}\end{bmatrix} + \begin{bmatrix}\frac{2C_{af}}{m}\\ 0\\ \frac{2\ell_f C_{af}}{I_z}\end{bmatrix}\delta$$

- Global Y position:

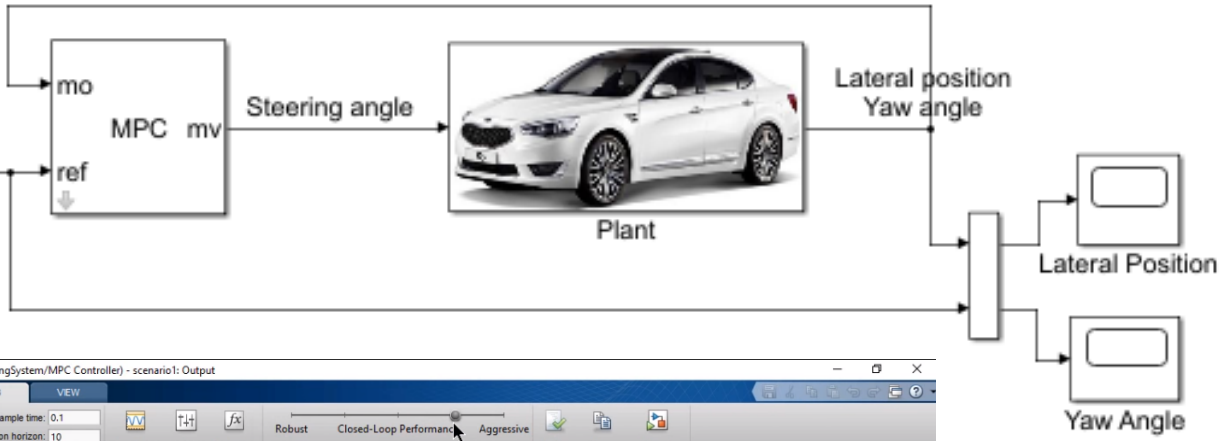
$$\dot{Y} = V_x\psi + V_y$$

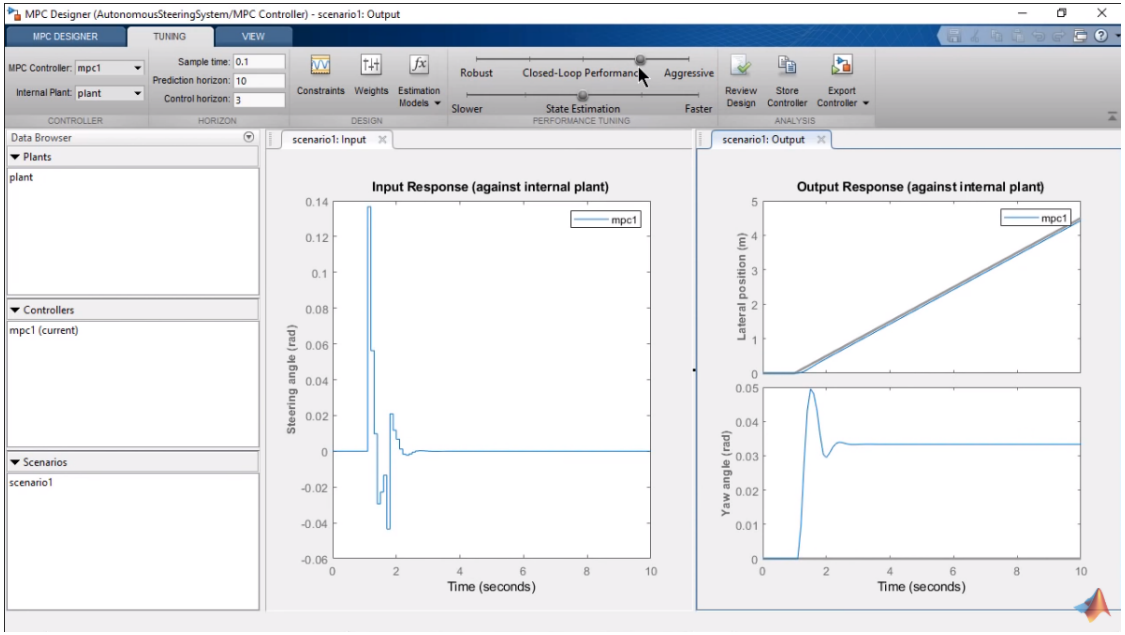
V_x	Longitudinal velocity at center of gravity of vehicle
m	Total mass of vehicle
I_z	Yaw moment of inertia of vehicle
l_f	Longitudinal distance from center of gravity to front tires
l_r	Longitudinal distance from center of gravity to front tires
C_α	Cornering stiffness of tire
δ	Front steering angle
y	Lateral position
ψ	Yaw angle

4. How to Design an MPC Controller with Simulink



Reference





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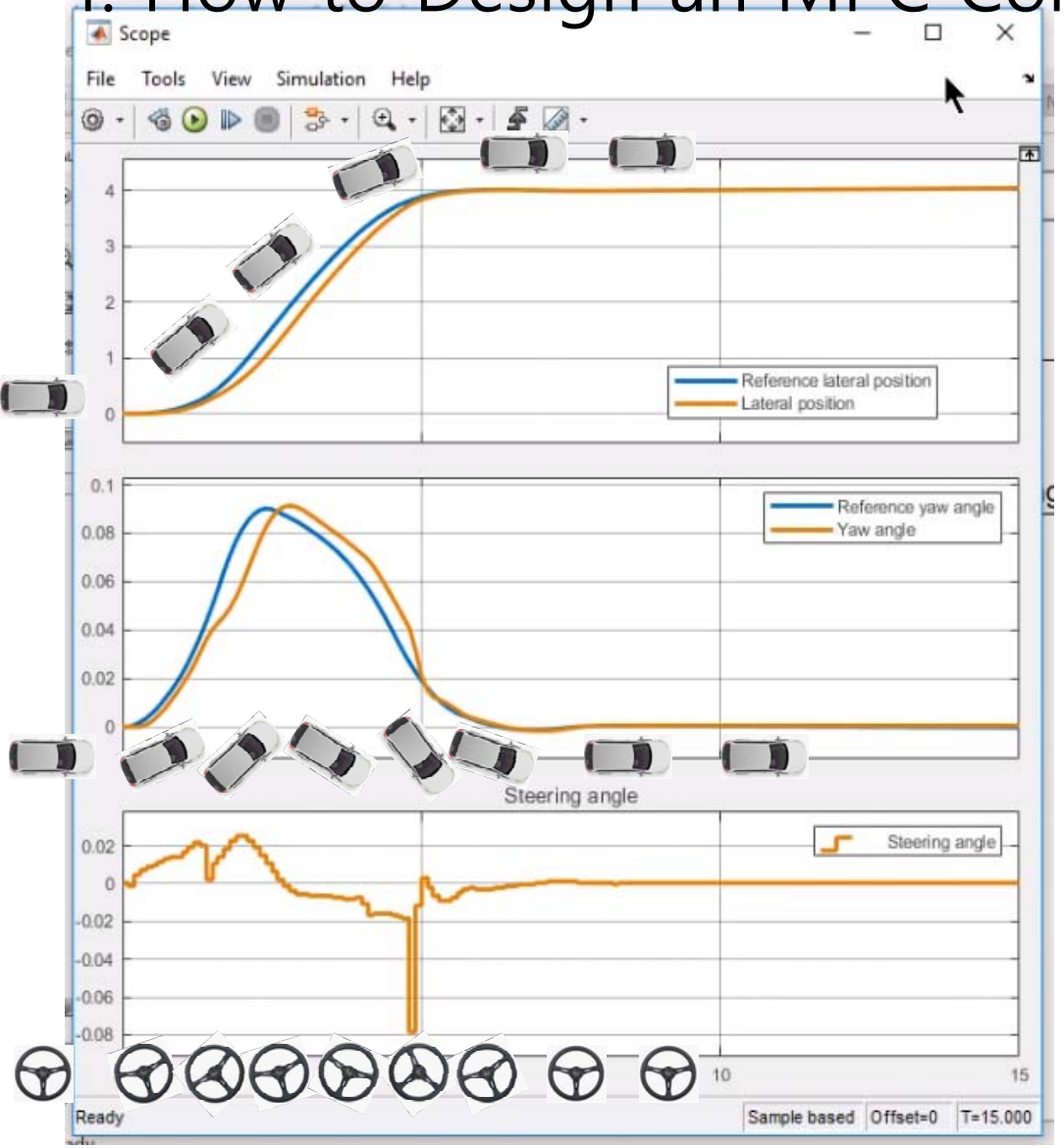

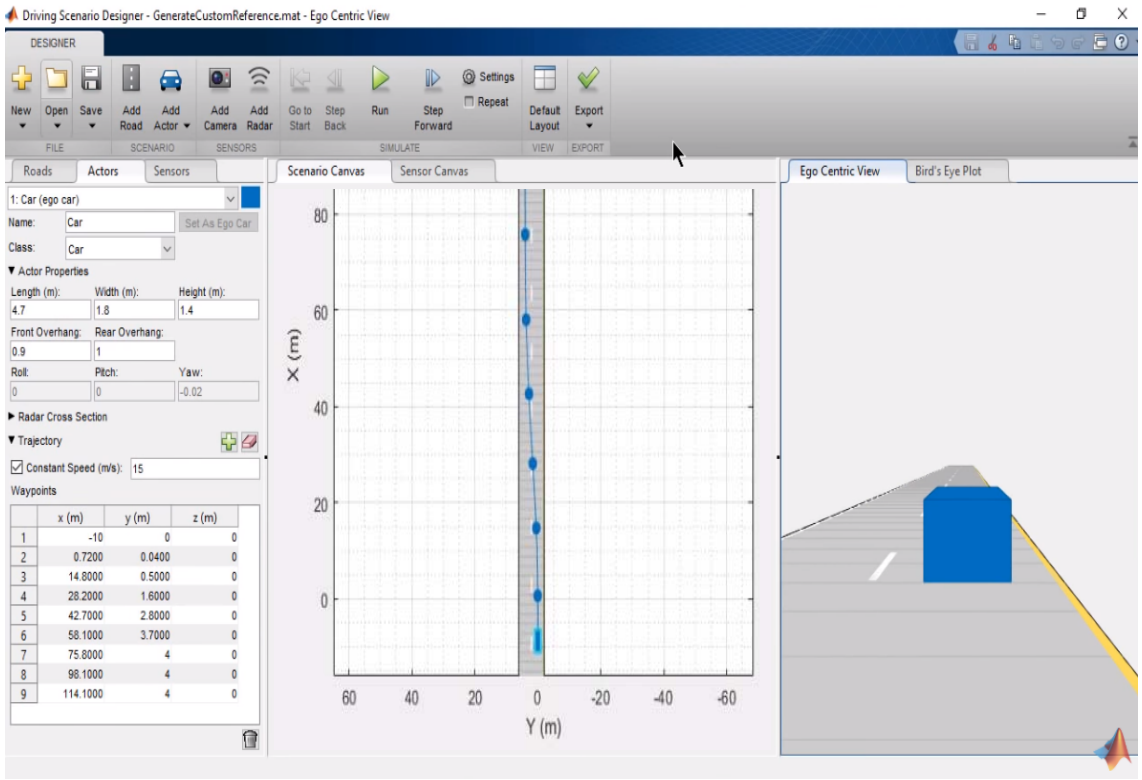

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