Homework Assignment 1

Exercise 1. Let $X_1, X_2, \ldots$ be independent Bernoulli r.v.'s r.v with

$P(X_n = 1) = p$ and $P(X_n = 0) = q = 1 - p$ for all n. The collection of r.v.'s $\{X_n, n \geq 1\}$ is a stochastic process, and it is called a Bernoulli process.

a) Describe the Bernoulli process.

b) Construct a typical sample sequence of the Bernoulli process.

c) Determine the probability of occurrence of the sample sequence obtained in part b).

Exercise 2. Let $Z_1, Z_2, \ldots$ be independent identically distributed r.v.'s with $P(Z_n = 1) = p$ and $P(Z_n = -1) = q = 1 - p$ for all n. Let

$$X_n = \sum_{i=1}^{n} Z_i,\ n = 1, 2, \ldots$$

and $X_0 = 0$. The collection of r.v.'s $\{X_n, n > 0)$ is a stochastic process, and it is called the simple random walk X_n.

a) Describe the simple random walk X_n.

b) Construct a typical sample sequence (or realization) of X_n.

c) Derive the first-order probability distribution of the simple random walk X_n

d) Find the probability that $X_n = -2$ after four steps.

e) Find the mean and variance of the simple random walk X_n.

f) Find the autocorrelation function $R_X(n, m)$ of the simple random walk X_n

g) Now let the random process $X(t)$ be defined by

$$X(t) = X_n,\ n \leq t < n + 1$$

Describe X(t). Construct a typical sample function of $X(t)$.

Exercise 3. Consider a random process $X(t)$ defined by

$$X(t) = Y cos\omega t,\ t \geq 0$$

where ω is a constant and Y is a uniform r.v. over $(0, 1)$.

a) Describe $X(t)$.

b) Sketch a few typical sample functions of $X(t)$.

c) Find E[X(t)].

d) Find the autocorrelation function $R_X(t,s)$ of $X(t)$.

e) Find the autocovariance function $K_X(t,s)$ of $X(t)$.

Exercise 4. Consider a discrete-parameter random process $X(n) = \{X_n, n \geq 1\}$ where the X_i's are iid r.v.'s with common cdf $F_X(x)$, mean p, and variance σ^2.

a) Find the joint cdf of $X(n)$.

b) Find the mean of $X(n)$.

c) Find the autocorrelation function $R_X(m,n)$ of $X(n)$.

d) Find the autocovariance function $K_X(m,n)$ of $X(n)$.