**Problem Set 6**

For each of the following problems, imagine that you are on a strange and unusual island (Logic Island), the natives of which are either Knights or Knaves. Knights may only tell the truth, whereas Knaves may only tell falsehoods. (Consequently, no one can be both a knight and a knave.) Each native wears medieval armor, and upon the breastplate of their armor, they have a single letter emblazoned (e.g., A, B, C, ….). Thus, the natives can be identified by the letter emblazoned on their breastplate.

You are a logic consultant for an investment firm that is aiming to develop business relations on the island. For each part, you should respond to your manager’s question in the form of a business email – meaning that you need a salutation and a valediction, and you need to maintain respectful language. That means you must solve the problem, but you must also CLEARLY explain your reasoning to your manager in normal prose. Each part should take no more than a paragraph. In your emails you may reference your previous emails (from earlier parts) to convince your boss that your reasoning is correct.

**Part 1 (one point total).**

Hello <your name>,

I need your help. I am at Logic Island. As you know we are trying to cultivate good business relations with the folks here. I recently encountered two of the inhabitants – A and B. A said to me, “At least one of us is a knave.” I can’t figure this out, but hey, that’s we hired you, right? Here are my questions:

Is A a knight or a knave?

How about B?

I am looking forward to your analysis. Thanks in advance.

Sincerely,

Terry

Hello Tery,

I think that A is a knight and B is a knave. I hope that my analysis may help you recognize them. If A is a knave, then what A said is a falsehood. So there are no knave among A and B. This is a contradiction since A is a knave. Therefore, A is knight. Thus what A said is right, there are at least one knave among them, and this is B.

I hope it works for you. Email me if you have any further question.

Sincerely,

Duy Pham

**Part 2 (one point total).**

Hello <you name>,

Thanks for the last email. Knowing which of the locals are knights and which are knaves is tremendously helpful. I’ve met three more of the locals – C, D, E. Here’s a transcription of their initial dialogue:

C: All of us are knaves.

D: Exactly one of us is a knight.

Is C a knight or a knave?

Can you tell me anything about D?

How about E?

Keep up the good work. I look forward to your response.

Sincerely,

Terry

Hello Terry,

In my opinion, C was supposing that C is a knave, so obviously C cannot be a knight who always tell the truth. C is knave and it also means it is not right for all of them are knaves. So either D or E is a knight and the other is a knave, or both D and E are knights. However, if both D and E are knights, then D did not tell the truth, since there are more than one knight. So either D or E is a knight and the other is a knave, which means there is exactly one knight among them. So D is a knight, he told the truth. And E is a knave.

I hope it works for you. Email me if you have any further question. Good luck!

Sincerely,

Duy Pham

**Part 3 (one point total).**

Hi <your name>,

I cannot tell you how much your analysis has helped me. I’m not here to judge their lifestyles, but knowing whether they are knights or knaves has really helped facilitate fruitful conversations. For that matter even knowing that I don’t know enough to determine whether they are knights or knaves is helpful. I’ve had some great conversations with A, B, C, D, and E thanks to your hard work. We are very close to making a deal. However, I met three new inhabitants – F, G, and H. They seem like nice people, but once again I don’t know whether they are knights or knaves or even whether I have enough information to determine that. You are so much better at this than I am. So, can you help me out? This is what F and G said about F, G, and H:

F: All of us are knaves.

G: Exactly one of us is a knave.

What can you tell me about F?

Can you tell me anything about G?

How about H?

Thanks again. I feel like we are really close to closing a deal.

Sincerely,

Terry

Hi Terry,

I am glad as my analysis could help you. I hope this time would be useful as well. So the same as the second conversation among C, D and E, F is definitely a knave, and either there is exactly one knight between G and H, or both of them are knights. However, this time is more different than the two last cases. I think there are two possible cases. First, both G and H are knights, then G told the truth, everything is proper. The second case is that G is a knave and H is a knight, so all of what F and G said are falsehoods. Of course, the case G is a knight and H is knave is not true, because there are more than knave, which means G is not honest.

So there are two possible case, I think if H said something, it is easier for us to completely determine them. Once again, good luck to you! Hope everything would go well!

Sincerely,

Duy Pham

**Part 4 (one point total).**

Hey <your name>,

I’m in a bit of a pickle. I just got off the phone with our CEO. It turns out we can’t make a deal unless we know whether they can supply enough gold with their mining operation. I can’t get a straight answer from A-H. The CEO is tired of throwing money at this. We need to know whether they are wasting our time. From talking to the others, I was able to determine that their mining manager, some person named J, was the only one who could answer my question. I straight up asked J, “Can your mines supply enough gold? The deal depends upon a straight answer.” J responded “We can supply enough gold if and only if I am a knight.” This is more of that “Logic-ese” that you’re so good at. Help me out, the CEO wants an answer YESTERDAY.

Send me an email as soon as you figure it out. Can they supply enough gold or not? Or are they wasting our time? I won’t be sleeping until I hear from you.

Sincerely,

Terry

Hello Terry,

I see your stress. Please, do not worry! Because they are not wasting our time, and they can supply enough gold. So you are amazed about why I am sure about it, aren't you? This is really interesting. If J is a knight then what J said is true, so as what J told, they can supply enough gold. On the other hand, if J is a knave, we suppose P is a statement “They can supply enough gold” and Q is a statement “J is a knight”. Since J is a knave, we have ~Q. Besides, what J said is not true, which means we also have ~(P↔Q).

By negated condition and De Morgan, just say,

~( P↔Q) = ~(P→Q & Q→P) = ~(P→Q) ∨ ~(Q→P) = (P&~Q) ∨ (Q&~P). However, we have ~Q, so it is obvious that (Q&~P) is not true. That shows that (P&~Q) must be true. Then P is true, which means they can supply enough gold.

Summary, no matter J is knight or knave, they can supply enough gold. Satisfy with that, hope that we can have a great deal.

Email me when you make a proper deal.

Sincerely,

Terry