**Problem Set 3**

**Part 1 (one point total).** *Use the following proof to answer the questions below.*

1 (1) P→Q&R A

2 (2) P A **(a)**

1,2 **(**3) Q&R 1,2 MP **(b)**

1,2 (4) Q 3 Simp. **(c)**

1,2 (5) R 3 Simp.

1,2 (6) P&R 2,5 Conj. **(d)**

1,2 (7) Q&(P&R) 4,6 Conj.

1. What rule is used in the line marked with **(a)**?
2. What assumption(s) are in the assumption set for the line marked **(a)**?
3. What is the line number for line marked **(a)**?
4. What is the annotation of line marked with **(b)**?
5. What is the assumption set of the line marked with **(b)**?
6. What rule is used in the line marked **(b)**?
7. What is the line number of the line marked with **(c)**?
8. What is the rule used on the line marked **(c)** and what line does the rule depend upon?
9. What rule is used in the line marked **(d)** and what line(s) is(are) that rule used on?
10. Explain how the assumption set from line **(d)** is determined.

Answer:

1. Assumption rule
2. Assumption set 2
3. The line number 2
4. The annotation is 1,2 MP
5. Assumption set 1,2
6. Modus Ponens
7. The line number 4
8. It used simplification rule, which depend upon the line number 3.
9. It used conjunction rule, which is used on the lines 2 and 5.
10. The rule is used on the lines 2 and 5. Since the assumption sets of lines 2 and 5 are 2 and 1,2, respectively, the assumption set from line (d) is the union of them. This is 1,2.

**Part 2 (one point total).** *Complete the following proof, by filling in the lettered blanks.*

1 (1) P→(Q→R) A

(a) (2) ~T∨(B→P∨S) A

3 (3) B&~S A

4 (4) Q&T A

4 (5) Q \_\_\_(b)\_\_

4 (6) \_\_\_(c)\_\_\_\_\_ 4 SIM

\_\_(d)\_ (7) B 3 SIM

3 (8) ~S \_\_(e)\_\_\_

2,4 (9) B→P∨S \_\_(f)\_\_\_\_

2,3 (10) P∨S \_\_(g)\_\_\_

2,3 (11) \_\_\_(h)\_\_\_\_\_ 8,10 DS

1,2,3 (12) ­­­\_\_\_(i)\_\_\_\_\_\_ 1,11 MP

1,2,3 (13) R \_\_\_(j)\_\_\_

Solution:

1. 2
2. 4 SIM
3. T
4. 3
5. 3 SIM
6. 2,6 DS
7. 7,9 MP
8. P
9. Q→R
10. 5,12 MP

**Part 3 (one point total).** *For each of the following sequents, provide a proof that demonstrates their validity. These proofs can be done with rules of implication and do not require the use of working assumptions.*

1. P, Q ├ (P&Q)
2. P, Q├ (P&Q)∨R
3. P, Q├ (Q&P)∨R
4. P, Q├ (P&Q)&(Q∨R)
5. P, Q├ (P∨R)&(Q∨R)
6. P→Q , P├ Q
7. P→Q , P├ (Q∨R)&(P∨S)
8. P→Q , P∨T, ~T├ (Q∨R)&(P∨S)
9. P→Q , P∨T, ~T├ ((Q∨R)&(P∨S))&~T
10. P→Q , P∨T, ~T├ ((Q∨R)&(P∨S))&((~T&P)∨H)

Solution:

1. (1) P A

(2) Q A

(3) P&Q 1,2 Conj

1. (1) P A

(2) Q A

(3) P&Q 1,2 Conj

(4) (P&Q)∨R 3 InDis

1. (1) P A

(2) Q A

(3) Q&P 1,2 Conj

(4) (Q&P)∨R 3 InDis

1. (1) P A

(2) Q A

(3) P&Q 1,2 Conj

(4) Q∨R 2 InDis

(5) (P&Q)&(Q∨R) 3,4 Conj

1. (1) P A

(2) Q A

(3) P∨R 1 InDis

(4) Q∨R 2 InDis

(5) (P∨R)&(Q∨R) 3,4 Conj

1. (1) P→Q A

(2) P A

(3) Q 1,2 MP

1. (1) P→Q A

(2) P A

(3) Q 1,2 MP

(4) Q∨R 3 InDis

(5) P∨S 2 InDis

(6) (Q∨R)&(P∨S) 4,5 Conj

1. (1) P→Q A

(2) P∨T A

(3) ~T A

(4) P 2,3 DS

(5) Q 1,4 MP

(6) Q∨R 5 InDis

(7) P∨S 4 InDis

(8) (Q∨R)&(P∨S) 6,7 Conj

1. (1) P→Q A

(2) P∨T A

(3) ~T A

(4) P 2,3 DS

(5) Q 1,4 MP

(6) Q∨R 5 InDis

(7) P∨S 4 InDis

(8) (Q∨R)&(P∨S) 6,7 Conj

(9) ((Q∨R)&(P∨S))&~T 3,8 Conj

1. (1) P→Q A

(2) P∨T A

(3) ~T A

(4) P 2,3 DS

(5) Q 1,4 MP

(6) Q∨R 5 InDis

(7) P∨S 4 InDis

(8) (Q∨R)&(P∨S) 6,7 Conj

(9) ~T&P 3,4 Conj

(10) (~T&P)∨H 9 InDis

(11) ((Q∨R)&(P∨S))&((~T&P)∨H) 8,10 Conj

**Part 4 (one point total).** *For each of the following sequents, provide a proof that demonstrates their validity. These proofs can be done with rules of implication and do not require the use of working assumptions.*

1. P&Q→R, ~R∨S, (P&T)&(Q&W) ├ P&Q
2. P&Q→R, ~R∨S, (P&T)&(Q&W) ├ R&Q
3. P&Q→R, ~R∨S, (P&T)&(Q&W) ├ R&S
4. P&Q→R, ~R∨S, (P&T)&(Q&W) ├ S&W
5. P&Q→R, ~R∨S, (P&T)&(Q&W) ├ (S&W)&(T∨Y)
6. (P→R)&(R→S) ├ P→S
7. (P→R)&(R→S), ~S├ ~P
8. (P→R)&(R→S), ~S ├ ~R&~P
9. (P→R)&(R→S), ~(R→S)∨~S ├ ~P∨Q
10. (P→R)&(R→S), ~(R→S)∨~S, P∨(Q&W)├ (Q&~R)&(W∨P)

Solution:

1. (1) P&Q→R A

(2) ~R∨S A

(3) (P&T)&(Q&W) A

(4) P&T 3 Simp

(5) Q&W 3 Simp

(6) P 4 Simp

(7) Q 5 Simp

(8) P&Q 6,7 Conj

1. (1) P&Q→R A

(2) ~R∨S A

(3) (P&T)&(Q&W) A

(4) P&T 3 Simp

(5) Q&W 3 Simp

(6) P 4 Simp

(7) Q 5 Simp

(8) P&Q 6,7 Conj

(9) R 1,8 MP

(10) R&Q 7,9 Conj

1. (1) P&Q→R A

(2) ~R∨S A

(3) (P&T)&(Q&W) A

(4) P&T 3 Simp

(5) Q&W 3 Simp

(6) P 4 Simp

(7) Q 5 Simp

(8) P&Q 6,7 Conj

(9) R 1,8 MP

(10) S 2,9 DS

(11) R&S 9,10 Conj

1. (1) P&Q→R A

(2) ~R∨S A

(3) (P&T)&(Q&W) A

(4) P&T 3 Simp

(5) Q&W 3 Simp

(6) P 4 Simp

(7) Q 5 Simp

(8) P&Q 6,7 Conj

(9) R 1,8 MP

(10) S 2,9 DS

(11) W 5 Simp

(12) S&W 10,11 Conj

1. (1) P&Q→R A

(2) ~R∨S A

(3) (P&T)&(Q&W) A

(4) P&T 3 Simp

(5) Q&W 3 Simp

(6) P 4 Simp

(7) Q 5 Simp

(8) P&Q 6,7 Conj

(9) R 1,8 MP

(10) S 2,9 DS

(11) W 5 Simp

(12) S&W 10,11 Conj

(13) T 4 Simp

(14) T∨Y 13 InDis

(15) (S&W)&(T∨Y) 12,14 Conj

1. (1) (P→R)&(R→S) A

(2) P→R 1 Simp

(3) R→S 1 Simp

(4) P→S 2,3 HS

1. (1) (P→R)&(R→S) A

(2) ~S A

(3) P→R 1 Simp

(4) R→S 1 Simp

(5) P→S 3,4 HS

(6) ~P 2,5 MT

1. (1) (P→R)&(R→S) A

(2) ~S A

(3) P→R 1 Simp

(4) R→S 1 Simp

(5) ~R 2,4 MT

(6) ~P 3,5 MT

(7) ~R&~P 5,6 Conj

1. (1) (P→R)&(R→S) A

(2) ~(R→S)∨~S A

(3) R→S 1 Simp

(4) ~S 2,3 DS

(5) ~R 3,4 MT

(6) P→R 1 Simp

(7) ~P 5,6 MT

(8) ~P∨Q 7 InDis

1. (1) (P→R)&(R→S) A

(2) ~(R→S)∨~S A

(3) P∨(Q&W) A

(4) R→S 1 Simp

(5) ~S 2,4 DS

(6) ~R 4,5 MT

(7) P→R 1 Simp

(8) ~P 6,7 MT

(9) Q&W 3,8 DS

(10) Q 9 Simp

(11) W 9 Simp

(12) Q&~R 6,10 Conj

(13) W∨P 11 InDis

(13) (Q&~R)&(W∨P) 12,13 Conj