**Problem Set 5**

**Part 1 (two points total).** *For each of the following sequents, provide a proof that demonstrates its validity. Make sure to use SL and you may use both rules of implication and replacement rules.*

1. ~(~R→~P)├ P
2. ~(~R→~P)├ P&~R
3. P→Q∨R, ~(~R→~P)├ P&Q
4. P→Q∨R, (~Q&S)∨(T→~P), ~(~R→~P)├ T→~P
5. P→Q∨R, (~Q&S)∨(T→~P), ~(~R→~P)├ ~T&Q
6. E→~(J∨H), J├ ~E
7. E→~(J∨H), ~S, J∨S├ ~E
8. C→E, E→~(J∨H), ~S, J∨S├ ~C
9. B→(C→E), E→~(J∨H), ~S, J∨S ├ B→~C
10. B→(C→E), E→~(J∨H), ~S, J∨S ├ ~B∨~C

Solution:

1. 1 (1) ~(~R→~P) A

1 (2) ~(R ∨ ~P) 1 Impl

1 (3) ~R & P 2 DM

1 (4) P 3 SIM

1. 1 (1) ~(~R→~P) A

1 (2) ~(R ∨ ~P) 1 Impl

1 (3) P&~R 2 DM

1. 1 (1) P→Q∨R A

2 (2) ~(~R→~P) A

2 (3) ~(R ∨ ~P) 2 Impl

2 (4) ~R & P 3 DM

2 (5) P 4 SIM

1,2 (6) Q∨R 1,5 MP

2 (7) ~R 4 SIM

1,2 (8) Q 6,7 DS

1,2 (9) P&Q 5,8 Conj

1. 1 (1) P→Q∨R A

2 (2) (~Q&S)∨(T→~P) A

3 (3) ~(~R→~P) A

3 (4) ~(R ∨ ~P) 3 Impl

3 (5) ~R & P 4 DM

3 (6) P 5 SIM

1,3 (7) Q∨R 1,6 MP

3 (8) ~R 5 SIM

1,3 (9) Q 7,8 DS

4 (10)~Q&S A (working for RAA) prove: ~(~Q&S)

4 (11) ~Q 10 SIM

1,3 (12) ~(~Q&S) 9,11 RAA discharge 10

1-3 (13) T→~P 2,12 DS

1. 1 (1) P→Q∨R A

2 (2) (~Q&S)∨(T→~P) A

3 (3) ~(~R→~P) A

3 (4) ~(R ∨ ~P) 3 Impl

3 (5) ~R & P 4 DM

3 (6) P 5 SIM

1,3 (7) Q∨R 1,6 MP

3 (8) ~R 5 SIM

1,3 (9) Q 7,8 DS

4 (10)~Q&S A (working for RAA) prove: ~(~Q&S)

4 (11) ~Q 10 SIM

1,3 (12) ~(~Q&S) 9,11 RAA discharge 10

1-3 (13) T→~P 2,12 DS

1-3 (14) ~T 6,13 MT

1-3 (15) ~T&Q 9,14 Conj

1. 1 (1) E→~(J∨H) A

2 (2) J A

3 (3) E A(for working RAA)

1,3 (4) ~(J∨H) 1,3 MP

1,3 (5) ~J&~H 4 DM

1,3 (6) ~J 5 SIM

1,2 (7) ~E 2,6 RAA discharge 3

1. 1 (1) E→~(J∨H) A

2 (2) ~S A

3 (3) J∨S A

4 (4) E A(for working RAA)

1,4 (5) ~(J∨H) 1,4 MP

1,4 (6) ~J&~H 5 DM

1,4 (7) ~J 6 SIM

2,3 (8) J 2,3 DS

1-3 (9) ~E 7,8 RAA discharge 4

1. 1 (1) C→E A

2 (2) E→~(J∨H) A

3 (3) ~S A

4 (4) J∨S A

5 (5) C A(working for RAA)

1,5 (6) E 1,5 MP

1,2,5 (7) ~(J∨H) 2,6 MP

1,2,5 (8) ~J&~H 7 DM

1,2,5 (9) ~J 8 SIM

1,2,4,5 (10) S 4,9 DS

1-4 (11) ~C 3, 10 RAA discharge 5

1. 1 (1) B→(C→E) A

2 (2) E→~(J∨H) A

3 (3) ~S A

4 (4) J∨S A

5 (5) B A(working for CP) prove: ~C

1,5 (6) C→E 1,5 MP

6 (7) C A(working for RAA)

1,5,6 (8) E 6,7 MP

1,2,5,6 (9) ~(J∨H) 2,8 MP

1,2,5,6 (10) ~J&~H 9 DM

1,2,5,6 (11) ~J 10 SIM

1,2,4,5,6 (12) S 4,11 DS

* 1. (13) ~C 3, 12 RAA discharge 7

1-4 (14) B→~C 13 CP discharge 5

1. 1 (1) B→(C→E) A

2 (2) E→~(J∨H) A

3 (3) ~S A

4 (4) J∨S A

5 (5) ~(~B∨~C) A(working for RAA)

5 (6) B&C 5 DM

5 (7) B 6 SIM

1,5 (8) C→E 1,7 MP

5 (9) C 6 SIM

1,5 (10) E 8,9 MP

1,2,5 (11) ~(J∨H) 2,10 MP

1,2,5 (12) ~J&~H 11 DM

1,2,5 (13) ~J 12 SIM

1,2,4,5 (14) S 4,13 DS

1-4 (15) ~B∨~C 3,14 RAA discharge 5

**Part 3 (two points total).** *For each of the following theorems, provide a proof that demonstrates its validity.*

1. ├ P→P
2. ├ P→ P∨Q
3. ├ P→ P∨(P&Q)
4. ├ P∨(P&Q)→P
5. ├ P ↔ P∨(P&Q)
6. ├ P&Q→Q&P
7. ├ ~(~P∨~Q)→Q&P
8. ├ ~(~P∨~Q)→ ~(Q→~P)
9. ├ ~(Q→~P)→ ~(~P∨~Q)
10. ├ ~~(Q&P)↔ ~(P→~Q)

Solution:

1. 1 (1) P A (working for CP)

(2) P→P 1 CP discharge 1

1. 1 (1) P A (working for CP)
2. (2) P∨Q 1 inDis

(3) P→ P∨Q 2 CP discharge 1

1. 1 (1) P A (working for CP)

1 (2) P∨(P&Q) 1 inDis

(3) P→ P∨(P&Q) 2 CP discharge 1

1. 1 (1) P∨(P&Q) A(working for CP)

2 (2) ~P A(working for RAA) prove: P

1,2 (3) P&Q 1,2 DS

1,2 (4) P 1,2 SIM

1 (5) P 2,4 RAA discharge 2

(6) P∨(P&Q)→P 5 CP discharge 1

1. By the result of part 3 ├ P→ P∨(P&Q)

By the result of part 4 ├ P∨(P&Q)→P

├ (P→ P∨(P&Q)) & (P∨(P&Q)→P)

├ P ↔ P∨(P&Q) ME

1. 1 (1) P&Q A (working for CP)

1 (2) P 1 SIM

1 (3) Q 1 SIM

1 (4) Q&P 2,3 Conj

(5) P&Q→Q&P 4 CP discharge 1

1. 1 (1) ~(~P∨~Q) A(working for CP)

1 (2) P&Q 1 DM

1 (3) P 2 SIM

1 (4) Q 2 SIM

1 (5) Q&P 3,4 Conj

(6) ~(~P∨~Q)→Q&P 5 CP discharge 1

1. 1 (1) ~(~P∨~Q) A(working for CP)

1 (2) P&Q 1 DM

1 (3) ~(Q→~P) 2 NCon

(4) ~(~P∨~Q)→ ~(Q→~P) 3 CP discharge 1

1. 1 (1) ~(Q→~P) A(working for CP)

1 (2) Q&P 1 NCon

1 (3) ~(~Q∨~P) 2 DM

(4) ~(Q→~P)→ ~(~P∨~Q) 3 CP discharge 1

1. 1 (1) ~~(Q&P) A(working for CP) prove: ~(P→~Q)

1 (2) Q&P 1 Double Denial

1. (3) P&Q

1 (4) ~(P→Q) 3 NCon

(5) ~~(Q&P) → ~(P→Q) 4 CP discharge 1

2 (6) ~(P→Q) A(working for CP) prove: ~~(Q&P)

2 (7) P&Q 6 NCon

1. (8) (Q&P)

2 (9) ~~(Q&P) 8 Double Denial

(10) ~(P→Q) → ~~(Q&P) 9 CP discharge 6

(11) (~~(Q&P) → ~(P→Q)) & (~(P→Q) → ~~(Q&P)) 5,10 Conj

(12) ~~(Q&P)↔ ~(P→~Q) 11 ME