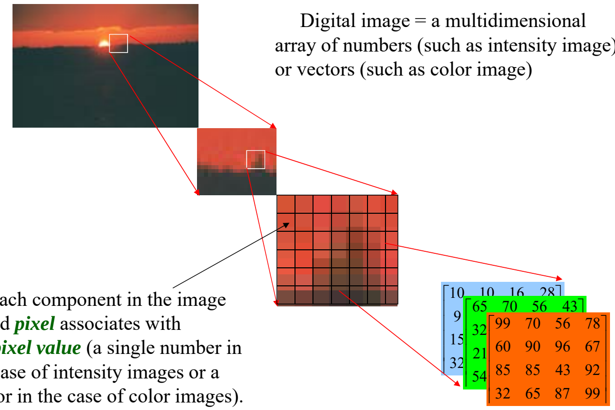
DIP Chapter 2:
Digital Image Fundamentals

Digital Image

Digital image = a multidimensional array of numbers (such as intensity image) or vectors (such as color image)



Each component in the image called ***pixel*** associates with the ***pixel value*** (a single number in the case of intensity images or a vector in the case of color images).

Human Eye Structure

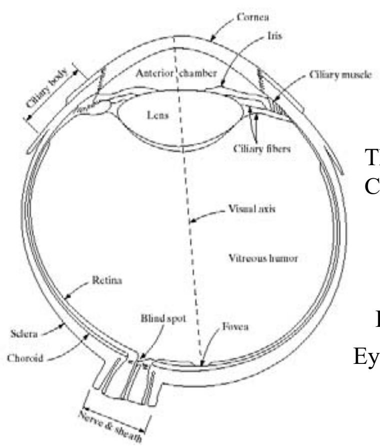


FIGURE 2.1 Simplified diagram of a cross section of the human eye.

Three membranes enclose the eye:
Cornea and sclera, Choroid, Retina

ciliary body   iris diaphragm

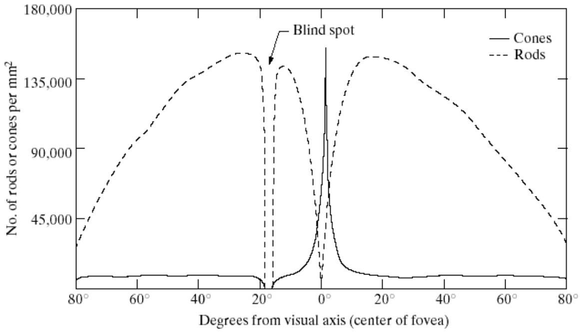
Pupil size: 2-8mm

Eye color: melanin (pigment) in iris

Visual Perception: Human Eye (cont.)

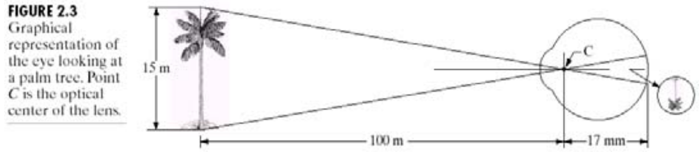
1. The ***lens*** contains 60-70% water, 6% of fat.
2. The ***iris*** diaphragm controls amount of light that enters the eye.
3. Light receptors in the ***retina***
   - About 6-7 millions ***cones*** for *bright light vision* called ***photopic***
     - Density of cones is about 150,000 elements/mm^2.
     - Cones involve in color vision.
     - Cones are concentrated in ***fovea*** about 1.5x1.5 mm^2.
   - About 75-150 millions ***rods*** for *dim light vision* called ***scotopic***
     - Rods are sensitive to low level of light and are not involved in color vision.
4. ***Blind spot*** is the region of emergence of the optic nerve from the eye.

Distribution of Rods and Cones in the Retina



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Image Formation in the Eye



Focal length: 14-17mm   Length of tree image≅2.55mm

For distant objects (>3m), lens exhibits the least refractive power (flattened)

For nearby objects (<1m), lens is most strongly refractive (curved)

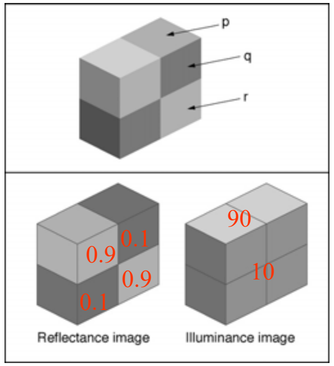
Lightness Perception: Objective Quantities

- *Luminance* is the amount of visible light that comes to the eye from a surface.
- *Illuminance* is the amount of light incident on a surface.
- *Reflectance* (also called *albedo*) is the proportion of incident light that is reflected from a surface.
  - varies from 0% to 100% where 0% is ideal black and 100% is ideal white. In practice, typical black paint is about 5% and typical white paint about 85%.

Lightness Perception: Subjective Quantities

- *Lightness* is the perceived reflectance of a surface. It represents the visual system's attempt to extract reflectance based on the luminances in the scene.
- *Brightness* is the perceived intensity of light coming from the image itself, rather than any property of the portrayed scene. Brightness is sometimes defined as perceived luminance.

Checker-block Illustration*

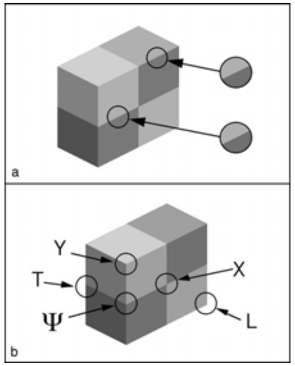


Patches p and q have the same reflectance, but different luminances.

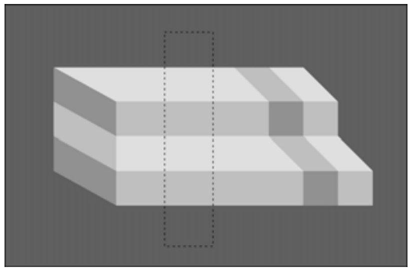
Patches q and r have different reflectances and different luminances; they share the same illuminance.

Patches p and r happen to have the same luminance, because the lower reflectance of p is counterbalanced by its higher illuminance.

Importance of Visual Context

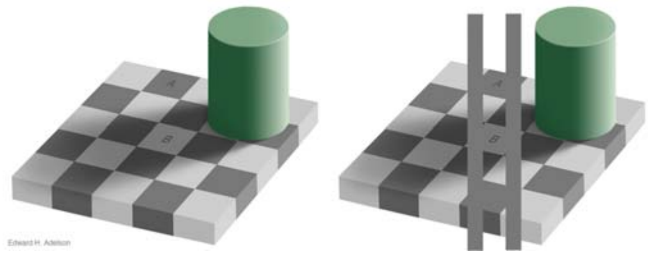


Lightness Illusion



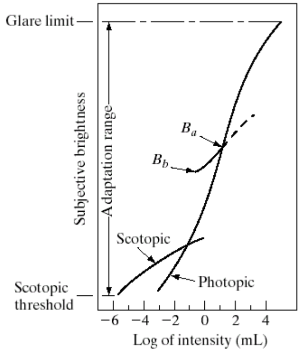
If we cover the right side of the figure and view the left side, it appears that the stripes are due to paint (reflectance). If we cover the left side and view the right, it appears that the stripes are due to different lighting on the stair steps (illumination).

Another Lightness Illusion



Range of Relative Brightness Sensation

FIGURE 2.4 Range of subjective brightness sensations showing a particular adaptation level.



Human visual system cannot operate over such a high dynamic range simultaneously, But accomplish such large variation by changes in its overall sensitivity, a phenomenon called "brightness adaptation"

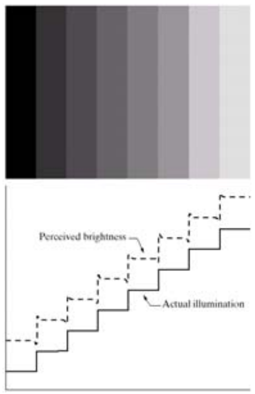
Simultaneous range is smaller than total adaptation range

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Scotopic Vision

- **Scotopic vision** is the *monochromatic* vision of the eye in dim light. Since *cone cells* are nonfunctional in low light, scotopic vision is produced exclusively through *rod cells* so therefore there is no colour perception.
  - Scotopic vision occurs at luminance levels of 10-2 to 10-6 cd/m^2.
- **Mesopic vision** occurs in intermediate lighting conditions (luminance level 10-2 to 1 cd/m^2) and is effectively a combination of *scotopic* and *photopic* vision.
  - This however gives inaccurate visual acuity and colour discrimination.
- In normal light (luminance level 1 to 106 cd/m^2), the vision of *cone cells* dominates and is **photopic vision**.
  - There is good visual acuity (VA) and colour discrimination.

Mach Band Effect



Perceived brightness is not a simple function of intensity.

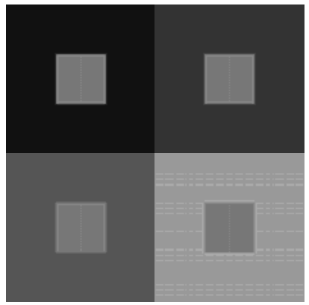
Intensities of surrounding points effect perceived brightness at each point.

In this image, edges between bars appear brighter on the right side and darker on the left side.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

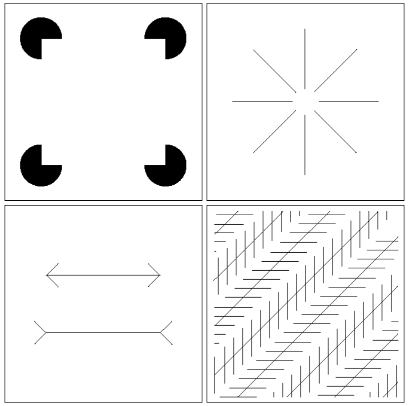
Brightness Adaptation of Human Eye : Simultaneous Contrast

A region's perceived brightness does not depend simply on its intensity



Simultaneous contrast. All small squares have exactly the same intensity but they appear progressively darker as background becomes lighter.

Optical illusion



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Visible Spectrum

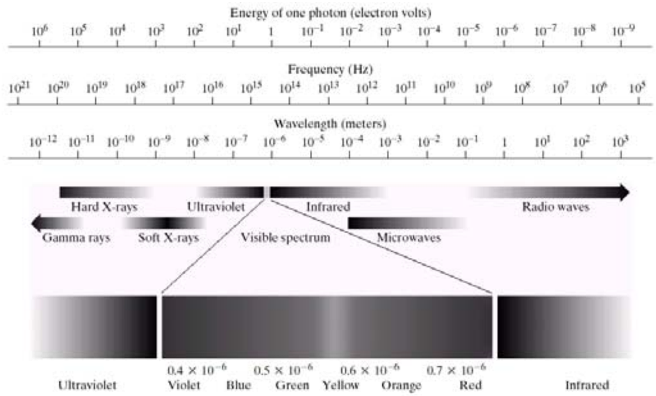


FIGURE 2.10 The electromagnetic spectrum. The visible spectrum is shown zoomed to facilitate explanation, but note that the visible spectrum is a rather narrow portion of the EM spectrum.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

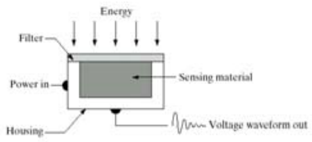
Light: the Visible Spectrum

- Visible range: 0.43µm(violet)-0.78µm(red)
- Six bands: violet, blue, green, yellow, orange, red
- The color of an object is determined by the nature of the light *reflected* by the object
- Monochromatic light (gray level)
- Three elements measuring chromatic light
  - ***Radiance*** : total amount of energy that flows from the light source(Watt)
  - ***Luminance*** : measure of the amount of energy an observer perceives from a light source(lumens, lm)
  - ***Brightness*** : a subjective descriptor of light perception
  - Ex) light emitted in far infrared region with large radiance, but no luminance

Beyond Visible

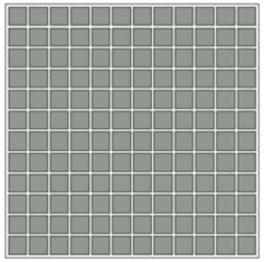
- Gamma-ray and X-ray: medical and astronomical applications
- Infrared (thermal imaging): near-infrared and far-infrared
- Microwave imaging:
- Radio-frequency: MRI and astronomic applications

Image Sensors



Single sensor

Line sensor



Array sensor

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Image Sensors : Single Sensor

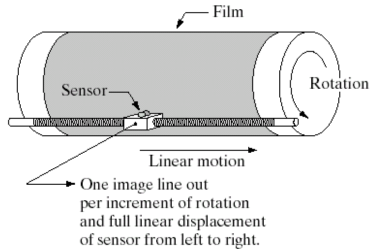
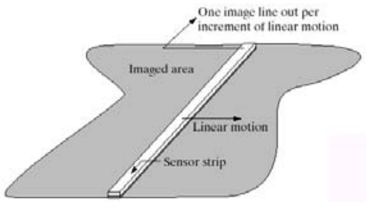


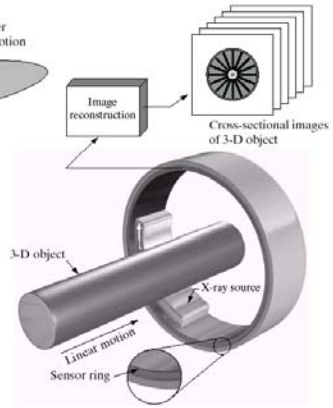
FIGURE 2.13 Combining a single sensor with motion to generate a 2-D image.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Image Sensors : Line Sensor



Fingerprint sweep sensor

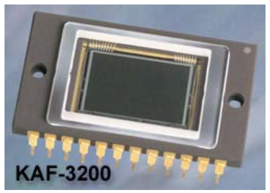


Computerized Axial Tomography

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Image Sensors : Array Sensor

Charge-Coupled Device (CCD)



CCD KAF-3200E from Kodak.
(2184 x 1472 pixels,
Pixel size 6.8 microns^2)

- Used to convert a continuous image into a digital image
- Contains an array of light sensors
- Converts photon into electric charges accumulated in each sensor unit

Fundamentals of Digital Images

- **An image**: **a multidimensional function of spatial coordinates**.
- **Spatial coordinate**: (x,y) for 2D case such as photograph,
  (x,y,z) for 3D case such as CT scan images
  (x,y,t) for movies
- The function f may represent intensity (for monochrome images) or color (for color images) or other associated values.

Digital Images

Digital image: an image that has been discretized both in Spatial coordinates and associated value.

- Consist of 2 sets:(1) a point set and (2) a value set
- Can be represented in the form
  $I = \{(x,a(x)): x \in X, a(x) \in F\}$
  where X and F are a point set and value set, respectively.
- An element of the image, $(x,a(x))$ is called a ***pixel*** where
  - x is called the pixel location and
  - $a(x)$ is the pixel value at the location x

Conventional Coordinate for Image Representation

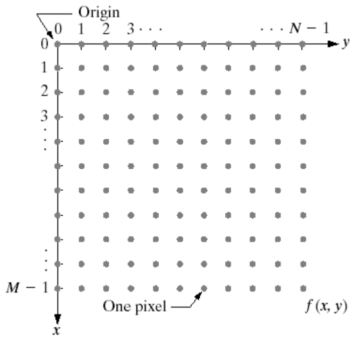


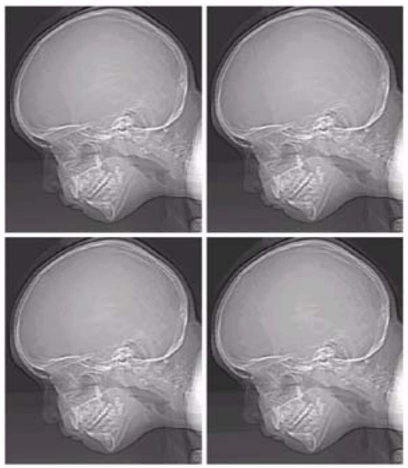
FIGURE 2.18 Coordinate convention used in this book to represent digital images.

Spatial resolution

Bit-depth resolution

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Bit-depth Resolution



a b
c d

FIGURE 2.21 (a) 452×374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

Bit-depth Resolution (Con'd)

e f
g h

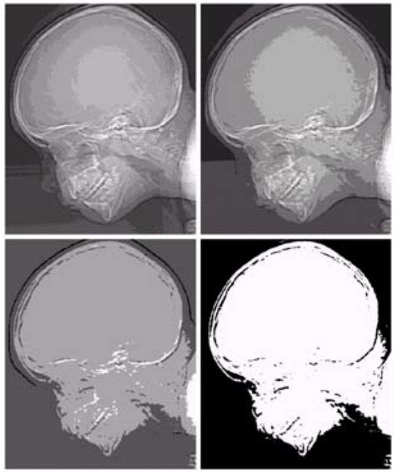
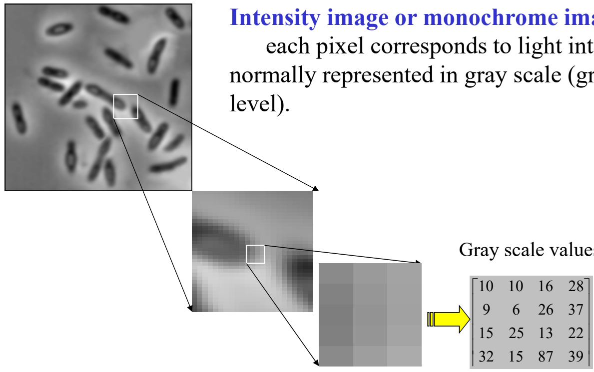


FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

Digital Image Types : Intensity Image

Intensity image or monochrome image
each pixel corresponds to light intensity normally represented in gray scale (gray level).



Digital Image Types : RGB Image

Color image or RGB image:
each pixel contains a vector representing red, green and blue components.

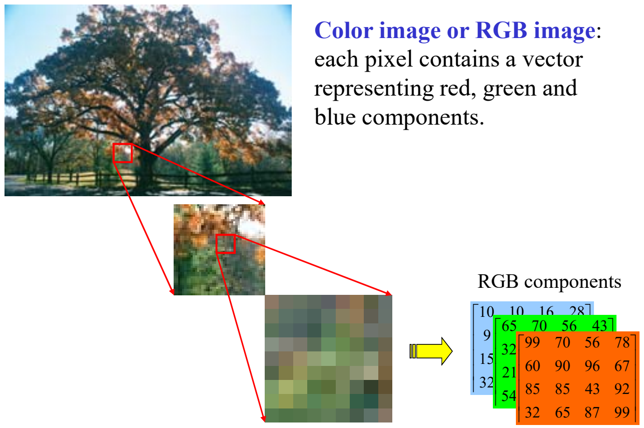


Image Types : Binary Image

Binary image or black and white image
Each pixel contains one bit :
1 represent white
0 represents black

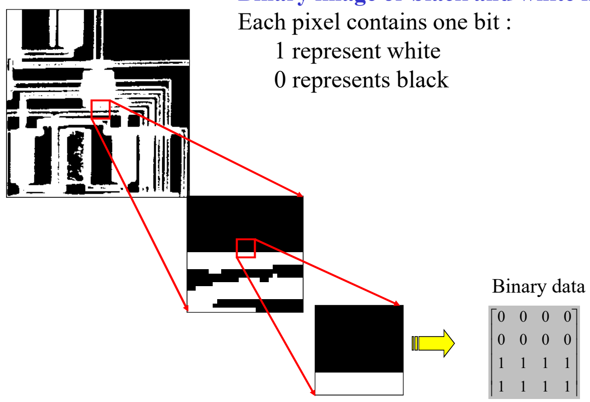
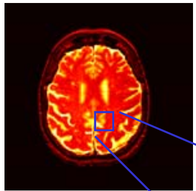


Image Types : Index Image

Index image
Each pixel contains index number pointing to a color in a color table



$$\begin{bmatrix} 1 & 4 & 9 \\ 6 & 4 & 7 \\ 6 & 5 & 2 \end{bmatrix}$$

Index value

Color Table

Index No.	Red component	Green component	Blue component
1	0.1	0.5	0.3
2	1.0	0.0	0.0
3	0.0	1.0	0.0
4	0.5	0.5	0.5
5	0.2	0.8	0.9
…	…	…	…

Digital Image Acquisition Process

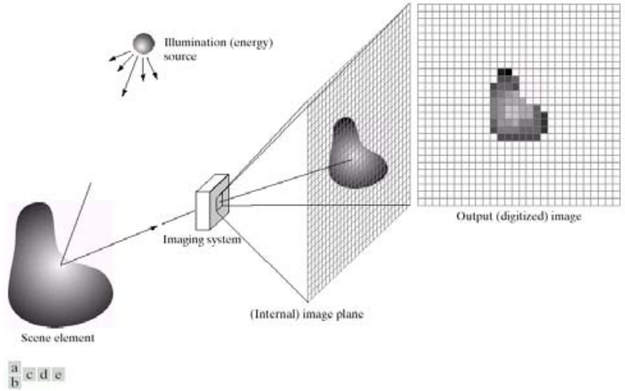


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Image Formation Model

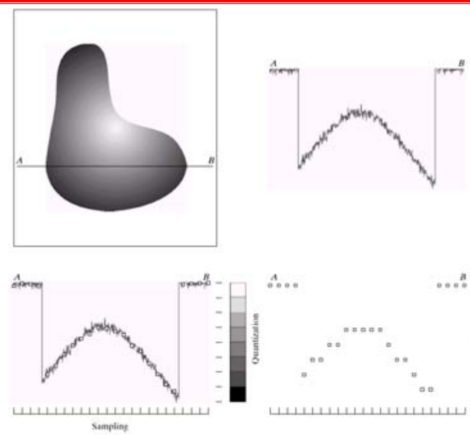
$$f(x,y)=i(x,y)r(x,y)$$

$0<f(x,y)<\infty$ Intensity – proportional to energy radiated by a physical source

$0<i(x,y)<\infty$ illumination

$0<r(x,y)<1$ reflectance ("intrinsic images")

Generating a Digital Image

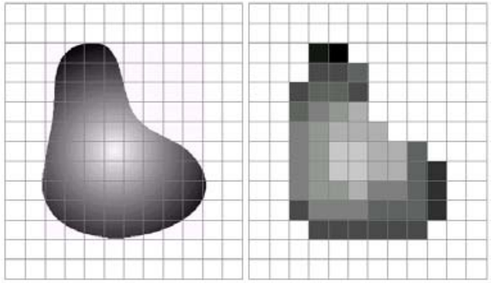


(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

a b
c d

FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Image Sampling and Quantization



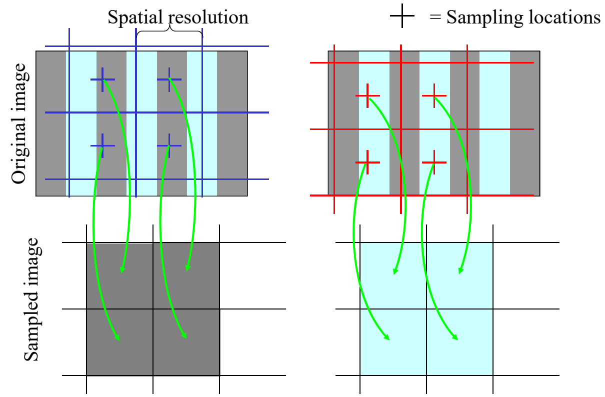
a b
FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

Image sampling: discretize an image in the spatial domain

Spatial resolution / image resolution: pixel size or number of pixels

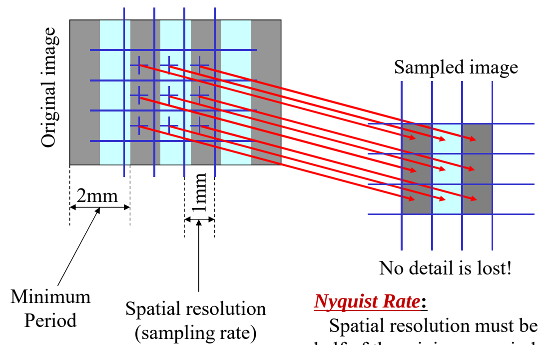
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

How to choose the spatial resolution



Under sampling, we lost some image details!

How to choose the spatial resolution : Nyquist Rate



= Sampling locations

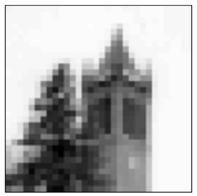
***Nyquist Rate*:**
Spatial resolution must be less or equal half of the minimum period of the image or sampling frequency must be greater or Equal twice of the maximum frequency.

Effect of Spatial Resolution

256x256 pixels

128x128 pixels

64x64 pixels



32x32 pixels

Effect of Spatial Resolution

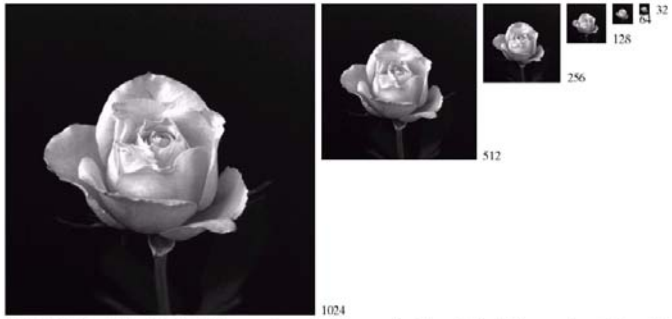
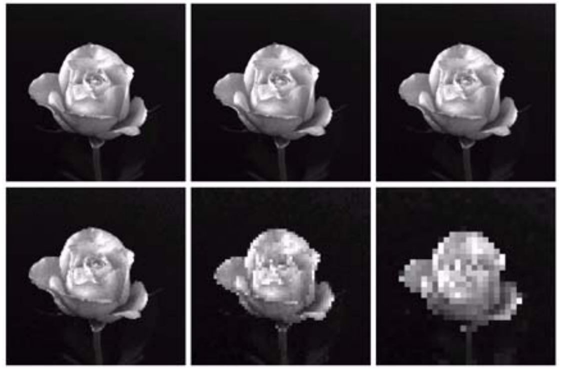


FIGURE 2.19 A 1024 × 1024, 8-bit image subsampled down to size 32 × 32 pixels. The number of allowable gray levels was kept at 256.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Effect of Spatial Resolution

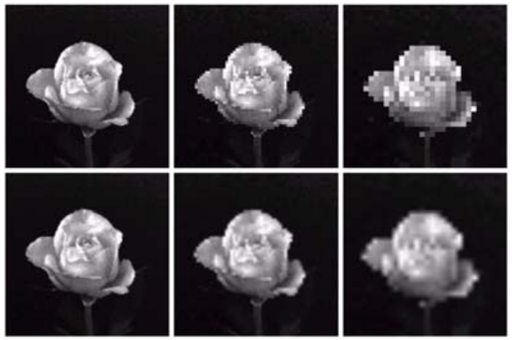


a b c
d e f

FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Can we increase spatial resolution by interpolation ?



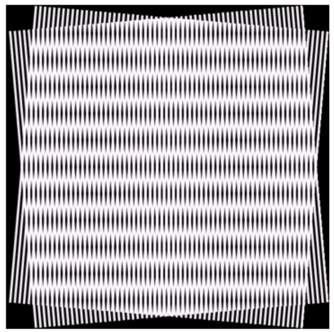
a b c
d e f

FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

Down sampling is an irreversible process.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Moire Pattern Effect : Special Case of Sampling

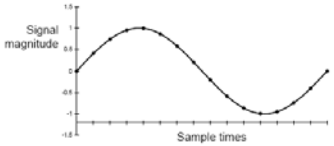


Moire patterns occur when frequencies of two superimposed periodic patterns are close to each other.

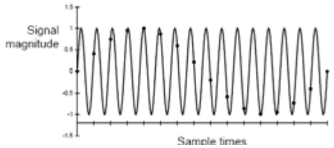
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Alias Frequency

It would normally be expected that the input signal was:



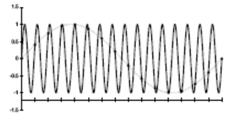
However, the input could have been:



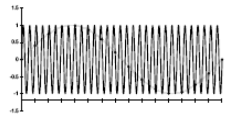
If the reconstructed signal is different to the original, it is said to be an **alias**

Alias Frequency

More examples of aliasing:



One cycle between samples



Two cycles between samples

If fact there is an infinite number of possible alias frequencies:

Aliasing is caused by not taking enough samples of the waveform per cycle.

Image Quantization

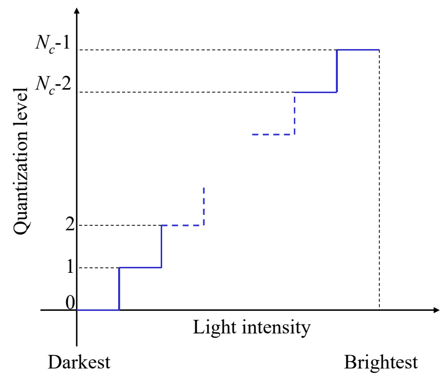
Image quantization:
discretize continuous pixel values into discrete numbers

Color resolution/ color depth/ levels:
- No. of colors or gray levels or
- No. of bits representing each pixel value
- No. of colors or gray levels N_c is given by

$$N_c = 2^b$$

where b = no. of bits

Quantization function



Effect of Quantization Levels


256 levels


128 levels


64 levels

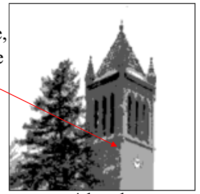

32 levels

Effect of Quantization Levels (cont.)


16 levels


8 levels

In this image, it is easy to see false contour.


4 levels


2 levels

How to select the suitable size and pixel depth of images

The word “suitable” is subjective: depending on “subject”.


Low detail image

Lena image


Medium detail image

Cameraman image


High detail image

To satisfy human mind

1. For images of the same size, the low detail image may need more pixel depth.
2. As an image size increase, fewer gray levels may be needed.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Isopreference curve

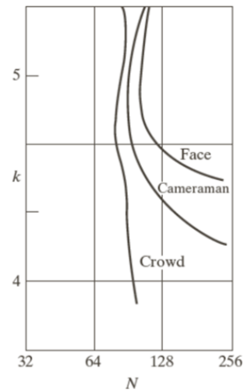
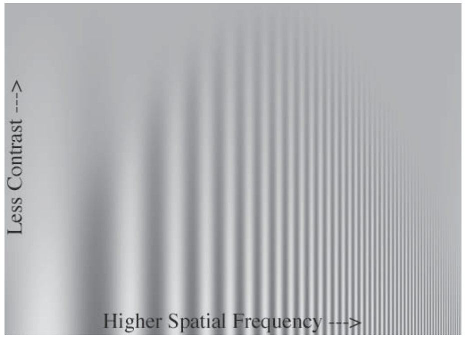
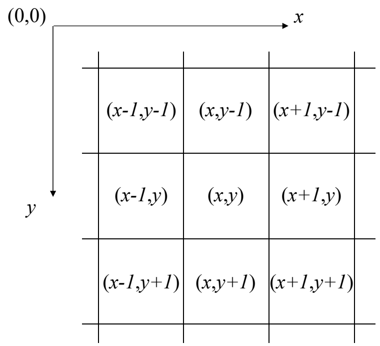


FIGURE 2.23 Typical isopreference curves for the three types of images in Fig. 2.22.

Human vision: Spatial Frequency vs Contrast



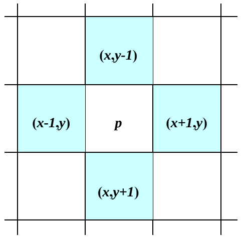
Basic Relationship of Pixels



Conventional indexing method

Neighbors of a Pixel

Neighborhood relation is used to tell adjacent pixels. It is useful for analyzing regions.



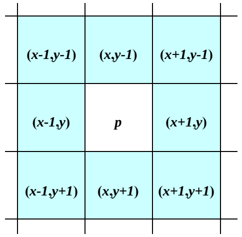
4-neighbors of *p*:

$$N_4(p) = \left\{ \begin{array}{c} (x-1,y) \\ (x+1,y) \\ (x,y-1) \\ (x,y+1) \end{array} \right\}$$

4-neighborhood relation considers only vertical and horizontal neighbors.

Note: $q \in N_4(p)$ implies $p \in N_4(q)$

Neighbors of a Pixel (cont.)

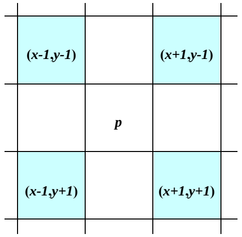


8-neighbors of *p*:

$$N_8(p) = \left\{ \begin{array}{c} (x-1,y-1) \\ (x,y-1) \\ (x+1,y-1) \\ (x-1,y) \\ (x+1,y) \\ (x-1,y+1) \\ (x,y+1) \\ (x+1,y+1) \end{array} \right\}$$

8-neighborhood relation considers all neighbor pixels.

Neighbors of a Pixel (cont.)



Diagonal neighbors of *p*:

$$N_D(p) = \begin{Bmatrix} (x-1,y-1) \\ (x+1,y-1) \\ (x-1,y+1) \\ (x+1,y+1) \end{Bmatrix}$$

Diagonal -neighborhood relation considers only diagonal neighbor pixels.

Connectivity

- Connectivity is adapted from neighborhood relation.
- Two pixels are connected if
  - they are in the same class (i.e. the same color or the same range of intensity) and
  - they are neighbors of one another.

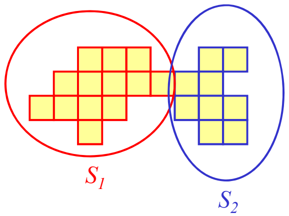
For p and q from the same class

- 4-connectivity: p and q are 4-connected if $q \in N_4(p)$
- 8-connectivity: p and q are 8-connected if $q \in N_8(p)$
- mixed-connectivity (m-connectivity):

  p and q are m-connected if $q \in N_4(p)$ or
  $q \in N_D(p)$ and $N_4(p) \cap N_4(q) = \varnothing$

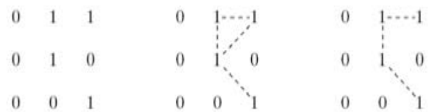
Adjacency

A pixel p is *adjacent* to pixel q is they are connected. Two image subsets S_1 and S_2 are adjacent if some pixel in S_1 is adjacent to some pixel in S_2



We can define type of adjacency: 4-adjacency, 8-adjacency or m-adjacency depending on type of connectivity.

multiple (ambiguous) 8-adjacency,



a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

Path

A ***path*** from pixel p at (x,y) to pixel q at (s,t) is a sequence of distinct pixels:

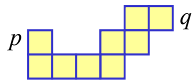
$$(x_0,y_0),\ (x_1,y_1),\ (x_2,y_2),\ldots,\ (x_n,y_n)$$

such that

$$(x_0,y_0) = (x,y) \text{ and } (x_n,y_n) = (s,t)$$

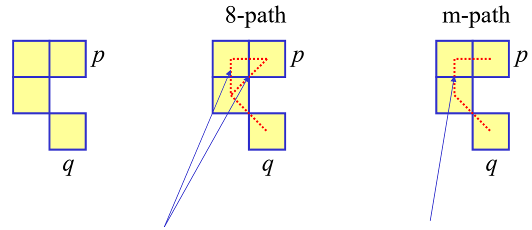
and

$$(x_i,y_i) \text{ is adjacent to } (x_{i-1},y_{i-1}), \qquad i = 1,\ldots,n$$



We can define type of path: 4-path, 8-path or m-path depending on type of adjacency.

Path (cont.)



8-path from p to q results in some ambiguity

m-path from p to q solves this ambiguity

Distance

For pixel p, q, and z with coordinates (x,y), (s,t) and (u,v), D is a ***distance function*** or ***metric*** if

- $D(p,q) \geq 0$ $\quad$ ($D(p,q) = 0$ if and only if $p = q$)
- $D(p,q) = D(q,p)$
- $D(p,z) \leq D(p,q) + D(q,z)$

Example: Euclidean distance

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

Euclidean distance *(2-norm)*

$2\sqrt{2}$	$\sqrt{5}$	2	$\sqrt{5}$	$2\sqrt{2}$
$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{5}$
2	1	0	1	2
$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$	$\sqrt{5}$
$2\sqrt{2}$	$\sqrt{5}$	2	$\sqrt{5}$	$2\sqrt{2}$

Distance (cont.)

D_4-distance (*city-block distance*) is defined as

$$D_4(p,q) = |x-s| + |y-t|$$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

Pixels with $D_4(p) = 1$ is 4-neighbors of p.

Distance (cont.)

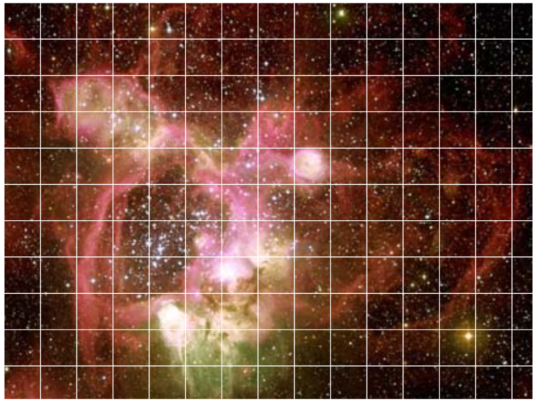
D_8-distance (*chessboard distance*) is defined as

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Pixels with $D_8(p) = 1$ is 8-neighbors of p.

Block-based Processing



Linear and Nonlinear Operations

H is said to be a *linear* operator if, for any two images *f* and *g* and any two scalars *a* and *b*,

$$H(af + bg) = aH(f) + bH(g)$$