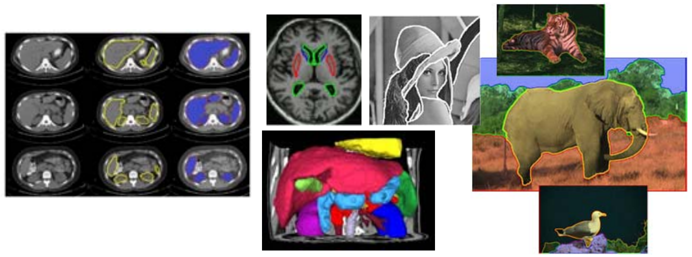
Digital Image Processing
Chapter 10:
Image Segmentation
-Binarization

Image Segmentation

- The goal of image segmentation is to cluster pixels into salient image regions, i.e., regions corresponding to individual surfaces, objects, or natural parts of objects.



The goals of segmentation

- Separate image into coherent “objects”

image | human segmentation

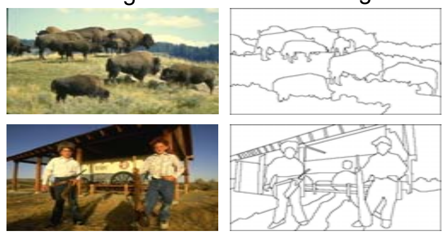
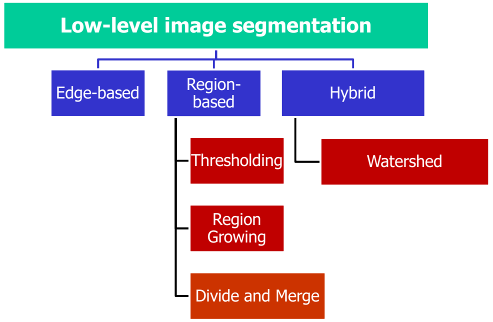
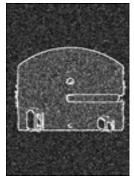


Image Segmentation



Edge-based Segmentation

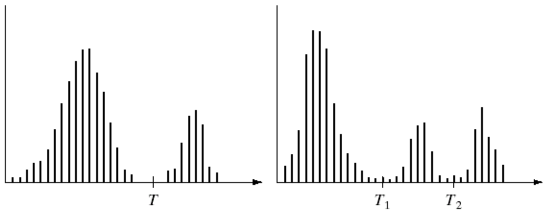
- Use Filters that find edges.
  - Prewitt operator
  - Roberts Cross-Difference Operator
  - Sobel Edge Detector
  - Canny Edge Detector
- Challenges:
  - Noise sensitive
  - None closed edges
  - Consider all edges



Thresholding

image with dark background and a light object

image with dark background and two light objects



a b

FIGURE 10.26 (a) Gray-level histograms that can be partitioned by (a) a single threshold, and (b) multiple thresholds.

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Multilevel thresholding

- A point (x,y) belongs to
  - to an object class if $T_1 < f(x,y) \leq T_2$
  - to another object class if $f(x,y) > T_2$
  - to background if $f(x,y) \leq T_1$
- T depends on
  - only $f(x,y)$: only on gray-level values ⇨ Global threshold
  - both $f(x,y)$ and $p(x,y)$: on gray-level values and its neighbors ⇨ Local threshold

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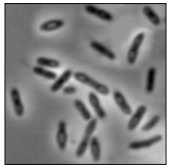
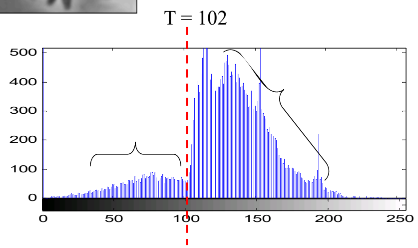
Pixel Oriented Image Segmentation: Thresholding

Intensity Thresholding



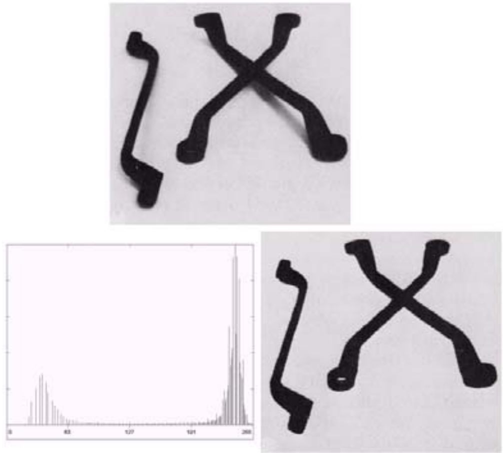
$$g(x,y)=\begin{cases}1 & f(x,y) > T\\ 0 & f(x,y) < T\end{cases}$$

Histogram multimodal



After thresholding

Intensity Thresholding Example



a
b c
FIGURE 10.28 (a) Original image. (b) Image histogram. (c) Result of global thresholding with T midway between the maximum and minimum gray levels.

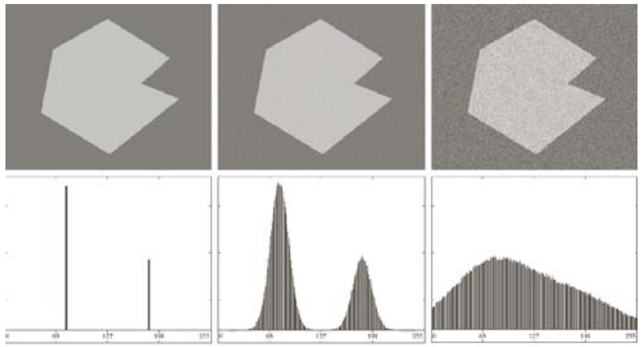
Different results with thresholds

Original

Threshold = 50

Threshold = 75

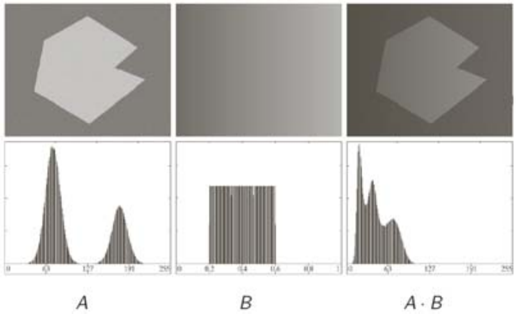
Histogram with noises



No noise 10% noise 50% noise

Illumination and reflection role in thresholding

f(x,y) = i(x,y) r(x,y)



Reflectance **Illumination** **Difficult to segment**

10.3.2 Basic Global Thresholding

The major problem of intensity thresholding is to find a good threshold level

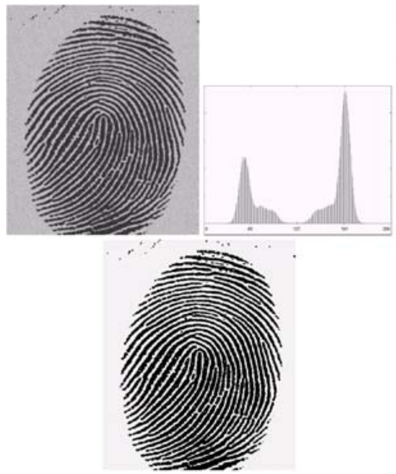
Algorithm: effective for bimodal histogram

1. Set initial value of T
2. $T_1 = Average(\, p(x,y) \mid p(x,y) > T\,)$
3. $T_2 = Average(\, p(x,y) \mid p(x,y) \le T\,)$
4. $T = \dfrac{T_1 + T_2}{2}$
5. Repeat step 2

Matlab Implementation

```
im = imread('boy.jpg');
I = rgb2gray(im);
I=double(I);
T=opthr(I);
bim=(I>T);
subplot(1,2,1), imshow(I, gray(256));
subplot(1,2,2), imshow(bim)
```

Automatic Threshold Level Selection Example



a b
c

FIGURE 10.29 (a) Original image. (b) Image histogram. (c) Result of segmentation with the threshold estimated by iteration. (Original courtesy of the National Institute of Standards and Technology.)

Optimal Thresholding

- In *optimal thresholding*, a criterion function is devised that yields some measure of separation between regions.
  - A criterion function is calculated for each intensity and that which maximizes this function is chosen as the threshold.
- *Otsu's thresholding* chooses the threshold to minimize the intraclass variance of the thresholded black and white pixels.
  - Formulated as *discriminant analysis*: a particular criterion function is used as a measure of statistical separation.

10.3.3. Optimum Global Thresholding Using Otsu's Method

- Segmentation is based on "region homogeneity".
  - Region homogeneity can be measured using variance (i.e., regions with high homogeneity will have low variance).
- Otsu's method selects the threshold by minimizing the within-class variance.
- Maximizes the between-class variance

Preliminaries

$$\sum_{i=0}^{L-1} p_i = 1,\ p_i \geq 0 \qquad P_1(k) = \sum_{i=0}^{k} p_i \qquad P_2(k) = \sum_{i=k+1}^{L-1} p_i = 1 - P_1(k)$$

$$m_1(k) = \sum_{i=0}^{k} iP(i/C_1) = \sum_{i=0}^{k} iP(C_1/i)P(i)/P(C_1) = \frac{1}{P_1(k)}\sum_{i=0}^{k} ip_i$$

$$P(A/B) = P(B/A)P(A)/P(B)$$

$$m_2(k) = \sum_{i=k+1}^{L-1} iP(i/C_2) = \frac{1}{P_2(k)}\sum_{i=k+1}^{L-1} ip_i$$

$$m(k) = \sum_{i=0}^{k} ip_i \qquad m_G = \sum_{i=0}^{L-1} ip_i \qquad P_1 m_1 + P_2 m_2 = m_G$$

$$P_1 + P_2 = 1$$

Metric for Otsu's Method

- Normalized dimensionless metric to evaluate the goodness of the threshold k
- σ_G^2 is the global variance
- σ_B^2 is the between-class variance, measure of separability between classes

$$\eta = \frac{\sigma_B^2}{\sigma_G^2} \qquad \sigma_G^2 = \sum_{i=0}^{L-1} (i - m_G)^2 p_i$$

$$\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2$$

$$\sigma_B^2 = P_1 P_2 (m_1 - m_2)^2 = \frac{(m_G P_1 - m)^2}{P_1(1 - P_1)}$$

Optimum threshold for Otsu's Method

- For all integer values of k, evaluate the measure η

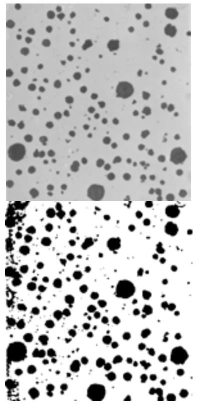
$$\eta(k) = \frac{\sigma_B^2(k)}{\sigma_G^2} \qquad \sigma_B^2(k) = \frac{\left[m_G P_1(k) - m(k)\right]^2}{P_1(k)\left[1 - P_1(k)\right]}$$

$$\sigma_B^2(k^*) = \max_{0 \le k \le L-1} \sigma_B^2(k) \qquad 0 \le \eta(k^*) \le 1$$

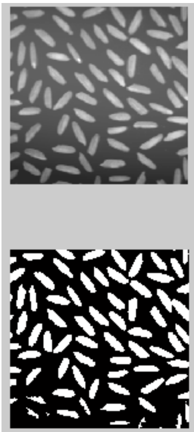
Matlab function for Otsu's method

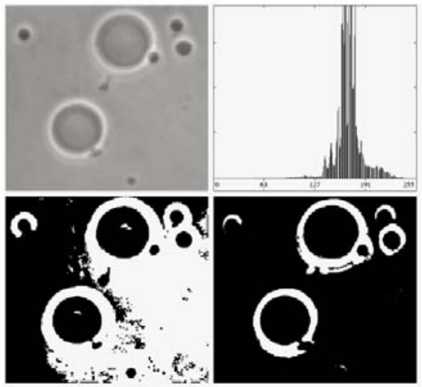
- function level = graythresh(I)
- GRAYTHRESH Compute global threshold
- using Otsu's method. Level is a normalized
- intensity value that lies in the range [0, 1].

```
>>n=imread('nodules1.tif');
>> tn=graythresh(n)
tn = 0.5804
>> imshow(im2bw(n,tn))
```



```
r=imread('rice.tif');
graythresh;
subplot(211);
imshow;
subplot(212);
imshow(im2bw(r,tr))
```





a b
c d

FIGURE 10.39 (a) Original image. (b) Histogram (high peaks were clipped to highlight details in the lower values). (c) Segmentation result using the basic global algorithm from Section 10.3.2. (d) Result obtained using Otsu's method. (Original image courtesy of Professor Daniel A. Hammer, the University of Pennsylvania.)

Drawbacks of the Otsu's method

- The method assumes that the histogram of the image is bimodal (i.e., two classes).
- The method breaks down when the two classes are very unequal (i.e., the classes have very different sizes)
  - In this case, may have two maxima.
  - The correct maximum is not necessary the global one.
- The method does not work well with variable illumination.

Multiple Thresholds

- Otsu's method can be extended to a multiple thresholding method
- Between-class variance can be reformulated as

$$\sigma_B^2 = \sum_{k=1}^{K} P_k (m_k - m_G)^2 \qquad (10.3\text{-}21)$$

$$P_k = \sum_{i \in C_k} p_i \qquad (10.3\text{-}22)$$

$$m_k = \frac{1}{P_k} \sum_{i \in C_k} i p_i \qquad (10.3\text{-}23)$$

Multiple Thresholds

- The K classes are separated by K-1 thresholds and these optimal thresholds can be solved by maximizing

$$\sigma_B^2(k_1^*, k_2^*, \ldots, k_{K-1}^*) = \max_{0 \le k_1 \le k_2 \le \ldots k_{n-1} \le L-1} \sigma_B^2(k_1, k_2, \ldots, k_{K-1})$$

For example (two thresholds)

$$\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2 + P_3(m_3 - m_G)^2 \qquad (10.3\text{-}25)$$

$$P_1 = \sum_{i=0}^{k_1} p_i, \quad P_2 = \sum_{i=k_1+1}^{k_2} p_i, \quad P_3 = \sum_{i=k_2+1}^{L-1} p_i \qquad (10.3\text{-}26)$$

$$m_1 = \frac{1}{P_1} \sum_{i=0}^{k_1} i p_i, \quad m_2 = \frac{1}{P_2} \sum_{i=k_1+1}^{k_2} i p_i, \quad m_3 = \frac{1}{P_3} \sum_{i=k_2+1}^{L-1} i p_i \qquad (10.3\text{-}27)$$

Multiple Thresholds

- The following relationships hold:

$$P_1 m_1 + P_2 m_2 + P_3 m_3 = m_G \text{ , where } P_1 + P_2 + P_3 = 1 \qquad (10.3\text{-}28)$$

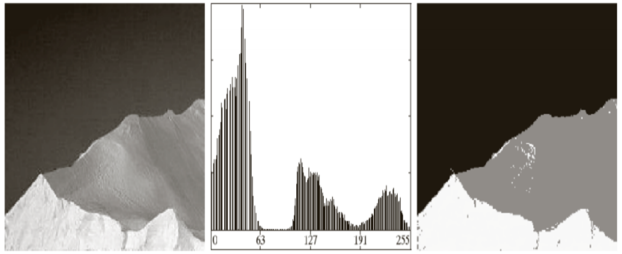
- The optimum thresholds can be found by :

$$\sigma_B^2(k_1^*, k_2^*) = \max_{0 \le k_1 \le k_2 \le L-1} \sigma_B^2(k_1, k_2) \qquad (10.3\text{-}30)$$

- The image is then segmented by

$$g(x,y) = \begin{cases} a, & \text{if } f(x,y) \le k_1^* \\ b, & \text{if } k_1^* < f(x,y) \le k_2^* \\ c, & \text{if } f(x,y) > k_2^* \end{cases} \qquad \eta(k_1^*, k_2^*) = \frac{\sigma_B^2(k_1^*, k_2^*)}{\sigma_G^2}$$

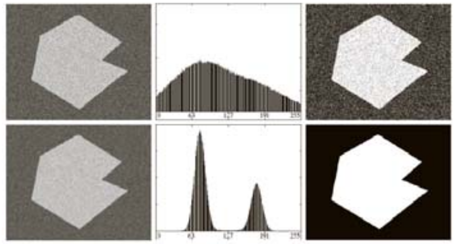
(10.3-32)



a b c

FIGURE 10.45 (a) Image of iceberg. (b) Histogram. (c) Image segmented into three regions using dual Otsu thresholds. (Original image courtesy of NOAA.)

10.3.4 Image smoothing to improve global thresholding



a b c
d e f

FIGURE 10.40 (a) Noisy image from Fig. 10.36 and (b) its histogram. (c) Result obtained using Otsu's method. (d) Noisy image smoothed using a 5 × 5 averaging mask and (e) its histogram. (f) Result of thresholding using Otsu's method.

10.3.5. Using Edges to improve Global thresholding

- Improving the shape of the histogram : consider only pixels around the edges between the object and the background
- Histogram would be less dependent on the relative size of the object, and also it improves the symmetry and deepens the valley

10.3.5. Using Edges to improve Global thresholding

- The chances of selecting a good threshold are increased if the histogram peaks are:
  - Tall
  - Narrow
  - Symmetric
  - Separated by deep valleys
- One way to improve the shape of histograms is to consider only those pixels that lie on or near the boundary between objects and the background.
  - Thus, histograms would be less dependent on the relative sizes of objects and the background.

Threshold Selection Based on Boundary Characteristics

- Problem:
  - The assumption that the boundary between objects and background is known.

- Solution:
  - An indication of whether a pixel is on an edge may be computed by its gradient.
  - The Laplacian yields information on whether a pixel lies on the dark or light side of an edge.
  - The average value of the Laplacian is 0 at the transition of an edge, so deep valleys are produced in the histogram.

Threshold Selection Based on Boundary Characteristics

- In essence:

$$s(x,y)=\begin{cases} 0 & \text{if} \quad \nabla f < T \\ + & \text{if} \quad \nabla f \geq T \quad \text{and} \quad \nabla^2 f \geq 0 \\ - & \text{if} \quad \nabla f \geq T \quad \text{and} \quad \nabla^2 f < 0 \end{cases}$$

- In the image $s(x,y)$:
  - pixels that are not on an edge are labeled 0
  - pixels on the dark side of an edge are labeled +
  - pixels on the light side of an edge are labeled –

- Light background/dark object:

(…) (-,+) (0 or +) (+,-) (…)

0 1 0

Threshold Selection Based on Boundary Characteristics

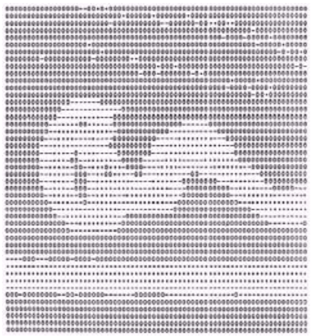


FIGURE 10.36 Image of a handwritten stroke coded by using Eq. (10.3-16). (Courtesy of IBM Corporation.)

Threshold Selection Based on Boundary Characteristics

a
b

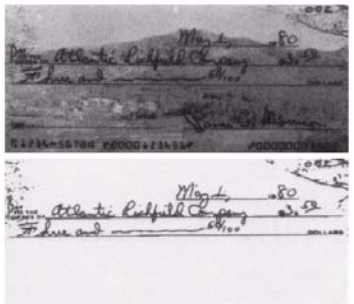


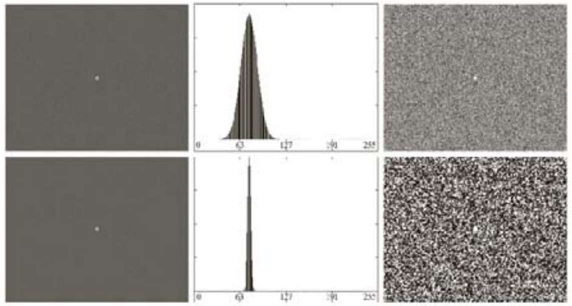
FIGURE 10.37 (a) Original image. (b) Image segmented by local thresholding. (Courtesy of IBM Corporation.)

10.3.5. Using Edges to improve Global thresholding

The preceding discussion is summarized in the following algorithm, where $f(x, y)$ is the input image:

1. Compute an edge image as either the magnitude of the gradient, or absolute value of the Laplacian, of $f(x, y)$ using any of the methods discussed in Section 10.2.
2. Specify a threshold value, T.
3. Threshold the image from Step 1 using the threshold from Step 2 to produce a binary image, $g_T(x, y)$. This image is used as a *mask image* in the following step to select pixels from $f(x, y)$ corresponding to "strong" edge pixels.
4. Compute a histogram using only the pixels in $f(x, y)$ that correspond to the locations of the 1-valued pixels in $g_T(x, y)$.
5. Use the histogram from Step 4 to segment $f(x, y)$ globally using, for example, Otsu's method.

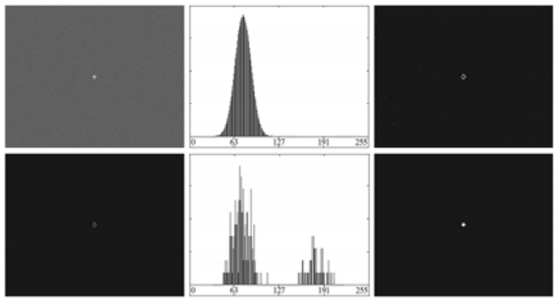
10.3.4 Image smooting to improve global thresholding



a b c
d e f

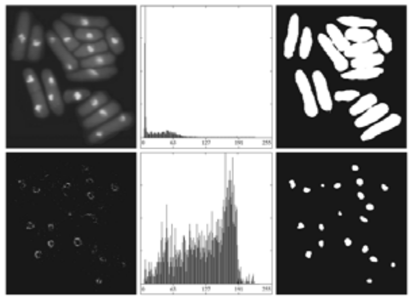
FIGURE 10.41 (a) Noisy image and (b) its histogram. (c) Result obtained using Otsu's method. (d) Noisy image smoothed using a 5×5 averaging mask and (e) its histogram. (f) Result of thresholding using Otsu's method. Thresholding failed in both cases.

Using Edges to improve Global thresholding



a b c
d e f

FIGURE 10.42 (a) Noisy image from Fig. 10.41(a) and (b) its histogram. (c) Gradient magnitude image thresholded at the 99.7 percentile. (d) Image formed as the product of (a) and (c). (e) Histogram of the nonzero pixels in the image in (d). (f) Result of segmenting image (a) with the Otsu threshold based on the histogram in (e). The threshold was 134, which is approximately midway between the peaks in this histogram.



$$\eta = \frac{\sigma_B^2(k)}{\sigma_G^2}$$

99.5%, Th=115, η=0.762

a b c
d e f

FIGURE 10.43 (a) Image of yeast cells. (b) Histogram of (a). (c) Segmentation of (a) with Otsu's method using the histogram in (b). (d) Thresholded absolute Laplacian. (e) Histogram of the nonzero pixels in the product of (a) and (d). (f) Original image thresholded using Otsu's method based on the histogram in (e). (Original image courtesy of Professor Susan L. Forsburg, University of Southern California.)

FIGURE 10.44 Image in Fig. 10.43(a) segmented using the same procedure as explained in Figs. 10.43(d)–(f), but using a lower value to threshold the absolute Laplacian image.

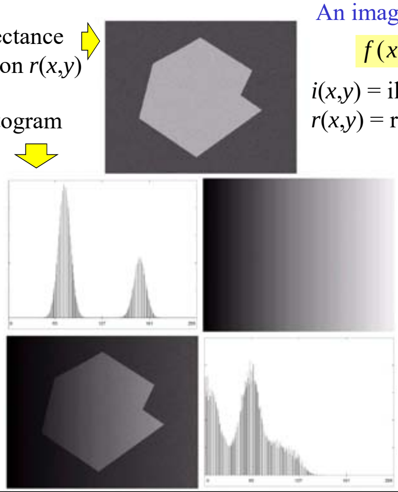
95%

Nonuniform Illumination Problem

An image can be expressed as

$$f(x,y) = i(x,y)r(x,y)$$

$i(x,y)$ = illumination component
$r(x,y)$ = reflectance component



Reflectance Function $r(x,y)$

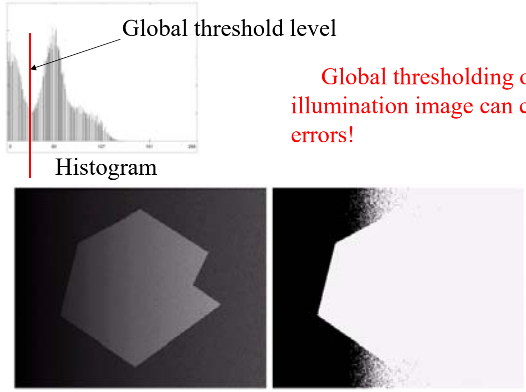
Histogram

Illumination Function $i(x,y)$

Image histogram

$f(x,y)$

Nonuniform Illumination and Global Thresholding



Global threshold level

Histogram

Global thresholding of nonuniform illumination image can cause huge errors!

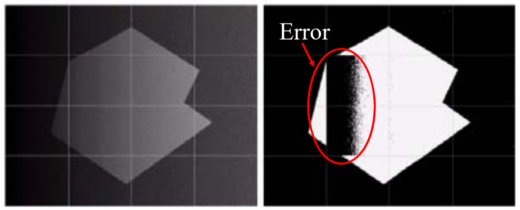
Nonuniform illumination image

Global thresholding result

Nonuniform Illumination and Local Thresholding

Local thresholding:

1. Divide an image into subimages.
2. Threshold each subimage independently
   2.1 Compute histogram of each subimage and select a suitable threshold value for each subimage
   2.2 threshold each subimage using a threshold value in 2.1
   2.3 Combine all local thresholding results



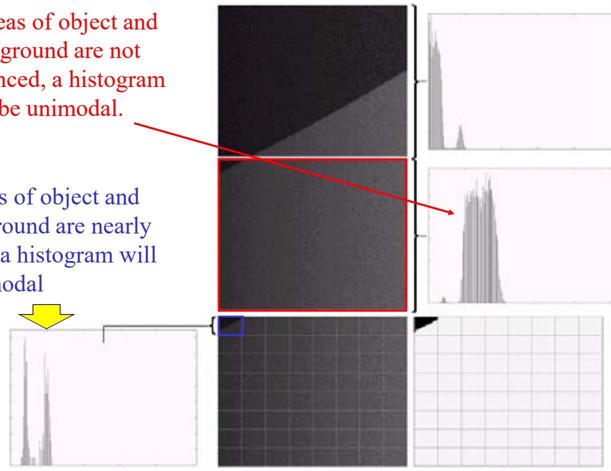
16 subimages

Result of local thresholding

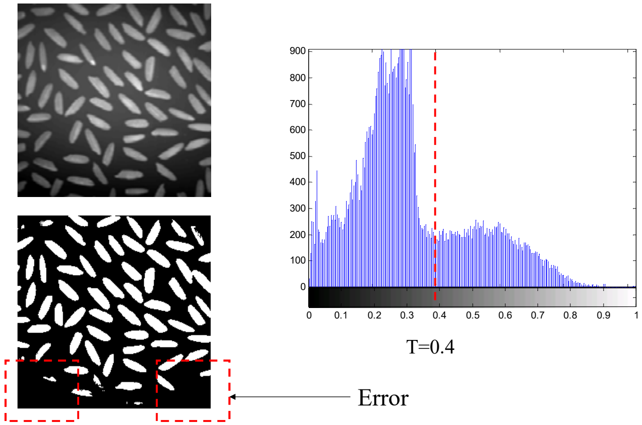
Histogram of Subimages and Local Thresholding

If areas of object and background are not balanced, a histogram will be unimodal.

If areas of object and background are nearly equal, a histogram will be bimodal



Example of Nonuniform Illumination Problem



10.3.6 Variable thresholding using local statistics

- Local properties (e.g., statistics) based criteria can be used for adapting the threshold. For example:

$$T_{xy} = a\sigma_{xy} + bm_{xy}$$
$$T_{xy} = a\sigma_{xy} + bm_G$$

- The segmentation is operated using a suitable predicate, Qxy :

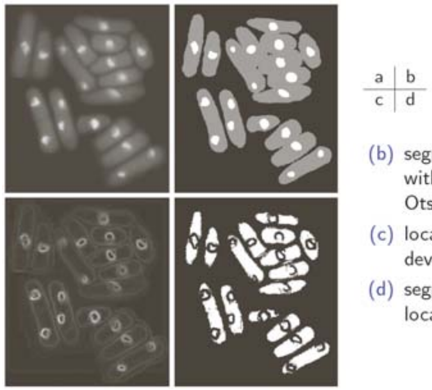
$$g(x, y) = \begin{cases} 1, & \text{if } Q_{xy} \\ 0, & \text{otherwise} \end{cases}$$

where Qxy can be, for instance:

  – This technique can be easily generalized to multiple thresholds segmentation.

$$f(x, y) > T_{xy}$$
$$f(x, y) > a\sigma_{xy} \text{ AND } f(x, y) > bm_{xy}$$

Local Thresholding



(b) segmentation of (a) with double threshold Otsu;
(c) local (3×3) standard deviation;
(d) segmentation with local thresholding.

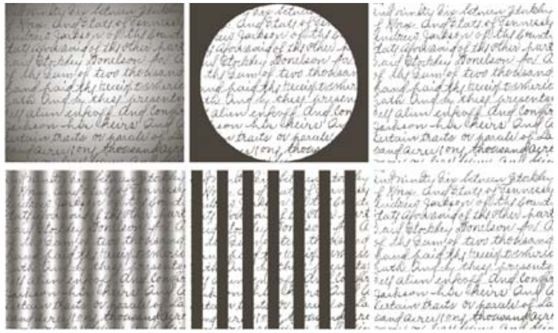
10.3.7 Using Moving Averages

- It is based on computing a moving average along scan lines of an image.

$$m(k+1) = \frac{1}{n}\sum_{i=k+2-n}^{k+1} z_i = m(k) + \frac{1}{n}(z_{k+1} - z_{k-n})$$

- z_{k+1} denote the intensity of the point at step k+1. n denote the number of point used in the averaging.
- $m(1) = z_1/n$ is the initial value.
- $T_{xy} = bm_{xy}$, where b is constant and m_{xy} is the moving average at point (x,y).

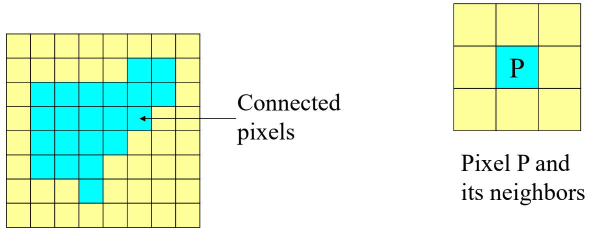
Moving averages thresholding



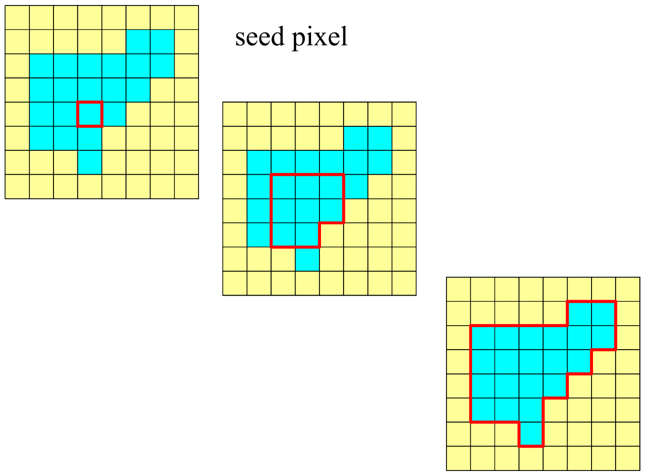
Region Oriented Image Segmentation

Region oriented image segmentation

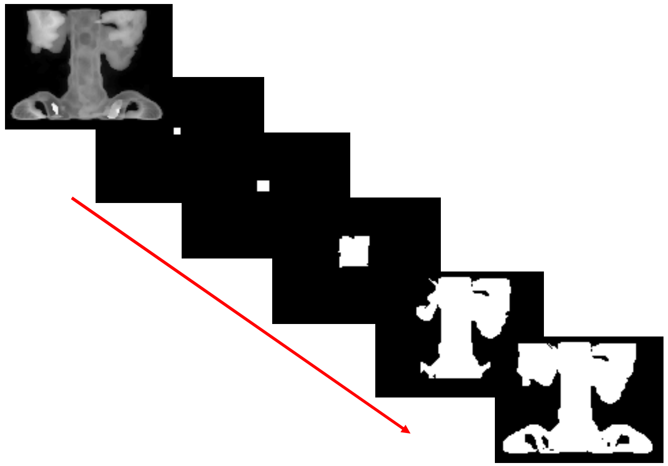
1. Region Growing
2. Region Splitting and Merging



Region Growing Algorithm

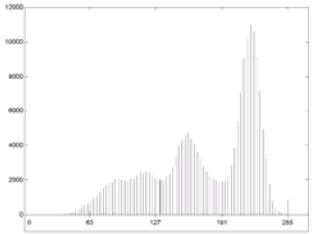


Region Growing Image Segmentation Example



Region Growing Image Segmentation Example

X-ray image of defective weld

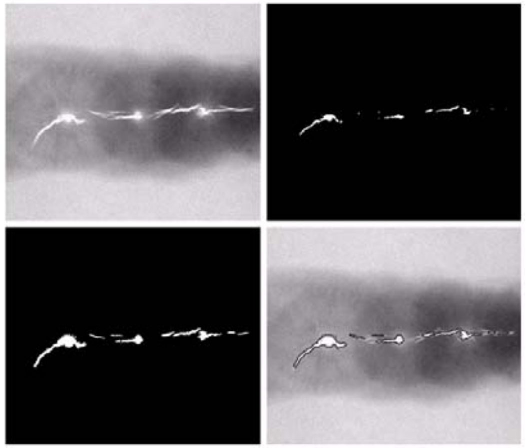


Histogram

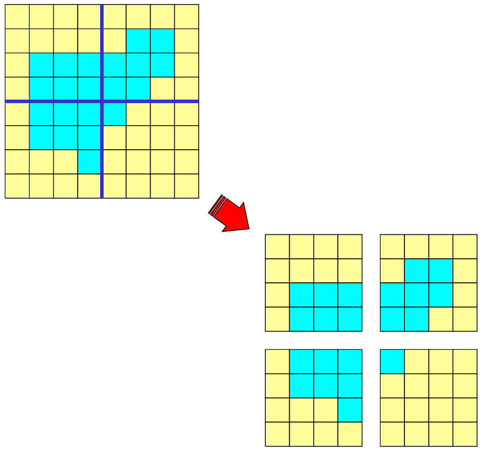
Region Growing Image Segmentation Example

a b
c d

FIGURE 10.40 (a) Image showing defective welds. (b) Seed points. (c) Result of region growing. (d) Boundaries of segmented defective welds (in black). (Original image courtesy of X-TEK Systems, Ltd.).



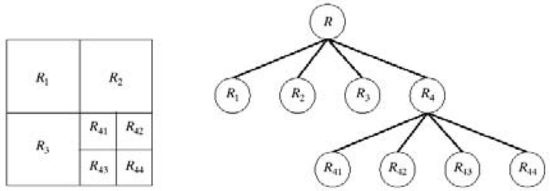
Region Splitting and Merging Algorithm



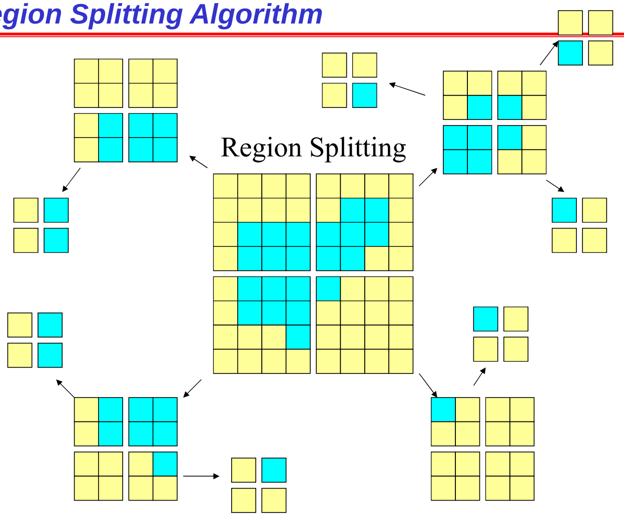
Quadtree for Region Splitting Representation

a b

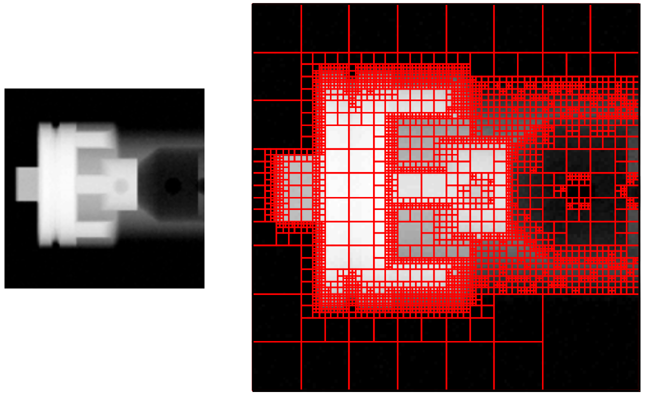
FIGURE 10.42 (a) Partitioned image. (b) Corresponding quadtree.



Region Splitting Algorithm



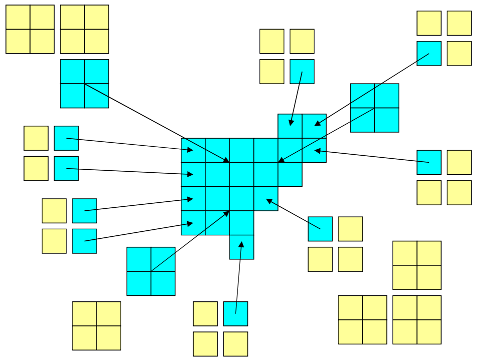
Region Splitting Example



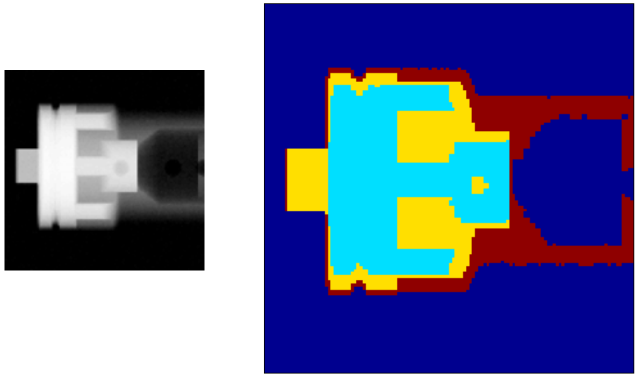
Region Spliting
standard deviation 5

Region Merging Algorithm

2. Merging

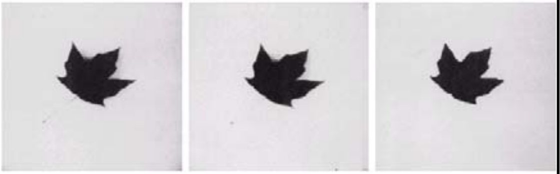


Region Merging Example



Region Merging

Region Splitting and Merging Example



a b c

FIGURE 10.43 (a) Original image. (b) Result of split and merge procedure. (c) Result of thresholding (a).