Digital Image Processing
Chapter 9:
Morphological Image Processing

What are Morphological Operations?

Morphology : a branch of biology that deals with the form and the structure of animals and plants

Mathematical morphology : a tool for extracting image components that are useful in the representation and description of region shape, such as boundaries, sleketons, and the convex hull.

The language of mathematical morphology is *set theory* :
Sets represent *objects* in an image
Ex) the set of all black pixels in a binary image, the set is in 2-D integer space Z^2

What are Morphological Operations?

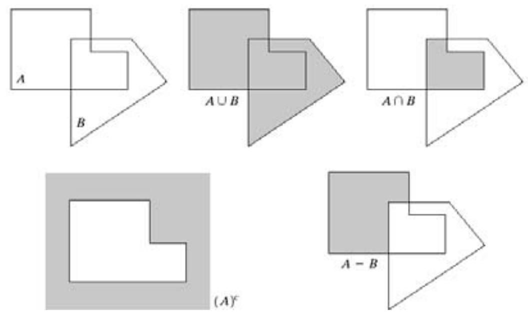
Morphological operations come from the word "morphing" in Biology which means "changing a shape".

Image morphological operations are used to manipulate object shapes such as thinning, thickening, and filling.

Binary morphological operations are derived from set operations.

Basic Set Operations

Concept of a set in binary image morphology:
Each set may represent one object. Each pixel (x,y) has its status: *belong to a set* or *not belong to a set*.

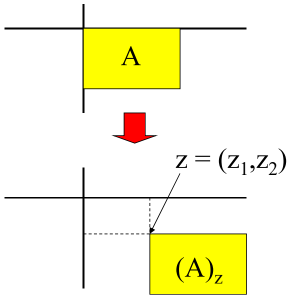


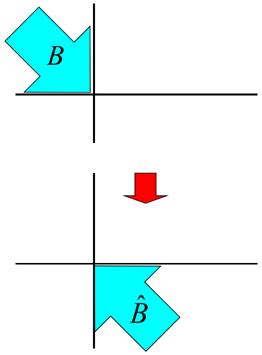
a b c
d e

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Translation and Reflection Operations

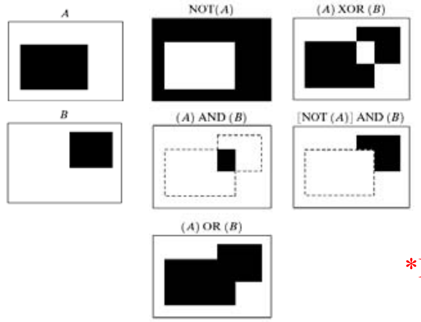
Translation	Reflection
$(A)_z = \{c \mid c = a + z, \ \text{for } a \in A\}$	$\hat{B} = \{w \mid w = -b, \ \text{for } b \in B\}$





Logical Operations*

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0



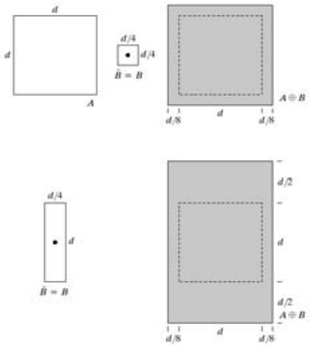
*For binary images only

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Dilation Operations

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \phi\},$$

where $(\hat{B})_z$ is the reflection of B with shift z



ϕ = Empty set

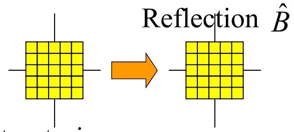
Dilate means "extend"

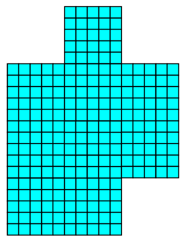
A = Object to be dilated

B = Structuring element

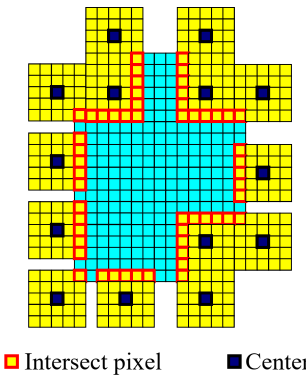
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Dilation Operations (cont.)





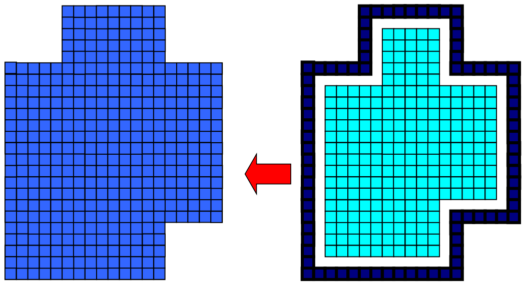
Original image (A)



Intersect pixel

Center pixel

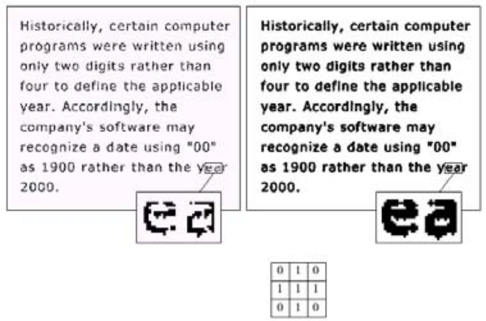
Dilation Operations (cont.)



Result of Dilation

Boundary of the “center pixels” where $(\hat{B})_z$ intersects A

Example: Application of Dilation



a c
b

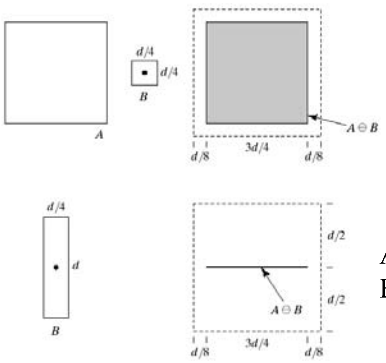
FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

“Repair” broken characters

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Erosion Operation

$$A \ominus B = \left\{ z \middle| (\hat{B})_z \subseteq A \right\}$$



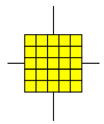
Erosion means “trim”

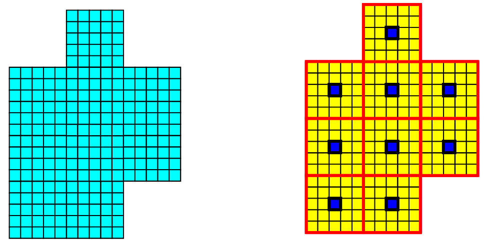
A = Object to be eroded
B = Structuring element

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Erosion Operations (cont.)

Structuring Element (B)

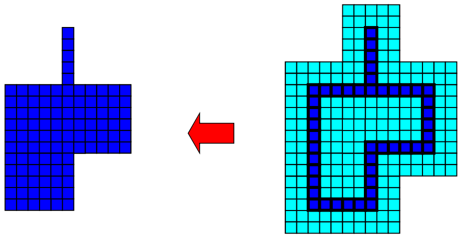




Original image (A)

Intersect pixel ■ Center pixel

Erosion Operations (cont.)



Result of Erosion

Boundary of the "center pixels" where B is inside A

Example: Application of Dilation and Erosion

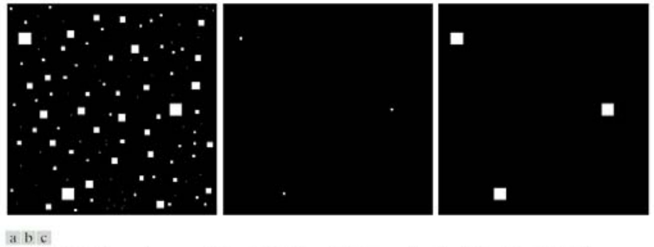


FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Remove small objects such as noise

Duality Between Dilation and Erosion

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

where c = complement

Proof:

$$\begin{aligned}(A \ominus B)^c &= \{z \mid (B)_z \subseteq A\}^c \\ &= \{z \mid (B)_z \cap A^c = \phi\}^c \\ &= \{z \mid (B)_z \cap A^c \neq \phi\} \\ &= A^c \oplus \hat{B}\end{aligned}$$

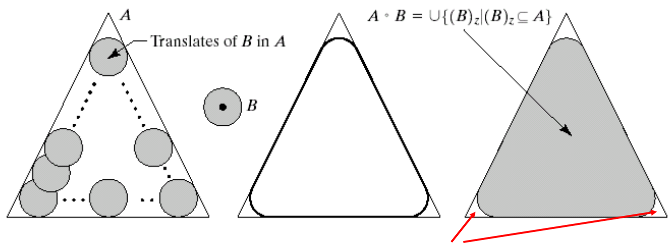
Opening Operation

$$A \circ B = (A \ominus B) \oplus B$$

or

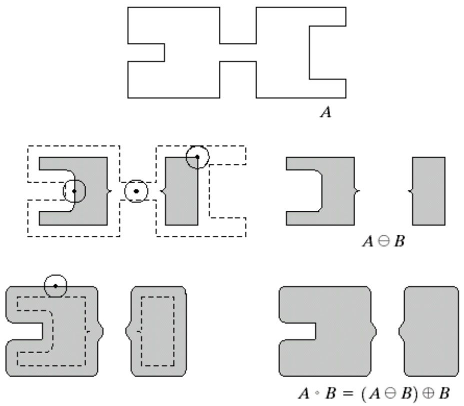
$$A \circ B = \cup\{(B)_z \mid (B)_z \subseteq A\}$$

= Combination of all parts of A that can completely contain B



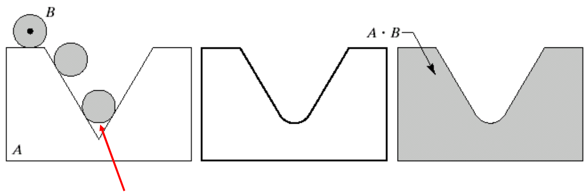
Opening eliminates narrow and small details and corners.

Example of Opening



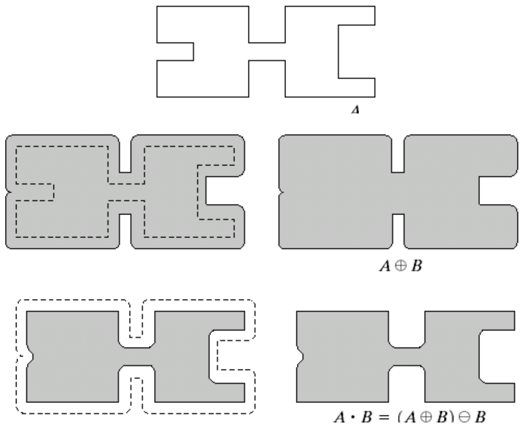
Closing Operation

$$A \bullet B = (A \oplus B) \ominus B$$



Closing fills narrow gaps and notches

Example of Closing



Duality Between Opening and Closing

$$(A \bullet B)^c = \left(A^c \circ \hat{B}\right)$$

Properties Opening

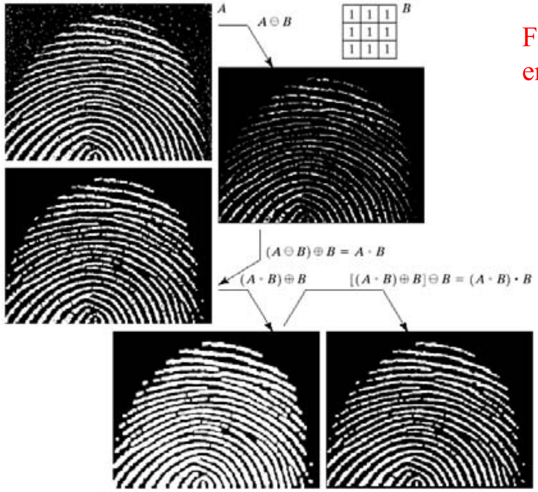
1. $A \circ B \subseteq A$
2. If $C \subset D$ then $C \circ B \subset D \circ B$
3. $(A \circ B) \circ B = A \circ B$

Properties Closing

1. $A \subseteq A \bullet B$
2. If $C \subset D$ then $C \bullet B \subset D \bullet B$
3. $(A \bullet B) \bullet B = A \bullet B$

Idempotent property: can't change any more

Example: Application of Morphological Operations



Finger print enhancement

FIGURE 9.11 (a) Noisy image. (b) Structuring element. (c) Eroded image. (d) Opening of A. (e) Dilation of the opening. (f) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Hit-or-Miss Transformation

$$A \circledast X = (A \ominus X) \cap [A^c \ominus (W - X)]$$

where X = shape to be detected, W = window that can contain X

Find the location of the shape X

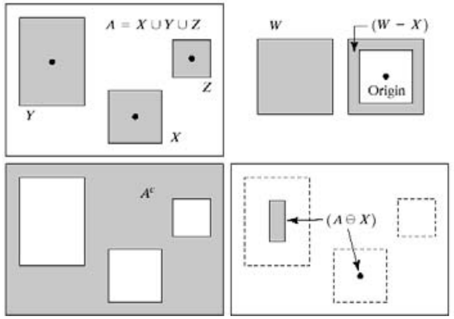
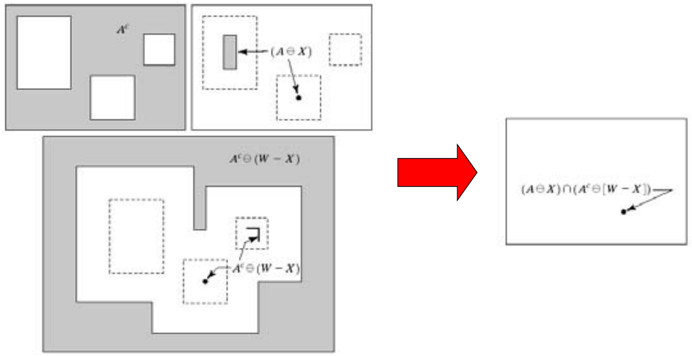


FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$. (c) Complement of A. (d) Erosion of A by X. (e) Erosion of A^c by $(W - X)$. (f) Intersection of (d) and (e), showing the location of the origin of X, as desired.

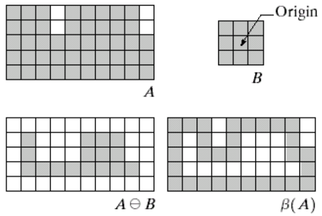
Hit-or-Miss Transformation (cont.)

$$A \circledast B = (A \ominus B) \cap [A^c \ominus (W - X)]$$



Boundary Extraction

$$\beta(A) = A - (A \ominus B)$$



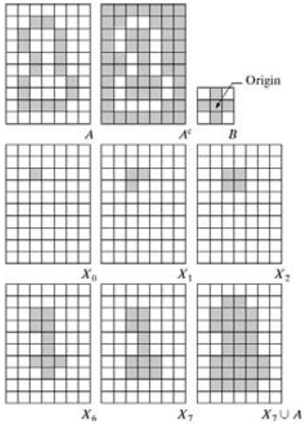
Original image

Boundary

Region Filling

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

where X_0 = seed pixel p



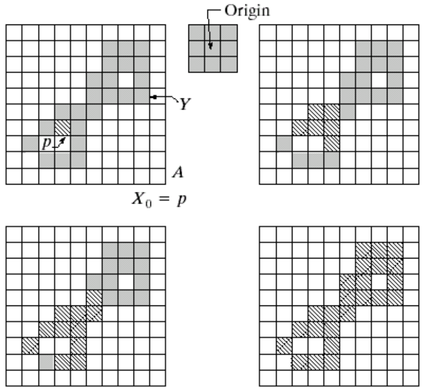
Original image

Results of region filling

Extraction of Connected Components

$$X_k = (X_{k-1} \oplus B) \cap A$$

where X_0 = seed pixel p

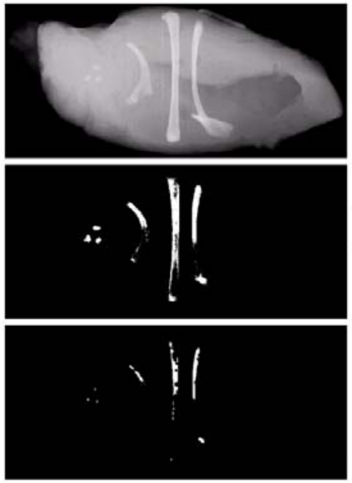


Example: Extraction of Connected Components

X-ray image of bones

Thresholded image

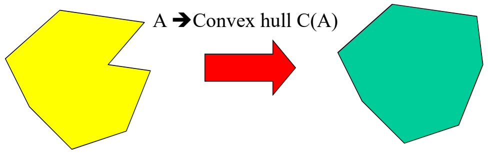
Connected components



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

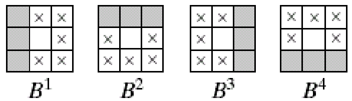
Convex Hull

Convex hull H=C(A) has no concave part.

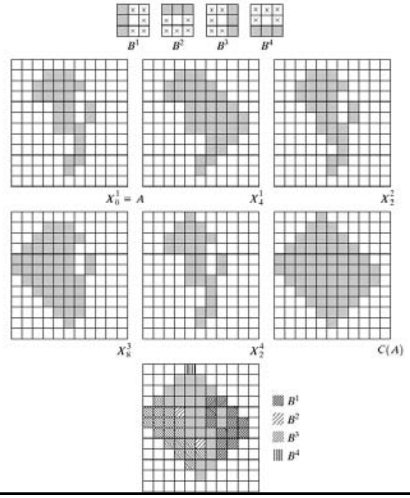


Algorithm: $$C(A) = \bigcup_{i=1}^{4} D^i$$ where $$D^i = X^i_{conv}$$

$$X_k^i = (X_{k-1} \circledast B^i) \cup A, \quad i = 1,2,3,4$$



Example: Convex Hull

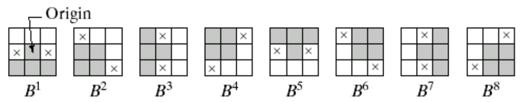


(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Thinning

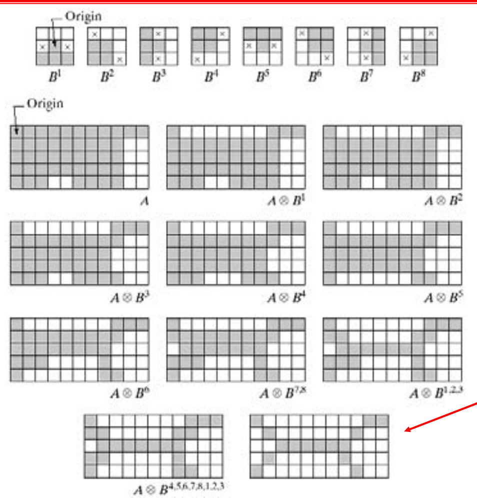
$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c$$

$$A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Example: Thinning



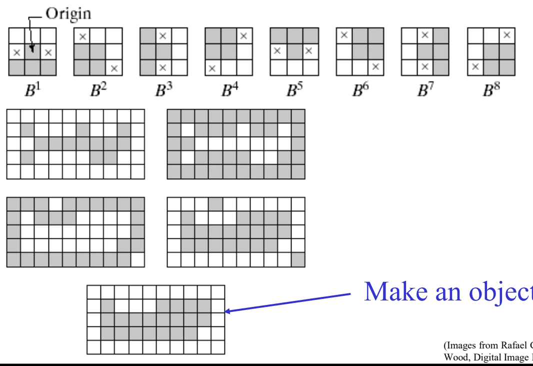
Make an object thinner.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Thickening

$$A \odot B = A \cup (A \circledast B)$$

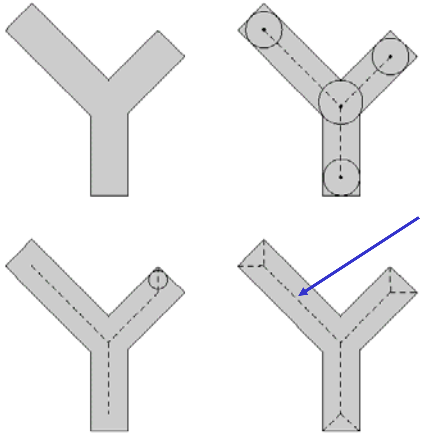
$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$



Make an object thicker

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Skeletons



Dot lines are skeletons of this structure

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Skeletons (cont.)

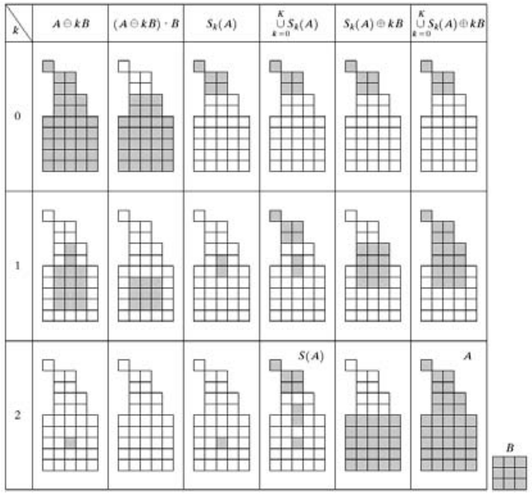
$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with $S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$

where $(A \ominus kB) = \underbrace{(...(A \ominus B) \ominus B) \ominus ...) \ominus B}_{k \text{ times}}$

and $K = \max\{k | (A \ominus kB) \neq \phi\}$

Skeletons



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

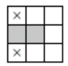
Pruning

$X_1 = A \otimes \{B\}$ = thinning

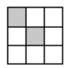
$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ = finding end points

$X_3 = (X_2 \oplus H) \cap A$ = dilation at end points

$X_4 = X_1 \cup X_3$ = Pruned result



B^1, B^2, B^3, B^4 (rotated 90°)



B^5, B^6, B^7, B^8 (rotated 90°)

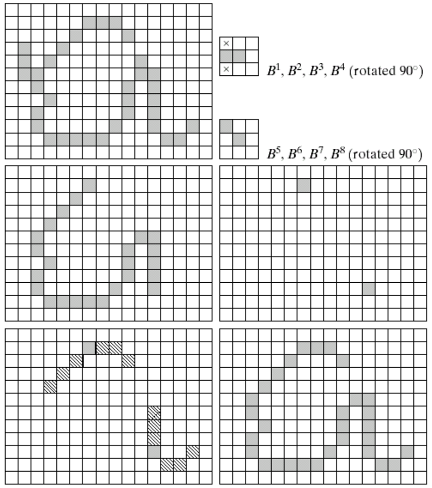
(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Example: Pruning

Original image

After Thinning 3 times

Dilation of end points



End points

Pruned result

Summary of Binary Morphological Operations

TABLE 9.2
Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \text{ for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

(Tables from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Summary of Binary Morphological Operations (cont.)

Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \ldots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$.	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

(Tables from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Summary of Binary Morphological Operations (cont.)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} = ((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} = ((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

TABLE 9.2
Summary of morphological results and their properties. *(continued)*

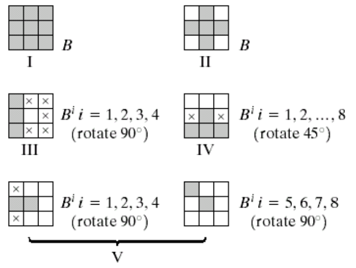
(Tables from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Summary of Binary Morphological Operations (cont.)

Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.

(Tables from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Basic Types of Structuring Elements



x = *don't care*

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Gray-Scale Dilation

1-D Case $f \oplus b = \max\{f(s-x) + b(x) \mid (s-x) \in D_f \text{ and } x \in D_b\}$

2-D Case $f \oplus b = \max\{f(s-x, t-y) + b(x,y) \mid (s-x),(t-y) \in D_f; (x,y) \in D_b\}$

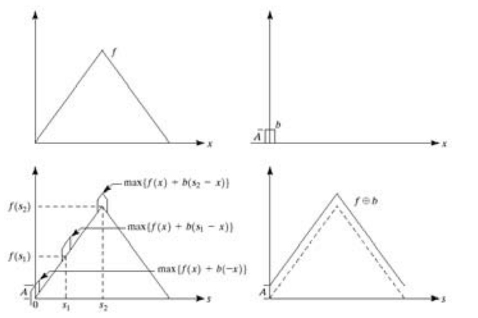
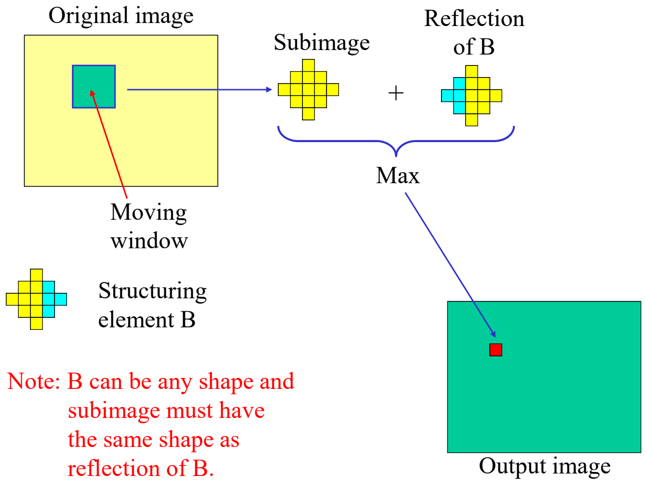


FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).

Gray-Scale Dilation (cont.)



Note: B can be any shape and subimage must have the same shape as reflection of B.

Gray-Scale Erosion

1-D Case

$$f \ominus b = \min\{f(s+x) - b(x) \mid (s+x) \in D_f \text{ and } x \in D_b\}$$

2-D Case

$$f \ominus b = \min\{f(s+x, t+y) - b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b\}$$

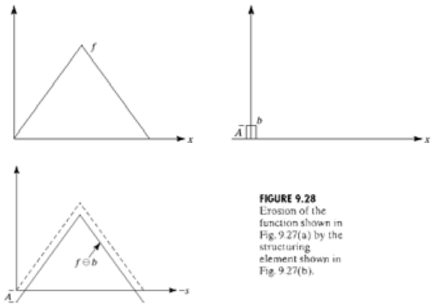
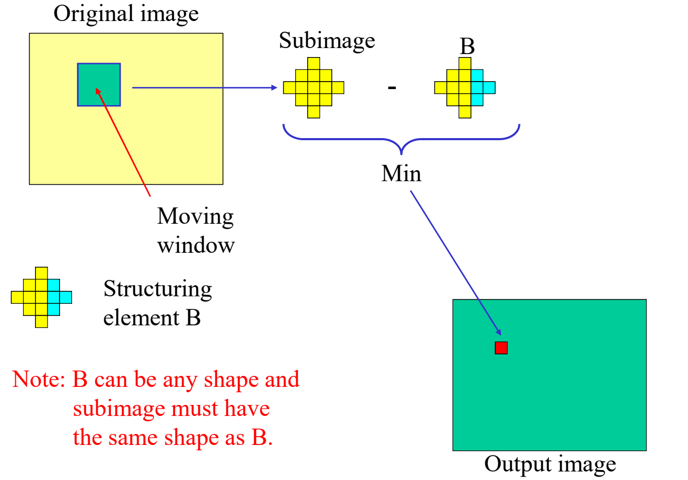
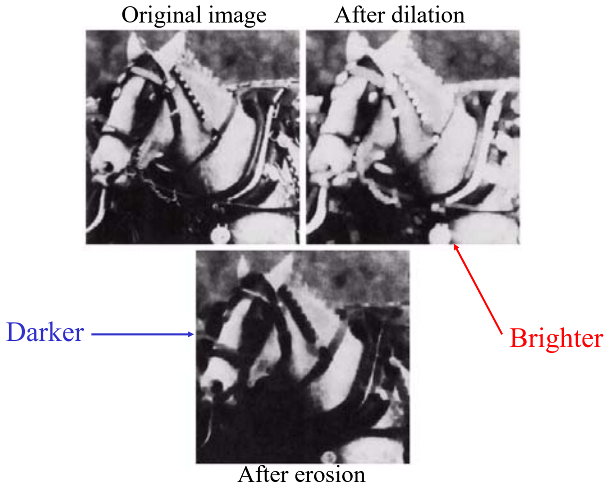


FIGURE 9.28 Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).

Gray-Scale Erosion (cont.)

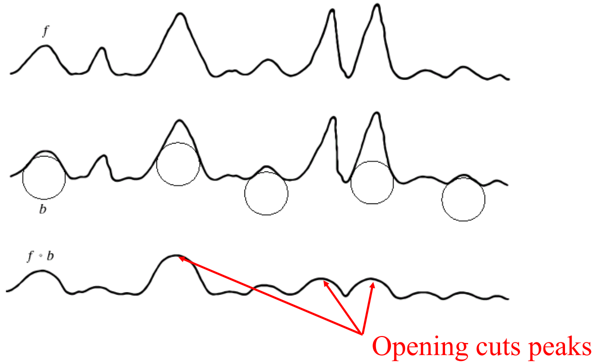


Example: Gray-Scale Dilation and Erosion



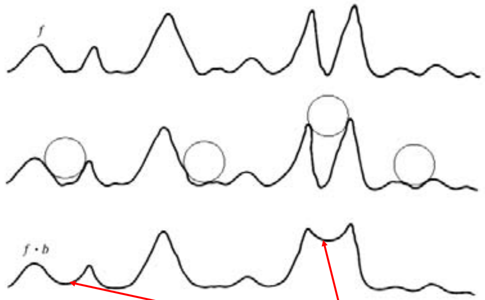
Gray-Scale Opening

$$f \circ b = (f \ominus b) \oplus b$$



Gray-Scale Closing

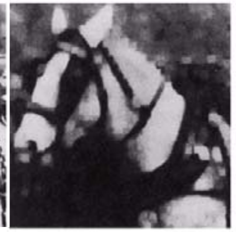
$$f \bullet b = (f \oplus b) \ominus b$$



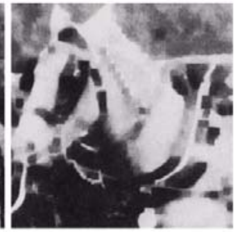
Closing fills valleys

Example: Gray-Scale Opening and Closing

Original image



After opening



After closing

Reduce white objects

Reduce dark objects

Gray-scale Morphological Smoothing

Smoothing: Perform opening followed by closing

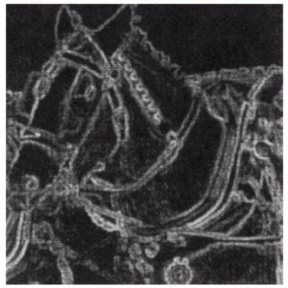
Original image

After smoothing

Morphological Gradient

$$g = (f \oplus b) - (f \ominus b)$$

Original image



Morphological Gradient

Top-Hat Transformation

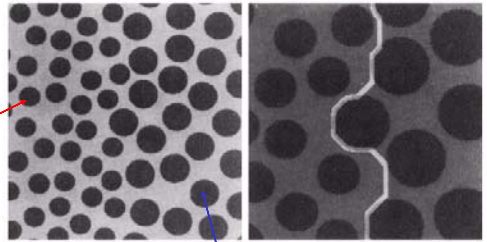
$$h = f - (f \circ b)$$

Original image

Results of top-hat transform

Example: Texture Segmentation Application

Small blob



Original image Segmented result

Large blob

Algorithm:

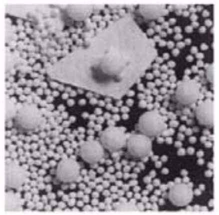
1. Perform closing on the image by using successively larger structuring elements until small blobs are vanished.
2. Perform opening to join large blobs together
3. Perform intensity thresholding

Example: Granulometry

Objective: to count the number of particles of each size

Method:

1. Perform opening using structuring elements of increasing size
2. Compute the difference between the original image and the result after each opening operation
3. The differenced image obtained in Step 2 are normalized and used to construct the size-distribution graph.



Original image

Size distribution graph

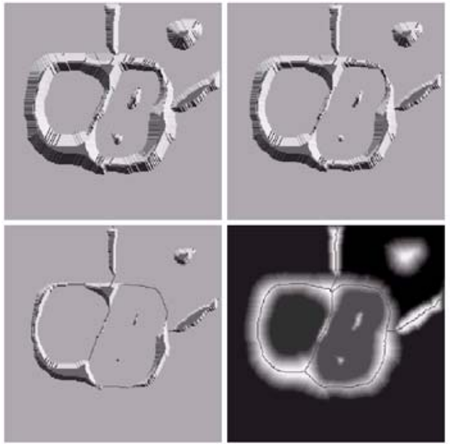
FIGURE 9.36
(a) Original image consisting of overlapping particles; (b) size distribution. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Morphological Watershads

a b
c d

FIGURE 10.44
(a) Original image. (b) Topographic view. (c)–(d) Two stages of flooding.

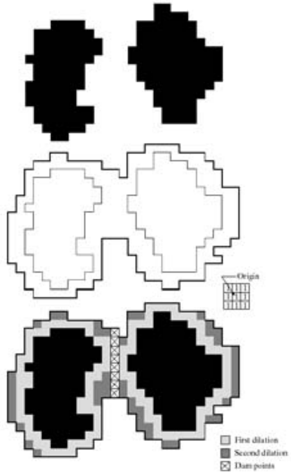
Morphological Watershads



c f
g h

FIGURE 10.44 *(Continued)* (e) Result of further flooding. (f) Beginning of merging of water from two catchment basins (a short dam was built between them). (g) Longer dams. (h) Final watershed (segmentation) lines. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

Morphological Watershads

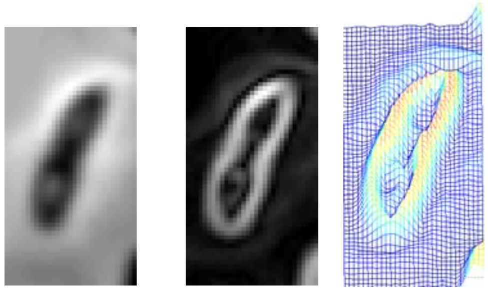


a
b c
d

FIGURE 10.45 (a) Two partially flooded catchment basins at stage $n-1$ of flooding. (b) Flooding at stage n, showing that water has spilled between basins (for clarity, water is shown in white rather than black). (c) Structuring element used for dilation. (d) Result of dilation and dam construction.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Gradient Image



Original image

$|\nabla P|$

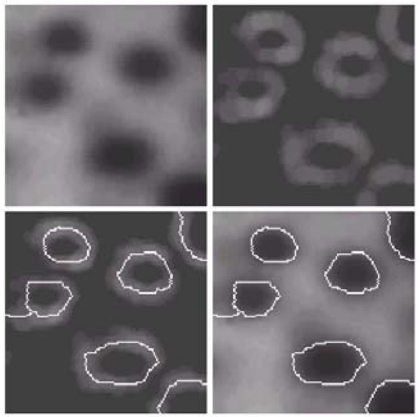
Surface of $|\nabla P|$

$|\nabla P|$ at edges look like mountain ridges.

Morphological Watershads

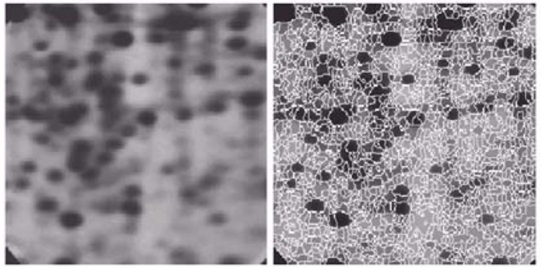
a b
c d

FIGURE 10.46 (a) Image of blobs. (b) Image gradient. (c) Watershed lines. (d) Watershed lines superimposed on original image. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)



(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

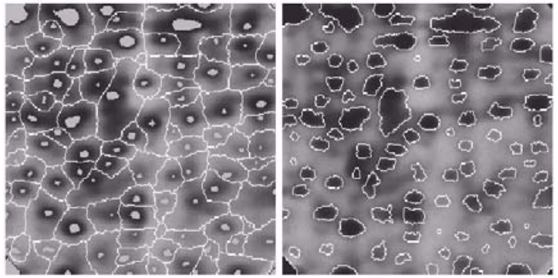
Morphological Watershads



a b
FIGURE 10.47 (a) Electrophoresis image. (b) Result of applying the watershed segmentation algorithm to the gradient image. Oversegmentation is evident. (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Morphological Watershads



a b
FIGURE 10.48 (a) Image showing internal markers (light gray regions) and external markers (watershed lines). (b) Result of segmentation. Note the improvement over Fig. 10.47(b). (Courtesy of Dr. S. Beucher, CMM/Ecole des Mines de Paris.)

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.

Convex Hull

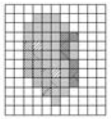


FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

(Images from Rafael C. Gonzalez and Richard E. Wood, Digital Image Processing, 2nd Edition.