

Digital Image Processing Introduction

Image restoration & Noise reduction

Reduction of Noise

Classification of **noise** is based upon:

- the shape of probability density function (analog case of noise)
- Histogram (discrete case of noise)

Uncorrelated noise is defined as the **random graylevel variations** within an image that have no spatial dependences from image to image

Typical image noise models

- Typical image noise models are
 - Uniform
 - Gaussian (normal)
 - Salt-and-Pepper (impulse)
 - Gamma noise
 - Rayleigh distribution
 - ...

Uniform noise

Uniform noise can be analytically described by:

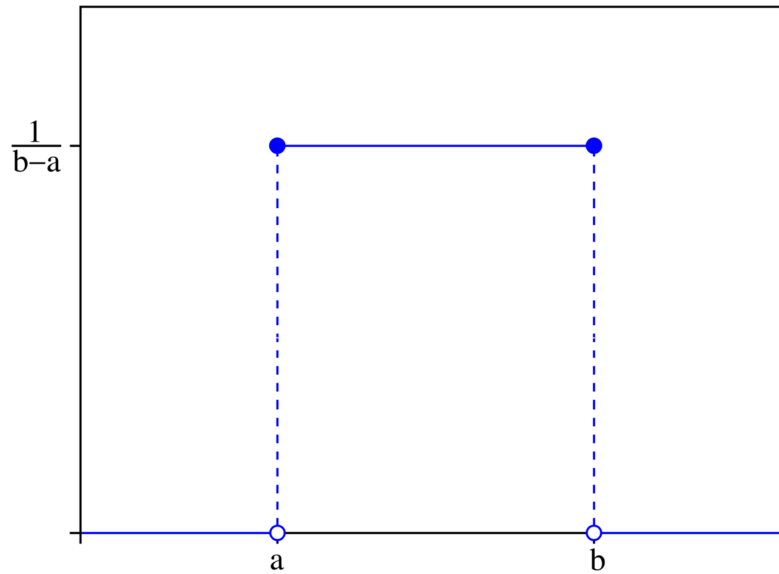
$$\text{HISTOGRAM}_{\text{Uniform}} = \begin{cases} \frac{1}{b-a} & \text{for } a \leq g \leq b \\ 0 & \text{elsewhere} \end{cases}$$

- The gray level values of the noise are evenly distributed across a specific range
- Quantization noise has an approximately uniform distribution

$$\text{mean} = \frac{a+b}{2}$$

$$\text{variance} = \frac{(b-a)^2}{12}$$

Histogram of Uniform Noise



Example of Uniform Noise

Original image:

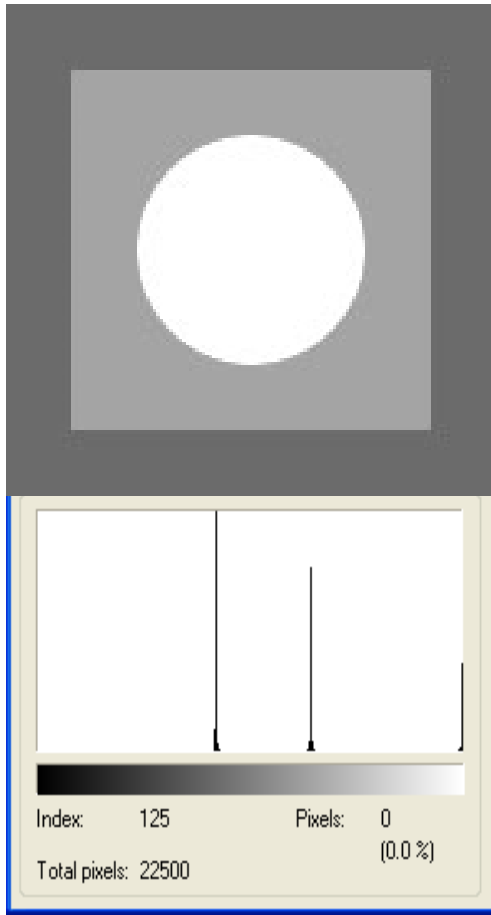
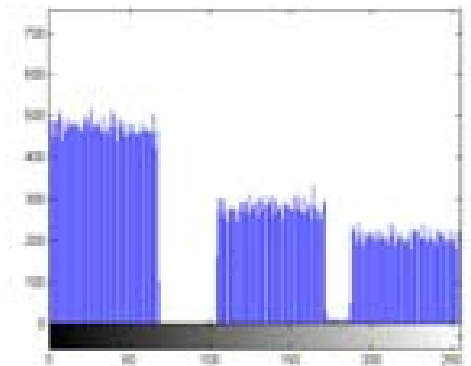
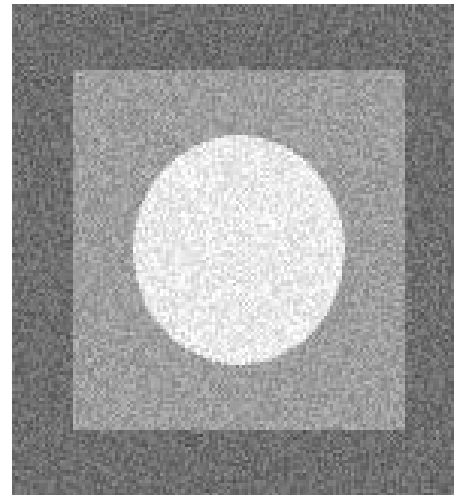


image disturbed by uniform noise:



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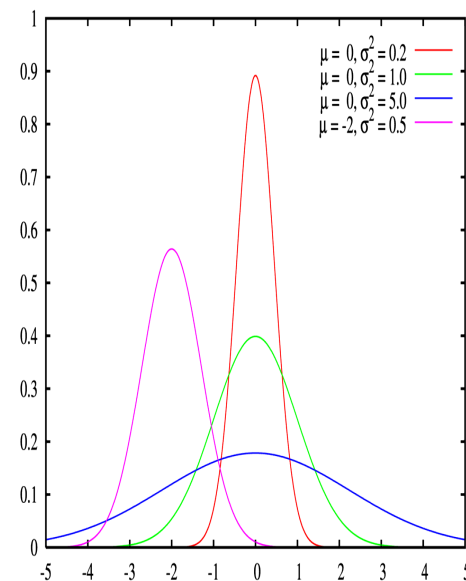
Uniform noise

- Uniform noise can be used to generate any other type of noise distribution, and is often used to degrade images for the evaluation of image restoration algorithms since it provides the most unbiased or neutral noise model

Gaussian noise (Amplifier noise)

- ... is statistical noise that has a probability density function (pdf) of the normal distribution (also known as Gaussian distribution).
- ...is a major part of the "read noise" of an image sensor, that is, of the constant noise level in dark areas of the image.

PDF (Propability density function)



Example of Gaussian Noise

Original image:

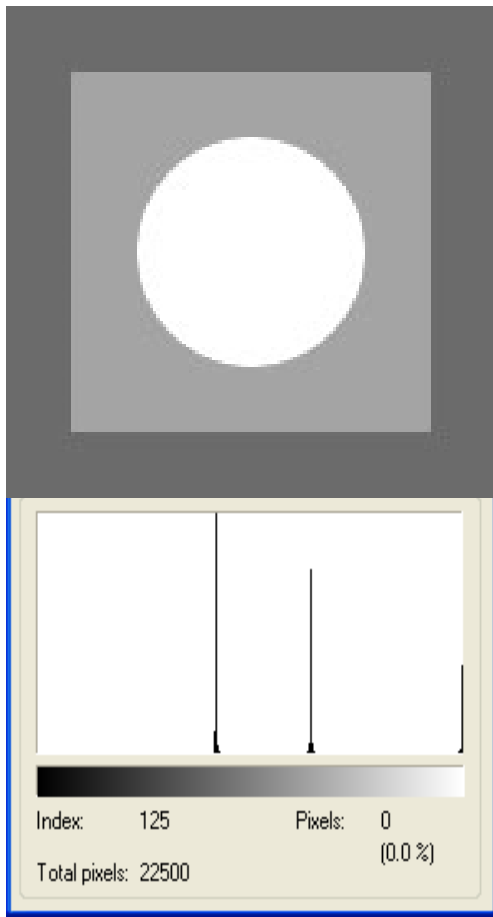
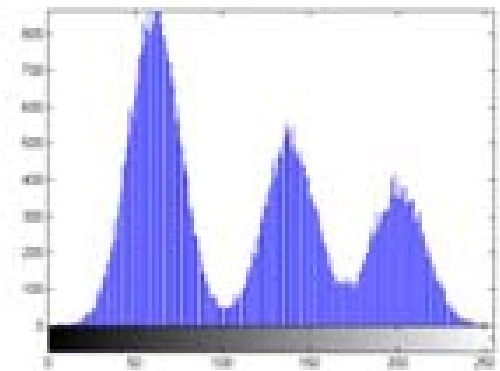
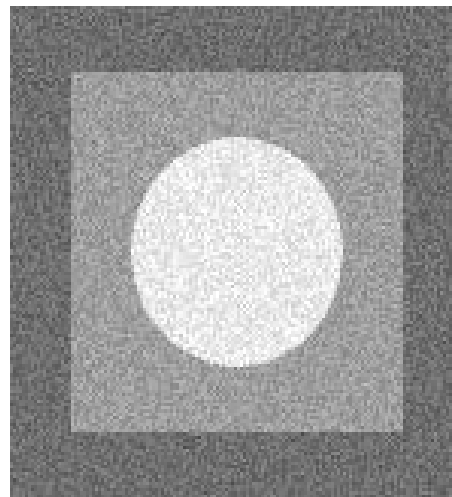


image disturbed by uniform noise:



Salt and Pepper noise

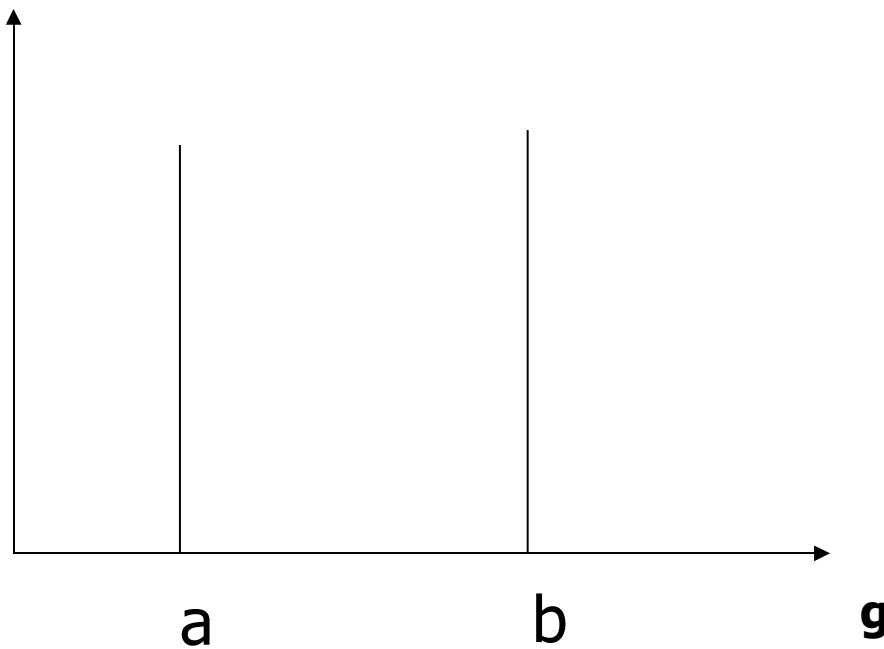
Salt and Pepper noise can be analytically described by:

$$HISTOGRAM_{Salt \& Pepper} = \begin{cases} A & \text{for } g = a \text{ ("pepper")} \\ B & \text{for } g = b \text{ ("salt")} \end{cases}$$

- There are only two possible values, a and b , and the probability of each is typically less than 0.2 – with numbers greater than this the noise will swamp out the image.
- For an 8-bit image, the typical value for pepper-noise is 0, and 255 for salt-noise

Salt and Pepper noise

Histogram



Salt and Pepper noise

- The salt-and-pepper type noise (also called impulse noise, shot noise or spike noise) is typically caused by malfunctioning pixel elements in the camera sensors, faulty memory locations, or timing errors in the digitization process

Rayleigh noise

- Rayleigh distribution is defined as:

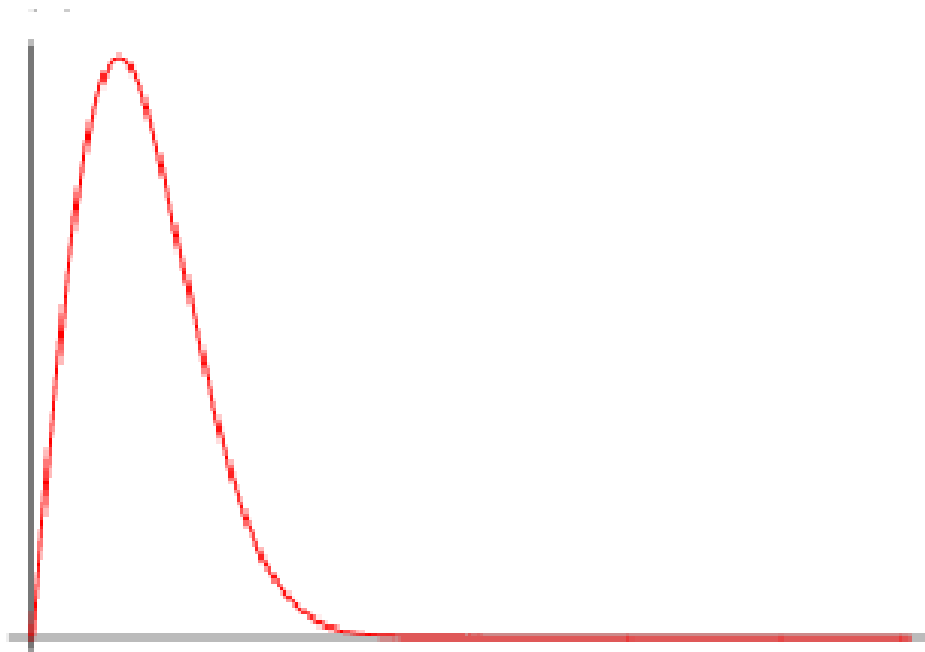
$$HISTOGRAM_{Rayleigh} = \frac{2g}{\alpha} e^{-g^2/\alpha}$$

$$where : mean = \sqrt{\frac{\pi \alpha}{4}}$$

$$variance = \frac{\alpha(4 - \pi)}{4}$$

Rayleigh distribution

PDF (Propability density function)



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Rayleigh noise

- *Radar range and velocity images* typically contain noise that can be modeled by the Rayleigh distribution

Gamma noise

- *Gamma noise* can be obtained by lowpass filtering of laser-based images
- The equation for gamma noise is:

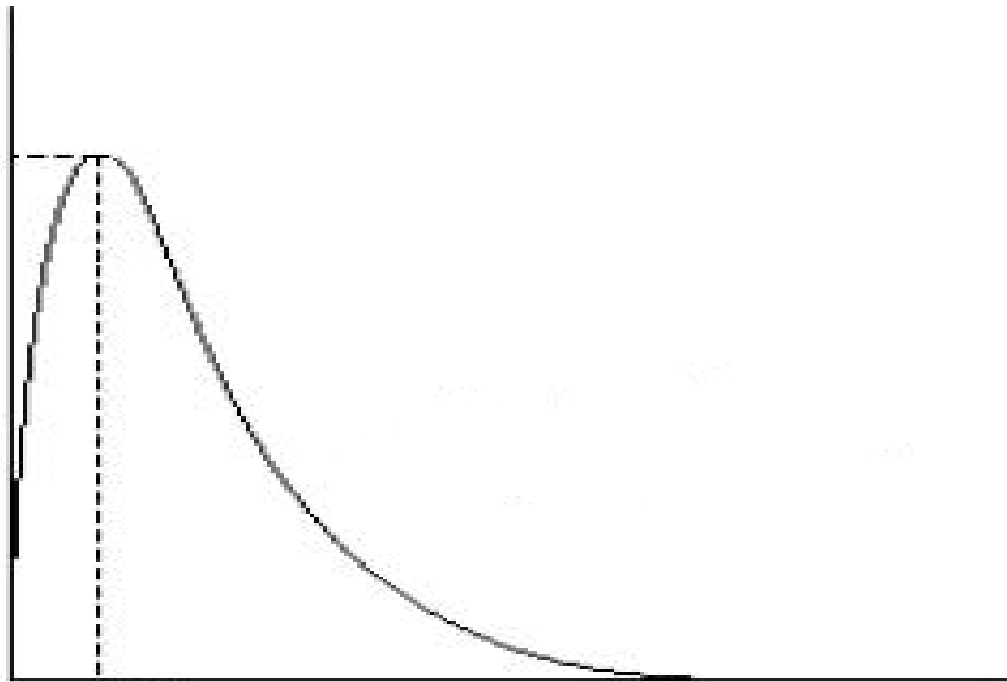
$$\text{HISTOGRAM}_{\text{Gamma}} = \frac{g^{\alpha-1}}{(\alpha-1)! \alpha^{\alpha}} e^{-g/\alpha}$$

where : $\text{mean} = \alpha \alpha$

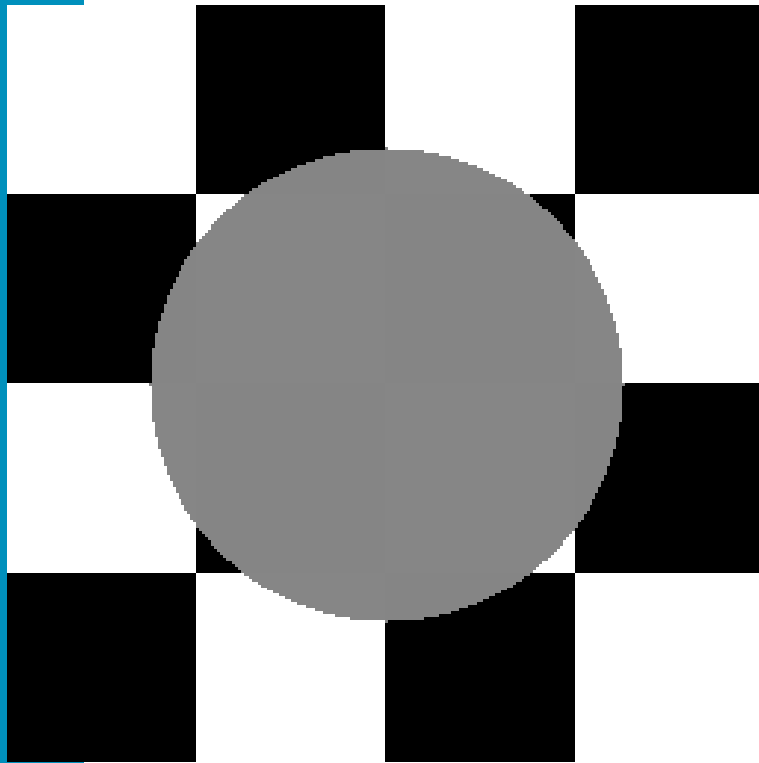
$\text{variance} = \alpha^2 \alpha$

Gamma noise

PDF (Propability density function)



Example of Gaussian Noise



Original image without noise

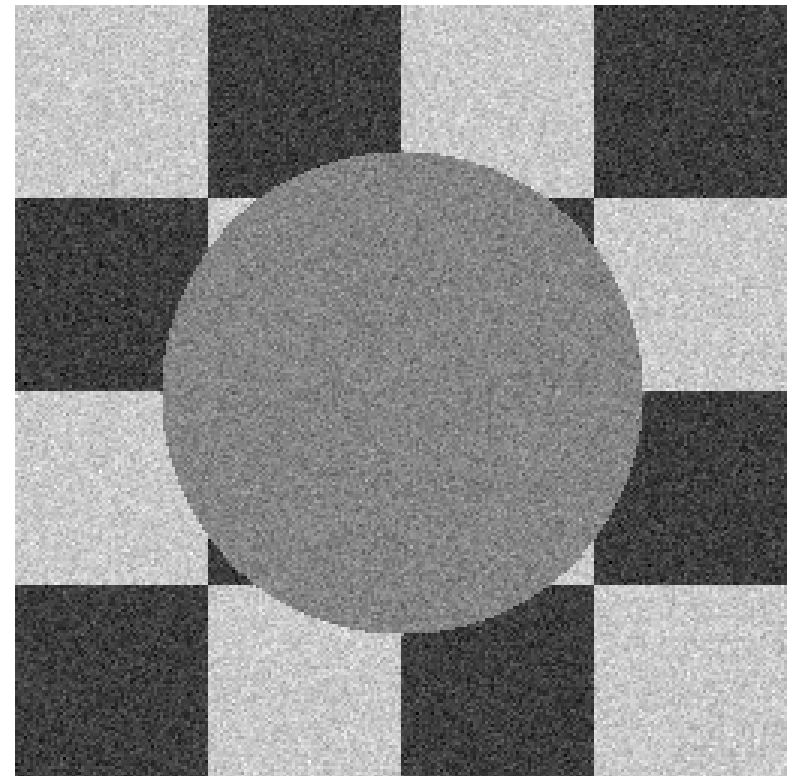


Image with added Gaussian noise

with mean = 0 and variance = 600

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Examples of Uniform Noise and Salt-and-pepper Noise

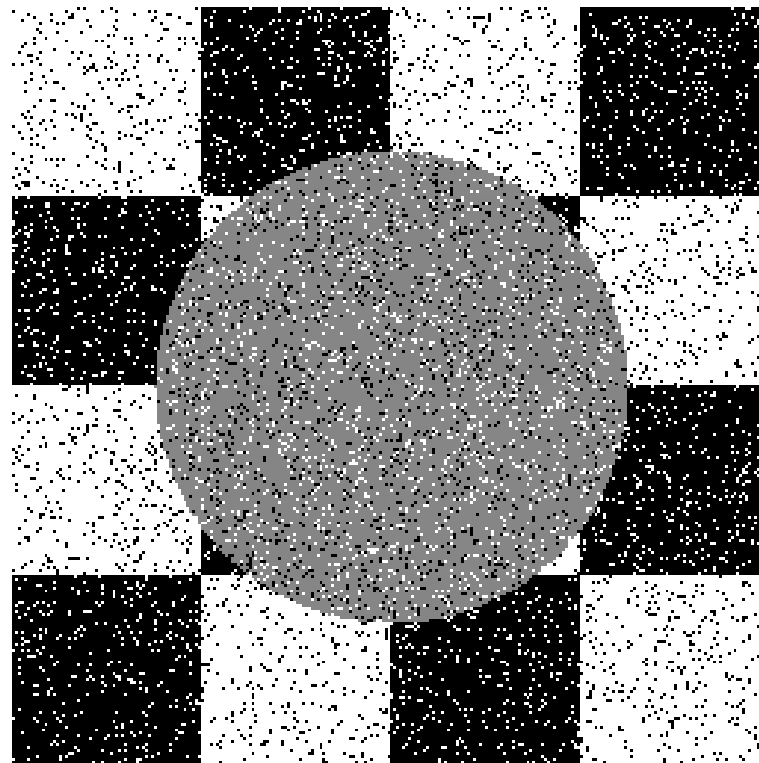
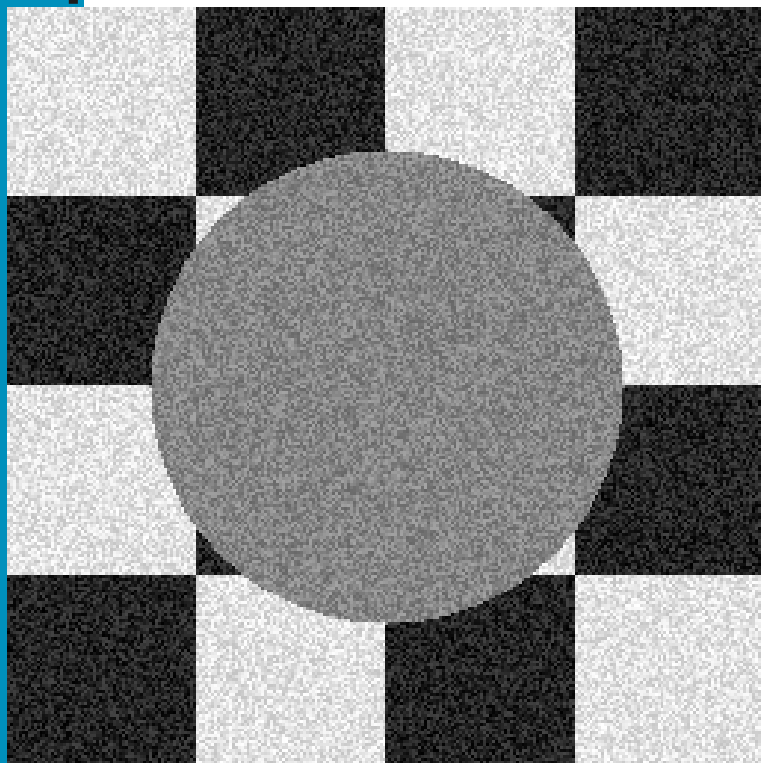


Image with added uniform noise
with mean = 0 and variance = 600

Image with added salt-and-pepper
noise with the probability of each 0.08

Examples of Rayleigh noise and Gamma noise

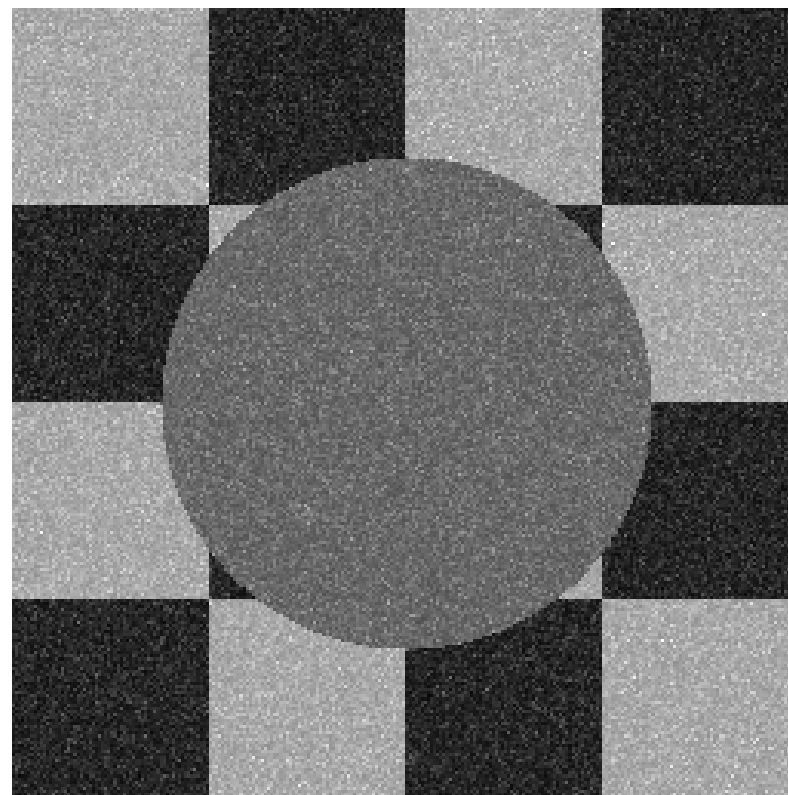
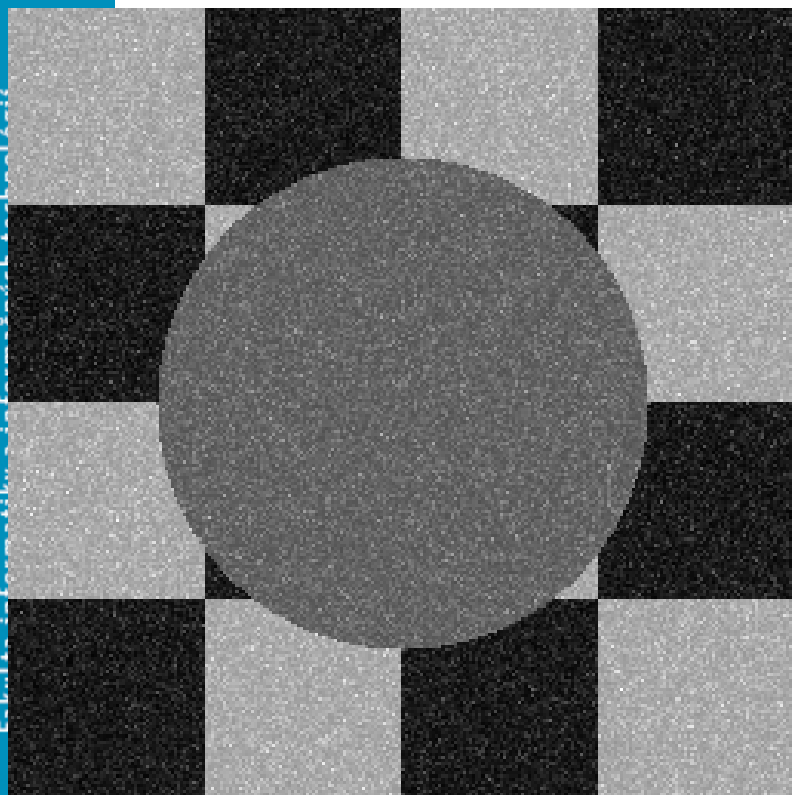


Image with added Rayleigh
noise with variance = 600

Image with added gamma noise
with variance = 600 and $\alpha = 6$

Methods of noise reduction

- Spatial / frequency filter
 - Low-Pass filter
 - Order filter
 - Specific noise -> tailor-made filter...

Image restoration using median filtration



Spectral Filtering

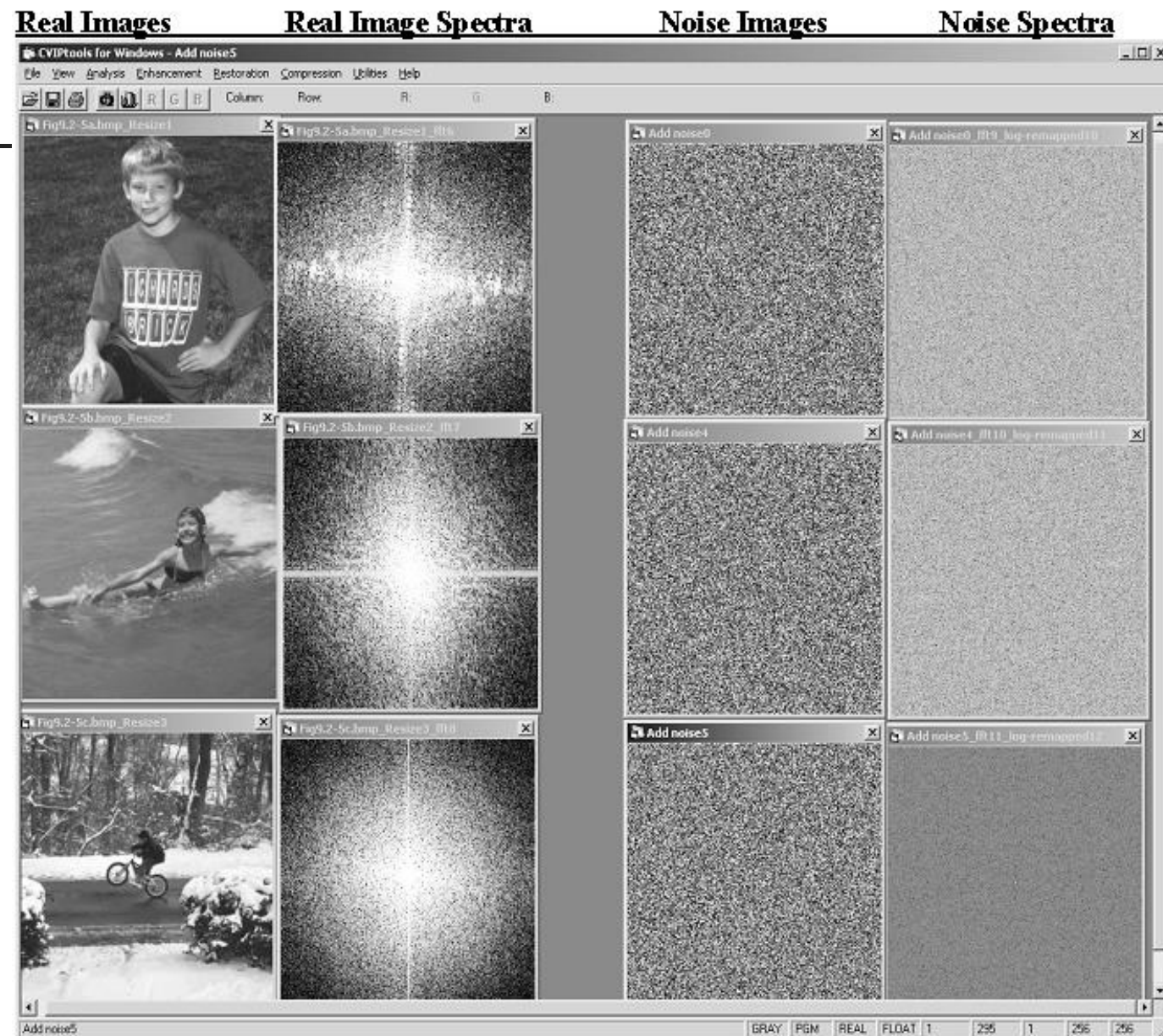
- In most **noiseless images** the spatial frequency energy is concentrated in the low frequencies

In an **image with added noise**, much of the **high frequency** content is due to noise

- This information is useful in the development of models for noise removal

Fourier Spectra of Real Images and Fourier Spectra of Noise Images.

On the left are three real images and their Fourier spectra. On the right are three noise only images and their Fourier spectra. Note that in real images the energy is concentrated in the low frequency areas, but in the noise images it is fairly evenly distributed

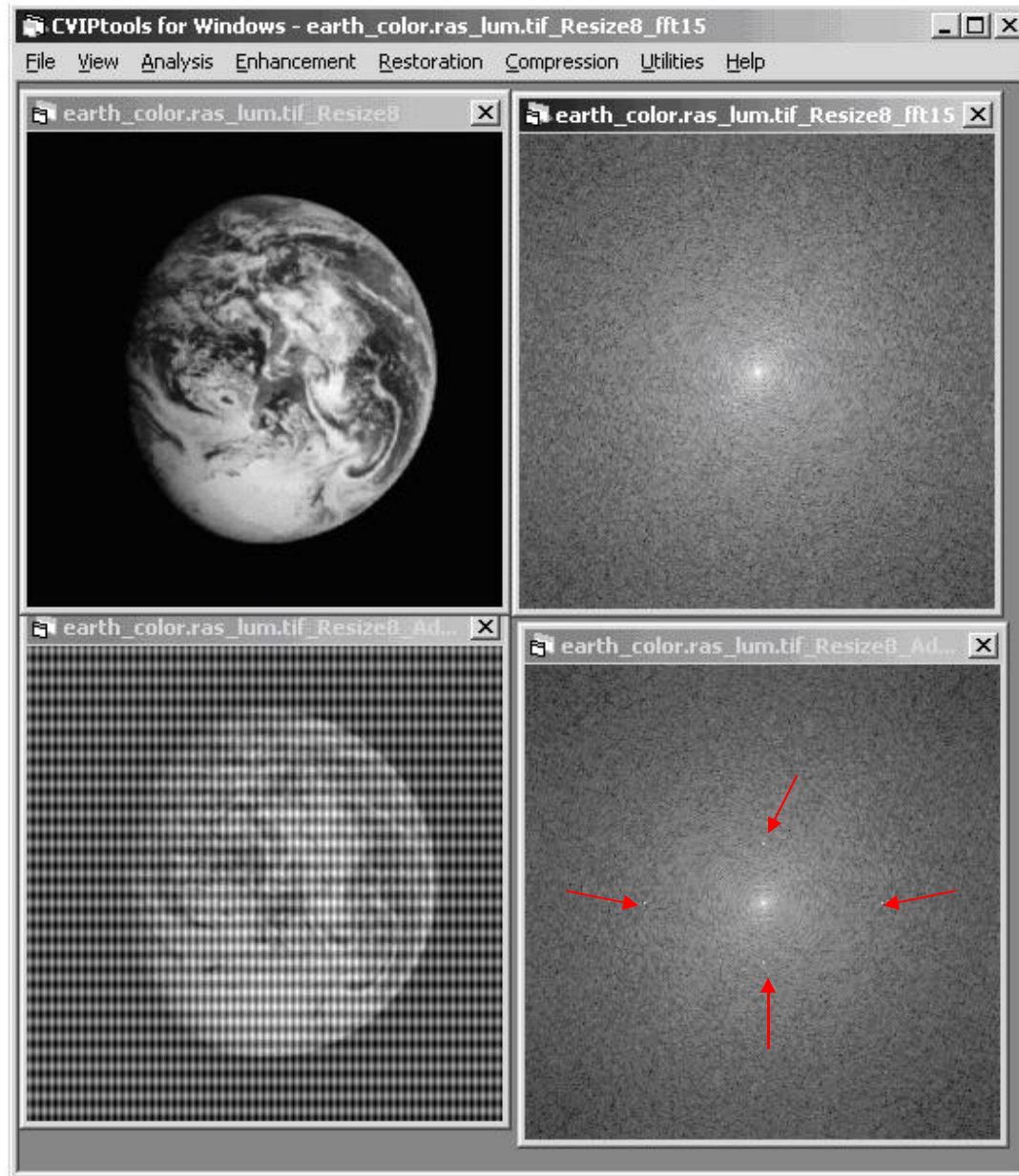


Periodic Noise

- Periodic noise in images is typically caused by electrical and/or mechanical systems, such as mechanical jitter (vibration) or electrical interference in the system during image acquisition
- It appears in the frequency domain as impulses corresponding to sinusoidal interference
- It can be removed with band reject and notch filters

Image Corrupted by Periodic Noise.

On the top are the original image and its spectrum; under it are the image with additive sinusoidal noise, and its spectrum. Note the four impulses corresponding to the noise appearing as white dots – two on the vertical axis and two on the horizontal axis



Estimation of Noise

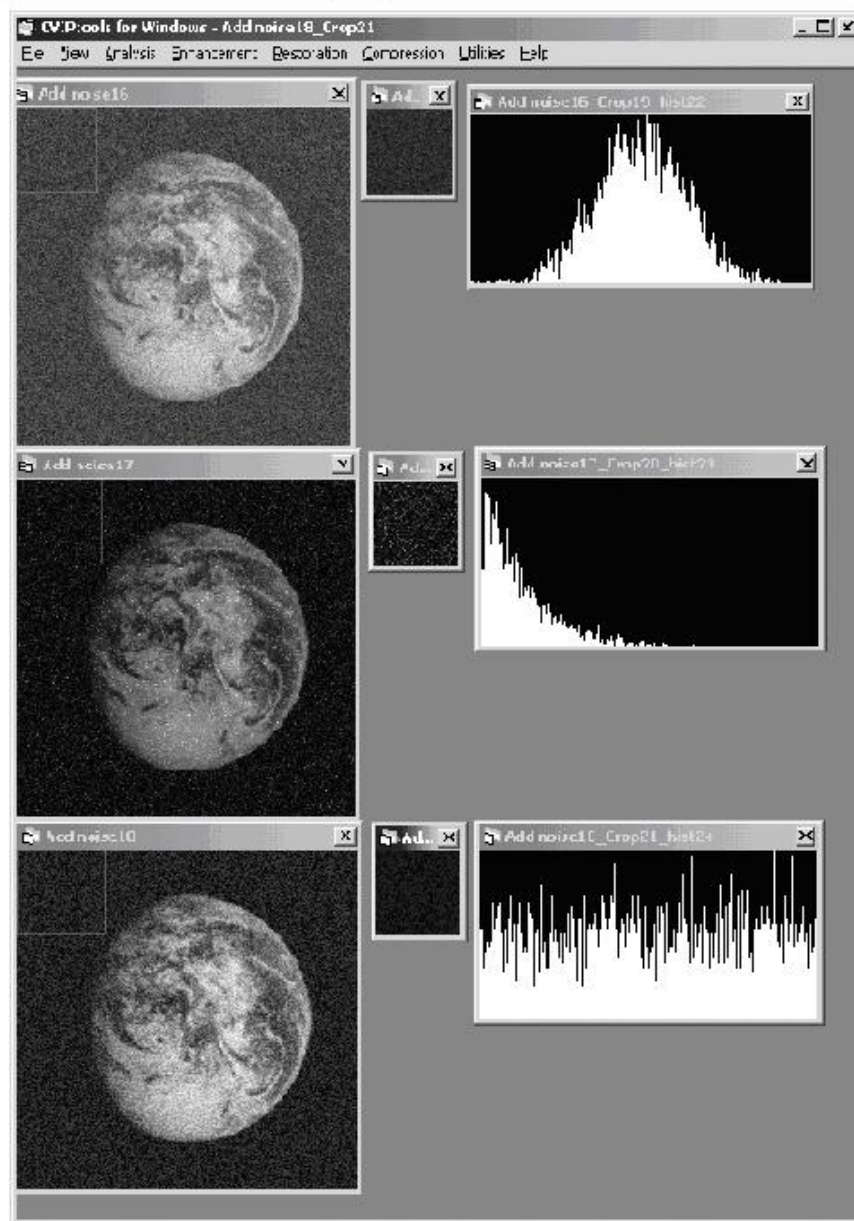
- Consists of finding an image (or sub-image) that contains only noise, and then using its histogram for the noise model
- Noise only images can be acquired by aiming the imaging device (e.g. camera) at a blank wall

Estimation of Noise

- In case we cannot find "noise-only" images, a portion of the image is selected that has a known histogram, and that knowledge is used to determine the noise characteristics
- After a portion of the image is selected, we subtract the known values from the histogram, and what is left is our noise model
- To develop a valid model many sub-images need to be evaluated

Estimating the Noise with Crop and Histogram

On the left are three images with different noise types added. The upper left corner is cropped from the image and is shown in the middle. The histogram for the cropped subimage is shown on the right. Although the noise images look similar, the histograms are quite distinctive – Gaussian, negative exponential and uniform



Spatial Filtering

The degradation model used, assumes that $h(r,c)$ causes no degradation, so the only corruption to the image is caused by additive noise :

$$d(r,c) = I(r,c) + n(r,c)$$

Where $d(r,c)$ is degraded image

$I(r,c)$ is original image

$n(r,c)$ is additive noise function

The two primary categories of spatial filters for noise removal are

- *Order filters*
- *Linear filters*

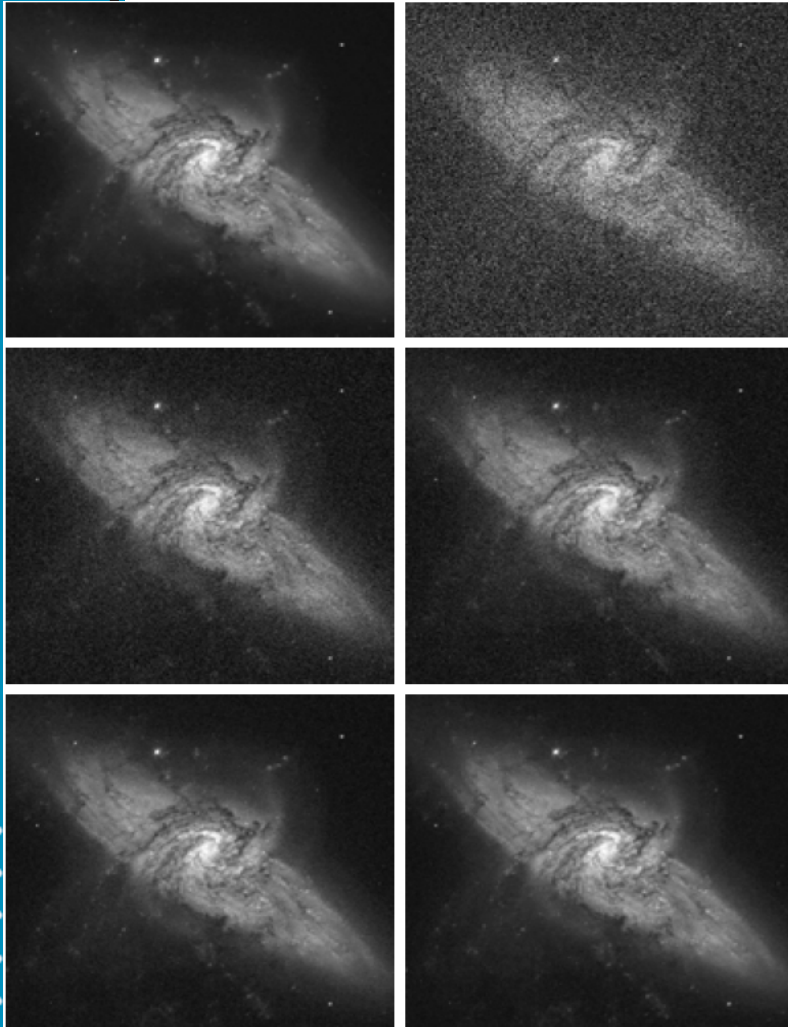
Order filters

- Implemented by arranging the neighborhood pixels in order from smallest to largest gray level value, and using this ordering to select the "correct" value
- ❖ Order filters such as the median can be used to smooth images
- ❖ Order filters work best with salt-and-pepper, negative exponential, or Rayleigh noise

Linear filters

- ❖ Linear filters measure some form of average value
- ❖ The linear filters work best with Gaussian or uniform noise
- ❖ The linear filters have the disadvantage of blurring the image edges, or details
- ❖ Essentially lowpass filters which can be used to mitigate noise effects

Linear filters - example



(a) Image of Galaxy Pair NGC 3314.

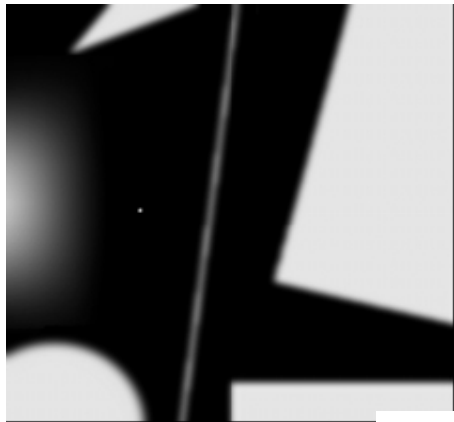
(b) Image corrupted by additive Gaussian noise with zero mean and a standard deviation of 64 gray levels.

(c)–(f) Results of averaging $K=8, 16, 64,$ and 128 noisy images.

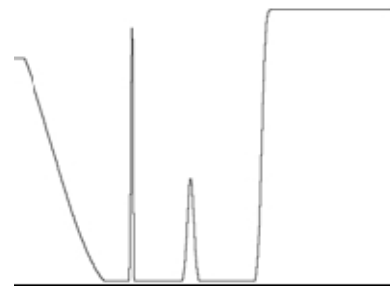
Sharpening Spatial Filters

- The principal objective of sharpening is to highlight fine detail in an image or
- to enhance detail that has been blurred, either in error or as a natural effect of a particular method of image acquisition.

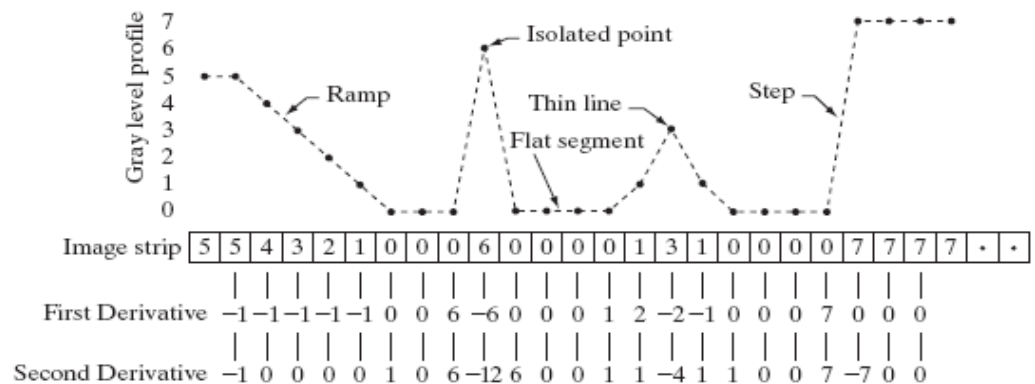
First and Second Derivative



A simple image.



1-D horizontal graylevel profile along the center of the image and including the isolated noise point.



Simplified profile (the points are joined by dashed lines to Simplify interpretation).

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First/Second-order derivatives

- (1) First-order derivatives generally produce thicker edges in an image.
- (2) Second-order derivatives have a stronger response to fine detail, such as thin lines and isolated points.
- (3) Firstorder derivatives generally have a stronger response to a gray-level step.
- (4) Second- order derivatives produce a double response at step changes in gray level.

Second derivation

Two –dimensional Laplacian

Laplacian for a function (image) $f(x, y)$ of two variables is defined as:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}.$$

There are several ways to define a digital Laplacian using neighborhoods.

One of the most used definitions:

$$\nabla^2 f = [f(x + 1, y) + f(x - 1, y) + f(x, y + 1) + f(x, y - 1)] - 4f(x, y).$$

Laplacian

0	1	0	1	1	1
1	-4	1	1	-8	1
0	1	0	1	1	1

0	-1	0	-1	-1	-1
-1	4	-1	-1	8	-1
0	-1	0	-1	-1	-1

Filter mask used to implement the digital Laplacian, as defined above

(b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

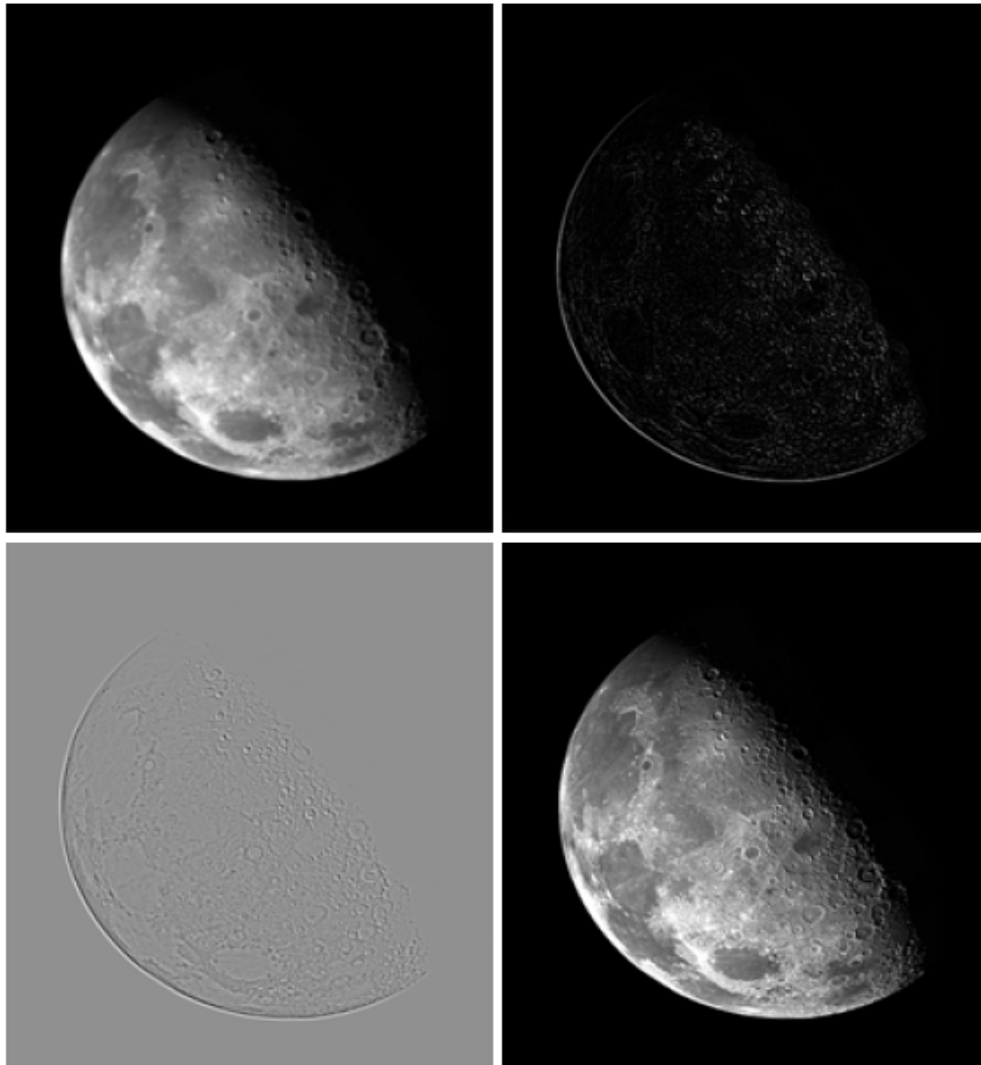
Add/Subtract the original and Laplacian images

If the definition used has a negative center coefficient, then we *subtract*, rather than add

Thus, the basic way in which we use the Laplacian for image enhancement is as follows:

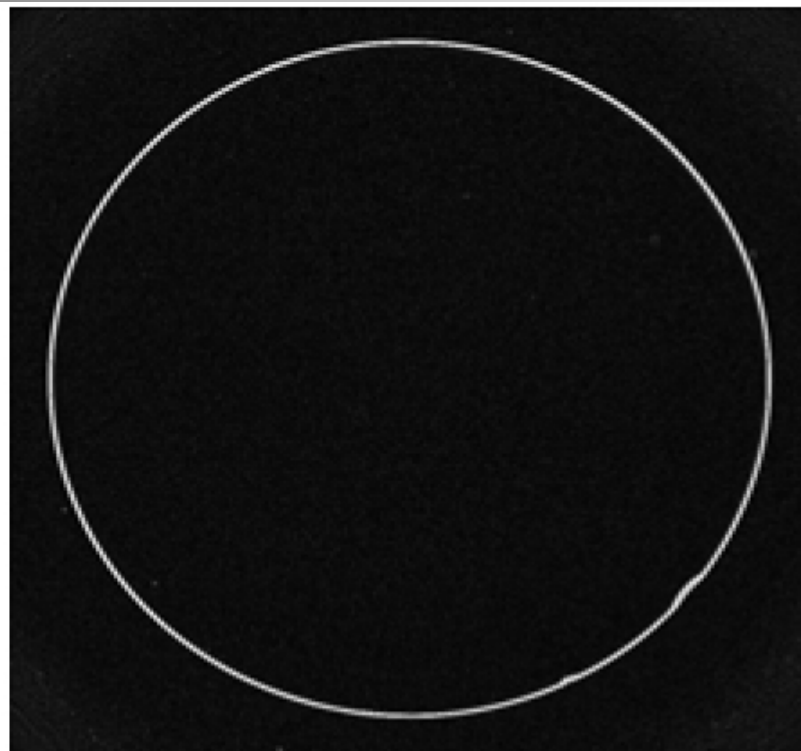
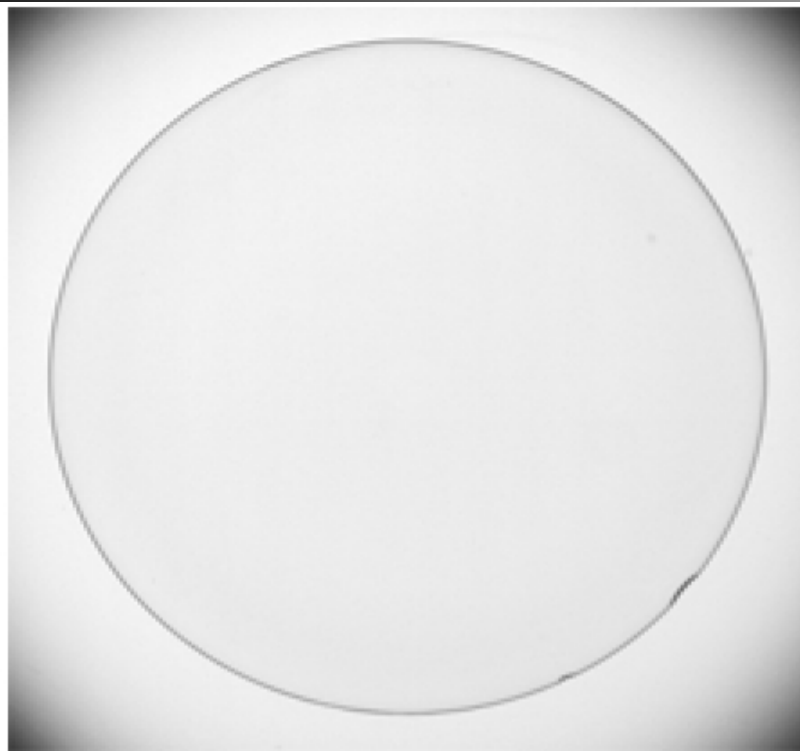
$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is negative} \\ f(x, y) + \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is positive.} \end{cases}$$

Example of Laplacian



- (a) Image of the North Pole of the moon.
- (b) Laplacianfiltered image.
- (c) Laplacian image scaled for display purposes.
- (d) Imageenhanced by using Eq. above

Use of First Derivatives for Enhancement—The Gradient



(a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock).

(b) Sobel gradient.

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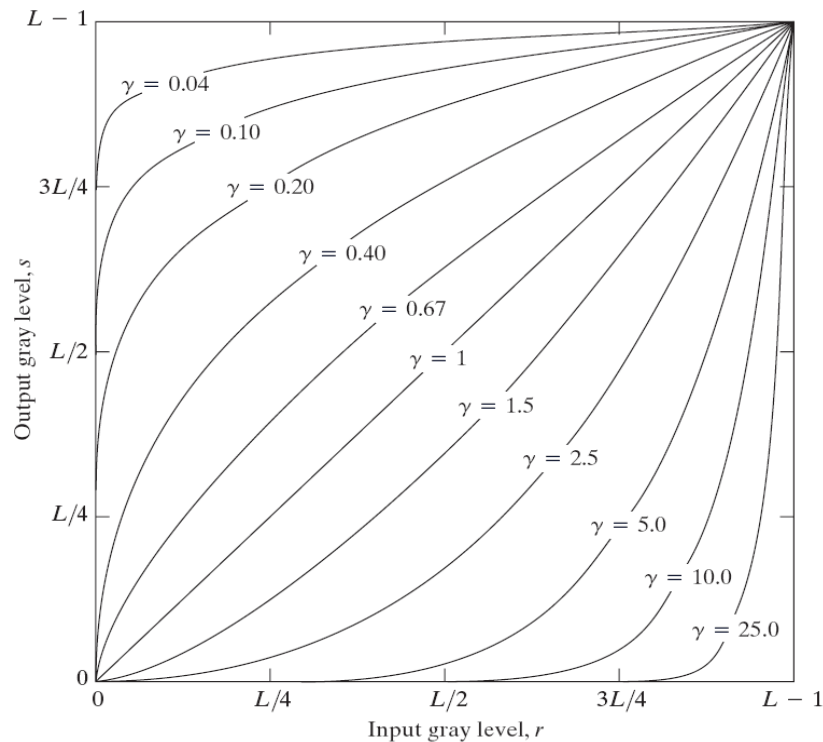
Power-Law Transformations

- Power-law transformations have the basic form:

$$s = cr^\gamma$$

where c and g are positive constants.

Power-Law Transformations



Plots of the equation

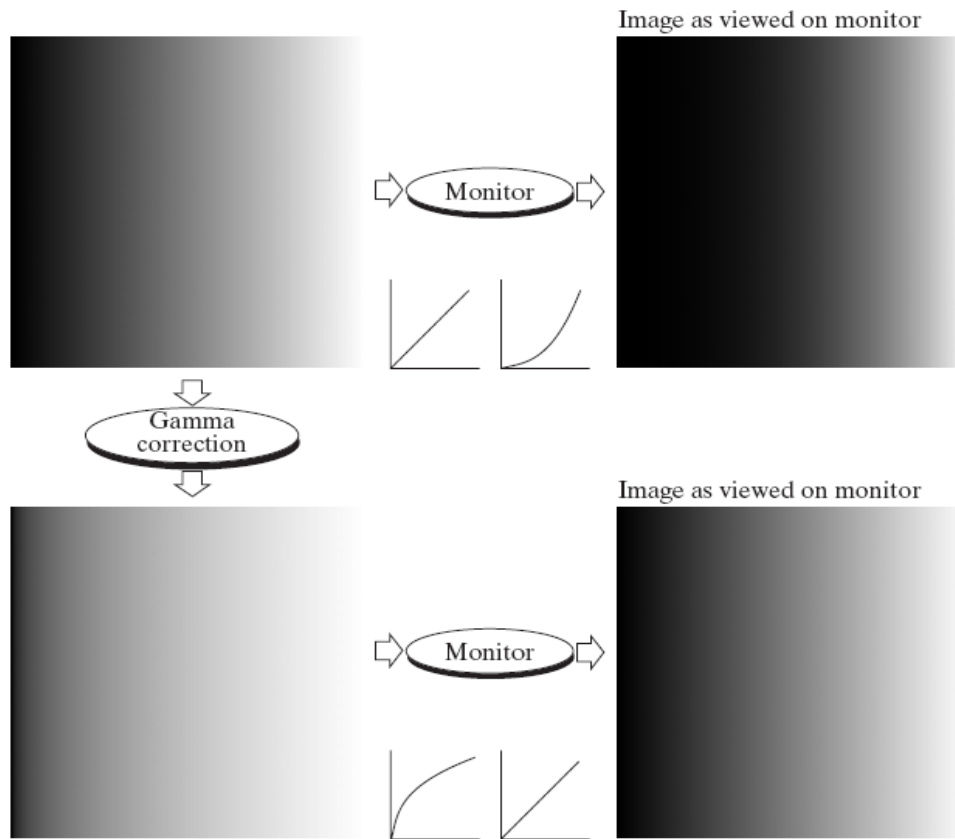
$$s = cr^\gamma$$

various values of γ ($c=1$ in all cases).

Power-Law Transformations

- The process of used the Power-Law Transformations is called ***gamma correction***.
- For example, cathode ray tube (CRT) devices have an intensity-to-voltage response that is a power function, with exponents varying from approximately 1.8 to 2.5

Gamma Correction



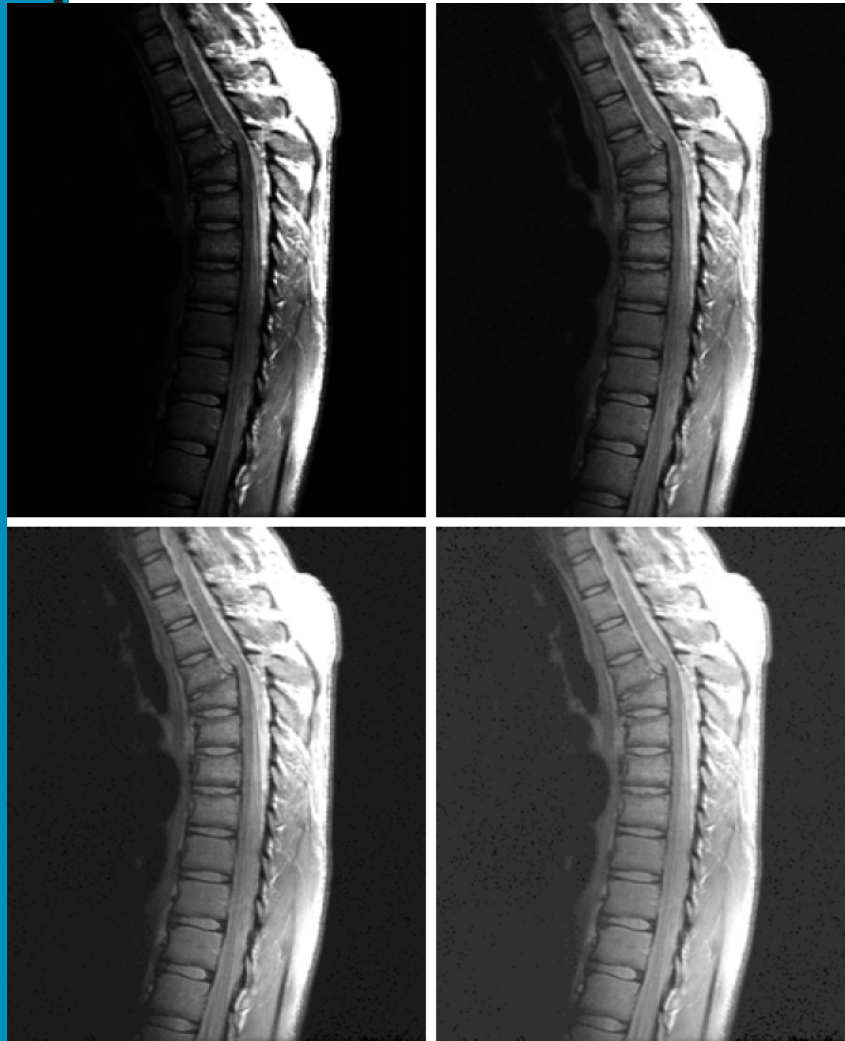
(a) Linear-wedge gray-scale image.

(b) Response of monitor to mean wedge.

(c) Gammacorrected wedge.

(d) Output of monitor.

Power-Law Transformations



(a) Magnetic resonance (MR) image of a fractured human spine.

(b)–(d) Results of applying the power-law transformation with $c=1$ and $g=0.6, 0.4,$ and 0.3 , respectively.

Adaptive filtration

- Adaptive filtration applies a linear filter to an image *adaptively*, tailoring itself to the local image variance. Where the variance is large, then performs little smoothing. Where the variance is small, then performs more smoothing.

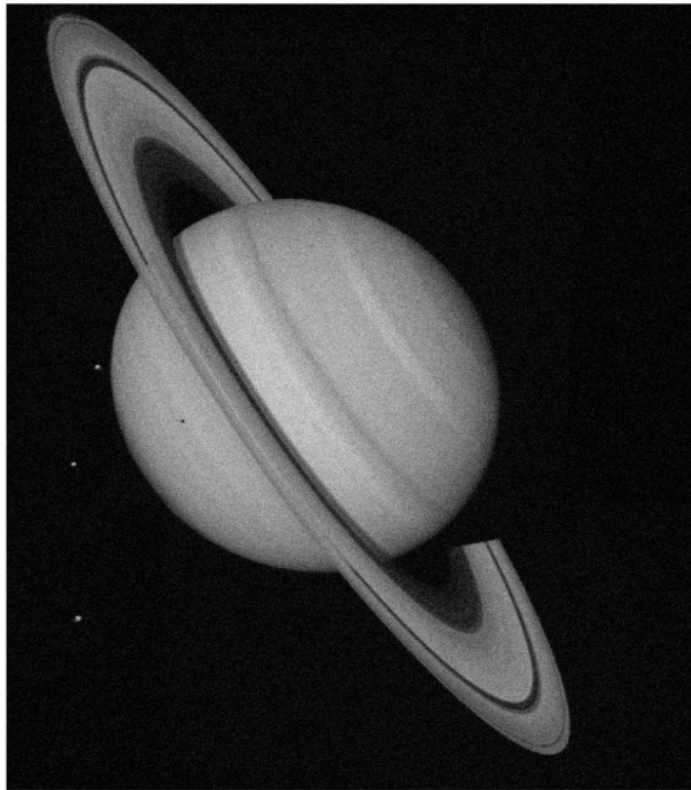
Adaptive filtration

- This approach often produces better results than linear filtering.
- The adaptive filter is more selective than a comparable linear filter, preserving edges and other high-frequency parts of an image.

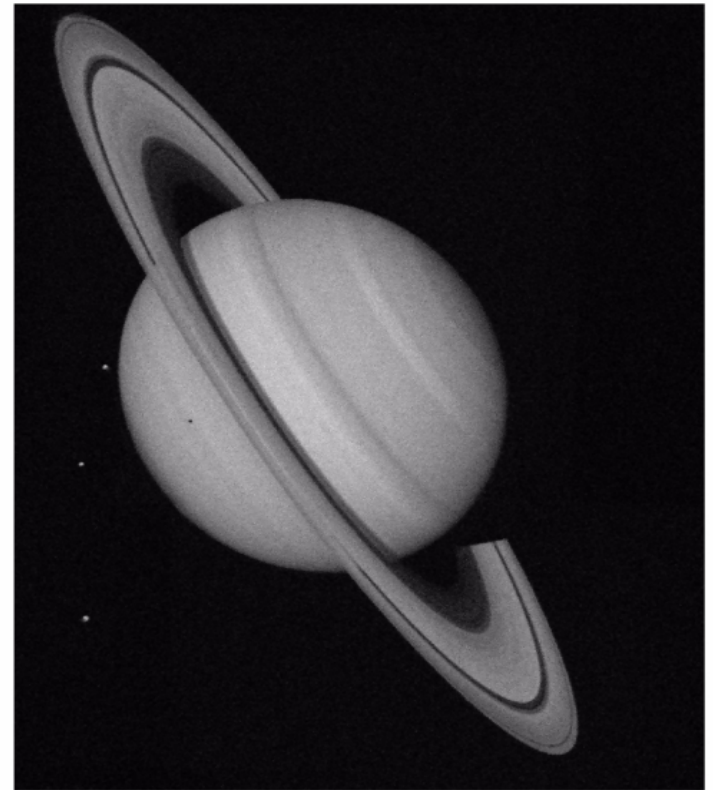
Example of adaptive filtration

- The example below applies adaptive filtration using the Wiener filter to an image of Saturn that has had Gaussian noise added.

Example of adaptive filtration using Wiener filtration

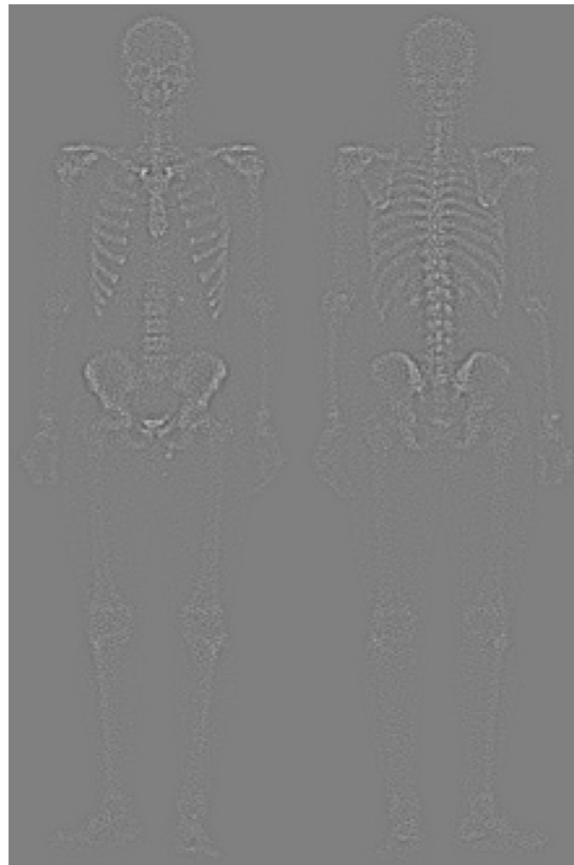
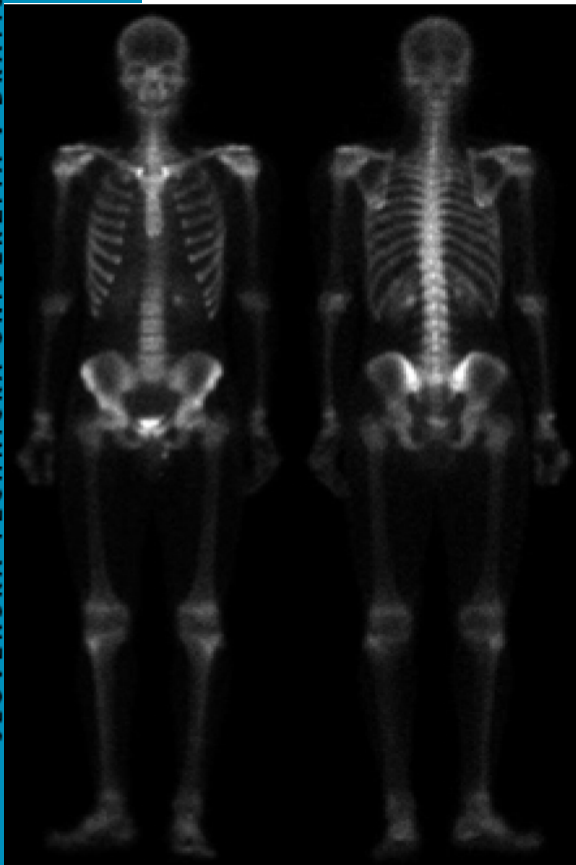


Original Image Courtesy of NASA



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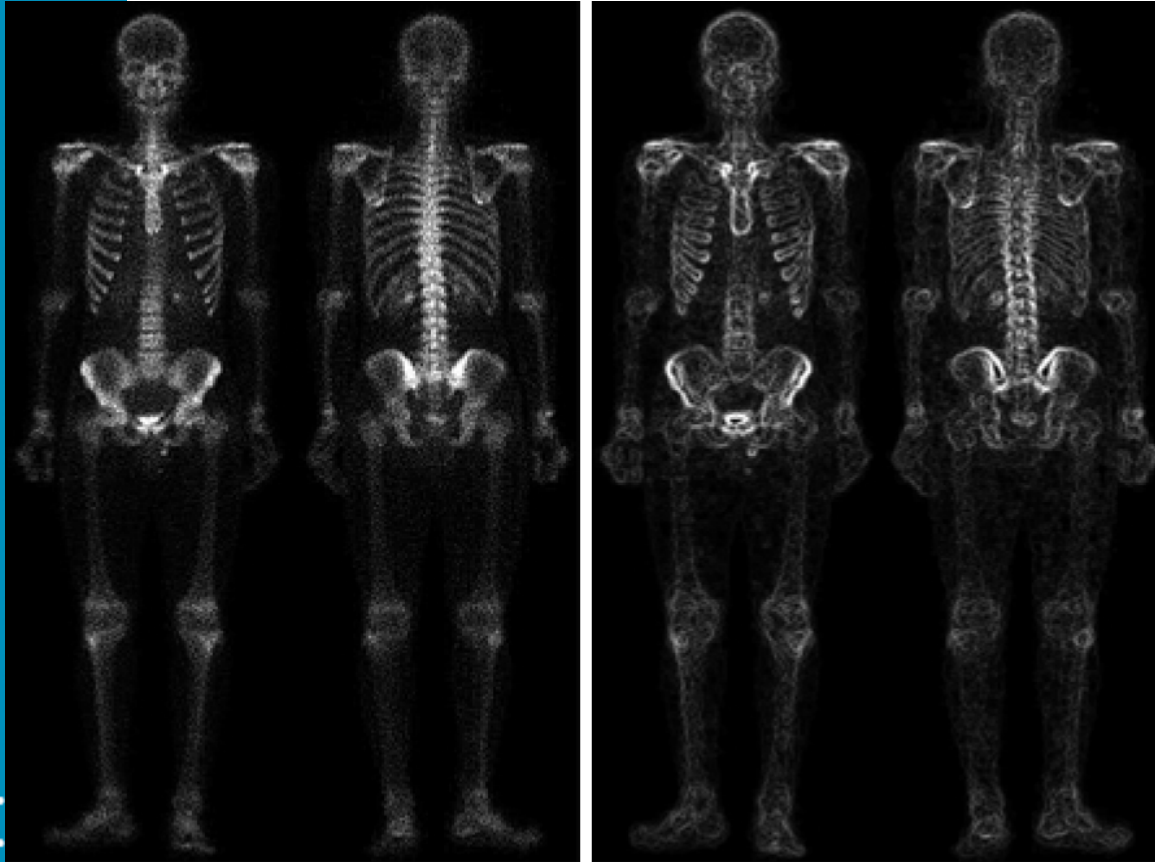
Example of Combining Spatial Enhancement Methods



(a) Image of whole body bone scan.

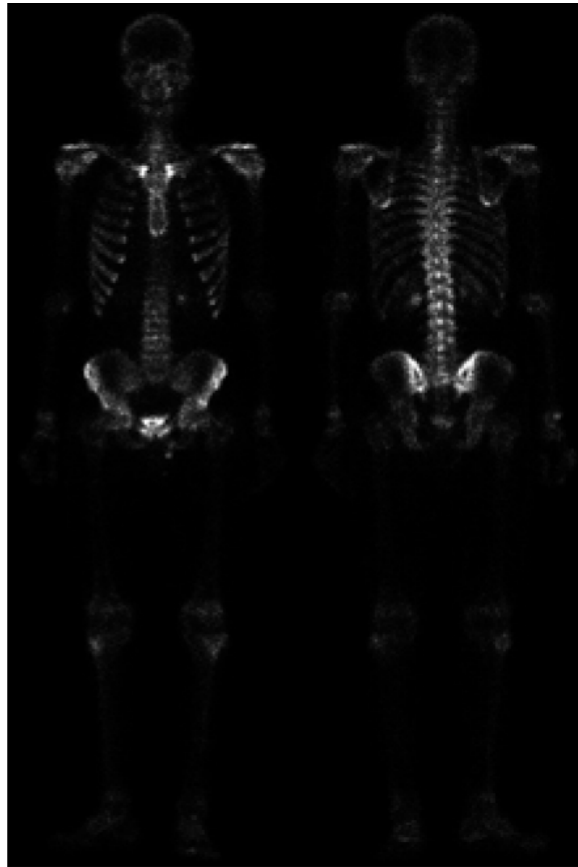
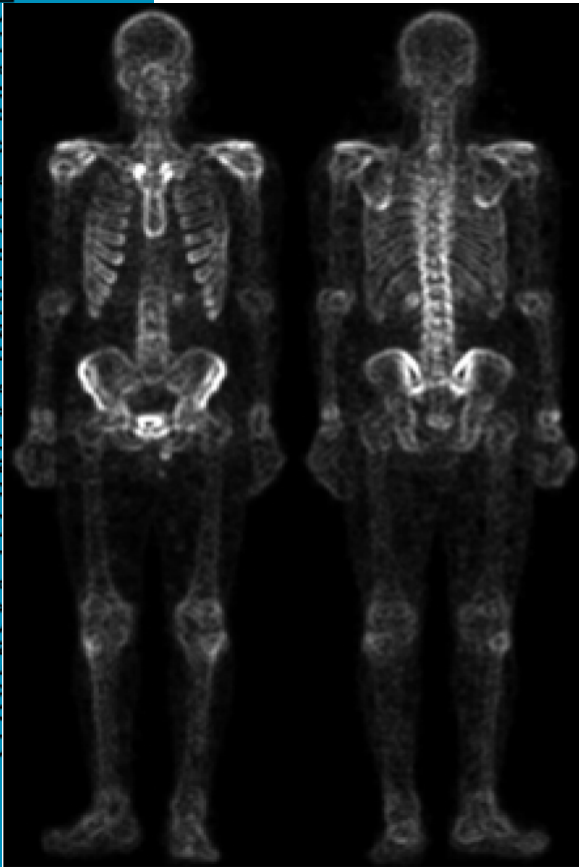
(b) Laplacian of (a)

Example of Combining Spatial Enhancement Methods



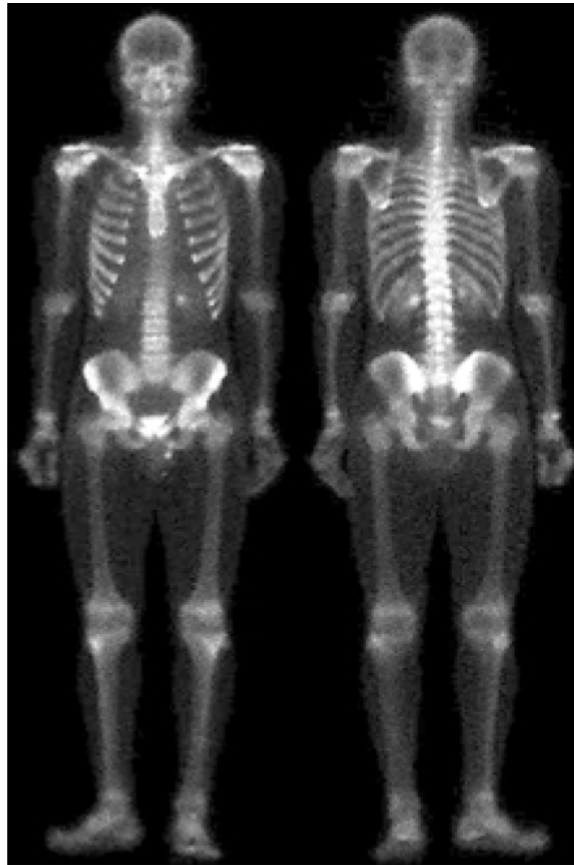
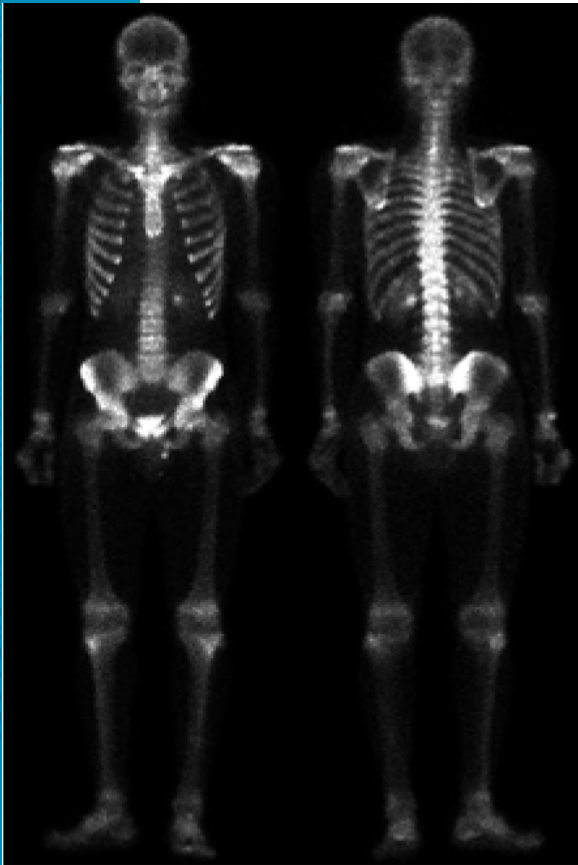
(c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a)

Example of Combining Spatial Enhancement Methods



(e) Sobel image smoothed with a 5×5 averaging filter. (f) Mask image formed by the product of (c) and (e).

Example of Combining Spatial Enhancement Methods



(g) Sharpened image obtained by the sum of (a) and (f).

(h) Final result obtained by applying a power-law transformation to (g).

Image Deblurring

- The blurring, or degradation, of an image can be caused by many factors:
- Movement during the image capture process, by the camera or, when long exposure times are used, by the subject
- Out-of-focus optics, use of a wide-angle lens, atmospheric turbulence, or a short exposure time, which reduces the number of photons captured

Image Deblurring

- A blurred or degraded image can be approximately described by this equation

$$\mathbf{g} = \mathbf{H}\mathbf{f} + \mathbf{n}, \text{ where}$$

g The blurred image

- **H** The distortion operator, also called the *point spread function* (PSF). The distortion operator, when convolved with the image, creates the distortion.
- **f** The original true image
- **n** Additive noise, introduced during image acquisition, that corrupts the image

Image Deblurring

- Based on this model, the fundamental task of deblurring is to deconvolve the blurred image with the PSF that exactly describes the distortion. Deconvolution is the process of reversing the effect of convolution.
- **Note!** The quality of the deblurred image is mainly determined by knowledge of the PSF.

Example of Image Deblurring

Original Image



Blurred Image



Restored, True PSF

