

PROJET

CHAINE COMPLÈTE DE COMPRESSION D'IMAGES

Réalisé par

TRAN Thanh Duy

Master 1 ICONE

Part 1: Overview

The main purpose of this project is to present how to combine the JPEG/JPEG2000 compression/decompression as well as some coding/decoding methods.

Before we start, I would like to present a little bit the main workflow of our project

- It has 2 main phases / process: Compression Coding , Decompression Decoding (naturally, these 2 process are opposite)
- For the "Compression Coding", firstly, the input image (extension .jpg, .jpeg, *.png, *.tif, etc) is "pushed" into "our model" to generate a compressed image. This compressed image will be saved into a file (I think binary file is a good choice I applied this type in the last TP to store the compressed image)
- The "Decompression Decoding" process will try to rebuild the original image from the compressed image (binary file). Of course, there are a missing rate between of 2 process (I will try calculate it)

Phase 1 Image brute Modèle de représentation Codage Image compressée Phase 2 Comparaison Image reconstruite Reconstruction Décodage Image compressée

Figure 1 – Schéma global du projet

Additionally, I also compress/decompress with many cases of scalability (spatial scalability and quality of scalability) to evaluate our model. For example, for spatial scalability, I will chose different values (QCIF, CIF, 4CIF, etc); for quality of scalability, I also choose different values of noise (to generate an image with different noises, I will use Gaussian noise).

Lastly, I also detect ROI (Region of Interested) by applying the Trial-and-Error method to detect a set of "ideal" values (of course, for each image, a set of "ideal" values is different, and I will detect it manually).

Lastly, I will evaluate the performance of model through different rate (e.g. entropy, compression ratio, compression curve, histogram, etc).

Part 2: Preliminary of version 1

For the TP of third weeks of Codage d'Information, I implemented many steps/methods that I think they are necessary for this version (Huffman and RLE compression/decompression image, Store encoded image into a binary file, etc). Therefore, for this version, I will reuse them to resolve the main goal, as well as provide some new methods (that I think they are important). If I have time, I will present the improvement of my implementation of last TP (my implementation is not really effective when applied to RGB images, I will try to fix it by using OpenCV).

To implement this version, I use the programming language Python version 3 (Python3). In addition, to well manage and install relevant packages/libraries, I used Pip version 3 (pip3), so you can install by using sudo apt-get install python3-pip. Some packages/libraries I used in this project are:

- Pillow
- Scipy (I used the version 1.1.0 to run and test something in this project)
- Numpv
- Matplotlib
- Skimage (you have to install scikit-image instead of skimage)
- Pandas
- Math
- Struct
- Os

Some library can not be available in Windows, so my advice is to code in Ubuntu (I used the version 18.04).

Moreover, I also reuse some functions defined in JPEG and JPEG2000 packages (provided by professor Renaud Peteri ©).

Part 3: Implementation of version 1

Before we start, I would like to present Spatial Scalability and Quality Scalability as well as some results that I obtained.

1. **Spatial Scalability** (Slide 81), can be simply understood as the frame size of an image (in this case, we just consider image) based on its height and width (pixels). Of course, there are many definitions/concepts of spatial scalability, but I use the most simple definition. There are many kinds/types of spatial scalability, e.g: SQCIF, QCIF, SIF(525), CIF/SIF(625), 4SIF(525), 4CIF/4SIF(625), 16CIF.



2. **Quality Scalability** (Slide 81), can be simply understood as the quality of an image based on SNR. That means the quality of an image is good or bad, it depends on the noise level. There are many types/kinds of noise, e.g. Gaussian, Salt-Pepper, Speckle, etc)



So, for spatial scalability, I will build the function spatial Scalable () with 4 parameters:

```
    img : the input image
    smallFrame : the "low" spatial level (I prefer to use "small frame")
    mediumFrame : the medium spatial level (I prefer to use "medium frame")
    largeFrame : the "high" spatial level (I prefer to use "large frame")
```

Although the frame size of each level can be customized, I prefer to use QCIF (176 x 144)) for smallFrame, CIF (352 x 288) for mediumFrame, and 4CIF (704 x 576) for largeFrame.

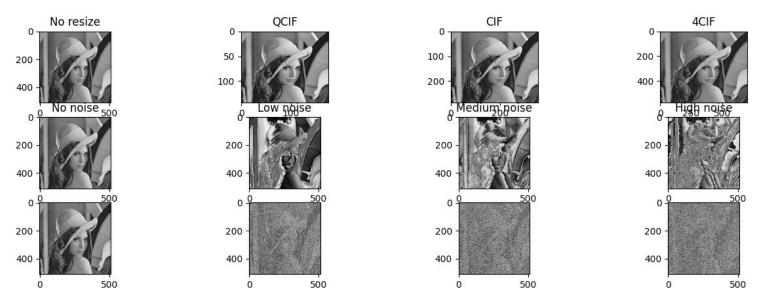
The body of this method have 4 steps:

- 1. Load an input image by using Image.open()
- Resize the image by use resize()
- 3. Store the output into a numpy array (npy file). Instead of store output image into JPG/JPEG/PNG, I decide to store into numpy array to easy reload later to calculate some relevant informations
- 4. Lastly, return a dict of results (I prefer to return a dict instead of simple value)

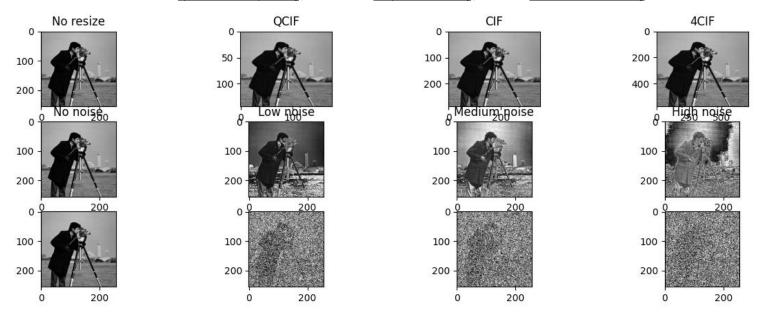
Next, for the quality scalability, I will add the noise into the input image. I will show how I did with 2 approaches (I called "the simple way" and "the complex way"). These two methods are .simpleQualityScalable() and .qualityScalable(), each functions has 4 parameters: img, lowNoise, mediumNoise, highNoise

```
simpleQualityScalable(img, lowNoise, mediumNoise, highNoise):
    image = misc.imread(img, mode='L')
    low image
                = image * lowNoise
    medium image = image *
                           mediumNoise
    high image
                = image * highNoise
    return {"low": low_image,
                "medium": medium image,
                    "high": high image}
def qualityScalable(img, lowNoise, mediumNoise, highNoise):
    image = Image.open(img)
    image = np.array(image)
    noise_low_gaussian = util.random_noise(image, mode="gaussian", var=lowNoise)
   noise medium gaussian = util.random_noise(image, mode="gaussian", var=mediumNoise)
    noise high gaussian = util.random noise(image, mode="gaussian", var=highNoise)
    return {"low":noise low gaussian,
                "medium": noise medium gaussian,
                    "high": noise high gaussian}
```

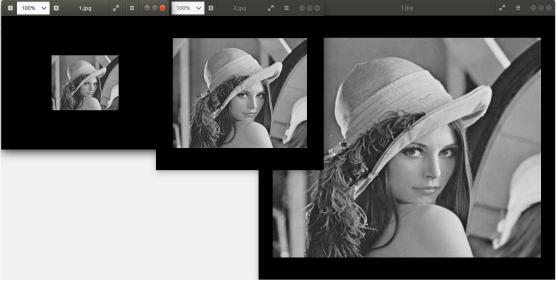
```
_name__ == '__main
image = "Lena.jpg"
          == ' main ':
qCif = (176, 144)
Cif = (352, 288)
fCif = (704, 576)
spatialDict = spatialScalable(image, qCif, Cif, fCif)
simpleQualityDict = simpleQualityScalable(image, 2, 4, 8)
qualityDict = qualityScalable(image, 2, 4, 8)
f, axarr = plt.subplots(3, 4)
axarr[0,0].imshow(Image.open(image), cmap=plt.get_cmap('gray'))
axarr[0,0].set_title("No resize")
axarr[0,1].imshow(spatialDict.get("small"), cmap=plt.get_cmap('gray'))
axarr[0,1].set_title("QCIF")
axarr[0,2].imshow(spatialDict.get("medium"), cmap=plt.get_cmap('gray'))
axarr[0,2].set_title("CIF")
axarr[0,3].imshow(spatialDict.get("high"), cmap=plt.get_cmap('gray'))
axarr[0,3].set_title("4CIF")
axarr[1,0].imshow(Image.open(image), cmap=plt.get_cmap('gray'))
axarr[1,0].set_title("No noise")
axarr[1,1].imshow(simpleQualityDict.get("low"), cmap=plt.get_cmap('gray'))
axarr[1,1].set_title("Low noise")
axarr[1,2].imshow(simpleQualityDict.get("medium"), cmap=plt.get cmap('gray'))
axarr[1,2].set_title("Medium noise")
axarr[1,3].imshow(simpleQualityDict.get("high"), cmap=plt.get_cmap('gray'))
axarr[1,3].set_title("High noise")
axarr[2,0].imshow(Image.open(image), cmap=plt.get_cmap('gray'))
axarr[2,1].imshow(qualityDict.get("low"), cmap=plt.get_cmap('gray'))
axarr[2,2].imshow(qualityDict.get("medium"), cmap=plt.get_cmap('gray'))
axarr[2,3].imshow(qualityDict.get("high"), cmap=plt.get_cmap('gray'))
plt.show()
```



First line : Spatial Scalability Coding / Second line : Simple noise adding / Third line : Gaussian noise adding



First line: Spatial Scalability Coding / Second line: Simple noise adding / Third line: Gaussian noise adding





Now, we will come to the main implementation of this project, compression-coding/decompression-decoding of image. I decided to choose <u>Huffman</u> algorithm and <u>Run-Length Encoding</u> algorithm for one-dimension (1D) for this project.

Before implementing these 2 algorithms, I implement 2 methods to save (saveEncodedImage) and load file (loadEncodedImage)

In addition, I also built 2 functions bitMapping and reverseBitMapping.

- 1. bitMapping convert a number of bits to a binary string (e.g bitMapping(10, 20) returns 01010)
- reverseBitMapping return an integer corresponding a byte (e.g: 101010101010 = 2730)

```
### convert a number of bits -> a binary string, e.g bitMapping(10, 20) -> 01016

def bitMapping(bitNumber, bitsMax):
    if(bitsMax < bitNumber):
        print("bitsMax to encode < bitNumbers >> Error to encoded")
    val = "{}".format(bitNumber)
    valMax = "{}".format(bitsMax)
    maxNum = "{}".format(bin(int(valMax,10))[2:])
    maxFormat = "{}".format(bin(int(val,10))[2:])
    bitNumber = ""
    for i in range(len(maxFormat),len(maxNum)):
        bitNumber += maxFormat
    return bitNumber

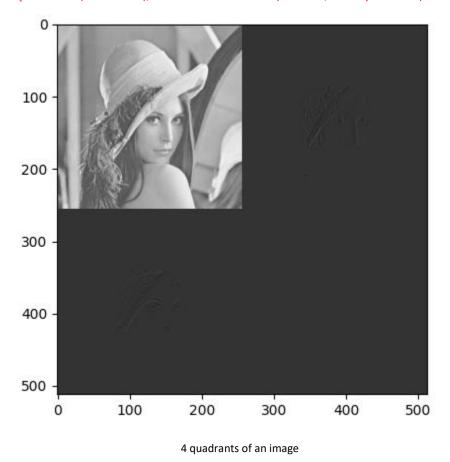
### convert a string bit (encoded) -> a integer

def reverseBitMapping(bitNumber):
    return int(str(bitNumber),2)
```

Subsequently, I will split an image into 4 quadrants (decompose), each quadrant is transformed into numpy array

```
### fourQuadrantSplit original image -> 4 quadrants, each quadrant is transformed into a numpy array
### return a dict

def fourQuadrantSplit(image):
    image1 = np.array([image[x][:int(len(image)/2)] for x in range(int(len(image)/2))]).astype(int)
    image3 = np.array([image[x][int(len(image)/2):] for x in range(int(len(image)/2))]).astype(int)
    image2 = np.array([image[x][int(len(image)/2):] for x in range(int(len(image)/2), len(image))]).astype(int)
    image4 = np.array([image[x][:int(len(image)/2)] for x in range(int(len(image)/2), len(image))]).astype(int)
    return {"first": image1, "second": image2, "third": image3, "fourth":image4}
```



Version 1 : TRAN Thanh Duy

To implement Huffman algorithm, I will setup 4 methods as belows:

1. huffmanTreeInsert(list, element) return a new list after inserting a new element

```
### insert an element into huffman tree
#### input : a list + a element => return a list after inserting
def huffmanTreeInsert(list, element):
    rang = 0
    while(rang < len(list) and (element[0] > list[rang][0]) ):
        rang += 1
    list.insert(rang, element)
    return list
```

2. buildHTree (occursIndexList, occursValList) return a Huffman tree (type: List)

3. huffmanTreeFinding (huffmanTree, pix (it is pixel), string) finds a pixel in a string into Huffman tree. This function return the result or NIL

4. HuffmanEncode (image, inputSize) returns an encoded image (the encoded result is a string). This function have 2 parameters: image (input image) and inputSize (how many bit you want to encode? In this case, I will hardcode 16000000). The output of Huffman encode is a binary string (0,1) and the size of this string

```
def HuffmanEncode(image, inputSize):
    beginBitNumber = len(bitMapping(inputSize,inputSize))
    reshapedImg = np.reshape(image, (-1,len(image)*len(image)))[0] # reshape input image
    FramedImg = pd.DataFrame(reshapedImg)
    occursList = FramedImg[0].value counts() # len of tabular (horizontal)
    occursValList = occursList.values.tolist()
    occursIndexList = occursList.index.tolist()
    huffmanTree = buildHTree(occursIndexList, occursValList)
    indexMax = max(occursIndexList)
    maxVal = max(occursValList)
    indexBitNumber = len(bitMapping(indexMax, indexMax))
    valueBitNumber = len(bitMapping(maxVal, maxVal))
    finalString = bitMapping(indexBitNumber, inputSize)
    finalString += bitMapping(valueBitNumber, inputSize)
    finalString += bitMapping(beginBitNumber*3+(indexBitNumber+valueBitNumber)*len(occursIndexList), inputSize)
    for rang in range(len(occursIndexList)):
        finalString += bitMapping(occursIndexList[rang], indexMax)
finalString += bitMapping(occursValList[rang], maxVal)
    res = [huffmanTreeFinding(huffmanTree, reshapedImg[x], "") for x in range(len(reshapedImg)) ]
    result = "".join(map(str, res))
    result = finalString + result
    return _result # result is a string -> store to bin file
```

5. HuffmanDecode (_result, inputSize) returns a decoded image (the decode result is an array, and it will be load by using np.array). This function also have 2 parameters: image (input image) and inputSize (how many bit you want to decode?). The output of this function is a list of integer value.

```
beginBitNumber = len(bitMapping(inputSize,inputSize))
indexBit = reverseBitMapping(_result(!beginBitNumber])
indexVal = reverseBitMapping(_result(!beginBitNumber:beginBitNumber*2])
minVal = reverseBitMapping(_result[beginBitNumber*2:beginBitNumber*3])
occursIndexList
occursValList = []
         range(beginBitNumber*3, minVal,indexBit+indexVal):
    occursIndexList.append(reverseBitMapping(_result[i:i+indexBit]))
   occurs ValList.append (reverse Bit Mapping (\_result[(i+indexBit):(i+indexBit+indexVal)])) \\
huffmanTree = buildHTree(occursIndexList, occursValList) #build a huffman tree using index list and value list
image = [] # output is a list that I will use np.load() to load
clonedHuffmanTree = huffmanTree
bit = minVal
    if(len(clonedHuffmanTree) > 0):
        if(len(clonedHuffmanTree) > 1):
           if(_result[bit] == "0"):
                  clonedHuffmanTree = clonedHuffmanTree[1]
                  bit += 1
                   if(len(clonedHuffmanTree) > 2 and _result[bit] == "1"):
                      clonedHuffmanTree = clonedHuffmanTree[2]
if(not(isinstance(clonedHuffmanTree[1], list)) ) :
    image.append(clonedHuffmanTree[1])
  turn image # image is a list of integer value -> loaded with np.array
```

Now, the Huffman algorithm (3 activities: finding, building, inserting and 2 phases: encoding, decoding) is finished. We will come to Run-Length Encoding (RLE) algorithm, in this project, I will use one-dimensional (1D) approach with 2 methods: HorizontalEncode() and HorizontalDecode(). Actually, RLE algorithm is similar to Huffman, so its implementation is quite simple.

 HorizontalEncode (image, inputSize) returns an encoded image (the encoded result is a string). This function have 2 parameters: image (input image) and inputSize (how many bit you want to encode? In this case, I will hardcode 16000000). The output of RLE encode is a binary string (0.1) and the size of this string

```
########### like Huffman Encode, this method will return an encoded image in form of a binary string ###########
HorizontalEncode(image, inputSize):
image = np.reshape(image, (-1,len(image)*len(image)))[0]
nbFact = 0
pixList = []
 orderList = []
        (i < len(image)):
        hile (i < len(image) - 1 and image[i] == image[i + 1]):
      pixList.append(int(image[i]))
      orderList.append(i-count+1)
      nbFact += i-count+1
minVal = -min(pixList)
pixList = [i+minVal for i in pixList]
maxVal = 10*int(mean(orderList))
while(i < len(pixList)):</pre>
      if(orderList[i] > maxVal):
           orderList.insert(i+1, orderList[i]-maxVal)
orderList[i] = maxVal
           pixList.insert(i+1, pixList[i])
 beginBitNumber = len(bitMapping(inputSize,inputSize))
 indexMax = max(orderList)
 maxVal = max(pixList)
 indexBitNumber = len(bitMapping(indexMax, indexMax))
valueBitNumber = len(bitMapping(maxVal, maxVal))
stringFinale = bitMapping(indexBitNumber, inputSize)
stringFinale += bitMapping(valueBitNumber, inputSize)
stringFinale += bitMapping(minVal, inputSize)
            range(len(orderList)):
      stringFinale += bitMapping(orderList[j], indexMax)
stringFinale += bitMapping(pixList[j], maxVal)
     urn stringFinale
```

2. RLEDecode (_result, inputSize) returns a decoded image (the decode result is an array, and it will be load by using np.array). This function also have 2 parameters: image (input image) and inputSize (how many bit you want to decode?). The output of this function is a list of integer value.

Lastly, I would like to present how to implement Wavelet Transform. To implement it, I relied the section of "Seuillage des Coefficient d'Ondelettes" of JPEG2000. From that, I rebuild 4 methods as follows:

1. applyContourByBias (image, discretZone, deathZone) aims to apply the contour with a bias (bias = seuil) to an image

```
def applyContourByBias(image, discretZone, deathZone):
    contour = np.array(image[int(image.shape[0]/3):2*int(image.shape[0]/3),int(image.shape[0]/3):2*int(image.shape[0]/3)])
    for rangX in range(len(image)):
        if (image[rangX][rangY] != 0.0):
        image[rangX][rangY] = int(image[rangX][rangY]/discretZone)*discretZone
    image = np.where(np.abs(image) > deathZone, image, 0.0)
    image[int(image.shape[0]/3):2*int(image.shape[0]/3),int(image.shape[0]/3):2*int(image.shape[0]/3)] = contour
    return image # return an image applied by contour with bias
```

2. biasDecomposing (img, LO_D, HI_D, compteur, deathZone, discretZone) aims to decompose an image which is applied Wavelet Transform. This method is quite similar to the function .decomposer() (predefined in JPEG2000)

- 3. decomposeImageWithWavelet(cpt,cheminImage,deathZone,discretZone) aims to load an input image with Wavelet Transform. This function returns a decomposed image (e.g. quadrant) transformed by Wavelet Transform. This function has 3 steps:
 - a. Load an image and store in "img" variable
 - b. Apply function of (2) to decompose an image to many sub-images
 - c. Return a decompose of image

```
##### decompose Image with Wavelet -> load input image + decompose #####

def decomposeImageWithWavelet(cpt, cheminImage, deathZone, discretZone):
    img = np.double(plt.imread(cheminImage))
    img = biasDecomposing(img, LO_D, HI_D, cpt, deathZone, discretZone)
    return img # return a decomposed image -> many sub-images (quadrants)
```

- 4. recomposeImageWithoutWavelet(cpt,compressedImage) aims to recompose a decomposed image (e.g. 4 quadrants >> 1 image) without Wavelet Transform. This function returns a recomposed image (full image) without Wavelet Transform. This function has 2 steps:
 - a. Recompose compressed image (image after decoding). *Image after decoding is the form of "quadrant". This step will "join" all to only one image
 - b. Return the "original" image (we can use np.load to load it)

Before to implement the test / the deployment, I would like to present some ways / criteria to evaluate the model. In this project, I used entropy (before encoding and after encoding) as well as compression ratio (before encoding and after encoding). Actually, you can use 1 in 2 these criteria, but I prefer to use both of them.

1. Firstly, to calculate entropy, I will import scipy.stats with the function entropy (e.g. from scipy.stats import entropy). I will calculate the entropy of image before and after encoding. Before to do that, I will store the decompressed image into numpy array (I save this array into npy file to load)

```
# 2 ways
# save rebuiltImaage -> jpg (using scipy.misc)
scipy.misc.imsave('outfile.jpg', np.array(rebuiltImg, dtype = np.float32))
# save rebuiltImage -> numpy array
np.save("rebuiltImage.npy", np.array(rebuiltImg, dtype = np.float32))
```

Next, I calculate the entropy after and before compressing. The way of calculation of both of them is similar (I only change the image after and before compressing). Lastly, I calculate the difference between these two entropies (to make sure, you can use ABS to eliminate all negative values). I will show you some results in the next part.

```
imgReshaped = Image.open(image)
  = imgReshaped.size[0]
  = imgReshaped.size[0]
imag = np.array(imgReshaped.getdata()).reshape(X,Y)
sumDCT = np.array([np.zeros(8) for x in range(8)])
for x in range(0,X,8):
    for y in range(0,Y,8):
        sumDCT+=abs(jpeg.DCT(imag[x:x+8,y:y+8]))
sumDCT = sumDCT / np.sum(sumDCT)
print("Entropy before encoding", entropy(np.reshape(sumDCT, (64))))
imgAfter = Image.fromarray(np.load("rebuiltImage.npy"))
XAfter = imgAfter.size[0]
YAfter = imgAfter.size[1]
imagAfter = np.array(imgAfter.getdata()).reshape(XAfter,YAfter)
sumDCTAfter = np.array([np.zeros(8) for x in range(8)])
for x in range(0,XAfter,8):
    for y in range(0,YAfter,8):
        sumDCTAfter+=abs(jpeg.DCT(imagAfter[x:x+8,y:y+8]))
sumDCTAfter = sumDCTAfter / np.sum(sumDCTAfter)
print("Entropy after encoding", entropy(np.reshape(sumDCTAfter, (64))))
print("Entropy BEFORE - Entropy AFTER", entropy(np.reshape(sumDCT, (64)))-entropy(np.reshape(sumDCTAfter, (64)));
```

2. Secondly, I also calculate compression ratio with the formula Ratio = Before_Compressing / After_Compressing

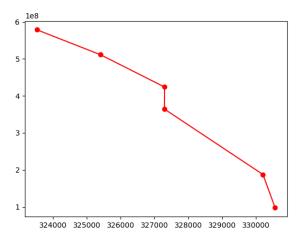
The compression ratio is quite simple, I just apply the simple formula Ration = Size_Before / Size_After (with Size is the file size of two images after and before compressing)

3. Lastly, I will give the compression curve for different bias (e.g. I choose 20, 50, 100, 200, 500, 1000). Since the curve is made from matplotlib.pyplot, it requires to calculate the plot size (Axis X, Axis Y). Hence, I have to modify a little bit the function decomposeImageWithWavelet() and biasDecomposing() by adding the size parameter (as well as return its value).

```
biasDecomposing(img, LO_D, HI_D, compteur, deathZone, discretZone, size, energie):
   compteur -= 1
if(compteur == 0):
       a1 = jp2.decompose(LO_D, LO_D, img)
       al, size, energie = biasDecomposing(jp2.decompose(LO_D, LO_D, img), LO_D, HI_D, compteur, deathZone, discretZone, size, energie)
       size += size
energie += energie
    im_out=np.double(np.zeros(img.shape))
   d1 = jp2.decompose(HI_D, HI_D, img)
   h1 = jp2.decompose(LO_D, HI_D, img)
   v1= jp2.decompose(HI D, LO D, img)
   h1 = applyContourByBias(h1, discretZone, deathZone)
   v1 = applyContourByBias(v1, discretZone, deathZone)
   d1 = applyContourByBias(d1, discretZone, deathZone)
   size += (h1.size - np.count_nonzero(h1) + v1.size - np.count_nonzero(v1) + d1.size - np.count_nonzero(d1))
   energie += ((h1**2).sum() + (v1**2).sum() + (d1**2).sum())
   im_out[0:a1.shape[0],0:a1.shape[1]] = a1
    im_out[0:a1.shape[0],a1.shape[0]:2*a1.shape[0]] = h1
    im_out[a1.shape[0]:2*a1.shape[0],0:a1.shape[1]] = v1
   im_out[a1.shape[0]:2*a1.shape[0],a1.shape[0]:2*a1.shape[0]] = d1
        r<mark>n im_out, size, energie</mark> # im_out is a string -> but you can store the returned value by array -> return [a1, h1, v1, d1]
def decomposeImageWithWavelet(cpt, cheminImage, deathZone, discretZone):
    img = np.double(plt.imread(cheminImage))
    img, size, energie = biasDecomposing(img, LO_D, HI_D, cpt, deathZone, discretZone, 0, 0)
      turn img,size,energie    # return a decomposed image -> many sub-images (quadrants)
```

The implementation of compression curve will be:

I will show you the curve of 6 levels (20, 50, 100, 200, 500, 1000) (of course, you can change the parameter to generate a new curve)

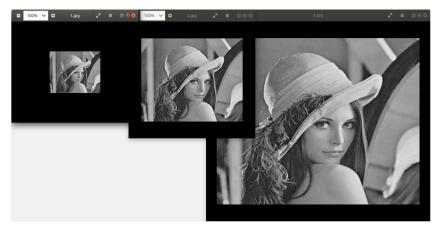


Compression curve

The results of ROI (Region of Interest) will be shown in the next part, the value of wavelet (that you chose) affects on the ROI (for example: I just detect the face of Lena)

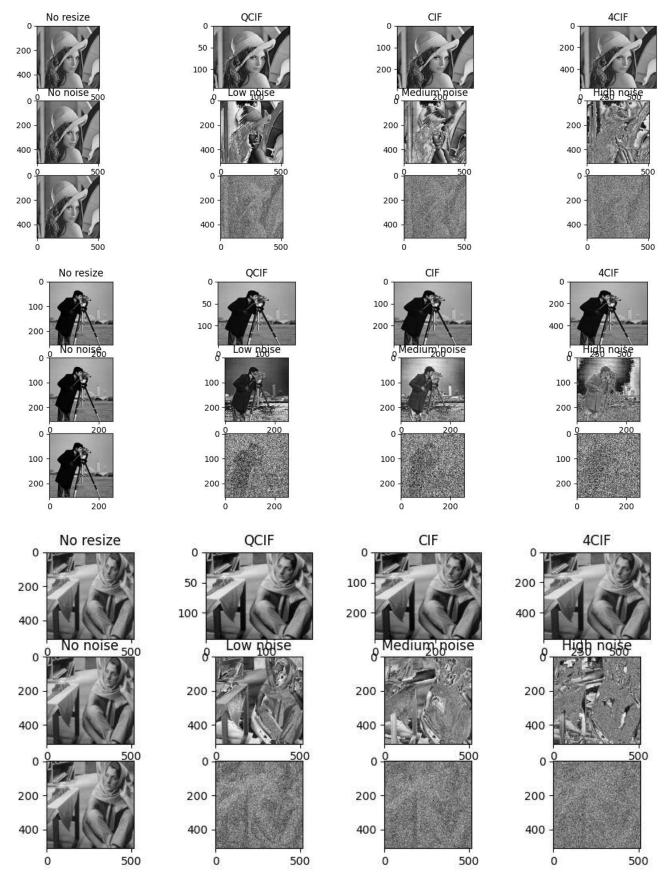
Part 4: Results of version 1

3 levels of spatial scalability (I make the test with 3 pictures: "Lena.jpg", "cameraman.tif", "barbara.jpg")

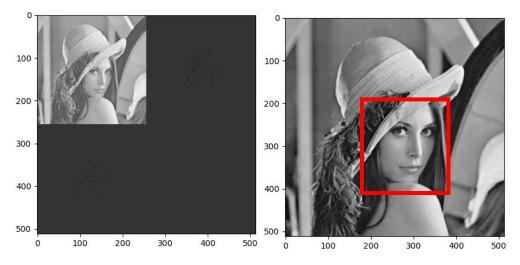




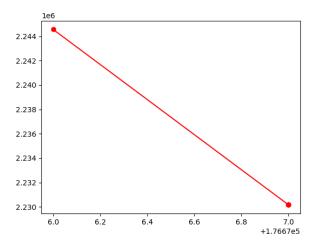




Firstly, I set the number of 4 quadrants recursive is 1, it means 4^1 = 4 (quadrants) et let's see what happens?



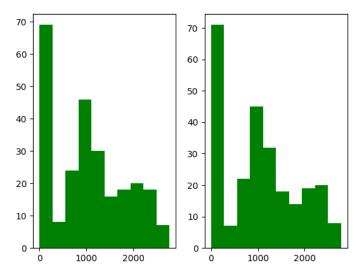
The decompressed image has the high quality 😊, and the red square is ROI (right image)



The compression curve is linear

Entropy before encoding 1.140041037165215
Entropy after encoding 1.1883335138751765
Entropy BEFORE - Entropy AFTER -0.04829247670996151
compression ratio : 3.3409168822264377%

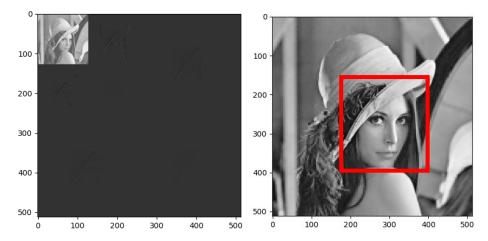
Entropy (before and after coding) as well as the compression ratio (3,34%)



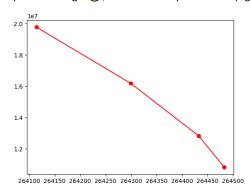
The histogram before (left) and after (right)

voluton i . man man day

Now, I would like to change some parameter (coefficient) and to see what happens. Firstly, I changed the multipleQuandrantRecursive to 2 (we will get 4^2 = 16 quadrants). All parameters remaining are fixed.



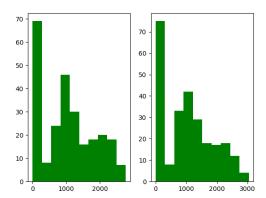
The decompressed image (a), and the red square is ROI (right image)



The compression curve (quite linear, but not too many)

```
Entropy before encoding 1.140041037165215
Entropy after encoding 0.9209429218999929
Entropy BEFORE - Entropy AFTER 0.21909811526522205
compression ratio : 12.229251721881141%
```

Entropy (before and after coding) as well as the compression ratio (12,22%)



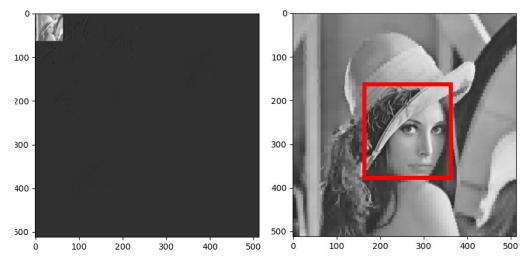
The histogram before (left) and after (right)

When I select multipleQuandrantRecursive = 1, the decoded image has a better quality than when I selected multipleQuandrantRecursive = 2. So multipleQuandrantRecursive affects to the quality of compression. Actually, this parameters shows how many compression times (when you compress an image many times, of course, its quality is not good).

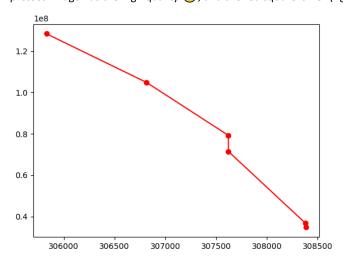
What happens next? All parameters remaining are fixed.

So, the compression curve which show the compression energy is linear. Moreover, the compression ratio is lower (3,34 % vs 12,22 %). So, when I decrease multipleQuandrantRecursive, the quality of decoded image is better, but the compression ration is decreased.

Case 1 : Value "3" => 4^3 = 64 quadrants



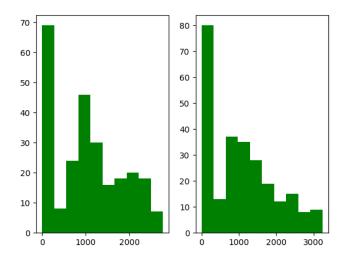
The decompressed image has the high quality 😊, and the red square is ROI (right image)



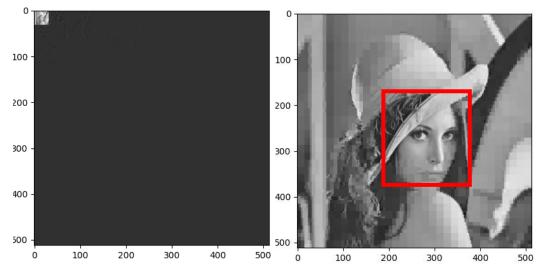
The compression curve

Entropy before encoding 1.140041037165215 Entropy after encoding 0.6040720369002065 Entropy BEFORE - Entropy AFTER 0.5359690002650085 compression ratio : 52.46361454794777%

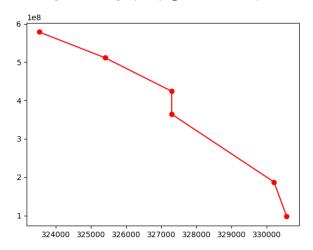
The entropy before and after as well as the compression ratio (52,46%)



The histogram before (left) and after (right)



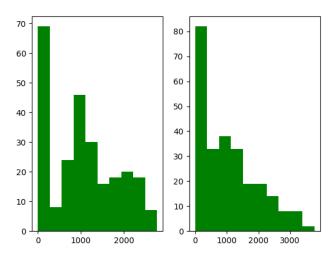
The decompressed image has the high quality (a), and the red square is ROI (right image)



The compression curve

Entropy before encoding 1.140041037165215
Entropy after encoding 0.603903660171036
Entropy BEFORE - Entropy AFTER 0.536137376994179
compression ratio : 53.61672951662667%

The entropy before and after as well as the compression ratio (53,61%)



The histogram before (left) and after (right)

So, I can summary as follows

multipleQuandrantRecursive	Number of quadrants (using ABS)	Compression ratio (%)
0	NULL/ERROR	NULL/ERROR
1	4	3,34
2	16	12,22
3	64	52,46
4	256	53,61
5	1024	53,96
6	2048	53,97

For 3 and 4,5,6 and above, the compression ratio is quite similar (53,46% vs 53,96% vs 53,97%). So, I think 1,2,3 is a three ideal levels (1 is the high, 2 is the medium, 3 is the low).