

Bayesian Statistics Workbook

Vu Nguyen Quang Duy

1 Bayes' Rules

1.1 Buidling up to Bayes' Rule

Excercise 1. *Comparing the prior and posterior*

For each scenario below, you're given a pair of events, A and B . Explain what you believe to be the relationship between the posterior and prior probabilities of B : $P(B|A) > P(B)$ or $P(B|A) < P(B)$

- a) A = you just finished reading Lambda Literary Award-winning author Nicole Dennis-Benn's first novel, and you enjoyed it! B = you will also enjot Benn's newest novel.
- b) A = it's 0 degrees Fahrenheit in Minnesota on a January day. B = it will be 60 degrees tomorrow.
- c) A = the authors only got 3 hours of sleep last night. B = the authors make several typos in their writing today.
- d) A = your friend includes three hashtags in their tweet. B = the tweet gets retweeted.

Solution

- a) **Answer:** $P(B|A) > P(B)$
 - The prior probability, $P(B)$: The general probability of enjoying Benn's newest novel before reading any of her previous work.
 - The posterior probability, $P(B | A)$: The updated probability of enjoying Benn's newest novel, given that her first novel was read and enjoyed.

The event A (enjoying the first novel) is positive evidence that provides a reason to increase belief in event B (enjoying the newest novel). A favorable experience with the author's work makes the updated belief (the posterior) stronger and therefore higher than the initial belief (the prior).

b) **Answer:** $P(B|A) < P(B)$

- The prior probability, $P(B)$: The general probability that it will be 60 degrees tomorrow.
- The posterior probability, $P(B | A)$: The updated probability that it will be 60 degrees tomorrow, given that it was 0 degrees Fahrenheit yesterday.

The event A (a temperature of 0°F yesterday) is negative evidence that provides a reason to decrease the belief in event B (a temperature of 60°F tomorrow). A temperature of 0°F makes it significantly less likely that the temperature will be a relatively mild 60°F the next day. This new information acts as negative evidence, causing a decrease in the belief of event B .

c) **Answer:** $P(B|A) > P(B)$

- The prior probability, $P(B)$: The general probability that the authors will make several typos in their writing today.
- The posterior probability, $P(B | A)$: The updated probability that the authors will make several typos, given they only got 3 hours of sleep last night.

The event A (only 3 hours of sleep) is positive evidence that increases the probability of event B (making typos). Lack of sleep is a well-known factor that impairs cognitive function and attention to detail, making errors like typos more probable. The updated belief is therefore higher than the initial belief.

d) **Answer:** $P(B|A) > P(B)$

- The prior probability, $P(B)$: The general probability that the tweet will be retweeted. This is the baseline likelihood without knowing anything about the tweet's content or format.
- The posterior probability, $P(B | A)$: the updated probability that the tweet will be retweeted, given that it includes three hashtags.

The event A (including three hashtags) is positive evidence that increases the probability of event B (the tweet being retweeted). Research on social media engagement shows that tweets with hashtags, especially a moderate number, tend to have wider reach and higher engagement, which includes retweets. Therefore, the updated belief is higher than the initial belief.