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Heterogeneous Agents and Unemployment in a New-Keynesian General Equilibrium Model

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Abstract

I develop a Heterogeneous Agent New Keynesian (HANK) model featuring a Search and Matching (SAM) labor market, which subjects households to cyclical unemployment risk, and study the resulting behavior. I first analyze the general properties emerging from a standard heterogeneous agents model w.r.t transitory shocks to income and interest rates, and discuss the relevance of a one-asset HANK model against the two-asset alternative. I proceed to carefully analyze the heterogeneous responses in general equilibrium. Among my findings are 1) Wealth-poor households experience significantly larger welfare losses when subjected to adverse shocks in general equilibrium 2) Unemployment risk becomes a leading determinant of cyclical consumption as one moves down the wealth distribution, 3) A reform lowering unemployment benefits has no effect on stabilization in response to shocks if the reform is revenue-neutral, but induces significant amplification if the reform reduces government debt and household assets. Finally I attempt to replicate the results of Broer et al. (2020a) who show that there exists a powerful propagation mechanism when vacancy creation is costly, job separations are endogenous and households save precautionarily due to endogenous unemployment risk. I estimate the model w.r.t the parameters governing costly vacancy creation and endogenous separations and find that the first two mechanisms alone generate long-lived responses, but endogenous precautionary savings have no effect.

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Contents

1 Introduction

The field of macroeconomics have in recent decades experienced an increased focus on the importance of household heterogeneity at the micro level in shaping aggregate outcomes. I take this to heart by formulating and analyzing a Heterogeneous Agent New-Keynesian (HANK) model. The household side of the model builds on a Aiyagari-Bewley-Hugget-Imrohoroglu incomplete markets model, where household heterogeneity arise due to uninsurable idiosyncratic earnings risk. The supply side contains the basic New-Keynesian features (monopolistic competition and price rigidities etc.) augmented with a search and matching (SAM) labor market. This addition generates equilibrium unemployment due to matching frictions. I solve the model using newly developed methods in Auclert et al. (2019).

In a standard representative agent New-Keynesian (RANK) model the role of assets is to smooth consumption over time and insure against aggregate risk. In my HANK model assets have the additional purpose of insuring against idiosyncratic risk and unemployment risk. I calibrate the model to a realistic wealth distribution using discount factor heterogeneity. This implies the existence of households with low amounts of assets - either due an unlucky income path or subjective impatience - who in turn are poorly insured against risk. I find that these households in particular respond strongly to changes in aggregates such as wages, employment and unemployment risk, while households at the top of the wealth distribution respond primarily to interest rate changes. Consequently, the effects on welfare of an adverse supply shock in general equilibrium are significantly different across the wealth distribution. Households in the poorest wealth decile suffer welfare losses 18 times the losses of the richest decile and 4 times the losses of the median household as measured by consumption equivalent variation. Furthermore, unemployed households tend to have higher welfare losses compared to their employed counterparts and this owes primarily to drops in the job-finding rate which triggers a precautionary savings motive, since their unemployment spell might be prolonged severely.

I next tap explicitly into the importance of insurance by performing a policy experiment where I cut the level of unemployment benefits to investigate the stabilizing properties of unemployment benefits on the cycle. I find that if the policy change is revenue-neutral households manage to self-insure against unemployment through assets, and this implies that the cyclical proprieties of the model are roughly unchanged. On the other hand, if the policy change is not revenue-neutral and the government permanently lowers public debt in response to lower expenditures adverse shocks are severely amplificated. The mechanism is that public debt is an asset to households, and when this is cut simultaneously with unemployment benefits the degree of insurance against risk declines. The decline in wealth simultaneously increases households' marginal propensities to consume, and this increased sensitivity to changes in income and employment is what generates amplification.

The final chapter of the thesis investigates the feedback loop uncovered in Broer et al. (2020a), in which costly vacancy creation, endogenous separations and precautionary savings interact to propagate shocks. Endogenous separations imply that unemployment increases more rapidly in response to an adverse shock, while costly vacancy creation insures that the job-finding rate remains suppressed for longer. A prolonged increase in unemployment risk triggers households' precautionary savings motive and reduces aggregate demand, hence exacerbating the crisis. I implement the theory of endogenous separations from their paper and add costly vacancy creation in the spirit of Fujita and Ramey (2007). I find that costly vacancy creation generates long-lived, hump-shaped responses for employment and output, while the presence of endogenous separations amplify these responses. Notably, I find no interaction between these prolonged responses and precautionary savings. I conjecture that this is because the household model deliver only short-lived consumption responses to changes in unemployment risk, and discuss alternate modeling choices in this regard.

Before moving to the main matter of the thesis I briefly discuss the existing literature on the topic of HANK models and unemployment risk.

1.1 Literature Review

Household heterogeneity has a long tradition in macroeconomics, but has only recently moved seriously into the analyses of business cycles.[1,2] The first generation of heterogeneous agent models, which modern HANK models derive from, can be found in the seminal papers of Bewley (1976), Imrohoroğlu (1989), Huggett (1993), Aiyagari (1994) and Krusell and Smith (1998). The defining feature of these heterogeneous agent models is that they assume incomplete markets. For households this imply that there is not a complete set of Arrow-Debreu state contingent claims to fully insure against risk. If house-

[1] I do not count two-agent models featuring one Ricardian agent and one rule-of-thumb agent as in Campbell and Mankiw (1989) as heterogeneous agent models.

[2] Inspired by Ben Moll's interview with David Beckworth found here. See also this rundown by Beatrice Cherrier.

holds are subjected to different shocks this implies different consumption/savings paths when conditioning on the same initial conditions. The latter two papers widened the literature by also implementing heterogeneous households in general equilibrium models. The recent advances that merge these household models with the New-Keynesian models often used for business cycle analysis, attempt to formulate models that are consistent with the emergence of rich microeconomic evidence on households' balance sheets and behavior. The introduction of heterogeneous households has a great deal of implications for general equilibrium NK models. These include predictions related to marginal propensities to consume (MPCs) and fiscal policy, the risk-free rate, Ricardian equivalence, equilibrium determinacy, precautionary savings and credit constraints. Furthermore, the introduction of heterogeneity allows for the analysis of distributional effects, something which representative agent models per definition cannot do. HANK models have already been applied to a wide amount of central macroeconomic questions such as the transmission of monetary policy (Kaplan et al. (2018), Auclert et al. (2020)), the distributional effects of monetary policy (Gornemann et al. (2016), Auclert (2019)) fiscal policy (Hagedorn et al. (2019)) and automatic stabilizers (McKay and Reis (2016b)).

My paper is not the first to merge a HANK model with theories of unemployment such as the Diamond–Mortensen–Pissarides (DMP) search and matching model - a HANK&SAM model, if you will. Gornemann et al. (2016) uses a HANK&SAM model to evaluate the welfare effects of monetary policy. Challe et al. (2017) also adopts the HANK&SAM framework[3] and focus on the interaction between aggregate demand and precautionary savings. They find that this interaction has lead to strong propagation in historic recessions. Ravn and Sterk (2016) study the theoretical properties of a HANK&SAM model using an analytical approach. They discuss indeterminacy that may arise from countercyclical earnings risk and show that nominal rigidities and incomplete markets reinforce each other in the propagation of shocks.

2 The Model

This section presents the main model. The backbone of the model is the HANK-framework, which merges the often applied NK models with a household sector con-

[3] Challe et al. (2017) do not assume incomplete markets for all household, but rather assume risk sharing between employed households, but not between employed and unemployed households. They have no additional idiosyncratic earnings risk.

taining heterogeneous agents. The NK characteristics include investment adjustment costs, price rigidities and monopolistic competition. Additionally, the model features a basic SAM labor market where households inelasticity supply labor which is matched with vacancy posting firms through a matching function. The model features one asset for households to save in, unlike the recent work of Kaplan & Violante (Kaplan and Violante (2014), Kaplan et al. (2018) - henceforth the KV model). I will return to the implications of this choice in later sections. Household assets are managed by a mutual fund who can invest in either government bonds (which are in positive supply) or firm equity shares. This setup is similar to Gornemann et al. (2016), Auclert et al. (2020).

Similar models have been applied in the recent literature. Gornemann et al. (2016) presents the perhaps closest resembling model in featuring discount factor heterogeneity and a search and matching labor market in addition to the main HANK components. Their model features aggregate uncertainty, which allows them to calibrate to secondary business cycle moments. The model of Hagedorn et al. (2019) is also closely related, with key differences being that their model features endogenous labor supply, no search and matching labor market, and a constant interest rate rule. McKay and Reis (2016b) applies a medium-sized HANK model to analyze automatic stabilizers in the US. Their model also features uninsurable risk, though only for a fraction of the population. The main model contains only exogenous transitions between employment states, but the authors let the Markov chain governing the transition depend on aggregate shocks. Comparably, the addition of a search-and-matching labor market in this paper adds an endogenous theory of unemployment as well as being resistant to the Lucas critique.

2.1 Households

There is a continuum of infinitely-lived single-member households of mass 1. Households are heterogeneous in several dimensions, and the expected lifetime utility of a household i depends on a basket of non-durable goods $c_{i,t}$. Lifetime utility is given by:

$$U_{i,t} = \sum_t \beta_i^t \mathbb{E}_{i,t} \frac{c_{i,t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}, \tag{1}$$

where the expectation is taken over earnings $e_{i,t}$, which are subject to idiosyncratic risk. There are no aggregate shocks, and, appealing to a law of large numbers, there is no aggregate uncertainty in the economy. I assume that flow utility is of the CRRA-form with σ denoting the intertemporal elasticity of substitution. The discount factor β varies

across households, but is constant over time to the individual household. The variation in this parameter across the population is used to match the empirical wealth distribution, see Krusell and Smith (1998), Krueger et al. (2016), Carroll et al. (2017) for similar approaches. The special case with only two values of β results in the patient/impatient household setup used in a variety of models. The heterogeneity in discount factors is a convenient, though ad hoc way, to capture not only subjective preferences but also characteristics such as age, education, gender and so forth which are absent in the household model but important determinants in the accumulation of assets.
Households maximize utility subject to the budget constraint and the borrowing constraint:

$$c_{i,t} + a_{i,t} = (1 + r_t^a(a_{i,t-1}))a_{i,t-1} + I_{i,t} - \tau^I(I_{i,t}) + T_t \tag{2}$$

$$a_t \geq \underline{a} \tag{3}$$

The left hand side of budget constraint are expenditures: The household can spend on either consumption or save in real, end-of-period assets $a_{i,t}$. The right hand side is current net-worth. This is composed of last periods asset stock plus real returns, $(1 + r_t^a(a_{i,t-1}))\,a_{i,t-1}$ and income net taxes $I_{i,t} - \tau^I(I_{i,t})$. Finally T_t denotes lump sum transfers to the household from the government. The income tax function $\tau^I(\bullet)$ is fitted to danish tax data and therefore progressive. Households are allowed to borrow up to the limit $\underline{a}$, but are subject to additional interest κ_{r^a} if in debt. $r_t^a(a_{i,t-1})$ is hence the relevant interest rate for a household with assets $a_{i,t-1}$ where $r_t^a(a_{i,t-1}) = r_t^a$ if $a_{i,t-1} \geq 0$ and $r_t^a + \kappa_{r^a}$ if $a_{i,t-1} < 0$.

The income of households depend on employment status and idiosyncratic earnings $e_{i,t}$, which are specific to the individual household. Letting k be an indicator for whether the household is currently employed ($k = N$) or unemployed ($k = U$) income is given by:

$$I_{i,t} = \begin{cases} w_t e_{i,t}, & k = N \\ b e_{i,t}, & k = U \end{cases} \tag{4}$$

An employed household inelastically supplies one unit of labor and receives the going wage rate w_t. If the household is unemployed it receives benefits b from the government.

In both states income is assumed proportional to earnings risk e.[4] Earnings $e_{i,t}$ follow a persistent AR(1) process in logs with normally distributed innovations:

$$\log e_{i,t} = \rho \log e_{i,t-1} + \sigma^e \varepsilon_{i,t}, \qquad \varepsilon \backsim \mathcal{N}(0,1) \tag{5}$$

This is the standard way in the literature of introducing heterogeneity. I describe this process in more detail in section 3, which also explains how I numerically solve the households' problem.

I preface this section by presenting the Euler equations governing the allocation of consumption over time here to provide intuition. Non-constrained households, understood as households for which the credit constraint $a_t \geq \underline{a}$ does not bind, choose consumption in accordance with the following Euler equations, depending on whether they are currently employed or not:

$$\left(c_{i,t}^{k=N}\right)^{-\frac{1}{\sigma}} = \beta_i \mathbb{E}_{i,t} R_{t+1}^a \left[\left(1-\delta\left(1-q_{t+1}\right)\right)\left(c_{i,t+1}^{k=N}\right)^{-\frac{1}{\sigma}} + \delta\left(1-q_{t+1}\right)\left(c_{i,t+1}^{k=U}\right)^{-\frac{1}{\sigma}}\right] \tag{6}$$

$$\left(c_{i,t}^{k=U}\right)^{-\frac{1}{\sigma}} = \beta_i \mathbb{E}_{i,t} R_{t+1}^a \left[q_{t+1}\left(c_{i,t+1}^{k=N}\right)^{-\frac{1}{\sigma}} + \left(1-q_{t+1}\right)\left(c_{i,t+1}^{k=U}\right)^{-\frac{1}{\sigma}}\right], \tag{7}$$

where q, δ^N denote respectively the probability of finding a job if unemployed, and the probability of being separated from a job if employed, and $R_{t+1}^a \equiv 1 + r_{t+1}^a(a_{i,t})$. Abstracting from employment dynamics for a second ($q = 1, \delta^N = 0$) the equations reduce to the standard Euler equations where households choose consumption today so as to equate the marginal utility of consumption with the expected marginal utility of the next period times the relative price $\beta_i R_{t+1}^a$. The main difference compared to standard NK models then lies in the expectational term taken over the idiosyncratic risk in (4). Concavity of the utility function implies that households are risk-averse, and this generates a precautionary savings motive. Hence they save more compared to the case with no risk to build up a buffer which they can rely on in case a bad state of earnings is realized in the future.[5] Assuming $0 < q < 1, 0 < \delta^N < 1$ adds unemployment risk to the households problem. This adds a second precautionary savings motive, as employed households save to ensure against potential future unemployment. Similarly, unemployed households save more than usual in the event that their unemployment spell is prolonged. This channel is endogenous over the cycle due to movements in the job finding rate q.

[4] A similar assumption is used in McKay and Reis (2016b). This is a simple way of capturing that the benefits households are entitled to usually depend on prior work duration and wages.

[5] Simple NK models can feature precautionary savings but this is usually induced by aggregate risk. Since idiosyncratic risk is usually larger than aggregate risk the degree of precautionary savings is usually larger in HA models.

2.2 Firms

The Final good Firm. There is a representative firm which assembles intermediate goods $y_{j,t}$ into a final good Y_t using CES technology:

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\epsilon_P - 1}{\epsilon_P}} dj \right)^{\frac{\epsilon_P}{\epsilon_P - 1}},$$

where ϵ_p is the elasticity of substitution between intermediate goods. The representative firm minimizes its cost, and the resulting demand for $y_{j,t}$ is given by:

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\epsilon_P} Y_t, \tag{8}$$

where P_t is the price associated with Y_t, and $p_{j,t}$ with $y_{j,t}$.

Intermediate goods firms. There exists a continuum of firms who produces a variety of intermediate goods $y_{j,t}$. Firm j produce using using capital $k_{j,t-1}$ and labor $n_{j,t}$ with Cobb-Douglas technology:

$$y_{j,t} = Z_t k_{j,t-1}^{\alpha} n_{j,t}^{1-\alpha}, \tag{9}$$

where α denotes the output elasticity of capital and Z_t denotes exogenous, deterministic total factor productivity (TFP). Each firm produces a slightly differentiated good which gives rise to monopolistic competition. Thus firms maximize dividends taking into account the demand schedule in (8). As is defining in Keynesian models nominal rigidities are present, the primary one being rigid prices. I follow Rotemberg (1982) in assuming there is a quadratic cost associated with changing prices from the long run equilibrium value of gross inflation $\overline{\Pi}$:

$$\Phi^P \left(p_{j,t-1}, p_{j,t} \right) = \frac{\kappa_P}{2} \left[\frac{p_{j,t}}{p_{j,t-1}} - \overline{\Pi} \right]^2 Y_t,$$

where the parameter κ_P determines how large the adjustment cost is.
Labor and capital are rented from labor and capital firms respectively at rates r_t^N and r_t^K. Resulting flow dividends of the intermediate goods firms are given by:

$$div_{j,t} = \frac{p_{j,t}}{P_t} y_{j,t} - r_t^N n_{j,t} - r_t^K k_{j,t-1} - \Phi^P \left(p_{j,t-1}, p_{j,t} \right)$$

Appendix A show that, in a symmetric equilibrium where all firms choose to set the same prices and demand the same amounts of capital and labor, the first-order condition for price-setting yields the New-Keynesian Philips-curve:

$$(1-\epsilon_p)+\epsilon_p mc_t-\kappa_P\left(\pi_t-\bar{\Pi}\right)\pi_t+\frac{\kappa_P}{1+r_{t+1}}\left(\pi_{t+1}-\bar{\Pi}\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t}=0 \tag{10}$$

where mc_t denotes the real marginal cost.[6] The first-order condition for labor and capital equates the rental rates with marginal products:

$$MPL_t=(1-\alpha)\,mc_t\frac{Y_t}{N_t}=r_t^N \tag{11}$$

$$MPK_t=\alpha mc_t\frac{Y_t}{K_{t-1}}=r_t^K \tag{12}$$

Labor service firms. Intermediate goods firm hire labor from labor service firms who acts as intermediaries between households, who supply labor, and intermediate goods firms who demand labor. Labor service firms post vacancies V_t which are filled with probability m_t, in which case they pay the real wage rate w_t to the matched household. Existing matches are destroyed at an exogenous rate δ^N. The value of a match to a labor firm is given by the Bellman equation:

$$\begin{aligned}\mathcal{J}_t^m&=\left(r_t^N-w_t\right)+\frac{\left(1-\delta^N\right)}{1+r_{t+1}}\mathcal{J}_{t+1}^m\\&=\left(MPL_t-w_t\right)+\frac{\left(1-\delta^N\right)}{1+r_{t+1}}\mathcal{J}_{t+1}^m\end{aligned} \tag{13}$$

I assume a recurrent cost of posting vacancies κ_V which must be payed in each period if firms wish to continue posting vacancies. The value of an unfilled vacancy is then:

$$\mathcal{J}_t^V=-\kappa_V+m_t\mathcal{J}_t^m$$

i.e. the value of a match, obtained with probability m_t, minus the cost. Free entry in the labor firm sector implies that the value of a vacancy $\mathcal{J}_t^V$ must be zero. Accordingly, the value of match is $\mathcal{J}_t^m=\frac{\kappa_V}{m_t}$, which combined with the value function in (13) generates the following labor demand schedule:

$$\frac{\kappa_V}{m_t}=\left(MPL_t-w_t\right)+\frac{\left(1-\delta^N\right)}{1+r_{t+1}}\frac{\kappa_V}{m_{t+1}} \tag{14}$$

[6]The marginal cost is defined as $mc_t\equiv\partial\Xi_t\left(y_{j,t}\right)/\partial y_{j,t}$ where $\Xi_t\left(y_{j,t}\right)=\min_{k_{j,t},n_{j,t}}div_{j,t}$ subject to the production function (9).

with resulting dividends $div_t^N = \left(r_t^N - w_t\right) N_t - \kappa_V$.

Capital firms. There is a representative capital firm which may create capital by investing units of the final goods in accordance with the law of motion:

$$K_t = (1-\delta) K_{t-1} + Z_{t-1}^I I_{t-1}, \tag{15}$$

where δ^K denotes the depreciation rate of capital, I_t denotes flow investment in capital and Z^I is investment specific technology. Capital accumulation features time-to-build which manifests in lagged investment entering (15). Adjusting the flow of investments is subject to adjustment costs $\phi_I\left(\frac{I_t}{I_{t-1}}\right) I_t$, such that dividends/profits are given by the revenue obtained from selling capital to the intermediate goods firms $r_t^K K_{t-1}$ minus costs of investing in capital and adjusting investments:

$$div_t^K = r_t^K K_{t-1} - I_t \left(1 + \phi_I\left(\frac{I_t}{I_{t-1}}\right)\right) \tag{16}$$

The function ϕ_I is assumed to be of the usual quadratic form:

$$\phi_I\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa_I}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2,$$

where κ_I measures the size of the adjustment costs. The first-order condition for investment reflect standard Q theory, and is given by:

$$1 + \frac{I_t}{I_{t-1}}\phi_I'\left(\frac{I_t}{I_{t-1}}\right) + \phi_I\left(\frac{I_t}{I_{t-1}}\right) = Q_{t+1}Z_t^I + \frac{1}{1+r_{t+1}}\phi_I'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2,$$

where Q_t obeys:

$$Q_t = \frac{1}{1+r_{t+1}}\left[(1-\delta) Q_{t+1} + r_{t+1}^k\right]$$

In steady state where $I_t = I_{t-1} = I$ and hence $\phi_I\left(\frac{I_t}{I_{t-1}}\right) = 0$, $Q = 1$ the price of capital r^K equals the steady state user-cost $r + \delta$. Outside the steady state investment and capital responds to changes in the price r_t^K and the discount rate r_{t+1}, but adjustment costs ensure that the period-by-period changes in investment are reduced and spread out over the duration of the shock.

Aggregate Dividends. Aggregate dividends, which are payed out to households who hold equity shares, are given by the sum of dividends from intermediate good firms, labor service firms, and the capital firm net a fixed cost Φ^F plus government transfers T^F.

$$\begin{aligned} div_t &= \int div_{j,t} dj + div_t^K + div_t^L - \Phi^F \\ &= Y_t - w_t N_t - \kappa_V - I_t \left(1 + \phi_I \left(\frac{I_t}{I_{t-1}}\right)\right) - \Phi^P \left(P_{t-1}, P_t\right) - \Phi^F + T^F \end{aligned}$$

2.3 Mutual Fund

It is common to assume that dividends are payed out to households lump sum. Given incomplete markets the distribution of these dividends to households can be an important determinant for aggregate demand, see Bilbiie (2008), Broer et al. (2020b), Werning (2015). Hence the author must argue for the allocation rule chosen, though often somewhat arbitrarily constructed. I treat dividends as being payed out to households who own shares in firms. Since there is wealth heterogeneity this imposes an implicit distribution rule for dividends which is proportional to wealth.

The setup, which closely resembles that of Auclert et al. (2020), is as follows: Household assets are administered by a mutual fund, where the aggregate stock of assets A_t is invested in either government bonds B_t or firm equity $\varsigma_{j,t}$ and pays a collective return r^a_{t+1} in the following period. Government bonds pays r_{t+1} which is the rate set by the central bank. Firm equity shares are priced at $p^e_{j,t}$. Households that hold shares $\varsigma_{j,t}$ are entitled to dividends $div_{j,t+1}$ per share. The absolute return to an equity share $\varsigma_{j,t}$ is then given by $div_{j,t+1} + p^e_{j,t+1}$. The mutual fund solves the following maximization problem:

$$V_t^{MF} \equiv \max_{\varsigma_{j,t}, B_t} \int \left(div_{j,t+1} + p^e_{j,t+1}\right) \varsigma_{j,t} dj + \left(1 + r_{t+1}\right) B_t - \left(1 + r^a_{t+1}\right) A_t$$

$$s.t.$$

$$A_t = B_t + \int p^e_{j,t} \varsigma_{j,t} dj,$$

where the constraint states that total value of household assets must equal the value of government bonds plus the value of firm equity in each period. Since this is a linear problem equilibrium is only compatible with zero profits. Hence the first-order conditions are standard no-arbitrage conditions:

$$r^a_{t+1} = r_{t+1} \tag{17}$$

$$1 + r^a_{t+1} = \frac{p^e_{t+1} + div_{t+1}}{p^e_t} \tag{18}$$

In addition, since firms are symmetrical and equity shares sum to 1 ($\int p^e_{j,t} \varsigma_{j,t} dj = p^e_t$) we have:

$$A_t = B_t + p^e_t,$$

The central bank supplies nominal cash reserves which are net zero in supply. This follows the usual Paradigm for NK models in modeling the cashless limit of Woodford (1998). Holding assets from the nominal stock in period t pays the nominal interest rate i_t set by the central bank. No-arbitrage implies that the real return $1 + r_{t+1}$ must obey the Fisher equation:

$$(1 + r_{t+1})(1 + \pi_{t+1}) = (1 + i_t)$$

The above setup has a small mechanical nuisance: In the presence of an unexpected shock the no-arbitrage condition (18) will fail, and the mutual fund will have non-zero profits in the impact period $t = 0$. As in Hagedorn et al. (2019) I assume that profits/deficit incurred by the fund is transferred to households through asset returns:

$$r^a_{t=0} = r_{t=0} + \frac{(div_{t=0} + p^e_{t=0}) + (1 + r_{t=0}) B_{t=-1} - (1 + r_{t=0}) A_{t=1}}{A_{t=-1}}$$

In all future periods the no-arbitrage condition holds and profits are zero.

Note from (18) that firm dividends are discounted using the rate r^a_{t+1} which per (17) equals the rate r_{t+1} . This explains why I use this rate to discount firm dividends in the prior sections, see for instance the Philips-curve in (10).

2.4 Labor Market

The labor market is a simple DMP search and matching labor market. Labor is supplied inelastically by households and households make no search decisions. Hence all households who are not employed at the start of a given period t search for a job, with the aggregate number of searchers S_t being given by:

$$S_t = 1 - N_{t-1}\left(1 - \delta^N\right), \tag{19}$$

i.e. the aggregate population minus workers who kept their job from the last period. Similarly, aggregate employment follows the law of motion:

$$N_t = N_{t-1}\left(1 - \delta^N\right) + M_t, \tag{20}$$

where M_t denotes the number of new matches made at the beginning of the period. The number of matches is determined by a constant returns-to-scale matching function, with the functional form attaining from Den Haan et al. (2000):

$$M_t = \frac{V \cdot S}{\left(S^\xi + V^\xi\right)^{\frac{1}{\xi}}} \tag{21}$$

The following definition holds relating the number of new matches, searchers, and vacancies:

$$M_t = q_t S_t = V_t m_t,$$

where q_t and m_t denote respectively the job-finding rate (probability of finding a job) and the matching probability (probability of filling a vacancy). Defining $\theta_t = \frac{V_t}{S_t}$ as a measure of labor market tightness, these can be expressed as:

$$q_t = \frac{\theta_t}{\left(1 + \theta_t^\xi\right)^{\frac{1}{\xi}}}, \qquad m_t = \frac{q_t}{\theta_t}$$

The functional form of the matching function ensure that these are well defined probabilities in the sense that $q_t \to 0, m_t \to 1$ as $\theta_t \to 0$, and similarly $q_t \to 1, m_t \to 0$ for $\theta_t \to \infty$. Unemployment is defined as the number of households without a job at the end of the period:

$$U_t = 1 - N_t,$$

and thus differs from the number of searchers by exactly the number of matches M_t made in the period. Note that the matching friction embedded in the matching function ensures the presence of involuntary unemployment is equilibrium since full employment requires a job-finding rate of 1, which only occurs when $\theta_t \to \infty$ which is infeasible.

2.5 Wage Formation

To close the labor market a model of wage formation is needed. It is common in the literature to assume that workers and firms engage in a Nash bargaining game over the matching surplus. The presence of matching frictions implies that a large bargaining set of wages exists (Hall (2005)). In general, any wage within this set could be the equilibrium bargaining wage under Nash bargaining. Given this indeterminacy - along with poor cyclical properties (see Shimer (2005)) - recent papers tend to calibrate the steady state wage and verify that this lies within the bargaining set, and assume simple, ad-hoc wage rules that determine the wage response outside of steady state (examples include Gornemann et al. (2016), McKay and Reis (2016a), Den Haan et al. (2018) and many more). In the baseline model I apply a wage rule of the form:

$$w_t = w^{ss} \left(\frac{\theta_t}{\theta^{ss}} \right)^{\eta}, \tag{22}$$

where θ measures labor market tightness, and η is the elasticity of wages w.r.t market tightness. This simple ad hoc rule implies that as the labor market becomes more tight, reflecting that firms finds it harder to obtain matches given vacancy posting, an upwards pressure is put on wages. For the calibration I choose a relatively low value of η such that wages move relatively little over the cycle, hence mimicking observed wage rigidities.

Given that the wage rate is not generated from a Nash bargaining game it can potentially, if calibrated inappropriately, lie outside the bargaining set of solutions. This would render it suboptimal for either workers or firms to engage in matching, and hence the equations describing the labor market would be incorrect. The Nash bargaining set is characterized as follows. The lower limit of the set satisfies that workers are indifferent being employed and unemployed:

$$\begin{aligned} \underline{w} e_{i,t} &= b e_{i,t} \\ \Leftrightarrow \underline{w} &= b \end{aligned}$$

The upper limit of the set equals firms reservation wage. The reservation wage is the highest wage under which the firm would still find it profitable to operate. This implies that the value of a match must be zero when evaluated at the reservation wage. Hence, the upper limit of the set is given by $\overline{w} = MPL_t$. The complete bargaining set is given by $[b, J_t]$. I verify numerically, conditional on calibration (see section 4), that the wage is in the aggregate bargaining set at all times during simulations.

2.6 Government Policy

The government raises income taxes which funds public consumption, unemployment benefits and interest payments on debt. Furthermore, it supplies bonds to households and pay interest rate r on these. With G denoting public consumption the government's real budget constraint is given by:

$$\begin{aligned} &\tau^I (w_t, N_t, b) + B_t \\ &= G_t + (1 - N_t) b + T_t + T^F + B_{t-1} (1 + r_t) \end{aligned} \tag{23}$$

The left hand side is revenue: It is composed of raised taxes, and the revenue that the supply of bonds generate. The term $T^I (w, N)$ constitutes aggregate income taxes given by:

$$\tau^I (w_t, N_t, b) = \int N_t \tau^I (w_t e) + (1 - N_t) \tau^I (be) \, d\mathcal{D}_t^e,$$

where $\mathcal{D}_t^e$ denotes the distribution of household over earnings e. The right-hand side of (23) are aggregate government expenses. It consists of public consumption, unemployment benefits, lump sum transfers to households (T), transfers to firms T^F and debt repayments plus interests. Transfers to households are zero in steady state and are reserved to achieve fiscal stability over the cycle. Transfers to firm T^F are instead used in steady state ensure long-run budget balance at a realistic debt-to-GDP ratio. Outside of steady state T^F is constant and hence cannot be identified from the fixed cost Φ^F.[7] The level of unemployment benefits b is exogenous and hence independent of wages.[8]

It is well known that the choice of financing public expenditures using either debt or taxes does not matter for economic outcomes under certain circumstances (Ricardian equivalence - Barro (1974)). This equivalence between debt and taxes applies in simple New-Keynesian models with representative, infinitely lived agents, but not in heterogeneous agent economics such as HANK models. Hence an argued choice must be made

[7] I avoid lump sum transfers/taxes on households in steady state since too large taxes can leave some households with negative income under uniform distribution rules.

[8] In practice unemployment befits are indexed to wage growth, but usually with a lag. In Denmark this occurs through the *satsregulering*, where public transfers are indexed to wage growth with a two year lag.

between letting debt or taxes take the adjustment in response to changes in the budget. I assume that in response to temporary shocks bonds B_t takes the initial adjustment in the budget constraint, and the government afterwards adjusts transfers to ensure non-explosive debt dynamics:[9] Hence B solves (23) at all times and T_t follows the transfer rule from McKay and Reis (2016b):

$$T_t = T^{ss} - \gamma^T \ln\left(\frac{B_{t-1}}{B^{ss}}\right), \tag{24}$$

2.7 Central Bank

Monetary policy follows a Taylor rule which features interest rate smoothing but abstracts from specific output stabilization:

$$i_t = i_{t-1}\rho^{MP} + \left(1-\rho^{MP}\right)\left(i^* + \phi^{MP}\pi_t\right),$$

where i^* is the target interest rate preferred by the central bank, ρ^{MP} is the interest rate smoothing parameter, and ϕ^{MP} determines how strongly interest rates react to changes in inflation. The rule implicitly assumes that the preferred inflation rate is 0.

2.8 Equilibrium

A dynamic general equilibrium of the model is a collection of paths for prices $\left\{r_t, r_t^k, w_t, p_t^e\right\}_{t\geq 0}$, aggregate quantities $\{Y_t, A_t, K_t, N_t, div_t, S_t, V_t\}_{t\geq 0}$, household decision rules $\{c_t\left(r_t^a, w_t, P_t, q_t\right), a_t\left(r_t^a, w_t, P_t, q_t\right)\}_{t\geq 0}$, a distribution $\mathcal{D}_t$ of households over earnings, discount factors, employment states and assets, and government policies $\{B_t, G_t, T_t, b_t\}_{t\geq 0}$ at every t such that:

(i) Households solve their dynamic programming problem by maximizing (1) subject to (2) and (3),

(ii) The distribution of households over earnings, discount factors and assets evolves in a manner consistent with optimal decision rules and the exogenous Markov chain of earnings risk and discount factors and the endogenous Markov chain of employment (described in-depth in section 3),

[9] The initial idea to obtain fiscal balance in the long run was to let bonds adjust freely in response to the temporary shock, and afterwards conduct a shock to transfers the ensures long run fiscal balance. This is similar to Hagedorn et al. (2019), and aids the analysis in the sense that the short run dynamics can be analyzed with only small effects from public transfers. However, the solution applied (see section 3.4) does not abide to this procedure since explosive government debt implies equilibrium indeterminacy.

(iii) The Final goods firm behave according to (8), intermediate goods firms obey optimality conditions (10), (11), (12), capital firms maximize (16) subject to (15), and labor service firms follow (14),

(iv) The mutual fund invests such that the no-arbitrage conditions (17), (18) are satisfied,

(v) Labor market stocks and flows obey (19), (20), the number of matches is determined by the matching function (21) and the wage follows (22),

(vi) The government satisfies its budget constraint (23),

(vii) All open markets clear.

There are 3 markets open in the economy that must clear. The market for assets clear when the stock of real household assets $A_t = \int a_t d\mathcal{D}_t$, equals the real value of government bonds plus firm equity:

$$A_t = B_t + p_t^e$$

The labor market attains equilibrium when, at a given level of market tightness θ, firms labor demand in (14) implies vacancies such that:

$$S_t q_t = m_t V_t,$$

Given the above, goods market equilibrium is implicitly imposed through Walras's law:

$$Y_t = C_t + I_t + G_t + \kappa_V + \Phi^P\left(p_{j,t-1}, p_{j,t}\right) + \phi_I\left(\frac{I_t}{I_{t-1}}\right) I_t - \kappa_{r^a} \int_{\underline{a}}^{0} a_{i,t} d\mathcal{D}_t + \Phi^F$$

which states that the production of the final good Y can be spend on private or public consumption, investment and 1) The recurring cost of posting vacancies 2) Price adjustment costs 3) Investment adjustment costs 4) Excess interest payments for households in debt and 5) Firms fixed cost.

3 Numerical Implementation

In this section I describe and discuss the numerical methods I use to solve and analyze the dynamic equilibrium model described in the prior chapter. Sections 3.1-3.3 focus on the solution to the households' dynamic programming problem, while section 3.4 describe how I apply the sequence-space Jacobian method of Auclert et al. (2019) to solve for the dynamic general equilibrium path of the model.

All relevant parts have been coded in python using packages from a wide array of authors, all of which I am thankful towards for making their work publicly available.[10] The code for this thesis is available at Github.[11]

3.1 Solving the Households problem

This section reviews the dynamic programming problem of the households briefly introduced earlier, and how to solve it numerically. The state variables of the household's optimization problem are $(k, \beta_i, e_i, a_i, \Lambda)$, i.e. employment states, the discount factor, earnings and assets, which is the endogenous state. In addition $\Lambda = (w, r^a, q, T)$ contains aggregate states such as wages and interest rates, which are the same to all households. Let $\mathbb{E}_t^k, \mathbb{E}_t^e$ denote expectations taken over employment states and earnings respectively conditional on the information set available at time t. Furthermore, let $\mathbb{E}_t$ denote the "aggregate" expectation taken over both states. No expectation is taken with respect to the aggregate states in Λ since the model contains no aggregate uncertainty. The general problem of the household is to maximize lifetime utility (1) by choosing consumption and assets subject to the budget and borrowing constraints. For a given employment state k, we can express this problem as the Bellman equation:

$$V_t^k\left(\beta, e_t, a_{t-1}, \Lambda_t\right) = \max_{c_t, a_t} \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \beta \mathbb{E}_t V_{t+1}^k\left(\beta, e_{t+1}, a_t, \Lambda_{t+1}\right),$$

subject to the constraints:

$$c_{i,t} + a_{i,t} = \left(1 + r^a\right) a_{i,t-1} + I_{i,t}^k + T_t - \tau\left(I_{i,t}^k\right)$$
$$a_t \geq \underline{a}$$

For intuition, it is useful to write up explicitly the expectation over unemployment risk.[12] Since employment shocks are persistent the expected continuation value depends

[10] This includes the code for the paper Auclert et al. (2019), code from the QuantEcon project, from my advisor Jeppe Druedahl's NumEconCopenhagen project and many more.

[11] https://github.com/nWaldstrom/HANK_Thesis.

[12] The expectations over earnings and employment states can be taken independently of each other since idiosyncratic earnings and employment do not correlate in the model.

on current employment status. If the household is employed at time t we have:

$$\mathbb{E}_t\left[V_{t+1}^k\left(\beta, e_{t+1}, a_t, \Lambda_{t+1}\right)|k_t = N\right] = \mathbb{E}_t^e\left[\left(1-\delta\left(1-q_{t+1}\right)\right)V_{t+1}^{k=N} + \delta\left(1-q_{t+1}\right)V_{t+1}^{k=U}\right]$$

The continuation value of unemployment is weighted with the probability $\delta\left(1-q\right)$, which states that with probability δ the job is destroyed, and with probability $1-q$ a new job is not obtained. The continuation value of employment is weighted with the complementary probability. Similarly, the continuation value if unemployed in period t is:

$$\mathbb{E}_t\left[V_{t+1}^k\left(\beta, e_{t+1}, a_t, \Lambda_{t+1}\right)|k_t = U\right] = \mathbb{E}_t^e\left[q_{t+1}V_{t+1}^{k=N} + \left(1-q_{t+1}\right)V_{t+1}^{k=U}\right]$$

Euler Equations I here derive the Euler equations governing intertemporal consumption/savings decision of the households. I do so using the Lagrange method, though they can as easily be derived from the above value functions by application of the Envelope theorem. The Lagrangian of an agent in employment state k with discount factor β is:

$$\mathcal{L}^k = \mathbb{E}_t\sum_t \beta^t \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \lambda_t^1\left[\left(1+r_t^a\right)a_{t-1} + I_t^k + T_t - \tau\left(I_t^k\right) - c_t - a_t\right] + \lambda_t^2\left[a_t - \underline{a}\right]$$

The first-order/Karush-Kuhn-Tucker conditions are:

$$\begin{aligned}
\frac{\partial\mathcal{L}^k}{\partial c_t} &= \beta^t c_t^{-\frac{1}{\sigma}} - \lambda_t^1 = 0\\
\frac{\partial\mathcal{L}^k}{\partial a_t} &= -\lambda_t^1 + \mathbb{E}_t\lambda_{t+1}^1 R_{t+1}^a + \lambda_t^2 = 0\\
\lambda_t^2\left[a_t - \underline{a}\right] &= 0
\end{aligned}$$

where $R_{t+1}^a = 1 + r_{t+1}^a$. In the derivative $\frac{\partial\mathcal{L}^k}{\partial a_t}$ I assume a degree of myopic behavior since households do not take into account that the real interest rate depends on their current level of assets though an interest rate differential is present. This simplifies the solution of the problem since the function $R_{t+1}^a(a_t)$ is not differentiable at $a_t = 0$.

If the borrowing constraint is violated $(a_t < \underline{a})$ then $\lambda_t^2 \neq 0$ and consumption/savings follows from the budget constraint with $a_t = \underline{a}$. If the borrowing constraint is slack then $\lambda_t^2 = 0$, and optimal consumption is determined by the Euler equation:

$$c_t^{-\frac{1}{\sigma}} = \beta R_{t+1}^a \mathbb{E}_t c_{t+1}^{-\frac{1}{\sigma}}$$

This general expression holds for agents in all states k. However, the exact expectation differs since it depends on today's state. The specific Euler equations for each of the employment states are:

$$\left(c_t^{k=N}\right)^{-\frac{1}{\sigma}} = \beta R_{t+1}^a \mathbb{E}_t^e \left[\left(1-\delta\left(1-q_{t+1}\right)\right)\left(c_{t+1}^{k=N}\right)^{-\frac{1}{\sigma}} + \delta\left(1-q_{t+1}\right)\left(c_{t+1}^{k=U}\right)^{-\frac{1}{\sigma}}\right]$$
$$\left(c_t^{k=U}\right)^{-\frac{1}{\sigma}} = \beta R_{t+1}^a \mathbb{E}_t^e \left[q_{t+1}\left(c_{t+1}^{k=N}\right)^{-\frac{1}{\sigma}} + \left(1-q_{t+1}\right)\left(c_{t+1}^{k=U}\right)^{-\frac{1}{\sigma}}\right],$$

3.2 The Endogenous Grid Method

To solve the dynamic programming problem numerically I apply the endogenous grid method (EGM) of Carroll (2006). EGM is preferable to, for instance, the standard method of value function iteration since it avoids root-finding operations by exploiting the analytical first-order conditions. The problem is solved as follows: Define grids for the state variables β, e_t, a_{t-1} as:

$$\beta \in \left\{\beta^1, \beta^2, ..., \beta^{\#_\beta}\right\}$$
$$e_t \in \left\{e^1, e^2, ..., e^{\#_e}\right\}$$
$$a_{t-1} \in \left\{a^1, a^2, ..., a^{\#_a}\right\}$$

The grid for β is calibrated by fitting the wealth distribution to data. The grid for e_t is set to reflect the variation in earnings - see the calibration section for details. The grid for a_{t-1} is chosen to to reflect the interval in which households can realistically accumulate assets given the calibration of the economy. The lower bound of this grid reflects the borrowing limit, while the upper bound is chosen based on trial and error[13]. I use a non-linear grid for assets, with a more dense grid at lower values of a since more households will lie in this region when calibrating to an empirical wealth distribution. The number of points in each grid are $\#_\beta = 5, \#_e = 11, \#_a = 300$.

The solution procedure works as follows for each employment state $k \in \{N, U\}$. Given an initial guess for the marginal utility of consumption $\left(c_{t+1}^k\right)^{-\frac{1}{\sigma}}$, conduct the following steps:

1. Apply the Euler equation to obtain the marginal utility of consumption today $\left(c_t^k\left(\beta, e_t, a_t, \Lambda_t\right)\right)^{-\frac{1}{\sigma}} = \beta \mathbb{E}_t R_{t+1}^a \left(c_{t+1}^k\left(\beta, e_{t+1}, a_t, \Lambda_{t+1}\right)\right)^{-\frac{1}{\sigma}}$.
   Compute this as: $\left(c_t^k\left(\beta, e_t, a_t, \Lambda_t\right)\right)^{-\frac{1}{\sigma}} = \beta R_{t+1}^a \sum_{e'}^{\#_e} \mathcal{D}^e\left(e_t^{'} | e_t\right)\left(c_{t+1}^k\left(\beta, e_{t+1}, a_t, \Lambda_{t+1}\right)\right)^{-\frac{1}{\sigma}}$
   where $\mathcal{D}^e\left(e_t^{'} | e_{t+1}\right)$ is the probability of transitioning to state e' from state e.

[13] In practice I use an upper bound of 50 which is roughly 100 times the calibrated, average wage rate

2. Invert the marginal utility of consumption to obtain optimal consumption: $\tilde{c}_t\left(\beta, e_t, a_t, \Lambda_t\right) = \left[c_t^{-\frac{1}{\sigma}}\left(\beta, e_t, a_t, \Lambda_t\right)\right]^{-\sigma}$.

3. Calculate the endogenous grid by first defining cash-on-hand as $m\left(\beta, e_t, a_t, \Lambda_t\right) = \tilde{c}_t\left(\beta, e_t, a_t, \Lambda_t\right) + a_t$. Shift optimal consumption to the previous period asset grid a_{t-1} by interpolating $\tilde{c}_t\left(\beta, e_t, a_t, \Lambda_t\right)$ against $m\left(\beta, e_t, a_t, \Lambda_t\right)$ and evaluating at $\left(1+r_t^a\right) a_{t-1} + I_{i,t}^k + T_t - \tau\left(I_t^k\right)$ to obtain $c^*\left(\beta, e_t, a_{t-1}, \Lambda_t\right)$.

4. Check whether borrowing constraints bind. If $a^* < \underline{a}$ set $a^* = \underline{a}$ and recalculate consumption as $c^*\left(\beta, e_t, a_{t-1}, \Lambda_t\right) = m_t$.

5. Calculate the marginal utility of consumption for further backwards iteration. This is done for each employment state:

$$\left(c_t^{k=N}\left(\beta, e_t, a_{t-1}, \Lambda_t\right)\right)^{-\frac{1}{\sigma}} = \left(1-\delta\left(1-q_t\right)\right)\left(c^{*,k=N}\right)^{-\frac{1}{\sigma}} + \delta\left(1-q_t\right)\left(c^{*,k=U}\right)^{-\frac{1}{\sigma}}$$
$$\left(c_t^{k=U}\left(\beta, e_t, a_{t-1}, \Lambda_t\right)\right)^{-\frac{1}{\sigma}} = q_t\left(c^{*,k=N}\right)^{-\frac{1}{\sigma}} + \left(1-q_t\right)\left(c^{*,k=U}\right)^{-\frac{1}{\sigma}}$$

Implementing this in an infinite horizon setting involves iteration from the initial guess until the difference $\left|c_{t+1}^*\left(\beta, e_t, a_t, \Lambda_t\right) - c_t^*\left(\beta, e_t, a_{t-1}, \Lambda_t\right)\right|$ is less than $\epsilon = 1 \times 10^{-8}$ for all c^* in the grid, afterwards which the problem is assumed to have converged.

3.3 The Distribution of Households over States

When aggregating the model households must be assigned to one of the first 4 states in $(k, \beta_i, e_i, a_i, \Lambda)$ which are none-aggregate states. These are employment, discount factors, earnings and assets. For the employment state the distribution of households is governed by the job-finding rate q_t and the job separation rate δ^N. Together these pin an unemployment rate U, which is the share of households in state $k = U$. Accordingly, the number of households in employment, $k = N$, is $1 - U$.

Regarding the earnings state recall from (5) that earnings obey an AR(1) process with mean 1 and normal innovations. The process is discretized as a Markov chain using Rouwenhorst's method (Rouwenhorst) with $n^e = 11$ points. The discretization results in a vector of states e of size n^e, a stochastic transition matrix $\mathcal{D}^e$ of size $n^e \times n^e$. Iterating on the transition matrix yields the ergodic distribution vector d^e (of size $1 \times n^e$), where each entry is the share of households in a given state. Since the process has mean 1 we have $d^e \cdot e' = 1$.

The distribution of discount factors is assumed to lie in the interval $\left[\bar{\beta}-\Delta^{\beta}, \bar{\beta}+\Delta^{\beta}\right]$ with $\bar{\beta}$ being the mean discount factor and Δ^{β} measuring the dispersion. The parameters of the process generating the discount factors of the population consists then of $\bar{\beta}$, Δ^{β} and a vector of weights d^{β}, with $d^{\beta}\left(\beta^{i}\right)$ being the share of households with discount factor β^{i}. These parameters are calibrated.

The above yields a distribution of households over the three exogenous states k, e, β. The distribution of households over the endogenous state a is obtained through iteration on the policy function. Given a transition matrix for earnings $\mathcal{D}^{e}\left(e^{i}, e^{i'}\right)$ and employment states $\mathcal{D}^{N}\left(k^{l}, k^{l'}\right)$[14] the probability of transitioning from state $h=(i, l)$ to $h'=(i', l')$ is $\mathcal{D}^{e}\left(e^{i}, e^{i'}\right) \cdot \mathcal{D}^{N}\left(k^{l}, k^{l'}\right)$, or in general the Kronecker product of the two $\mathcal{D}^{T}=\mathcal{D}^{e} \otimes \mathcal{D}^{N}$. To obtain the general distribution in the population I apply the law of motion:

$$\mathcal{D}_{t+1}\left(\beta^{k}, e^{i}, k^{l}, a^{j}\right)=\sum_{h'} \mathcal{D}^{T}\left(h, h'\right) \sum_{j'=1}^{n^{a}} \mathcal{D}_{0}\left(\beta^{k}, e^{i}, k^{l}, a^{j}\right) \omega\left(a^{*}\left(\beta^{k}, e^{i}, k^{l}, a^{j'}\right), a^{j}\right), \tag{25}$$

where the function $\omega\left(a^{*}\left(\beta^{k}, e^{i}, k^{l}, a^{j'}\right), a^{j}\right)$ interpolates the associated grid point and its two closest neighbors when the optimal asset choices are between grid points:

$$\omega\left(a^{*}, \underline{a}, \tilde{a}, \overline{a}\right)=1\left\{a^{*} \in[\underline{a}, \overline{a}]\right\}\begin{cases}\frac{\overline{a}-a^{*}}{\overline{a}-\tilde{a}}, & a \geq \tilde{a} \\ \frac{a^{*}-\underline{a}}{\overline{a}-\underline{a}}, & a<\tilde{a}\end{cases}$$

with $\underline{a}=a^{\max\{j-1,1\}}$ and $\overline{a}=a^{\min\{j+1, n^{a}\}}$. As an initial guess for the iteration procedure I assume a uniform prior on the asset distribution:

$$\mathcal{D}_{0}\left(\beta^{k}, e^{i}, k^{l}, a^{j}\right)=d^{\beta}\left(\beta^{k}\right) d^{e}\left(e^{i}\right) d^{N}\left(k^{l}\right) \frac{1}{n^{a}}$$

Forward iteration of the law of motion until convergence gives the ergodic, steady state distribution $\mathcal{D}_{ss}$. Outside of steady state the law of motion can be used to determine the dynamic path of distribution of households over states given a policy function and using $\mathcal{D}_{ss}$ as the initial $\mathcal{D}_{0}$.

[14] The transition matrix for the earnings process is the outcome of applying Rouwenhorst's method to the AR(1) process in (5). For employment dynamics the transition matrix is $\mathcal{D}_{t}^{N}=\begin{bmatrix}1-\delta\left(1-q_{t+1}\right) & q_{t+1} \\ \delta\left(1-q_{t+1}\right) & 1-q_{t+1}\end{bmatrix}$.

3.4 General Equilibrium - The Sequence-Space Jacobian Method

Having solved the households' problem in the previous section I now turn to the general equilibrium. I solve for the steady state using a combination of analytical and numerical methods. In particular, some blocks of the model can be solved analytically conditional on calibration and targets (parts of the firm block for instance), while other blocks require numerical solvers.

General Equilibrium Jacobians. To determine dynamic responses away from the steady state I apply the sequence-Space method proposed by Auclert et al. (2019). Alternative methods include the often used Krussel-Smith algorithm (Krusell and Smith (1998)) and the algorithm of Reiter (2009), who both solves for the (approximate) dynamic equilibrium path in the presence of aggregate shocks. To build intuition for the sequence-space Jacobian method, consider aggregate assets:[15]

$$\int a^* \left(\Lambda_t, \beta, e_t, a_{t-1}\right) d\mathcal{D}_t,$$

which equals optimal assets a^* (i.e. the policy that solves the households' problem) aggregated over the various states with the distribution matrix $\mathcal{D}_t$ from (25). The budget constraint implies that the policy a^* is a function of the paths of aggregates $\{r_s, w_s, N_s, q_s\}_{s\geq t}$, which in turn together with an initial stationary distribution $D_0 = D_{ss}$ determines the path $\{D_s\}_{s\geq t}$. For expositional simplicity, assume that we can deduce the paths $\{N_s, q_s\}_{s\geq t}$ from the paths $\{r_s, w_s\}_{s\geq t}$ through the firms problem, which are then sufficient to write aggregate assets in sequence space as:

$$\mathcal{A}_t \left(\{r_s, w_s\}\right) = \int a^* \left(k, \beta, e_t, a_{t-1}\right) dD_t,$$

This produces a mapping between the aggregate asset sequence into the sequences for wages and the interest rate. To derive impulse responses, let us for a minute assume we are interested in the response following an exogenous shock to productivity Z_t, of which

[15]This outline follows *A Note on Solving Heterogenous Agent General Equilibrium Models* by Jeppe Druedahl and Christoffer Jessen Weissert. For the full exposition I encourage readers to pick up the original paper Auclert et al. (2019).

r, w are functions. The model solution then solves the equation:

$$H_t(\boldsymbol{A}, \mathbf{Z}, D_0) \equiv \mathcal{A}_t\left(\{r(Z_s, A_{s-1}), w(Z_s, A_{s-1})\}_{s\geq 0}, D_0\right) - A_t = 0, \quad t = 0, 1, \ldots$$

where $\boldsymbol{A} = (A_0, A_1, ...)$ and $\boldsymbol{Z} = (Z_0, Z_1, ...)$ are the time-stacked vectors of assets and productivity. Stacking the equilibrium condition over time yields:

$$\boldsymbol{H}(\boldsymbol{A}, \mathbf{Z}, D_0) = 0$$

Totally differentiating this condition gives $\boldsymbol{H_A} d\boldsymbol{A} + \boldsymbol{H_Z} d\boldsymbol{Z} = 0$, where $\boldsymbol{H_A}$ is the Jacobian of H w.r.t. $\boldsymbol{A}$ defined by:

$$\boldsymbol{H_A} = \begin{bmatrix} \frac{\partial H_0}{\partial A_0} & \frac{\partial H_0}{\partial A_1} & \cdots \\ \frac{\partial H_1}{\partial A_0} & \ddots & \ddots \\ \vdots & \ddots & \ddots \end{bmatrix}$$

and similarly for $\boldsymbol{H_Z}$. Restating the above equation gives the impulse of $\boldsymbol{A}$ w.r.t $\boldsymbol{Z}$:

$$d\boldsymbol{A} = -\boldsymbol{H_A}^{-1} \boldsymbol{H_Z} d\boldsymbol{Z}$$

To calculate the impulse response of $\boldsymbol{A}$ only the jacobians $\boldsymbol{H_A}, \boldsymbol{H_Z}$ need to be determined. Applying the chain rule to $\boldsymbol{H}(\boldsymbol{A}, \mathbf{Z}, D_0) = 0$ yields:

$$\boldsymbol{H_A} = \mathcal{J}^{\mathcal{A},r} \mathcal{J}^{r,A} + \mathcal{J}^{\mathcal{A},w} \mathcal{J}^{w,A} - \boldsymbol{I},$$
$$\boldsymbol{H}_Z = \mathcal{J}^{\mathcal{A},r} \mathcal{J}^{r,Z} + \mathcal{J}^{\mathcal{A},w} \mathcal{J}^{w,Z},$$

where $\mathcal{J}^{\mathcal{A},r}$ is the Jacobian of $\mathcal{A}$ w.r.t. r (and similarly for the other Jacobians), and $\boldsymbol{I}$ is the identity matrix. Using the Jacobians the impulse for any variable $x(Z, A)$ in response to Z can be calculated as:

$$d\boldsymbol{X} = \boldsymbol{G}^{X,Z} d\boldsymbol{Z},$$
$$\boldsymbol{G}^{X,Z} = \mathcal{J}^{X,Z} + \mathcal{J}^{X,A} \frac{d\boldsymbol{A}}{d\boldsymbol{Z}},$$

where $\frac{d\boldsymbol{A}}{d\boldsymbol{Z}}$ was obtained earlier as $-\boldsymbol{H_A}^{-1}\boldsymbol{H_Z}$. If the variable is not a direct function of the shock, for instance $x\left(h\left(A\right)\right)$, the above method is just applied repeatedly:

$$\boldsymbol{G}^{X,Z} = \mathcal{J}^{h,Z}\frac{d\boldsymbol{h}}{d\boldsymbol{Z}}$$

where $\frac{d\boldsymbol{h}}{d\boldsymbol{Z}} \equiv \boldsymbol{G}^{h,Z} = \mathcal{J}^{h,A}\frac{d\boldsymbol{A}}{d\boldsymbol{Z}}$ and $\frac{d\boldsymbol{A}}{d\boldsymbol{Z}} = -\boldsymbol{H_A}^{-1}\boldsymbol{H_Z}$ from above.

The above method computes only a first-order approximation of a dynamic system. Auclert et al. (2019) also show that how this method can be used to speed up the computation of the full non-linear solution to a dynamic system. In the remainder of the thesis I apply mostly the first-order approximations to compute impulse responses due to the sizeable computational gain compared with the estimation of non-linear impulse responses. The main limitation to the linear approximation is their invariance to shock sizes and signs. Furthermore, since investment and price adjustment costs are quadratic and zero in steady state, these are zero in the first-order approximation of the model around the steady state.

Directed acyclic graph. The majority of Auclert et al. (2019) presents various tools and methods to speed up the computation of the Jacobians. One such method is the use of formulating the underlying model in $\mathbf{H}$ as a *directed acyclic graph* (DAG). Imagine two blocks of equations b, b' and that b takes inputs from b', $b' \to b$. If all the blocks of equations in the model $\mathbf{H}$ has a weak ordering $b^1, ..., b^N$ such that the inputs to block b^i is the output of blocks earlier $b^1, ...b^{i-1}$ the blocks of the model form a DAG.

The computational gain from this is straightforward: One can start by calculating the Jacobian of b^1, and if the model forms a DAG, b^2 will depend on the outputs of b^1, and hence the Jacobian of b^2 can be calculated. Continued application of this reasoning eventually yields all jacobians. The trick from here in Auclert et al. (2019) is to view this procedure as a mapping between an initial set of unknowns to a set of targets that must be zero, and solve the resolving system w.r.t the unknowns. For the model in this paper the DAG representation is presented in figure 1.

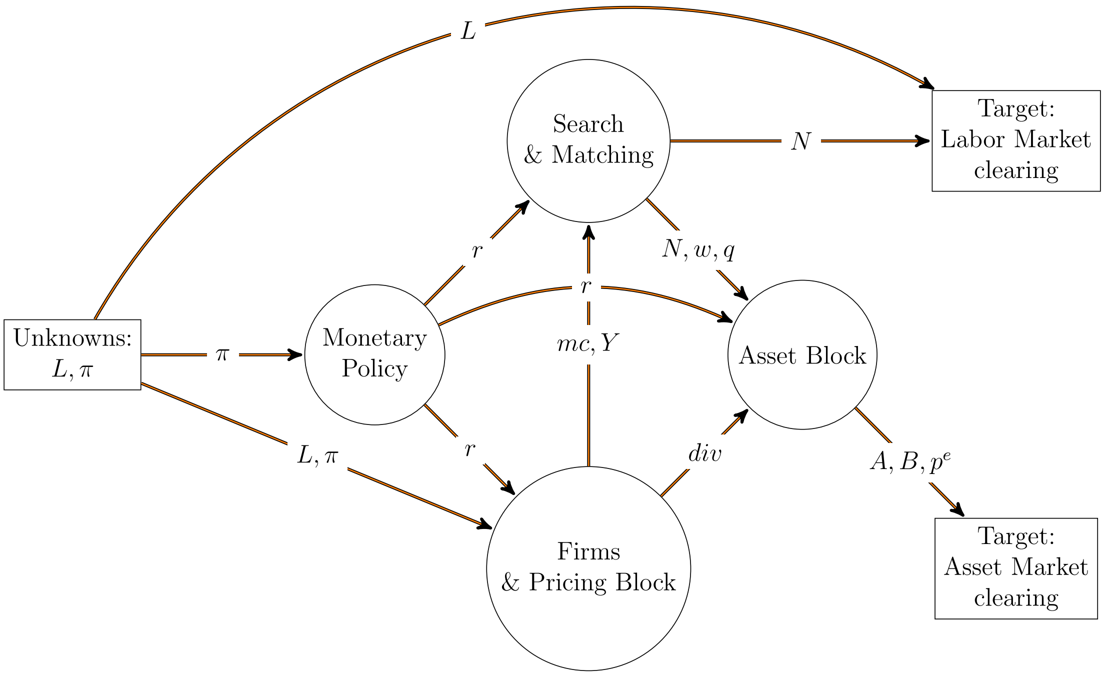


Figure 1: Directed acyclic graph of the main model.

For complicated models where a DAG is not readily constructable, the framework allows for smaller DAG blocks to be constructed first. Thus, the blocks "Labor Block", "Firms & Pricing Block", and "Asset Block" are themselves DAGs that are solved through forward accumulation first.

4 Calibration

Table 1 displays the parameters in the model and how they are chosen or calibrated. A large number of the parameters are calibrated in the steady state to match certain targets and statistics. For a reasonable amount of parameters, this can be done analytically. The main computational hurdle in the calibration lies in fitting the wealth distribution using the discount factors along while simultaneously obtaining asset market equilibrium. In practice I calibrate all parameters that can be calibrated independently of the household problem first. This yields bonds and firm equity B_{ss}, p^e_{ss}, which through the mutual fond sum to aggregate household assets A_{ss}. I then minimize the following objective function:[16]

$$\left[(B_{ss} + p^e_{ss} - A^*_{ss})^2 + \sum_{10,40,50} \left(\omega_i \left(\bar{\beta}, \Delta^\beta, \gamma^\beta\right) - \omega_i\right)^2 + \left(\frac{\int_{\underline{a}}^0 a d\mathcal{D}_{ss}}{A_{ss}} - 0.15 \right)^2 \right]$$

The first term ensures asset market equilibrium. This is primarily targeted with $\bar{\beta}$. The second term fits the wealth distribution to the observed distribution in the data. In particular, it matches the share of wealth held by the bottom 10%, the bottom 50%, and the middle 40%. These moments are "targeted" with the dispersion of discount factors in the population Δ^β and the distribution parameter γ^β.[17] In practice Δ^β and γ^β are poorly identified from each other, but including γ^β has the advantage that if β_{n^β} is close to the upper bound for convergence $1/(1+r)$ one can increase γ^β to increase asset holdings since this increases the share of households with high discount factors. The last term sets the borrowing wedge κ_{r^a} such that 15% of households are in debt.

Households. I set the elasticity of intertemporal substitution σ to 0.5 (corresponding to a CRRA-parameter of 2), which is standard in the literature. The persistence parameter ρ^e for the earnings process e_t is picked from Floden and Lindé (2001) who estimates the earnings process for Sweden resulting in $\rho^e = 0.81^{\frac{1}{4}} = 0.95$ at the quarterly level. The standard error is set to 0.7 from the same source. This is between the estimates used in Kaplan et al. (2018), Auclert et al. (2018) (roughly 0.94), and Hagedorn et al. (2019), Gornemann et al. (2016) (roughly 0.2). Note that the choice of standard error has two effects in the model: It determines the dispersion of income in the population and how strong the precautionary savings channel is w.r.t earnings. The importance of the first channel works primarily through the wealth distribution which affects the distribution of MPCs. However, since I calibrate to the wealth distribution using discount factors anyway, this channel is less important, though a more realistic income distribution of course reduces the amount of dispersion needed in discount factors. Thus the primary difference is in the degree of precautionary savings. The calibration of the household

[16] After minimizing the objective function I apply a root finder to ensure exact asset market equilibrium by using the previous value of $\bar{\beta}$ from the minimization problem as starting value.

[17] Discount factors are distributed according to $\mathcal{D}^\beta = \left\{ d^\beta_1, d^\beta_2, ..., d^\beta_{n^\beta} \right\}$ where d^β_j is the share of population with associated discount factor β_j in $\beta = \{\beta_1, \beta_2, ..., \beta_{n^\beta}\}$, where β is generated from a non-linear grid with points closer as β_j increases. The weights are parametrized as $d^\beta_j = \frac{j^{\gamma^\beta - 0.05}}{\sum_j j^{\gamma^\beta}}$ for $j = 1, 2, ..., n^\beta$ such that $d^\beta = 0.05$ implies that discount factors are uniformly distributed, $\gamma^\beta \to \infty$ implies that the entire population has the highest discount factor β_{n^β} and so forth.

block results in a mean discount factor $\bar{\beta}$ of 0.968 with dispersion $\Delta^{\beta} = 0.02$. Figure 2a plots the estimated density of discount factors in the population.
The aggregate stock of net wealth A^*_{ss} is calibrated such that the ratio between wealth and disposable income is 300% annually (Nationalbanken). This results in a quarterly level of $A^*_{ss} = 4.2$.

As per the model section I explicitly leave out capital as part of households' asset stock, the reason being that else the wealth-to-income ratio is vastly overstated since capital-output (GDP) ratios are usually in the range 8-12 for quarterly data. As explained in section 5.2 this choice has significant implications for obtaining an initially positive response of consumption to drops in the interest rate, the reason being that the negative effect on consumption from lower financial income is not too strong. For the distribution of aggregate assets by bonds and firm equity I use the statics on net financial assets from Isaksen et al. (2014). Here, bonds and firm equity account for roughly equal parts of aggregate net financial assets and I apply this calibration in steady state.

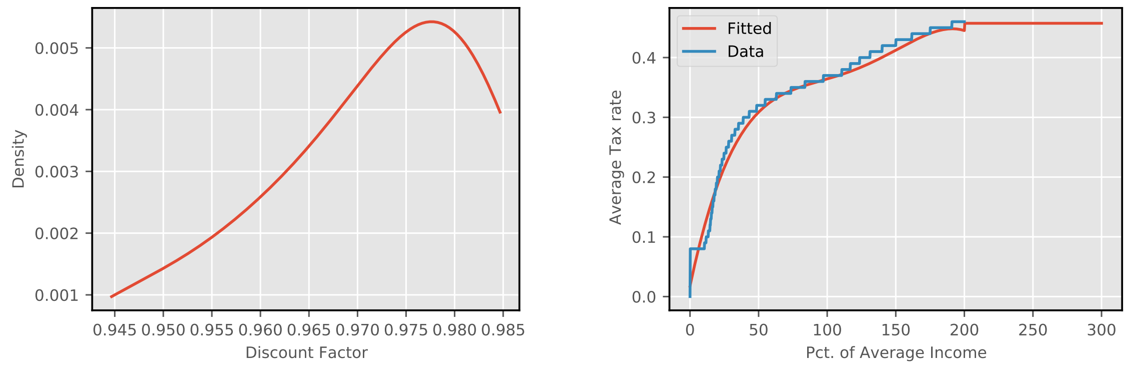
Firms. I calibrate the steady state level of output Y to 1 using TFP Z. Consequently, aggregate consumption and investment etc. can readily be interpreted as shares of output. The output elasticity of capital α is set to 0.35, which is standard. The steady-state markup is set to 1.1 which implies an elasticity of substitution of 11 in the underlying CES demand. I fix the level of capital to match a quarterly K/Y-ratio of 8. Given a real interest rate and markup this pins the deprecation rate of capital and the investment rate. These are respectively $\delta^K = 3.4\%$ and $I = 27\%$. I calibrate the steady state wage rate w to be 95% of the marginal product of labor as in Hall (2005). This pins the recurring cost of vacancy posting through the labor service firms' first-order condition. The resulting value is $\kappa_V = 0.077$. Total cost associated with hiring a new worker amounts to $\frac{\kappa_V/m}{w} = 18\%$ percent of wages, which is significant, but not unreasonably higher than the 7% argued by Christiano et al. (2016) following the evidence in Silva et al. (2009). Finally, I calibrate the fixed cost, which comes out of firm dividends, such that firm equity equals 50% of the aggregate stock of assets.

Central Bank. The steady state features zero inflation, $\pi_{ss} = 0, \Pi_{ss} = 1$. The steady-state interest and the central banks targeted interest rate r^* is set to 0.5% at the quarterly level. Coefficients in the Taylor rule are standard ($\phi^{MP} = 1.3$, $\rho^{MP} = 0.8$).

Government. As explain above I set the stock of bonds B to make up half of aggregate assets. This implies $B = 2.1$, which is higher than the quarterly debt-to-GDP ratio of 160% implied by the data from Statistics Denmark. Conditional on B, I calibrate the lump sum transfer to firms T^F to satisfy the government budget constraint in steady state. For the income tax rate, I fit a cubic spline to the average income tax rates obtained from the report *Dansk Økonomi, efterår 2018* by the Danish Economic Council. Before fitting, I convert the income to percent of average income (39.000 DKK monthly for households in 2013 - see Statistic (2014), Statistics Denmark). Figure 2b displays the data and the fitted tax function. Above a threshold of 200% of average income I impose a constant income tax rate of 46%. For the transfer rule (24), which ensures stable debt dynamics in response to shocks, I set $\gamma^T = 0.1$. This is chosen to maximize the use of bonds relative to transfers over the cycle while still maintaining stable debt dynamics.

Labor Market. I fix the job separation rate δ^N at 0.1, which is standard for a quarterly model. I set the unemployment rate to 5%. These two calibrations imply a steady-state job finding rate of 65%. The probability of filling a vacancy (the matching probability) m_t is set to 0.7 following Den Haan et al. (2000) and Ravenna and Walsh (2008). The elasticity of the matching function ξ is then calculated residually to satisfy the matching function. The resulting value is $\xi = 1.4$, which is between the calibrated value of 1.27 in Den Haan et al. (2000) and the 1.7 of Gornemann et al. (2016).

Business Cycle parameters. The adjustment cost parameter of investments κ_I is set to 6 as in Pedersen and Ravn (2013). I choose the price adjustment cost parameter κ_P to match a slope of 0.03 of the NK Philips curve. This slope is within the usual calibrations, and consistent with the empirical estimates from Galı and Gertler (1999). This implies $\kappa_P = 11/0.03 = 366$. The elasticity of wages w.r.t market tightness η is chosen to reflect that wages move little over the cycle, such that the majority of the adjustment is taken by employment. It turns out that $\eta = 0.005$ is a reasonable value of this purpose.



(a) Density of Discount factors.

(b) Fitted tax function.

Figure 2: Calibrated discount factors and fitted tax function.

a) Kernel density of calibrated discount factors with average = 0.968. b) Tax function for average tax rates from the Danish Economic Council (*Data*) and fit with cubic spline (*Fitted*).

Table 1: Calibration

Symbol	Desc.	Value	Target	Source
		Households		
σ	EIS	0.5	-	Standard
$\bar{\beta}$	Mean of Beta dist.	0.968	Wealth-to-Income ratio = 300% p.a.	Nationalbanken
$\Delta^{\beta}, \gamma^{\beta}$	Span and weights for Beta dist.	0.04, 0.06	Wealth Distribution Moments	Balestra and Tonkin (2018)
ρ	Persistence of earnings	0.95	-	Floden and Lindé (2001)
σ	SE of earnings	0.7	-	Floden and Lindé (2001)
κ_{r^a}	Borrowing Wedge	0.05	15% of Households indebted	Kaplan et al. (2018)
		Firms		
α	Output-elasticity of capital	0.35	-	Standard
K	Capital	8	Quarterly K/Y-ratio of 8	Standard
ϵ_p	Elasticity of substitution in demand	11	Markup = 10%	Standard
κ_P	Price Adj. Cost	366	NKPC slope = 0.03	Standard
κ_I	Investment adjustment cost	6	-	Pedersen and Ravn (2013)
κ^V	Vacancy posting cost	0.07	calibrated	-
Φ^F	Fixed Cost	3% of Y	$\frac{p^e}{A} = 50\%$	-
		Labor Market		
ξ	Matching Elasticity	1.4	Unemployment rate = 5%	Statistics Denmark
w	Steady state wage	$0.95 \times MPL$	95% of Match Output	Hall (2005)
m	Matching Probability	0.7	-	Christiano et al. (2016)
δ^N	Job Destruction Rate	0.1	-	Shimer (2005)
η_t	Wage elasticity	0.005	-	-
		Government		
G	Public Consumption	0.24	24% of GDP	Statistics Denmark
B	Public Debt	210% of Y	$\frac{B}{A} = 50\%$	-
b	Unemployment Benefits	0.29	After-tax Replacement rate of 51%	Schindler and Aleksynska (2011)
γ^T	Debt adjustment coef.	0.1	-	-
ϕ^{π}	Inflation coefficient	1.3	-	Standard
ρ^{MP}	Interest rate smoothing	0.8	-	Standard

4.1 Steady State Distributional Statistics

Table 2 shows how well the model fits the data in terms of the distribution of wealth. The targets (Middle 40%, Bottom 50%, Bottom 10% and share of households in debt) all fit the data well, and by construction the top 10% share of wealth also matches reasonably.

The share of the top 1% wealthiest is significantly underestimated, but this should have little implication for further analysis and is common to this type of model.[18] The Gini coefficients of income are close to empirical estimates, but the redistribution from taxes and transfers is heavily underestimated. This is unsurprising as the model features only income taxes and unemployment benefits.

Table 2: Wealth Distribution - Model Fit vs. Danish Estimates

Share of Wealth of:	Data	Model
Top 1%	14.7%	6.7%
Top 10%	47.4%	43.3%
Middle 40%	47.6%	51.7%
Bottom 50%	5%	5%
Bottom 10%	-2.2%	-2.4%
Share in debt	15%	16.4%
Share Constrained	-	0.9%
Wealth Gini	0.69	0.67
Income Gini (pre taxes & transfers)	0.44	0.38
Income Gini (post taxes & transfers)	0.25	0.31

See Balestra and Tonkin (2018) for all wealth distribution statistics. Gini coefficients for income are obtained from Neamtu and Westergaard-Nielsen (2014).

Table 3 displays characteristics across the wealth and income distributions respectively. Accumulated wealth exhibits a strong, positive correlation with the degree of patience β. This in turn implies a strong correlation with marginal propensities to consume, which are decreasing in wealth across the population. The last two columns in panel A shows that the income distribution is off less importance in the determination of the wealth distribution. There is a tendency for higher skill/earnings and lower unemployment rates as one move up the wealth ladder, though the correlation is not strictly monotone. Earnings are roughly the same for the top 30% of the wealth distribution, and the primary determination of wealth is the degree of patience. For the poorest households, it seems that income is an equally important determinant of wealth since households in this group have earnings significantly below the average and higher unemployment rates. This is also reflected in panel B of the table, which shows similar statistics for the income distribution. The calibration implies a positive correlation be-

[18]To match the wealth share of the top wealthiest households entrepreneurial risk as in Benhabib et al. (2014) is usually needed.

tween income (skill percentile) and MPCs, but not nearly as strong as for the wealth distribution.

Table 3: Distributional statistics for Households

Panel A: Wealth Distribution Characteristics

	MPC	Avg. β	Unemployment Rate	Skill Percentile
Top 1%	0.02	0.994	5.01%	94.8
Top 10%	0.03	0.991	5.09%	94.9
70th-90th	0.06	0.981	4.98%	94.8
50th-70th	0.13	0.968	4.97%	83.6
30th-50th	0.24	0.955	4.79%	83.6
10th-30th	0.44	0.945	4.54%	63.9
Bottom 10%	0.69	0.928	6.37%	45.1
Bottom 1%	0.99	0.918	8.13%	39.9

The table shows the average MPC, discount factor, unemployment rate and skill percentile for various groups in the wealth distribution. Top 1% refers to the 1% holding the most assets, and similarly for Top 10%, Bottom 10%, Bottom 1%. 70th-90th refers to the group between the 70th percentile in the wealth distribution and the 90th percentile, and similarly for 50th-70th, 30th-50th, and 10th-30th.

Panel B: Income Distribution Characteristics

	MPC	Avg. Income Tax rate	Unemployment Rate
Top 1%	0.06	46%	4.5%
Top 10%	0.09	46%	4.9%
70th-90th	0.18	45%	4.8%
50th-70th	0.25	37%	4.7%
30th-50th	0.29	30%	5%
10th-30th	0.32	26%	5.8%
Bottom 10%	0.32	11%	5.1%
Bottom 1%	0.23	8%	5.3%

Definitions are similar to those in panel A.

5 Consumption Analysis

Since the primary novelty of HANK models compared with simpler NK models lie in the consumption/savings dynamics it stands to reason to review these dynamics. I first focus on the response of consumption to transitory shocks to income and to the real interest rate in partial equilibrium, since these are the main drivers of consumption over the cycle in traditional NK models. I discuss the implications for consumption responses of adding heterogeneity, and show how a simple decomposition can split the aggregate HA consumption response into the response from a two-agent (TA) model and a part relating to distributional dynamics. Sections 5.1-5.5 concerns only the partial equilibrium of the HA model. This consists basically of the dynamic programming problem described in 3.1. At the end of the section I move to the general equilibrium and analyze the transmission of a persistent markup shock.[19]

For the majority of the partial equilibrium consumption analysis, I discuss the response of the HA model relative to the responses from representative agent or two-agent models as I consider these "anchor points" in the literature. I first state these models explicitly for future reference and proceeds to the analysis of the HA model afterwards.

Canonical Household models. The representative agent model is characterized by complete markets. The presence of these markets imply that households can perfectly insure against all but aggregate risk.[20] Appendix B.2 contains the details, but optimization implies that consumption then obeys the basic Euler equation:

$$\left(C_t^R\right)^{-\frac{1}{\sigma}} = R_{t+1}^a \beta \left(C_{t+1}^R\right)^{-\frac{1}{\sigma}}$$

where C^R denotes consumption of the representative (or Ricardian) agent. This results in permanent-income behavior where transitory income shocks have only small effects on current consumption, i.e. a low MPC. To counter this empirical oddity, Campbell and Mankiw (1989) added Hand-to-Mouth households to the model, the resulting model being the two-agent (TA) model. HtM households holds no savings and consume their entire income each period, with resulting MPC equal to 1. Let λ denote the share of

[19]The majority of this chapter focuses on the partial equilibrium of the household model. I refer to this as the HA model, with HANK being reserved for the general equilibrium case.

[20]More formally the assumption of complete markets assume the existence and availability of a complete set of Arrow-Debreu state contingency claims such that households can insure against every possible state. Trading these claims amongst themselves imply that all households obtain the same marginal utility of consumption, and are hence symmetric.

HtM household. Consumption of these household is:

$$C_t^{HtM} = \left(\lambda I_t - \tau\left(\lambda I_t\right) + \lambda T_t\right),$$

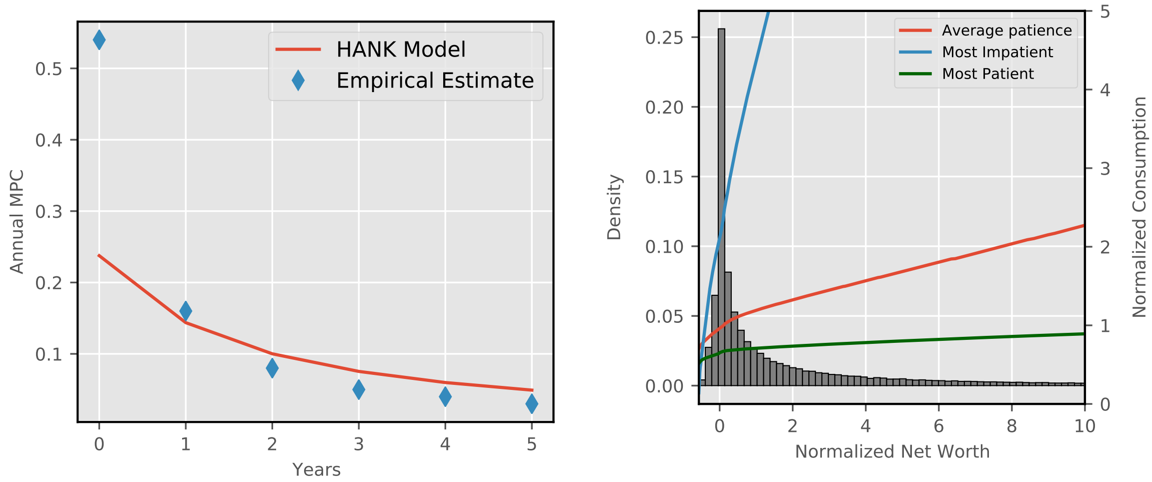
with resulting aggregate consumption $C_t = C_t^R + C_t^{HtM}$. The share of HtM households λ allows the model to capture two extreme states: For $\lambda = 0$ consumption responses to transitory shocks are very persistent since Ricardian households smooth consumption extensively, but MPCs are only marginal. For $\lambda = 0$ the aggregate MPC is equal to 1, but shocks have no persistence since HtM households are not forward looking and respond only to contemporaneous changes to income.

5.1 Income shocks and Marginal Propensities to Consume

I now proceed to the evaluation of the calibrated HA model. I first focus on the models ability to match empirical MPCs, here defined as the change in consumption relative to the change in after-tax income, $MPC_t \equiv \frac{dC}{d(I_t - \tau(I_t) + T_t)}$.

To evaluate how well the calibrated model fits empirical consumption responses, I consider an experiment where households are subjected to a transitory income shock where all households receive a lump sum transfer corresponding to 1% of the steady state wage. The shock lasts only for one period, so as to match the shock carried out in Auclert et al. (2018), since this allows for a comparison with empirical estimates of dynamic consumption responses. Figure 3a plots the annual marginal propensity to consume following the shock along with the empirical estimates from Fagereng et al. (2019). The authors here use lottery prizes in Norway as proxies for transitory income shocks to estimate the dynamic consumption response for households. The model fails to match the high initial MPC, but the persistence afterwards is reasonably well captured. The contemporaneous, annual MPC of 0.25 is not consistent with the Norwegian evidence, but matches other empirical estimates in the literature well (Souleles (1999), Shapiro and Slemrod (2003), Johnson et al. (2006), Shapiro and Slemrod (2009), Sahm et al. (2010), Broda and Parker (2014)). Overall the model matches the micro moment of intertemporal MPCs well, which Auclert et al. (2018) argue is a key moment for general equilibrium models with heterogeneous agents.

The second panel (3b) plots the consumption functions for different values of β along with the distribution of net worth. The figure shows clearly the effect of discount factors on MPCs (the slope of the consumption function), especially when taking into consideration the correlation between β and wealth. For the most impatient households the consumption function is steep, reflecting a high MPC. Since this type of household tends to be in the range with low net worth this further boosts the aggregate MPC. The slope of the consumption function for the most impatient is relatively more steep than in Carroll et al. (2017), which is due to the low value of β I obtain for these (0.913 vs 0.9867 in Carrol et. al).[21]



(a) Annual model MPCs vs. data.

(b) Consumption Functions and the distribution of Net worth.

Figure 3: Consumption Behavior

a) Annual MPCs (calculated as $\sum_{t=0}^{3}$ Quarterly $\text{MPC}_t = \sum_{t=0}^{3} \frac{dC_t}{dI_{t_0}}$) in the calibrated HA model versus evidence from Fagereng et al. (2019) b) Consumption function for consumers with lowest, average and highest β respectively and histogram of cash on hand $m_{ss} = a_{ss} + c_{ss}$. In panel (b), all variables are normalized by income.

Note that the presence of discount factor heterogeneity is a necessity to obtain empirically found MPCs in the one-asset HANK. This is not the case with the two-asset KV model since the transaction cost associated with changing the illiquid asset stock can be calibrated to match the wealth distribution and MPCs.

5.2 Consumption and Interest rates

Turning to the sensitivity of consumption w.r.t the real interest rate, Figure 4 presents the response of consumption to a persistent monetary policy loosening in the HA model. The shock is similar to that carried out in Kaplan et al. (2018). To contextualize the response, recall that in the RA model consumption obeys the Euler equation, and a drop

[21] One reason why I estimate a lower discount factor is that they assume a uniform distribution over discount factors, which I do not. The share of population on the lowest discount factors are 7% vs. 17% for my and Carrol et. al's calibrations respectively. This matters when calibrating to aggregate wealth moments.

in the interest rate hence stimulates consumption "today" through a drop in the relative price of consumption between periods. After the initial stimulus there is monotone convergence back to the steady state value of consumption. In the HA model the relative price effect also implies an initial increase in consumption, but the effect is weaker than the corresponding effect in the RA model. Additionally, there is a persistent drop in consumption following the initial stimulus.

The persistent decline in consumption is caused by a drop in households net worth owing to lesser interest on existing assets, i.e. lower financial income. Since households respond more aggressively to changes in income and net worth in the HA setup there is a large negative effect on consumption from this channel. This is also what diminishes the initial stimulus compared to the RA response.[22] This consumption response is not in line with the impulses from the KV two-asset model where aggregate consumption declines only marginally in the later periods.

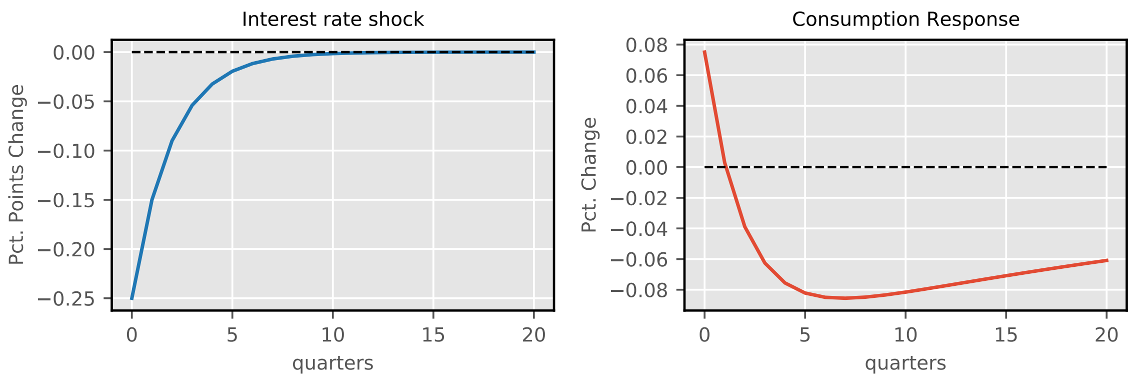


Figure 4: Impulses to a temporary interest rate shock analogues to Kaplan et al. (2018).
Note: Response to a -0.25 percentage point drop in the real interest rate with persistence 0.6.

Liquid vs. Illiquid assets. What are the differences between the two HANK models that imply so wildly different consumption responses? The model of Kaplan et. al. is a two-asset model featuring one liquid and one illiquid asset. Their calibration implies that aggregate household wealth is composed of 10% liquid assets and 90% illiquid assets. Thus, a shock to the interest rate on liquid assets affects only a small part of overall household wealth, whereas in the one-asset HANK model it affects the entirety of wealth. Since the effect of the interest rate on net worth is proportional to the relevant asset stock

[22]This is among the arguments for choosing a calibration with a low level of aggregate wealth. If the stock of aggregate wealth is large enough, the negative effect from lower financial income can dominate the substitution effect implying that consumption drops in response to a monetary policy loosening. This in turn can generate indeterminacy in general equilibrium.

the effect on net worth is significantly smaller in the two-asset setup.[23] As an example of this compositional effect arising in the KV two-asset model, consider the following example: Imagine that assets $a_{i,t}$ is composed of 10% liquid assets which yields return r^a set by the central bank, and 90% percent illiquid assets $\bar{a}_i$ which yields exogenous return $\bar{r}$, which equals the steady state return. Assume further that the Euler equation uses the rate r. Figure B.1 in the appendix shows consumption responses under this calibration. The response moves closer to the RA response, and closely resembles that from Kaplan et. al. Note though, that in the KV-model changes in the liquid return set by the central bank also affect the return to illiquid assets and hence generate changes in financial income. This, however, occurs only in general equilibrium where monetary policy affects the return to capital and firm equity.[24]

These considerations tie directly into the discussion of whether the calibration of single asset macroeconomic models should reflect overall net worth or only liquid assets, see in particular Carroll et al. (2017). Carrol et. al. show that their "Beta-dist" model - which closely resembles the household model in this paper - matches empirical MPCs well, and does so in a simpler setting than the KV model. However, as shown here the weakness of the "beta-dist" model is that it generates large changes in financial income following shocks to the real interest rate, and this yields unrealistic consumption responses. On the other hand, the two-asset HANK model excels in this regard through a more realistic description of households' balance sheets.

Empirical Interest Rate Responses. Recent micro data evidence in Holm et al. (2020) finds that consumption responses to changes in the interest rate differs wildly across the wealth distribution. The authors find that households at the bottom of the wealth distribution increase consumption in response to a monetary loosening while households in the middle of the wealth distribution decrease savings. For households at the top of distribution, their response is primarily determined by changes in financial income since they hold large amounts of assets. In particular, these households initially decrease both consumption and savings, hence showing less RA-like behavior. Foreshadowing the next section, the robustness analysis in 5.3 shows that the HA model has some issues replicating these heterogeneous effects. For instance, Holm et al. find that

[23] Given the budget constraint $m_t = I_t + (1 + r_t^a)a_{t-1}$ where $m_t = c_t + a_t$ measures net worth, the mechanical effect of a change in the interest on net worth is $\frac{dm_t}{dr_t^a} = a_{t-1}$.

[24] In the model of Kaplan et. al the liquid return is determined by the central bank. The return to illiquid assets equals the return to capital, which by no-arbitrage argument also equals the the price of a firm equity share.

the financial income channel should affect households at the bottom of the distribution strongly, though I find that they respond primarily to the relative price change. I also find that the most patient households respond primarily to changes in financial income since they hold vast amounts of wealth, which is consistent with the results from Holm et al.

5.3 Dynamic Consumption Responses And Discount Factor Heterogeneity

Figure 5 considers the effect of heterogeneous discount factors on consumption responses by plotting the average consumption responses for each of the different discount factor groups to an income and interest rate shock respectively. Through a close correlation with the wealth distribution (recall table 2), the model displays a wide amount of heterogeneity in responses by discount factors. For the income shock the contemporaneous (annual) MPC varies from 0.05 to roughly 0.6 with the min/max MPCs being represented by the lowest and highest discount factor groups respectively. Furthermore, the persistence of the income shock declines monotonously in discount factors. To this end, recall that in the limiting case where households are so impatient that they are financially constrained they act as HtM agents and consumption exhibit zero persistence.

For the interest rate shock the responses are also prone to heterogeneity. For patient households the on-impact effect of a lower interest rate is relatively low, and almost negative for the most patient households. The reason, as argued earlier, is that the drop in financial income generates a persistent consumption drop which dominates the relative price effect from the Euler equation. For impatient households, who tend to hold less wealth, the financial income channel is weak and their response more closely resemble that of a representative agent where the on-impact effect is governed primarily by the drop in the relative price of consumption. Note also that for impatient households a larger share of households tend to be indebted and a drop in the interest rate hence reduces interest payments thus increasing consumption.

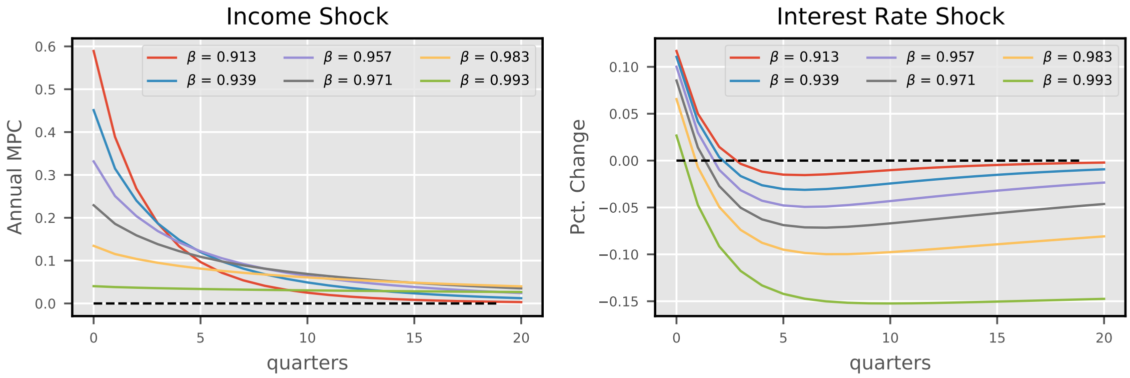


Figure 5: Consumption responses to income and interest rate shocks by discount factors.

Section B.1 in the appendix considers the consumption responses by different values of the standard deviation of earnings and the intertemporal elasticity of substitution, conditional on the wealth distribution, as robustness checks. I find that the standard deviation matters little when calibration to the wealth distribution, but that the value of the intertemporal elasticity matters greatly, but as the value of this parameter is relatively well established in the literature I consider this to be less of an issue.

5.4 Unexpected and Expected Shocks

The interest rate shock analyzed is a persistent shock meaning that the consumption response contains both a static effect and a forward looking, expectational effect. The static effect occurs in period 0 and involves no forward-looking behavior as the shock is unexpected, and consists only of a drop in financial income occurring through a lower interest rate. The forward-looking channel occurs both directly through changes to the future interest rate (since the shock is persistent, and the interest rate appearing in the Euler equations is that of the next period) as well as through changes to future marginal utilities of consumption. The second channel can potentially be strong, such as the case of forward guidance in NK models. In general equilibrium this can imply that weak, but persistent shocks can lead to strong propagation since households precautionarily cut consumption today in expectation of having to smooth consumption over the duration of the shock.

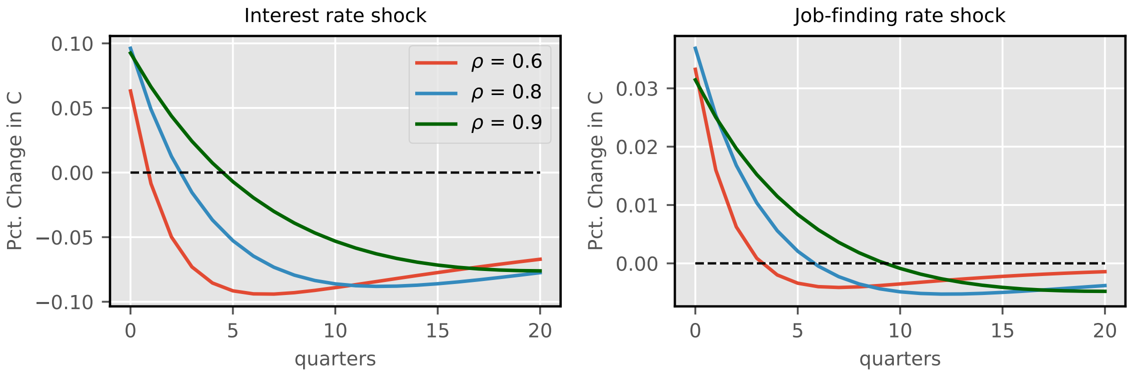


Figure 6: Consumption responses to interest- and job-finding rate shocks with differing persistent.

Note: Each panel plots the consumption response to an AR(1) shock to either the real interest rate or the job-finding rate. The baseline shocks have persistence $\rho = 0.6$ with the on-impact drop being 0.25 pct.points and 5% for the two shocks respectively. The shocks with $\rho = 0.8, 0.9$ have been re-scaled such that the cumulative shocks are the same across the shocks.

I investigate this channel in the partial equilibrium for an interest rate shock and a shock to the job-finding rate. I focus on the latter since this is a key mechanism in the final part of the thesis, which investigates persistent labor market dynamics. Figure 6 plots the consumption responses to interest rate and finding-rate shocks of varying degree of persistence.[25] In interpreting the shocks there are two things to note. Since the Euler equations are forward looking in the interest rate an unexpected, one-period change affects contemporaneous consumption only through changes in financial income, not intertemporal substitution. Similarly an unexpected change in the job-finding rate has no affect on consumption*when keeping employment fixed*, again due to the forward-looking nature of the Euler equations. Hence precautionary motives from unemployment risk are only induced when shocks to the finding rate are persistent and expected.

For both shocks it is the case that even though the most persistent shock is roughly 5 times as small on-impact as the least persistent shock, the effect on contemporaneous consumption is roughly the same across the shocks. In conducting monetary policy this implies that forward guidance is a strong tool. For the general job-finding rate shock the consumption responses are small, reflecting that households are already well-insured against unemployment risk (recall that the steady state consumption wedge is roughly 5%).

[25]Figure B.3 in the appendix plots the shocks.

5.5 Representative-Heterogeneous Agent Equivalence

Above I reference the RA/TA models extensively, and several papers have formally compared the two in calibrated models (Kaplan and Violante (2018) etc.). While the HA model contains the RA model as a special case when there is no income risk, the full HA model can also be decomposed to a part relating to the RA/TA models. The difference between this RA/TA part and the full consumption response then identifies exactly the contribution of the HA model conditional on calibration.

Suppressing dependencies of the consumption function $c_t^*(k_{i,t}, e_{i,t}, \beta_i, a_{i,t}, \Lambda_t)$ aggregate consumption is given by $C_t = \int c_t^* d\mathcal{D}_t$. A first-order perturbation around the steady state yields:

$$dC_t = \underbrace{\int dc_t^* d\mathcal{D}_{ss}}_{\text{Individual Effect}} + \underbrace{\int c_{ss}^* d\mathcal{D}_t}_{\text{Distributional Effect}} \tag{26}$$

The firm term, which I dub the "Individual effect", captures how individual consumption changes *conditional on placement in the earnings and asset distribution.* Since placement in the ergodic distribution is determined by the dynamics of the budget constraint, the Individual effect - which holds constant this channel - corresponds to the dynamics of the RA (if no households are constrained in the steady state) and TA models (if there exists constrained households in the steady state): For rational, forward looking agents the Euler equation governs all dynamic responses, and for hand-to-mouth agents the budget constraint contains by definition no dynamics. Hence the difference between RA/TA and HA models in the partial equilibrium can be captured by the term $\int c_{ss}^* d\mathcal{D}_t$ to the first-order. Appendix B.3 contains a formal proof that the Individual effect term aggregates to a RA/TA setup.[26]

Figure 7 plots the decomposition in (26) for the income and interest rate shock discussed earlier. Perhaps unsurprisingly, but nonetheless interesting, it shows that changes in the income and wealth distribution of households accounts for the majority of persistence in the partial equilibrium impulses. For the income shock the persistence is generated by the fact that the model features low-wealth households who simultaneously have high MPCs and are forward looking and thus smooth consumption. For the interest rate shock persistence is generated by a transitory drop in financial income. This result

[26] Post-writing I realized that this decomposition is actually the basis for the fake-news algorithm developed by Auclert et al. (2019), used to efficiently calculate Jacobians of heterogeneous agent blocks. I adopt their labeling for the two channels.

suggests that household heterogeneity is one way to microfound persistent consumption responses, as opposed to the more traditional method of adding habit formation to the utility function. The disadvantage of the HA response is that it fails in delivering hump-shaped consumption responses, which habit formation can produce.[27]

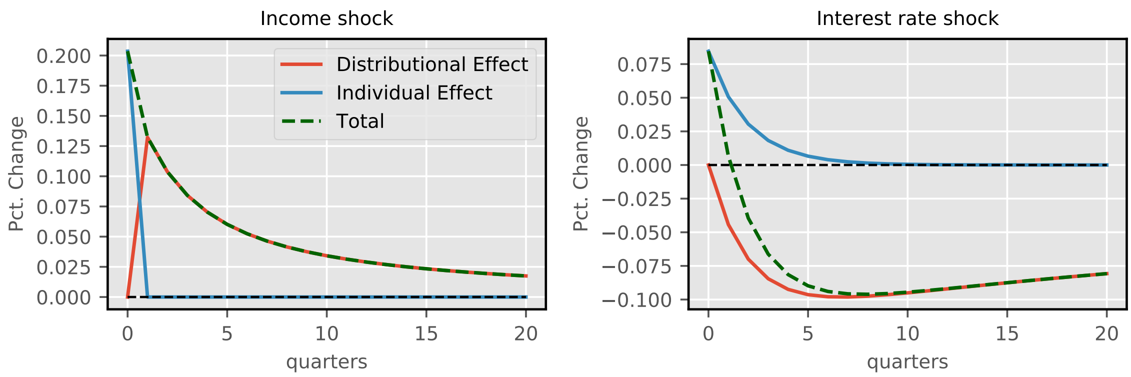


Figure 7: HA responses decomposed in individual and distributional effects.

Note: The individual effect $\int dc_t^* d\mathcal{D}_{ss}$ in response to a shock keeps constant the distribution of households across states, while the distributional effect $\int c_{ss}^* d\mathcal{D}_t$ keeps constant consumption decisions. To a first-order approximation the total effect is the sum of the two effects. The exogenous shocks are the transitory income and interest rates shocks analyzed earlier.

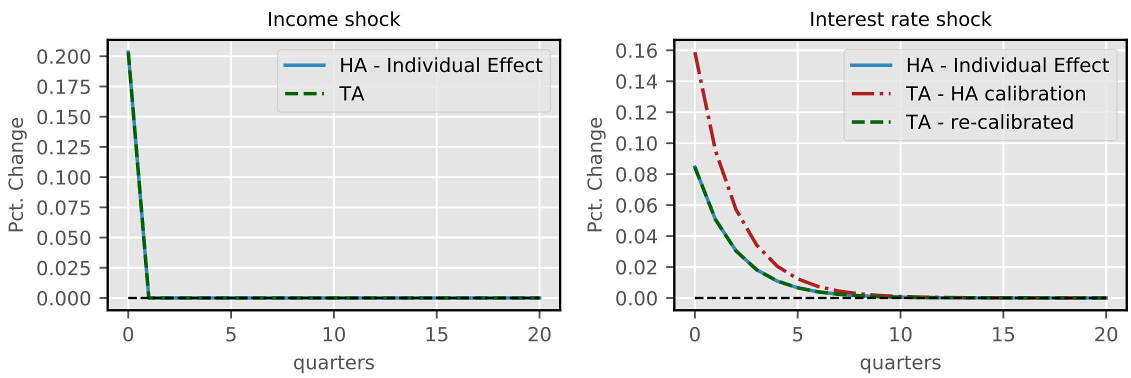


Figure 8: Individual effect from HA vs. TA responses.

Note: Each panel plots the individual response $\int dc_t^* d\mathcal{D}_{ss}$ from the HA model against a calibrated TA model. The TA model's share of HtM consumers is calibrated to match the initial MPC of the HA model. The 'TA - HA calibration' model refers to a TA-model with the steady state interest rate from the HA model, while 'TA - re-calibrated' re-calibrates the interest rate and discount factor to match the initial response of consumption to an interest rate shock in the HA model.

Figure 8 shows the extent of the equivalence. The left panel plots the individual effect from the HA model $\int dc_t^* d\mathcal{D}_{ss}$ against a standard TA model for the income shock, with the share of HtM consumers in the TA model calibrated to match the initial MPC of the HA model.[28] The two responses intersect exactly.

[27] This is the main subject of Auclert et al. (2020). They show that the HANK model can generate hump-shaped responses for consumption in response to monetary policy shocks if sticky household expectations are added.

[28] The RA/TA models are described in appendix B.2.

The right side panel conducts a similar exercise for the interest rate shock. However, here calibration matters to a larger extend. Applying the same interest rate as in the HA model implies a discount factor of $\beta^{TA} = \frac{1}{1+r_{ss}^{HA}}$ in the TA model. Due to precautionary savings, this TA discount factor is significantly higher than the average discount factor in the HA model. This explains the majority of the difference between the HA and TA (HA calibration) responses in the panel. To this end, I re-calibrate the real interest rate in the TA model (and hence the discount factor) to match the initial response of the HA model. Again, the two curves intersect exactly. Thus the HA model contains the TA model as a special case when $d\mathcal{D}_t = 0$, *conditional on calibration.*

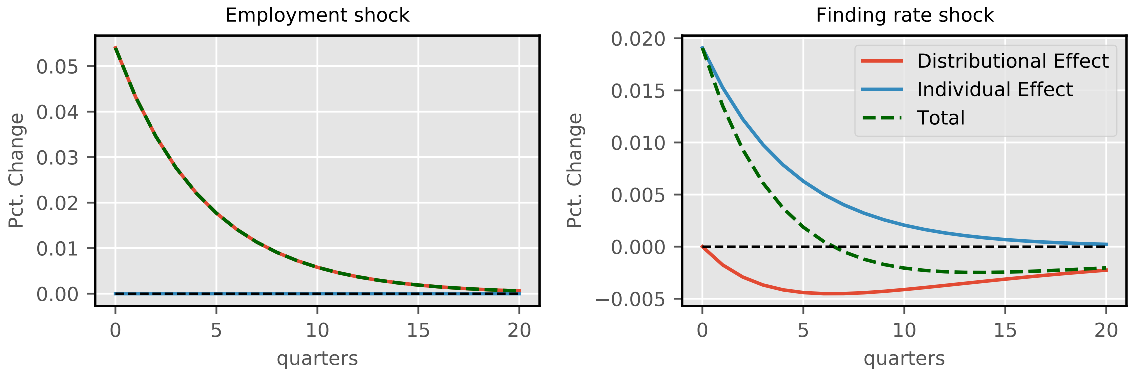


Figure 9: HA responses decomposed in individual and distributional effects for a positive employment and job finding rate shock respectively.

Note: Each panel decomposes the total response to a shock in a distributional and a individual response. The left panel considers a 1% increase in employment with persistence 0.8, and the right panel considers a positive job-finding rate shock with the same proprieties.

Employment and job-finding rates. Figure 9 shows the distributional/individual decomposition for a positive employment shock and a positive job-finding rate shock. The employment shock consists only of the distributional effect of shifting households from an unemployed state to an employed state since I keep the job-finding rate constant.[29] Note that the contemporaneous effect on consumption from higher employment equals exactly the average difference between consumption of employed and unemployed households in steady state times the size of the employment shock[30] Due to households

[29] In general equilibrium employment rarely moves without the job-finding rate moving too, though that is the experiment the figure considers. Harmenberg and Öberg (2020) provides the following intuition for this decomposition: The consumption response to a positive job-finding rate shock (holding still employment) answers the question (rephrased) "*What is the consumption response if the economy is in steady state but households believe they are in an expansion?*". Similarly the employment shock answers the question "*What is the consumption response if the economy is in an expansion but households believe they are in steady state?*"

[30] $dC_t = C_{ss}^{k=N} dN_t - C_{ss}^{k=U} dN_t \Leftrightarrow \frac{dC_t}{C_{ss}} = \frac{C_{ss}^{k=N} - C_{ss}^{k=U}}{C_{ss}} dN_t$.

self-insuring against unemployment risk this consumption wedge is significantly less than the replacement ratio of 51% (roughly 5%), and the impact of employment changes is lessened. For the job-finding rate shock the individual effect through the Euler equation dominates the total response, with the distributional effects being minor, though persistent.

5.6 Transmission of a Cost-push Shock in General Equilibrium

Shocking TFP, Markups or Investments? Before moving to the general equilibrium it is worthwhile to first discuss the characteristics of different shocks in the basic New-Keynesian framework. Though not documented formally, the general equilibrium responses of the HANK model displays a rich amount of heterogeneity w.r.t different shocks. A TFP shock generates procyclical responses for investments and inflation but a countercyclical response of employment and market tightness.[31] The mechanism is that a drop in TFP initially lowers the marginal product of labor, but for given wages and vacancy costs firms simply replace the lost production from lower TFP by hiring more workers.[32] This increases marginal costs and prices, but since firms are subject to price adjustment costs prices moves less than the marginal costs and the markup drops. Markups are hence procyclical for TFP shocks.

For direct a shock to the markup - in reality a shock to the elasticity of substitution in the CES demand since the markup is endogenous - markups are countercyclical and so is employment. This kind of shock affects primarily inflation and interest rates and is usually included in DSGE models to capture inflation dynamics over the cycle. A shock to investment-specific technology also induces a procyclical response for employment and a countercyclical markup. The cyclicality of these shocks is generally consistent with simpler NK models, but not necessarily with empirical evidence cf. Nekarda and Ramey (2013). Since one goal of the thesis is to analyze the interaction of unemployment and the associated risk with the business cycle the shock should be able to generate procyclical employment. To this end I focus on a markup shock, since this does this in a simple way and affects capital and labor equally unlike a shock to investment-specific technology. Additionally the markup shock - unlike the investment shock - is not subject to a "divine coincidence"-scenario since inflation and output moves opposite each other.

[31] I here define cyclicality with respect to how variables co-move with output in response to a shock.

[32] Recall the FOC $\frac{\kappa_V}{m_t} = (MPL_t - w_t) + \frac{(1-\delta^N)}{1+r_{t+1}} \frac{\kappa_V}{m_{t+1}}$ where $MPL_t = mc_t Z_t (1-\alpha) K_{t-1}^{\alpha} N_t^{-\alpha}$.

Baseline Impulse Responses. I now move to the general equilibrium model described in detail in section 2. For this purpose, I subject the model to an unanticipated, 5% positive markup shock (interpreted as a decrease in the CES elasticity of substitution such that the firm markup increases) with persistence 0.6. The initial shock corresponds to a decrease of the elasticity of substitution in the CES demand from 11 to ≈ 7.5. The literature often refers to this type of shock as a cost-push shock since it mimics a shock to the marginal costs incurred by firms. Figure 10 shows the impulse responses.[33]

The cost-push shock initially increases the monopoly power of intermediate goods firms and they raise prices further above their marginal costs. Higher prices reduces demand, and firms respond by cutting production by employing less labor and capital. Drops in employment and wages further reduces demand through household income which initiates the well-known Keynesian multiplier effect. In addition to this, consumption also declines for precautionary reasons as the job-finding rate drops. The central bank responds to the increase in inflation by increasing the real interest rate through the nominal rate.[34] This further reduces investment and consumption.

The middle panel of Figure 10 displays the response of asset variables (Household assets A, government bonds B and firm equity p^e). Government debt initially declines marginally due to lower interest payments. However, in the subsequent periods the drop in income taxes, resulting from lower wages and employment, along with increased expenses to unemployment benefits and interest payments forces the government to issue more bonds. For firms the economic downturn imply a decrease in profits and dividends. Since firm equity equals the discounted stream of future dividends the value of firm shares drop. Overall, households stock of wealth decreases due to a sharp decline in firm equity, but as the government builds up more debt this drop is slightly alleviated. Compared to models which features only firm equity and/or capital as assets Keynes' paradox of thrift is partially alleviated here since the government issues debt over the cycle to fill the gap from lower tax revenue, thus acting as a buffer for household savings.

[33]Earlier versions of the paper considered productivity shocks instead. However, to obtain procyclical employment responses unrealistic calibration were needed. Hence I now consider a markup shock which better fit the cyclical properties from the literature as explained above. I addition to this it is not clear empirically what the sign of employment to technology signs should be. Gali (1999) argues that there is a near-zero correlation between TFP and employment, and this paper along with Basu et al. (2006), Collard and Dellas (2007) finds, using SVAR-approaches, that employment responds negatively to positive TFP shocks.

[34]The real interest rate declines on impact even though inflation increases and the Taylor principle is satisfied. This occurs since the shock is unforeseen by the central bank, which in combination with the fisher equation $1 + r_t = \frac{1+i_{t-1}}{1+\pi_t}$ implies that the central bank can not affect the real interest rate on impact.

In models with only capital/firm equity the paradox of thrift implies that household savings increase in the partial equilibrium for precautionary reasons but decrease in the general equilibrium since the drop consumption decreases demand and hence firm profits/dividends and capital, leading to a drop in the aggregate stock of assets.

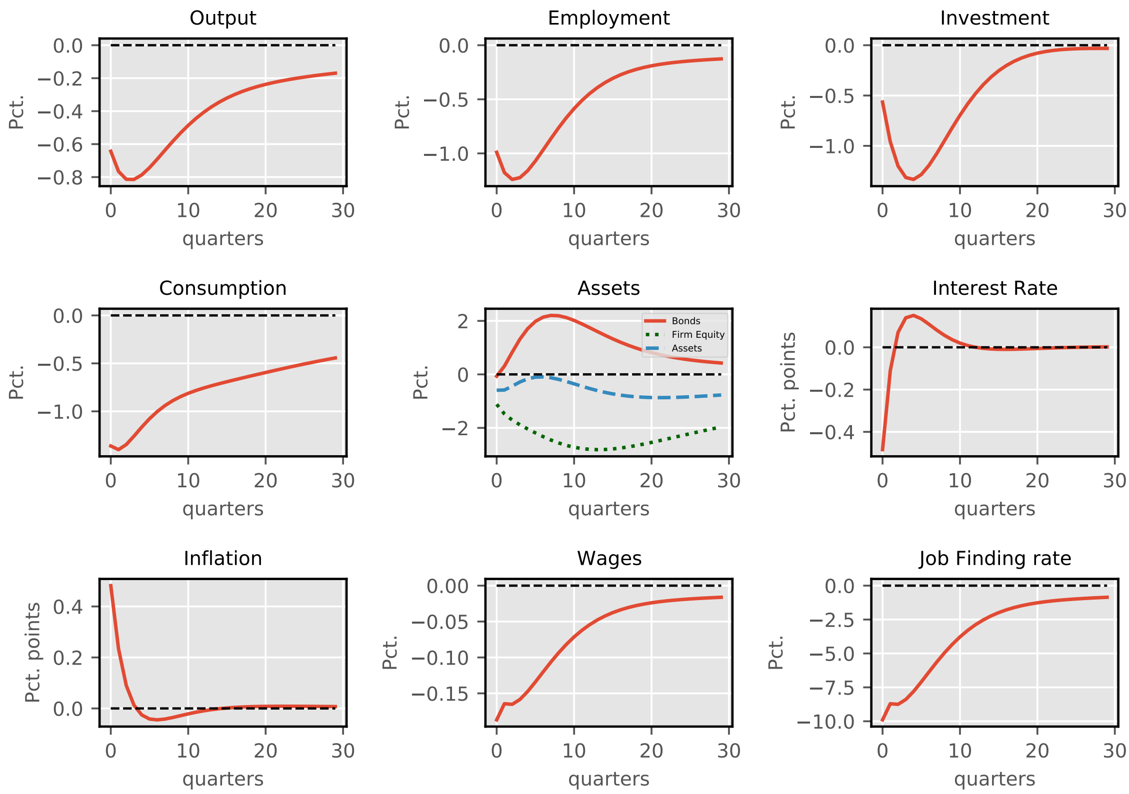


Figure 10: Impulse responses to a cost-push shock.
Note: Impulse responses to an increase in firm markups by 5% with persistence 0.6.

Decomposing Aggregate Consumption. Aggregate consumption C_t is a function of paths $\{w_t, T_t, r_t, q_t, N_t\}_{t\geq 0}$, and given a particular general equilibrium path $\{x_t\}_{t\geq 0} \in \{w_t, T_t, r_t, q_t, N_t\}_{t\geq 0}$ the effect of this particular x on C can be obtained by a first-order approximation. Figure 11 shows the resulting decomposition. The initial drop in consumption is driven equally by a higher future interest rate, as well as the expectation of higher taxes (lower transfers), which are imminent to pay of increases in government debt. In the general equilibrium the real interest rate returns quickly to the long run level, and the effect on consumption has faded after roughly 10 quarters. The drop in employment accounts for a significant share of the persistence in the response since aggregate employment remains below the steady state level for many quarters. The drop in the job-finding rate, which implies an increase in unemployment risk implies a drop in consumption of 0.2% initially, but this precautionary measure fades rapidly.

The majority of the persistence is caused by the government cutting transfers to avoid explosive debt dynamics. Because the debt dynamics are rather rigid and slow moving so are the changes to transfers which explain the extensive persistence they account for. That is, the consumption response to changes in transfers is persistence not only because of household behavior but also because transfers themselves are persistent. Wages move little in the equilibrium, which combined with high income taxes implies only a marginal effect on consumption.

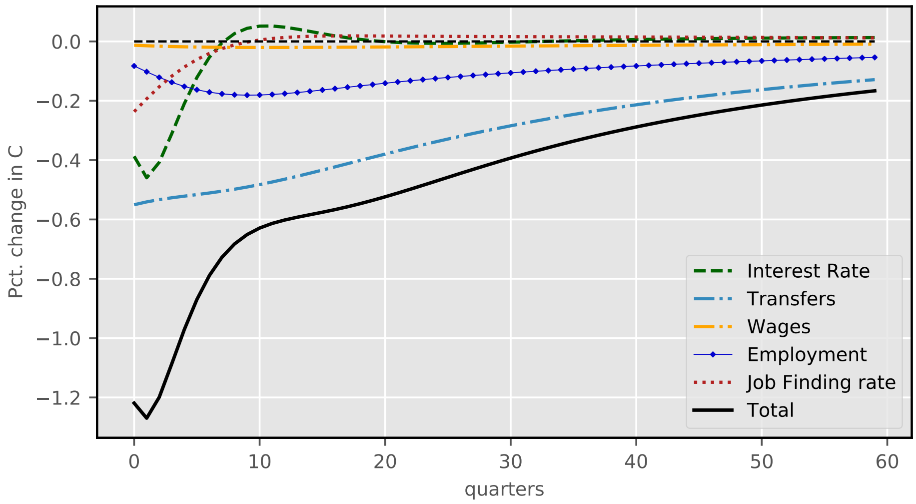


Figure 11: Decomposition of Consumption in response to a cost-push shock.
Note: Decomposed responses sum to total.

5.7 Heterogeneity In Consumption Responses

Before evaluating the welfare implications of the shock I first consider the disaggregated consumption responses. The left panel of Figure 12 displays the on-impact drop in consumption by wealth deciles, along with a decomposition which serves to identify the drivers of consumption across the wealth distribution. Future higher taxes[35] seem to be the most prominent driver of lower consumption, with the job-finding rate and interest rate also being prominent dependent on placement in the wealth distribution. The relative declines in consumption are more or less monotone in wealth deciles, with the exception being the 2nd wealth decile. For the poorest households in the 1st and 2nd wealth deciles respectively it is the case that the majority of these hold negative wealth - recall that the calibration implies 15% of households in debt. Hence they are subject to a higher interest rate due to the interest rate wedge, and movements in the

[35]The chosen transfer rule implies that taxes/transfers never move on impact, so the observed consumption responses arise purely from forward looking behavior.

interest rate have less of an impact in the Euler equation. However, as only 15% of households in debt the wealthiest half of the 2nd wealth decile has positive wealth and hence responds more to changes in the interest rate. This generates the non-monotone consumption behavior.

For the poorest households higher unemployment risk and lower transfers account for the entire change in consumption. As one moves up the wealth deciles unemployment risk affects consumption less because households are better insured, and the main determinant of consumption becomes movements in the interest rate along with transfers. For the wealthiest households changes in unemployment risk matters very little because their asset holdings are so large that marginal utilities across employment states are always equalized. Accordingly, they almost only react to changes in the marginal rate of substitution between present and future consumption.

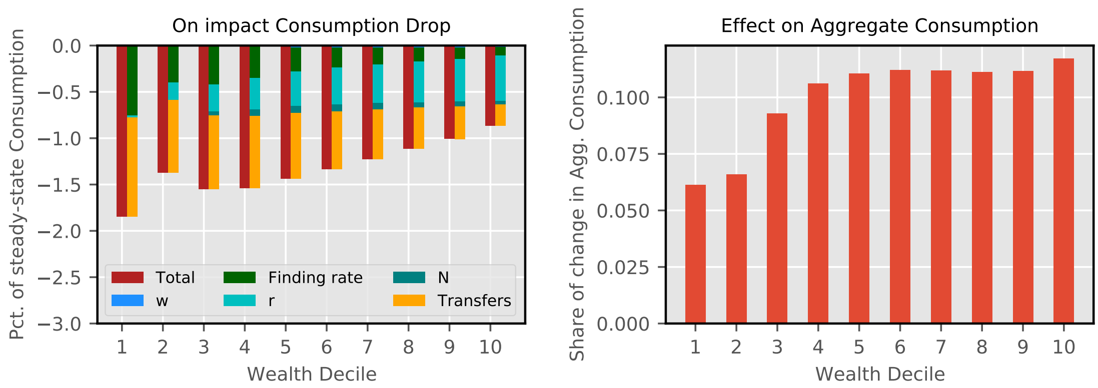


Figure 12: Decomposition of Consumption by Wealth Deciles.
Note: The decomposed column might not add to the total due to approximation errors.

While the left panel of Figure 12 might lead one to think that stabilizing consumption is roughly equivalent with stabilizing consumption of households in bottom tail of the wealth distribution, the right panel shows that this is not necessarily the case.[36] The change in aggregate consumption dC_t can be written as an additive decomposition across wealth deciles, where each decile's contribution to the aggregate is given by $\frac{C_{i,ss}}{C_{ss}}\frac{dC_{i,t}}{C_{i,ss}}$. That is, the product of a static share $\frac{C_{i,ss}}{C_{ss}}$ (wealth decile i's share of aggregate consumption in steady state) and the relative change in the decile's consumption $\frac{dC_{i,t}}{C_{i,ss}}$ following the shock.[37] The left panel shows the share of the aggregate consumption response in period 0 that each decile accounts for. This is the product of the left panel and

[36] When discussion stabilization in terms of stimulus to groups with high MPCs these consideration of course do not apply since MPCs are an absolute quantity, not a relative.

[37] $dC_t = \sum_i dC_{i,t} \Leftrightarrow \frac{dC_t}{C_{ss}} = \sum_i \frac{1}{C_{ss}} dC_{i,t} = \sum_i \frac{C_{i,ss}}{C_{ss}} \frac{dC_{i,t}}{C_{i,ss}}$.

the steady state consumption shares (see appendix Figure B.4 for the steady state distribution of consumption by wealth deciles). Despite having the largest relative decline in consumption the poorest wealth decile accounts for the smallest effect on aggregate consumption, whereas the richest households - whose consumption move less - have the largest effect on aggregate consumption. Because households in different parts of the distribution respond to different incentives - unemployment risk vs. interest rates for instance - this compositional effect matters in terms of stabilization.

Figure 13 disaggregates even further by considering the on-impact consumption change by employment state within each wealth decile.[38] Since the majority of the population are employed (95%) the left panel of the figure is roughly identical to the one from Figure 12. For unemployed households the drop in consumption is roughly 1.6% on average compared with 1.3% for employed households. This difference is primarily driven by unemployed households' strong precautionary behavior where they reduce consumption and increase savings in expectation of a prolonged unemployment spell. Again this mechanism is strongest at the bottom of the wealth distribution where households have not self-insured sufficiently against unemployment risk and are more exposed during bad spells.

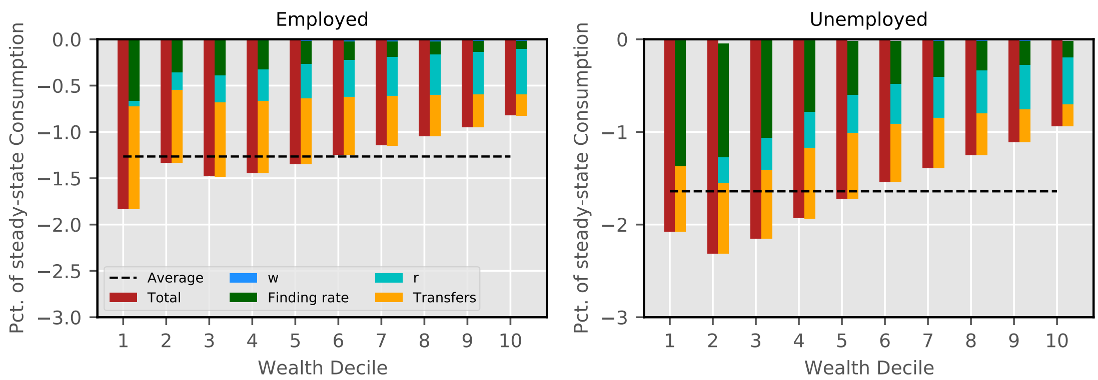


Figure 13: Decomposition of Consumption by Wealth Deciles and Employment.
Note: The decomposed column might not add to the total due to approximation errors.

Welfare. I follow Gornemann et al. (2016) in evaluating the welfare implications of shocks. They compute lifetime consumption equivalent welfare gains/losses, defined as the amount of consumption that a given household is willing to give up to experience

[38] That is, I first calculate the aggregate wealth deciles in the entire population and afterwards divide households by employment status *within* each wealth decile.

a certain event, here a persistent, cost-push shock.[39] Figure 14 displays the lifetime consumption-equivalent welfare losses following the cost-push shock by wealth deciles and employment states. Employed households are on average willing to trade off 0.62% of steady state consumption to avoid the shock, while the corresponding figure for unemployed households is 0.60%. The fact that employed households seem to be worse of compared to unemployed households is driven solely by the poorest households in the 1st wealth decile. Section B.5 in the appendix investigates this oddity, and concludes that the primary reason for this is that unemployed households in this decile have larger asset buffers compared to employed households. Unemployed households rely on this buffer in response to changes in transfers and interest rates to smooth consumption which is reflected in their smaller welfare loss.

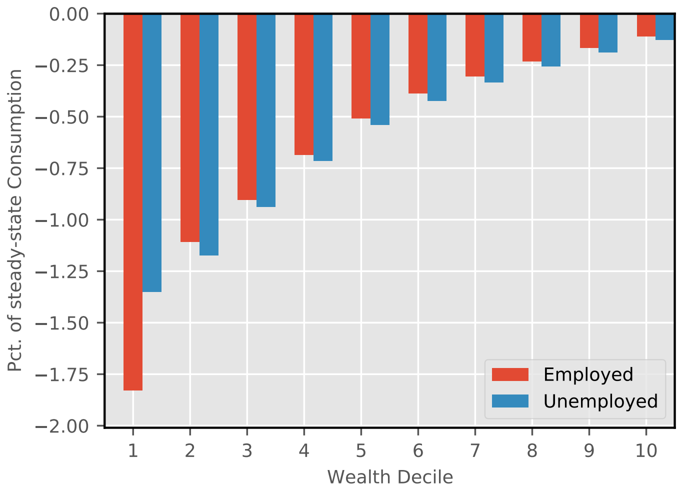


Figure 14: Consumption-equivalent Welfare Loss by Type

Aggregate Inequality. Figure B.5 (appendix) spells out the effects on aggregate inequality following the shock. Overall wealth inequality declines, primarily due to the large losses rich households suffer through the initial drops in interest rates.[40] Consumption inequality increases which is relatively unsurprising as wealth-poor households cut consumption due to unemployment risk.

[39] Let $U_{t_0}^k\left(\{c_{i,t}\}_{t=t_0}^T\right) = \int \sum_{t=t_0}^T \beta_i^t u\left(c_{i,t}\right) d\mathcal{D}_{t,k}$ denote discounted welfare for some group of households k (percentiles, quantiles, employed, unemployed etc.) from time t_0 to T. The lifetime consumption equivalent of this group in response to the shock solves $U_{ss}^k\left(\{(1+x)\, c_{i,ss}\}\right) = U_{t_0}^k\left(\{c_{i,t}\}_{t=t_0}^T\right)$ for x. I use a horizon of 300 quarters, with t_0 being the impact period of the shock.

[40] The mutual fund also suffers a loss in the first period of the shock through failing arbitrage conditions. This is is reflected in the period 0 return on assets, and also contributes to declining wealth inequality.

6 Applying HANK in Analyzing Unemployment Risk

The interaction of incomplete markets and matching frictions generates countercyclical precautionary savings as households attempt to self-insure against unemployment risk over the cycle. In this section I take a closer look at how unemployment benefits - which act as compulsory insurance against unemployment shocks - affects the economy when subjected to an adverse cost-push shock. This includes both effects on aggregates in the general equilibrium and distributional/welfare effects.

A new Steady State I conduct the following experiment: The government announces and implements a permanent drop in the unemployment benefit rate b, which is phased in slowly over 100 quarters. After 300 quarters the model is assumed to have converged to a new, counterfactual steady state which mimics the exogenous properties of the original steady state, with the exception of having a lower unemployment benefit rate. The excess revenue generated from lower unemployment benefits is transferred to households lump sum such that the reform is revenue-neutral.[41] Accordingly, the debt-to-GDP ratio is the same as in the original steady state. To evaluate the stabilizing properties of unemployment benefits I subject the original and the new steady state to a cost-push shock and compare.

I consider a shock that lowers benefits b by 50%, phased in over 100 quarters.[42] Figure C.1 in the appendix shows the responses to the permanent benefits shock for both the linear approximation and the full, non-linear solution. Non-linearities matter for output, investment, employment and, in particular, the long run values of consumption and assets. This is not surprising as it is a large shock I consider, and I hence prefer the non-linear solution in this case. The new steady state is in itself of interest, as it touches upon the literature concerning the relation between unemployment benefits, employment, production and wages (does unemployment benefits stimulate the economy in the long run?). However, in that regard the model misses the moral hazard channel of unemployment benefits affecting the extensive labor market margin, which is a major channel in this discussion. Note also that had the model included Nash-bargaining over wages,

[41] I consider the scenario where households are in some way compensated for the drop in unemployment benefits to be the most realistic.

[42] McKay and Reis (2016b) considers an 80% reduction in unemployment and poverty benefits, so a shock of this size is not unheard of in the literature.

the level of unemployment benefits would have a direct effect on the bargained wage through its interpretation as the outside option. This channel is omitted in the model. The level of unemployment benefits then only affects aggregate quantities and prices through household behavior and the dynamics of the government budget constraint.

Figure C.1 shows that the drop in unemployment benefits implies an initial drop in consumption as households self-insure by increasing savings. This demand shock reduces output, employment and investment in the short run. As the lower level of unemployment benefits is phased in transfers to households increases to clear the government's budget constraint and aggregate consumption increases in the long run. This permanent increase in demand stimulates employment and increases market tightness and hence wages. Since wages adjust endogenously to the drop in unemployment benefits the replacement ratio b/w is in itself endogenous, and hence can be either higher or lower than the 25% implied by the calibration and the drop in benefits of 50%. However, wages increase only marginally and the replacement rate drops by roughly the size of the shock, 50%.

Relative to the old steady state the aggregate stock of assets increase by 0.5%, primarily for precautionary reasons. Figure 15 shows that this increase is driven entirely by employed households. Average income in this group increases due to marginally higher wages and increased transfers from the government. Hence, they can afford to increase their insurance against unemployment through assets. This channel is strongest for households in the bottom of the wealth distribution since these are particularly poor insured to begin with. Unemployed households respond to the drop in benefits by drawing on their buffer of assets to increase consumption such that assets drop. Unemployed households at the bottom of the wealth distribution has a smaller decrease in assets due to borrowing constraints. The fact that these constrained households cannot insure to the extent of their wish potentially imply large welfare losses.

The right panel of the figure shows the percentage difference between consumption when employed vs. unemployed (the *consumption wedge*) for the new and old steady state respectively. The consumption wedge measures exactly the mechanical effect of moving an employed household to unemployment, other things equal. In the baseline steady state this wedge averaged to 6%, which is to be compared to a new average wedge of 11.2%. Figure 15 shows that the increase in this wedge is driven primarily by by wealth-poor households, with the the poorest households - who are indebted - almost experiencing a doubling in the wedge from 25% to 55%. As we move up the wealth distribution this figure drops rapidly, and the consumption wedges eventually converges as households accumulate sufficient wealth to self-insure. Overall consumption of employed households increase by 1.75%, driven by higher public transfers, while consumption of unemployed households drop by -3.3%.

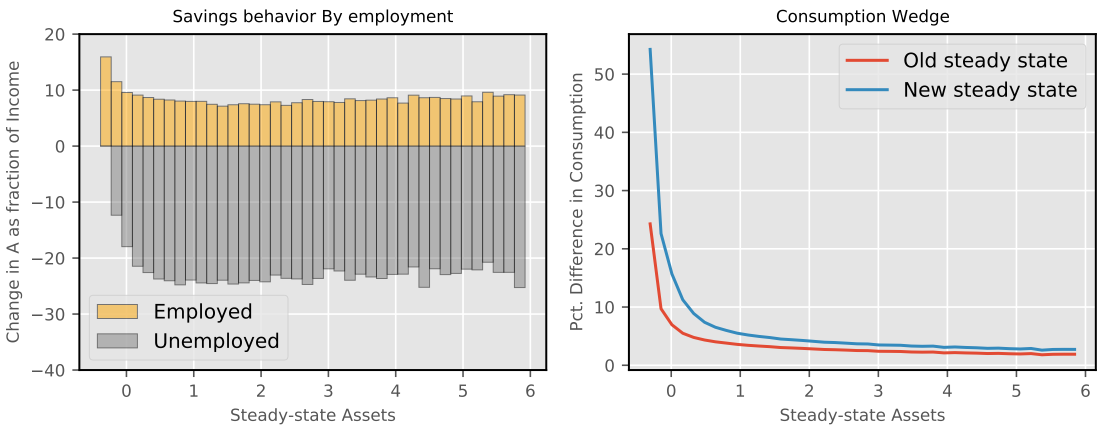


Figure 15: Change in Assets and Consumption Wedge by initial steady state assets.
Note: The left panel displays the percentage change in the asset income ratio by assets in the original steady state. The right panel shows the percentage difference between $c^{k=N}$ and $c^{k=U}$ by assets for the new and original steady state.

Unemployment Benefits as a Stabilizer Having numerically obtained the new steady state with a lower level of unemployment benefits, I now investigate the stabilizing properties of these benefits. Figure 16 compares a cost-push shock in the baseline model and the model with lower unemployment benefits. The overall impulses are roughly identical, and the level of unemployment benefits does not seem to have any cyclical effects.

My results mirror those of McKay and Reis (2016b) in analyzing the effect of transfers as automatic stabilizers. In the context of their paper transfers are to be understood as unemployment benefits and safety net programs such as food stamp programs. They find that an 80% reduction of transfers *reduces* the volatility of aggregate consumption since households self-insure through assets. These assets can in turn be used to smooth against other kinds of shocks too, not just unemployment risk, and hence reduce the overall volatility of consumption. For aggregate output and hours worked they finds increasing volatility which stems from the fact that since transfers are lower less taxes have to be raised over the cycle and wealth effects make households increase labor supply along the intensive margin. The latter effect is not present in my model since I abstract from labor supply decisions.

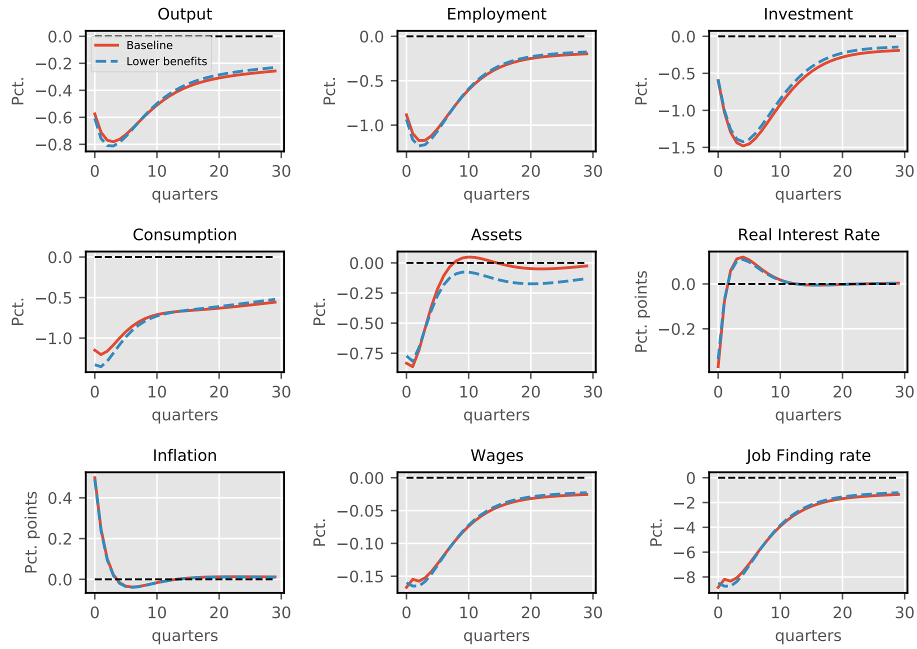


Figure 16: Impulses to a cost-push shock with standard and low unemployment benefits.

Note: impulses to a 5% increase in the firm markup μ_p with persistence 0.6.

Figure 17 decomposes the aggregate response of consumption. The figure reveals that the level of unemployment benefits is important for the response of consumption to changes in employment and job-finding opportunities. The drop in job-finding rates has a strong, immediate negative effect on consumption (roughly double the effect on impact) under lower benefits due to a stronger precautionary motive: For employed households lower job-finding rates imply that if they drop into unemployment exogenously their odds of bouncing back into employment diminishes. Hence they buffer up on assets by cutting consumption to ensure against this increased risk. For unemployed households future employment prospects drop, and they increase savings to accommodate a longer unemployment spell. Similarly the effect of lower employment on consumption also increases due to an increase in the aggregate consumption wedge, though the effect is still quantitatively small. The response of consumption to changes the interest is roughly the same. Consumption responds less to transfers, which can either be because transfers simply move less over the cycle since government expenditures are less volatile with lower benefits rates, or because household has accumulated more wealth, hence decreasing MPCs and diminishing the effects of income on consumption. Figure C.3 (appendix) shows that it is the only the first mechanism that matters. Hence, while consumption respond more to employment and the associated risk, transfers move less and the overall response is the same. Essentially, the reform has little effect exactly because it is revenue neutral.

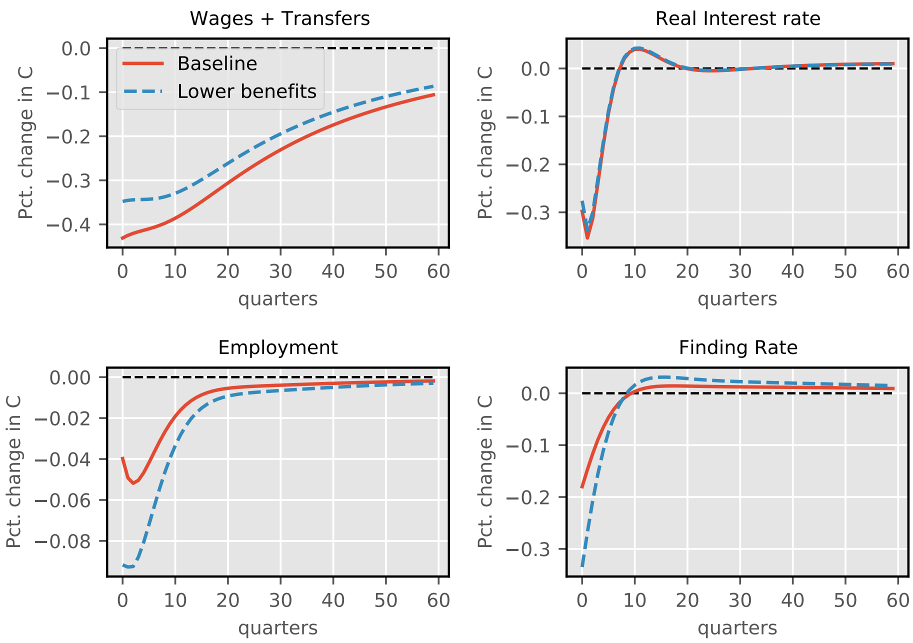


Figure 17: Decomposition of Consumption.

Welfare Implications. Figure 18 shows the consumption equivalent welfare losses between the shocks. That is, it measures how much of steady state consumption a given household is willing to give up to experience the cost-push shock under lower unemployment benefits than the cost-push shock under the baseline level of unemployment benefits. The tendency is that households in the lower part of the wealth distribution experience a welfare loss following the policy change since these are poorly insured. Households in the upper half of the wealth distribution are not dependent on the level of unemployment benefits and experience a welfare gain since public transfers decline less. At a fundamental level, the reform redistributes funds from a state dependent transfer (unemployment benefits) to a uniform, state-independent transfer, and whether households gain or loose dependent on whether they have the means to smooth consumption across both states. For all wealth deciles it is the case that unemployed households have lower consumption equivalents.

Stated differently, households who have the "cognitive ability" to self-insure (as captured by the discount factors) gain following the reform, but households that rely on compulsory unemployment insurance lose since they do not display the required discipline to save otherwise. Still, the welfare changes are minor compared to the size of the reform. This point could change radically under different modeling choices such as introducing bounded rationality (Gabaix (2014)) where agents solve a simpler "sparse" problem, and might understate unemployment risk and hence underinsure.

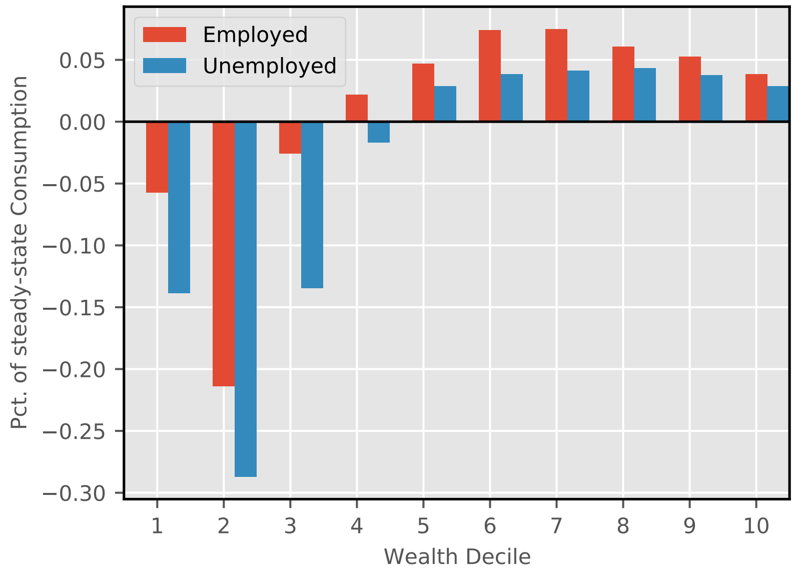


Figure 18: Consumption equivalent welfare losses from cutting unemployment benefits.

Aggregate Inequality. Figure C.4 in the appendix compares the changes in aggregate inequality. Consumption inequality increases for the first couple of periods as households in the lower part of the wealth distribution cut consumption for precautionary reasons, but the change in aggregate inequality is overall neglectable.

6.1 Paying Off Government Debt

The prior section showed that if the government cut unemployment benefits in a revenue neutral manner, the was no effect on the business cycle. I here take the alternative route and consider the implications of cutting unemployment benefits and using the resulting revenue to pay off government debt. Since households own the debt owed by the government this is essentially a one time transfer from from the government to households. In the long run this translates into higher consumption, but a lower level of aggregate assets and hence insurance. The average consumption wedge in the new steady state 9.2% - slightly less than for the prior experiment. However, the decrease in aggregate asset holdings affects greatly the average, annual MPC which increases from 0.24 to 0.35. Figure 20 shows the resulting impulses analogues to Figure 16. With households being overall more poorly insured against risk the cost-push shock is considerably propagated compared to 16. Figure C.6 (appendix) decomposes the consumption response and shows that this is driven by, in particular, employment and transfers. Precautionary savings have large negative effect initially, but fades rapidly and is not the source of the propagated shock. It is rather the fact that MPCs increase overall due to lower insurance, and changes in transfers and employment are then propagated. This reveals an important points which I return to in chapter 7: In the one-asset HANK model changes in unemployment risk generates strong, but short-lived changes in consumption, while the mechanical effect of moving households from employment to unemployment yields less strong, but more persistent responses. Figure 20 shows that the welfare effects are magnitudes larger in this alternate scenario: All households prefer not to implement the lower level of unemployment benefits, primarily because it involves a decline in the aggregate asset stock.

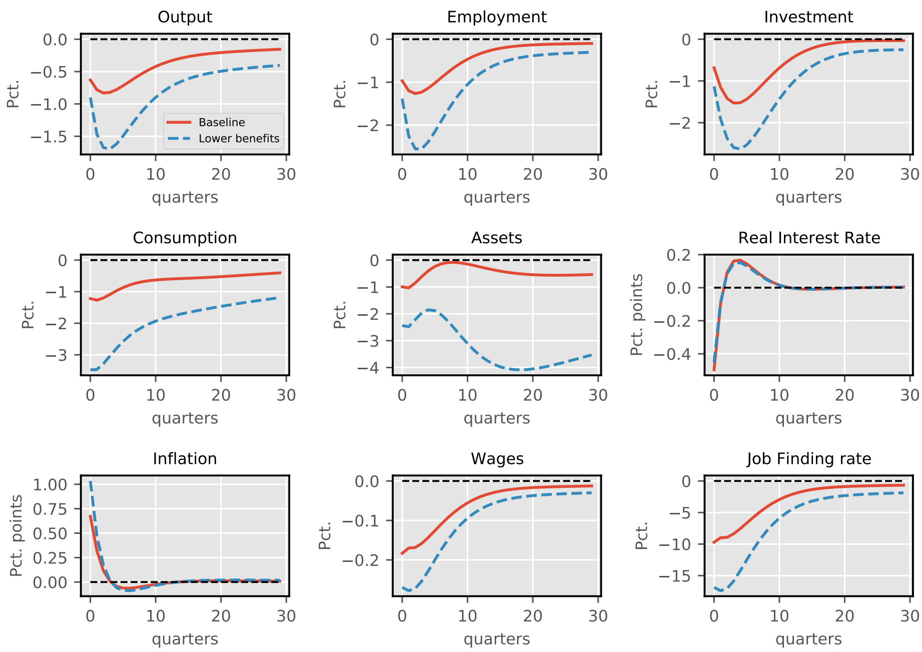


Figure 19: Importance of unemployment benefits with non revenue-neutral policy.

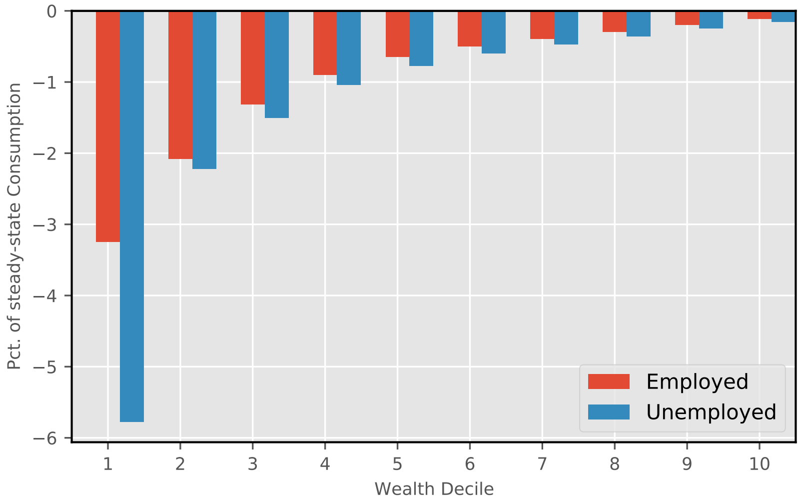


Figure 20: Consumption equivalent welfare losses from cutting unemployment benefits.

7 Propagation in Labor Market Dynamics

The baseline search and matching model implicitly assumes free-entry for labor service firms, such that value of posting a vacancy is zero in equilibrium. This implies that the dynamic response of vacancies tend to be highly volatile and short lived, the reason being that any increase in the value of a match over the cycle is immediately met by increased vacancy creation to exploit this surplus due to free entry, which resultingly drives down the value of a vacancy to zero. This is generally considered to be incompatible with empirical evidence for vacancies and unemployment rates, cf. Elsby et al. (2015).

Figure 21 shows the empirical Beveridge curve for the US 1951-2003. Conditional on historical time periods the figure displays a clear negative correlation between the vacancy rate and the unemployment rate. In the context of SAM models this relationship derives essentially from the matching function: When firms increase vacancy posting the number of matches increases. This implies higher job-finding rates and lower unemployment. This generates a negative relationship between vacancy posting and the unemployment rate.[43] Figure D.1 (appendix) shows the vacancy response and the implied Beveridge curve for the baseline HANK mode following the cost-push shock earlier examined. While the model delivers a downward sloped curve, the overall negative correlation is dubious due to the low persistence of vacancies.[44]

[43]More causal evidence that vacancies and market tightness in general acts sluggishly over the cycle has been presented by Blanchard et al. (1990), Fujita and Ramey (2007).

[44]It is also notable that the linear fit is bad because unemployment displays a hump-shaped response whereas vacancies do not. This is at odds with Fujita and Ramey (2007).

This chapter reviews and discusses different variants of the SAM model. I introduce sunk costs into the vacancy creation process, in turn converting vacancies V_t from a jump variable to a predetermined stock. I investigate the properties in both a partial equilibrium and in the general equilibrium framework developed earlier, as well as providing sensitivity analysis to central parameters to exactly determine the underlying mechanisms. In addition to costly vacancy creation, I endogenize the separation rate in a simple way. I consider both the case where a separation rate that destroys matches is endogenous and the case where a separation rate that destroys jobs/positions (vacancies + matches) is endogenous.

The chapter builds up to a replication of Broer et al. (2020a), who shows that the presence of sluggish vacancy creation, endogenous separations and precautionary savings behavior can lead to persistent, demand driven recessions following transitory shocks. I conduct a small-scale estimation of the full HANK model w.r.t the central labor market parameters and find that the first two mechanisms generate prolonged, persistent recessions but I find no propagation through precautionary savings. I discuss several extensions of the HANK model that might generate propagation.

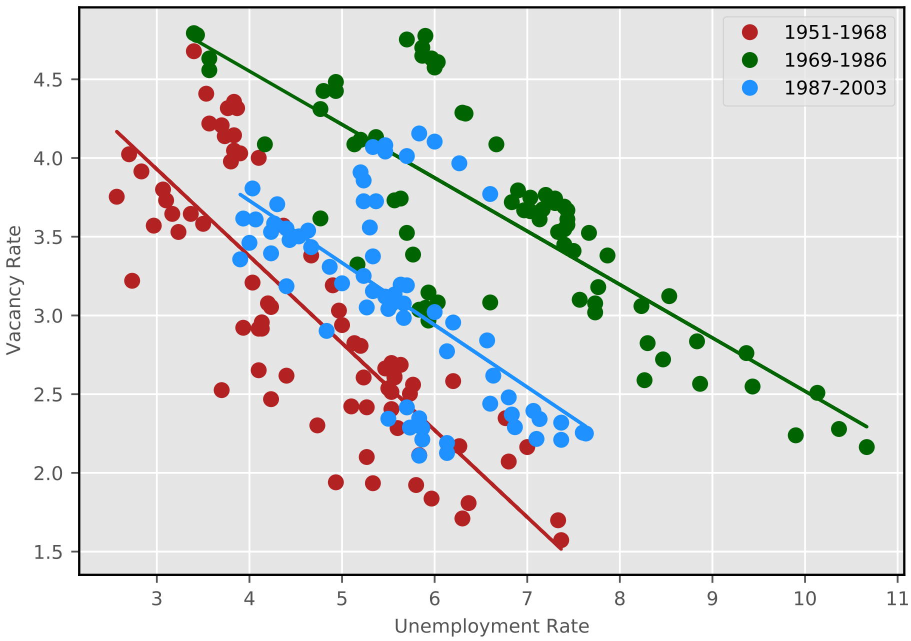


Figure 21: US Beveridge Curves 1951-2003

The vacancy rate is proxied by the *Composite Help-Wanted Index* from Barnichon (2010). The unemployment rate is obtained from FRED. [45]

[45] I thank my advisor for easy access to the data.

7.1 Sunk Costs in Vacancy Creation

To closer match the persistence observed in the data I follow Fujita and Ramey (2007) (henceforth Fujita-Ramey or FR) in modifying the vacancy creation process. Firms that want to enter the labor service sector must pay an additional cost $\kappa_x x_t$ where x_t is the number of new, open positions in the economy at time t. Since the cost of creating a single job is proportional to the overall amount of jobs created there are increasing marginal costs to the individual firm. The purpose of vacancies are to fill open positions. Positions can be open either because they have been created recently or because an existent match has been terminated through exogenous job destruction. Positions exist until they are terminated through *obsolescence*. Let δ^O denote the rate of job obsolescence. Let δ^{NO} denote the probability that a match is destroyed but the position not terminated (i.e a normal separation). The job destruction rate that prevails at the aggregate level is given by $\delta^N = \delta^O + \left(1 - \delta^O\right) \delta^{NO}$. That is, the sum of the probability that a job is destroyed by obsolesce δ^O and the probability that a match is destroyed given that the position is not made obsolete $\left(1 - \delta^O\right) \delta^{NO}$.

Households' Euler equations. For households that are unemployed at time t the probability of obsolescence imply that the Euler equation now reads:

$$\left(c_{i,t}^{k=U}\right)^{-\frac{1}{\sigma}} = \beta \mathbb{E}_{i,t} R_{t+1} \left[\left(1 - \delta^O\right) q_{t+1} \left(c_{i,t+1}^{k=N}\right)^{-\frac{1}{\sigma}} + \left(1 - \left(1 - \delta^O\right) q_{t+1}\right) \left(c_{i,t+1}^{k=U}\right)^{-\frac{1}{\sigma}}\right] \quad (27)$$

which reads that the households obtains employment with probability q_{t+1} times the probability that the position is not made obsolete. The probability of remaining in unemployment is the complementary probability. Hence the new addition is that an unlucky share $\left(1 - \left(1 - \delta^O\right) q_{t+1}\right)$ of time $t+1$ searchers will find a job in the subsequent period, but this job will be made obsolete immediately thus sending the households back into unemployment.[46] The Euler equation for employed households (6) is unchanged though the job destruction rate δ^N has been redefined per the above. The law of motion for employment now reads:

$$N_t = \left(1 - \delta^N\right) N_{t-1} + S_t q_t \left(1 - \delta^O\right), \quad (28)$$

[46] One can discuss whether this is realistic for a model calibrated to a quarterly level. I keep this timing assumption to remain as close to Fujita and Ramey (2007) as possible.

which is only modified by the term $\left(1-\delta^{O}\right)$. Compared to the canonical SAM model, where vacancies is a flow/jump variable, the above assumptions introduce a few additional dynamics for vacancies. Period t vacancies V_t is given by:

$$V_t = \left(1-\delta^{O}\right)\left(\left(1-m_{t-1}\right)V_{t-1}+\delta^{NO}N_{t-1}\right)+x_t \tag{29}$$

which states that the stock of vacancies equals previous vacancies not matched $\left(1-m_{t-1}\right)V_{t-1}$ plus vacancies created for positions where the match was destroyed but the position was not $\delta^{NO}N_{t-1}$ all net of obsolescence, $\left(1-\delta^{O}\right)$. In addition, each new position created x_t also creates an associated vacancy.

The value of a match $\mathcal{J}_t^m$ is modified to include the continuation value of a match with the probability that neither the match or the position is destroyed $\left(1-\delta^{O}\right)\left(1-\delta^{NO}\right)$, and the value of the associated vacancy if the match is destroyed but the position still remains:

$$\mathcal{J}_t^m = \left(MPL_t - w_t\right)+\frac{\left(1-\delta^{O}\right)\left(1-\delta^{NO}\right)}{1+r_{t+1}}\mathcal{J}_{t+1}^m+\frac{\left(1-\delta^{O}\right)\delta^{NO}}{1+r_{t+1}}\mathcal{J}_{t+1}^V \tag{30}$$

Similarly, the value of an unfilled vacancy now takes into account that vacancies associated with jobs not made obsolete in period t now persist into period $t+1$ where they might match with a searching household. Hence a continuation value is introduced in $\mathcal{J}_t^V$:

$$\mathcal{J}_t^V = -\kappa_V + m_t\mathcal{J}_t^m+\frac{\left(1-m_t\right)\left(1-\delta^{O}\right)}{1+r_{t+1}}\mathcal{J}_{t+1}^V \tag{31}$$

The free entry condition equations the value of vacancy posting with associated costs:

$$\mathcal{J}_t^V = \kappa_x x_t \tag{32}$$

This setting has the potential to generate more persistence in the response of employment dynamics since it combines backwards looking dynamics, introduced by converting vacancies to a stock variable, with forward looking firm behavior. Intuitively, the presence of the sunk cost implies that entrant firms smooth out the creation of vacancies due to increasing costs. That is, during an economic boom the number of new positions created x_t is high and which implies a high sunk cost $\kappa_x x_t$ and firms have an incentive to enter the market at a later (or earlier) time.

A simple Fujita-Ramey style model. Bodart et al. (2006) considers the Fujita-Ramey model in a simpler setting which does not distinguish between the destruction of positions and matches. They assume that the stock of vacancies at time t equals vacancies not matched in the previous period plus vacancies needed for new job openings x_t:

$$V_t = (1 - m_{t-1})\, V_{t-1} + x_t \tag{33}$$

The Bellman equations for vacancies and matches are:

$$\mathcal{J}_t^V = -\kappa_V + m_t \mathcal{J}_t^m + \frac{(1 - m_t)}{1 + r_{t+1}} \mathcal{J}_{t+1}^V, \tag{34}$$

$$\mathcal{J}_t^m = (MPL_t - w_t) + \frac{\left(1 - \delta^N\right)}{1 + r_{t+1}} \mathcal{J}_{t+1}^m, \tag{35}$$

with free entry implying $\mathcal{J}_t^V = \kappa_x x_t$. I dub this model the *simple sunk cost* model. Note that the primary difference between this model and the full FR model is the rate at which vacancies are destroyed. In addition, in the FR model the value of match takes into account that even though the match is destroyed vacancies still have value since the underlying job position might not be destroyed.

Calibration. Fujita and Ramey (2007) finds that normal job destruction and obsolescence each account for roughly half of aggregate job destruction. I use this relation ship and set $\delta^O = \frac{1}{2} \cdot \delta^N$, where $\delta^N = 0.1$ is the aggregate job destruction rate used in the baseline model. The normal job destruction rate is then found residually as $\delta^{NO} = \frac{\delta^N - \delta^O}{1 - \delta^O}$. The remaining calibration of the labor market is the same as earlier (same steady state unemployment rate and matching probability), though the calibrated job-finding rate changes slightly since the flow out of unemployment is affected by the job obsolescence rate. For the recurring cost κ_V and the sunk cost κ_x Fujita and Ramey (2007) finds that to fit the empirical standard deviation of employment the marginal sunk cost need to be roughly 5 times the recurring cost, $\frac{\kappa_x x_t}{\kappa_V} = 5$. This condition along with the relevant value function for a match pins κ_x, κ_V in both of the sunk cost models. Section D.1 (appendix) discusses the importance of the sunk cost vs. the recurring vacancy cost as well the importance of the labor wedge $MPL - w$.

Partial Equilibrium Evaluation. Figure 22 compares the basic model which has only a recurring cost of posting vacancies with the simple model from Bodart et al. (2006) and the full model from Fujita and Ramey (2007). The figure shows the response of employment, vacancies and the job-finding rate in the three models when subjected to a persistent, negative productivity shock.[47] As opposed to earlier, I consider a more discrete shock where TFP drops by 1% for 10 quarters, and afterwards reverts to the baseline instantly.[48] I consider this type of shock to more easily evaluate the degree of persistence each model generates. The standard model generates a large initial impulse since vacancies is a jump variable that ensures zero value of posting a vacancy, with monotone decay to zero afterwards. The simple sunk cost model produces hump-shaped responses for employment and job-finding rates, but for employment the overall responses are similar during the shock. The Fujita-Ramey sunk cost model also produces humps-shaped responses, but vacancies and employment move less due to the more extensive vacancy dynamics obtained via the obsolescence-mechanic. Both sunk cost models induce a degree of persistence in employment dynamics which is not present in the baseline model.

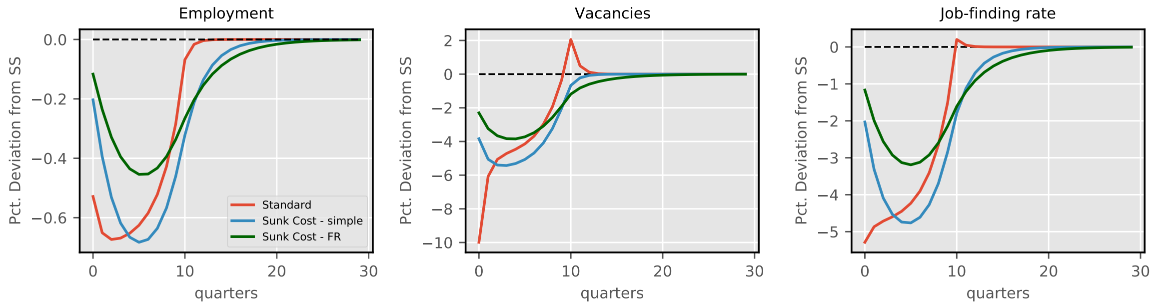


Figure 22: Partial Equilibrium comparison of different vacancy models.

The partial equilibrium model includes all relevant labor market equations along with the relevant vacancy/matching value functions and free entry conditions. The shock is a 1% negative TFP shock lasting 10 quarters. *Standard* denotes the model with a recurring vacancy cost and $\mathcal{J}_t^V = 0$. *Sunk Cost - simple* refers to the model from Bodart et al. (2006), while *Sunk Cost - FR* refers to the model from Fujita and Ramey (2007).

General Equilibrium. Figure D.7 shows the responses in general equilibrium following a cost-push shock for different vacancy posting models.[49] The introduction of the sunk cost imply that the creation of vacancies is smoothed extensively compared to the standard model. These models hence generate significant, hump-shaped responses. The simple sunk-cost model creates a response between the two extremes: The shape of the response largely resembles that of the FR model, but due to diminished vacancy dy-

[47] A negative shock to the real marginal costs, which closer resembles the markup shock earlier analyzed, yields identical results since both Z and mc enter linearly in the marginal product of labor.

[48] $dZ_t = \begin{cases} -0.01 \cdot Z_{ss}, & 0 \leq t \leq 10 \\ 0 & t > 10 \end{cases}$.

[49] I consider a shock similar to the productivity shock for the partial equilibrium where the markup μ_p increases by 1% for 10 periods and then returns to the steady state level.

namics the response is more volatile during the shock and this generates long lasting effects.

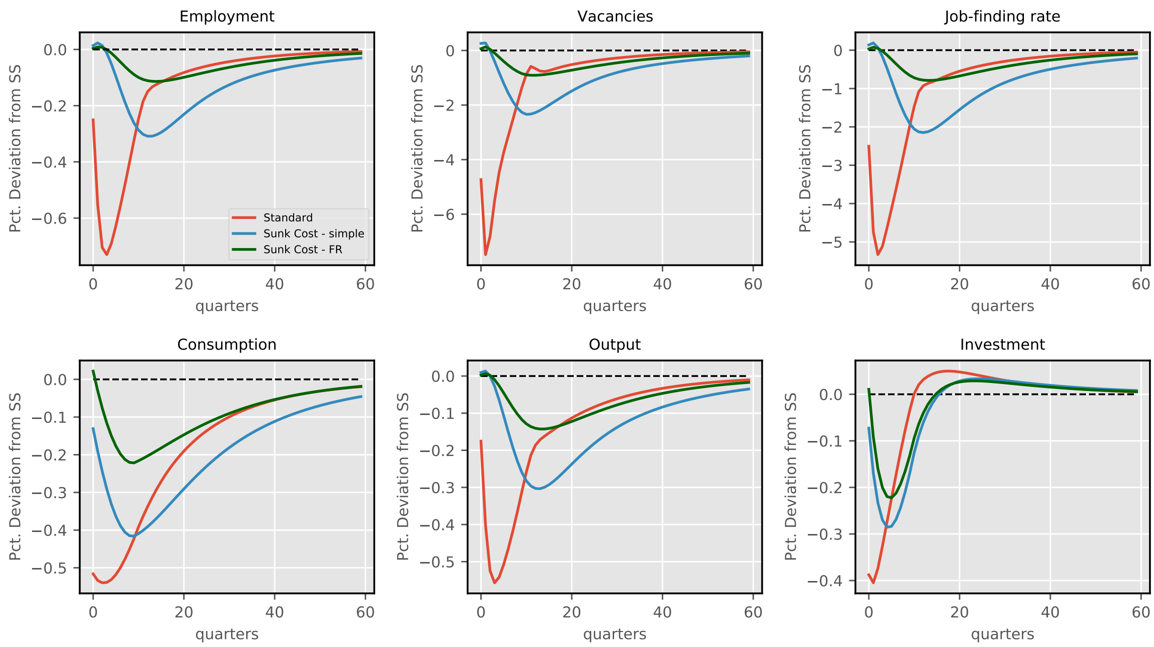


Figure 23: Responses for labor markets variables in to cosh-push shock in general equilibrium for different vacancy posting models.

7.2 Separation Rate shock

I next consider how the dynamics of the model is affected by job separations, this being the third factor in generating propagation as per Broer et al. (2020a) after sunk costs and precautionary savings. To this end I first consider an exogenous disturbance in the separation rate for the different models. For the full FR model I carry out shocks to both the normal separation rate δ^{NO} and the obsolesce rate δ^{O}. Figure 24 shows the responses in the partial equilibrium to a 1 pct. point increase in the aggregate job separation rate for 10 quarters. In the standard model increased job separations decrease employment for the duration of the shock and sharply increases vacancy creation due to free entry. This effect dominates the increase in the number of searchers and the job finding rate slightly increases. The responses show no persistence and returns to the steady state level abruptly as the shock disappears.

Adding sunk costs diminishes the response of vacancies, in turn generating a drop in the job-finding rate as per the simple FR model: Firms initially increase vacancies to make up for the drop in labor caused by a higher separation rate. However, firms correctly anticipate that the separation rate returns to the steady state level in the future and prematurely decrease the flow of new jobs to smooth out costs. A premature drop in vacancies occurring while the separation rate is still elevated generates unemployment. This effect adds some persistence in the aftermath of the shock.

Moving to the full FR model, the shock to the obsolescence rate δ^O generate even more persistence, while a shock to normal separation rate δ^{NO} generates a response almost identical to that of the standard model. Thus an important observation is that persistence is only obtained in the FR model if the shock destroyed both vacancies and matches (obsolescence) and not only matches ("normal" separations).

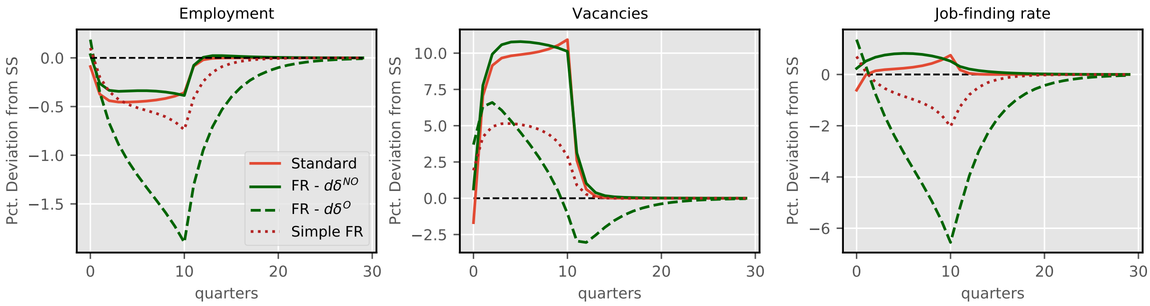


Figure 24: Responses to an increase in job separations lasting for 10 quarters.

7.3 Endogenous Job Separation

I now formally endogenize the obsolescence rate in the spirit of Broer et al. (2020a) in the FR framework. I relegate the extension of an endogenous, normal separation rate δ^{NO} to the appendix, see section D.4. The main conclusion for this extension is, as partially suggested by Figure 24, that the effects of changes in the normal separation rate have negligible impact both in the partial and general equilibrium.

For the endogenous obsolescence rate the model is as follows: At the end of each period firms must pay a continuation cost χ_t to keep jobs active to the next period, or else they are destroyed. In this case both the match and the associated vacancy are destroyed. The continuation cost is stochastic and has cumulative distribution function G. Firms pay the continuation cost if the expected value of keeping the job position outweighs the cost, $\frac{1}{1+r_{t+1}}\mathbb{E}_t\mathcal{J}^x_{t+1} > \chi_t$, where $\mathcal{J}^x$ denotes the value of a job to the firm. The value of a job $\mathcal{J}^x$ depends on whether it is currently filled or not. If filled it generates value $\mathcal{J}^m$ to the firm. If it is not filled it generates value $\mathcal{J}^V$. The share of currently matched positions is $j_t \equiv \frac{N_t}{N_t+V_t}$, and the share of unfilled positions is $1 - j_t = \frac{V_t}{N_t+V_t}$. Hence the aggregate value of a job at time t is $j_t\mathcal{J}^m_t + (1 - j_t)\,\mathcal{J}^V_t$. Since there is no additional heterogeneity for firms, all firms that draw a cost above some cutoff $\chi_{c,t}$ will choose not to pay the cost and hence separate, whereas firms that draw costs below the cutoff will choose to keep their jobs. The cutoff is then defined as the point where the cost equals the expected value of a job in the next period:

$$\chi_{c,t} = \frac{1}{1+r_{t+1}} \left[j_{t+1} \left(1-\delta^{NO}\right) \mathcal{J}^m_{t+1} + \left(1 - j_{t+1}\left(1-\delta^{NO}\right)\right) \mathcal{J}^V_{t+1} \right]$$

where the presence of δ^{NO} captures that current matches might be severed by normal job destruction. Optimally entails the Bellman equations:

$$\begin{aligned}
\mathcal{J}^V_t &= -\kappa_V + m_t \mathcal{J}^m_t + \int^{\chi_{c,t}} \left[\frac{(1-m_t)}{1+r_{t+1}} \mathcal{J}^V_{t+1} - (1-j_t)\,\chi_t \right] dG(\chi_t) \\
\mathcal{J}^m_t &= (MPL_t - w_t) + \int^{\chi_{c,t}} \left[\frac{\left(1-\delta^{NO}\right)}{1+r_{t+1}} \mathcal{J}^m_{t+1} + \frac{\delta^{NO}}{1+r_{t+1}} \mathcal{J}^V_{t+1} - j_t \chi_t \right] dG(\chi_t)
\end{aligned}$$

Firms that draw costs above the cutoff $\chi_{c,t}$ choose to have their jobs/positions destroyed, and the aggregate number of jobs made obsolete in this manner is hence given by $1 - G(\chi_{c,t})$. I define this object as the endogenous obsolescence rate $\delta^O_t = 1 - G(\chi_{c,t})$. Letting $\mu_t = \int^{\chi_{c,t}} \chi_t dG(\chi_t)$ denote the average continuation cost the Bellman equations can be restated as:

$$\begin{aligned}
\mathcal{J}^V_t &= -\kappa_V - (1-j_t)\,\mu_t + m_t \mathcal{J}^m_t + \frac{\left(1-\delta^O_t\right)(1-m_t)}{1+r_{t+1}} \mathcal{J}^V_{t+1} \\
\mathcal{J}^m_t &= (MPL_t - w_t) - j_t \mu_t + \frac{\left(1-\delta^O_t\right)\left(1-\delta^{NO}\right)}{1+r_{t+1}} \mathcal{J}^m_{t+1} + \frac{\left(1-\delta^O_t\right)\delta^{NO}}{1+r_{t+1}} \mathcal{J}^V_{t+1}
\end{aligned}$$

To implement this extension as simply as possible, I keep the assumption from Broer et al. (2020a) that G is a mixture of a point mass and a Pareto distribution with parameter ϵ_x:

$$G(\chi_t) = \begin{cases} 0 & \chi_t < \underline{\chi} \\ 1-p & \underline{\chi} \leqslant \chi_t < \bar{\chi} \\ (1-p) + p\left(1 - \left(\frac{\chi_t}{\bar{\chi}}\right)^{-\epsilon_x}\right) & \chi_t \geqslant \bar{\chi} \end{cases}$$

Suitable calibration ensures that $\underline{\chi}, \bar{\chi}, p$ ensures that the steady state cost μ_t is zero and that $\delta^{NO}_t = \delta^O_{ss}$, where δ^O_{ss} is the obsolescence rate from the baseline calibrations. Appendix D.2 shows that the following relations govern the obsolescence rate and the continuation cost respectively:

$$\delta_t^O = \delta_{ss}^O \left(\frac{1 + r_{ss}}{1 + r_{t+1}} \frac{\mathcal{J}_{t+1}^x}{\mathcal{J}_{ss}^x} \right)^{-\epsilon_x},$$

$$\mu_{\rm t} = \delta_{ss}^O \frac{\epsilon_x}{\epsilon_x - 1} \frac{\left(1 - \delta^O\right) \mathcal{J}_{ss}^x}{(1 + r_{ss})} \left(1 - \left(\frac{1 + r_{ss}}{1 + r_{t+1}} \frac{\mathcal{J}_{t+1}^x}{\mathcal{J}_{ss}^x} \right)^{1-\epsilon_x} \right),$$

where $\mathcal{J}_{t+1}^x$ measures the expected value of a job:

$$\mathcal{J}_{t+1}^x = \frac{1}{1 + r_{t+1}} \left[j_t \left(1 - \delta^{NO}\right) \mathcal{J}_{t+1}^m + \left(1 - j_t \left(1 - \delta^{NO}\right)\right) \mathcal{J}_{t+1}^V \right]$$

Under this formulation ϵ_x measures exactly the elasticity of the obsolescence rate δ_t^O to the future value of a job $\mathcal{J}_{t+1}^x$. For now I keep the calibration from section 7.1, and for the new parameter ϵ_m I adopt the calibration from Broer et al. (2020a) with $\epsilon_\delta = 0.5$. Section 7.4 considers the validity of this calibration by estimating the model w.r.t this elasticity and the sunk cost parameter κ_x.

Section D.3 contains the equations for the labor market model featuring a time-varying separation rate. The main take away is that the various separation rates $\delta^O, \delta^{NO}, \delta^N$ are end-of-period separation rates and hence enters backwards looking equations with a lag.

Partial Equilibrium. Figure 25 display the partial equilibrium impulses for different values of ϵ_x, with $\epsilon_x = 0$ corresponding to exogenous obsolescence. The main take away is that in a partial equilibrium the presence of endogenous obsolescence induce strong amplification since but has only marginal effects on the persistence of variables. This is is in contrast with the effect of sunk costs which - recall Figure 22 - introduce persistence but reduce the peak drop in employment.

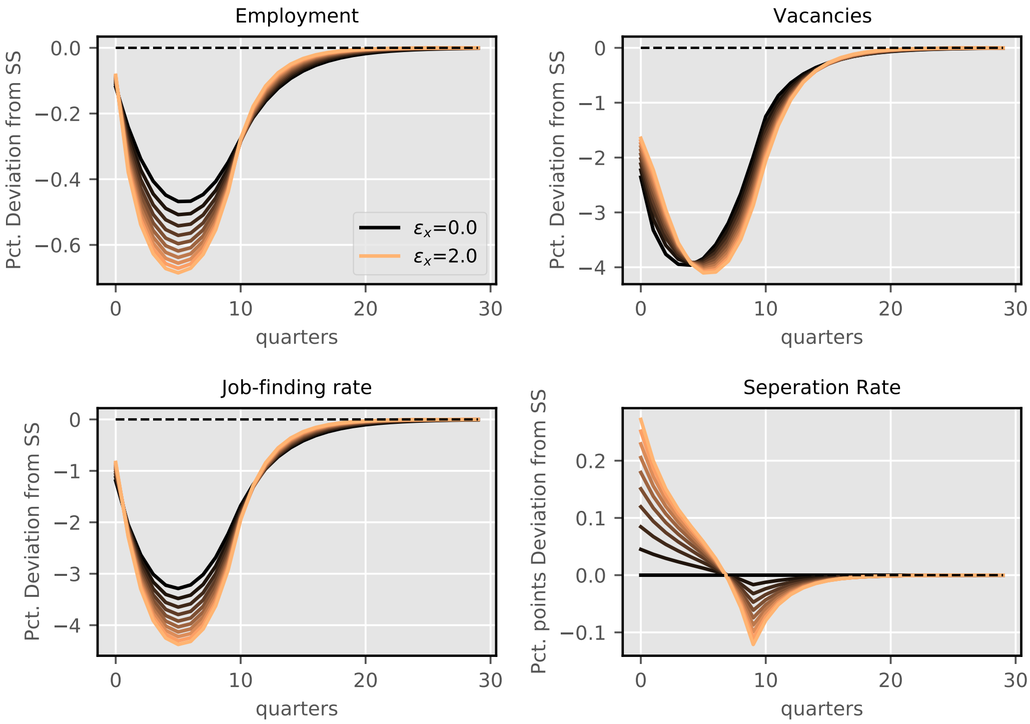


Figure 25: Partial Equilibrium: Sensitivity w.r.t ϵ_x (volatility in separation rate δ^O).

General Equilibrium. Figure 26 compares the general equilibrium responses for the baseline HANK model along with the FR model with exogenous and endogenous separations respectively. With endogenous separations through obsolescence the responses of the FR model are slightly amplificated, reflecting that that as the value of a job drops firms choose not to pay the continuation cost to keep the position and unemployment hence rise. With sunk costs present in the creation of vacancies this rise in unemployment is not met by an immediate increase in new positions and this generates a slightly elevated employment response. Still, the overall responses are small but this warrants little interpretation until the parameters of the labor market model have been estimated in a more systematic manner. I turn to this in the next section.

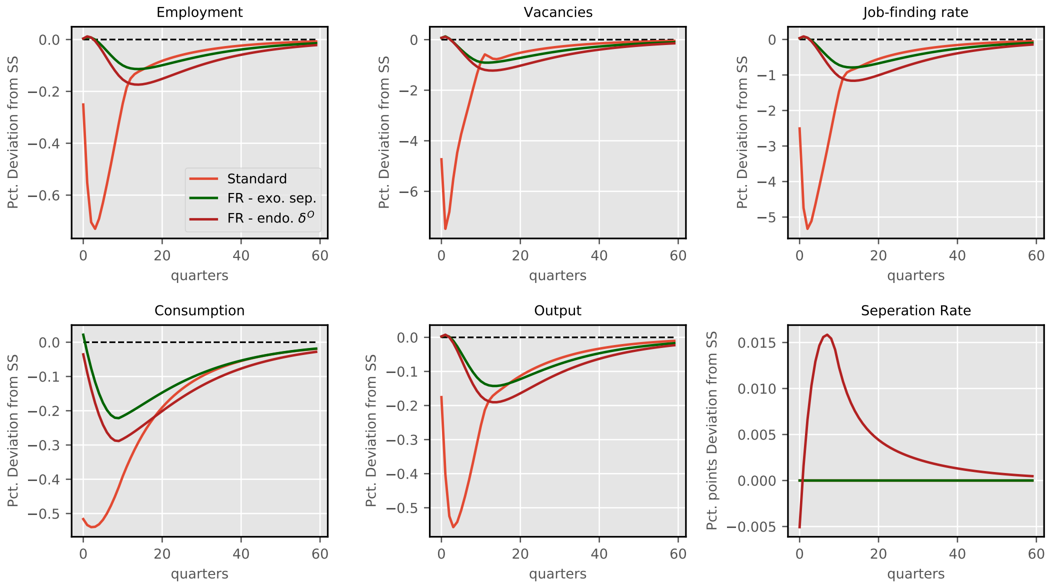


Figure 26: General Equilibrium responses for the baseline model and the Fujita-Ramey model with exogenous and endogenous separations respectively.

7.4 Estimation.

The parameterization so far has been rather unsatisfactory in the sense that the size of the sunk cost κ_x has been taken directly from Fujita and Ramey (2007), though their framework is radically different being only a SAM model. Similarly, the value of ϵ_x (elasticity of separations to job value) has been taken from Broer et al. (2020a), though I do provided some robustness checks for both parameters.

To this end I conduct a simple estimation procedure where I estimate the parameters κ_x, ϵ_x. In steady state different values of κ_x affects the calibrated value of the recurring cost κ_V, and I instead resort to estimating the cost ratio $\mathcal{R} = \frac{\kappa_x x_{ss}}{\kappa_V}$, which is the one Fujita and Ramey (2007) estimated to roughly 5. The implied values of κ_x, κ_V can be recovered afterwards given $\mathcal{R}$.

I conduct the estimation by introducing aggregate risk in the form of a supply shock, a cost-push shock and a demand shock. I assume that aggregate TFP, the markup and the average discount factor follow stationary AR(1) process with independent, normally distributed innovations and estimate these processes simultaneously with the parameters $\mathcal{R}, \epsilon_x$. I use quarterly data for the period 1953-2003 with the cyclical component filtered using a HP filter and target three time series: real GDP, the unemployment rate, and the vacancy rate.[50]

[50]The vacancy rate is the Composite Help-Wanted Index from Barnichon (2010), while the unemployment rate and aggregate real GDP is obtained from FRED.

The last two time series together imply I estimate using the Beveridge curve from Figure 21. I estimate the model using maximum likelihood since this is the simplest to implement, and leave a full Bayesian estimation for the future. Appendix D.5 presents details on how I construct the likelihood function using the methods in Auclert et al. (2019) and how I conduct inference.

Table 4 presents the estimates.[51] The estimate of the sunk-to-recurring costs ratio $\mathcal{R}$ of 3 is lower than the baseline value of 5 from Fujita and Ramey, and also significantly different. On the other hand, the estimated elasticity of obsolescence ϵ_x is roughly three times the size of the baseline value of 0.5. This implies that more amplification and less persistence is needed in the model to match the correlations in the data.

Table 4: Results of Maximum Likelihood Estimation of HANK Model

Shock/Parameter		Estimate	std. dev of Est.	95% CI
Markup	AR(1)-Coef.	0.988	(0.001)	$[0.986, 0.991]$
	s.d.	0.0117	(0.001)	$[0.011, 0.014]$
TFP	AR(1)-Coef.	0.544	(0.062)	$[0.432, 0.677]$
	s.d.	0.007	(0.001)	$[0.006, 0.008]$
Preference	AR(1)-Coef.	0.472	(0.160)	$[0.149, 0.779]$
	s.d.	0.062	(0.024)	$[0.016, 0.112]$
$\mathcal{R} = \frac{\kappa_x x_{ss}}{\kappa_V}$		3.040	(0.640)	$[1.785, 4.295]$
ϵ_x		1.633	(0.021)	$[1.228, 2.040]$
Log-likelihood			793.271	

Estimates from maximum likelihood estimation of the full HANK model including sunk costs for vacancy creation and endogenous separation. During maximization of the log-likelihood function I restrict standard errors of the AR(1) processes to be positive and persistent parameters to lie between 0 and 1 by penalizing guesses outside these intervals. As a robustness check I have also maximized the likelihood function with a Simulated annealing method, which is more robust in finding a global maximum. The results are similar to the standard Nelder–Mead method. Standard errors of estimates are computed using the sandwich variance estimator. The log-likelihood is positive because I estimate only few parameters, and that a reasonable amount of these are estimated tightly (low variance).

7.5 Propagation In the Full Model.

Consider Figure 27. It shows the impulses to a markup shock in the full, estimated model with sunk costs, endogenous obsolescence and precautionary savings from unemployment risk - the three channels which in Broer et al. (2020a) generate considerable propagation.[52] In addition it plots the impulses obtained when removing each of the

[51] Figure D.8 plots the likelihood function in $(\mathcal{R}, \epsilon_x)$ space, holding constant the remaining estimated parameters at their estimated value as an informal identification check.

[52] To keep the impulses comparable with the earlier work I use the "discrete" 10-quarter shocks and not the estimated ones.

previously mentioned channels turn by turn. The full, estimated model delivers a reasonable amount of propagation, with the peak arriving roughly 5 quarters after the shock reverts. This delayed peak is primarily driven by the presence of sunk costs in vacancy creation: Without sunk-costs the value of a vacancy is zero and firms can relatively cheaply adjust the flow of vacancies. Hence they reduce vacancy posting severely for the duration of the shock, but as soon as the shock disappears vacancy creation reverts to the steady-state value and there is no significant persistence.

The presence of endogenous separations accounts for the majority of the amplification occurring: Without cyclical separations employment moves significantly less as only movements in hiring affect cyclical employment. With endogenous job separation employment drops more as firms find it infeasible to pay the continuation cost, and this generates significant amplification through Keynesian multiplier effects.

Abstracting from precautionary savings arising from unemployment risk has a small effect on consumption, which declines slightly less, but no effect on the remaining aggregates. This is unlike Broer et al. (2020a), but unsurprising given the earlier results in this thesis which showed that unemployment risk has a medium initial effect on consumption, but ultimately fails to deliver any persist response - see in particular section 6.1. Figure 28 makes this point explicitly by decomposing the consumption response from Figure 27. Precautionary savings account for a 0.1% drop in consumption on impact, but otherwise the effect is weak. The main driver of a more persistent consumption response is the mechanical effect of employment along with reduced government transfers arising from higher expenditures to unemployment benefits and lower income taxes.

The weak propagation from precautionary savings could also arise from what Challe et al. (2017) characterize as a "stabilizing aggregate supply effect": Precautionary savings increases the supply of assets and this puts a downwards pressure on the real interest rate, which in turn stimulates investments and output. Figure D.9 shows that this mechanism is not the source of "missing" propagation by reconstructing Figure 28 with inelastic investments, obtained by setting a sufficiently high investment adjustment cost.

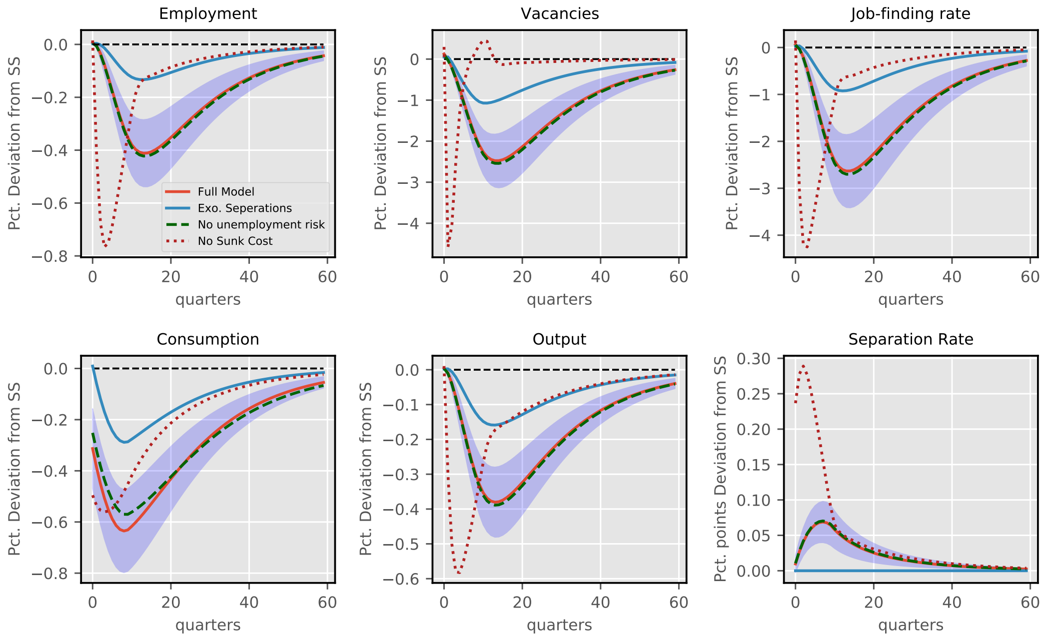


Figure 27: Markup shock.

GE impulses to a 1% increase in price markups for 10 quarters. "Full model" denotes the HANK model featuring sunk-costs and endogenous separations using the estimated parameters. The confidence interval is 90% and constructed using the Delta method. "Exo. Separations" sets $\epsilon_x = 0$. "No Sunk Cost" sets $\kappa_x = 0$. "No unemployment risk" keeps constant the job-finding rate and separation rate in households' Euler equation.

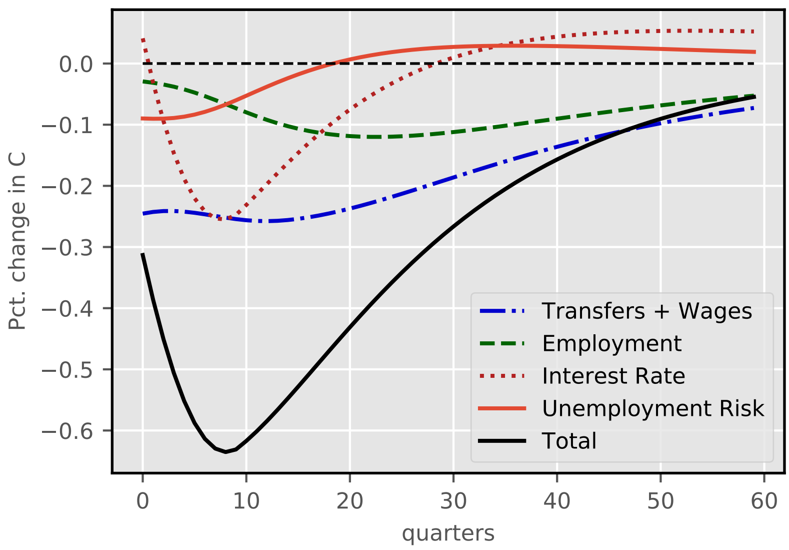


Figure 28: Decomposed consumption in response to cost-push shock (*Full model*).

Appendix D.7 goes through the remaining two estimated shocks (TFP and preference shock). The conclusions are the same as for the cost-push shock.

7.6 Discussion: Precautionary Savings and Unemployment Risk

Precautionary savings in response to cyclical unemployment risk generates a small drop in consumption initially, but does not yield any noteworthy propagation in the one-asset HANK model. The main reason is that when households expect higher unemployment risk in the future they adjust consumption immediately on-impact using their asset stock, and the risk shock generates little persistence. This in turn shuts down the feedback mechanism from Broer et al. (2020a) where precautionary savings are the main driver of demand. This mechanism does not appear in Broer et al. (2020a) since they assume that assets are in zero net supply and households hence cannot use them to adjust consumption immediately as in the full HANK model.

I propose two avenues for future work. The first is to deviate from the assumption of rational expectations and follow Auclert et al. (2020) in assuming sticky households expectations. Per construction this delays the response of consumption to higher unemployment risk, and hence allow for a feedback mechanism to occur between precautionary savings and sluggish labor market dynamics.

The second path is the one that I have continuously discussed in the thesis so far: Adopting a full two-asset HANK model à la Kaplan et al. (2018). Harmenberg and Öberg (2020) constructs a model with durable and non-durable consumption and show that, in a partial equilibrium, non-durable consumption respond little to changes in unemployment risk, and more to mechanical changes in employment, which is consistent with my results since consumption in the one-asset HANK should be interpreted as a non-durable good. The crux of the matter is that they show that *durable* consumption respond much more to changes in unemployment risk since this is subject to adjustment costs: When households are faced with higher income risk through unemployment risk they postpone durable-consumption in order to smooth out the expected adjustment costs that come with adjusting durable consumption. This generates more long-lived responses to changes in income risk, and can potentially interact with the sluggish labor market dynamics I have discussed in detail.

8 Bibliography

References

S Rao Aiyagari. Uninsured idiosyncratic risk and aggregate saving. *The Quarterly Journal of Economics*, 109(3):659–684, 1994.

Adrien Auclert. Monetary policy and the redistribution channel. *American Economic Review*, 109(6):2333–67, 2019.

Adrien Auclert, Matthew Rognlie, and Ludwig Straub. The intertemporal keynesian cross. Technical report, National Bureau of Economic Research, 2018.

Adrien Auclert, Bence Bardóczy, Matthew Rognlie, and Ludwig Straub. Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. Technical report, National Bureau of Economic Research, 2019.

Adrien Auclert, Matthew Rognlie, and Ludwig Straub. Micro jumps, macro humps: monetary policy and business cycles in an estimated hank model. Technical report, National Bureau of Economic Research, 2020.

Carlotta Balestra and Richard Tonkin. Inequalities in household wealth across oecd countries. 2018.

Regis Barnichon. Building a composite help-wanted index. *Economics Letters*, 109(3): 175–178, 2010.

Robert J Barro. Are government bonds net wealth? *Journal of political economy*, 82(6): 1095–1117, 1974.

Susanto Basu, John G Fernald, and Miles S Kimball. Are technology improvements contractionary? *American Economic Review*, 96(5):1418–1448, 2006.

Jess Benhabib, Alberto Bisin, and Shenghao Zhu. The wealth distribution in bewley models with investment risk. Technical report, National Bureau of Economic Research, 2014.

Truman Bewley. The permanent income hypothesis: A theoretical formulation. Technical report, HARVARD UNIV CAMBRIDGE MASS, 1976.

Florin O Bilbiie. Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. *Journal of economic theory*, 140(1):162–196, 2008.

Olivier Jean Blanchard, Peter Diamond, Robert E Hall, and Kevin Murphy. The cyclical behavior of the gross flows of us workers. *Brookings Papers on Economic Activity*, 1990 (2):85–155, 1990.

Vincent Bodart, Gregory De Walque, Olivier Pierrard, Henri R Sneessens, and Rafael Wouters. Nominal wage rigidities in a new keynesian model with frictional unemployment. *National Bank of Belgium Working Paper*, (97), 2006.

Christian Broda and Jonathan A Parker. The economic stimulus payments of 2008 and the aggregate demand for consumption. *Journal of Monetary Economics*, 68:S20–S36, 2014.

Tobias Broer, Jeppe Druedahl, Karl Harmenberg, and Erik Oberg. Labor-market hysteresis and persistent paradox-of-thrift recessions. 2020a.

Tobias Broer, Niels-Jakob Harbo Hansen, Per Krusell, and Erik Öberg. The new keynesian transmission mechanism: A heterogeneous-agent perspective. *The Review of Economic Studies*, 87(1):77–101, 2020b.

John Y Campbell and N Gregory Mankiw. Consumption, income, and interest rates: Reinterpreting the time series evidence. *NBER macroeconomics annual*, 4:185–216, 1989.

Christopher Carroll, Jiri Slacalek, Kiichi Tokuoka, and Matthew N White. The distribution of wealth and the marginal propensity to consume. *Quantitative Economics*, 8 (3):977–1020, 2017.

Christopher D Carroll. The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics letters*, 91(3):312–320, 2006.

Edouard Challe, Julien Matheron, Xavier Ragot, and Juan F Rubio-Ramirez. Precautionary saving and aggregate demand. *Quantitative Economics*, 8(2):435–478, 2017.

Lawrence J Christiano, Martin S Eichenbaum, and Mathias Trabandt. Unemployment and business cycles. *Econometrica*, 84(4):1523–1569, 2016.

Fabrice Collard and Harris Dellas. Technology shocks and employment. *The Economic Journal*, 117(523):1436–1459, 2007.

Wouter J Den Haan, Garey Ramey, and Joel Watson. Job destruction and propagation of shocks. *American economic review*, 90(3):482–498, 2000.

Wouter J Den Haan, Pontus Rendahl, and Markus Riegler. Unemployment (fears) and deflationary spirals. *Journal of the European Economic Association*, 16(5):1281–1349, 2018.

Michael WL Elsby, Ryan Michaels, and David Ratner. The beveridge curve: A survey. *Journal of Economic Literature*, 53(3):571–630, 2015.

Andreas Fagereng, Martin Blomhoff Holm, and Gisle James James Natvik. Mpc heterogeneity and household balance sheets. *Available at SSRN 3399027*, 2019.

Martin Floden and Jesper Lindé. Idiosyncratic risk in the united states and sweden: Is there a role for government insurance? *Review of Economic dynamics*, 4(2):406–437, 2001.

Shigeru Fujita and Garey Ramey. Job matching and propagation. *Journal of Economic dynamics and control*, 31(11):3671–3698, 2007.

Xavier Gabaix. A sparsity-based model of bounded rationality. *The Quarterly Journal of Economics*, 129(4):1661–1710, 2014.

Jordi Gali. Technology, employment, and the business cycle: do technology shocks explain aggregate fluctuations? *American economic review*, 89(1):249–271, 1999.

Jordi Galı and Mark Gertler. Inflation dynamics: A structural econometric analysis. *Journal of monetary Economics*, 44(2):195–222, 1999.

Nils Gornemann, Keith Kuester, and Makoto Nakajima. Doves for the rich, hawks for the poor? distributional consequences of monetary policy. 2016.

Marcus Hagedorn, Iourii Manovskii, and Kurt Mitman. The fiscal multiplier. Technical report, National Bureau of Economic Research, 2019.

Robert E Hall. Employment fluctuations with equilibrium wage stickiness. *American economic review*, 95(1):50–65, 2005.

Karl Harmenberg and Erik Öberg. Consumption dynamics under time-varying unemployment risk. *Journal of Monetary Economics*, 2020.

Martin Blomhoff Holm, Pascal Paul, and Andreas Tischbirek. The transmission of monetary policy under the microscope. Federal Reserve Bank of San Francisco, 2020.

Mark Huggett. The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of economic Dynamics and Control*, 17(5-6):953–969, 1993.

Ayşe Imrohoroğlu. Cost of business cycles with indivisibilities and liquidity constraints. *Journal of Political economy*, 97(6):1364–1383, 1989.

Jacob Isaksen, Paul Lassenius Kramp, Louise Punch Sørensen, and Søren Vaster Sørensen. Household balance sheets and debt: An international country study. In *A flow-of-funds perspective on the financial crisis*, pages 257–270. Springer, 2014.

David S Johnson, Jonathan A Parker, and Nicholas S Souleles. Household expenditure and the income tax rebates of 2001. *American Economic Review*, 96(5):1589–1610, 2006.

Greg Kaplan and Giovanni L Violante. A model of the consumption response to fiscal stimulus payments. *Econometrica*, 82(4):1199–1239, 2014.

Greg Kaplan and Giovanni L Violante. Microeconomic heterogeneity and macroeconomic shocks. *Journal of Economic Perspectives*, 32(3):167–94, 2018.

Greg Kaplan, Benjamin Moll, and Giovanni L Violante. Monetary policy according to hank. *American Economic Review*, 108(3):697–743, 2018.

Loukas Karabarbounis. The labor wedge: Mrs vs. mpn. *Review of Economic Dynamics*, 17(2):206–223, 2014.

Dirk Krueger, Kurt Mitman, and Fabrizio Perri. Macroeconomics and household heterogeneity. In *Handbook of Macroeconomics*, volume 2, pages 843–921. Elsevier, 2016.

Per Krusell and Anthony A Smith, Jr. Income and wealth heterogeneity in the macroeconomy. *Journal of political Economy*, 106(5):867–896, 1998.

Alisdair McKay and Ricardo Reis. Optimal automatic stabilizers. Technical report, National Bureau of Economic Research, 2016a.

Alisdair McKay and Ricardo Reis. The role of automatic stabilizers in the us business cycle. *Econometrica*, 84(1):141–194, 2016b.

Monika Merz. Search in the labor market and the real business cycle. *Journal of monetary Economics*, 36(2):269–300, 1995.

Nationalbanken. Household wealth and debt. https://www.nationalbanken.dk/en/knowledge_bank/themes/Pages/Household-wealth-and-debt.aspx. Accessed: 02-09-2020.

Ioana Neamtu and Niels Westergaard-Nielsen. chapter 9 sources and impact of rising inequality in denmark. *Changing Inequalities and Societal Impacts in Rich Countries: Thirty Countries' Experiences*, page 196, 2014.

Christopher J Nekarda and Valerie A Ramey. The cyclical behavior of the price-cost markup. Technical report, National Bureau of Economic Research, 2013.

Jesper Pedersen and Søren Hove Ravn. What drives the business cycle in a small open economy? evidence from an estimated dsge model of the danish economy. Technical report, Danmarks Nationalbank Working Papers, 2013.

Federico Ravenna and Carl E Walsh. Vacancies, unemployment, and the phillips curve. *European Economic Review*, 52(8):1494–1521, 2008.

Morten O Ravn and Vincent Sterk. Macroeconomic fluctuations with hank & sam: An analytical approach. 2016.

Michael Reiter. Solving heterogeneous-agent models by projection and perturbation. *Journal of Economic Dynamics and Control*, 33(3):649–665, 2009.

Julio J Rotemberg. Sticky prices in the united states. *Journal of Political Economy*, 90 (6):1187–1211, 1982.

Geert Rouwenhorst. K.,(1995).'asset pricing implications of equilibrium business cycle models' chapter 10, frontiers of business cycle research, t. cooley.

Claudia R Sahm, Matthew D Shapiro, and Joel Slemrod. Household response to the 2008 tax rebate: Survey evidence and aggregate implications. *Tax Policy and the Economy*, 24(1):69–110, 2010.

Mr Martin Schindler and Ms Mariya Aleksynska. *Labor market regulations in low-, middle-and high-income countries: A new panel database*. Number 11-154. International Monetary Fund, 2011.

Matthew D Shapiro and Joel Slemrod. Consumer response to tax rebates. *American Economic Review*, 93(1):381–396, 2003.

Matthew D Shapiro and Joel Slemrod. Did the 2008 tax rebates stimulate spending? *American Economic Review*, 99(2):374–79, 2009.

Robert Shimer. The cyclical behavior of equilibrium unemployment and vacancies. *American economic review*, 95(1):25–49, 2005.

José Ignacio Silva, Manuel Toledo, et al. Labor turnover costs and the cyclical behavior of vacancies and unemployment. *Macroeconomic Dynamics*, 13(S1):76–96, 2009.

Nicholas S Souleles. The response of household consumption to income tax refunds. *American Economic Review*, 89(4):947–958, 1999.

Danmarks Statistic. Indkomster 2014. 2014.

Iván Werning. Incomplete markets and aggregate demand. Technical report, National Bureau of Economic Research, 2015.

Michael Woodford. Doing without money: controlling inflation in a post-monetary world. *Review of Economic Dynamics*, 1(1):173–219, 1998.

Appendix

A Model Derivations

A.1 Firm First-order Conditions

Pricing. Let η be the Lagrange multiplier for the Lagrange function $\mathcal{L}$ associated with the constraint $y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\frac{\epsilon_p}{\epsilon_p-1}} Y_t$. The first-order condition for prices $p_{j,t}$ is:

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial p_{j,t}} = &\left\{ \frac{y_{j,t}}{P_t} - \kappa_P \left[\frac{p_{j,t}}{p_{j,t-1}} - \overline{\Pi}\right] Y_t \frac{1}{p_{j,t-1}} \right. \\ &\left. + \frac{1}{1+r_{t+1}} \left[-\kappa_P \left[\frac{p_{j,t+1}}{p_{j,t}} - \overline{\Pi}\right] Y_{t+1} \left(-\frac{p_{j,t+1}}{p_{j,t}^2}\right)\right]\right\} \\ &- \eta_t \left[- \left(-\frac{\epsilon_p}{\epsilon_p - 1}\right) \left(\frac{p_{j,t}}{P_t}\right)^{-\frac{\epsilon_p}{\epsilon_p-1}-1} \frac{Y_t}{P_t}\right] = 0\end{aligned}$$

Define $mc_t \equiv 1 - \eta_t$ and simplify, to get the forward looking, New Keynesian Philips curve:

$$(1-\epsilon_p) + \epsilon_p mc_t - \kappa_P \left[\pi_{t+1} - \overline{\pi}\right](1+\pi_t) + \frac{\kappa_P}{1+r_{t+1}} \frac{Y_{t+1}}{Y_t} \left[\pi_{t+1} - \overline{\pi}\right](1+\pi_{t+1}) = 0$$

where $\overline{\pi} = \overline{\Pi} - 1$.

Capital and Labor Demand. The first-order condition for capital is:

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial k_{j,t}} = \left\{ \frac{p_{j,t}}{P_t} \frac{\partial y_{j,t}}{\partial k_{j,t}} - r_t^k \right\} - \eta_t \left[\frac{\partial y_{j,t}}{\partial k_{j,t}}\right] = 0 \\ \Leftrightarrow \alpha \frac{Y_t}{K_t} mc_t = r_t^k\end{aligned}$$

and similarly for labor:

$$\begin{aligned}\frac{\partial \mathcal{L}_t}{\partial n_{j,t}} = \left\{ \frac{p_{j,t}}{P_t} \frac{\partial y_{j,t}}{\partial n_{j,t}} - r_t^N \right\} - \eta_t \left[\frac{\partial y_{j,t}}{\partial n_{j,t}}\right] = 0 \\ \Leftrightarrow (1-\alpha)\, mc_t \frac{Y_t}{N_t} = r_t^N\end{aligned}$$

Capital firms. Capital firms solve the following dynamic programming problem:

$$V_t^K = \max_{K_t, I_t} r_t^k K_{t-1} - I_t \left(1 + \phi_I \left(\frac{I_t}{I_{t-1}}\right)\right) + \frac{1}{1+r_{t+1}} V_{t+1}^K$$

subject to capital accumulation $K_t = (1-\delta) K_{t-1} + I_t$. Let Q_t denote the Lagrange multiplier for this constraint. The first-order condition for investment I_t is:

$$-\left(1 + \frac{I_t}{I_{t-1}} \phi_I' \left(\frac{I_t}{I_{t-1}}\right) + \phi_I \left(\frac{I_t}{I_{t-1}}\right)\right) + Q_{t+1} Z_t^I + \frac{1}{1+r_{t+1}} \phi_I' \left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 = 0$$

$$\Leftrightarrow 1 + \frac{I_t}{I_{t-1}} \phi_I' \left(\frac{I_t}{I_{t-1}}\right) + \phi_I \left(\frac{I_t}{I_{t-1}}\right) = Q_{t+1} Z_t^I + \frac{1}{1+r_{t+1}} \phi_I' \left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2$$

The first-order condition for capital is:

$$r_{t+1}^k \frac{1}{1+r_{t+1}} - Q_t + (1-\delta) Q_{t+1} = 0$$

$$\Leftrightarrow Q_t = \frac{1}{1+r_{t+1}} \left[(1-\delta) Q_{t+1} + r_{t+1}^k\right]$$

B Consumption Analysis

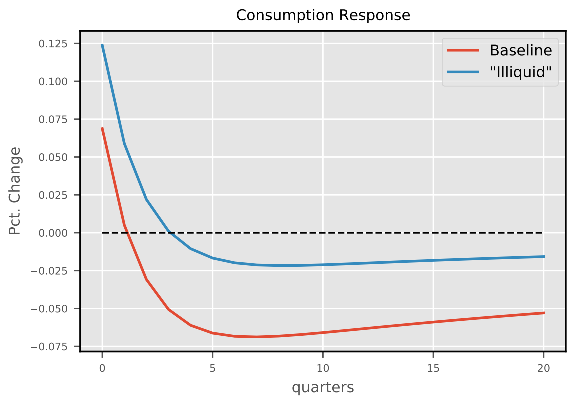


Figure B.1: Responses with "illiquid" assets (reduced unhedged interest rate exposure).

B.1 Robustness check: Household Parameters

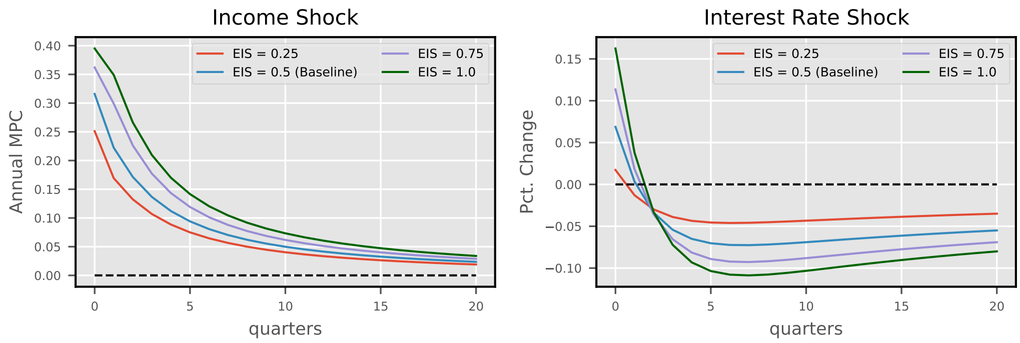
The responses of consumption to changes in income and the interest rate are central to the impulse responses of the model. In the standard RANK setup the household setup is sufficiently transparent such that the implication of different parameterizations are relatively easy to analyze and test. In models with heterogeneous consumers a similar robustness check is more complicated since a change in a given parameter, say, the intertemporal elasticity of substitution σ (EIS), has two effects: 1) A different EIS affects the curvature of the Euler equation hence affecting the dynamic consumption response shape, and 2) A different EIS affect the amount of assets individuals choose to hold in steady-state, and hence the wealth distribution changes. To overcome the interplay between these two channels I do the following: To investigate the effect of the first channel I consider the change in aggregate consumption dC calculated as $\sum_i \mathcal{D}_{i,t}^{Base} c_{i,t}$ where $\sum_i d\mathcal{D}_{i,t}^{Base} dc_{i,t}$ is the distribution matrix from the *baseline* calibration and $dc_{i,t}$ is the change in individual consumption under the alternative parameterization. Hence I avoid distributional effects from parameter changes by fixing the distribution to the one from the baseline calibration.[53]

Figure B.2a shows the consumption (MPC) responses to the earlier described income and interest rate shocks (i.e. the partial equilibrium responses) for different values of the intertemporal elasticity of substitution, with $\sigma = 0.5$ being the baseline value. The figure reveals that, conditional on wealth and income distributions, the dynamic consumption response is very sensitive to the choice of the EIS. Recall that a higher EIS imply less curvature in the utility of consumption, with the limiting cases $\sigma \to 0, \sigma \to \infty$ corresponding to current and future consumption being perfect complements and perfect substitutes respectively. For the income shock a higher EIS imply that households spend more of the transitory income gain today, with the result being a higher contemporaneous MPC, and higher persistence. For the interest rate shock, the response becomes signficantly more volatile as the elasticity increases since households are more willing to substitute between current and future consumption. Neither of the EIS parametriations match the response from Kaplan et al. (2018), which resembles more closely the RANK response.

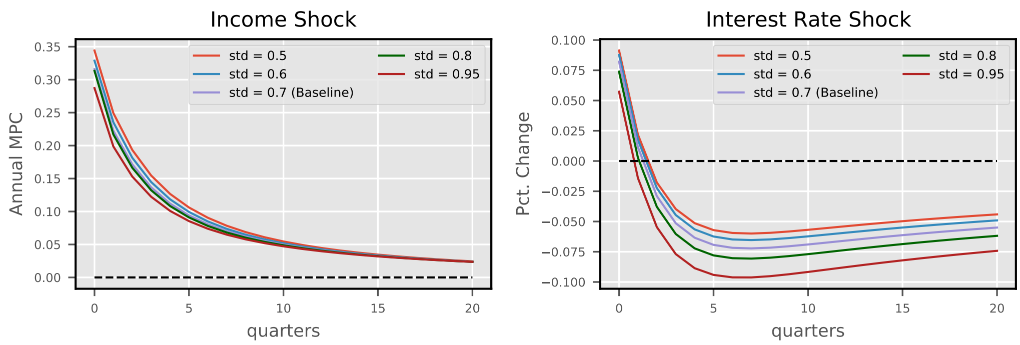
Figure B.2b shows the effect of varying the standard deviation of the earnings process. Note that since the standard deviation determines income dispersion it primarily affects the model through the wealth distribution, which is kept fixed here. The remaining effect from the earnings dispersion is marginal: MPCs are roughly invariant towards

[53]keeping the distribution $\mathcal{D}$ fixed is not sufficient to match the exact same steady-state distribution since asset levels can change endogenously, but it does ensure that the distributions are very similar. Still, the marginal differences can have large effects if they change the number of constrained consumers significantly. To avoid this, I compute a reshuffling of $\mathcal{D}$ by removing a small share of consumers ϵ of the constrained consumers and adding them uniformly to all other groups with the aim being the same share of constrained consumers as in the baseline.

the dispersion, while for the interest rate shock it primarily matters for the long run persistence of the shock. The marginal differences here could, however, be due to slight changes in the wealth distribution.



(a) Varying the intertemporal elasticity of substitution.



(b) Varying the standard deviation of earnings.

Figure B.2: Consumption responses to income and interest rate shocks under various calibrations.

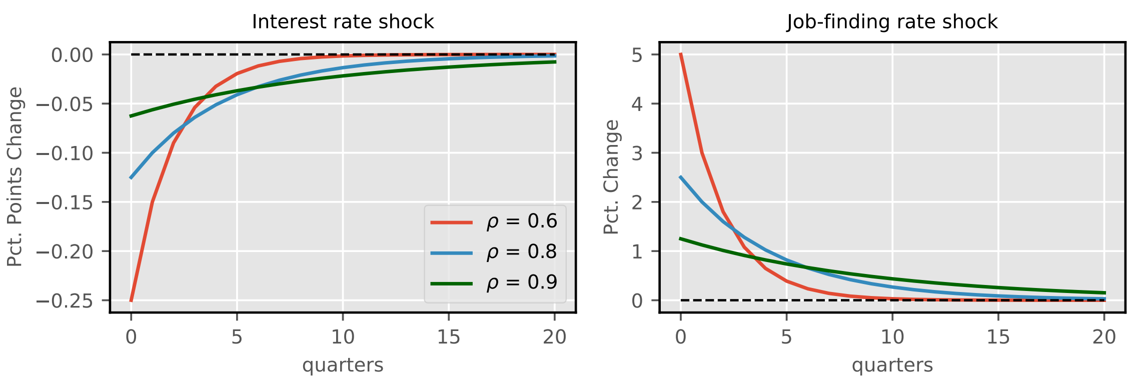


Figure B.3: Shocks to interest- and job-finding rates with differing AR coefficient ρ.

B.2 Details on RA/TA household models

Canonical RA model. Assuming the representative family construct of Merz (1995) implies that there is no idiosyncratic risk or unemployment risk. Hence only aggregate uncertainty matters, which there is none-off in the baseline model. The representative agent solves:

$$V_t = \max_{C_t^R, A_t, \ell_t} \frac{C_t^{R 1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \varphi \frac{\ell_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} + \beta \mathbb{E}_t V_{t+1}$$

$$s.t.$$

$$(1+\tau^c) C_t^R + A_t = (1+r) A_{t-1} + I_t + T_t - \tau(I_t),$$

where $I_t = w_t \ell_t N_t + b_t (1-N_t)$. This results in the standard consumption Euler equation $\left(C_t^R\right)^{-\frac{1}{\sigma}} = R_{t+1}\beta \left(C_{t+1}^R\right)^{-\frac{1}{\sigma}}$.

Canonical TA model. The canonical two-agent household model of Campbell and Mankiw (1989) assumes that a fraction λ of households obey a rule-of-thumb of consuming their entire income each period, and hence holds no assets. Consumption obeys:

$$C_t^{HtM} = \frac{1}{1+\tau^c}\left(\lambda I_t - \tau(\lambda I_t) + \lambda T_t\right),$$

$$C_t = C_t^R + C_t^{HtM},$$

where C_t^R is defined as above in the RA's problem, but with income flows scaled by the share of Ricardian agents $1-\lambda$.

B.3 Aggregation Result: Proof

As in the main text I consider the aggregate change in consumption $dC_t = \int dc_t^* d\mathcal{D}_{ss} + \int c_{ss}^* d\mathcal{D}_t$, and specifically the term $\int dc_t^* d\mathcal{D}_{ss}$. Since we condition on the steady state distribution of households the share of constrained households λ is fixed. Let c_t^R denote consumption of non-constrained ("Ricardian") agents and c_t^{HtM} consumption of constrained ("Hand-to-Mouth") agents. Then:

$$\int dc_t^* d\mathcal{D}_{ss} = (1-\lambda)\int dc_t^R d\mathcal{D}_{ss}^R + \lambda \int dc_t^{HtM} d\mathcal{D}_{ss}^{HtM}$$

The budget constraint for a constrained household is:

$$c_{i,t}^{HtM} + \underline{a} = (1 + r_t^a)a_{i,t-1} + I_{i,t} - \tau^I\left(I_{i,t}\right) + T_t$$

Assume that wages are fixed and consider a first-order perturbation:

$$dc_{i,t}^{HtM} = (1 + r_{ss}^a)da_{i,t-1} + \underline{a}dr_t^a + dT_{i,t}$$

since $a_{i,t-1}$ is fixed when conditioning on the initial distribution we have $da_{i,t-1} = 0$ such that $dc_{i,t}^{HtM} = \underline{a}dr_t^a + dT_{i,t}$, which aggregates:

$$\int dc_{i,t}^{HtM} d\mathcal{D}_{ss}^{HtM} = \int \underline{a}dr_t^a d\mathcal{D}_{ss}^{HtM} + \int dT_{i,t} d\mathcal{D}_{ss}^{HtM}$$
$$\Leftrightarrow dC_t^{HtM} = \underline{a}dr_t^a + dT_t$$

Rational household obey the linearized Euler equation:

$$dc_{i,t}^R = \beta_i R_{ss}\left(E_{i,t}dc_{i,t+1}^R - \sigma R_{t+1}c_{i,ss}^R\right)$$

This disregards unemployment risk since I do not consider shock to the separation rate or the job-finding rate in this setting. Iterating once yields:

$$\Leftrightarrow dc_t^R = R_{ss}\beta_i E_{i,t}\left(R_{ss}\beta_i E_{i,t+1}dc_{i,t+2}^R - \sigma dr_{t+2}^a c_{ss}^R\right) - \sigma dr_{t+1}^a c_{ss}^R$$

Applying the law of iterated expectations for some stochastic variable x gives $E_{i,t}\left(E_{i,t+1}\left(x_t \left|\mathcal{I}_{t+1}, \mathcal{I}_t\right.\right)\left|\mathcal{I}_t\right.\right) = E_{i,t}\left(x_t \left|\mathcal{I}_t\right.\right)$ where the latter is defined as $E_{i,t}\left(x_t\right)$ such that $E_{i,t}\left(E_{i,t+1}dc_{i,t+2}^R\right) = E_{i,t}dc_{i,t+2}^R$. Hence in the terminal period we have:

$$\Leftrightarrow dc_t^R = \left(R_{ss}\beta_i\right)^T E_{i,t}dc_{i,t+T}^R - \sigma c_{ss}^R \sum_{j=1}^{T} dr_{t+j}^a$$

Since we condition on the steady-state distribution the expectation is the same every period and can be omitted. Together with the budget constraint $dc_{i,t}^R + da_{i,t} = dr_t^a a_{i,ss} + dT_t$, which importantly conditions on the initial distribution and hence hold existing assets fixed, implies that the household act as a permanent-income agent.

B.4 Figures for cost-push shock analysis.

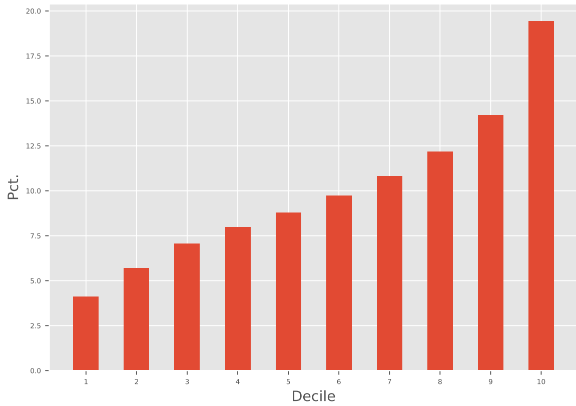


Figure B.4: Share of aggregate consumption by wealth deciles in steady-state.

Figure B.5: Inequality responses following cost-push shock.

Note: The first three panels measure wealth, consumption and income inequality over the cycle by the change in the variance of the log distribution relative to steady-state. The final panel computes the change in the wealth share of the top 5% wealthiest households relative to steady-state.

B.5 Welfare Analysis: Moving from Static to Dynamic Responses.

The picture in 13 does not translate one-to-one into the welfare losses in Figure 14, the most notable difference being that employed households in the 1st wealth decile experience a lesser consumption drop on impact but a larger, overall welfare loss compared to unemployed households. This is because the decomposition of consumption across the wealth distribution is a static picture, while the consumption-equivalent welfare is intertemporal in the sense that it takes into account the entire discounted path of consumption. Figure B.6 plots consumption trajectories by the poorest, middle and wealthiest wealth decile and by employment status. For the 5th and 10th wealth deciles the point of Figure 13 remains since the consumption responses are monotone from period 0 and onwards between employed and unemployed. However, for the poorest wealth decile the picture is different. While unemployed households in this decile initially suffers a larger consumption drop compared to employed households this is not true for the subsequent periods, and this explains the welfare losses in 14.

Figure B.7 decomposes the consumption response of the 1st wealth decile by employment. It shows that unemployed households in this decile initially suffers a larger consumption drop due to precautionary savings induced by a lower job-finding rate. The larger drop in employed households' consumption in the subsequent periods are driven by both the interest rate, transfers and wages, though the effect of wages is marginal.

The different interest rate responses are driven by the fact unemployed have accumulated severely more debt compared to the unemployed in steady state. This is occurs because the precautionary savings motive from unemployment risk is significantly stronger for unemployed, as per the figure. Thus employed households are affected more when the interest rate increases as interest payments increase. Additionally employed households also respond more aggressively to transfers for the duration of the shock. The source of this is the same as for the interest rate: Employed households hold less assets (more debt) than unemployed households, and unemployed households are hence more capable of smoothing consumption for the duration of the shock.

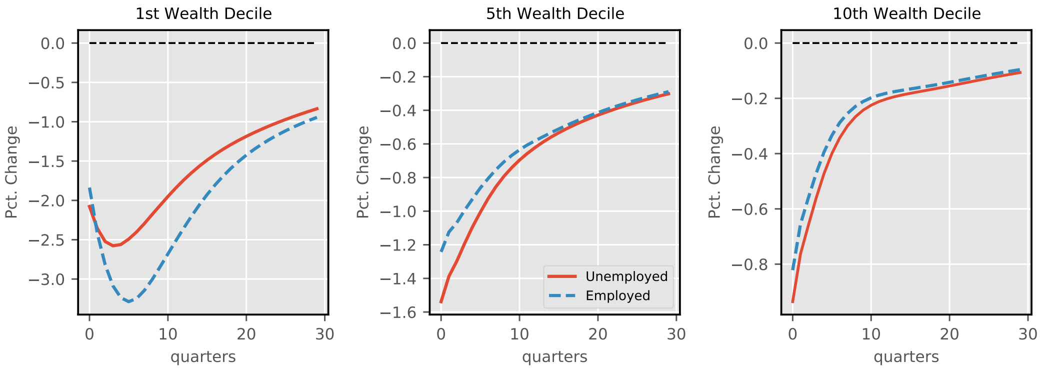


Figure B.6: Consumption Paths for bottom, mid and top wealth deciles by employment.

Figure B.7: Decomposed consumption for poorest wealth decile by employment.

C Automatic Stabilizers- Unemployment Benefits

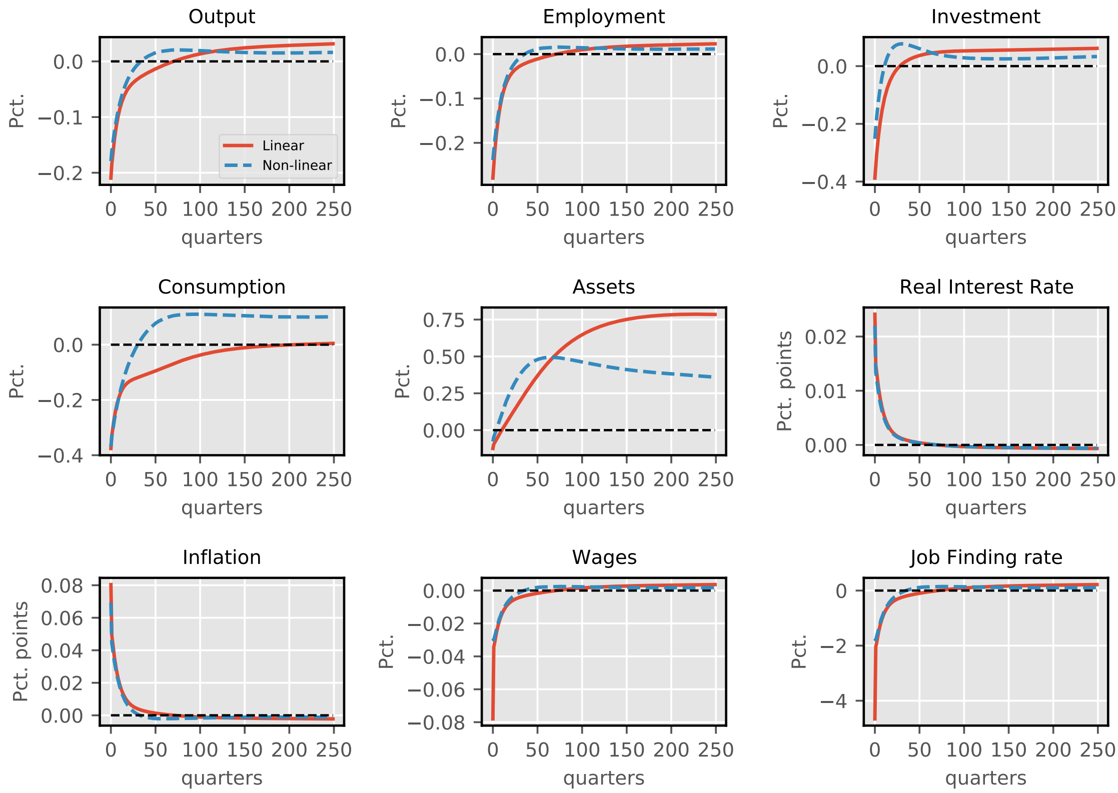


Figure C.1: Transitional Dynamics following a lower unemployment benefit rate - linear and non-linear impulses.

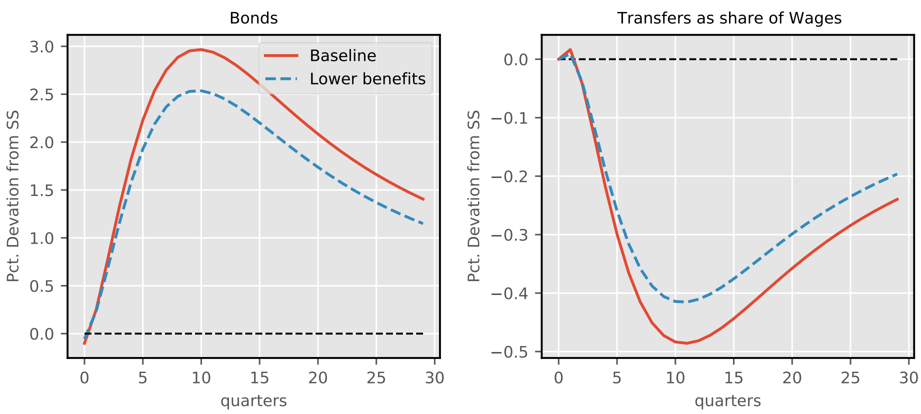


Figure C.2: Responses of Bonds and Transfers.

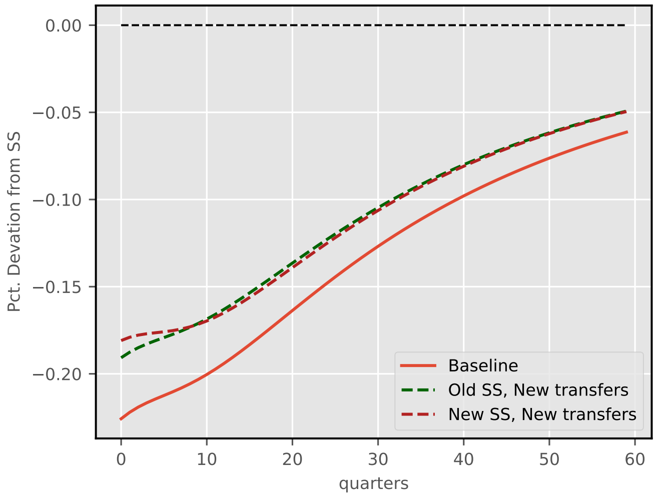


Figure C.3: Response of Consumption to transfers under different scenarios.

Figure C.4: Inequality responses following cost-push shock - comparison between baseline and model with lower unemployment benefits.

Note: The first three panels measure wealth, consumption and income inequality over the cycle by the change in the variance of the log distribution relative to steady-state. The final panel computes the change in the wealth share of the top 5% wealthiest households relative to steady-state.

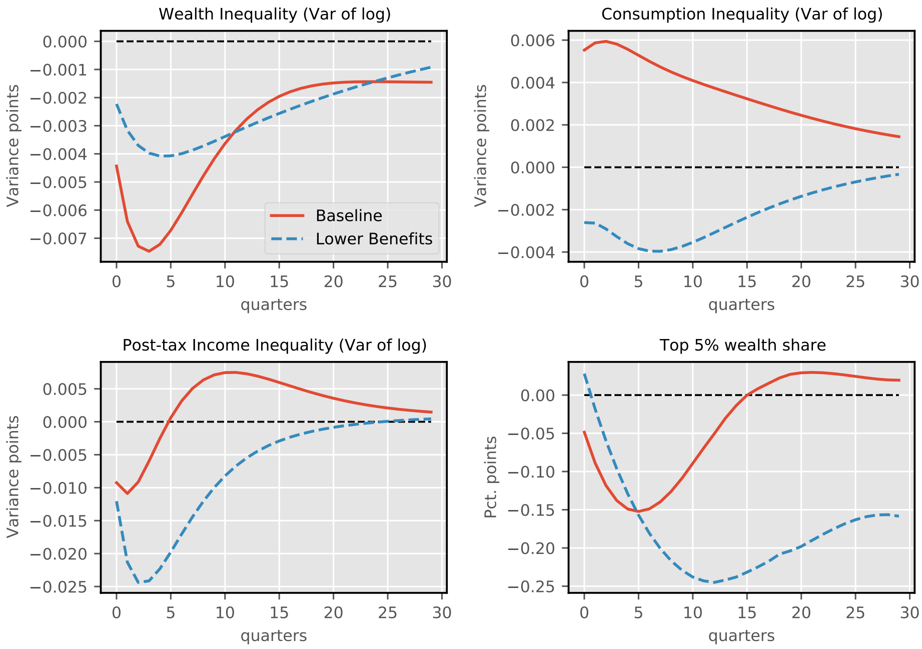


Figure C.5: Inequality responses following cost-push shock - comparison between baseline and model with lower unemployment benefits.

Note: The first three panels measure wealth, consumption and income inequality over the cycle by the change in the variance of the log distribution relative to steady-state. The final panel computes the change in the wealth share of the top 5% wealthiest households relative to steady-state.

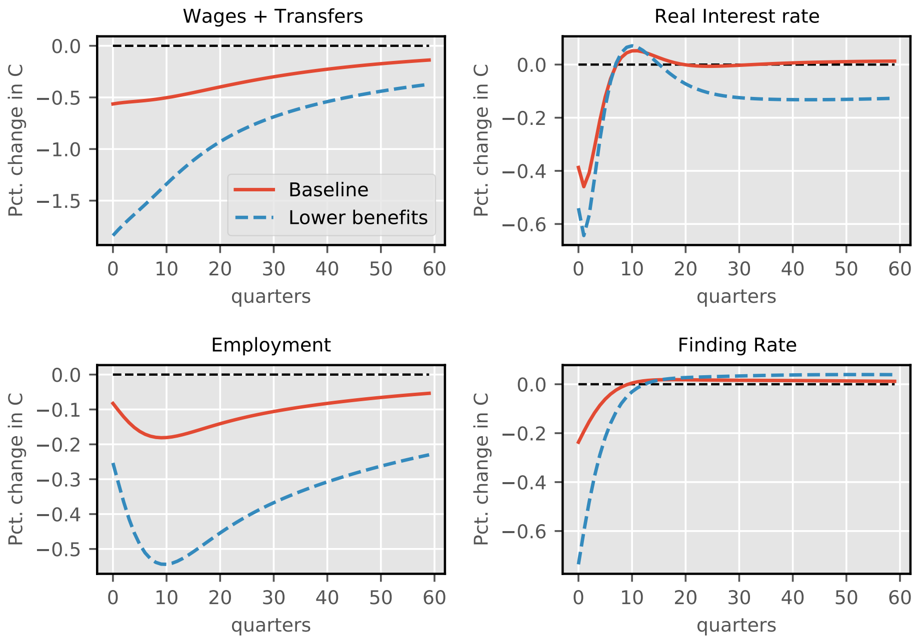


Figure C.6: Decomposition of consumption response when policy change to unemployment benefits is financed with bonds.

D Labor Market Dynamics

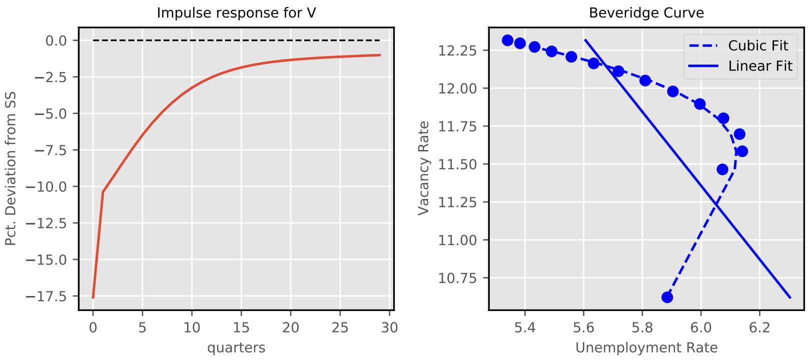


Figure D.1: Response of vacancies in the baseline HANK model (left panel) and the implied Beveridge curve following a cost-push shock.

D.1 Calibration With Sunk Costs

Recurring vs sunk costs. Figure D.2 plots the labor market responses in the Fujita-Ramey model for different combinations of the recurring vacancy posting cost κ_V and the sunk cost κ_x. The special case where $\kappa_x = 0$ corresponds to the baseline model. The figure suggests that the presence of the sunk cost strongly reduces the initial volatility of employment and vacancies in the first 10 quarters, by smoothing out the creation of vacancies. Accordingly, there are longer lasting effects on employment and the job-finding rate, which shows that the introduction of the sunk cost have the desired effect, at least in a partial equilibrium.

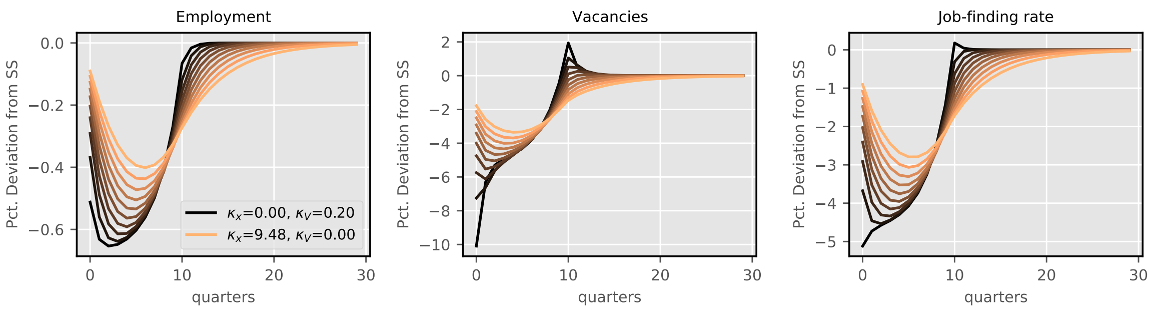


Figure D.2: Responses for the Fujita-Ramey Sunk-cost model for different combinations of the recurring cost κ_V and the sunk cost κ_x

Note: Each curve within a plot corresponds to a calibration where the sunk cost accounts for a share λ of total vacancy costs for $\lambda = \frac{x_t \kappa_x}{x_t \kappa_x + \kappa_v}$ for $\lambda = 0, 0.1, ..., 1$.

Labor Wedge. Figure D.3 shows the importance of the calibrated labor wedge $MPL - w$ for the result.[54] Obviously the bare existence of a labor wedge is necessary to introduce any persistence. Beyond this, a larger labor wedge strongly reduces volatility for the duration of the shock, but has only marginal effects on persistence. The main driver of this is that a large labor wedge implies the presence of large vacancy costs and per Figure D.2 larger sunk costs reduces volatility.

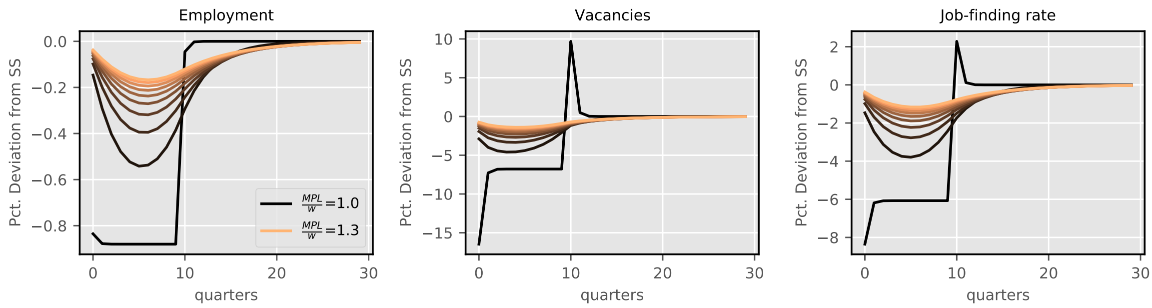


Figure D.3: Responses for the Fujita-Ramey Sunk-cost model for different combinations of the "labor we

Note: Each curve within a plot corresponds to a calibration where the "labor wedge" $\frac{MPL_t}{w_t}$ equals λ where $\lambda = \frac{x_t \kappa_x}{x_t \kappa_x + \kappa_v}$ for $\lambda = 0, 0.03, 0, 06, ..., 0.3$.

D.2 Stochastic Continuation Cost Derivations

Assume that the stochastic continuation cost χ_t follows a distribution G. For this distribution I assume the same density as in Broer et al. (2020a) since this admits analytical aggregation:

$$G(\chi_t) = \begin{cases} 0 & \chi_t < \underline{\chi} \\ 1 - p & \underline{\chi} \leqslant \chi_t < \bar{\chi} \\ (1-p) + p\left(1 - \left(\frac{\chi_t}{\bar{\chi}}\right)^{-\epsilon_x}\right) & \chi_t \geqslant \bar{\chi} \end{cases}$$

The cutoff $\chi_{c,t}$ must satisfy $\chi_{c,t} = \frac{1}{1+r_{t+1}} \mathcal{J}^x_{t+1}$. Assuming $\chi_{c,t} > \bar{\chi}$ we can write the separation rate as:

$$\delta^O_t = 1 - G(\chi_{c,t}) = p - p\left(1 - \left(\frac{\chi_{c,t}}{\bar{\chi}}\right)^{-\epsilon_x}\right) = p\left(\frac{\chi_{c,t}}{\bar{\chi}}\right)^{-\epsilon_x}$$

[54] I am aware that the literature sometimes refer to the labor wedge as the difference between the marginal product of labor and the marginal rate of substitution between labor and consumption, e.g. Karabarbounis (2014).

We can choose parameters $p = \delta^O_{ss}, \bar{\chi} = \chi_{c,ss} = \frac{\mathcal{J}^x_{ss}}{(1+r_{ss})}$ and insert for the cutoff to arrive at the equation in the main text:

$$\delta^O_t = \delta^O_{ss}\left(\frac{1+r_{ss}}{1+r_{t+1}}\frac{\mathcal{J}^x_{t+1}}{\mathcal{J}^x_{ss}}\right)^{-\epsilon_x}$$

The average cost paid to keep matches μ_t can be written as:

$$\mu_{\mathrm{t}} = (1-p)\,\underline{\chi} + p\frac{\epsilon_x}{\epsilon_x - 1}\bar{\chi}^{\epsilon_x}\left(\bar{\chi}^{1-\epsilon_x} - \chi^{1-\epsilon_x}_{c,t}\right)$$

Using $\bar{\chi} = \chi_{c,ss}$ from above this simplifies to:

$$\begin{aligned}
\mu_{\mathrm{t}} &= \left(1-\delta^O_{ss}\right)\underline{\chi} + \delta^O_{ss}\frac{\epsilon_x}{\epsilon_x - 1}\chi^{\epsilon_x}_{c,ss}\left(\chi^{1-\epsilon_x}_{c,ss} - \chi^{1-\epsilon_x}_{c,t}\right) \\
&= \left(1-\delta^O_{ss}\right)\underline{\chi} + \delta^O_{ss}\frac{\epsilon_x}{\epsilon_x - 1}\chi_{c,ss}\left(1 - \left(\frac{\chi_{c,t}}{\chi_{c,ss}}\right)^{1-\epsilon_x}\right)
\end{aligned}$$

Since $\chi_{c,t} = \chi_{c,ss}$ in steady state we can set $\underline{\chi} = 0$ to obtain $\mu_{ss} = 0$. Substitute in for the cutoffs to arrive at:

$$\mu_{\mathrm{t}} = \delta^O_{ss}\frac{\epsilon_x}{\epsilon_x - 1}\frac{\left(1-\delta^O\right)\mathcal{J}^x_{ss}}{(1+r_{ss})}\left(1 - \left(\frac{1+r_{ss}}{1+r_{t+1}}\frac{\mathcal{J}^x_{t+1}}{\mathcal{J}^x_{ss}}\right)^{1-\epsilon_x}\right)$$

D.3 Labor Market model with Endogenous Separations

With a time-varying separation rate - either - δ^{NO} or δ^O - the interpretation of the time t separation rate δ^N_t is that this is is the share of employed households that keep their jobs for the next period $t+1$. Hence the appropriate law of motion for employment reads:

$$N_t = \left(1-\delta^N_{t-1}\right)N_{t-1} + S_t q_t\left(1-\delta^O_{t-1}\right),$$

And by extension the number of searchers is:

$$S_t = 1 - N_{t-1}\left(1-\delta^N_{t-1}\right),$$

For the Fujita-Ramey model, the law of motion for vacancies reads:

$$V_t = \left(1-\delta^O_{t-1}\right)\left((1-m_{t-1})\,V_{t-1} + \delta^{NO}_{t-1}N_{t-1}\right) + x_t$$

For households the Euler equations are:

$$\left(c_{i,t}^{k=U}\right)^{-\frac{1}{\sigma}} = \beta \mathbb{E}_{i,t} R_{t+1} \left[\left(1-\delta_t^O\right) q_{t+1} \left(c_{i,t+1}^{k=N}\right)^{-\frac{1}{\sigma}} + \left(1 - \left(1-\delta_t^O\right) q_{t+1}\right) \left(c_{i,t+1}^{k=U}\right)^{-\frac{1}{\sigma}} \right]$$
$$\left(c_{i,t}^{k=N}\right)^{-\frac{1}{\sigma}} = \beta \mathbb{E}_{i,t} R_{t+1} \left[\left(1-\delta_t^N \left(1-q_{t+1}\right)\right) \left(c_{i,t+1}^{k=N}\right)^{-\frac{1}{\sigma}} + \delta_t^N \left(1-q_{t+1}\right) \left(c_{i,t+1}^{k=U}\right)^{-\frac{1}{\sigma}} \right]$$

The Bellman equations governing the values of vacancies and matches respectively are:

$$\mathcal{J}_t^V = -\kappa_V - \left(1-j_t\right)\mu_t + m_t \mathcal{J}_t^m + \frac{\left(1-\delta_t^O\right)\left(1-m_t\right)}{1+r_{t+1}} \mathcal{J}_{t+1}^V$$
$$\mathcal{J}_t^m = \left(MPL_t - w_t\right) - j_t \mu_t + \frac{\left(1-\delta_t^O\right)\left(1-\delta_t^{NO}\right)}{1+r_{t+1}} \mathcal{J}_{t+1}^m + \frac{\left(1-\delta_t^O\right)\delta_t^{NO}}{1+r_{t+1}} \mathcal{J}_{t+1}^V$$

D.4 Endogenous "Normal" Job Separation

I here present the results obtained in the Fujita-Ramey model with endogenous "Normal" separations. For the simple FR model this implies simply endogenizing the aggregate separation rate δ^N, while for the full FR model this involves endogenizing the "normal" separation rate δ^{NO}. This is done in the same way as for the obsolescence rate δ^O in the main text, though there are some key differences in constructing the value of a job vs. the value of a match in the full FR model.

Similar to the main text, firms must pay a continuation cost χ_t to keep its match (not job as in the main text), or else it is destroyed. Firms are willing to pay the cost if the expected value of keeping the match exceeds the cost, $\frac{1}{1+r_{t+1}} \mathbb{E}_t \mathcal{J}_{t+1}^M > \chi_t$. Optimality entails that the value of a match is given by the Bellman equation:

$$\mathcal{J}_t^m = \left(MPL_t - w_t\right) + \int^{\chi_{c,t}} \left[\frac{1-\delta^O}{1+r_{t+1}} \mathcal{J}_{t+1}^m - \chi_t \right] dG\left(\chi_t\right),$$

where $\chi_{c,t} = \frac{1}{1+r_{t+1}} \mathbb{E}_t \mathcal{J}_{t+1}^m$ is the cutoff above which firms do not pay the cost, and matches are destroyed. Conducting identical operations as in the main text yields:

$$\mathcal{J}_t^m = \left(MPL_t - w_t - \mu_t\right) + \frac{\left(1-\delta^O\right)\left(1-\delta_t^{NO}\right)}{1+r_{t+1}} \mathcal{J}_{t+1}^m,$$

where $\mu_t = \int^{\chi_{c,t}} \chi_t dG\left(\chi_t\right)$ and $\delta_t^{NO} = 1 - G\left(\chi_{c,t}\right)$. The separation rate and the associated cost is then given by:

$$\delta_t^{NO} = \delta_{ss}^{NO} \left(\frac{1 + r_{ss}}{1 + r_{t+1}} \frac{\mathcal{J}_{t+1}^m}{\mathcal{J}_{ss}^m} \right)^{-\epsilon_m},$$

$$\mu_{\mathrm{t}} = \delta_{ss}^{NO} \frac{\epsilon_m}{\epsilon_m - 1} \frac{\left(1 - \delta^O\right) \mathcal{J}_{ss}^m}{(1 + r_{ss})} \left(1 - \left(\frac{1 + r_{ss}}{1 + r_{t+1}} \frac{\mathcal{J}_{t+1}^m}{\mathcal{J}_{ss}^m} \right)^{1-\epsilon_m} \right)$$

To obtain the relevanet equations for the simple FR model, simply set $\delta^N = \delta^{NO}$ and $\delta^O = 0$.

Partial Equilibrium. Figure D.4 considers the TFP shock in the partial equilibrium with endogenous job separations. Introducing endogenous job separation imply that when the TFP shock hits the economy and the value of a match declines the separation rate increases since some firms now find it infeasible to pay the continuation cost of keeping a match. This implies slightly less volatility in the posting of vacancies since a higher separation rate acts as a substitute to separate from employees. There are ambiguous effects on the job-finding rate since the number of searchers increase through a higher separation rate, but vacancies are stabilized. I turns out that the vacancy effect dominates and the job-finding rate drops marginally less. For both models the increase in separations slightly increases the drop in employment for the duration of the shock. As the TFP shock disappears the value of a match increases and the separation rate return to the steady state level, and the persistence of the responses are identical to the situation with exogenous separations. The very marginal effect that the normal separation rate has on the cycle stems from the fact that it separates matches while allowing the jobs and associated vacancies to continue existing. Hence when the value of a match drops and firms choose not to pay the continuation cost they simply allow the match to be destroyed and wait for the job and vacancy to be filled again next period.

Figure D.5 also establishes this point by varying the elasticity of match destruction $_m$ in the partial equilibrium model. The elasticity affects strongly the separation rate as is expected. Increased separations imply that firms create less new jobs and the number of vacancies drop less in response to the shock as the elasticity increases. However, the effect on equilibrium employment and job-finding rate is marginal since the increase in separations also strongly affects the number of searchers, cf. Figure D.6.

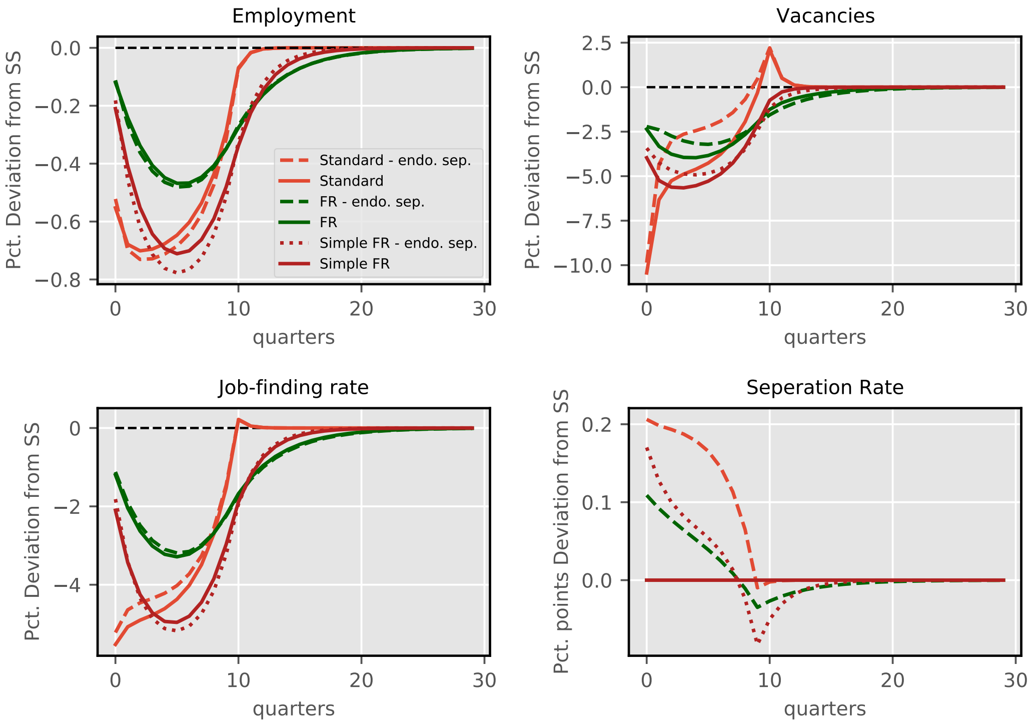


Figure D.4: Partial Equilibrium responses with endogenous "normal" separations (δ^{NO}).

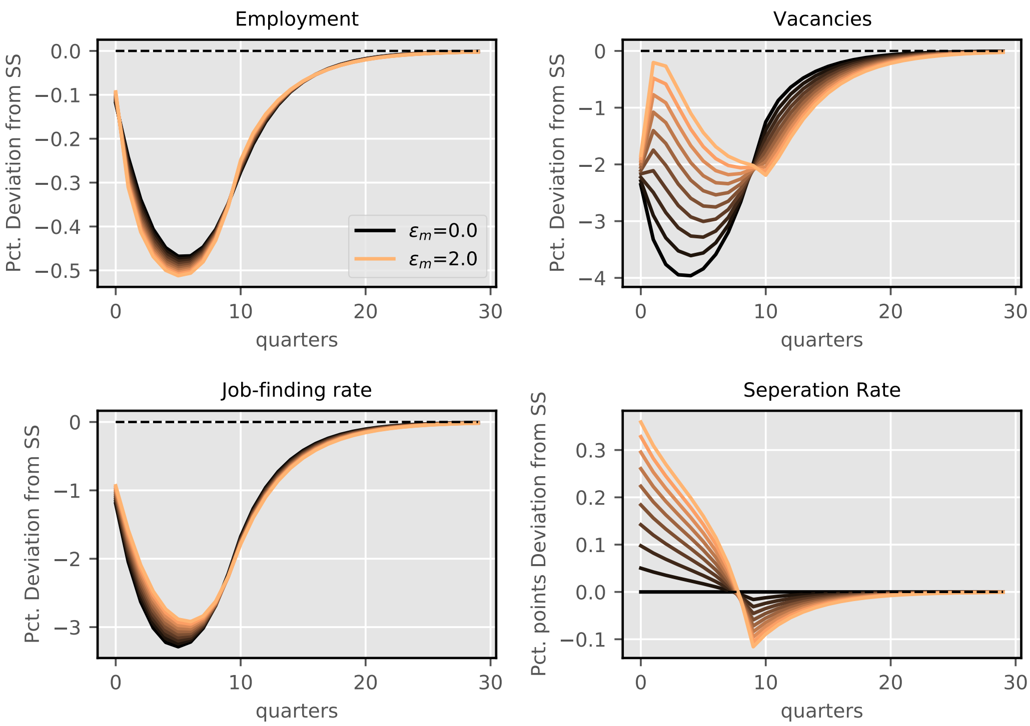


Figure D.5: Partial Equilibrium: Sensitivity w.r.t ϵ_m in the FR model (elasticity of separation rate δ^{NO} to value of a match).

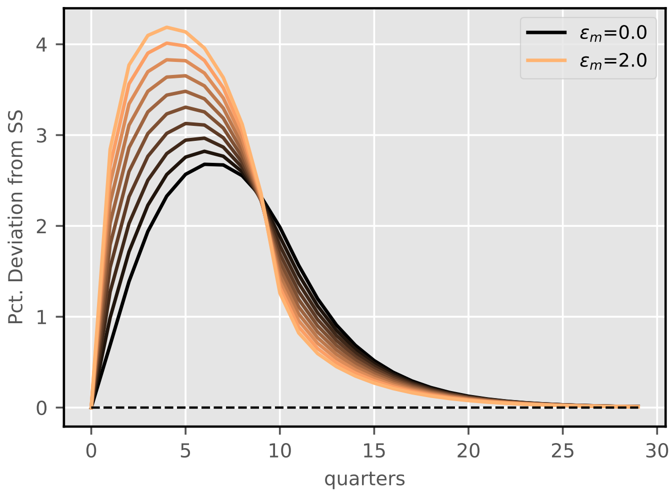


Figure D.6: Response of searchers for different ϵ_m in the Fujita-Ramey model with endogenous separations δ^{NO}.

General Equilibrium. Figure D.7 considers the effects of endogenizing δ^{NO} in the general equilibrium. Similarly to the partial equilibrium case the impulses are roughly the same.

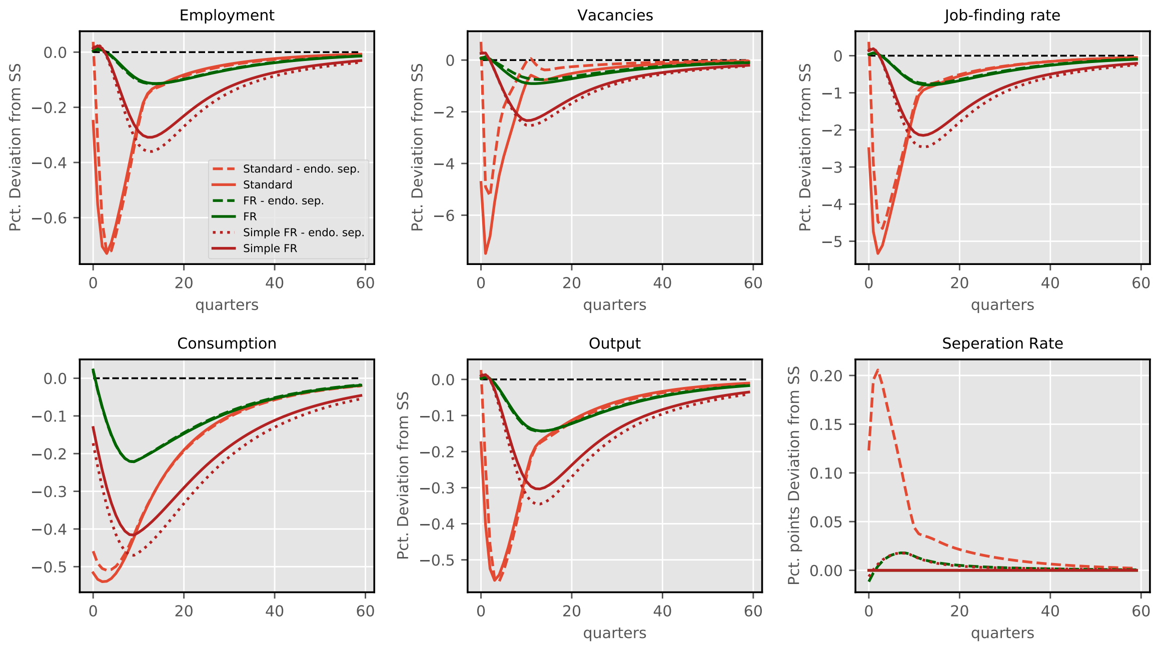


Figure D.7: Responses for labor markets variables in to cosh-push shock in general equilibrium for different vacancy posting models with endogenous separations δ^{NO}.

D.5 Estimation Using SHADE Methods.

Constructing the Likelihood Function. I briefly outline the method for estimation of HA general equilibrium models presented in Auclert et al. (2019). Let the exogenous shocks of the model be written as a $MA(\infty)$ process $d\tilde{Z}_t^z = \sum_{s=0}^{\infty} m_s^z \epsilon_{t-s}^z$ where ϵ_{t-s}^z are I.I.D standard normally distributed innovations and m_s^z are the coefficients of the process. Let $d\tilde{X}_t^o$ be the response of a variable X in the general equilibrium model with aggregate risk. Since each of the shocks in the model are independent $MA(\infty)$ the response $\tilde{X}_t^o$ is also a $MA(\infty)$ process: This can be written as:

$$d\tilde{X}_t^o = \sum_{s=0}^{\infty} \sum_{z} m_s^{o,z} \epsilon_{t-s}^z, \tag{36}$$

The coefficients $m_s^{o,z}$ of this process are objects that depend on the general equilibrium Jacobians and the moving average coefficients of the exogenous processes m_s^z. The idea to extract the coefficients $m_s^{o,z}$, who determines IRFs in the stochastic model, is to utilize a certainty equivalence property of linearization. This implies that to a first-order, the impulses to the stochastic model equals those from the associated perfect foresight model. Stacking the coefficients over time in vectors $\mathbf{m}^{0,z}, \mathbf{m}^z$ the coefficients can be computed as:

$$\mathbf{m}^{o,z} = \mathbf{G}^{o,z}\mathbf{m}^z \tag{37}$$

where $\mathbf{G}^{o,z}$ is the general equilibrium Jacobian variable $\tilde{X}_t^o$ w.r.t z. This Jacobian can be rapidly computed using the Fake-news algorithm as per the earlier sections. MA coefficients map directly to autocovariances:

$$\text{Cov}\left(d\tilde{\mathbf{X}}_t, d\tilde{\mathbf{X}}_{t'}\right) = \sum_{s=0}^{T-(t'-t)} \left[\mathbf{m}_s^{o,z}\right] \left[\mathbf{m}_{s+t'-t}^{o,z}\right]' \tag{38}$$

Stacking these autocovariances into a complete variance-covariance matrix $\mathbf{V}$ the log likelihood function can, up to a constant, be written as:

$$\log \mathcal{L}(\theta) = -\frac{1}{2}\log\det\mathbf{V} - \frac{1}{2}\left[d\tilde{\mathbf{X}}^{obs}\right]' \mathbf{V}^{-1}\left[d\tilde{\mathbf{X}}^{obs}\right] \tag{39}$$

where $d\tilde{\mathbf{X}}^{obs}$ contain demeaned data for the variables targeted in the estimation and θ is a vector of parameters to be estimated. Coefficients of the shock processes and other parameters that affect dynamics responses outside of the steady state can then be estimated by maximizing the likelihood function (39).

Computational Speed. The log-likelihood function depends only on observable time series $\tilde{\mathbf{X}}^{obs}$ and the variance-matrix $\mathbf{V}$. The variance matrix in turn depends on the parameters of the underlying stochastic processes and the general equilibrium Jacobians. Since to a first-order the Jacobian does not depend on the stochastic processes the likelihood function can be evaluated for multiple values of $\sum_z m_s^{o,z}$ while only calculating the Jacobian once. Estimation of parameters that does not alter the Jacobian can then be carried out in a few seconds.

For estimation of parameters such as κ_x, ϵ_x, which do affect the Jacobian, this object must be recomputed at each new guess in the optimization process and estimation time is significantly increased. Still, if the parameters in question do not affect the jacobian of the heterogeneous-agent part of the model this Jacobian can be saved and reused at each step and this generates a significant speed-up since this is the costliest Jacobian to compute.

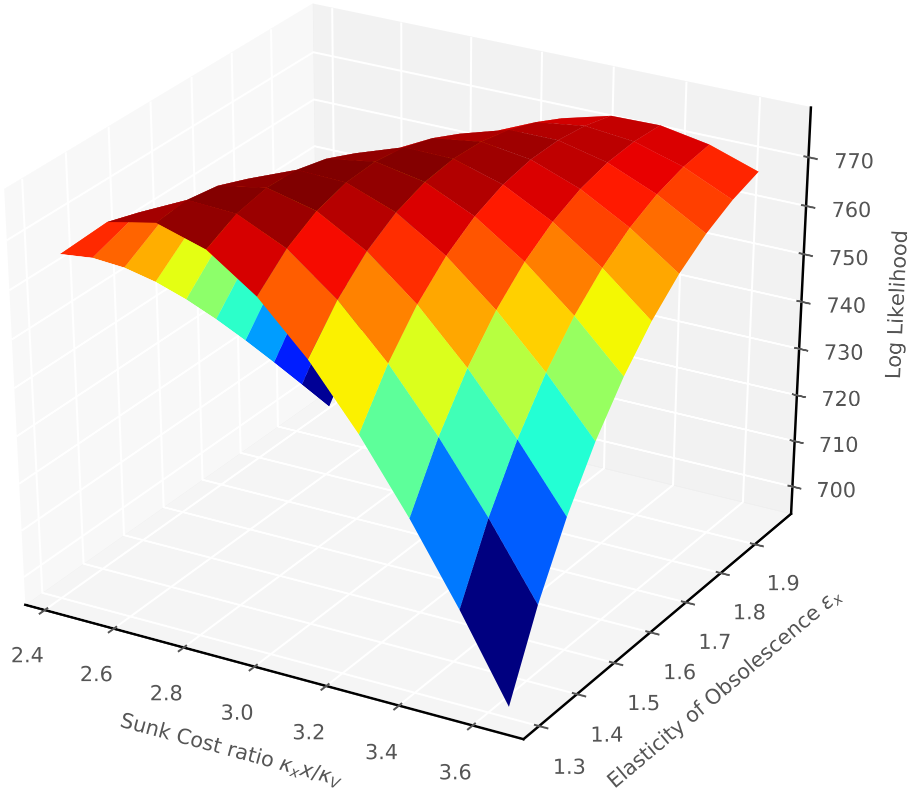


Figure D.8: Log Likelihood Evaluated on grid of κ_x, ϵ_x for identification purposes.

Inference. Since this is a simple I estimate standard errors in the simplest way possible relying on asymptotic properties. I estimate the variance-covariance matrix of the estimates by the inverse of the Fisher information evaluated at the estimate:

$$\text{Var}\left(\hat{\theta}\right) = \mathcal{I}\left(\hat{\theta}\right)^{-1} \tag{40}$$

The Fisher information is approximated as the minus the Hessian of the log-likelihood function:

$$\mathcal{I}\left(\hat{\theta}\right) = -\frac{\partial^2 \log \mathcal{L}\left(\theta\right)}{\partial\theta\partial\theta'}\left(\hat{\theta}\right) \tag{41}$$

where the right-hand side can be evaluated using numerical differentiation. Taking the square-root of the diagonal of (40) then yields the standard errors of the estimates in $\hat{\theta}$.

To obtain confidence bands for impulse responses I apply the Delta-method. Let $d\tilde{X}_t^o\left(\hat{\theta}\right)$ be the impulse of a variable $\tilde{X}_t^o$ and note that this is a function of the estimated parameters in $\hat{\theta}$. Using a first-order approximation the variance of the impulse is:

$$\operatorname{Var}\left(d\tilde{X}_t^o\left(\hat{\theta}\right)\right) = \nabla d\tilde{X}_t^o\left(\hat{\theta}\right) \operatorname{Var}\left(\hat{\theta}\right) \nabla d\tilde{X}_t^o\left(\hat{\theta}\right)' \tag{42}$$

where $\nabla d\tilde{X}_t^o\left(\hat{\theta}\right)$ is the gradient of the impulse w.r.t to the parameters in θ, evaluated at the estimates. Taking the square-root yields the standard-error of the IRF, and symmetric confidence bands can be constructed using asymptotic normality.

D.6 Propagation With Inelastic Investments

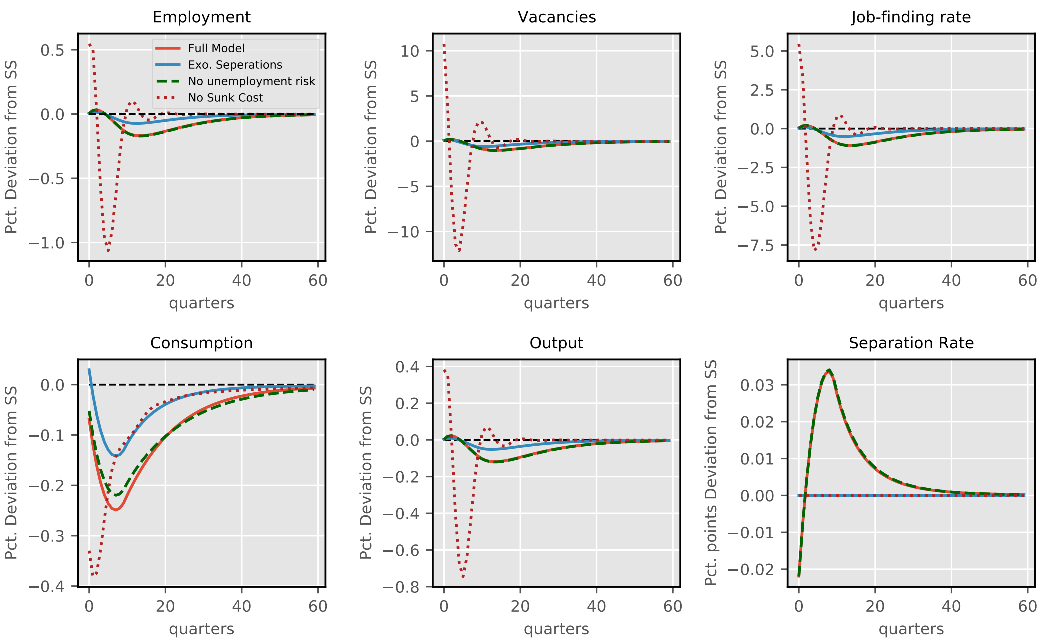


Figure D.9: Model Comparison for Markup shock With Inelastic Investment

GE impulses to a 1% increase in price markups for 10 quarters. "Full model" denotes the HANK model featuring sunk-costs and endogenous separations using the estimated parameters. The confidence interval is 90% and constructed using the Delta method. "Exo. Separations" sets $\epsilon_x = 0$. "No Sunk Cost" sets $\kappa_x = 0$. "No unemployment risk" keeps constant the job-finding rate and separation rate in households' Euler equation. Investment has been made inelastic by setting the investment adjustment cost κ_I sufficiently high.

D.7 Propagation to TFP shocks and Preference Shocks

Figures D.10 and D.11 reconstructs Figure 27 for the remaining two estimated shocks, a TFP shock and a preference shock. For the TFP shock the conclusions are largely unchanged, though the propagation mechanism is slightly less alleviated, with peak response occurring roughly after 10 quarters when the shock disappears. Still, the main point from Broer et al. (2020a) holds. The preference shock differs notably from the other two shocks in being a demand shock. This naturally implies less persistence following shocks, especially since output and inflation co-move such that the monetary policy authority can stabilize both simultaneously. Still, the observations from the markup shock hold: The presence of sunk costs pushes the peak responses of employment and output and precautionary savings and endogenous separations amplify this delayed peak significantly. The effect on consumption from this mechanism is lesser compared to the other models, but as mentioned this is exactly because the real interest rate drops more.

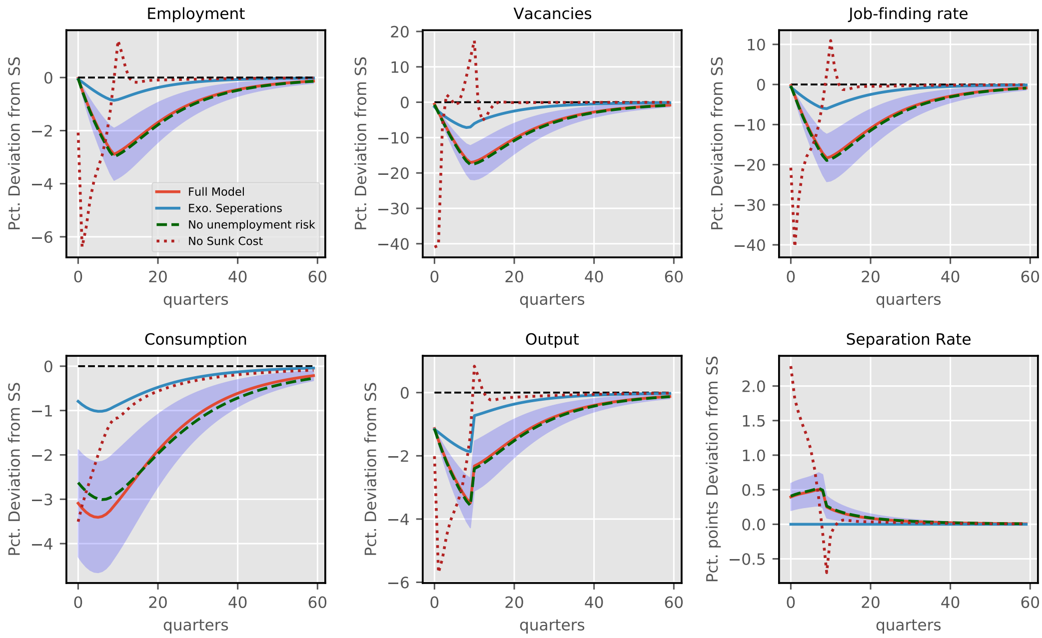


Figure D.10: TFP shock.

GE impulses to a 1% decrease in TFP for 10 quarters. See Figure 27 for further details.

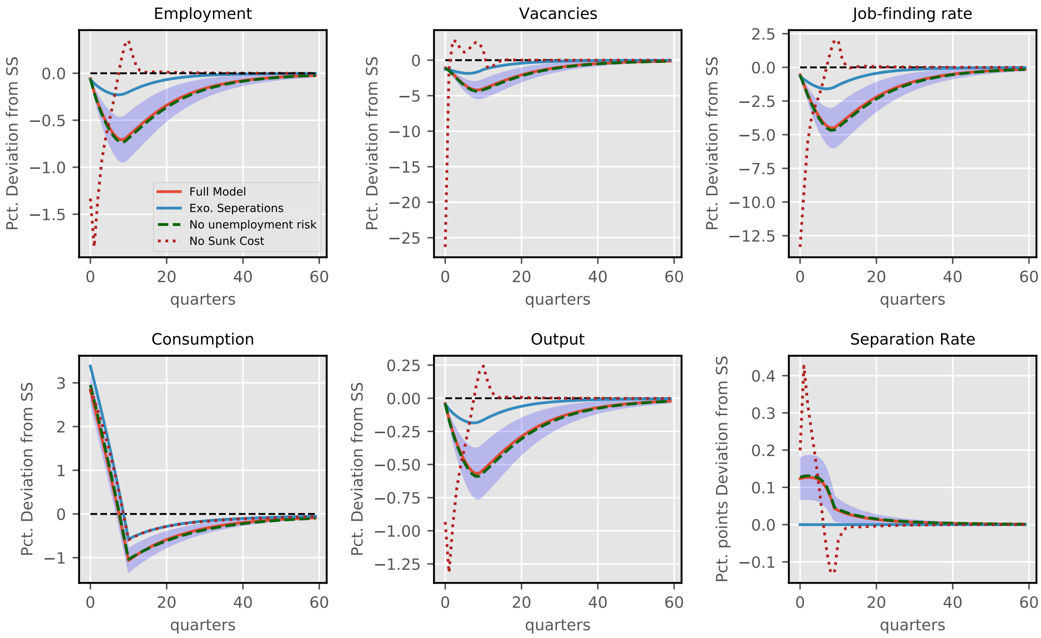


Figure D.11: Preference shock.

GE impulses to a uniform decrease in households' discount factors by 0.005 (on average 0.5%) for 10 quarters. See Figure 27 for further details.