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Tax Bunching, Income Shifting and Self-employment

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TAX BUNCHING, INCOME SHIFTING AND SELF-EMPLOYMENT*

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Abstract

This paper proposes a dynamic extension to Saez (2010) bunching formula that allows us to distinguish bunching based on real responses and income shifting. We provide direct evidence of income shifting and pronounced bunching in taxable income for the case of Danish self-employed. If income shifting was neglected in this case, we would conclude that taxable incomes were highly sensitive to changes in marginal tax rates. We show, however, that more than half of the observed bunching in taxable income for the self-employed is driven by intertemporal income shifting, implying a structural elasticity in the range of 0.14-0.20.

KEYWORDS: Self-employment, tax bunching, retained profits, tax avoidance, income shifting.

JEL-CLASSIFICATION: H20, J20.

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1 Introduction

Since Feldstein (1995, 1999), the behavioral response of taxable income to changes in marginal tax rates has been seen as the central parameter in the formulation of tax and transfer policies. A large empirical literature has therefore focused on estimating the taxable income elasticity.¹ In a seminal contribution to this literature, Saez (2010) shows that the compensated elasticity of reported taxable income can be estimated directly from the amount of bunching around the tax cutoffs. It is well-known, however, that tax avoidance and tax evasion among tax-filers are not only empirically relevant, they also bias estimates of behavioral response to tax changes, cf. Slemrod (1994, 2007). Since Saez' bunching method is based on a positive one-to-one relationship between the elasticity and observed bunching in taxable income, the method may well result in an upward-biased elasticity estimate if neglected evasion and avoidance imply more bunching.

We therefore propose an extension to Saez' bunching formula that allows us to distinguish bunching based on real responses and pure income shifting. We apply this bunching method to the case of Danish self-employed who can legally shift income intertemporally by retaining earned profits in the firm. Our empirical application provides direct evidence of substantial tax avoidance and pronounced bunching in taxable income. The application is a clear example where tax avoidance cannot be neglected, but using our extension to Saez' method we are able to quantify the relative importance of pure tax avoidance and real behavioral responses to taxes.

Saez' method has recently received a lot of attention and has already been used in several applications (see Saez (2010), Kleven et al. (2011), Chetty et al. (2011), Bastani and Selin (2011), and Kleven and Waseem (2011)). A common finding is that the largest excess mass, and thereby the highest observed elasticity, is found for individuals with self-employment income, whereas bunching for workers is much less pronounced.

We investigate sources that drive the massive amount of bunching for self-employed. More specifically, we ask the following question: Can we interpret the pronounced bunching in *taxable* income for the self-employed as a real behavioral response in *earned* income, or is bunching for the self-employed primarily driven by income shifting and reporting effects?

Several papers have empirically documented income shifting using indirect measures such as expenditure on food, but there is only little direct evidence of income shifting.² We observe income shifting directly in the data: Tax planning for Danish self-employed consists of deferring taxes through retaining earnings in the firm, transfers to assisting spouses, pension contributions, and classification of personal income as capital income. We show that the key margin

¹See Saez et al. (2012) for a comprehensive review of this literature

²Slemrod (1996) and Gordon and Slemrod (2000) provide US evidence on shifting between personal and corporate tax bases whereas Kleven and Waseem (2011) provide evidence on shifting between wage income and self-employment income in Pakistan.

facilitating bunching is retained earnings.

The institutional feature allowing Danish self-employed to retain earnings in the firm is an important smoothing device for self-employed as they face much more uncertainty and earnings fluctuations compared to workers. Although it may be hard for self-employed to precisely adjust *earned income* to the tax thresholds, they can easily adjust *taxable income* using retained earnings to smooth variations in earned income across years - in part to reduce tax liability. In other words, retained earnings provide self-employed with the possibility to locate themselves exactly at the kinks of the tax system - without adjusting their efforts to earn profits.

Our bunching formula is explicitly derived from a simple dynamic model of income shifting. The model extends the standard static model of consumption and labor supply under progressive income taxation, allowing self-employed to use retained earnings to legally transfer firm profits across years. To capture that a substantial share of income fluctuations seems to be independent of efforts, we model income fluctuations by including a time-varying, exogenous income component. Two central predictions of the model are that i) tax-filers will aim at holding their marginal tax rates constant over time by smoothing variations in taxable income by the use of retained earnings, and ii) we will observe bunching even when taxes have no effects on earned profits.

We derive two ways of identifying the structural elasticity that fully indexes the behavioral response. Both ways of estimating the elasticity require that we can disentangle the shocks to earned profits over time. This is achieved by exploiting the panel structure of the data, and by observing both taxable income and earned income. The first method is simply to apply Saez' bunching formula on earned income adjusted for income shocks, whereas the other method combines bunching in taxable income and areas of the adjusted earned income density where income shifters locate.

Using high quality Danish individual tax register data from 1994-2009, we analyze self-employed taxpayers' bunching at the kink points of the personal income tax schedule. We find clear evidence of bunching around the largest kink points in the tax schedule for both workers and self-employed. Compared to wage earners, self-employed display substantial tax bunching at the kinks of the Danish progressive tax-system. While the excess mass around the largest kink is 0.2 percent for wage earners, the excess mass for self-employed is 7.2 percent at this kink.

Tax bunching is concentrated among the self-employed who either retain or withdraw earnings from the firm. On average, 20.8 percent of the self-employed that retained earnings in the period 1994-2009, were located within a window of ± 500 DKK ($1\$ \simeq 5$ DKK) around the top kink. About half of this group is *exactly* at the top kink ($\pm 1\$$).³ In contrast to this, tax bunching is very limited for the group that neither retains nor withdraws earnings. The fact

³If we widen the window to ± 7500 DKK the excess mass at the top bracket is 28.6 percent.

that taxable income for self-employed is much more responsive to changes in the marginal tax rate reflects that self-employed can adjust taxable income at almost zero marginal cost.

We find that using Saez (2010) method directly on either earned income or taxable income results in estimated elasticities of 0.0 and 0.5 respectively. Although these estimates can be interpreted as lower and upper bounds of the structural elasticity of interest, the interval is clearly too wide to be very informative. However, using the two estimators derived from our dynamic model, we estimate that 50 – 70 percent of the bunching in taxable income is due to income shifting; implying a structural elasticity of 0.14 – 0.20. Hence, our empirical application to the case of Danish self-employed illustrates the importance income shifting and the potential consequences of neglecting it.

While our bunching method and empirical application could seem specific to the intertemporal income shifting case of the Danish self-employed, we can interpret our model of income shifting more broadly. The bunching method can be adapted to the case of income shifting between persons (transfers to assisting spouses, or joint taxation of couples), shifting between different tax bases (capital income vs. labor income) and intertemporal income shifting, although our focus is specifically on the latter.

The rest of the paper is organized as follows. In section 2, we describe the data and provide some institutional background. In section 3, we formulate a stylized two period model of consumption, supply of efforts to earn profits and intertemporal income shifting. Section 4 presents our empirical results. Section 5 concludes.

2 Data and Institutional Background

We have access to a high quality panel data set covering the entire Danish population in the period 1994-2009. The data set, compiled by Statistics Denmark, is mainly based on the Income Tax Register which contains highly reliable and detailed information on incomes and tax returns. Besides this, we have access to a large set of socioeconomic variables from the IDA database (Integrated Database for Labor Market Research).

Only persons aged 25-59 years are considered. Unless we explicitly state the opposite we only consider persons whose main occupation is self-employment. Unfortunately, we only have information on transfers to assisting spouses for 1994-2001. For self-employed, whose spouses are not self-employed, we can uncover part of these transfers for 2002-2009. Therefore, for the entire sample 1994-2009, we restrict attention to self-employed whose spouses' primary or secondary occupation is not self-employment.

We neither observe the distance to the various tax cutoffs, nor the marginal tax rates - at least not directly. We therefore construct our own tax simulator taking a number of special deductions and joint taxation of couples into account. Using this simulator, we can replicate

actual tax payments very precisely. For 95 percent of all individuals in our sample, the simulated tax payments are within a distance of ± 5 DKK from the actual tax payments.

In the period under study, personal income was taxed according to a piecewise linear tax system with five brackets before 1996 and four brackets from 1996 and onwards. In 2001, for example, the marginal tax rate begins at approximately 8 percent for incomes lower than 33,400 DKK. Above this level, a bottom tax is levied, increasing the marginal tax rate to approximately 44 percent for incomes lower than 179,900 DKK.⁴ For incomes above this level an additional *middle tax* is levied, such that the marginal tax rate increases to 49 percent. Finally, personal incomes exceeding 279,900 DKK are taxed with an additional *top tax*, thereby increasing the marginal tax rate to 63 percent. Before 1996 there were two separate middle taxes that were consolidated into a single middle tax in subsequent years. The taxes relevant for our analysis, the top tax and the middle taxes, have separate tax bases. While the top tax base depends only on individual income for most individuals, the middle tax base is a function of household income. In 2009 the cutoff level of the middle tax was increased to the same amount as the top tax, although the tax bases stayed different.

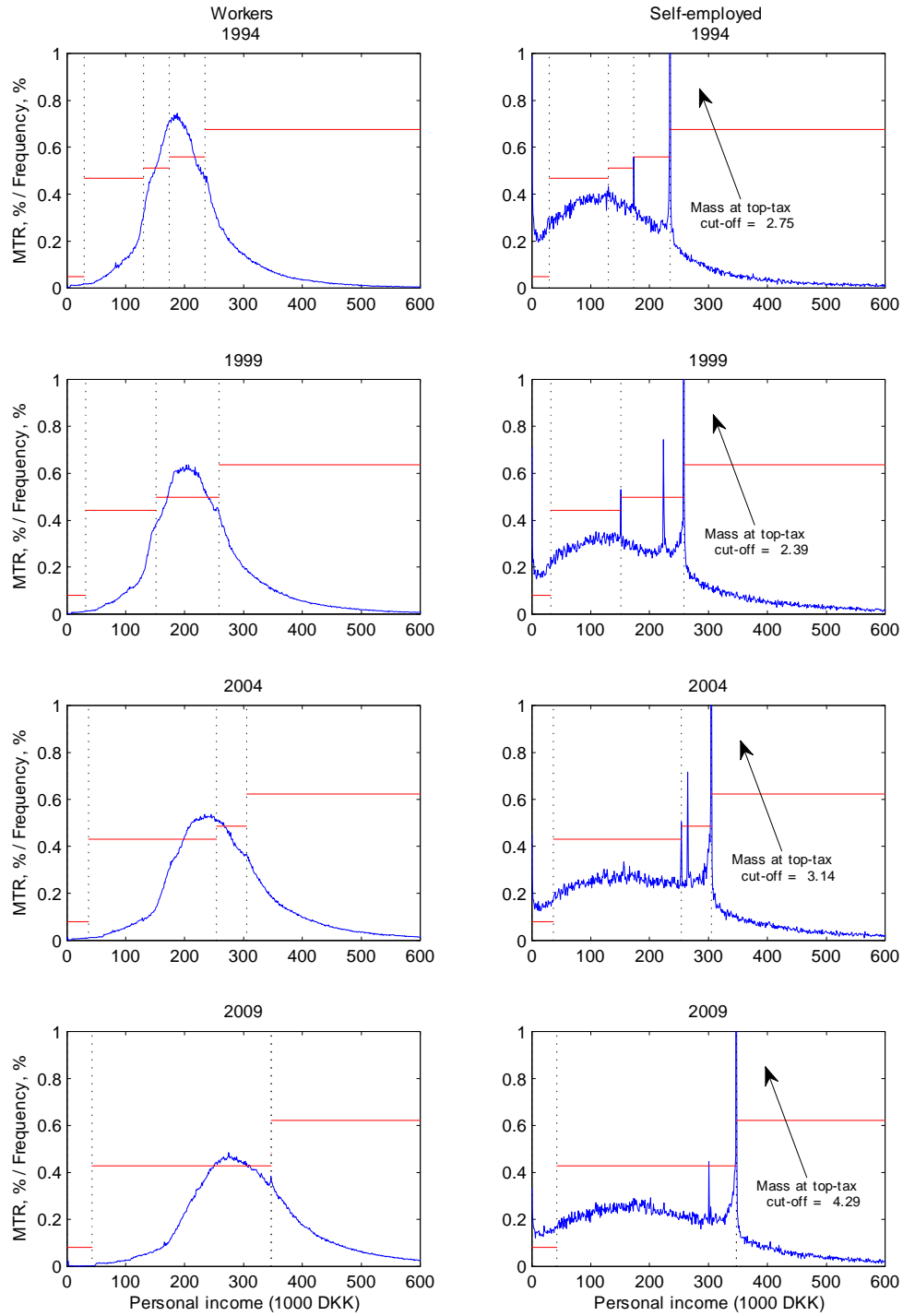
Figure 1 shows the cross-sectional distribution of personal income for workers and self-employed respectively. The figure also displays the location of the tax bracket cutoffs (the dotted vertical lines) and the corresponding marginal tax rates (the horizontal lines). The empirical distribution is presented as percentage frequency plots, where the sample is divided into 1000 DKK bins. For each bin we plot the frequencies in percentage of the relevant subpopulation. This is done for each of the years 1994, 1999, 2004, and 2009.

Looking at the empirical distributions of personal income, three things are worth noting: First, the distribution of personal income for self-employed exhibits massive bunching exactly at each of the tax cutoffs whereas the distribution of personal income for workers is much smoother around the cutoffs. Second, for self-employed the spike is located exactly at the cutoffs in each of the years, 1994-2009, despite the fact that the location of both the top and middle bracket cutoffs significantly move over time (also in real terms). Third, the size of the spike is much larger at the kink point for the top bracket compared to the middle bracket. In fact, the spike is so large, that we have truncated the scale on vertical axis at one percent to be able to inspect the details of the rest of the distribution. For example, in 2009 as much as 4.3 percent of all self-employed was located on the top tax cutoff within a window of ± 500 DKK.

Figure 1 plotted the distribution of *personal income* and marked the location of each of the tax brackets with vertical lines at fixed levels of personal income. However, the effective location of the cutoffs is not the same for all individuals due to joint-taxation of couples and special adjustments for different tax bases (such as deductions of pension contributions). Figure 1 disregard this aspect, but in Figure 2 we depict the distribution of taxable income measured

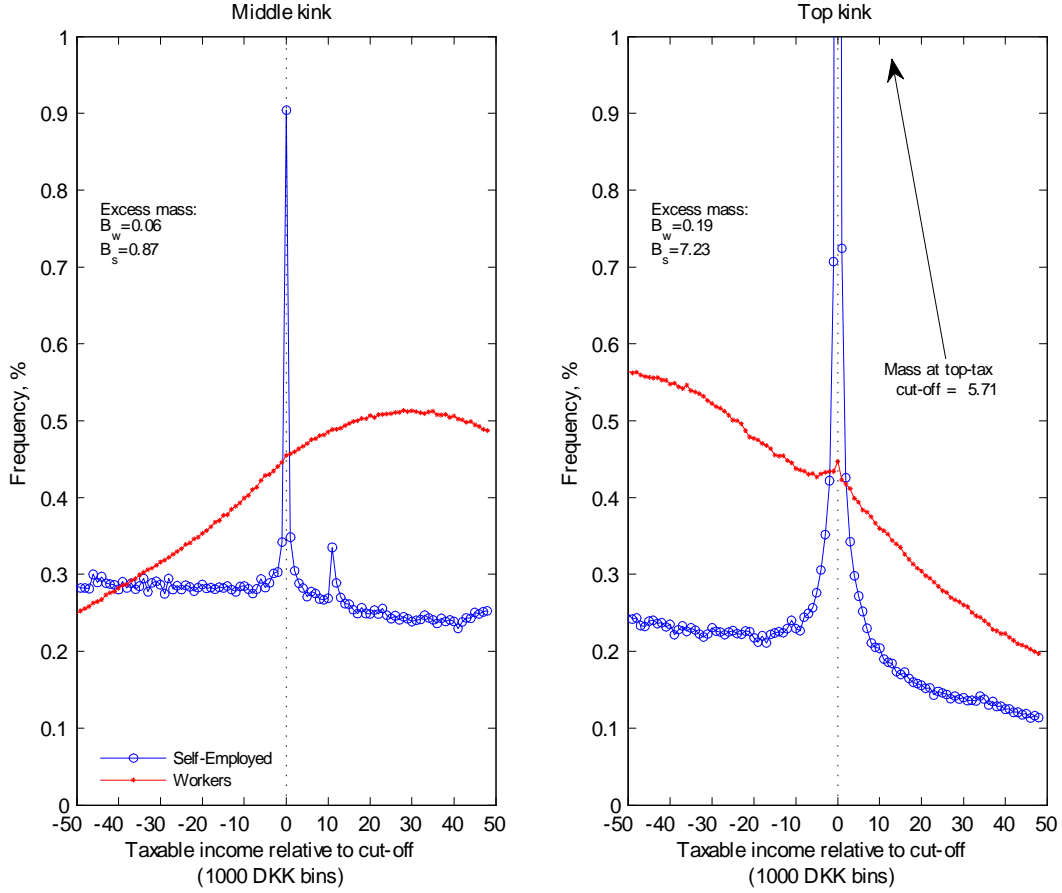
⁴The exact marginal tax rate depends on the municipality in which the person lives.

FIGURE 1: DISTRIBUTION OF PERSONAL INCOME - WORKERS AND SELF-EMPLOYED



Notes: These figures plot the empirical distribution of personal income for workers (left panels) and for self-employed with self-employment as main occupation (right panels) for 1994, 1999, 2004 and 2009. Personal income is defined as the sum of wage earnings, declared firm profits (net of retained earnings), remunerations, provisions, alimony, etc. minus deductions.

FIGURE 2: DISTRIBUTION OF TAXABLE INCOME CENTERED AROUND KINK, 1994-2009



Notes: These figures plot the empirical distribution of taxable income for wage earners (red curve) and self-employed (blue curve) as primary occupation. Taxable income is centered relative to the middle tax cutoff (left panel) and the top tax cutoff (right panel). Taxable income (horizontal axis) is divided into small 1000 DKK bins. A symbol is plotted at each midpoint. Each point shows the percentage frequency of individuals with taxable income in a given 1000 DKK bin.

relative to the effective cutoff. That is, for each individual we calculate the difference between the actual taxable income and the taxable income needed to reach the tax bracket under study. The conclusion from Figure 2 is very clear: While the bunching for self-employed is massive and sharp, there is only a very small excess mass for wage earners.⁵

2.1 Three Tax Schemes for the Self-employed

In Denmark, self-employed can choose between three different tax schemes: i) the personal income tax scheme, ii) the capital returns scheme, or iii) the firm tax scheme. The three different tax schemes lead to a different division of the firm's profits between personal income

⁵The small excess mass next to the middle kink is self-employed bunching at the top tax kink with a particular combination of deductions that bring the effective thresholds of the middle and the top tax kinks close in the end of the sample period.

and capital income. Furthermore, the two latter schemes have a business cycle compensation scheme that allows the self-employed to retain earnings in the firm.

We begin by describing the personal income tax scheme which is the default tax scheme also faced by wage earners. Personal income is defined as the sum of wage earnings, firm profits, remunerations, provisions, alimony, etc. minus deductions. Interest income is not part of personal income, but is regarded as capital income which is taxed by lower rates and with less progression. Thus, if taxed according to the personal income tax scheme, interest expenses only give a deduction in capital income. This means that the firm's profits in, say, 2001 may be taxed at about 44-63 percent depending on the level of personal income, while the firm's interest costs alone only have a tax deduction worth about 33 percent. Furthermore, the high level of progressivity in the tax system, punishes self-employed with profits that fluctuate a lot over time.

Self-employed running personally owned businesses and partnerships can choose to use special tax rules allowing them i) to retain earnings in the firm, ii) to fully deduct interests from the firm's taxable profits, and iii) to classify part of the profits as capital income.

The first set of such taxation rules, the so-called firm tax scheme, came into force after a major tax reform in 1987, where the separate taxation of personal income and capital income were introduced. From 1993 and onwards self-employed were also given possibility to use a simpler set of rules in the so-called capital returns scheme.

The objectives and mechanisms of the capital return scheme and firm tax scheme are largely the same, and we will therefore treat them as identical schemes in our empirical analysis in section 4.⁶ Both the firm tax scheme and the capital returns scheme have a business-cycle compensation scheme that allows self-employed to retain part of firm profits in the firm. Retained profits must be located on a separate bank account and withdrawals are only allowed for business purposes and cannot be used to finance e.g. private consumption without being subject to final taxation. Retained profits are taxed with a temporary tax equal to the firm tax rate in the given year, which is much lower than personal income taxes (in 1994 it was 34 percent and since then it has decreased gradually to 25 percent in 2009). When retained profits are transferred from the firm economy to the private economy, the provisional tax is reimbursed and withdrawn earnings are subject to final taxation according to the personal income tax rules. Hence, self-employed taxed according to these schemes can legally use retained earnings to transfer firm profits across years and thereby smooth variations in income.

⁶Compared to the firm tax scheme, the capital return scheme is administratively simpler, more schematic and better suited for very small businesses, including small housing rental firms, leased farming and the like. The capital returns scheme has some limitations in terms of limited ability to deduct losses from the firm in other sources of income (e.g. wage income). Moreover, firms with large debts facing interest rates higher than the tax-assessed rates of return may benefit less from the capital return scheme. If taxed according to the firm tax scheme the firm's profits are determined after deducting the actual interest payments. Additionally, returns to the net value of business assets (assets-debts) may also be deducted from the firm's profits. The capital returns are then regarded as capital income.

TABLE 1: INDIVIDUALS WITH FIRM INCOME BY INDUSTRY AND TAX SCHEME IN 2001

	Firm tax		Capital returns		Personal tax	
	Number of indv.	Profits mill. DKK	Number of indv.	Profits mill. DKK	Number of indv.	Profits mill. DKK
<i>Self-employment as main occupation</i>						
Primary sector	27,693	6,251	4,270	391	3,134	218
Manufacturing	7,078	2,091	1,676	179	5,209	434
Electricity, gas and water supply	316	60	59	4	84	6
Construction	11,180	3,550	2,122	344	6,913	1,072
Wholesale and retail trade	21,368	6,533	4,458	606	16,925	1610
Hotel, restaurants, and transport	6,404	2,216	1,066	218	3,584	562
Professionals	27,584	12,929	4,990	859	30,554	4,086
Unknown industry	6,103	1,555	5,027	249	12,971	910
Total	107,726	35,184	23,668	2,851	79,374	8,897
<i>Self-employment as secondary occupation</i>						
Employed workers	45,305	-932	28,746	95	66,817	674
Retired workers	5,665	-20	14,965	286	22,084	141
Other workers	255	6	231	2	791	9
Total	51,225	-946	43,942	383	89,692	825
<i>All self-employed</i>						
Total	158,951	34,238	67,610	3,234	169,066	9,722

Notes: Self-employed are categorized into primary or secondary occupation self-employed using the variables, *pstill2* and *sstill2*, constructed by Statistics Denmark.

As can be seen in Table 1, the firm tax scheme was the most popular tax scheme for self-employed in 2001 despite the additional administrative burdens. About 227,000 self-employed individuals with non-zero income from a personally-driven business (corresponding to 57 percent of the self-employed) used either the firm tax scheme (40 percent) or the capital returns scheme (17 percent). Among those who have self-employment as their main occupation, the proportion using the firm tax scheme is even higher (51 percent). For individuals with self-employment as primary occupation, the proportion that used either of these two tax schemes is ranging from more than four out of five for self-employed in the primary sector (agriculture, horticulture, fisheries, etc.) to slightly more than half in most other industries.

Compared to Chetty et al. (2011) we use a more narrow definition of self-employment in which we only include individuals whose primary occupation is self-employment. For wage earners whose secondary occupation is self-employment it is not unlikely that hours constraints and search frictions imply that they, to a lesser extent, can adjust taxable income to locate themselves at the top tax cutoff. It turns out that the excess mass at the top tax cutoff is not

TABLE 2: DIVISION OF TAXABLE PROFITS UNDER THE FIRM TAX SCHEME 2001

	Million DKK	No. of Persons
Net profits primary operations	54.529	158.951
+ Interest income	1.799	98.956
+ Interest expenses	20.207	144.190
- Transfers to assisting spouse	1.884	14.338
Net profits according to firm tax scheme	34.238	158.951
Positive net profits	38.929	114.683
Negative net profits	-4.691	44.268
Division of net profits		
Retained earnings	11.268	44.226
Profits paid to owner	27.707	114.683
Capital income	3.128	114.683
Personal income and retained earnings	24.579	114.683
Losses deducted in owners income	-4.737	44.268
Withdrawn retained earnings	3.994	25.758

Notes: We use population register data for 2001, which is the last year we can observe the transfers to assisting spouses. Only self-employed whose primary occupation is self-employment and who use the firm tax scheme are included.

really affected by use of the narrow definition. However, the density surrounding the top tax cutoff is higher when also including persons whose secondary occupation is self-employment, presumably because this group is more likely to be constrained from adjusting income, and thus less likely to bunch.

The firm tax scheme is primarily chosen by high income individuals. In 2001, 75 percent of total taxable profits, generated by individuals with self-employment as their primary occupation, originated from self-employed who was taxed according to firm tax scheme. This group has almost three times higher average profits (326,606 DKK) than those using the capital returns scheme (120,458 DKK), or those not using any of the schemes (112,090 DKK). This is not surprising given that the benefits in terms reduced tax liability seem to increase with income.

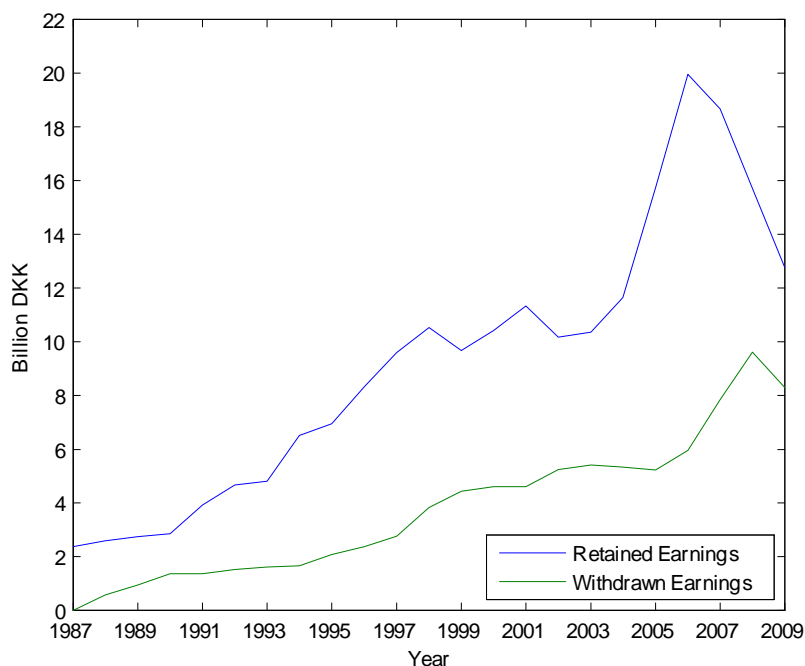
Table 2 illustrates how the firms' taxable gross profits are divided into different tax concepts. As mentioned above, interest expenses and transfers to assisting spouses can be deducted directly in gross profits in line with other operational cost like wages, purchases of materials, rental expenses and the like. Net profits can either be retained in the firm or be directly transferred to the owner, either as capital income or personal income. Since capital income is taxed at a lower rate, there are limits on the amount that can be transferred as capital income.⁷

The use of retained earnings is substantial. In 2001, roughly 44,000 self-employed in the firm tax scheme, or 28 percent with primary occupation being self-employment, used the cycle

⁷The maximum amount of capital income that can be transferred to the private economy is determined as the product of a long term bond rate and the net value of business assets.

offsetting option of retaining earnings in the firm. The total amount of retained profits for provisional firm taxation in 2001, amounted to a total of approximately 11.3 billion DKK or on average approximately 255,000 DKK per person that retain earnings. Hence, a substantial fraction (33 percent) of total net profits (34.2 bill. DKK) are retained in the firm.

FIGURE 3: RETAINED AND WITHDRAWN EARNINGS, 1987-2009



Notes: We use population register data for 1994-2009. For the period 1987-1993 we use the Tax Ministry's figures on basis of their 3.3 percentage representative sample. Both primary and secondary occupation self-employed are included.

In fact, since the option to retain earnings became available, the total annual amount of retained earnings has been much larger than the total annual withdrawn earnings. Figure 3 shows the development of retained and withdrawn earnings for the period 1987-2009. It is clear that this development implies that a substantial accumulation of retained earnings has taken place over more than two decades. Much of the development can be explained by increased use of the firm tax scheme up to 2001 and the last decade shows more fluctuations with the general economic activity.

3 Theory

In this section, we present a stylized two-period model of consumption, efforts and intertemporal income shifting. We consider self-employed individuals whose incomes are subject to progressive

taxation, but unlike workers in regular employment they have the possibility to retain earnings in the firm and thereby shift income between the two periods. Based on this model, we then derive a dynamic extension to Saez' bunching formula that allows us to distinguish bunching based on real responses and pure income shifting.

3.1 A Simple Model of Tax Bunching and Income shifting

The model builds on the canonical two-period labor-leisure model. The self-employed individual exerts efforts, e_t , and derives utility from consumption and disutility of exerting efforts $\psi(e_t)$. In addition to efforts, earned income also consists of an additive component, η_t , which is unaffected by efforts in either of the two periods. We refer to η_t as income shocks although they are perfectly foreseen. In period t , the self-employed individual earns

$$z_t = e_t + \eta_t$$

The self-employed individual can retain earnings in the firm by transferring a part of earned profits, m , to the following period by paying a shifting cost of $g(m)$. To simplify the analysis, we abstract from the intertemporal allocation of consumption, assume no liquidity constraints, no income uncertainty, and no discounting, such that the individual's problem is to maximize the sum of after tax income over the two periods less of disutility of efforts and shifting costs. We think of $g(m)$ as the reduced form utility loss associated with intertemporal income shifting from period 1 to period 2, that for example could arise when an intertemporal transfer results in deviating from the optimal intertemporal allocation of consumption, when risk averse individuals face income uncertainty, are impatient, have binding liquidity constraints, etc.

The individual chooses exerted effort levels, e_t , and how much income to transfer from period 1 to period 2, m . We assume that the income component unrelated to effort is greater in period 1 than in period 2, that is $\eta_1 \geq \eta_2$. Hence, self-employed facing a progressive tax system may have an incentive to transfer income from period 1 to period 2. In the set-up below, the timing of the two periods is irrelevant and the assumption that $\eta_1 \geq \eta_2$ is without loss of generality. Retained earnings are deductible in the year they are retained and taxable in the year they are withdrawn and thus taxable income in period 1 and 2 is given by

$$y_1 = z_1 - m = e_1 + \eta_1 - m$$

$$y_2 = z_2 + m = e_2 + \eta_2 + m$$

We consider a piecewise linear progressive tax system with only one kink in y^* such that incomes below y^* are taxed at the tax rate τ_0 whereas the marginal tax rate above the kink

point is $\tau_1 > \tau_0$ such that the tax function is

$$T(y_t) = \tau_0 \min(y_t, y^*) + \tau_1 \max(y_t - y^*, 0)$$

We can then summarize the individuals' decision problem as

$$\begin{aligned} \max_{e_1, e_2, m} \quad & u(e_1, e_2, m) = z_1 - m - T(z_1 - m) - \psi(e_1) + z_2 + m - T(z_2 + m) - \psi(e_2) - g(m) \\ \text{s.t.} \quad & z_t = e_t + \eta_t \end{aligned}$$

where the first order condition for the individuals' choice of m is

$$\tau_{mtr}(z_1 - m) - \tau_{mtr}(z_2 + m) \geq g'(m) \quad (1)$$

and where $\tau_{mtr}(y_t) = \tau_0 \mathbf{1}(y_t < y^*) + \tau_1 \mathbf{1}(y_t \geq y^*)$ is the marginal tax rate associated with $T(y_t)$. Assuming that $g'(m) \geq 0$, a self-employed will transfer income from a high income period to the low income period until the marginal shifting costs exceed the difference in tax rates at either side of the kink, y^* . At the extreme where $g'(m) = 0$ individuals will equalize marginal taxes across periods. With constant marginal costs, i.e. $g'(m) = \alpha$, a self-employed not bunching in taxable income in either of the periods will have no reasons to transfer income between the two periods and $m = 0$, but for a self-employed bunching in at least one period there are potential benefits from transferring income.

Since the marginal tax jumps discretely at the kink point, y^* , there will be a continuum of values of z_t , which is consistent with the reporting y^* . Hence, if z_t is continuously distributed in the population we could observe bunching at y^* . This result holds regardless of whether z_t is purely exogenous or whether it is determined by an optimal supply of efforts.

To consider the bunching behavior of self-employed in more detail we need to be explicit about the functional forms. We assume that the disutility of effort is iso-elastic

$$\psi(e_t) = \frac{1}{\gamma^{\frac{1}{\varepsilon}}} \frac{e_t^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}$$

where γ is a heterogenous ability parameter which follows the cumulative distribution function $F(\gamma)$ with a corresponding density being $f(\gamma) = F'(\gamma)$, whereas ε is the structural elasticity which is constant in the population.

With the assumed functional forms the first order conditions for effort imply that

$$\begin{aligned} e_t &= \gamma (1 - \tau_0)^\varepsilon & \text{when } y_t \leq y^* \\ e_t &= \gamma (1 - \tau_1)^\varepsilon & \text{when } y_t > y^* \end{aligned} \quad (2)$$

These effort levels are optimal whenever the self-employed does not bunch in period t .

Since earned income in either of the periods is a continuously increasing function of the ability parameter γ , there exists a continuum of values of γ for which the self-employed bunches in a given period. Denote the lower and upper bounds for these sets by γ_t^{low} and γ_t^{high} . As $z_1 \geq z_2$ and $g'(m) \geq 0$, individuals with $\gamma < \gamma_1^{low}$ will not bunch at the tax threshold in any of the periods because even in the high income period their incomes are below the threshold y^* . Similarly, individuals $\gamma > \gamma_2^{high}$ will have incomes in both periods which are above y^* , and hence, they will not bunch. Since persons with $\gamma < \gamma_1^{low}$ and $\gamma > \gamma_2^{high}$ face the same marginal tax rate without bunching they have no incentive to transfer income across periods.

Individuals with $\gamma = \gamma_1^{low}$ will only bunch in the high income period by supplying effort $e_1 = \gamma(1 - \tau_0)^\varepsilon$ such that $z_1 = y_1 = y^*$. Solving for the specific γ satisfying these two conditions yield

$$\gamma_1^{low} = \frac{y^* - \eta_1}{(1 - \tau_0)^\varepsilon}$$

The self-employed with $\gamma = \gamma_1^{low}$ will actually not transfer any income across periods, but for all values of $\gamma \in]\gamma_1^{low}, \gamma_2^{high}[$ the amount transferred from period 1 to period 2 will be strictly positive, that is $m > 0$.

Self-employed with $\gamma \in [\gamma_1^{low}, \gamma_2^{low}[$ have a sufficiently low income in period 2 that they will only bunch in period 1. However, self-employed with $\gamma \in [\gamma_2^{low}, \gamma_1^{high}]$ will bunch in both periods by transferring income from the high income period to the low income period such that $y_1 = y_2 = y^*$. We can solve for γ_2^{low} by setting $e_2 = \gamma(1 - \tau_0)^\varepsilon$ and $y_1 = y_2 = y^*$ and using that the first order conditions imply that $\psi'(e_1) - \psi'(e_2) = g'(m)$. If we further assume constant marginal costs of intertemporal income shifting such that $g'(m) = \alpha$, we find that $e_1 = \gamma((1 - \tau_0) - \alpha)^\varepsilon$ and we can express the lower limit for bunching in the second period as

$$\gamma_2^{low} = \frac{y^* - \bar{\eta}}{\frac{1}{2}((1 - \tau_0)^\varepsilon + ((1 - \tau_0) - \alpha)^\varepsilon)}$$

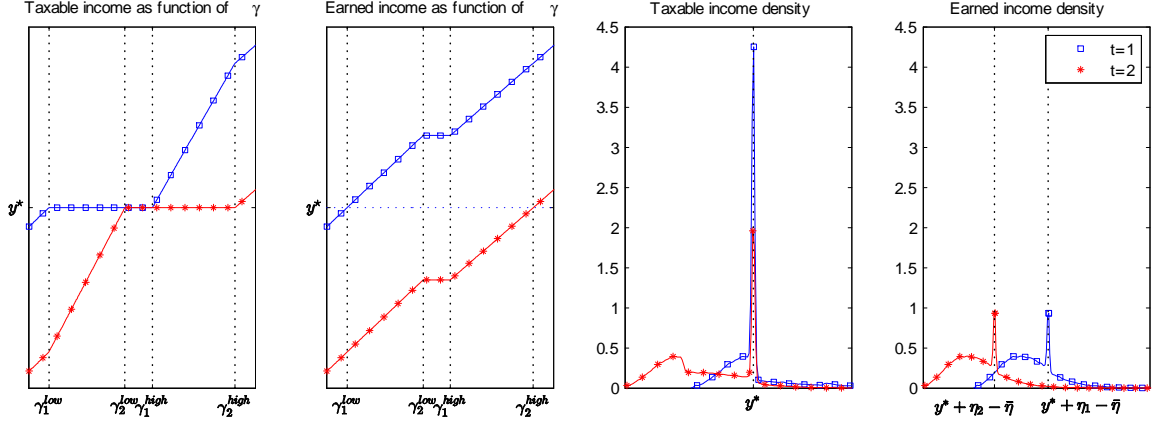
where $\bar{\eta} = \frac{1}{2}(\eta_1 + \eta_2)$.

In completely analogous ways we can also compute the upper limits for bunching in the two periods

$$\begin{aligned} \gamma_1^{high} &= \frac{y^* - \bar{\eta}}{\frac{1}{2}((1 - \tau_1)^\varepsilon + (\alpha + (1 - \tau_1))^\varepsilon)} \\ \gamma_2^{high} &= \frac{y^* - \eta_2}{(1 - \tau_1)^\varepsilon} \end{aligned}$$

It is useful first to consider the case where there are no costs of transferring income across time, that is $g'(m) = \alpha = 0$. This case is shown in Figure 4 for the structural elasticity $\varepsilon = 0.3$. When $g'(m) = 0$ the self-employed minimizes the sum of the disutilities of effort by setting

FIGURE 4: MODEL SOLUTION, $\varepsilon = 0.3$ AND $\alpha = 0$



Notes: The parameter values are: $\tau_0 = 0.5$, $\tau_1 = 0.625$, $\eta_1 = 1$, $\eta_2 = -1$, and $\log(\gamma) = N(0.5, 0.64)$.

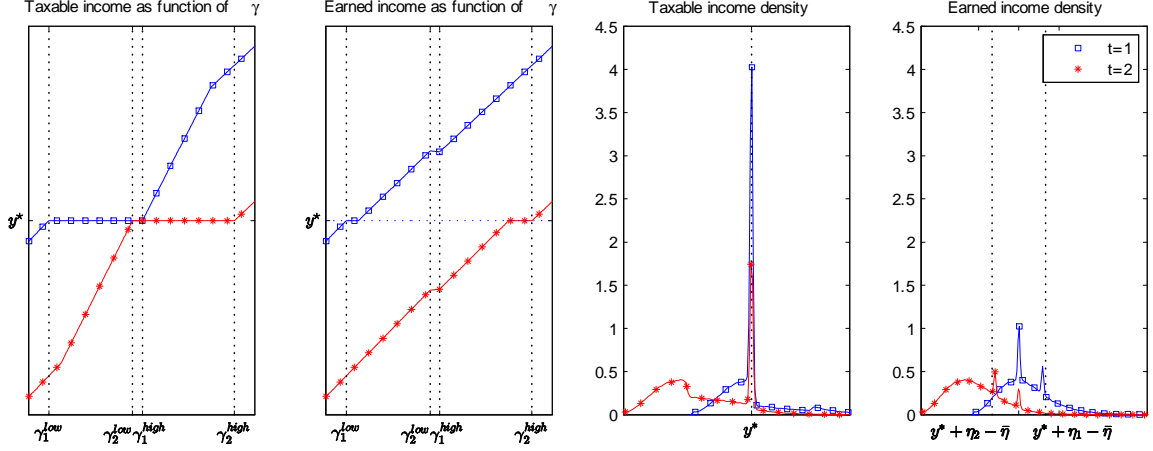
$e_1 = e_2$ and transfers an amount such that the tax rate is the same in the two periods. Hence, when the self-employed bunches in only one period the effort levels in equation (2) apply. This can be seen in Figure 4 as there is no kink in earned income as a function of γ below γ_2^{low} and above γ_1^{high} .

For individuals with $\gamma \in [\gamma_2^{low}, \gamma_1^{high}]$ earned income is constant and these individuals bunch in earned income. From the taxable income function it can be verified that this is exactly the group of self-employed with $\gamma \in [\gamma_2^{low}, \gamma_1^{high}]$ who bunch in both periods. In addition to this flat part, taxable income in period 1 is also flat for $\gamma \in [\gamma_1^{low}, \gamma_2^{low}]$ whereas taxable income in period 2 is flat for $\gamma \in [\gamma_1^{high}, \gamma_2^{high}]$ and individuals with γ in these two intervals bunch solely due to income shifting.

If we assume that γ follows a log normal distribution we can illustrate the implied densities of earned and taxable income in the two periods. It is immediately apparent that there is much more bunching in taxable income. Whereas bunching in taxable income occurs at the tax cutoff, y^* , bunching in earned income occurs at $y^* + \eta_1 - \bar{\eta}$ and $y^* + \eta_2 - \bar{\eta}$ in period 1 and 2 respectively.

When $g'(m) = \alpha > 0$ bunching in one period implies that the effort level in that particular period is not given by equation (2) simply because transferring income is costly so it is not optimal to set $e_1 = e_2$. Whereas γ_1^{low} and γ_2^{high} do not depend on the costs of transferring income α , both γ_2^{low} and γ_1^{high} depend on α . Since γ_1^{high} is decreasing in α and γ_2^{low} is increasing in α we have that the amount of bunching in both periods is decreasing in α . This can also be seen in Figure 5, where we consider the case where $\varepsilon = 0.3$ and $\alpha = 0.08$. Furthermore, for the range of values just above γ_1^{low} and for the range of values just below γ_2^{high} the earned income function is flat since these self-employed reduce effort in order to earn just y^* rather than transferring income to the second period. The reduction in effort is optimal as long as this

FIGURE 5: MODEL SOLUTION, $\varepsilon = 0.3$ AND $\alpha = 0.08$



Notes: The parameter values are: $\tau_0 = 0.5$, $\tau_1 = 0.625$, $\eta_1 = 1$, $\eta_2 = -1$, and $\log(\gamma) = N(0.5, 0.64)$

reduction is so small that the difference in marginal disutilities from efforts between the two periods is smaller than the marginal costs of transferring income, α .

We can express the range of persons, measured by γ , who bunch in at least one period as

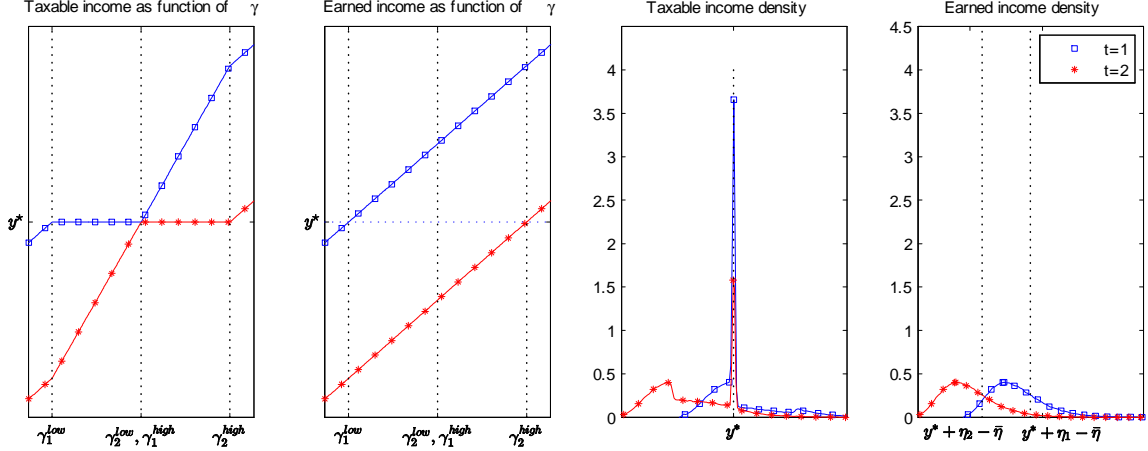
$$\gamma_2^{high} - \gamma_1^{low} = \frac{y^* - \eta_2}{(1 - \tau_1)^\varepsilon} - \frac{y^* - \eta_1}{(1 - \tau_0)^\varepsilon}$$

By straightforward differentiation it is apparent that more self-employed will bunch if τ_1 increases while holding τ_0 constant or if τ_0 decreases while holding τ_1 constant. Furthermore, for a given distance $\tau_1 - \tau_0$ a parallel increase in both tax rates also imply that a larger range of ability types will bunch. Whether such a parallel increase in both tax rates actually increases the observed bunching depends on the distribution of γ as well as η_t . When η_1 increases and η_2 decreases we will observe more bunching. Hence, more income volatility implies more bunching.

In the two cases considered above, we observe bunching in both earned and taxable income, but due to income shifting there is more bunching in taxable income. In Figure 6 we consider the case where the structural elasticity ε is zero. This case implies that $\gamma_2^{low} = \gamma_1^{high}$ and, hence, there is no interval of γ with bunching in taxable income in both periods. Nevertheless, self-employed with $\gamma \in [\gamma_1^{low}, \gamma_2^{low}]$ will bunch in taxable income in period 1 and self-employed with $\gamma \in [\gamma_2^{low}, \gamma_2^{high}]$ will bunch in taxable income in period 2. Hence, although we observe no bunching in earned income, taxpayers will bunch in taxable income due to income shifting - even when the true elasticity is zero.

The immediate conclusion from Figure 6 is that using Saez (2010) method on taxable income can only provide us with an upper bound of the structural elasticity. Furthermore, since

FIGURE 6: MODEL SOLUTION, $\varepsilon = 0.0$ AND $\alpha = 0$



Notes: The parameter values are: $\tau_0 = 0.5$, $\tau_1 = 0.625$, $\eta_1 = 1$, $\eta_2 = -1$, and $\log(\gamma) = N(0.5, 0.64)$.

the income shocks are presumably smoothly distributed in the population and in reality very heterogeneous we cannot use Saez (2010) method and focus on bunching in earned income $y^* + \eta_t - \bar{\eta}$. If costs of transferring income is sufficiently large we might be able to obtain a lower bound of the elasticity using Saez (2010) method focusing on bunching in earned income at y^* . However, as the empirical results will show the bounds we can obtain using Saez method directly on earned and taxable income at y^* are not informative.

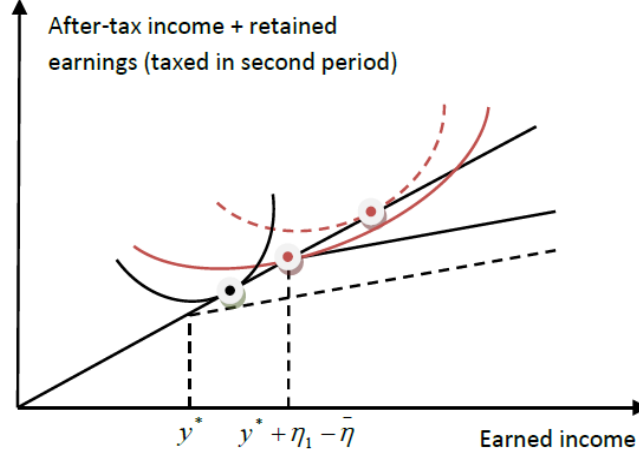
3.2 A Dynamic Extension to the Bunching Method

Using the model outlined in the previous section, we now extend Saez (2010) method to allow for income shifting. We focus on the special case where $\alpha = 0$, since this allows us to obtain closed form solutions. As we will show in the following section, this simplification seems to be a good approximation in the case of Danish self-employed.

The method not only allows us to identify the structural elasticity, ε , it also enables us to distinguish between two types of bunching in taxable income: bunching based on real responses and bunching due to pure income shifting. Figure 7 illustrates these two types responses. In the figure, we consider the effect of introducing a kink in the budget set. In absence of income shifting, an introduction of a higher marginal tax rate $\tau_1 > \tau_0$ for taxable income above y^* , will show up as a kink in the budget set at y^* . However, when taxpayers can transfer income from a high income period to a low income period, the effective kink in the budget set moves from y^* to $y^* + \eta_t - \bar{\eta}$.

The person located on the dashed, red indifference curve earns income above this effective threshold before the reform. Therefore, in response to the tax increase, he reduces earned

FIGURE 7: INDIFFERENCE CURVES AND BUNCHING



Notes: The figure shows the effect of increasing the tax rate at y^* for a person with a real response (red) and a person purely income shifting (black). The figure is drawn for individuals in the high income period, $t = 1$, where $\eta_t - \bar{\eta} > 0$.

income to locate himself exactly at the effective tax threshold, $y^* + \eta_t - \bar{\eta}$. In other words, this person makes a real response. Notice also that he will bunch in taxable income at y^* and completely off-set the income shock by income shifting, i.e. $m = \eta_t - \bar{\eta}$.

Earned income of the second individual (black indifference curve) is below the effective tax threshold and his efforts are thereby unaffected by the reform. Although this person is bunching in taxable income, he has no real response and is purely income shifting. In fact, income shifters with $\eta_t - \bar{\eta} > 0$, will always locate in between y^* and $y^* + \eta_t - \bar{\eta}$ and, hence, they will not completely off-set income shocks.

The figure illustrates important differences in the behavior of persons with a real response and persons income shifting. A person with a real response will always bunch and retain earnings for any $\eta_t - \bar{\eta} > 0$, whereas for the income shifter the income shock (relative to its mean) needs to be sufficiently large before he will earn more than y^* and, therefore, bunch and retain earnings. More generally, income shifters only retain and withdraw earnings when they experience sufficiently large negative or positive shocks, and there will be an interval of values around $\bar{\eta}$ for which income shifters will neither retain nor withdraw earnings. Therefore, while a person with a real response uses retained and withdrawn earnings symmetrically around y^* , this is not the case for income shifters.

Below we suggest two ways of estimating the structural elasticity. Figure 7 provides the intuition for the formulas while the more formal derivation has been relegated to the appendix. Figure 7 (and the right panel of Figure 4) show that self-employed bunching in taxable income due to a real labor supply response also bunch in earned income in $y^* + \eta_t - \bar{\eta}$. When allowing

$\eta_t - \bar{\eta}$ to be heterogenous in the population, we will not observe bunching in earned income in a specific point, and thus it is useful to define adjusted earned income as $z_t^{adj} \equiv z_t - (\eta_t - \bar{\eta})$. This removes income shocks in each period and makes sure that all self-employed bunching in taxable income due to real response also bunch in adjusted earned income at exactly y^* .

Assuming we have access to panel data, where we observe individual $i = 1, \dots, N$ at $t = 1, \dots, T_i$, we can simply use Saez (2010) method on adjusted earned income

$$B_{z^{adj}} \simeq \frac{1}{2\delta} y^* \left(\left(\frac{1-\tau_0}{1-\tau_1} \right)^\varepsilon - 1 \right) \sum_{i=1}^N \frac{1}{NT_i} \sum_{t=1}^{T_i} \left\{ \frac{\mathbf{1}(y^* - 2\delta \leq z_{it}^{adj} < y^* - \delta) + \mathbf{1}(y^* + \delta \leq z_{it}^{adj} < y^* + 2\delta)}{\left(\frac{1-\tau_0}{1-\tau_1} \right)^\varepsilon} \right\} \quad (3)$$

where δ is the bandwidth and $B_{z^{adj}}$ following Saez is estimated as

$$\hat{B}_{z^{adj}} = \sum_{i=1}^N \frac{1}{NT_i} \sum_{t=1}^{T_i} \left\{ \mathbf{1}(|z_{it}^{adj} - y^*| < \delta) - \mathbf{1}(y^* - 2\delta \leq z_{it}^{adj} < y^* - \delta) - \mathbf{1}(y^* + \delta \leq z_{it}^{adj} < y^* + 2\delta) \right\} \quad (4)$$

For the estimation strategy using equation (3) we need to precisely measure the bunching in adjusted earned income at y^* . Instead of trying to estimate the share of persons bunching due to real response, an alternative strategy is to measure the share of individuals purely income shifting. As illustrated in Figure 7, income shifters with $\eta_{it} - \bar{\eta}_i > 0$ will locate where $z_{it} \in [y^*, y^* + \eta_{it} - \bar{\eta}_i[$ which in terms of adjusted earned income corresponds to $z_{it}^{adj} \in [y^* - (\eta_{it} - \bar{\eta}_i), y^*]$. Symmetrically, income shifters with $\eta_{it} - \bar{\eta}_i < 0$ will locate where $z_{it}^{adj} \in]y^*, y^* - (\eta_{it} - \bar{\eta}_i)]$. The left hand side of equation (3) measures bunching in adjusted earned income, $B_{z^{adj}}$. If we add the mass of individuals in the two shifting areas we obtain the bunching mass in taxable income, B_y . Hence, we have the following extended bunching formula,

$$B_y \simeq \frac{1}{2\delta} y^* \left(\left(\frac{1-\tau_0}{1-\tau_1} \right)^\varepsilon - 1 \right) \sum_{i=1}^N \frac{1}{NT_i} \sum_{t=1}^{T_i} \left\{ \frac{\mathbf{1}(y^* - 2\delta \leq z_{it}^{adj} < y^* - \delta) + \mathbf{1}(y^* + \delta \leq z_{it}^{adj} < y^* + 2\delta)}{\left(\frac{1-\tau_0}{1-\tau_1} \right)^\varepsilon} \right\} \\ + \sum_{i=1}^N \frac{1}{NT_i} \sum_{t=1}^{T_i} \mathbf{1}(|y_{it} - y^*| < \delta) \left\{ \frac{\mathbf{1}(\eta_{it} - \bar{\eta}_i \geq 0) \mathbf{1}(y^* - (\eta_{it} - \bar{\eta}_i) - \delta \leq z_{it}^{adj} < y^* - \delta) + \mathbf{1}(\eta_{it} - \bar{\eta}_i < 0) \mathbf{1}(y^* + \delta < z_{it}^{adj} \leq y^* - (\eta_{it} - \bar{\eta}_i) + \delta)}{\left(\frac{1-\tau_0}{1-\tau_1} \right)^\varepsilon} \right\} \quad (5)$$

The first part of the right hand side equals bunching due to a real response and the second part bunching due to income shifting. The amount of bunching due to income shifting is increasing in the size of the income shock (compared to its average). In contrast, a larger income shock has no effect on the bunching due to a real response. In the limit where the income shocks go to zero the second term measuring income shifting disappears and the right hand side is identical to the formula in equation (3).

In theory, the two ways of estimating the elasticity should give the same parameter estimate and both estimations also provide us with the possibility of calculating the share of the observed bunching in taxable income which is due to real response and income shifting.⁸ However, any behavior deviating from the simple structure of the model is likely to imply that the two estimates differ.

Besides $\eta_{it} - \bar{\eta}_i$, all terms in equations (3) and (5) are observable variables, and ε has a closed-form solution in both cases. The only remaining problem is to obtain an estimate of $\eta_{it} - \bar{\eta}_i$. When estimating these income shocks, we need to take into account that we do not necessarily observe all individuals $i = 1, \dots, N$ for their entire planning horizon, T_i^p . Hence, the sample period may be smaller than planning horizon, i.e. $T_i \leq T_i^p$. We can, however, use the theoretical model to derive an estimate of $\eta_{it} - \bar{\eta}_i$ by noticing that whenever $\alpha = 0$ we must have that $e_{it} = \bar{e}_i$ for all $t = 1, \dots, T_i^p$. Therefore,

$$\eta_{it} - \bar{\eta}_i = z_{it} - \frac{1}{T_i^p} \sum_{t=1}^{T_i^p} z_{it}$$

To use this formula we need to know $\sum_{t=1}^{T_i^p} z_{it}$, but we only observe $\sum_{t=1}^{T_i} z_{it}$. However, as illustrated in Figure 7 we know that self-employed bunching due to real response use retained earnings to completely off-set variations in z_{it} . As it seems reasonable to assume that self-employed withdraw all previously retained earnings by the end of firm's lifetime we must have that $\sum_{t=1}^{T_i^p} m_{it} = 0$. In theory, we can therefore precisely uncover $\eta_{it} - \bar{\eta}_i$ for persons bunching due to real response. We therefore estimate $\eta_{it} - \bar{\eta}_i$ as

$$\widehat{\eta_{it} - \bar{\eta}_i} = z_{it} - \frac{1}{T_i} \sum_{t=1}^{T_i} z_{it} + \frac{1}{T_i} \sum_{t=1}^{T_i} m_{it}$$

For persons bunching due to income shifting we will not, in general, uncover the the true $\eta_{it} - \bar{\eta}_i$ although adding the term $\frac{1}{T_i} \sum_{t=1}^{T_i} m_{it}$ improves the estimate for income shifters as m_{it} contains a signal about $\eta_{it} - \bar{\eta}_i$. Therefore, we will most likely obtain imprecise estimates for persons never bunching. However, this will not seriously affect our results. What matters for both estimators is that the density of adjusted earned income is precisely estimated for a fairly small interval around the tax cutoff.

To uncover the true adjusted earned income for persons bunching due to a real response we actually only need to observe persons twice. It is likely that using several observations will give us a biased estimate of adjusted earned income for those in the interval around the tax cutoff. First, the identifying assumption that γ is individual specific and time constant is most likely only a good approximation for shorter samples. Second, the longer time span observed the higher the chance is that a self-employed makes an optimization error. Third, the theoretical

⁸We have confirmed this in several numerical simulations of the model.

model does not take liquidity constraints into account. Hence, it may not always be possible to perfectly bunch using retained earnings because of urgent needs for cash.

Obviously, these situations are more likely to take place for an individual we observe over 15 years. Therefore, when using a shorter estimation sample fewer observations are polluted with one extreme observation. Hence, we should expect that the estimated elasticity from bunching in adjusted earned income is negatively biased for longer estimation samples. Consequently, we expect that the estimation using Saez (2010) method on adjusted earned income on shorter time-periods will be more reliable.

Contrary to using Saez (2010) method directly on adjusted earned income, applying equation (5) by estimating the shifting areas to left and right of the excess mass in adjusted earned income seems to be more robust to the length of the estimation sample. Although the excess mass at the cutoff is estimated more precisely with shorter time periods, the adjusted earned income will be more dispersed since fewer observations are used when averaging out an extreme observation. This is likely to imply that the shifting areas are underestimated which would lead to an overestimated elasticity.

Tax evasion is usually unobserved in the data and it is important to examine how our estimators are affected by this. It is clear, that if tax evasion is the single mechanism used to bunch, our estimates will be upward biased. For our estimators, this problem is completely analogous to the case of neglecting income shifting and using Saez' static bunching formula on taxable income for self-employed.

Another potential concern could be if bunching is partially achieved by the use of tax evasion. To mimic this case, we model tax evasion as an unobserved amount that reduces both earned income and retained earnings. It turns out that both of our estimators are quite robust to this exercise. The important thing to notice is that adjusted earned income, z_{it}^{adj} , is unaffected by the parallel reduction in earned income and retained earnings. Therefore, the estimator based on bunching in adjusted earned income (equation (3)) is completely unaffected and is still unbiased in presence of tax evasion.

However, tax evasion affects the estimate of $\eta_{it} - \bar{\eta}_i$. As illustrated in Figure 7, $\eta_{it} - \bar{\eta}_i$ determines the length of the shifting intervals used in our alternative estimator in equation (5). Unobserved tax evasion therefore tightens the estimated shifting interval for income shifters who are bunching and retaining earnings. A smaller shifting interval implies a lower estimate of the overall share of shifters and thereby an upward biased elasticity estimate. However, it turns out that for a shifting observation to move outside the shifting interval, the amount of tax evasion needs to be larger than the amount of retained earnings if tax evasion was not possible. In other words, to bias the elasticity estimate, tax evasion needs to be large enough to change the sign of retained earnings (such that it becomes withdrawn earnings).⁹

⁹For income shifters who are bunching and withdrawing earnings, tax evasion widens the shifting interval.

4 Empirical results

In this section, we first provide graphical evidence to illustrate the relevance of our model and to investigate the sources that drive the massive amount of bunching for the self-employed. We then formally estimate the structural elasticity, as well as the share of bunching due to income shifting, using our extensions of Saez' bunching formula.

4.1 The role of retained earnings and other margins of income transfers

To provide a first check of what drives the massive bunching in taxable income documented in Figure 1 and 2, we break down the sample into three mutually exclusive groups: individuals who i) retain earnings, ii) withdraw earnings or iii) neither retain nor withdraw earnings. Figure 8 compares the cumulative distribution of taxable income for each of the three groups. As we would expect, we observe much more bunching around the top tax kink, where the decrease in the after-tax rate is largest. Obviously, this finding is both consistent with a static labor supply model as well as our model with income shifting. However, it is primarily individuals that retain or withdraw earnings who bunch around the cutoffs.

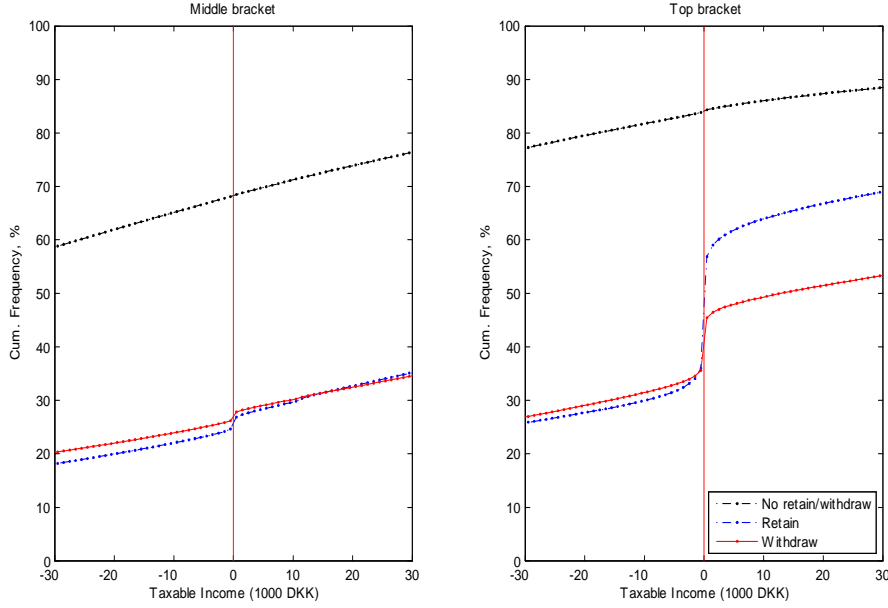
Although, we of course need to be careful when endogenously cutting the sample by those with/without retained earnings, it certainly does look like retained earnings is the key driving factor for the massive observed bunching in taxable income. In fact, we observe almost no bunching for individuals that neither retain nor withdraw earnings. In the light of the theoretical model, the latter fact suggests that either the structural elasticity or the costs of retaining earning must be relatively small. Otherwise, we would observe bunching for this group too (as illustrated in Figure 4-6).

In our theoretical model using retained earnings is the only margin at which taxpayers can adjust taxable income. In reality, however, it is possible for self-employed to use other means of income shifting. A second possibility is to transfer income to an assisting spouse. A third margin at which individuals can shift income to reduce tax payments is to increase contributions to capital pension accounts. Finally, self-employed can build up inventories or invest in equipment to reduce both earned and taxable income. While contributions to capital pension and transfers to assisting spouses are observed in our data, the last margin is not. However, such unobserved reductions in earned income work in a similar way as tax evasion. As argued in section 3.2 our estimators are quite robust to unobserved adjustments that reduce earned income and retained earnings in a parallel way.

We can easily interpret our model as a model of income shifting more broadly - including the two additional margins mentioned above. In the case of income transfers to assisting spouses,

This has no effect on the estimate, because we condition on bunching in taxable income, such that no additional individuals are erroneously labeled income shifters.

FIGURE 8: CUMULATIVE DISTRIBUTION OF TAXABLE INCOME, 1994-2009



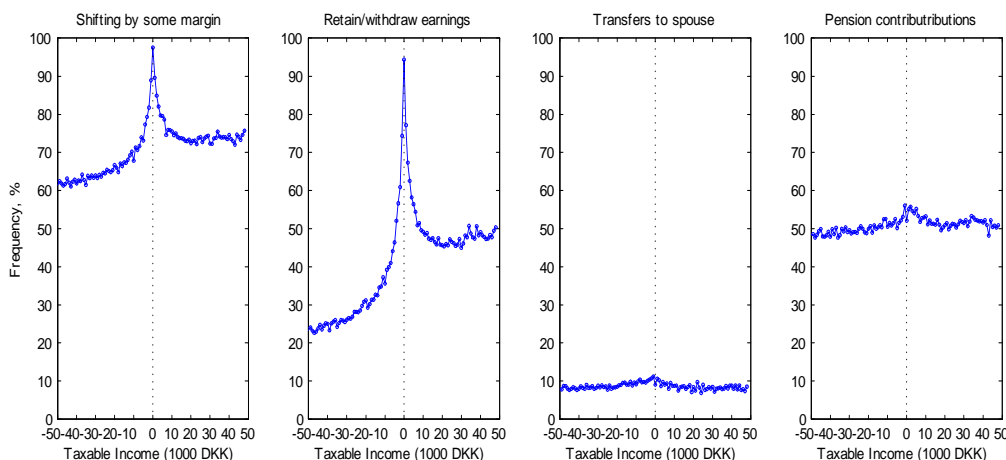
Notes: These figures plot the cumulative distribution of taxable income for different subpopulations: Individuals who i) retain earnings, ii) withdraw earnings, and iii) neither retain nor withdraw earnings. Taxable income is centered relative to the middle tax cutoff (left panel) and the top tax cutoff (right panel). The horizontal axis is divided into small 1000 DKK bin. Each point shows the midpoint of the percentage frequency of individuals with taxable income (relative to the cutoff) smaller than the right endpoint in a given 1000 DKK bin.

we could let t index persons in the household rather than time, such that $t = 1$ refers to the high income person in the household, and $t = 2$ refers to the low income person in the household. If we abstract from intra-household allocation of resources, the marginal cost of income shifting, $g'(m)$, is truly zero, since the transfer to spouse is available for consumption in the same period. Completely analogous to the intertemporal income shifting interpretation of the model, no income transfer will occur if the low-income spouse is already above top tax threshold.

Similar to retained earnings, the possibility of using transfers to assisting spouses is only available for self-employed and could thus contribute to explain why we observe more bunching among self-employed compared to workers. In contrast to retained earnings, however, there is a maximum amount per year that can be deducted in taxable income. In 2001 the maximal deductible amount was 171,100 DKK. Moreover, it is not possible to perfectly smooth taxable income over time using this scheme since only non-negative amounts can be transferred to the spouse.

Compared to retained earnings using pension contribution as a mean of income shifting is less flexible than retained earnings. First, since retirement may be far out in the future, the

FIGURE 9: INCOME SHIFTING AT DIFFERENT MARGINS VS. TAXABLE INCOME, 1994-2009



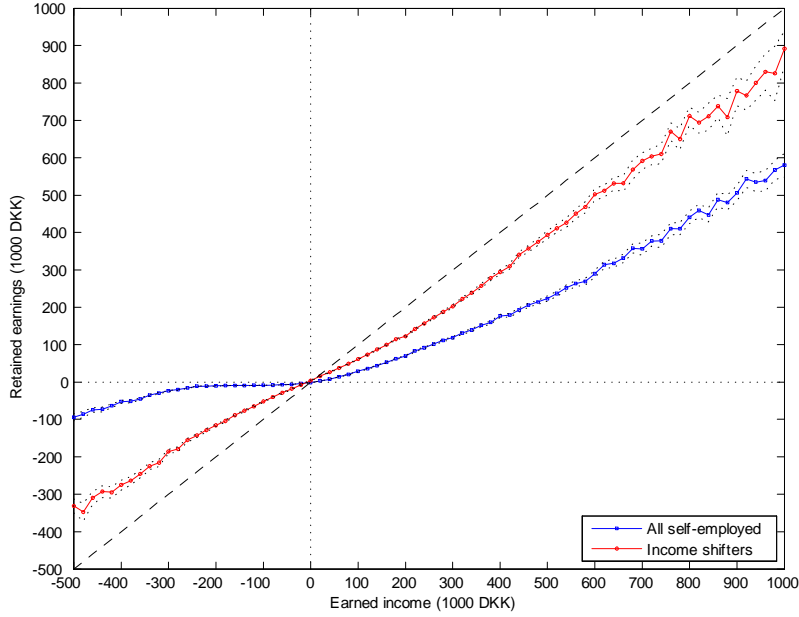
Notes: These figures depict the share of self-employed who shift income at different margins in a window of -50,000 to 50,000 DKK centered around the top tax cutoff. Taxable income is centered relative to the top tax cutoff.

utility loss from deviating from the optimal consumption-savings path may be non-negligible. Secondly, the decision to save is partially irreversible, since capital pensions withdrawn before the early retirement age are taxed with 60 percent compared to only 40 percent if withdrawn after retirement. Third, both retained earnings and transfers to spouses are fully deductible in all tax bases, whereas contributions to capital pensions are only fully deductible prior to a tax reform in 1999. From 1999 and onwards, contributions to capital pensions are no longer deductible in the top tax. Fourth, the maximum amount that can be deducted in personal income per year is small compared to observed intertemporal earned income volatility. In 1998 the maximal deductible amount was 33,100 DKK. Finally, since both workers and self-employed share this opportunity - we should observe much more bunching for workers if capital pensions were a key driving factor for bunching in taxable income.

As a backdrop, note that self-employed can also extract income from the firm as capital income. Using Danish data, Kleven and Schultz (2011) examine cross-tax effects in a regression framework and find evidence for weak substitutionability of labor and capital income. However, since capital income is generally taxed by a lower rate, most tax payers would like to declare as much capital income as legally possible. Therefore, we do not expect that taxpayers use capital income to bunch at the kink points.

Empirically, these secondary margins are indeed secondary as expected: For the group that neither retain nor withdraw earnings, the distribution of taxable income is very smooth around the middle tax cutoff and there is only a very limited bunching at top tax cutoff with an

FIGURE 10: RETAINED EARNINGS VS. EARNED INCOME, 1994-2009



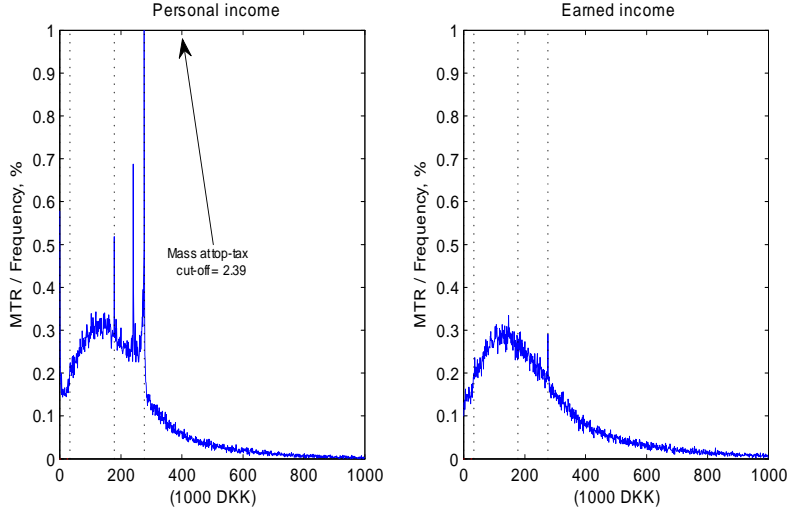
Notes: This figure plots a non-parametric regression of retained earnings against earned income for all self-employed (blue curve) and for the group of self-employed transferring income and whose average earned income is in between the top tax cutoff and 200,000 DKK above the cutoff (red curve). The dotted lines plotted around the regression curves mark the 95 percent pointwise confidence intervals. Earned income is centered around the top tax cutoff. The dotted line is the 45 degree line. Only self-employed whose primary occupation is self-employment are included.

excess mass less than 0.5 percent - despite this group could still adjust taxable income using transfers to assisting spouses, pension contributions or by declaring income as capital incomes. If we focus on the group that does not transfer income by any margin, the excess mass drops slightly, leaving us with almost no signs of bunching.¹⁰ The excess mass around the top kink is only 0.3 percent for this group. This is in sharp contrast to the self-employed who retain or withdraw their earnings. For example, 20.8 percent of the self-employed who increase their retained earnings in the firm have taxable income within a window of ± 500 DKK around the top bracket cutoff and the excess mass around the cutoff is 26.7 percent of this population.

Since the different ways of income shifting are not mutually exclusive, Figure 9 examines the relationship between bunching and the probability of income shifting using the different margins. Around the top tax cutoff we observe a sharp increase in the share of self-employed retaining/withdrawing earnings. About 95 percent of self-employed located at the top tax threshold either retain or withdraw earnings - and practically 100 percent shift income by some

¹⁰Results are not shown, but are available upon request.

FIGURE 11: DENSITY OF TAXABLE INCOME AND EARNED INCOME 2001



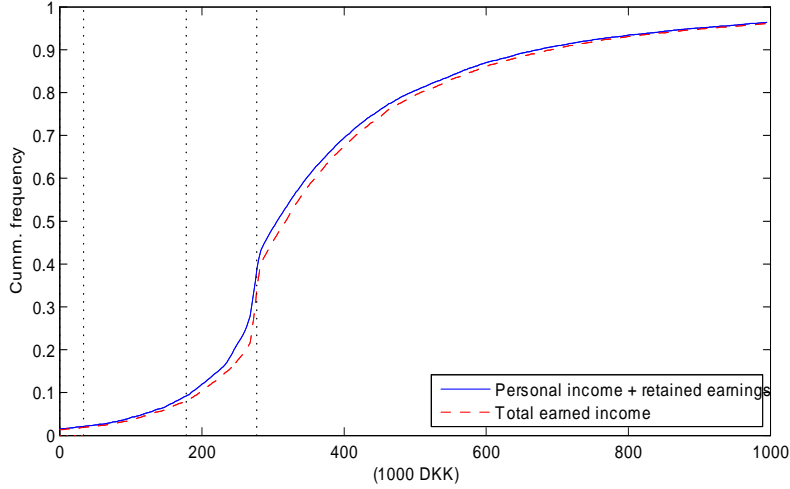
Notes: These figures plot the empirical distribution of personal income (left panel) and earned income (right panel) for self-employed with self-employment as main occupation. The tax cutoffs are marked with vertical dotted lines.

margin. It is also clear from Figure 9 that retaining/ withdrawing earnings is the most important margin at which income is adjusted. While the fraction of individuals that shifts income using pension contributions and transfers to assisting spouses is almost independent of the distance to the kink, the fraction of individuals using retained or withdrawn earnings increases sharply around the top tax kink.

In Figure 10 we plot the average retained income against earned income - centered around the top tax cutoff. Hence, if all individuals were perfectly bunching each year by using only the margin of retained earnings this curve would coincide with the 45 degree line. Looking at all self-employed, there is indeed a clear positive relationship between retained earnings and earned income, but the curve is above the 45 degree line for earned incomes below the top tax cutoff and below the 45 degree line when earned income is above the top tax cutoff. Hence, when Danish self-employed individuals earn more than the top tax cutoff, they retain less than one dollar each time they earn a dollar - on average. Conversely, when earned income is smaller than the top tax cutoff, previously retained earnings are withdrawn - but again not perfectly. This seems to support the theoretical finding that self-employed retain earnings when they experience a positive shock and withdraw earnings when they get a negative shock, whereby the volatility of earned income is higher than of taxable income.

For the group of self-employed (denoted as income shifters in Figure 10) with non-zero retained earnings and whose average earned income in the observed period is at least at the top tax cutoff, but less than 200,000 DKK in excess of the top tax threshold, we see that the use of

FIGURE 12: INCOME DISTRIBUTIONS FOR BUNCHING INDIVIDUALS, 2001



Notes: This figure plots the empirical distribution of personal income added retained earnings (blue curve) and total earned income (red dashed curve) for individuals with taxable income within a window of ± 2000 DKK around the top tax cutoffs. The dotted vertical lines mark the bottom, middle and top tax thresholds

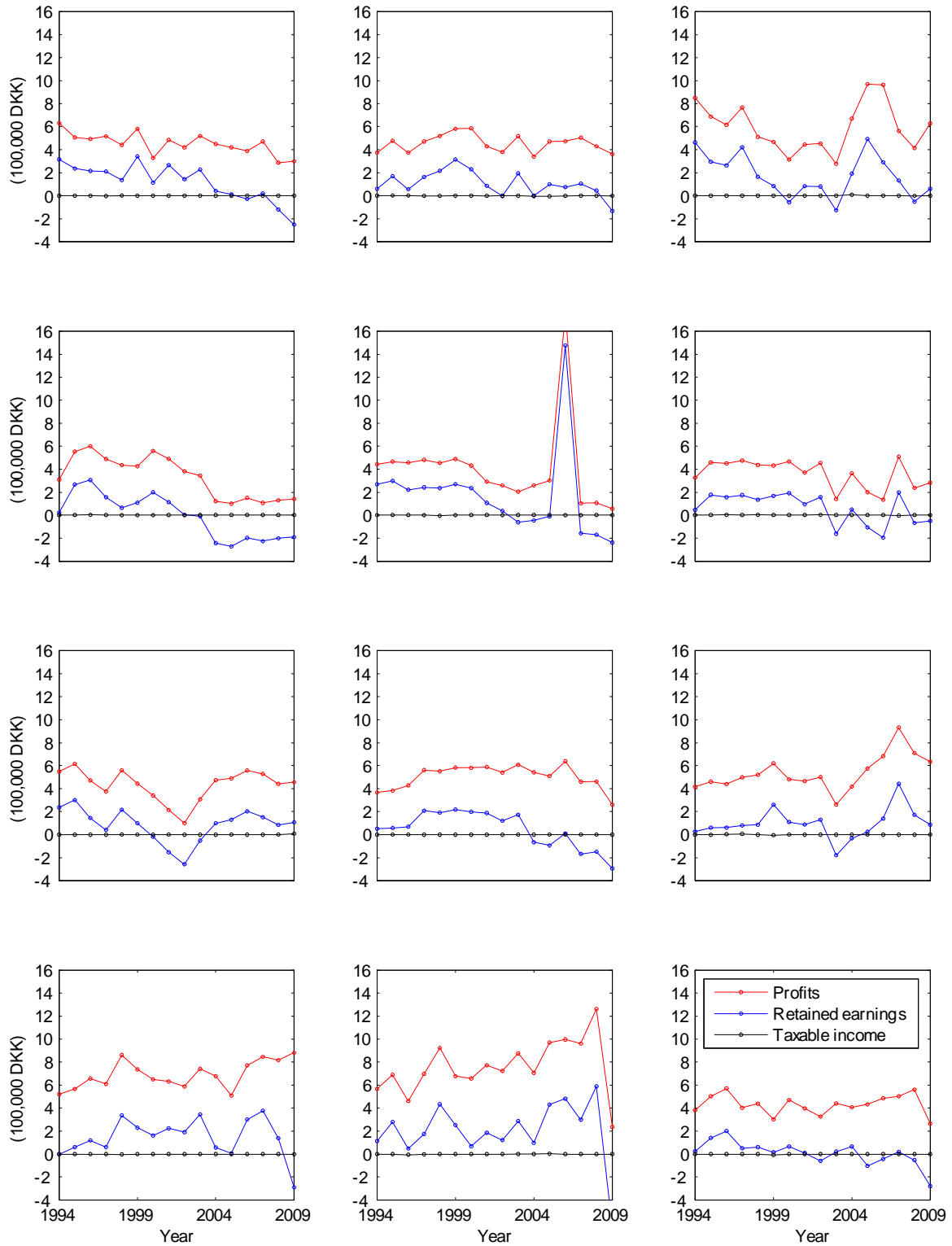
retained earnings is much closer to the 45 degree line.

For estimating the structural elasticity we assume that the costs of transferring income over time is zero. From Figure 8 we have already obtained some evidence supporting very small costs or that the elasticity is close to zero. Another way to obtain similar evidence is to depict the distribution of earned income along with personal income for all self-employed in 2001 as done in Figure 11. The very small excess mass in the top tax cutoff in earned income supports that the costs are small, but we cannot distinguish small costs of transferring income (together with heterogenous income shocks) from a low structural elasticity.

In Figure 12 we take a closer look at the amount of income shifting for individuals located at the top cutoff. We have limited the sample to individuals with taxable income within a window of ± 2000 DKK around the top-tax cutoffs. For this subsample, the we plot the cumulative distribution of i) personal income plus retained earnings and ii) total earned income. The difference between these two distributions, is that total earned income also includes transfers along other margins such as pension contributions and transfers to assisting spouses.

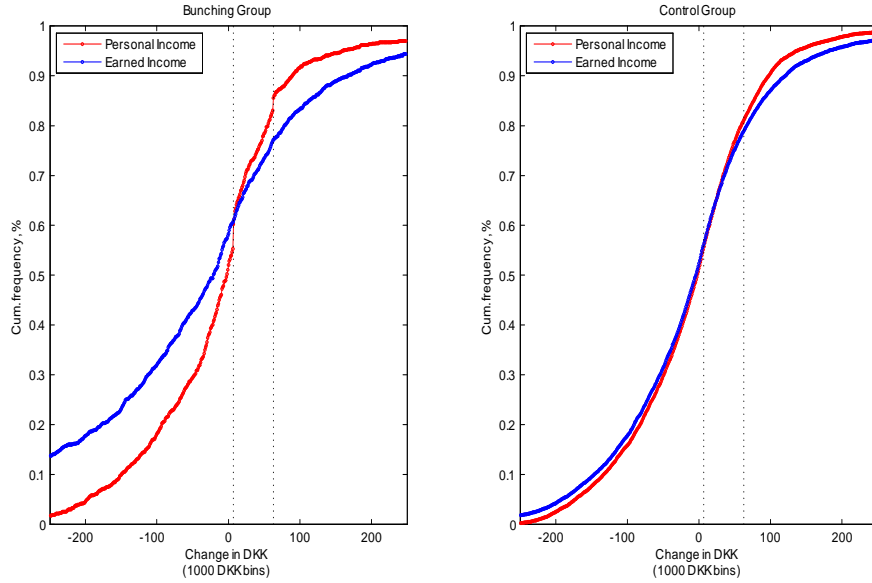
If no income was retained (or shifted by any other margin) the cumulative distribution should increase from 0 to 1 in the ± 2000 DKK window around the top tax cutoffs. This is clearly not the case. The figure shows that individuals bunching are transferring substantial amounts into the future. More than 20 percent of the individuals who are located at the top tax cutoff (± 2000 DKK) actually earn more than twice as much without paying top tax. Furthermore,

FIGURE 13: INCOME TRAJECTORIES FOR SELECTED INDIVIDUALS, 1994-2009



Notes: These figures illustrate the time-profiles of profits, retained earnings and taxable income for selected self-employed who are bunching at the top tax cutoff in all years in the sample window, 1994-2009.

FIGURE 14: DISTRIBUTIONS OF INCOME CHANGES



Notes: These figures plot the empirical distribution of income changes between 2007 and 2008 for two different groups of self-employed. In the left panel, we consider individuals that were bunching at the middle tax threshold in 2007. In the right, panel we consider self-employed whose taxable income was below the middle tax cutoff in 2007. In both panels, we plot changes in personal income (red curves) and earned income (blue curves) for the bunching and the control group respectively. Income changes (horizontal axis) is divided into small 1000 DKK bins. Both panels plot the cumulative percentage frequencies. The left vertical line marks the change in middle tax cutoff between 2007 and 2008. The right vertical line marks the difference between the middle tax cutoff in 2007 and the top tax cutoff in 2008

this figure also shows that the lion's share of the difference between taxable income and earned income is driven by retained earnings, whereas pension contributions and transfers to assisting spouses clearly play a smaller role.

In the period 1994-2009, on average 9.1 percent of the self-employed are located around the top kink within a window of ± 7500 DKK and the corresponding excess mass is 7.2 percent. Out of those who are observed to be self-employed all 16 years of the sample window, 60.4 percent bunch at the top kink in at least one year, 7.7 percent are observed to bunch at least 8 years, and 0.2 percent bunch all 16 years. Figure 13 shows income trajectories for individuals that bunch in each of the 16 observed years. Although this group is obviously not representative, the graph shows that in principle it is possible to use retained earnings to bunch 16 years in a row. Despite substantial fluctuations in earned income these, self-employed keep bunching at the same tax threshold such that fluctuations in earned income are completely off-set by adjusting retained earnings.

Figure 14 depicts the distribution of changes in personal and earned incomes from 2007 to

2008. We consider two specific groups; i) those who bunch at the middle tax in the first year, whom we refer to as the bunching group, and ii) those who were located below the middle tax cutoff, whom we refer to as the control group. In the left panel we can see how the two groups changed their taxable income from 2007 to 2008. Looking at the right panel of Figure 14 we see that for the controls the distributions of changes in taxable and earned income are almost identical, whereas this - as shown in the left panel - is not the case for the bunching group. It seems to be the case that for those whose earned income did not change, personal income neither changed. However, for those whose earned income increased more than the distance to the following year's top tax cutoff the taxable income increased less than the earned income. Additionally, for those whose earned income decreased, the taxable income decreased less. Hence, the self-employed use retained income to smoothen out taxable income such that earned income over time has a larger variance than taxable income as the theoretical model also predicts.

4.2 Elasticity estimates

Based on the evidence in the preceding sections, it is difficult to tell how much of the tax bunching is due to pure income shifting and how much is due to real labor market responses, but the theoretical model provides us with a possibility of estimating the share of the bunching which is due to income shifting as well as of estimating the structural elasticity. This elasticity is not equal to the taxable income elasticity, but is the earned income elasticity when the income shock is zero.

In table 3 we report our main estimation results for different sub-periods. The first part of the table reports the excess mass at the top tax cutoff for three measures of income: taxable income, earned income and adjusted earned income. Regarding the first two measures, the picture from the descriptive evidence above is confirmed: We observe a massive amount of bunching in taxable income with excess masses ranging from 6.83 to 9.22 percent, whereas the excess mass in earned income is always below 0.3 percent.

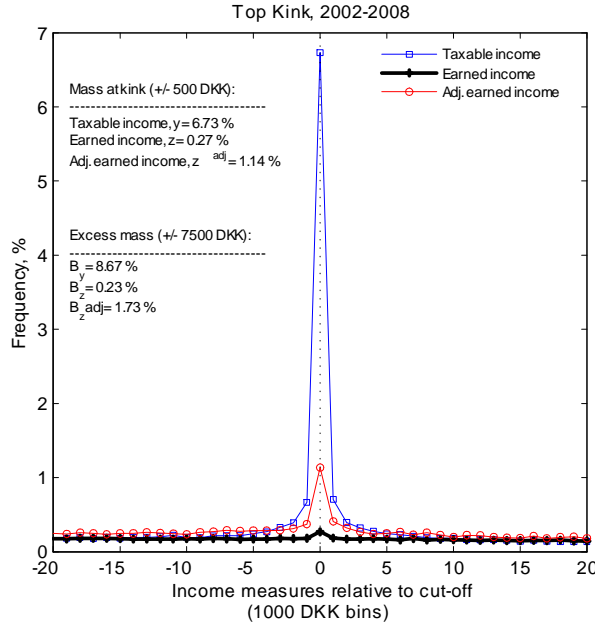
The observed bunching in taxable income reflects both a real response and pure income shifting. Therefore, it is not possible to separately identify these two effects focusing solely on the bunching in taxable income. Earned income on the other hand, presumably gives a clearer signal about the amount of exerted efforts, but does not take into account that the effective cutoff depends on income fluctuations that are not completely controlled by efforts. Therefore, we will not be able to clearly observe bunching in the distribution of earned income in the presence of income shifting and income shocks. Using Saez' method directly on either of the two income measures give rise to very different elasticity estimates of 0.45 – 0.53 and 0.01 – 0.02 for taxable and earned income respectively. Even though these elasticity estimates can be interpreted as upper and lower bounds they are clearly not very informative.

TABLE 3: ELASTICITY ESTIMATES

Excess mass, B	1996-2001	2002-2008	1996-1998	1999-2001	2002-2004	2005-2008
Taxable income, y	0.0683 (0.0004)	0.0867 (0.0004)	0.0687 (0.0007)	0.068 (0.0007)	0.0798 (0.0007)	0.0922 (0.0006)
Earned income, z	0.0015 (0.0003)	0.0023 (0.0003)	0.001 (0.0004)	0.002 (0.0005)	0.0019 (0.0004)	0.0026 (0.0004)
Adj. earned income	0.0117 (0.0008)	0.0173 (0.0008)	0.0255 (0.001)	0.0255 (0.0009)	0.0338 (0.001)	0.0301 (0.0009)
Taxable and earned income elasticity						
Taxable income, y	0.4521 (0.0050)	0.518 (0.0054)	0.4759 (0.0082)	0.4279 (0.0079)	0.4958 (0.009)	0.5326 (0.0074)
Earned income, z	0.011 (0.0021)	0.0158 (0.0019)	0.0076 (0.0033)	0.0144 (0.0033)	0.0136 (0.0032)	0.0174 (0.0025)
Adjusted earned income elasticity						
Bunching method	0.0663 (0.0058)	0.0831 (0.0052)	0.1543 (0.0091)	0.1412 (0.008)	0.1783 (0.0088)	0.1413 (0.0074)
Shifting method	0.1481 (0.0034)	0.1957 (0.0037)	0.2029 (0.0064)	0.1844 (0.0056)	0.2459 (0.0069)	0.2371 (0.0056)
Shifting share						
Bunching method	0.8285 (0.0118)	0.8004 (0.0087)	0.6281 (0.0124)	0.6251 (0.0121)	0.5769 (0.0102)	0.6733 (0.0094)
Shifting method	0.6161 (0.0040)	0.5288 (0.0029)	0.5108 (0.0051)	0.5101 (0.0051)	0.4159 (0.0042)	0.451 (0.0036)

Notes: This table gives the estimation results using Saez' bunching formula on taxable income and earned income as well as the two bunching formulas using adjusted earned income as outlined in the theory section. The bandwidth is set to 7,500 DKK. The bunching method refers to the estimation of equation (3) whereas the shifting method refers to the estimation of equation (5). The tax cutoffs used differ between each of the 6 periods considered. In each case they are computed as the simple average. Marginal tax rates are calculated as the sample average of marginal tax rates based on our tax-simulator. This is done for the middle bracket as well as the top bracket separately for each the periods considered. For each estimation sample we only include self-employment observed at least three times. Only self-employed whose primary occupation is self-employment are included.

FIGURE 15: BUNCHING IN ADJUSTED EARNED INCOME



Notes: This figure shows frequency plots of taxable, earned and adjusted earned incomes in a window of -20,000 to 20,000 DKK centered around the top tax cutoff. The horizontal axis is divided into small 1000 DKK bins.

The excess masses and elasticity estimates using taxable income are larger for the second half of the sample period. Besides a possible higher elasticity in the second half of the sample there are two factors which could explain this. First, the annual growth rate of earnings have exceeded the growth rate in the top tax cutoff such that more persons are located around the top tax cutoff. Abstracting from income effects¹¹ this should only affect the excess mass and not the estimated elasticity. Second, the financial crisis has implied larger income fluctuations in the second half of the sample and according to the theory this is likely to increase bunching as a result of income shifting.

We should also notice that the elasticity estimates using bunching in taxable income are roughly twice the size of the estimates in Chetty et al. (2011) and Kleven et al. (2011) since we use a more narrow definition of self-employed: We only include persons whose primary occupation is self-employment. By excluding wage earners whose secondary occupation is self-employment we are less likely to obtain elasticities that are biased downwards because of hours constraints and search frictions and we can focus on income shifting vs. real response. Abstracting from persons whose secondary occupation is self-employment has almost no effect on the excess mass, but it affects the surrounding density such that the resulting elasticity estimate

¹¹Bastani and Selin (2011) show analytically that income effects do not affect Saez' bunching estimator when the tax change is small. For larger tax changes their simulations show that the elasticity estimate is only slightly attenuated in presence of income effects.

is lower.

By applying Saez' bunching formula on adjusted earned income we obtain elasticities lower than 0.1 when only dividing the sample period into two, that is 1996-2001 and 2002-2008.¹² In Figure 15 the bunching mass in adjusted earned income at the top tax cutoff is compared to the bunching in taxable and earned incomes for the latter sample period. It is clear that by correcting for income shocks we find more bunching than in earned income, but less than in taxable income. Nevertheless, as we noted in the theory section, the estimator using the bunching mass in adjusted earned income is likely to be downward biased for longer samples. Even if a self-employed was bunching as a real response in every year, a single deviating observation implies no observed bunching in adjusted earned income in all years. Cutting the sample period into four periods delivers elasticity estimates in between 0.14 and 0.18, which we consider more trustworthy.

The alternative estimation which uses the shifting mass on both sides of the cutoff suggests that the elasticity is 0.15 in the first half of the sample and 0.20 in the second half of the sample. As expected we obtain only slightly higher elasticities when cutting the sample into four, since this estimator does not crucially rely on precisely estimating the excess mass in adjusted earned income. However, as mentioned in the in section 3.2, we are likely to obtain upward biased estimates in shorter samples with this estimator, since extreme observations are less likely to be averaged out.

This lead us to conclude that the structural elasticity lies in the range 0.14 – 0.20 and that the share of the bunching due to pure income shifting is in the interval between 52 and 68 percent.

Our bunching formula is derived under the assumption of zero costs of transferring income. Suppose instead that these costs are non-zero. As shown in section 3.1, higher shifting costs would imply less bunching as fewer persons would bunch in both periods. Therefore, for a given observed bunching mass in taxable income, allowing for non-zero costs would imply a higher estimated elasticity. However, the combination of elasticities higher than 0.14-0.20 and significantly higher costs seem to contradict the findings of very limited bunching in earned income as well as the fact that 95 per cent of those bunching use retained earnings. In sum, this suggests that the costs of transferring income is indeed low.

Kleven et al. (2011) compare incomes before and after the auditing by the Danish tax authorities. Contrary to wage earners whose incomes typically are third-party reported, Kleven et al. observe substantial tax evasion among self-employed. Out of self-employed bunching at the top tax threshold, 45 percent underreport incomes such that bunching is at least partly achieved by underdeclaring. However, self-employed caught in tax evasion were not allowed to

¹²We have cut the sample in two with the aim of obtaining two periods with fairly stable tax systems over time. Besides this, transfers to assisting spouses are imputed for the second period, 2002-2008.

re-optimize by increasing retained earnings. Therefore, it is not unlikely the amount evaded would simply have been retained in the firm if it was allowed to adjust retained earnings after being caught.

As argued in section 3.2, our bunching estimators are quite robust to tax evasion. Only when tax evasion is the only margin used to bunch in taxable (and earned) income, both estimators are upward-biased. This problem seems to be negligible since more than 95 percent of self-employed individuals, located at the top kink, either retain or withdraw earnings. This is also reflected in the very limited bunching in earned income.

If the amount evaded is so large, that it makes taxpayers withdraw earnings (rather than retaining earnings) in high income years, the method using shifting areas results in upward-biased elasticity estimates, whereas the estimator using bunching in adjusted earned income stays unbiased. Hence, this may explain part of the divergence between the estimates using the two methods.

5 Conclusion

In this paper we have documented a massive bunching in taxable income for Danish self-employed. At first sight, this would suggest that taxable incomes are highly sensitive to changes in marginal tax rates (especially at the top tax bracket) and thus large efficiency costs would seem to be associated with taxing high income self-employed individuals. In fact, estimating the elasticity using Saez (2010) method on taxable income gives us an estimate of 0.50.

We show, however, that more than half of the observed bunching in taxable income for the self-employed is driven by pure income shifting. We examine the different margins by which self-employed can shift income and find that in particular the possibility of using retained earnings facilitates a lot of the bunching. As much as 95 percent of the self-employed who bunch at the top tax cutoff use retained earnings.

The pronounced bunching in taxable income is in stark contrast to earned income where we find almost no bunching. Using Saez' method on earned income gives elasticities lower than 0.02. This estimate is clearly downward biased. Although earned income presumably gives a clearer signal about the amount of exerted efforts, using Saez' method directly does not take into account that the effective cutoff depends on income fluctuations that are not completely controlled by efforts.

Since it is likely that a non-negligible share of the bunching mass in taxable income is due to pure income shifting, we set-up a simple model to analyze the intertemporal labor supply and tax planning decisions. We find that self-employed use retained earnings to smooth variations in earned income in order to reduce tax liability. We show that observed bunching in taxable income and absence of bunching in earned income is consistent with a true elasticity of zero.

However, since the effective cutoff depends on income shocks that are heterogenous, we should not expect to observe bunching in earned income even with a positive elasticity if the marginal cost of shifting income is zero.

We use our theoretical model to extend Saez' method to accommodate the possibility of retaining earnings in the firm and show how it is possible to identify the structural elasticity. Our bunching formula divides bunching in taxable income into two separate terms. The first term measures bunching solely due to income shifting whereas the other term measures bunching due to real responses. The resulting estimates suggest that 50 – 70 percent of the bunching is due to pure income shifting and that the structural elasticity is in the range of 0.14 – 0.20.

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Appendix 1: Derivation of bunching formula

To simplify the derivation of (3) and (5) we begin by assuming that $\eta_1 > 0$ and $\eta_2 < 0$, but in our final bunching formula we allow that η_t can take both positive and negative values by pooling the two periods. To allow η_t being heterogeneously distributed in the population, we now let $F(\gamma)$ denote the distribution of γ conditional on η_t . For a given value of η_t , let the counterfactual cumulative distribution function of earned income in absence of the kink be given by $H_0(z_t|\eta_t) = \Pr(\gamma(1-\tau_0)^\varepsilon + \eta_t \leq z_t|z_t, \eta_t) = F\left(\frac{z_t - \eta_t}{(1-\tau_0)^\varepsilon}\right)$ where $t = 1, 2$. Differentiating with respect to z_t gives us the conditional density $h_0(z_t|\eta_t) = \frac{1}{(1-\tau_0)^\varepsilon} f\left(\frac{z_t - \eta_t}{(1-\tau_0)^\varepsilon}\right)$.

In period, persons with $\gamma \in [\gamma_1^{low}, \gamma_2^{low}[$ only bunch due to income shifting and, therefore, only bunch in taxable income. We can also express this interval in terms of counterfactual earnings in case the only tax levied is τ_0 , $z_1^{CF} \in [y^*, y^* + \eta_1 - \bar{\eta}[$. Those bunching in both earned and taxable incomes in period 1 have $\gamma \in [\gamma_2^{low}, \gamma_1^{high}]$ which in terms of counterfactual earnings correspond to $z_1^{CF} \in \left[y^* + \eta_1 - \bar{\eta}, \left(\frac{1-\tau_0}{1-\tau_1}\right)^\varepsilon (y^* - \bar{\eta}) + \eta_1\right]$. In period 2, persons bunching in both taxable and earned incomes have $\gamma \in [\gamma_2^{low}, \gamma_1^{high}]$ which corresponds to $z_2^{CF} \in \left[y^* + \eta_2 - \bar{\eta}, \left(\frac{1-\tau_0}{1-\tau_1}\right)^\varepsilon (y^* - \bar{\eta}) + \eta_2\right]$. Finally, those only bunching in taxable income have $\gamma \in \left[\gamma_1^{high}, \gamma_2^{high}\right]$ and counterfactual earned income $z_2^{CF} \in \left[\left(\frac{1-\tau_0}{1-\tau_1}\right)^\varepsilon (y^* - \bar{\eta}) + \eta_2, \left(\frac{1-\tau_0}{1-\tau_1}\right)^\varepsilon (y^* - \eta_2) + \eta_2\right]$.

Since we do not observe the counterfactual earning densities we need to relate the observed distributions of earned income to the counterfactual counterparts. Let the realized conditional distribution of z_t be given by $H(z_t|\eta_t)$. In period t there is no bunching in earned income for persons earning less than $z_t < y^* + \eta_t - \bar{\eta}$ and, therefore, $h(z_t|\eta_t) = h_0(z_t|\eta_t)$. For $z_t \geq y^* + \eta_t - \bar{\eta}$ we have that $H(z_t|\eta_t) = F\left(\frac{z_t - \eta_t}{(1-\tau_1)^\varepsilon}\right)$ with the corresponding probability density function being $h(z_t|\eta_t) = \frac{1}{(1-\tau_1)^\varepsilon} f\left(\frac{z_t - \eta_t}{(1-\tau_1)^\varepsilon}\right)$. Using that both the counterfactual and realized earnings densities, $h_0(\cdot)$ and $h(\cdot)$, are related to $f(\cdot)$ we can for $z_t \geq y^* + \eta_t - \bar{\eta}$ write that

$$h_0(z_t|\eta_t) = h\left(\left(\frac{1-\tau_1}{1-\tau_0}\right)^\varepsilon (z_t - \eta_t) + \eta_t \middle| \eta_t\right) / \left(\frac{1-\tau_0}{1-\tau_1}\right)^\varepsilon$$

Following Saez (2010) we can relate the observed bunching in taxable income in period 1, B_{y_1} , to the counterfactual earned income density in period 1

$$\begin{aligned} B_{y_1} &= \int_0^\infty \int_{y^*}^{\left(\frac{1-\tau_0}{1-\tau_1}\right)^\varepsilon (y^* - \bar{\eta}) + \eta_1} dH_0(z_1|\eta_1) d\Upsilon(\eta_1) \\ &= \int_0^\infty \left(\int_{y^*}^{y^* + \eta_1 - \bar{\eta}} h_0(z_1|\eta_1) dz_1 + \int_{y^* + \eta_1 - \bar{\eta}}^{\left(\frac{1-\tau_0}{1-\tau_1}\right)^\varepsilon (y^* - \bar{\eta}) + \eta_1} h_0(z_1|\eta_1) dz_1 \right) v(\eta_1) d\eta_1 \end{aligned} \quad (6)$$

where η_t follows the distribution function $\Upsilon(\cdot)$ with the corresponding density $v(\cdot) = \Upsilon'(\cdot)$. The first part of the right hand side equals bunching due to income shifting and the second part bunching due to a real response. Since $\eta_1 > \eta_2$ self-employed bunching due to income shifting in the first period earn between y^* and $y^* + \eta_1 - \bar{\eta} > y^*$ regardless of the increase in the marginal tax rate from τ_0 to τ_1 as illustrated in Figure 7. For counterfactual incomes above $y^* + \eta_1 - \bar{\eta}$

self-employed cannot set $y_1 = y_2 = y^*$ without reducing efforts. Therefore, the second term measures the bunching due to real response.

Next, we substitute the counterfactual earned income density with the observed income density and use the trapezoid approximation for the second integral in equation (6)

$$B_{y_1} \simeq \int_0^\infty [H(y^* + \eta_1 - \bar{\eta}|\eta_1)_- - H(y^*|\eta_1)_-] v(\eta_1) d\eta_1 \\ + \frac{1}{2} y^* \left(\left(\frac{1-\tau_0}{1-\tau_1} \right)^\varepsilon - 1 \right) \int_0^\infty \left(h(y^* + \eta_1 - \bar{\eta}|\eta_1)_- + h(y^* + \eta_1 - \bar{\eta}|\eta_1)_+ \right) / \left(\frac{1-\tau_0}{1-\tau_1} \right)^\varepsilon v(\eta_1) d\eta_1$$

where $H(z|\eta_1)_-$ is the distribution function evaluated just below z , and $h(z|\eta_1)_-$ $h(z|\eta_1)_+$ are the densities evaluated just below and above z .

For the second period we can express the bunching mass as

$$B_{y_2} = \int_{-\infty}^0 \left(\int_{y^* - \bar{\eta} + \eta_2}^{\left(\frac{1-\tau_0}{1-\tau_1} \right)^\varepsilon (y^* - \bar{\eta}) + \eta_2} h_0(z_2|\eta_2) dz_2 + \int_{\left(\frac{1-\tau_0}{1-\tau_1} \right)^\varepsilon (y^* - \bar{\eta}) + \eta_2}^{\left(\frac{1-\tau_0}{1-\tau_1} \right)^\varepsilon (y^* - \eta_2) + \eta_2} h_0(z_2|\eta_2) dz_2 \right) v(\eta_2) d\eta_2 \\ \simeq \frac{1}{2} y^* \left(\left(\frac{1-\tau_0}{1-\tau_1} \right)^\varepsilon - 1 \right) \int_{-\infty}^0 \left(h(y^* + \eta_2 - \bar{\eta}|\eta_2)_- + h(y^* + \eta_2 - \bar{\eta}|\eta_2)_+ \right) / \left(\frac{1-\tau_0}{1-\tau_1} \right)^\varepsilon v(\eta_2) d\eta_2 \\ + \int_{-\infty}^0 [H(y^*) - H(y^* + \eta_2 - \bar{\eta})_-] v(\eta_2) d\eta_2$$

We can then express bunching in taxable income over all observed periods in terms of adjusted earned income

$$B_y \simeq \frac{1}{2\delta} y^* \left(\left(\frac{1-\tau_0}{1-\tau_1} \right)^\varepsilon - 1 \right) \sum_{i=1}^N \frac{1}{NT_i} \sum_{t=1}^{T_i} \left\{ \begin{array}{l} \mathbf{1} \left(y^* - 2\delta \leq z_{it}^{adj} < y^* - \delta \right) + \\ \mathbf{1} \left(y^* + \delta \leq z_{it}^{adj} < y^* + 2\delta \right) / \left(\frac{1-\tau_0}{1-\tau_1} \right)^\varepsilon \end{array} \right\} \\ + \sum_{i=1}^N \frac{1}{NT_i} \sum_{t=1}^{T_i} \mathbf{1}(|y_{it} - y^*| < \delta) \left\{ \begin{array}{l} \mathbf{1}(\eta_{it} - \bar{\eta}_i \geq 0) \mathbf{1} \left(y^* - (\eta_{it} - \bar{\eta}_i) - \delta \leq z_{it}^{adj} < y^* - \delta \right) + \\ \mathbf{1}(\eta_{it} - \bar{\eta}_i < 0) \mathbf{1} \left(y^* + \delta < z_{it}^{adj} \leq y^* - (\eta_{it} - \bar{\eta}_i) + \delta \right) \end{array} \right\} \quad (7)$$

Furthermore, subtracting the second term (measuring income shifters) on both sides gives equation (3)