

Advanced Macroeconomics II

Handout 6 - Speeding Up VFI

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Short recap

Prototypical DP problem:

$$\begin{aligned} V(z, k) &= \max_{\{c, k'\}} u(c) + \beta E \left[V(z', k') | z \right] \\ \text{s.t. } c + k' &= f(z, k) \\ z' &= h(z, \eta); \eta \text{ stochastic} \end{aligned}$$

- ▶ We are looking for functions V, g^c, g^k : We cannot solve this.

We need to solve an approximate problem:

- ▶ Approximate continuous function: **Interpolation**
 - ▶ Requires “exact” solution of maximization problem: **Optimization**
 - ▶ Requires computing expectations: **Integration**

Why is VFI so costly?

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- ▶ Optimizing requires *a lot* of function evaluations
 - ▶ Each evaluation requires expectations, interpolations and often derivatives

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Key idea:

- ▶ Can we bypass the maximization step?
- ▶ Focus on the Euler equation

Carroll (2006)

Maximization requires satisfying FOC:

$$u'(c) = \beta E \left[V_k(z', k') | z \right] \quad c + k' = f(z, k)$$

Usual approach:

- ▶ Fix (z, k) and solve for (k', c)
- ▶ Consumption is immediately given k' : $c = f(z, k) - k'$
- ▶ Problem is to try a bunch of k' to solve

$$u'(f(z, k) - k') = \beta E \left[V_k(z', k') | z \right]$$

Carroll (2006)

Maximization requires satisfying FOC:

$$u'(c) = \beta E \left[V_k(z', k') | z \right] \quad c + k' = f(z, k)$$

Carroll's approach:

- Fix (k', z) and solve for k ! Hence the endogenous grid name
- Problem is to solve:

$$f(z, k) = \underbrace{(u')^{-1} \left(\beta E \left[V_k(z', k') | z \right] \right)}_{\text{Known given } (k', z)} + k'$$

- This is a nonlinear equation, but a simple one to solve (often analytically)

Key: Expectation and derivatives only taken once! No interpolations!

Standard algorithm

Algorithm 1: EGM: Standard Method

Function EGM($V, \vec{k}, \vec{z}, parameters$):

for $i=1:n_z$ do

for $j=1:n_k$ do

$$F(x) = f(\vec{z}_i, x) - \vec{k}_j - (u')^{-1} \left(\beta E \left[V_k \left(z', \vec{k}_j \right) | \vec{z}_i \right] \right)$$

Find $[k_min, k_max]$, check corners, further bracket zero

$$k_endo[j] = \text{Roots}(F, k_min, k_max)$$

$$V_endo[j] = u(f(\vec{z}_i, k_endo[j]) - \vec{k}_j) + \beta E \left[V \left(z', \vec{k}_j \right) | \vec{z}_i \right]$$

Interpolate value function to exogenous grid

$$V_new(i,:) = \text{Interpolation}(k_endo, V_endo, \vec{k})$$

return V_new

Change of variable - Know your states

- ▶ A change of variable makes things easier
- ▶ Define Y as total income, or cash on hand: $Y = f(z, k)$

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$$\begin{aligned} V(z, Y) &= \max_{\{k'\}} u(Y - k') + \beta E[V(z', Y') | z] \\ \text{s.t. } Y' &= f(z', k') \\ z' &= h(z, \eta); \eta \text{ stochastic} \end{aligned}$$

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- ▶ Define Y as total income, or cash on hand: $Y = f(z, k)$

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- ▶ Control variable k' (partially) determines future state
- ▶ We still need to hang onto z as a state, why?

Change of variable - Know your states

Note that Y' is a function of k' and z' so we can write

$$\mathbb{V}(z, k') = \beta E \left[V \left(Y' \left(z', k' \right), z' \right) | z \right]$$

$$\mathbb{V}_k(z, k') = \beta E \left[V_Y \left(Y' \left(z', k' \right), z' \right) \frac{\partial Y \left(z', k' \right)}{\partial k} | z \right]$$

Change of variable - Know your states

Note that Y' is a function of k' and z' so we can write

$$\mathbb{V}(z, k') = \beta E \left[V(Y'(z', k'), z') | z \right]$$
$$\mathbb{V}_k(z, k') = \beta E \left[V_Y(Y'(z', k'), z') \frac{\partial Y(z', k')}{\partial k} | z \right]$$

Now the problem is:

$$\begin{aligned} V(z, Y) &= \max_{\{k'\}} u(Y - k') + \mathbb{V}(z, k') \\ \text{s.t. } Y' &= f(z', k') \\ z' &= h(z, \eta); \eta \text{ stochastic} \end{aligned}$$

Modified EGM

Algorithm 2: EGM: Change of State Method

Function EGM($V, \vec{k}, \vec{z}, parameters$):

Note: You already know Y for any (z, k) , let $Y_{ij} = Y(\vec{z}_i, \vec{k}_j)$

for $i=1:n_z$ **do**

for $j=1:n_k$ **do**

$\mathbb{V}_j = \beta E \left[V \left(Y(z', \vec{k}_j), z' \right) \mid \vec{z}_i \right]$

$\vec{c_endo} = (u')^{-1} \cdot (\mathbb{V}_k)$ (Note: Evaluating whole vector)

$\vec{Y_endo} = \vec{c_endo} + \vec{k}$

$\vec{V_endo} = u(\vec{c_endo}) + \mathbb{V}$

 # Interpolate value function to exogenous grid

$V_new[i,:] = \text{Interpolation}(\vec{Y_endo}, \vec{V_endo}, Y[i,:])$

Change variable back to k . Note: $V_{ji} = V(Y(\vec{z}_i, \vec{k}_j), z_i) = V(\vec{z}_i, \vec{k}_j)$

Some comments

The method still has some flexibility

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1. How to compute derivatives (you can get it from interpolation step)
2. How to compute expectations (no interpolation if z is discrete)
3. How to judge convergence (standard practice is to pass \mathbb{V} along and judge convergence with it)
4. How to map to capital after convergence
 - 4.1 Interpolation of $V(Y_{endo})$ to $V(Y_{exo})$: Y_{exo} maps to k by construction
 - 4.2 Keep Y_{endo} . Solve for $\vec{k}(z)$ s.t. $Y_{endo}(z) = f(z, \vec{k}(z))$

Labor Supply

Labor supply adds an equation

$$\begin{aligned} V(z, k) &= \max_{\{c, k'\}} u(c, \ell) + \beta E \left[V(z', k') \mid z \right] \\ \text{s.t. } c + k' &= f(z, k, \ell) \\ z' &= h(z, \eta); \eta \text{ stochastic} \end{aligned}$$

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FOC:

$$u_c(c, \ell) = \beta E \left[V_k(z', k') | z \right] \quad -u_\ell(c, \ell) = f_\ell(z, k, \ell) \quad c + k' = f(z, k, \ell)$$

Attempting EGM - Problems

Change of variable:

$$Y(z, k) = f(z, k, \ell(z, k)) = zk^{\alpha} \ell(z, k)^{1-\alpha} + (1 - \delta)k$$

- ▶ Cannot define exogenous grid for Y . Grid depends on policy function.

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Euler equation:

$$u_c(c, \ell(z, k)) = \beta E \left[V_k(z', k') | z \right]$$

- ▶ In general, cannot invert this equation for c .
 - ▶ Special case for additively separable preferences: $u_c(c, \ell) = u_c(c)$

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If only we knew $\ell(z, k)$ we could almost use EGM!

Barillas & Fernandez-Villaverde (2007)

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1. Fix a policy function for labor $\ell_0(z, k)$ (a good guess is $\ell_0(z, k) = \ell_{ss}$)
2. Conduct N steps of EGM given $\ell_0(z, k)$ (say $N = 10$)
 - ▶ EGM has to be modified to include labor.
 - ▶ Solve non-linear equation $u_c(c, \ell_0(z, k)) = \beta E \left[V_k(z', k') | z \right]$

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3. Conduct M steps of VFI (say $M = 1$) on the exogenous capital grid.
4. Replace $\ell(z, k)$ with the output of step 3 and conduct step 2.

Algorithm 3: EGM: Fixed labor supply

Function $\text{EGM}(V, \vec{k}, \vec{z}, \ell(z, k), k'(z, k), \text{parameters})$:

for $i=1:n_z$ do

for $j=1:n_k$ do

1. Solve for \tilde{k}_{ij} s.t. $k'(\tilde{k}_{ij}) = \vec{k}_j$

2. Evaluate for labor from old policy: $\tilde{\ell}_{ij} = \ell(\vec{z}_i, \tilde{k}_{ij})$

3. Evaluate expected value: $\mathbb{V} = \beta E \left[V(z', \vec{k}_j) \mid \vec{z}_i \right]$

4. Recover consumption: $u_c(\tilde{c}_{ij}, \tilde{\ell}_{ij}) = \mathbb{V}_k$

5. Find endogenous capital \hat{k}_{ij} s.t.: $\tilde{c}_{ij} + \vec{k}_j = f(\vec{z}_i, \hat{k}_{ij}, \tilde{\ell}_{ij})$

6. Update value at endogenous grid:

$$V(\vec{z}_i, \hat{k}_{ij}) = u(\tilde{c}_{ij}, \tilde{\ell}_{ij}) + \mathbb{V}$$

7. Interpolate to exogenous grid: $V_{\text{new}}[i,:] = \text{Interp}(\hat{k}, V, \vec{k})$

Algorithm 4: EGM: Fixed labor supply II

Function $\text{EGM}(V, \vec{k}, \vec{z}, \ell(z, k), \text{parameters})$:

for $i=1:n_z$ **do**

for $j=1:n_k$ **do**

 1. Evaluate expected value: $\mathbb{V} = \beta E \left[V(z', \vec{k}_j) \mid \vec{z}_i \right]$

 2. Define consumption given labor: $u_c(\hat{c}(\ell), \ell) = \mathbb{V}_k$

 3. Find endogenous capital \hat{k}_{ij} s.t.:

$$\hat{c}(\ell(\vec{z}_i, \hat{k}_{ij})) + \vec{k}_j = f(\vec{z}_i, \hat{k}_{ij}, \ell(\vec{z}_i, \hat{k}_{ij}))$$

 (Requires root finding and interpolation of ℓ at guess \hat{k})

 4. Update value at endogenous grid:

$$V(\vec{z}_i, \hat{k}_{ij}) = u(\hat{c}(\ell(\vec{z}_i, \hat{k}_{ij})), \ell(\vec{z}_i, \hat{k}_{ij})) + \mathbb{V}$$

 5. Interpolate to exogenous grid: $V_new[i,:] = \text{Interp}(\hat{k}, V, \vec{k})$

Some comments

1. In step 1 you can get the inverse from an interpolation routine
 - ▶ One option is to use a root finder
 - ▶ If you are using your own routine (say Cubic Splines) you can code your own inverse function

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 - ▶ If you are using your own routine (say Cubic Splines) you can code your own inverse function
2. In step 5 we can be more ambitious and also update labor.
 - 5'. Find endogenous capital \hat{k}_{ij} s.t.: $\tilde{c}_{ij} + \vec{k}_j = f\left(\vec{z}_i, \hat{k}_{ij}, \ell\left(\vec{z}_i, \hat{k}_{ij}\right)\right)$
 - ▶ Doing this implies adding an interpolation step to the root finding
 - ▶ It also provides a better update of the value function
 - 6'. Update value at endogenous grid: $V(\vec{z}_i, \hat{k}_{ij}) = u\left(\tilde{c}_{ij}, \ell\left(\vec{z}_i, \hat{k}_{ij}\right)\right) + \mathbb{V}$

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 - 6'. Update value at endogenous grid: $V(\vec{z}_i, \hat{k}_{ij}) = u\left(\tilde{c}_{ij}, \ell\left(\vec{z}_i, \hat{k}_{ij}\right)\right) + \mathbb{V}$
3. Step 4 can be simplified if utility is separable $u(c, \ell) = U(c) - H(\ell)$
 - ▶ In fact, all the algorithm gets easier
 - ▶ No need to carry around the policy function for labor

Algorithm 5: EGM: Endogenous labor supply with separable utility

Function EGM($V, \vec{k}, \vec{z}, parameters$):

for $i=1:n_z$ **do**

for $j=1:n_k$ **do**

 1. Evaluate expected value: $\mathbb{V} = \beta E \left[V \left(z', \vec{k}_j \right) \mid \vec{z}_i \right]$

 2. Recover consumption: $\tilde{c}_{ij} = (U')^{-1}(\mathbb{V}_k)$

 3. Find $(\hat{k}_{ij}, \tilde{\ell}_{ij})$ that solve FOC:

 3a. Define $\hat{k}(\ell)$ analytically from FOC: $\frac{-H'(\ell)}{U'(\tilde{c}_{ij})} = f_\ell(\vec{z}_i, \hat{k}(\ell), \ell)$

 3b. Solve for $\tilde{\ell}_{ij}$ numerically s.t.: $\tilde{c}_{ij} + \vec{k}_j = f \left(\vec{z}_i, \hat{k}(\tilde{\ell}_{ij}), \tilde{\ell}_{ij} \right)$

 3c. Assign endogenous grid point $\hat{k}_{ij} = \hat{k}(\tilde{\ell}_{ij})$

 4. Update value at endogenous grid: $V(\vec{z}_i, \hat{k}_{ij}) = u \left(\tilde{c}_{ij}, \tilde{\ell}_{ij} \right) + \mathbb{V}$

 5. Interpolate to exogenous grid: $V_new[i,:] = \text{Interp}(\hat{k}, V, \vec{k})$

Envelope Condition Method

Maliar & Maliar (2013)

First order conditions:

$$u_c(c, \ell) = \beta E \left[V_k(z', k') | z \right] \quad (1)$$

$$\frac{-u_\ell(c, \ell)}{u_c(c, \ell)} = f_\ell(z, k, \ell) \quad (2)$$

$$c + k' = f(z, k, \ell) \quad (3)$$

Envelope condition:

$$V_k(z, k) = u_c(c, \ell) f_k(z, k, \ell) \quad (4)$$

Combining (1) and (4) we get a recursive equation for value derivative:

$$V_k(z, k) = \beta f_k(z, k, \ell) E \left[V_k(z', k') | z \right] \quad (5)$$

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Key: EGM works by solving (1), (2) and (3). ECM solves (4), (2) and (3).
Equation (5) lets us update V_k directly without computing V

ECM - Algorithm - Inelastic labor supply

Algorithm 6: ECM: Inelastic labor supply

Function ECM($V_k, \vec{k}, \vec{z}, parameters$):

for $i=1:n_z$ do

for $j=1:n_k$ do

1. Get consumption analytically: $c_{ij} = (u')^{-1} \left(\frac{V_k(\vec{z}_i, \vec{k}_j)}{f_k(\vec{z}_i, \vec{k}_j)} \right)$

Note: We are using present k , not future k .

2. Get k' : $k'_{ij} = f(\vec{z}_i, \vec{k}_j) - c_{ij}$

3. Update V_k : $V_k^{new}(\vec{z}_i, \vec{k}_j) = \beta f_k(\vec{z}_i, \vec{k}_j) E[V_k(z', k'_{ij}) | \vec{z}_i]$

Note: This step requires interpolation inside expectation

return V_k^{new} ;

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2. Careful when updating derivatives directly
 - ▶ You lose the power of the contraction mapping theorem
 - ▶ Particularly you lose uniqueness ($V_k = 0$ is a fixed point)
 - ▶ Remember that your function is monotone, so $V_k > 0$!

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3. Alternative is to update with V as we do always:
 - 0'. Get derivative of V at grid nodes $V_k(\vec{z}_i, \vec{k}_j)$
 - 3'. Update value function: $V^{new}(z, k) = u(c_{ij}) + \beta E \left[V(z', k'_{ij}) | z \right]$
 - ▶ Note that you still need to do the interpolation inside the expectation

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 - ▶ Note that you still need to do the interpolation inside the expectation
4. Authors say they get better results with V_k
 - ▶ They reference another paper (Maliar & Maliar, 2012) that solves problems with 16 states using variants of the ECM

ECM - Labor supply

Algorithm 7: ECM: Endogenous labor supply, Separable utility

Function ECM($V_k, \vec{k}, \vec{z}, parameters$):

for $i=1:n_z$ do

for $j=1:n_k$ do

1. Get labor ℓ_{ij} numerically: $V_k(\vec{z}_i, \vec{k}_j) = \frac{H'(\ell_{ij})}{f_\ell(\vec{z}_i, \vec{k}_j, \ell_{ij})} f_k(\vec{z}_i, \vec{k}_j, \ell_{ij})$

Note: No interpolation or expectation

2. Get consumption analytically: $c_{ij} = (U')^{-1} \left(\frac{H'(\ell_{ij})}{f_\ell(\vec{z}_i, \vec{k}_j, \ell_{ij})} \right)$

3. Get k' : $k'_{ij} = f(\vec{z}_i, \vec{k}_j, \ell_{ij}) - c_{ij}$

3. Update V_k :

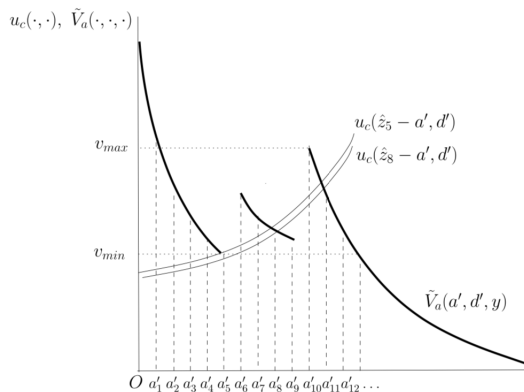
$$V_k^{new}(\vec{z}_i, \vec{k}_j) = \beta f_k(\vec{z}_i, \vec{k}_j, \ell_{ij}) E[V_k(z', k'_{ij}) | \vec{z}_i]$$

return V_k^{new} ;

Extensions

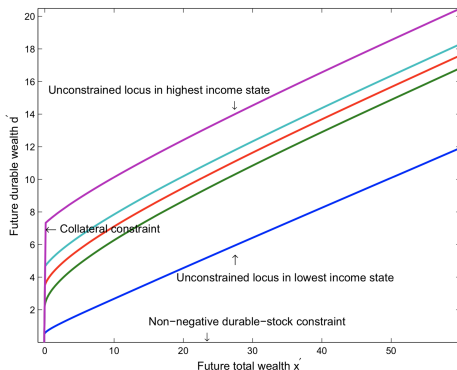
Non-Convex, Non-Smooth Problems - Fella (2014)

- ▶ Extend EGM to a problem with discrete state variable and continuous choices
- ▶ Discreteness is a problem because it generates kinks in the function
- ▶ Idea: EGM works away from the kinks!
- ▶ This is worth checking!



2 States+Borrowing Const. - Hintermaier & Koeniger (2010)

- ▶ Method for model with occasionally binding collateral constraints and non-separable utility in durable and non-durable consumption
- ▶ Good for applications with uninsurable income risk
- ▶ Idea: Solve the problem with a new state variable
 - ▶ x : Cash on hand or beginning of period wealth



Robustness

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 - ▶ Your EGM or ECM might send you out of bounds
 - ▶ This often means negative savings
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- ▶ Always build a safety check
 - ▶ Your EGM or ECM might send you out of bounds
 - ▶ This often means negative savings
 - ▶ Under-estimate curvature at the bottom → Over-estimate consumption
- ▶ If you are going out of bounds revert to traditional VFI